Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.1-Sine/79-4.1.7-d-trig-^m-a+b-c-sin-^n-^p

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September 27, 2022 Compiled on September 27, 2022 at 8:24am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [594]. This is test number [79].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.66 (592)	0.34 (2)
Mathematica	98.15 (583)	1.85 (11)
Maple	87.88 (522)	12.12 (72)
Fricas	79.80 (474)	20.20 (120)
Giac	60.44 (359)	39.56 (235)
Mupad	56.23 (334)	43.77 (260)
Maxima	55.89 (332)	44.11 (262)
Sympy	12.46 (74)	87.54 (520)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

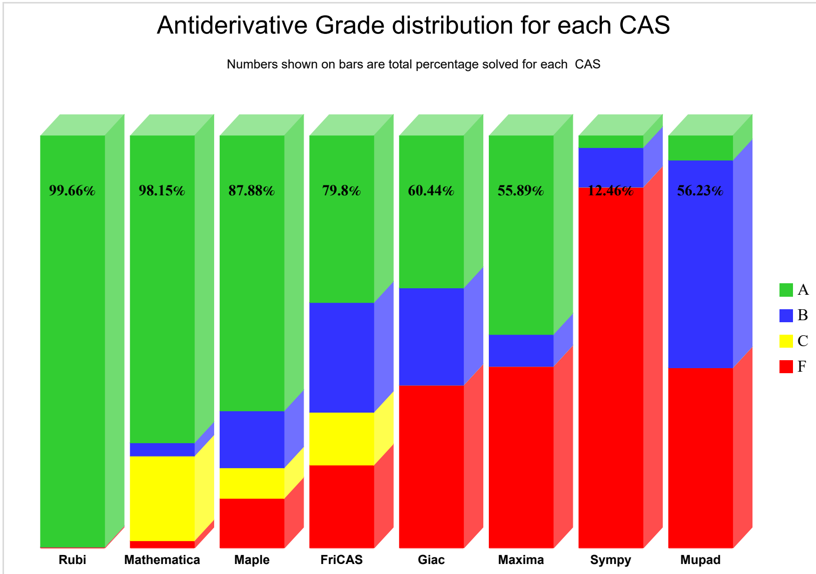
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

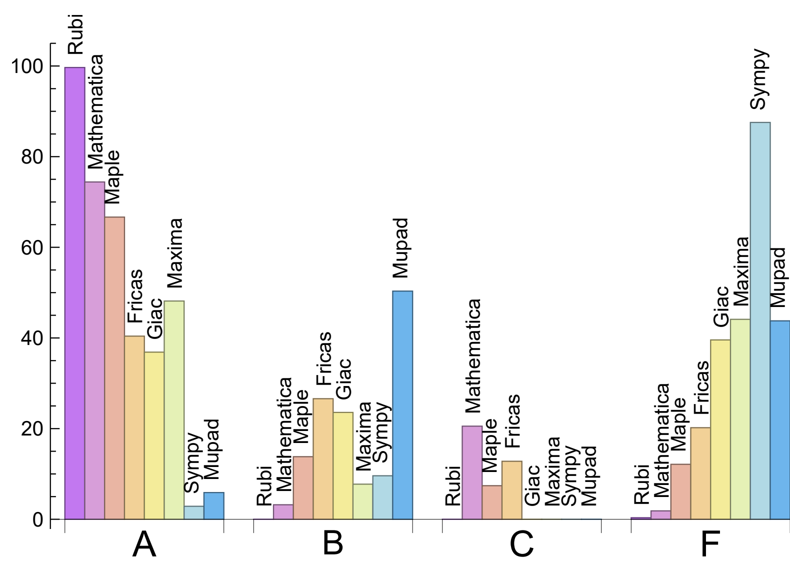
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.66	0.00	0.00	0.34
Mathematica	74.41	3.20	20.54	1.85
Maple	66.67	13.80	7.41	12.12
Maxima	48.15	7.74	0.00	44.11
Fricas	40.40	26.60	12.79	20.20
Giac	36.87	23.57	0.00	39.56
Mupad	N/A	50.34	0.00	43.77
Sympy	2.86	9.60	0.00	87.54

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	11	100.00 %	0.00 %	0.00 %
Maple	72	100.00 %	0.00 %	0.00 %
Fricas	120	85.00 %	7.50 %	7.50 %
Giac	235	84.68 %	2.13 %	13.19 %
Maxima	262	95.42 %	1.53 %	3.05 %
Sympy	520	55.00 %	35.58 %	9.42 %
Mupad	260	98.85 %	1.15 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

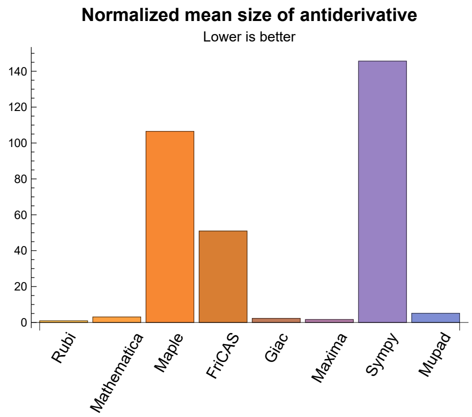
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

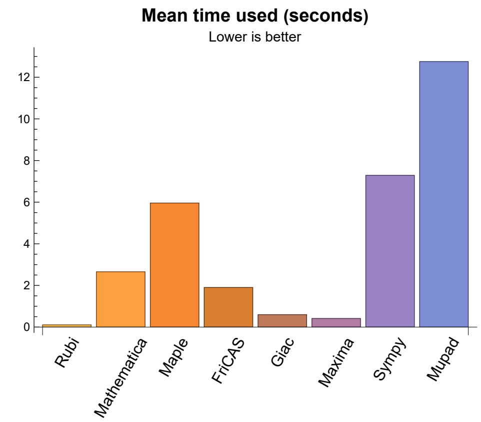
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.11	127.13	0.95	93.00	1.00
Mathematica	2.66	1003.03	3.06	98.00	0.97
Maple	5.96	8483.91	106.51	109.00	1.09
Maxima	0.41	135.66	1.64	72.00	1.10
Fricas	1.90	5373.66	50.96	305.00	3.56
Sympy	7.29	5409.65	145.64	151.50	3.13
Giac	0.60	283.43	2.28	90.00	1.40
Mupad	12.75	643.78	5.11	99.50	1.14

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
140
120
100
80
60
40
20
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
12
10
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{399, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 564, 569, 570, 571, 572, 573, 577, 578, 581, 582, 583, 586, 587, 588, 589, 590\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{399, 400, 401, 402, 403\}$

Maple $\{399, 400, 401, 402, 403\}$

Maxima $\{\}$

Fricas $\{400, 401, 402, 403\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{399, 400, 401, 402, 403\}$

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {212, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 239, 240, 241, 242, 243, 244, 245, 249, 251, 252, 254, 328, 339, 348, 356, 357, 366, 372, 373, 378, 379, 402, 403, 565, 566, 574, 575, 576, 594}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

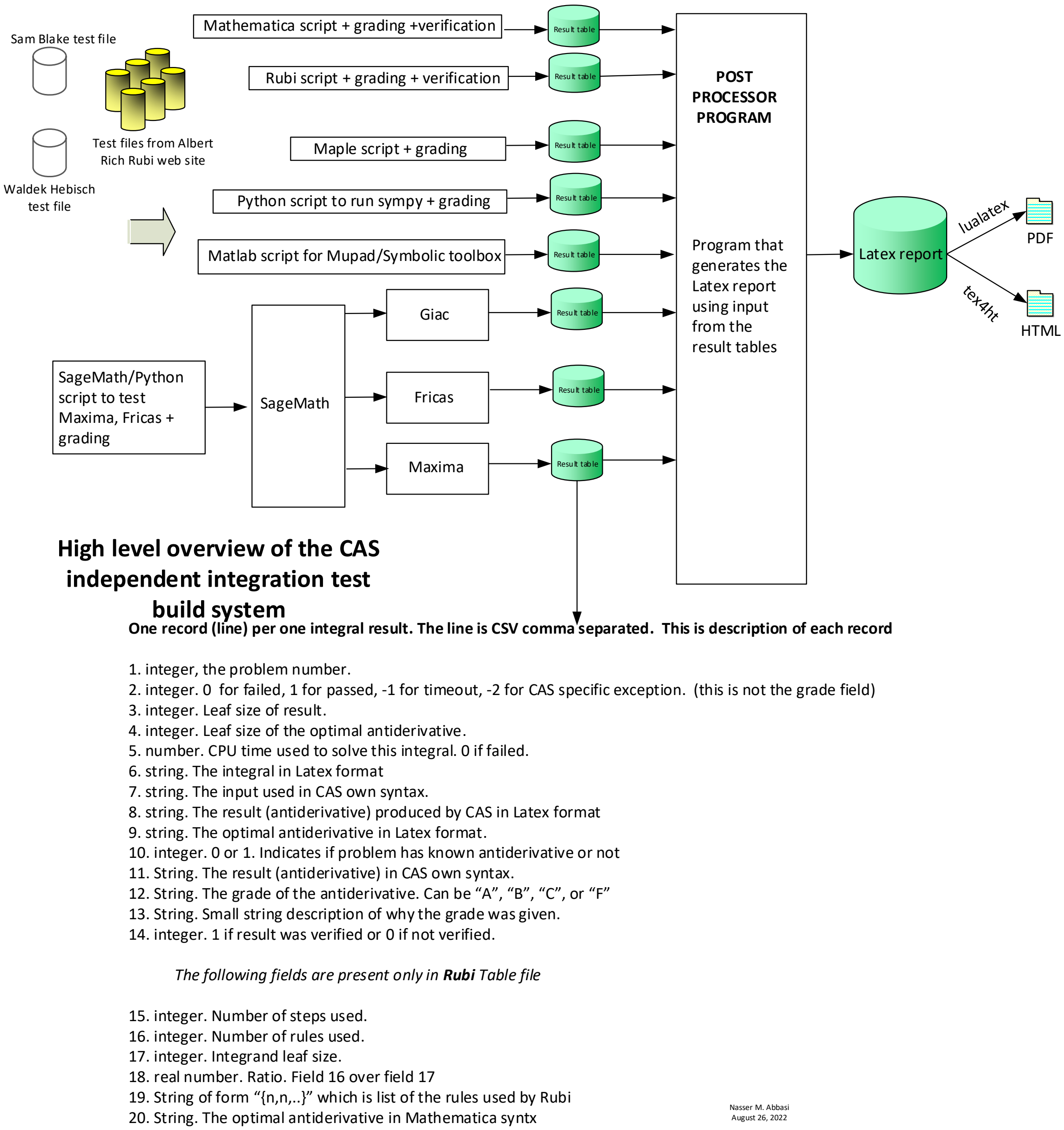
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594 }

B grade: { }

C grade: { }

F grade: { 391, 392 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 77, 85, 86, 87, 88, 89, 90, 91, 92, 93, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 178, 203, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 230, 231, 232, 233, 234, 235, 236, 256, 260, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 374, 375, 380, 384, 385, 395, 396, 399, 400, 401, 402, 403, 407, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 564, 567, 568, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592 }

B grade: { 40, 54, 66, 67, 119, 120, 179, 265, 270, 276, 277, 308, 310, 312, 378, 379, 565, 566, 593 }

C grade: { 76, 78, 79, 80, 81, 82, 83, 84, 94, 95, 96, 97, 98, 99, 113, 114, 115, 172, 173, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 212, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 294, 328, 338, 339, 348, 356, 357, 366, 373, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 404, 405, 406, 408, 409, 410, 484, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 561, 562, 563, 574, 575, 576, 591, 594 }

F grade: { 175, 176, 177, 180, 181, 376, 377, 381, 382, 423, 424 }

2.1.3 Maple

A grade: { 1, 2, 3, 6, 13, 14, 15, 16, 17, 18, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 116, 117, 118, 121, 127, 128, 129, 130, 131, 139, 140, 141, 142, 143, 148, 149, 151, 152, 154, 157, 158, 159, 160, 161, 162, 163, 164, 168, 170, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 246, 256, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 340, 341, 342, 344, 345, 346, 349, 350, 352, 353, 354, 355, 358, 359, 360, 361, 364, 365, 368, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 407, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 491, 492, 493, 494, 495, 496, 497, 498, 502, 503, 504, 505, 507, 508, 509, 513, 514, 515, 516, 519, 520, 524, 525, 526, 527, 528, 529, 530, 531, 541, 542, 552, 553, 554, 557, 564, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 4, 5, 34, 113, 115, 120, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 153, 155, 156, 165, 166, 167, 169, 237, 247, 259, 260, 298, 326, 327, 328, 336, 337, 338, 339, 343, 347, 348, 356, 357, 362, 363, 366, 367, 369, 370, 371, 406, 408, 448, 488, 489, 490, 499, 500, 501, 506, 510, 511, 512, 517, 521, 522, 523, 532, 533, 534, 535, 536, 537, 538, 539, 540, 562, 591, 593 }

C grade: { 7, 8, 9, 10, 11, 12, 119, 150, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 239, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 257, 258, 269, 281, 351, 388, 389, 390, 391, 392, 518, 594 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 240, 244, 245, 248, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 420, 421, 422, 423, 424, 431, 432, 433, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 558, 559, 560, 561, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 592 }

2.1.4 Maxima

A grade: { 3, 4, 13, 14, 15, 16, 17, 18, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 122, 123, 124, 132, 133, 134, 144, 145, 153, 154, 163, 164, 236, 256, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 334, 335, 336, 345, 346, 354, 355, 363, 364, 365, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 398, 404, 405, 406, 407, 408, 409, 410, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 464, 465, 466, 467, 468, 469, 472, 478, 479, 481, 482, 488, 489, 491, 492, 493, 499, 500, 502, 503, 504, 511, 513, 514, 515, 522, 524, 525, 526, 533, 535, 536, 537, 552, 553, 554, 555, 558, 559, 560, 564, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 5, 6, 34, 112, 119, 120, 121, 146, 155, 162, 165, 265, 270, 276, 277, 310, 312, 322, 323, 347, 356, 366, 441, 462, 463, 470, 471, 473, 474, 475, 476, 477, 480, 483, 484, 485, 486, 487, 490, 501, 510, 512, 521, 523, 532, 534 }

C grade: { }

F grade: { 1, 2, 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 147, 148, 149, 150, 151, 152, 156, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 327, 328, 329, 330, 331, 332, 333, 337, 338, 339, 340, 341, 342, 343, 344, 348, 349, 350, 351, 352, 353, 357, 358, 359, 360, 361, 362, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 399, 400, 401, 402, 403, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 431, 432, 433, 494, 495, 496, 497, 498, 505, 506, 507, 508, 509, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 556, 557, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 591, 592, 593, 594 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 13, 14, 15, 17, 18, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 97, 100, 116, 117, 118, 120, 121, 122, 125, 126, 132, 133, 136, 137, 147, 154, 163, 256, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 317, 319, 324, 328, 334, 338, 339, 355, 364, 365, 385, 395, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 451, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 499, 500, 501, 502, 503, 504, 510, 511, 512, 513, 514, 515, 521, 555, 558, 559, 560, 564, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 4, 16, 67, 84, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 123, 124, 134, 135, 144, 145, 146, 153, 155, 156, 162, 164, 165, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 251, 254, 260, 265, 270, 276, 277, 309, 311, 313, 314, 315, 316, 318, 320, 321, 322, 323, 325, 326, 327, 335, 336, 337, 345, 346, 347, 348, 354, 356, 357, 363, 366, 396, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 447, 448, 449, 450, 452, 453, 454, 455, 456, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 556, 557 }

C grade: { 7, 8, 9, 10, 11, 12, 130, 131, 142, 150, 151, 152, 159, 160, 161, 167, 168, 169, 170, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 250, 253, 332, 333, 344, 351, 352, 353, 360, 361, 362, 368, 369, 370, 371, 383, 384, 386, 387, 388, 390, 391, 392, 393, 394, 397, 398, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 552, 591 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 127, 128, 129, 138, 139, 140, 141, 143, 148, 149, 157, 158, 166, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 252, 255, 257, 258, 259, 329, 330, 331, 340, 341, 342, 343, 349, 350, 358, 359, 367, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 389, 399, 420, 421, 422, 423, 424, 431, 432, 433, 494, 495, 496, 497, 498, 505, 506, 507, 508, 509, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 592, 593, 594 }

2.1.6 Sympy

A grade: { 3, 31, 37, 70, 74, 269, 281, 288, 385, 400, 401, 437, 438, 577, 578, 583, 588 }

B grade: { 29, 32, 33, 34, 35, 36, 38, 42, 43, 44, 45, 49, 50, 51, 52, 55, 56, 57, 58, 61, 62, 63, 64, 65, 68, 69, 75, 76, 77, 81, 89, 236, 261, 262, 263, 264, 265, 267, 268, 273, 274, 275, 276, 277, 279, 280, 282, 285, 286, 287, 293, 294, 295, 307, 319, 396, 407 }

C grade: { }

F grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 39, 40, 41, 46, 47, 48, 53, 54, 59, 60, 66, 67, 71, 72, 73, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 266, 270, 271, 272, 278, 283, 284, 289, 290, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 579, 580, 581, 582, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 13, 14, 15, 16, 31, 32, 33, 34, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 72, 73, 74, 75, 76, 77, 80, 81, 85, 86, 87, 90, 91, 92, 93, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 120, 122, 123, 132, 133, 144, 153, 154, 163, 164, 237, 238, 261, 262, 263, 264, 266, 267, 268, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 334, 335, 345, 346, 354, 355, 364, 365, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 445, 446, 452, 453, 454, 458, 460, 465, 466, 467, 468, 469, 471, 476, 477, 478, 479, 480, 481, 483, 484, 487, 491, 502, 513, 524, 535, 552, 553, 554, 564, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 5, 6, 29, 35, 36, 39, 40, 41, 50, 53, 54, 66, 67, 71, 78, 79, 82, 83, 84, 88, 89, 94, 95, 98, 99, 112, 113, 115, 116, 117, 118, 121, 126, 136, 137, 145, 156, 162, 198, 199, 203, 204, 205, 206, 207, 208, 209, 210, 211, 215, 216, 218, 219, 220, 221, 222, 223, 227, 228, 230, 231, 232, 233, 234, 235, 236, 256, 259, 260, 265, 269, 270, 276, 277, 289, 310, 311, 312, 313, 322, 323, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 441, 442, 443, 444, 447, 448, 449, 450, 451, 455, 456, 457, 459, 461, 462, 463, 464, 470, 472, 473, 474, 475, 482, 485, 486, 488, 489, 490, 500, 501, 510, 511, 512, 515, 521, 522, 523, 525, 526, 532, 533, 534, 536, 537 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 119, 124, 125, 127, 128, 129, 130, 131, 134, 135, 138, 139, 140, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 155, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202, 212, 213, 214, 217, 224, 225, 226, 229, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 326, 327, 328, 329, 330, 331, 332, 333, 336, 337, 338, 339, 340, 341, 342, 343, 344, 347, 348, 349, 350, 351, 352, 353, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 420, 421, 422, 423, 424, 431, 432, 433, 492, 493, 494, 495, 496, 497, 498, 499, 503, 504, 505, 506, 507, 508, 509, 514, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 591, 592, 593, 594 }

2.1.8 Mupad

A grade: { 399, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 564, 569, 570, 571, 572, 573, 577, 578, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 3, 16, 17, 18, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 118, 154, 163, 164, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 335, 346, 355, 364, 365, 375, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 422, 433, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 470, 475, 476, 477, 478, 479, 480, 485, 486, 487, 552 }

C grade: { }

F grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 382, 420, 421, 423, 424, 431, 432, 460, 461, 462, 463, 464, 471, 472, 473, 474, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 579, 580, 584, 585, 591, 592, 593, 594 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	0	43	0	45	-1
N.S.	1	1.00	0.68	0.60	0.00	0.81	0.00	0.85	-0.02
time (sec)	N/A	0.018	0.026	3.797	0.000	0.390	0.000	0.513	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	0	29	0	24	-1
N.S.	1	1.00	0.76	0.71	0.00	0.85	0.00	0.71	-0.03
time (sec)	N/A	0.012	0.030	3.978	0.000	0.387	0.000	0.416	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	13	19	17	17	40
N.S.	1	1.00	1.00	1.14	0.93	1.36	1.21	1.21	2.86
time (sec)	N/A	0.006	0.005	2.300	0.504	0.404	0.108	0.450	13.638

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	30	49	26	70	0	15	-1
N.S.	1	1.00	1.76	2.88	1.53	4.12	0.00	0.88	-0.06
time (sec)	N/A	0.007	0.012	4.020	0.551	0.385	0.000	0.538	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	55	70	314	58	0	87	-1
N.S.	1	1.00	1.31	1.67	7.48	1.38	0.00	2.07	-0.02
time (sec)	N/A	0.014	0.063	6.583	0.565	0.381	0.000	0.568	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	77	89	931	78	0	126	-1
N.S.	1	1.00	1.26	1.46	15.26	1.28	0.00	2.07	-0.02
time (sec)	N/A	0.020	0.159	7.381	0.878	0.415	0.000	0.462	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	65	152	0	110	0	0	-1
N.S.	1	1.00	0.53	1.24	0.00	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.129	0.542	0.000	0.123	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	337	0	80	0	0	-1
N.S.	1	1.00	0.74	4.62	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.073	0.342	0.000	0.095	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	124	0	69	0	0	-1
N.S.	1	1.00	0.82	2.48	0.00	1.38	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.023	0.536	0.000	0.091	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	318	0	102	0	0	-1
N.S.	1	1.00	0.77	6.62	0.00	2.12	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.018	0.385	0.000	0.088	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	48	372	0	139	0	0	-1
N.S.	1	1.00	0.62	4.83	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.052	0.356	0.000	0.091	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	60	1349	0	209	0	0	-1
N.S.	1	1.00	0.49	10.97	0.00	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.159	0.345	0.000	0.099	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	53	63	85	82	0	57	-1
N.S.	1	1.00	0.40	0.48	0.64	0.62	0.00	0.43	-0.01
time (sec)	N/A	0.030	0.120	0.380	0.570	0.386	0.000	0.445	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	38	47	55	56	0	27	-1
N.S.	1	1.00	0.49	0.60	0.71	0.72	0.00	0.35	-0.01
time (sec)	N/A	0.018	0.076	0.231	0.513	0.383	0.000	0.421	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	25	33	22	36	0	15	-1
N.S.	1	1.00	0.69	0.92	0.61	1.00	0.00	0.42	-0.03
time (sec)	N/A	0.009	0.012	0.201	0.530	0.410	0.000	0.428	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	9	36	0	9	7
N.S.	1	1.00	1.00	0.94	0.56	2.25	0.00	0.56	0.44
time (sec)	N/A	0.009	0.005	0.170	0.589	0.389	0.000	0.519	13.712

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	29	23	74	0	0	44
N.S.	1	1.00	0.50	0.43	0.34	1.09	0.00	0.00	0.65
time (sec)	N/A	0.013	0.026	0.189	0.492	0.366	0.000	0.000	14.279

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	47	41	35	104	0	0	117
N.S.	1	1.00	0.40	0.35	0.30	0.88	0.00	0.00	0.99
time (sec)	N/A	0.020	0.044	0.236	0.506	0.375	0.000	0.000	16.558

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.121	0.120	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.080	0.086	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.049	0.103	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.050	0.096	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.073	0.085	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.071	0.079	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	0	0	14	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.18	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.040	0.089	0.000	0.432	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	0	0	17	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.052	0.217	0.000	0.408	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	0	0	26	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.053	0.239	0.000	0.387	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	65	0	0	27	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.060	0.325	0.000	0.396	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	0	16	58	384	36
N.S.	1	1.00	1.00	0.00	0.00	0.64	2.32	15.36	1.44
time (sec)	N/A	0.013	0.023	0.093	0.000	0.400	0.379	1.781	13.748

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	16	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.20	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.036	0.099	0.000	0.410	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	17	12	15	17	11
N.S.	1	1.00	1.00	1.12	1.06	0.75	0.94	1.06	0.69
time (sec)	N/A	0.006	0.003	0.053	0.286	0.391	0.010	0.408	13.575

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	43	40	28	110	25	33
N.S.	1	1.00	0.79	1.30	1.21	0.85	3.33	0.76	1.00
time (sec)	N/A	0.017	0.004	0.109	0.311	0.393	0.140	0.438	13.762

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	72	69	37	233	34	42
N.S.	1	1.00	0.74	1.57	1.50	0.80	5.07	0.74	0.91
time (sec)	N/A	0.020	0.003	0.217	0.296	0.408	0.318	0.441	13.685

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	105	104	46	376	43	51
N.S.	1	1.00	0.71	1.78	1.76	0.78	6.37	0.73	0.86
time (sec)	N/A	0.026	0.003	0.226	0.298	0.386	0.707	0.440	13.694

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	58	45	50	46	314	149	54
N.S.	1	1.00	0.94	0.73	0.81	0.74	5.06	2.40	0.87
time (sec)	N/A	0.055	0.043	0.237	0.364	0.379	12.161	0.445	13.648

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	35	40	36	143	105	38
N.S.	1	1.00	0.93	0.76	0.87	0.78	3.11	2.28	0.83
time (sec)	N/A	0.054	0.030	0.205	0.321	0.389	4.722	0.465	0.057

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	23	27	25	36	29	25
N.S.	1	1.00	0.93	0.85	1.00	0.93	1.33	1.07	0.93
time (sec)	N/A	0.045	0.025	0.178	0.287	0.387	1.685	0.538	0.045

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	15	15	34	15	15
N.S.	1	1.00	1.00	1.23	1.15	1.15	2.62	1.15	1.15
time (sec)	N/A	0.024	0.011	0.115	0.296	0.380	0.630	0.457	13.601

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	46	39	46	55	0	62	31
N.S.	1	1.00	1.59	1.34	1.59	1.90	0.00	2.14	1.07
time (sec)	N/A	0.041	0.032	0.221	0.304	0.392	0.000	0.439	0.084

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	146	63	70	98	0	149	55
N.S.	1	1.00	2.52	1.09	1.21	1.69	0.00	2.57	0.95
time (sec)	N/A	0.064	0.185	0.280	0.287	0.428	0.000	0.498	0.088

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	132	87	90	135	0	181	74
N.S.	1	1.00	1.61	1.06	1.10	1.65	0.00	2.21	0.90
time (sec)	N/A	0.063	2.783	0.288	0.294	0.401	0.000	0.443	0.096

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	57	72	56	1161	63	68
N.S.	1	1.00	0.60	0.78	0.99	0.77	15.90	0.86	0.93
time (sec)	N/A	0.061	0.130	0.209	0.517	0.383	7.849	0.432	13.718

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	44	49	45	502	50	45
N.S.	1	1.00	0.69	0.90	1.00	0.92	10.24	1.02	0.92
time (sec)	N/A	0.054	0.095	0.180	0.514	0.385	2.885	0.439	13.532

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	27	24	26	34	100	26	20
N.S.	1	1.00	1.35	1.20	1.30	1.70	5.00	1.30	1.00
time (sec)	N/A	0.044	0.010	0.152	0.534	0.386	1.010	0.497	13.737

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	21	41	13	13
N.S.	1	1.00	1.00	1.08	1.00	1.62	3.15	1.00	1.00
time (sec)	N/A	0.015	0.006	0.158	0.318	0.388	0.473	0.465	13.595

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	16	25	28	36	0	19	17
N.S.	1	1.00	0.57	0.89	1.00	1.29	0.00	0.68	0.61
time (sec)	N/A	0.048	0.023	0.217	0.300	0.375	0.000	0.451	13.504

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	35	42	56	0	42	38
N.S.	1	1.00	1.07	0.76	0.91	1.22	0.00	0.91	0.83
time (sec)	N/A	0.054	0.035	0.244	0.321	0.383	0.000	0.442	13.727

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	70	45	52	77	0	52	50
N.S.	1	1.00	1.13	0.73	0.84	1.24	0.00	0.84	0.81
time (sec)	N/A	0.057	0.030	0.254	0.303	0.374	0.000	0.413	13.955

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	47	52	46	156	57	48
N.S.	1	1.00	0.91	0.72	0.80	0.71	2.40	0.88	0.74
time (sec)	N/A	0.058	0.038	0.155	0.285	0.399	29.632	0.454	13.643

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	37	41	38	156	106	36
N.S.	1	1.00	0.89	0.79	0.87	0.81	3.32	2.26	0.77
time (sec)	N/A	0.047	0.026	0.204	0.294	0.418	12.872	0.441	0.054

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	29	28	28	156	28	26
N.S.	1	1.00	0.94	0.88	0.85	0.85	4.73	0.85	0.79
time (sec)	N/A	0.044	0.025	0.204	0.304	0.395	4.584	0.412	13.557

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	156	16	16
N.S.	1	1.00	1.00	0.94	0.89	0.89	8.67	0.89	0.89
time (sec)	N/A	0.028	0.010	0.149	0.314	0.368	2.152	0.515	13.588

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	61	49	59	70	0	107	41
N.S.	1	1.00	1.30	1.04	1.26	1.49	0.00	2.28	0.87
time (sec)	N/A	0.044	0.033	0.254	0.297	0.386	0.000	0.429	0.088

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	208	75	86	118	0	175	70
N.S.	1	1.00	2.67	0.96	1.10	1.51	0.00	2.24	0.90
time (sec)	N/A	0.060	0.317	0.306	0.339	0.406	0.000	0.443	13.761

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	46	56	64	59	1275	68	66
N.S.	1	1.00	0.67	0.81	0.93	0.86	18.48	0.99	0.96
time (sec)	N/A	0.059	0.148	0.142	0.529	0.391	21.951	0.452	13.808

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	34	37	49	551	44	31
N.S.	1	1.00	1.11	0.89	0.97	1.29	14.50	1.16	0.82
time (sec)	N/A	0.041	0.012	0.198	0.521	0.392	8.061	0.438	13.479

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	32	94	16	16
N.S.	1	1.00	1.00	0.94	0.89	1.78	5.22	0.89	0.89
time (sec)	N/A	0.047	0.013	0.170	0.290	0.379	3.265	0.433	13.376

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	25	34	238	25	24
N.S.	1	1.00	0.81	0.78	0.78	1.06	7.44	0.78	0.75
time (sec)	N/A	0.019	0.029	0.159	0.307	0.371	1.328	0.512	13.455

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	50	37	40	46	0	48	36
N.S.	1	1.00	1.06	0.79	0.85	0.98	0.00	1.02	0.77
time (sec)	N/A	0.054	0.033	0.257	0.301	0.360	0.000	0.461	13.575

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	47	52	72	0	34	48
N.S.	1	1.00	0.71	0.72	0.80	1.11	0.00	0.52	0.74
time (sec)	N/A	0.058	0.021	0.302	0.312	0.370	0.000	0.449	13.682

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	22	25	362	22	21
N.S.	1	1.00	1.07	0.69	0.76	0.86	12.48	0.76	0.72
time (sec)	N/A	0.016	0.005	0.133	0.291	0.368	2.309	0.523	13.410

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	24	28	31	675	28	33
N.S.	1	1.00	1.11	0.65	0.76	0.84	18.24	0.76	0.89
time (sec)	N/A	0.018	0.005	0.175	0.300	0.384	7.549	0.483	13.408

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	32	34	37	1083	34	43
N.S.	1	1.00	1.00	0.63	0.67	0.73	21.24	0.67	0.84
time (sec)	N/A	0.020	0.005	0.095	0.317	0.372	22.905	0.426	13.345

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	77	54	43	43	107	67	44
N.S.	1	1.00	1.51	1.06	0.84	0.84	2.10	1.31	0.86
time (sec)	N/A	0.031	0.025	0.244	0.279	0.388	0.259	0.426	13.366

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	54	34	34	27	58	40	27
N.S.	1	1.00	1.74	1.10	1.10	0.87	1.87	1.29	0.87
time (sec)	N/A	0.015	0.017	0.180	0.303	0.385	0.107	0.567	13.313

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	63	33	38	42	0	58	23
N.S.	1	1.00	2.42	1.27	1.46	1.62	0.00	2.23	0.88
time (sec)	N/A	0.017	0.023	0.187	0.291	0.429	0.000	0.422	13.366

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	118	59	58	95	0	121	42
N.S.	1	1.00	2.95	1.48	1.45	2.38	0.00	3.02	1.05
time (sec)	N/A	0.021	0.032	0.253	0.295	0.398	0.000	0.441	13.388

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	70	86	104	69	258	68	92
N.S.	1	1.00	0.79	0.97	1.17	0.78	2.90	0.76	1.03
time (sec)	N/A	0.035	0.085	0.276	0.534	0.401	0.434	0.479	13.944

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	65	74	50	158	43	68
N.S.	1	1.00	0.74	1.07	1.21	0.82	2.59	0.70	1.11
time (sec)	N/A	0.029	0.074	0.220	0.536	0.401	0.184	0.462	13.545

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	32	29	29	51	25	27
N.S.	1	1.00	1.10	1.07	0.97	0.97	1.70	0.83	0.90
time (sec)	N/A	0.010	0.024	0.140	0.285	0.396	0.074	0.409	13.401

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	23	32	0	39	16
N.S.	1	1.00	1.00	1.38	1.44	2.00	0.00	2.44	1.00
time (sec)	N/A	0.016	0.013	0.198	0.514	0.376	0.000	0.482	13.357

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	35	28	54	0	37	29
N.S.	1	1.00	1.14	0.81	0.65	1.26	0.00	0.86	0.67
time (sec)	N/A	0.025	0.025	0.260	0.292	0.390	0.000	0.457	13.368

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	95	56	45	81	0	61	49
N.S.	1	1.00	1.46	0.86	0.69	1.25	0.00	0.94	0.75
time (sec)	N/A	0.029	0.026	0.286	0.297	0.358	0.000	0.583	13.383

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	17	16	15	17	15
N.S.	1	1.00	1.00	0.89	0.89	0.84	0.79	0.89	0.79
time (sec)	N/A	0.006	0.004	0.075	0.292	0.385	0.010	0.532	13.310

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	42	39	47	110	42	44
N.S.	1	1.00	0.86	0.84	0.78	0.94	2.20	0.84	0.88
time (sec)	N/A	0.011	0.042	0.192	0.292	0.385	0.143	0.409	13.521

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	73	71	81	246	76	118
N.S.	1	1.00	0.92	0.84	0.82	0.93	2.83	0.87	1.36
time (sec)	N/A	0.056	0.073	0.207	0.288	0.400	0.321	0.417	14.129

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	113	110	108	123	410	118	147
N.S.	1	1.00	0.81	0.79	0.77	0.88	2.93	0.84	1.05
time (sec)	N/A	0.109	0.110	0.273	0.290	0.399	0.716	0.455	13.632

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	180	110	116	272	0	332	112
N.S.	1	1.00	1.70	1.04	1.09	2.57	0.00	3.13	1.06
time (sec)	N/A	0.075	0.961	0.365	0.499	0.419	0.000	0.498	0.163

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	150	70	88	218	0	173	72
N.S.	1	1.00	1.95	0.91	1.14	2.83	0.00	2.25	0.94
time (sec)	N/A	0.059	0.337	0.273	0.502	0.409	0.000	0.445	0.110

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	125	45	67	165	0	57	44
N.S.	1	1.00	2.40	0.87	1.29	3.17	0.00	1.10	0.85
time (sec)	N/A	0.045	0.163	0.245	0.590	0.410	0.000	0.417	0.097

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	97	29	50	117	367693	37	29
N.S.	1	1.00	2.62	0.78	1.35	3.16	9937.65	1.00	0.78
time (sec)	N/A	0.027	0.094	0.171	0.544	0.410	86.312	0.440	0.087

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	143	62	83	161	0	100	457
N.S.	1	1.00	2.60	1.13	1.51	2.93	0.00	1.82	8.31
time (sec)	N/A	0.042	0.191	0.279	0.546	0.425	0.000	0.546	13.727

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	224	107	120	327	0	196	592
N.S.	1	1.00	2.64	1.26	1.41	3.85	0.00	2.31	6.96
time (sec)	N/A	0.076	1.462	0.369	0.545	0.416	0.000	0.432	13.914

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	657	168	181	612	0	334	1105
N.S.	1	1.00	5.26	1.34	1.45	4.90	0.00	2.67	8.84
time (sec)	N/A	0.125	6.239	0.399	0.526	0.444	0.000	0.439	13.928

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	133	168	192	453	0	233	2244
N.S.	1	1.00	0.82	1.03	1.18	2.78	0.00	1.43	13.77
time (sec)	N/A	0.234	0.819	0.274	0.526	0.454	0.000	0.450	15.319

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	118	128	372	0	157	1892
N.S.	1	1.00	0.81	1.01	1.09	3.18	0.00	1.34	16.17
time (sec)	N/A	0.141	0.324	0.296	0.537	0.466	0.000	0.475	14.820

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	69	81	78	305	0	114	481
N.S.	1	1.00	0.90	1.05	1.01	3.96	0.00	1.48	6.25
time (sec)	N/A	0.074	0.219	0.239	0.511	0.417	0.000	0.400	13.974

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	48	46	260	0	81	104
N.S.	1	1.00	1.00	1.04	1.00	5.65	0.00	1.76	2.26
time (sec)	N/A	0.048	0.108	0.213	0.565	0.427	0.000	0.422	13.481

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	30	29	236	16298	64	33
N.S.	1	1.00	1.00	0.83	0.81	6.56	452.72	1.78	0.92
time (sec)	N/A	0.017	0.062	0.217	0.546	0.406	14.551	0.467	13.527

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	50	48	313	0	83	45
N.S.	1	1.00	1.00	0.94	0.91	5.91	0.00	1.57	0.85
time (sec)	N/A	0.048	0.217	0.305	0.542	0.421	0.000	0.534	13.475

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	119	69	69	451	0	111	68
N.S.	1	1.00	1.55	0.90	0.90	5.86	0.00	1.44	0.88
time (sec)	N/A	0.072	0.473	0.326	0.526	0.432	0.000	0.462	13.431

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	147	96	98	595	0	155	95
N.S.	1	1.00	1.35	0.88	0.90	5.46	0.00	1.42	0.87
time (sec)	N/A	0.083	1.020	0.358	0.528	0.439	0.000	0.479	13.798

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	137	130	137	789	0	215	130
N.S.	1	1.00	0.98	0.93	0.98	5.64	0.00	1.54	0.93
time (sec)	N/A	0.101	1.128	0.357	0.547	0.452	0.000	0.441	15.074

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	194	116	154	529	0	322	123
N.S.	1	1.00	1.52	0.91	1.20	4.13	0.00	2.52	0.96
time (sec)	N/A	0.125	1.065	0.310	0.512	0.449	0.000	0.552	0.217

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	172	90	131	427	0	342	95
N.S.	1	1.00	1.69	0.88	1.28	4.19	0.00	3.35	0.93
time (sec)	N/A	0.098	0.622	0.385	0.565	0.451	0.000	0.442	0.164

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	160	77	111	327	0	93	71
N.S.	1	1.00	1.93	0.93	1.34	3.94	0.00	1.12	0.86
time (sec)	N/A	0.060	0.319	0.307	0.534	0.421	0.000	0.456	13.490

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	149	65	98	282	0	79	62
N.S.	1	1.00	2.01	0.88	1.32	3.81	0.00	1.07	0.84
time (sec)	N/A	0.035	0.215	0.234	0.535	0.423	0.000	0.433	0.105

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	194	107	149	455	0	246	2039
N.S.	1	1.00	1.88	1.04	1.45	4.42	0.00	2.39	19.80
time (sec)	N/A	0.082	0.538	0.412	0.503	0.456	0.000	0.490	14.631

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	390	152	223	838	0	512	2338
N.S.	1	1.00	2.55	0.99	1.46	5.48	0.00	3.35	15.28
time (sec)	N/A	0.154	1.094	0.480	0.508	0.516	0.000	0.456	14.946

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	106	134	181	623	0	223	2295
N.S.	1	1.00	0.72	0.91	1.22	4.21	0.00	1.51	15.51
time (sec)	N/A	0.182	1.038	0.308	0.560	0.424	0.000	0.465	16.064

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	101	109	492	0	140	1959
N.S.	1	1.00	1.00	1.09	1.17	5.29	0.00	1.51	21.06
time (sec)	N/A	0.088	0.593	0.347	0.530	0.456	0.000	0.447	15.227

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	74	75	74	419	0	109	72
N.S.	1	1.00	0.95	0.96	0.95	5.37	0.00	1.40	0.92
time (sec)	N/A	0.053	0.368	0.280	0.510	0.416	0.000	0.468	13.433

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	84	87	89	463	0	113	79
N.S.	1	1.00	0.97	1.00	1.02	5.32	0.00	1.30	0.91
time (sec)	N/A	0.040	0.288	0.290	0.516	0.430	0.000	0.399	13.426

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	130	155	103	133	588	0	179	132
N.S.	1	1.02	1.22	0.81	1.05	4.63	0.00	1.41	1.04
time (sec)	N/A	0.098	0.815	0.398	0.546	0.467	0.000	0.444	13.599

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	202	122	172	843	0	174	164
N.S.	1	1.00	1.25	0.75	1.06	5.20	0.00	1.07	1.01
time (sec)	N/A	0.136	0.881	0.430	0.513	0.473	0.000	0.453	14.398

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	134	158	234	950	0	224	2500
N.S.	1	1.00	0.91	1.07	1.58	6.42	0.00	1.51	16.89
time (sec)	N/A	0.187	1.812	0.337	0.546	0.472	0.000	0.556	17.957

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	109	158	683	0	152	149
N.S.	1	1.00	0.88	0.99	1.44	6.21	0.00	1.38	1.35
time (sec)	N/A	0.067	0.891	0.240	0.534	0.430	0.000	0.482	13.689

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	112	131	191	771	0	191	159
N.S.	1	1.00	0.85	1.00	1.46	5.89	0.00	1.46	1.21
time (sec)	N/A	0.102	0.957	0.359	0.519	0.460	0.000	0.481	13.854

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	125	148	211	843	0	211	176
N.S.	1	1.00	0.87	1.03	1.47	5.85	0.00	1.47	1.22
time (sec)	N/A	0.105	0.845	0.382	0.549	0.430	0.000	0.450	13.817

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	214	160	270	1003	0	232	251
N.S.	1	1.00	1.09	0.82	1.38	5.12	0.00	1.18	1.28
time (sec)	N/A	0.176	1.157	0.518	0.504	0.470	0.000	0.470	15.241

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	201	230	378	1361	0	344	339
N.S.	1	1.00	0.98	1.12	1.83	6.61	0.00	1.67	1.65
time (sec)	N/A	0.204	0.978	0.468	0.560	0.450	0.000	0.521	15.436

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	312	336	588	2017	0	524	450
N.S.	1	1.00	1.12	1.20	2.11	7.23	0.00	1.88	1.61
time (sec)	N/A	0.362	1.282	0.460	0.542	0.553	0.000	0.425	16.104

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	29	33	10	57	0	25	18
N.S.	1	1.00	2.64	3.00	0.91	5.18	0.00	2.27	1.64
time (sec)	N/A	0.016	0.054	3.618	0.499	0.414	0.000	0.444	13.377

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	53	51	25	71	0	25	-1
N.S.	1	1.00	1.77	1.70	0.83	2.37	0.00	0.83	-0.03
time (sec)	N/A	0.018	0.035	5.515	0.561	0.413	0.000	0.471	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	39	57	11	94	0	48	-1
N.S.	1	1.00	2.60	3.80	0.73	6.27	0.00	3.20	-0.07
time (sec)	N/A	0.018	0.068	5.330	0.518	0.382	0.000	0.781	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	31	40	0	84	-1
N.S.	1	1.00	0.68	0.60	0.58	0.75	0.00	1.58	-0.02
time (sec)	N/A	0.034	0.015	2.796	0.562	0.381	0.000	0.479	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	17	26	0	57	-1
N.S.	1	1.00	0.76	0.71	0.50	0.76	0.00	1.68	-0.03
time (sec)	N/A	0.022	0.008	3.223	0.572	0.387	0.000	0.431	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	6	15	0	27	46
N.S.	1	1.00	1.00	1.15	0.46	1.15	0.00	2.08	3.54
time (sec)	N/A	0.015	0.004	1.236	0.576	0.399	0.000	0.445	0.216

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	20	38	65	0	0	-1
N.S.	1	1.00	2.88	1.25	2.38	4.06	0.00	0.00	-0.06
time (sec)	N/A	0.017	0.015	0.138	0.608	0.420	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	91	70	304	40	0	47	-1
N.S.	1	1.00	2.17	1.67	7.24	0.95	0.00	1.12	-0.02
time (sec)	N/A	0.023	0.044	6.525	0.628	0.401	0.000	0.489	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	72	89	933	49	0	129	-1
N.S.	1	1.00	1.18	1.46	15.30	0.80	0.00	2.11	-0.02
time (sec)	N/A	0.031	0.117	7.029	0.856	0.427	0.000	0.519	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	119	311	186	501	0	131	-1
N.S.	1	1.00	0.95	2.49	1.49	4.01	0.00	1.05	-0.01
time (sec)	N/A	0.088	0.296	7.186	0.540	0.660	0.000	0.765	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	93	182	74	433	0	83	-1
N.S.	1	1.00	1.19	2.33	0.95	5.55	0.00	1.06	-0.01
time (sec)	N/A	0.039	0.183	7.556	0.537	0.498	0.000	0.880	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	99	174	139	1158	0	0	-1
N.S.	1	1.00	1.19	2.10	1.67	13.95	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.081	10.362	0.529	0.603	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	100	227	0	338	0	0	-1
N.S.	1	1.00	1.19	2.70	0.00	4.02	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.184	8.167	0.000	0.453	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	127	379	0	520	0	962	-1
N.S.	1	1.00	0.89	2.65	0.00	3.64	0.00	6.73	-0.01
time (sec)	N/A	0.088	0.336	10.091	0.000	0.569	0.000	0.639	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	199	413	0	39	0	0	-1
N.S.	1	1.00	0.77	1.59	0.00	0.15	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.946	6.938	0.000	0.106	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	266	0	30	0	0	-1
N.S.	1	1.00	1.00	1.67	0.00	0.19	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.558	5.968	0.000	0.109	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	18	0	0	-1
N.S.	1	1.00	1.20	1.39	0.00	0.35	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.069	3.543	0.000	0.099	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	137	156	0	626	0	0	-1
N.S.	1	1.00	0.79	0.90	0.00	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.414	6.203	0.000	0.135	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	188	342	0	947	0	0	-1
N.S.	1	1.00	0.80	1.46	0.00	4.05	0.00	0.00	-0.00
time (sec)	N/A	0.169	2.128	6.832	0.000	0.172	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	152	446	262	579	0	197	-1
N.S.	1	1.00	0.90	2.64	1.55	3.43	0.00	1.17	-0.01
time (sec)	N/A	0.103	0.530	9.453	0.501	1.426	0.000	0.859	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	113	309	112	495	0	128	-1
N.S.	1	1.00	0.99	2.71	0.98	4.34	0.00	1.12	-0.01
time (sec)	N/A	0.052	0.269	8.800	0.495	0.646	0.000	0.791	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	141	255	189	1282	0	0	-1
N.S.	1	1.00	1.16	2.09	1.55	10.51	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.625	10.338	0.540	0.700	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	147	287	0	1449	0	0	-1
N.S.	1	1.00	1.15	2.24	0.00	11.32	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.779	11.000	0.000	0.726	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	114	376	0	484	0	960	-1
N.S.	1	1.00	0.89	2.94	0.00	3.78	0.00	7.50	-0.01
time (sec)	N/A	0.089	0.439	10.384	0.000	0.588	0.000	0.763	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	161	565	0	752	0	1708	-1
N.S.	1	1.00	0.82	2.87	0.00	3.82	0.00	8.67	-0.01
time (sec)	N/A	0.113	0.804	12.996	0.000	1.230	0.000	0.913	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	249	602	0	68	0	0	-1
N.S.	1	1.00	0.77	1.85	0.00	0.21	0.00	0.00	-0.00
time (sec)	N/A	0.301	1.758	7.951	0.000	0.139	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	201	429	0	47	0	0	-1
N.S.	1	1.00	0.92	1.97	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.201	0.936	6.720	0.000	0.110	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	156	266	0	18	0	0	-1
N.S.	1	1.00	1.01	1.73	0.00	0.12	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.531	6.493	0.000	0.109	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	141	174	0	45	0	0	-1
N.S.	1	1.00	0.78	0.96	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.917	6.458	0.000	0.132	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	201	408	0	969	0	0	-1
N.S.	1	1.00	0.85	1.73	0.00	4.11	0.00	0.00	-0.00
time (sec)	N/A	0.195	2.970	8.539	0.000	0.163	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	194	437	0	59	0	0	-1
N.S.	1	1.00	0.92	2.08	0.00	0.28	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.966	7.260	0.000	0.123	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	105	186	79	438	0	91	-1
N.S.	1	1.00	1.27	2.24	0.95	5.28	0.00	1.10	-0.01
time (sec)	N/A	0.063	0.215	7.691	0.569	0.506	0.000	0.911	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	53	99	25	370	0	83	-1
N.S.	1	1.00	1.29	2.41	0.61	9.02	0.00	2.02	-0.02
time (sec)	N/A	0.030	0.085	4.662	0.554	0.510	0.000	0.937	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	112	115	219	0	0	-1
N.S.	1	1.00	1.17	2.73	2.80	5.34	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.115	5.590	0.583	0.478	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	102	231	0	347	0	0	-1
N.S.	1	1.00	1.15	2.60	0.00	3.90	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.221	9.964	0.000	0.488	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	163	268	0	59	0	0	-1
N.S.	1	1.00	0.79	1.30	0.00	0.29	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.607	6.522	0.000	0.116	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	78	93	0	48	0	0	-1
N.S.	1	1.00	0.70	0.84	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.165	4.925	0.000	0.100	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	60	0	305	0	0	-1
N.S.	1	1.00	1.18	1.18	0.00	5.98	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.068	0.200	0.000	0.121	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	138	140	0	643	0	0	-1
N.S.	1	1.00	0.78	0.79	0.00	3.63	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.356	6.598	0.000	0.134	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	195	354	0	955	0	0	-1
N.S.	1	1.00	0.80	1.45	0.00	3.91	0.00	0.00	-0.00
time (sec)	N/A	0.176	2.634	7.662	0.000	0.148	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	96	156	86	564	0	113	-1
N.S.	1	1.00	1.22	1.97	1.09	7.14	0.00	1.43	-0.01
time (sec)	N/A	0.068	0.299	12.941	0.548	0.584	0.000	1.002	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	41	31	34	57	0	53	119
N.S.	1	1.00	1.21	0.91	1.00	1.68	0.00	1.56	3.50
time (sec)	N/A	0.030	0.082	4.879	0.293	0.446	0.000	0.626	15.179

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	93	165	174	422	0	0	-1
N.S.	1	1.00	1.18	2.09	2.20	5.34	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.230	12.068	0.538	0.496	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	134	274	0	634	0	520	-1
N.S.	1	1.00	1.00	2.04	0.00	4.73	0.00	3.88	-0.01
time (sec)	N/A	0.109	0.480	16.321	0.000	0.687	0.000	0.723	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	197	405	0	92	0	0	-1
N.S.	1	1.00	0.72	1.48	0.00	0.34	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.888	7.307	0.000	0.136	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	136	241	0	81	0	0	-1
N.S.	1	1.00	0.67	1.19	0.00	0.40	0.00	0.00	-0.00
time (sec)	N/A	0.131	0.470	8.819	0.000	0.113	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	138	191	0	780	0	0	-1
N.S.	1	1.00	0.90	1.25	0.00	5.10	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.292	8.408	0.000	0.165	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	938	0	0	-1
N.S.	1	1.00	0.89	1.02	0.00	9.29	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.100	6.241	0.000	0.178	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	170	199	0	1075	0	0	-1
N.S.	1	1.00	0.72	0.85	0.00	4.57	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.891	8.790	0.000	0.184	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	133	243	301	885	0	338	-1
N.S.	1	1.00	0.97	1.77	2.20	6.46	0.00	2.47	-0.01
time (sec)	N/A	0.095	0.546	18.345	0.493	0.872	0.000	0.775	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	56	129	137	0	149	176
N.S.	1	1.00	0.79	0.69	1.59	1.69	0.00	1.84	2.17
time (sec)	N/A	0.059	0.210	5.906	0.277	0.539	0.000	0.698	20.866

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	55	67	134	0	137	159
N.S.	1	1.00	0.82	0.75	0.92	1.84	0.00	1.88	2.18
time (sec)	N/A	0.037	0.125	6.435	0.282	0.495	0.000	0.615	20.795

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	127	249	322	752	0	0	-1
N.S.	1	1.00	0.98	1.93	2.50	5.83	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.411	19.383	0.517	0.672	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	192	698	0	130	0	0	-1
N.S.	1	1.00	0.67	2.45	0.00	0.46	0.00	0.00	-0.00
time (sec)	N/A	0.218	1.416	9.638	0.000	0.147	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	182	623	0	1432	0	0	-1
N.S.	1	1.00	0.68	2.32	0.00	5.32	0.00	0.00	-0.00
time (sec)	N/A	0.182	1.128	10.108	0.000	0.216	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	174	483	0	1400	0	0	-1
N.S.	1	1.00	0.79	2.19	0.00	6.33	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.998	7.925	0.000	0.221	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	172	547	0	1531	0	0	-1
N.S.	1	1.00	0.77	2.45	0.00	6.87	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.932	9.721	0.000	0.215	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	214	527	0	1719	0	0	-1
N.S.	1	1.00	0.66	1.64	0.00	5.34	0.00	0.00	-0.00
time (sec)	N/A	0.264	1.566	10.813	0.000	0.250	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	113	0	0	29	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.333	0.467	0.000	0.445	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	98	0	0	45	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.20	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.387	1.115	0.000	0.440	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	98	0	0	36	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.272	0.647	0.000	0.427	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	25	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.34	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.135	0.158	0.000	0.405	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.053	15.145	0.322	0.000	0.408	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.056	100.583	0.315	0.000	0.430	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.058	121.735	0.344	0.000	0.454	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	39	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.388	1.385	0.000	0.426	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	240	0	0	30	0	0	-1
N.S.	1	1.00	2.42	0.00	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.504	1.029	0.000	0.405	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.069	6.381	0.284	0.000	0.419	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.066	11.367	0.323	0.000	0.431	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	219	221	0	21338	0	0	1978
N.S.	1	1.00	0.65	0.66	0.00	63.70	0.00	0.00	5.90
time (sec)	N/A	0.501	0.369	0.685	0.000	1.684	0.000	0.000	15.241

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	255	141	0	29175	0	0	1962
N.S.	1	1.00	0.93	0.52	0.00	106.87	0.00	0.00	7.19
time (sec)	N/A	0.386	0.202	0.702	0.000	1.474	0.000	0.000	14.466

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	140	104	0	29221	0	0	1672
N.S.	1	1.00	0.54	0.40	0.00	112.82	0.00	0.00	6.46
time (sec)	N/A	0.327	0.137	0.583	0.000	1.442	0.000	0.000	14.901

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	172	78	0	18879	0	0	652
N.S.	1	1.00	0.64	0.29	0.00	70.71	0.00	0.00	2.44
time (sec)	N/A	0.192	0.117	0.638	0.000	1.501	0.000	0.000	16.121

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	264	96	0	29139	0	0	1439
N.S.	1	1.00	1.00	0.36	0.00	110.38	0.00	0.00	5.45
time (sec)	N/A	0.258	0.158	0.753	0.000	4.000	0.000	0.000	15.656

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	181	136	0	29431	0	0	1573
N.S.	1	1.00	0.63	0.47	0.00	102.55	0.00	0.00	5.48
time (sec)	N/A	0.305	0.294	0.963	0.000	14.643	0.000	0.000	15.035

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	290	196	0	21564	0	0	1560
N.S.	1	1.00	0.84	0.57	0.00	62.69	0.00	0.00	4.53
time (sec)	N/A	0.375	1.414	1.066	0.000	104.912	0.000	0.000	14.598

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	164	143	0	29350	0	0	1800
N.S.	1	1.00	0.56	0.49	0.00	100.17	0.00	0.00	6.14
time (sec)	N/A	0.301	0.204	0.707	0.000	1.531	0.000	0.000	14.584

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	186	104	0	21185	0	0	665
N.S.	1	1.00	0.66	0.37	0.00	75.39	0.00	0.00	2.37
time (sec)	N/A	0.324	0.180	0.687	0.000	1.570	0.000	0.000	15.072

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	231	76	0	25253	0	0	590
N.S.	1	1.00	0.96	0.32	0.00	105.22	0.00	0.00	2.46
time (sec)	N/A	0.198	0.140	0.580	0.000	1.449	0.000	0.000	15.652

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	126	83	0	25429	0	0	609
N.S.	1	1.00	0.51	0.34	0.00	103.79	0.00	0.00	2.49
time (sec)	N/A	0.197	0.087	0.523	0.000	1.425	0.000	0.000	15.884

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	196	114	0	21243	0	0	697
N.S.	1	1.00	0.70	0.41	0.00	75.60	0.00	0.00	2.48
time (sec)	N/A	0.337	0.204	0.761	0.000	1.552	0.000	0.000	14.417

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	333	162	0	29423	0	0	1503
N.S.	1	1.00	1.12	0.55	0.00	99.40	0.00	0.00	5.08
time (sec)	N/A	0.276	1.508	0.903	0.000	4.090	0.000	0.000	16.383

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	228	137	0	872	0	0	1067
N.S.	1	1.00	1.29	0.77	0.00	4.93	0.00	0.00	6.03
time (sec)	N/A	0.179	0.338	0.476	0.000	0.481	0.000	0.000	14.439

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	310	110	0	849	0	0	1119
N.S.	1	1.00	2.09	0.74	0.00	5.74	0.00	0.00	7.56
time (sec)	N/A	0.124	0.216	0.425	0.000	0.479	0.000	0.000	14.269

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	198	99	0	815	0	0	1001
N.S.	1	1.00	1.43	0.72	0.00	5.91	0.00	0.00	7.25
time (sec)	N/A	0.127	0.168	0.504	0.000	0.473	0.000	0.000	14.260

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	285	83	0	703	0	166	976
N.S.	1	1.00	2.48	0.72	0.00	6.11	0.00	1.44	8.49
time (sec)	N/A	0.081	0.128	0.344	0.000	0.460	0.000	0.721	0.509

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	183	87	0	703	0	183	361
N.S.	1	1.00	1.46	0.70	0.00	5.62	0.00	1.46	2.89
time (sec)	N/A	0.071	0.127	0.391	0.000	0.448	0.000	0.980	15.111

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	318	119	0	773	0	0	2031
N.S.	1	1.00	2.34	0.88	0.00	5.68	0.00	0.00	14.93
time (sec)	N/A	0.125	0.188	0.516	0.000	0.508	0.000	0.000	15.361

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	242	152	0	924	0	0	2779
N.S.	1	1.00	1.32	0.83	0.00	5.02	0.00	0.00	15.10
time (sec)	N/A	0.140	0.240	0.641	0.000	0.548	0.000	0.000	15.186

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	409	193	0	1089	0	0	2500
N.S.	1	1.00	1.79	0.84	0.00	4.76	0.00	0.00	10.92
time (sec)	N/A	0.176	0.813	0.646	0.000	0.602	0.000	0.000	15.517

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	172	209	0	1311	0	461	2500
N.S.	1	1.00	0.93	1.14	0.00	7.12	0.00	2.51	13.59
time (sec)	N/A	0.197	0.635	0.386	0.000	0.560	0.000	1.108	16.867

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	157	186	0	1275	0	695	1273
N.S.	1	1.00	1.01	1.20	0.00	8.23	0.00	4.48	8.21
time (sec)	N/A	0.140	0.584	0.365	0.000	0.605	0.000	1.256	16.401

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	143	163	0	1125	0	912	2991
N.S.	1	1.00	1.13	1.28	0.00	8.86	0.00	7.18	23.55
time (sec)	N/A	0.133	0.276	0.365	0.000	0.530	0.000	0.913	16.275

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	137	145	0	1087	0	397	443
N.S.	1	1.00	1.10	1.16	0.00	8.70	0.00	3.18	3.54
time (sec)	N/A	0.081	0.223	0.382	0.000	0.528	0.000	0.999	16.187

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	128	145	0	1079	0	361	671
N.S.	1	1.00	1.11	1.26	0.00	9.38	0.00	3.14	5.83
time (sec)	N/A	0.065	0.171	0.351	0.000	0.535	0.000	0.585	14.994

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	143	164	0	1229	0	672	371
N.S.	1	1.00	1.03	1.18	0.00	8.84	0.00	4.83	2.67
time (sec)	N/A	0.127	0.743	0.497	0.000	0.558	0.000	0.835	14.512

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	165	177	0	1365	0	938	1670
N.S.	1	1.00	1.11	1.19	0.00	9.16	0.00	6.30	11.21
time (sec)	N/A	0.132	1.076	0.536	0.000	0.577	0.000	0.874	15.554

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	174	195	0	1477	0	471	416
N.S.	1	1.00	0.98	1.10	0.00	8.30	0.00	2.65	2.34
time (sec)	N/A	0.143	3.144	0.529	0.000	0.580	0.000	0.778	15.256

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	277	213	0	1585	0	467	1704
N.S.	1	1.00	1.41	1.08	0.00	8.05	0.00	2.37	8.65
time (sec)	N/A	0.162	6.222	0.513	0.000	0.630	0.000	0.830	17.078

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	486	209	0	2649	0	0	2500
N.S.	1	1.00	2.06	0.89	0.00	11.22	0.00	0.00	10.59
time (sec)	N/A	0.337	0.820	1.059	0.000	0.727	0.000	0.000	16.004

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	565	214	0	2507	0	0	2500
N.S.	1	1.00	2.69	1.02	0.00	11.94	0.00	0.00	11.90
time (sec)	N/A	0.233	0.396	0.937	0.000	0.687	0.000	0.000	15.870

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	469	188	0	2507	0	0	2500
N.S.	1	1.00	2.16	0.87	0.00	11.55	0.00	0.00	11.52
time (sec)	N/A	0.193	0.443	0.875	0.000	0.658	0.000	0.000	16.630

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	345	204	0	2049	0	605	3060
N.S.	1	1.00	1.85	1.10	0.00	11.02	0.00	3.25	16.45
time (sec)	N/A	0.136	0.221	0.635	0.000	0.562	0.000	0.912	15.603

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	469	241	0	2269	0	693	2500
N.S.	1	1.00	2.12	1.09	0.00	10.27	0.00	3.14	11.31
time (sec)	N/A	0.196	0.280	1.063	0.000	0.660	0.000	0.789	16.794

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	600	243	0	2711	0	0	2500
N.S.	1	1.00	1.85	0.75	0.00	8.34	0.00	0.00	7.69
time (sec)	N/A	0.264	0.592	1.145	0.000	0.962	0.000	0.000	17.550

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	262	266	0	3544	0	1563	2500
N.S.	1	1.00	0.82	0.83	0.00	11.08	0.00	4.88	7.81
time (sec)	N/A	0.316	3.359	0.792	0.000	1.044	0.000	0.921	17.602

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	238	262	0	3135	0	1481	2500
N.S.	1	1.00	1.02	1.12	0.00	13.45	0.00	6.36	10.73
time (sec)	N/A	0.224	1.774	0.714	0.000	1.004	0.000	0.872	16.545

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	225	218	0	2796	0	1262	2980
N.S.	1	1.00	1.15	1.12	0.00	14.34	0.00	6.47	15.28
time (sec)	N/A	0.160	2.985	0.550	0.000	0.757	0.000	0.886	15.990

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	255	250	0	3445	0	1407	2500
N.S.	1	1.00	1.16	1.14	0.00	15.73	0.00	6.42	11.42
time (sec)	N/A	0.201	1.432	0.800	0.000	0.989	0.000	0.892	17.349

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	230	260	0	3477	0	1506	2500
N.S.	1	1.00	1.10	1.24	0.00	16.56	0.00	7.17	11.90
time (sec)	N/A	0.172	1.959	0.817	0.000	1.045	0.000	0.557	16.516

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	274	269	0	3648	0	1545	2500
N.S.	1	1.00	1.16	1.14	0.00	15.46	0.00	6.55	10.59
time (sec)	N/A	0.360	1.614	0.881	0.000	1.189	0.000	0.869	18.290

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	785	342	0	4640	0	0	2500
N.S.	1	1.00	2.49	1.09	0.00	14.73	0.00	0.00	7.94
time (sec)	N/A	0.387	1.070	1.754	0.000	1.140	0.000	0.000	19.294

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	630	303	0	4185	0	0	2500
N.S.	1	1.00	2.17	1.04	0.00	14.43	0.00	0.00	8.62
time (sec)	N/A	0.301	0.773	1.502	0.000	0.881	0.000	0.000	19.618

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	786	347	0	4524	0	0	2500
N.S.	1	1.00	2.51	1.11	0.00	14.45	0.00	0.00	7.99
time (sec)	N/A	0.319	0.933	1.790	0.000	1.058	0.000	0.000	20.153

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	631	403	0	4050	0	1076	2500
N.S.	1	1.00	2.19	1.40	0.00	14.06	0.00	3.74	8.68
time (sec)	N/A	0.342	0.738	1.447	0.000	0.925	0.000	1.289	19.278

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	784	430	0	4160	0	766	2500
N.S.	1	1.00	2.50	1.37	0.00	13.29	0.00	2.45	7.99
time (sec)	N/A	0.322	0.846	1.778	0.000	1.042	0.000	1.197	18.910

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	920	384	0	5020	0	0	2500
N.S.	1	1.00	1.49	0.62	0.00	8.14	0.00	0.00	4.05
time (sec)	N/A	0.578	2.932	2.010	0.000	2.240	0.000	0.000	20.835

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	331	374	0	5219	0	1989	2500
N.S.	1	1.00	1.04	1.17	0.00	16.36	0.00	6.24	7.84
time (sec)	N/A	0.348	2.946	1.155	0.000	1.475	0.000	1.321	19.648

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	350	432	0	5961	0	2231	2500
N.S.	1	1.00	1.02	1.26	0.00	17.38	0.00	6.50	7.29
time (sec)	N/A	0.514	2.582	1.729	0.000	2.084	0.000	1.271	20.337

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	316	372	0	5510	0	1986	2500
N.S.	1	1.00	1.01	1.19	0.00	17.60	0.00	6.35	7.99
time (sec)	N/A	0.459	3.420	1.102	0.000	1.466	0.000	1.414	19.499

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	343	422	0	6215	0	1184	2500
N.S.	1	1.00	0.99	1.22	0.00	17.91	0.00	3.41	7.20
time (sec)	N/A	0.462	5.373	1.649	0.000	2.426	0.000	1.408	19.905

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	333	416	0	6152	0	1131	2500
N.S.	1	1.00	1.04	1.30	0.00	19.29	0.00	3.55	7.84
time (sec)	N/A	0.424	2.097	1.658	0.000	2.601	0.000	0.710	19.666

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	357	437	0	6323	0	2203	2500
N.S.	1	1.00	1.00	1.22	0.00	17.71	0.00	6.17	7.00
time (sec)	N/A	0.874	3.514	1.776	0.000	2.746	0.000	1.370	20.801

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	45	24	18	17	43	724	51	17
N.S.	1	1.80	0.96	0.72	0.68	1.72	28.96	2.04	0.68
time (sec)	N/A	0.013	0.041	0.154	0.498	0.397	24.195	0.439	14.340

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	148	771	0	823	0	318	407
N.S.	1	1.00	0.30	1.58	0.00	1.69	0.00	0.65	0.84
time (sec)	N/A	0.774	0.224	0.334	0.000	0.549	0.000	0.617	15.178

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	45	190	0	3830	0	170	236
N.S.	1	1.00	0.15	0.61	0.00	12.39	0.00	0.55	0.76
time (sec)	N/A	0.136	0.056	0.438	0.000	18.116	0.000	0.581	14.352

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	47242	439	0	35	0	0	-1
N.S.	1	1.00	99.04	0.92	0.00	0.07	0.00	0.00	-0.00
time (sec)	N/A	0.261	61.215	18.470	0.000	0.110	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	118912	0	0	35	0	0	-1
N.S.	1	1.00	228.24	0.00	0.00	0.07	0.00	0.00	-0.00
time (sec)	N/A	0.442	41.545	1.325	0.000	0.156	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	47246	837	0	55	0	0	-1
N.S.	1	1.00	97.62	1.73	0.00	0.11	0.00	0.00	-0.00
time (sec)	N/A	0.279	61.223	20.277	0.000	0.126	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	89374	398	0	46	0	0	-1
N.S.	1	1.00	207.36	0.92	0.00	0.11	0.00	0.00	-0.00
time (sec)	N/A	0.211	51.300	20.519	0.000	0.096	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	13300	163	0	35	0	0	-1
N.S.	1	1.00	77.78	0.95	0.00	0.20	0.00	0.00	-0.01
time (sec)	N/A	0.074	45.064	17.546	0.000	0.112	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	63281	0	0	0	0	0	-1
N.S.	1	1.00	134.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	41.073	1.442	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	776	776	119171	0	0	37	0	0	-1
N.S.	1	1.00	153.57	0.00	0.00	0.05	0.00	0.00	-0.00
time (sec)	N/A	0.702	41.765	1.382	0.000	0.119	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	287	881	0	40	0	0	-1
N.S.	1	1.00	0.58	1.77	0.00	0.08	0.00	0.00	-0.00
time (sec)	N/A	0.462	32.076	46.237	0.000	0.203	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	304	396	0	28	0	0	-1
N.S.	1	1.00	1.88	2.44	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.054	16.176	29.469	0.000	0.096	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	432	0	0	37	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.08	0.00	0.00	-0.00
time (sec)	N/A	0.280	101.050	1.349	0.000	0.093	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	149	109	0	0	0	0	1515
N.S.	1	1.00	0.39	0.28	0.00	0.00	0.00	0.00	3.95
time (sec)	N/A	0.485	0.147	0.707	0.000	0.000	0.000	0.000	19.752

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	148	68	0	15501	0	0	513
N.S.	1	1.00	0.87	0.40	0.00	90.65	0.00	0.00	3.00
time (sec)	N/A	0.184	0.166	0.852	0.000	1.938	0.000	0.000	15.705

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	174	85	0	665483	0	0	816
N.S.	1	1.00	0.71	0.35	0.00	2716.26	0.00	0.00	3.33
time (sec)	N/A	0.338	0.221	0.482	0.000	6.293	0.000	0.000	16.948

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	149	109	0	0	0	0	1515
N.S.	1	1.00	0.39	0.29	0.00	0.00	0.00	0.00	4.00
time (sec)	N/A	0.335	0.135	0.639	0.000	0.000	0.000	0.000	20.141

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	148	71	0	16697	0	0	513
N.S.	1	1.00	0.85	0.41	0.00	95.41	0.00	0.00	2.93
time (sec)	N/A	0.180	0.125	0.920	0.000	1.944	0.000	0.000	16.069

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	174	88	0	643307	0	0	818
N.S.	1	1.00	0.82	0.41	0.00	3020.22	0.00	0.00	3.84
time (sec)	N/A	0.145	0.144	0.492	0.000	6.382	0.000	0.000	16.542

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	411	133	0	0	0	0	2500
N.S.	1	1.00	2.11	0.68	0.00	0.00	0.00	0.00	12.82
time (sec)	N/A	0.257	0.107	0.668	0.000	0.000	0.000	0.000	15.254

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	79	72	71	138	0	185	98
N.S.	1	1.00	0.77	0.70	0.69	1.34	0.00	1.80	0.95
time (sec)	N/A	0.073	0.124	0.217	0.503	0.421	0.000	0.414	14.226

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	129	141	71	0	0	0	0	945
N.S.	1	0.59	0.65	0.33	0.00	0.00	0.00	0.00	4.33
time (sec)	N/A	0.136	0.116	0.549	0.000	0.000	0.000	0.000	14.919

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	413	133	0	0	0	0	2500
N.S.	1	1.00	2.21	0.71	0.00	0.00	0.00	0.00	13.37
time (sec)	N/A	0.174	0.098	0.665	0.000	0.000	0.000	0.000	14.440

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	117	202	0	0	0	197	99
N.S.	1	1.00	1.65	2.85	0.00	0.00	0.00	2.77	1.39
time (sec)	N/A	0.097	0.216	0.309	0.000	0.000	0.000	0.477	14.208

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	64	206	0	3884	0	220	141
N.S.	1	1.00	0.72	2.31	0.00	43.64	0.00	2.47	1.58
time (sec)	N/A	0.055	0.132	0.386	0.000	18.059	0.000	0.560	14.038

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	26	28	27	580	28	34
N.S.	1	1.00	0.92	0.68	0.74	0.71	15.26	0.74	0.89
time (sec)	N/A	0.032	0.005	0.104	0.272	0.409	15.009	0.444	0.097

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	20	22	21	311	22	19
N.S.	1	1.00	0.93	0.69	0.76	0.72	10.72	0.76	0.66
time (sec)	N/A	0.033	0.004	0.147	0.273	0.397	6.489	0.437	0.076

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	14	14	13	124	14	16
N.S.	1	1.00	1.06	0.78	0.78	0.72	6.89	0.78	0.89
time (sec)	N/A	0.030	0.004	0.149	0.285	0.407	2.469	0.465	0.059

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	15	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	2.50	1.00	1.00
time (sec)	N/A	0.025	0.003	0.110	0.275	0.414	0.819	0.447	13.757

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	37	8	21	20	19	23	7
N.S.	1	1.00	5.29	1.14	3.00	2.86	2.71	3.29	1.00
time (sec)	N/A	0.016	0.005	0.089	0.280	0.400	0.080	0.428	13.598

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	61	52	51	46	0	47	31
N.S.	1	1.00	1.74	1.49	1.46	1.31	0.00	1.34	0.89
time (sec)	N/A	0.036	0.099	0.172	0.271	0.397	0.000	0.438	13.879

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	35	37	23	473	36	25
N.S.	1	1.00	0.79	1.06	1.12	0.70	14.33	1.09	0.76
time (sec)	N/A	0.032	0.004	0.163	0.505	0.413	4.066	0.466	13.614

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	23	21	12	153	24	13
N.S.	1	1.00	0.90	1.15	1.05	0.60	7.65	1.20	0.65
time (sec)	N/A	0.029	0.003	0.143	0.489	0.398	1.459	0.454	13.813

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	5	5	2	14	5
N.S.	1	1.00	1.00	1.60	1.00	1.00	0.40	2.80	1.00
time (sec)	N/A	0.026	0.001	0.137	0.501	0.392	0.453	0.450	13.811

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	45	36	37	37	0	38	25
N.S.	1	1.00	2.05	1.64	1.68	1.68	0.00	1.73	1.14
time (sec)	N/A	0.026	0.032	0.161	0.277	0.408	0.000	0.464	13.873

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	21	14	14	19	0	14	13
N.S.	1	1.00	1.17	0.78	0.78	1.06	0.00	0.78	0.72
time (sec)	N/A	0.029	0.004	0.157	0.280	0.368	0.000	0.462	13.868

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	22	25	0	22	21
N.S.	1	1.00	1.07	0.69	0.76	0.86	0.00	0.76	0.72
time (sec)	N/A	0.032	0.004	0.152	0.273	0.382	0.000	0.458	13.922

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	20	22	21	362	22	19
N.S.	1	1.00	0.93	0.69	0.76	0.72	12.48	0.76	0.66
time (sec)	N/A	0.031	0.004	0.086	0.274	0.425	32.983	0.409	14.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	14	14	13	144	14	16
N.S.	1	1.00	1.06	0.78	0.78	0.72	8.00	0.78	0.89
time (sec)	N/A	0.030	0.003	0.088	0.282	0.378	15.193	0.441	0.036

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	19	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	3.17	1.00	1.00
time (sec)	N/A	0.026	0.002	0.158	0.270	0.379	6.596	0.431	0.025

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	37	8	21	20	22	23	7
N.S.	1	1.00	5.29	1.14	3.00	2.86	3.14	3.29	1.00
time (sec)	N/A	0.025	0.004	0.158	0.272	0.380	2.318	0.430	0.056

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	45	36	41	37	117	38	30
N.S.	1	1.00	2.05	1.64	1.86	1.68	5.32	1.73	1.36
time (sec)	N/A	0.021	0.006	0.126	0.292	0.388	0.245	0.452	0.076

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	61	52	57	46	0	47	31
N.S.	1	1.00	1.74	1.49	1.63	1.31	0.00	1.34	0.89
time (sec)	N/A	0.030	0.008	0.161	0.286	0.390	0.000	0.447	13.985

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	35	43	23	549	31	29
N.S.	1	1.00	0.79	1.06	1.30	0.70	16.64	0.94	0.88
time (sec)	N/A	0.034	0.004	0.101	0.536	0.388	23.245	0.454	13.821

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	23	25	12	178	22	13
N.S.	1	1.00	0.90	1.15	1.25	0.60	8.90	1.10	0.65
time (sec)	N/A	0.029	0.003	0.092	0.519	0.376	10.241	0.448	13.926

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	5	5	3	5	5
N.S.	1	1.00	1.00	1.60	1.00	1.00	0.60	1.00	1.00
time (sec)	N/A	0.025	0.000	0.176	0.494	0.367	4.115	0.435	13.943

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	10	20	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.67	3.33	1.00	1.00
time (sec)	N/A	0.029	0.003	0.159	0.304	0.367	1.657	0.459	13.780

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	22	25	0	22	21
N.S.	1	1.00	1.07	0.69	0.76	0.86	0.00	0.76	0.72
time (sec)	N/A	0.032	0.004	0.174	0.282	0.432	0.000	0.436	13.799

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	24	28	31	0	28	33
N.S.	1	1.00	1.11	0.65	0.76	0.84	0.00	0.76	0.89
time (sec)	N/A	0.032	0.005	0.215	0.319	0.401	0.000	0.426	13.837

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	87	112	131	78	354	87	119
N.S.	1	1.00	0.80	1.03	1.20	0.72	3.25	0.80	1.09
time (sec)	N/A	0.044	0.218	0.479	0.532	0.400	0.918	0.525	15.222

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	64	92	104	63	250	67	91
N.S.	1	1.00	0.77	1.11	1.25	0.76	3.01	0.81	1.10
time (sec)	N/A	0.034	0.115	0.384	0.523	0.386	0.423	0.458	14.203

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	70	74	47	150	41	67
N.S.	1	1.00	0.81	1.23	1.30	0.82	2.63	0.72	1.18
time (sec)	N/A	0.028	0.065	0.246	0.524	0.388	0.179	0.479	13.655

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	32	31	29	51	26	27
N.S.	1	1.00	1.10	1.07	1.03	0.97	1.70	0.87	0.90
time (sec)	N/A	0.011	0.025	0.109	0.293	0.387	0.073	0.452	13.646

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	36	30	33	35	0	49	26
N.S.	1	1.00	2.00	1.67	1.83	1.94	0.00	2.72	1.44
time (sec)	N/A	0.021	0.014	0.266	0.504	0.389	0.000	0.450	13.643

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	41	46	29	38	0	38	31
N.S.	1	1.00	1.37	1.53	0.97	1.27	0.00	1.27	1.03
time (sec)	N/A	0.020	0.060	0.282	0.279	0.385	0.000	0.448	14.162

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	76	46	59	0	64	45
N.S.	1	1.00	1.28	1.52	0.92	1.18	0.00	1.28	0.90
time (sec)	N/A	0.029	0.140	0.306	0.284	0.386	0.000	0.480	13.895

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	86	104	64	77	0	88	59
N.S.	1	1.00	1.19	1.44	0.89	1.07	0.00	1.22	0.82
time (sec)	N/A	0.036	0.233	0.291	0.288	0.373	0.000	0.507	13.844

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	96	167	178	114	481	108	160
N.S.	1	1.00	0.62	1.07	1.14	0.73	3.08	0.69	1.03
time (sec)	N/A	0.088	0.243	0.504	0.494	0.399	0.955	0.509	15.490

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	79	134	134	85	314	84	120
N.S.	1	1.00	0.68	1.16	1.16	0.73	2.71	0.72	1.03
time (sec)	N/A	0.093	0.205	0.352	0.494	0.398	0.438	0.463	14.733

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	58	78	73	63	168	60	77
N.S.	1	1.00	0.81	1.08	1.01	0.88	2.33	0.83	1.07
time (sec)	N/A	0.014	0.095	0.227	0.292	0.397	0.184	0.415	14.487

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	48	87	81	68	0	99	74
N.S.	1	1.00	0.94	1.71	1.59	1.33	0.00	1.94	1.45
time (sec)	N/A	0.060	0.235	0.382	0.521	0.417	0.000	0.493	14.088

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	57	76	56	70	0	80	46
N.S.	1	1.00	1.27	1.69	1.24	1.56	0.00	1.78	1.02
time (sec)	N/A	0.039	0.272	0.395	0.482	0.416	0.000	0.489	13.759

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	67	101	58	83	0	86	44
N.S.	1	1.00	1.26	1.91	1.09	1.57	0.00	1.62	0.83
time (sec)	N/A	0.038	0.264	0.342	0.272	0.402	0.000	0.486	15.833

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	92	149	85	108	0	127	72
N.S.	1	1.00	1.15	1.86	1.06	1.35	0.00	1.59	0.90
time (sec)	N/A	0.051	0.355	0.368	0.278	0.433	0.000	0.481	15.137

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	107	195	108	128	0	168	94
N.S.	1	1.00	1.01	1.84	1.02	1.21	0.00	1.58	0.89
time (sec)	N/A	0.061	0.340	0.377	0.277	0.415	0.000	0.498	14.218

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	109	91	86	233	0	98	99
N.S.	1	1.00	1.40	1.17	1.10	2.99	0.00	1.26	1.27
time (sec)	N/A	0.057	0.200	0.328	0.502	0.428	0.000	0.472	0.115

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	96	114	312	0	131	1804
N.S.	1	1.00	0.91	1.10	1.31	3.59	0.00	1.51	20.74
time (sec)	N/A	0.124	0.141	0.291	0.501	0.434	0.000	0.439	15.451

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	84	54	52	159	0	58	65
N.S.	1	1.00	1.56	1.00	0.96	2.94	0.00	1.07	1.20
time (sec)	N/A	0.051	0.128	0.267	0.498	0.424	0.000	0.413	14.301

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	63	64	239	0	92	119
N.S.	1	1.00	0.93	1.07	1.08	4.05	0.00	1.56	2.02
time (sec)	N/A	0.072	0.118	0.263	0.496	0.420	0.000	0.462	14.666

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	30	101	0	30	28
N.S.	1	1.00	1.00	0.86	0.83	2.81	0.00	0.83	0.78
time (sec)	N/A	0.036	0.019	0.214	0.504	0.428	0.000	0.437	0.095

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	35	206	0	62	272
N.S.	1	1.00	1.00	0.97	0.90	5.28	0.00	1.59	6.97
time (sec)	N/A	0.039	0.042	0.172	0.485	0.440	0.000	0.430	14.713

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	16	78	66	16	17
N.S.	1	1.00	1.00	0.68	0.64	3.12	2.64	0.64	0.68
time (sec)	N/A	0.018	0.007	0.113	0.475	0.393	0.411	0.445	14.636

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	96	55	47	116	0	49	856
N.S.	1	1.00	2.40	1.38	1.18	2.90	0.00	1.22	21.40
time (sec)	N/A	0.033	0.097	0.197	0.500	0.415	0.000	0.448	14.695

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	37	255	0	45	39
N.S.	1	1.00	1.00	0.97	0.95	6.54	0.00	1.15	1.00
time (sec)	N/A	0.039	0.065	0.230	0.478	0.451	0.000	0.476	14.656

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	147	96	104	203	0	102	1139
N.S.	1	1.00	2.41	1.57	1.70	3.33	0.00	1.67	18.67
time (sec)	N/A	0.057	0.237	0.313	0.477	0.458	0.000	0.416	15.357

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	62	72	343	0	134	77
N.S.	1	1.00	1.00	1.05	1.22	5.81	0.00	2.27	1.31
time (sec)	N/A	0.057	0.160	0.270	0.480	0.416	0.000	0.448	15.025

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	214	154	199	327	0	177	832
N.S.	1	1.00	2.30	1.66	2.14	3.52	0.00	1.90	8.95
time (sec)	N/A	0.098	0.930	0.390	0.500	0.481	0.000	0.437	17.450

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	90	109	126	459	0	254	121
N.S.	1	1.00	1.03	1.25	1.45	5.28	0.00	2.92	1.39
time (sec)	N/A	0.072	0.284	0.346	0.485	0.474	0.000	0.445	13.990

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	90	100	150	491	0	175	463
N.S.	1	1.00	0.80	0.88	1.33	4.35	0.00	1.55	4.10
time (sec)	N/A	0.144	0.224	0.246	0.504	0.462	0.000	0.489	14.697

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	118	77	79	296	0	82	96
N.S.	1	1.00	1.64	1.07	1.10	4.11	0.00	1.14	1.33
time (sec)	N/A	0.077	0.259	0.322	0.482	0.425	0.000	0.476	14.402

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	75	80	367	0	109	533
N.S.	1	1.00	1.05	1.00	1.07	4.89	0.00	1.45	7.11
time (sec)	N/A	0.069	0.237	0.276	0.485	0.452	0.000	0.464	14.327

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	53	53	206	0	56	47
N.S.	1	1.00	1.00	0.90	0.90	3.49	0.00	0.95	0.80
time (sec)	N/A	0.039	0.054	0.255	0.484	0.412	0.000	0.509	0.136

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	51	49	313	0	77	50
N.S.	1	1.00	1.09	0.94	0.91	5.80	0.00	1.43	0.93
time (sec)	N/A	0.036	0.103	0.196	0.484	0.414	0.000	0.450	14.305

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	39	38	165	289	38	36
N.S.	1	1.00	1.00	0.81	0.79	3.44	6.02	0.79	0.75
time (sec)	N/A	0.023	0.044	0.147	0.506	0.394	3.436	0.444	15.377

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	130	79	115	354	0	109	2213
N.S.	1	1.00	1.78	1.08	1.58	4.85	0.00	1.49	30.32
time (sec)	N/A	0.064	0.386	0.319	0.484	0.485	0.000	0.482	15.835

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	81	119	505	0	113	123
N.S.	1	1.00	1.00	1.07	1.57	6.64	0.00	1.49	1.62
time (sec)	N/A	0.081	0.366	0.302	0.481	0.441	0.000	0.455	14.777

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	183	119	220	560	0	194	2009
N.S.	1	1.00	1.68	1.09	2.02	5.14	0.00	1.78	18.43
time (sec)	N/A	0.108	0.758	0.408	0.478	0.488	0.000	0.481	15.058

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	97	110	170	653	0	270	176
N.S.	1	1.00	1.01	1.15	1.77	6.80	0.00	2.81	1.83
time (sec)	N/A	0.094	0.695	0.371	0.486	0.440	0.000	0.479	14.297

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	125	144	127	511	0	96	-1
N.S.	1	1.00	1.07	1.23	1.09	4.37	0.00	0.82	-0.01
time (sec)	N/A	0.077	0.315	7.822	0.268	0.652	0.000	0.526	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	96	60	49	453	0	65	61
N.S.	1	1.00	1.33	0.83	0.68	6.29	0.00	0.90	0.85
time (sec)	N/A	0.036	0.189	0.133	0.273	0.485	0.000	0.525	14.477

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	129	150	133	1246	0	0	-1
N.S.	1	1.00	1.57	1.83	1.62	15.20	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.194	24.247	0.550	0.597	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	164	290	0	337	0	0	-1
N.S.	1	1.00	2.00	3.54	0.00	4.11	0.00	0.00	-0.01
time (sec)	N/A	0.061	1.520	24.378	0.000	0.494	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	669	570	0	443	0	0	-1
N.S.	1	1.00	4.68	3.99	0.00	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.093	10.753	26.531	0.000	0.692	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	260	199	432	0	27	0	0	-1
N.S.	1	1.18	0.90	1.96	0.00	0.12	0.00	0.00	-0.00
time (sec)	N/A	0.182	1.025	8.159	0.000	0.127	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	199	158	265	0	27	0	0	-1
N.S.	1	1.25	0.99	1.67	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.685	6.224	0.000	0.104	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	18	0	0	-1
N.S.	1	1.00	1.20	1.39	0.00	0.35	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.059	4.402	0.000	0.093	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	171	134	294	0	640	0	0	-1
N.S.	1	1.31	1.02	2.24	0.00	4.89	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.359	11.377	0.000	0.144	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	236	187	367	0	789	0	0	-1
N.S.	1	1.20	0.95	1.87	0.00	4.03	0.00	0.00	-0.01
time (sec)	N/A	0.144	1.322	13.518	0.000	0.182	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	149	198	186	577	0	134	-1
N.S.	1	1.00	0.95	1.26	1.18	3.68	0.00	0.85	-0.01
time (sec)	N/A	0.088	0.578	7.595	0.286	1.449	0.000	0.623	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	86	78	503	0	84	60
N.S.	1	1.00	0.89	0.83	0.75	4.84	0.00	0.81	0.58
time (sec)	N/A	0.044	0.361	0.127	0.283	0.656	0.000	0.575	14.539

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	233	417	178	1381	0	0	-1
N.S.	1	1.00	1.93	3.45	1.47	11.41	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.483	19.569	0.526	0.790	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	210	402	0	1471	0	0	-1
N.S.	1	1.00	1.65	3.17	0.00	11.58	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.667	21.859	0.000	0.802	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	63	406	0	413	0	0	-1
N.S.	1	1.00	0.52	3.33	0.00	3.39	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.089	17.539	0.000	0.742	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	938	693	0	545	0	0	-1
N.S.	1	1.00	4.81	3.55	0.00	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.114	11.689	21.061	0.000	2.323	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	247	590	0	44	0	0	-1
N.S.	1	1.00	0.77	1.84	0.00	0.14	0.00	0.00	-0.00
time (sec)	N/A	0.261	1.848	8.427	0.000	0.140	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	200	429	0	44	0	0	-1
N.S.	1	1.00	0.77	1.66	0.00	0.17	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.942	8.306	0.000	0.129	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	156	266	0	18	0	0	-1
N.S.	1	1.00	1.01	1.73	0.00	0.12	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.051	5.865	0.000	0.101	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	144	466	0	45	0	0	-1
N.S.	1	1.00	0.79	2.56	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.631	14.703	0.000	0.114	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	190	375	0	784	0	0	-1
N.S.	1	1.00	0.81	1.59	0.00	3.32	0.00	0.00	-0.00
time (sec)	N/A	0.169	1.473	12.935	0.000	0.172	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	93	73	461	0	71	-1
N.S.	1	1.00	1.00	1.18	0.92	5.84	0.00	0.90	-0.01
time (sec)	N/A	0.057	0.081	6.118	0.266	0.484	0.000	0.653	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	22	394	0	65	33
N.S.	1	1.00	1.00	0.89	0.58	10.37	0.00	1.71	0.87
time (sec)	N/A	0.028	0.013	0.117	0.285	0.463	0.000	0.684	14.741

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	105	111	240	0	0	-1
N.S.	1	1.00	1.00	2.50	2.64	5.71	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.026	16.033	0.490	0.463	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	436	360	0	361	0	0	-1
N.S.	1	1.00	4.79	3.96	0.00	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.071	8.875	17.962	0.000	0.495	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	208	170	316	0	47	0	0	-1
N.S.	1	1.24	1.01	1.88	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.628	8.470	0.000	0.109	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	153	83	111	0	47	0	0	-1
N.S.	1	1.34	0.73	0.97	0.00	0.41	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.150	6.186	0.000	0.107	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	60	0	305	0	0	-1
N.S.	1	1.00	1.18	1.18	0.00	5.98	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.061	0.242	0.000	0.119	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	180	141	278	0	632	0	0	-1
N.S.	1	1.29	1.01	1.99	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.440	13.322	0.000	0.146	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	252	205	405	0	808	0	0	-1
N.S.	1	1.19	0.97	1.91	0.00	3.81	0.00	0.00	-0.00
time (sec)	N/A	0.160	1.526	15.487	0.000	0.166	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	85	79	559	0	71	-1
N.S.	1	1.00	1.17	1.13	1.05	7.45	0.00	0.95	-0.01
time (sec)	N/A	0.062	0.125	7.848	0.278	0.534	0.000	0.692	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	29	49	0	29	117
N.S.	1	1.00	1.00	0.97	1.00	1.69	0.00	1.00	4.03
time (sec)	N/A	0.027	0.022	0.139	0.270	0.427	0.000	0.663	15.613

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	480	397	161	453	0	0	-1
N.S.	1	1.00	6.15	5.09	2.06	5.81	0.00	0.00	-0.01
time (sec)	N/A	0.060	7.068	28.670	0.530	0.494	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	224	3217	0	625	0	0	-1
N.S.	1	1.00	1.67	24.01	0.00	4.66	0.00	0.00	-0.01
time (sec)	N/A	0.114	4.126	68.895	0.000	0.729	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	184	415	0	69	0	0	-1
N.S.	1	1.00	0.67	1.51	0.00	0.25	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.829	7.965	0.000	0.178	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	139	269	0	69	0	0	-1
N.S.	1	1.00	0.69	1.33	0.00	0.34	0.00	0.00	-0.00
time (sec)	N/A	0.126	0.411	8.312	0.000	0.108	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	133	145	0	777	0	0	-1
N.S.	1	1.00	0.71	0.77	0.00	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.218	5.823	0.000	0.155	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	938	0	0	-1
N.S.	1	1.00	0.89	1.02	0.00	9.29	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.107	8.119	0.000	0.170	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	167	468	0	1079	0	0	-1
N.S.	1	1.00	0.70	1.95	0.00	4.50	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.873	14.682	0.000	0.177	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	128	383	222	799	0	0	-1
N.S.	1	1.00	0.98	2.95	1.71	6.15	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.585	23.858	0.293	1.176	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	120	115	107	0	58	183
N.S.	1	1.00	0.70	1.64	1.58	1.47	0.00	0.79	2.51
time (sec)	N/A	0.057	0.076	24.442	0.278	0.578	0.000	0.680	23.810

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	47	56	59	104	0	48	164
N.S.	1	1.00	0.72	0.86	0.91	1.60	0.00	0.74	2.52
time (sec)	N/A	0.033	0.036	0.158	0.276	0.533	0.000	0.748	21.839

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	1291	899	288	775	0	0	-1
N.S.	1	1.00	10.25	7.13	2.29	6.15	0.00	0.00	-0.01
time (sec)	N/A	0.100	8.443	36.230	0.518	0.716	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	283	194	712	0	109	0	0	-1
N.S.	1	1.16	0.80	2.93	0.00	0.45	0.00	0.00	-0.00
time (sec)	N/A	0.198	1.495	10.571	0.000	0.167	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	263	171	485	0	1343	0	0	-1
N.S.	1	1.18	0.77	2.17	0.00	6.02	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.988	10.000	0.000	0.232	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	257	175	549	0	1394	0	0	-1
N.S.	1	1.18	0.81	2.53	0.00	6.42	0.00	0.00	-0.00
time (sec)	N/A	0.154	1.017	10.240	0.000	0.220	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	172	547	0	1531	0	0	-1
N.S.	1	1.00	0.77	2.45	0.00	6.87	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.870	10.771	0.000	0.230	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	328	245	1082	0	1746	0	0	-1
N.S.	1	1.14	0.85	3.76	0.00	6.06	0.00	0.00	-0.00
time (sec)	N/A	0.230	2.327	17.941	0.000	0.270	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	228	0	0	29	0	0	-1
N.S.	1	1.00	1.98	0.00	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.637	0.428	0.000	0.463	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	191	0	0	27	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.13	0.00	0.00	-0.00
time (sec)	N/A	0.141	0.321	1.114	0.000	0.441	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	119	120	0	0	27	0	0	-1
N.S.	1	0.96	0.97	0.00	0.00	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.146	0.582	0.000	0.442	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	25	0	0	64
N.S.	1	1.00	1.00	0.00	0.00	0.37	0.00	0.00	0.96
time (sec)	N/A	0.029	0.019	0.154	0.000	0.413	0.000	0.000	15.217

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	25	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.048	4.805	0.364	0.000	0.384	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.053	9.840	0.285	0.000	0.411	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	199	0	0	27	0	0	-1
N.S.	1	1.00	2.21	0.00	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.420	1.606	0.000	0.402	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	195	0	0	27	0	0	-1
N.S.	1	1.00	2.17	0.00	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.392	0.574	0.000	0.434	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	145	0	0	18	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	0.20	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.374	0.300	0.000	0.399	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.058	4.988	0.331	0.000	0.421	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	27	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.056	10.840	0.361	0.000	0.419	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	203	264	210	3216	0	221	229
N.S.	1	1.00	0.93	1.21	0.96	14.68	0.00	1.01	1.05
time (sec)	N/A	0.185	0.179	1.105	0.499	1.203	0.000	0.451	15.019

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	139	133	159	2298	0	156	153
N.S.	1	1.00	0.83	0.80	0.95	13.76	0.00	0.93	0.92
time (sec)	N/A	0.098	0.112	0.701	0.508	40.619	0.000	0.498	15.025

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	116	115	121	401	184	137	123
N.S.	1	1.00	0.81	0.80	0.84	2.78	1.28	0.95	0.85
time (sec)	N/A	0.065	0.038	0.467	0.535	0.447	2.707	0.466	0.282

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	268	300	288	4396	0	309	600
N.S.	1	1.00	0.92	1.03	0.99	15.16	0.00	1.07	2.07
time (sec)	N/A	0.224	0.149	1.010	0.530	1.297	0.000	0.446	0.270

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	333	372	470	10135	0	510	898
N.S.	1	1.00	0.86	0.97	1.22	26.32	0.00	1.32	2.33
time (sec)	N/A	0.328	1.577	1.379	0.532	2.168	0.000	0.495	15.265

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	764	764	300	121	0	23437	0	0	2338
N.S.	1	1.00	0.39	0.16	0.00	30.68	0.00	0.00	3.06
time (sec)	N/A	1.053	0.115	1.676	0.000	4.643	0.000	0.000	18.041

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	231	83	0	0	0	0	951
N.S.	1	1.00	0.48	0.17	0.00	0.00	0.00	0.00	1.96
time (sec)	N/A	0.435	0.071	0.931	0.000	0.000	0.000	0.000	17.401

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	126	83	0	25429	0	0	609
N.S.	1	1.00	0.51	0.34	0.00	103.79	0.00	0.00	2.49
time (sec)	N/A	0.179	0.122	0.469	0.000	1.489	0.000	0.000	16.714

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	C	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	0	432	162	0	59362	0	0	2500
N.S.	1	0.00	1.44	0.54	0.00	198.54	0.00	0.00	8.36
time (sec)	N/A	0.028	0.197	2.125	0.000	3.801	0.000	0.000	18.524

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F(-2)	C	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1093	0	679	291	0	85064	0	0	2500
N.S.	1	0.00	0.62	0.27	0.00	77.83	0.00	0.00	2.29
time (sec)	N/A	0.027	1.223	4.962	0.000	11.213	0.000	0.000	25.850

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	402	310	263	6415	0	277	384
N.S.	1	1.00	1.40	1.08	0.91	22.27	0.00	0.96	1.33
time (sec)	N/A	0.220	2.442	1.295	0.565	95.600	0.000	0.482	0.424

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	258	284	213	3878	0	228	203
N.S.	1	1.00	1.08	1.19	0.89	16.29	0.00	0.96	0.85
time (sec)	N/A	0.155	0.754	0.882	0.515	63.735	0.000	0.481	14.982

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	184	165	163	665	0	169	172
N.S.	1	1.00	1.01	0.90	0.89	3.63	0.00	0.92	0.94
time (sec)	N/A	0.105	0.517	0.959	0.525	0.494	0.000	0.483	0.380

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	152	146	155	655	617	162	165
N.S.	1	1.00	0.86	0.83	0.88	3.72	3.51	0.92	0.94
time (sec)	N/A	0.075	0.335	0.537	0.542	0.492	122.542	0.462	15.004

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	503	389	483	10855	0	566	980
N.S.	1	1.00	0.86	0.66	0.82	18.49	0.00	0.96	1.67
time (sec)	N/A	0.465	3.083	1.483	0.537	2.697	0.000	0.500	15.173

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	747	747	657	483	788	15989	0	790	1605
N.S.	1	1.00	0.88	0.65	1.05	21.40	0.00	1.06	2.15
time (sec)	N/A	0.677	6.301	2.842	0.514	6.244	0.000	0.586	15.752

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	394	241	0	0	0	0	2431
N.S.	1	0.00	15.15	9.27	0.00	0.00	0.00	0.00	93.50
time (sec)	N/A	0.027	0.280	1.635	0.000	0.000	0.000	0.000	15.995

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	273	175	0	36403	0	0	1648
N.S.	1	0.00	10.50	6.73	0.00	1400.12	0.00	0.00	63.38
time (sec)	N/A	0.027	0.185	2.011	0.000	3.279	0.000	0.000	15.746

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	502	349	0	70185	0	0	1567
N.S.	1	0.00	29.53	20.53	0.00	4128.53	0.00	0.00	92.18
time (sec)	N/A	0.008	0.372	3.128	0.000	8.354	0.000	0.000	17.893

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	845	398	0	102913	0	0	2500
N.S.	1	0.00	32.50	15.31	0.00	3958.19	0.00	0.00	96.15
time (sec)	N/A	0.029	1.164	6.409	0.000	38.673	0.000	0.000	21.207

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	1158	525	0	133123	0	0	2500
N.S.	1	0.00	44.54	20.19	0.00	5120.12	0.00	0.00	96.15
time (sec)	N/A	0.027	1.263	8.928	0.000	104.276	0.000	0.000	25.437

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	207	176	177	1429	0	360	1931
N.S.	1	1.00	1.58	1.34	1.35	10.91	0.00	2.75	14.74
time (sec)	N/A	0.126	0.183	1.159	0.534	0.605	0.000	0.808	0.774

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	189	152	158	1041	0	311	1097
N.S.	1	1.00	1.67	1.35	1.40	9.21	0.00	2.75	9.71
time (sec)	N/A	0.109	0.145	0.834	0.504	0.563	0.000	0.776	15.784

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	160	136	121	631	0	280	489
N.S.	1	1.00	1.68	1.43	1.27	6.64	0.00	2.95	5.15
time (sec)	N/A	0.068	0.062	0.710	0.531	0.463	0.000	0.707	15.811

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	68	100	330	129	224	40
N.S.	1	1.00	0.76	0.96	1.41	4.65	1.82	3.15	0.56
time (sec)	N/A	0.039	0.018	0.304	0.502	58.432	2.595	0.696	0.105

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	184	185	167	1329	0	370	2500
N.S.	1	1.00	1.57	1.58	1.43	11.36	0.00	3.16	21.37
time (sec)	N/A	0.109	0.135	0.677	0.543	0.542	0.000	0.739	17.906

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	255	241	244	2529	0	475	2500
N.S.	1	1.00	1.46	1.38	1.39	14.45	0.00	2.71	14.29
time (sec)	N/A	0.139	0.726	1.262	0.520	0.891	0.000	0.780	19.192

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	317	322	363	3703	0	630	2500
N.S.	1	1.00	1.27	1.29	1.46	14.87	0.00	2.53	10.04
time (sec)	N/A	0.206	3.747	1.915	0.505	1.585	0.000	0.761	20.572

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	233	258	0	2948	0	896	2500
N.S.	1	1.00	0.92	1.02	0.00	11.70	0.00	3.56	9.92
time (sec)	N/A	0.315	0.632	1.436	0.000	1.536	0.000	0.871	18.799

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	200	224	0	2433	0	836	2500
N.S.	1	1.00	1.08	1.20	0.00	13.08	0.00	4.49	13.44
time (sec)	N/A	0.215	0.476	1.075	0.000	1.030	0.000	0.858	17.463

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	194	179	0	1751	0	995	2500
N.S.	1	1.00	1.25	1.15	0.00	11.30	0.00	6.42	16.13
time (sec)	N/A	0.188	0.333	0.779	0.000	0.766	0.000	0.802	18.042

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	171	148	0	1197	0	906	2500
N.S.	1	1.00	1.35	1.17	0.00	9.43	0.00	7.13	19.69
time (sec)	N/A	0.149	0.181	0.862	0.000	0.613	0.000	0.786	16.400

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	158	115	0	541	0	558	1409
N.S.	1	1.00	1.26	0.92	0.00	4.33	0.00	4.46	11.27
time (sec)	N/A	0.074	0.199	0.557	0.000	0.495	0.000	0.829	15.661

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	175	166	0	2589	0	1211	2832
N.S.	1	1.00	1.23	1.17	0.00	18.23	0.00	8.53	19.94
time (sec)	N/A	0.146	0.337	0.769	0.000	0.735	0.000	0.814	16.940

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	205	230	0	4113	0	2183	2500
N.S.	1	1.00	1.27	1.43	0.00	25.55	0.00	13.56	15.53
time (sec)	N/A	0.224	0.706	1.381	0.000	0.919	0.000	0.904	17.739

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	253	311	0	5587	0	3106	2500
N.S.	1	1.00	1.24	1.52	0.00	27.39	0.00	15.23	12.25
time (sec)	N/A	0.255	0.906	1.457	0.000	1.446	0.000	0.936	18.226

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.028	8.275	1.276	0.000	0.720	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	191	141	0	0	37	0	0	-1
N.S.	1	0.97	0.72	0.00	0.00	0.19	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.110	0.885	0.000	0.416	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	0	0	37	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.040	1.819	0.000	0.421	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	35	0	0	64
N.S.	1	1.00	1.00	0.00	0.00	0.52	0.00	0.00	0.96
time (sec)	N/A	0.029	0.013	0.423	0.000	0.458	0.000	0.000	15.619

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	35	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.105	8.358	0.557	0.000	0.414	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	0	0	0	37	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.15	0.00	0.00	-0.00
time (sec)	N/A	0.152	13.737	0.512	0.000	0.512	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.026	5.020	2.292	0.000	0.441	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.026	16.646	2.871	0.000	0.458	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	28	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.65	0.00	0.00	-0.06
time (sec)	N/A	0.008	2.269	0.595	0.000	0.405	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.026	9.385	0.532	0.000	0.436	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.026	12.991	0.541	0.000	0.484	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.031	8.275	0.325	0.000	0.424	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	155	0	0	25	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.11	0.00	0.00	-0.00
time (sec)	N/A	0.119	0.152	0.329	0.000	0.448	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	114	0	0	25	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.069	0.412	0.000	0.393	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	23	0	0	70
N.S.	1	1.00	1.00	0.00	0.00	0.33	0.00	0.00	1.01
time (sec)	N/A	0.033	0.018	0.086	0.000	0.417	0.000	0.000	15.906

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	23	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.024	6.819	0.236	0.000	0.404	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.032	13.774	0.265	0.000	0.428	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.031	35.366	0.450	0.000	0.413	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.032	21.537	0.315	0.000	0.406	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	16	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.94	0.00	0.00	-0.06
time (sec)	N/A	0.009	2.637	0.074	0.000	0.371	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.032	8.755	0.224	0.000	0.419	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.030	17.584	0.193	0.000	0.392	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	113	114	273	179	0	603	115
N.S.	1	1.00	0.88	0.89	2.13	1.40	0.00	4.71	0.90
time (sec)	N/A	0.090	0.217	0.492	0.339	0.619	0.000	3.291	14.340

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	78	83	159	118	0	393	90
N.S.	1	1.00	0.83	0.88	1.69	1.26	0.00	4.18	0.96
time (sec)	N/A	0.071	0.186	0.468	0.340	0.485	0.000	1.626	14.353

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	52	58	82	78	0	234	52
N.S.	1	1.00	0.81	0.91	1.28	1.22	0.00	3.66	0.81
time (sec)	N/A	0.051	0.073	0.357	0.343	0.432	0.000	0.759	14.282

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	42	43	37	0	110	28
N.S.	1	1.00	0.86	0.98	1.00	0.86	0.00	2.56	0.65
time (sec)	N/A	0.026	0.034	0.376	0.319	0.409	0.000	0.483	14.506

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	37	35	0	38	41
N.S.	1	1.00	1.00	0.92	0.97	0.92	0.00	1.00	1.08
time (sec)	N/A	0.030	0.018	0.265	0.284	0.416	0.000	0.474	14.430

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	50	101	56	91	0	108	69
N.S.	1	1.00	0.79	1.60	0.89	1.44	0.00	1.71	1.10
time (sec)	N/A	0.051	0.106	0.533	0.305	0.440	0.000	0.554	14.457

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	72	161	92	198	0	205	103
N.S.	1	1.00	0.81	1.81	1.03	2.22	0.00	2.30	1.16
time (sec)	N/A	0.062	0.360	0.542	0.321	0.449	0.000	0.524	14.504

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	100	253	137	371	0	353	138
N.S.	1	1.00	0.83	2.09	1.13	3.07	0.00	2.92	1.14
time (sec)	N/A	0.078	0.183	0.566	0.306	0.502	0.000	0.543	15.351

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	147	120	180	602	0	472	141
N.S.	1	1.00	1.22	1.00	1.50	5.02	0.00	3.93	1.18
time (sec)	N/A	0.090	1.588	0.663	0.538	0.445	0.000	4.925	14.780

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	111	121	130	472	0	296	112
N.S.	1	1.00	1.14	1.25	1.34	4.87	0.00	3.05	1.15
time (sec)	N/A	0.072	0.541	0.550	0.524	0.430	0.000	2.191	15.110

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	75	78	85	366	0	164	83
N.S.	1	1.00	1.01	1.05	1.15	4.95	0.00	2.22	1.12
time (sec)	N/A	0.066	0.232	0.522	0.536	0.449	0.000	1.008	15.184

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	51	51	300	0	86	53
N.S.	1	1.00	1.00	0.96	0.96	5.66	0.00	1.62	1.00
time (sec)	N/A	0.050	0.108	0.496	0.527	0.415	0.000	0.606	14.922

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	55	50	290	0	85	44
N.S.	1	1.00	1.00	1.06	0.96	5.58	0.00	1.63	0.85
time (sec)	N/A	0.043	0.134	0.522	0.508	0.413	0.000	0.501	14.715

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	72	79	76	402	0	120	64
N.S.	1	1.00	1.01	1.11	1.07	5.66	0.00	1.69	0.90
time (sec)	N/A	0.054	0.219	0.570	0.561	0.434	0.000	0.510	15.090

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	101	115	111	576	0	171	82
N.S.	1	1.00	1.05	1.20	1.16	6.00	0.00	1.78	0.85
time (sec)	N/A	0.067	0.616	0.574	0.511	0.434	0.000	0.527	16.150

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	135	155	154	834	0	238	100
N.S.	1	1.00	1.15	1.32	1.32	7.13	0.00	2.03	0.85
time (sec)	N/A	0.081	0.758	0.654	0.518	0.499	0.000	0.516	18.908

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	48	72	47	0	136	326
N.S.	1	1.00	0.80	0.75	1.12	0.73	0.00	2.12	5.09
time (sec)	N/A	0.071	0.070	14.026	0.308	0.394	0.000	1.554	19.711

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	48	34	0	39	69
N.S.	1	1.00	0.76	0.92	1.26	0.89	0.00	1.03	1.82
time (sec)	N/A	0.068	0.062	13.214	0.300	0.376	0.000	0.714	0.716

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	21	17	0	39	20
N.S.	1	1.00	1.00	1.11	1.11	0.89	0.00	2.05	1.05
time (sec)	N/A	0.042	0.029	0.140	0.537	0.374	0.000	0.492	15.252

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	52	74	57	0	55	-1
N.S.	1	1.00	1.10	1.04	1.48	1.14	0.00	1.10	-0.02
time (sec)	N/A	0.055	0.051	11.217	0.516	0.402	0.000	0.451	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	78	105	88	0	171	-1
N.S.	1	1.00	1.01	0.90	1.21	1.01	0.00	1.97	-0.01
time (sec)	N/A	0.082	0.301	11.072	0.547	0.392	0.000	0.566	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	75	120	2144	87	0	251	-1
N.S.	1	1.00	0.62	1.00	17.87	0.72	0.00	2.09	-0.01
time (sec)	N/A	0.094	0.232	7.111	2.121	0.413	0.000	1.972	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	84	910	77	0	211	-1
N.S.	1	1.00	0.60	0.92	10.00	0.85	0.00	2.32	-0.01
time (sec)	N/A	0.085	0.140	6.997	0.669	0.403	0.000	0.958	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	40	54	80	55	0	136	-1
N.S.	1	1.00	0.70	0.95	1.40	0.96	0.00	2.39	-0.02
time (sec)	N/A	0.069	0.034	7.428	0.560	0.413	0.000	0.597	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	43	45	42	0	90	88
N.S.	1	1.00	0.61	0.75	0.79	0.74	0.00	1.58	1.54
time (sec)	N/A	0.074	0.054	4.580	0.522	0.398	0.000	0.506	18.529

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	47	55	61	66	0	132	364
N.S.	1	1.00	0.52	0.60	0.67	0.73	0.00	1.45	4.00
time (sec)	N/A	0.080	0.057	3.918	0.500	0.405	0.000	0.532	18.418

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	67	65	73	86	0	174	555
N.S.	1	1.00	0.54	0.52	0.59	0.69	0.00	1.40	4.48
time (sec)	N/A	0.082	0.139	4.881	0.526	0.394	0.000	0.550	26.643

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	51	72	50	0	71	486
N.S.	1	1.00	0.66	0.78	1.11	0.77	0.00	1.09	7.48
time (sec)	N/A	0.073	0.062	14.434	0.304	0.389	0.000	1.606	23.610

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	41	48	40	0	57	100
N.S.	1	1.00	0.74	0.98	1.14	0.95	0.00	1.36	2.38
time (sec)	N/A	0.071	0.052	11.014	0.300	0.410	0.000	0.850	19.497

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	69	27	0	41	61
N.S.	1	1.00	1.00	1.11	3.83	1.50	0.00	2.28	3.39
time (sec)	N/A	0.043	0.021	0.128	0.519	0.404	0.000	0.579	0.390

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	40	54	84	0	32	-1
N.S.	1	1.00	1.58	1.29	1.74	2.71	0.00	1.03	-0.03
time (sec)	N/A	0.054	0.033	5.461	0.295	0.397	0.000	0.439	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	80	67	86	79	0	117	-1
N.S.	1	1.00	1.21	1.02	1.30	1.20	0.00	1.77	-0.02
time (sec)	N/A	0.079	0.127	9.309	0.576	0.402	0.000	0.630	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	103	1652	80	0	201	-1
N.S.	1	1.00	0.73	1.13	18.15	0.88	0.00	2.21	-0.01
time (sec)	N/A	0.094	0.095	11.082	0.916	0.432	0.000	1.136	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	43	65	575	67	0	169	-1
N.S.	1	1.00	0.69	1.05	9.27	1.08	0.00	2.73	-0.02
time (sec)	N/A	0.079	0.034	9.575	0.621	0.403	0.000	0.729	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	98	36	0	67	37
N.S.	1	1.00	1.00	1.28	3.92	1.44	0.00	2.68	1.48
time (sec)	N/A	0.069	0.021	0.305	0.570	0.386	0.000	0.569	15.063

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	37	44	571	58	0	99	118
N.S.	1	1.00	0.62	0.73	9.52	0.97	0.00	1.65	1.97
time (sec)	N/A	0.080	0.043	3.819	0.629	0.381	0.000	0.666	19.187

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	49	54	1344	79	0	128	491
N.S.	1	1.00	0.51	0.56	14.00	0.82	0.00	1.33	5.11
time (sec)	N/A	0.078	0.050	5.454	0.671	0.436	0.000	0.763	22.470

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	51	72	50	0	64	583
N.S.	1	1.00	0.75	0.75	1.06	0.74	0.00	0.94	8.57
time (sec)	N/A	0.082	0.070	13.605	0.325	0.391	0.000	0.818	33.909

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	34	41	50	40	0	44	389
N.S.	1	1.00	0.77	0.93	1.14	0.91	0.00	1.00	8.84
time (sec)	N/A	0.082	0.080	13.657	0.327	0.382	0.000	0.699	20.209

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	101	28	0	23	72
N.S.	1	1.00	1.00	1.00	4.81	1.33	0.00	1.10	3.43
time (sec)	N/A	0.049	0.023	0.106	0.558	0.425	0.000	0.661	18.310

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	75	77	58	0	59	-1
N.S.	1	1.00	1.04	1.42	1.45	1.09	0.00	1.11	-0.02
time (sec)	N/A	0.063	0.051	16.431	0.306	0.412	0.000	0.426	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	82	67	106	83	0	129	-1
N.S.	1	1.00	1.24	1.02	1.61	1.26	0.00	1.95	-0.02
time (sec)	N/A	0.086	0.123	8.811	0.285	0.432	0.000	0.731	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	59	104	1666	77	0	84	-1
N.S.	1	1.00	0.56	0.98	15.72	0.73	0.00	0.79	-0.01
time (sec)	N/A	0.107	0.067	8.653	0.867	0.399	0.000	0.733	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	65	240	66	0	70	-1
N.S.	1	1.00	0.70	1.03	3.81	1.05	0.00	1.11	-0.02
time (sec)	N/A	0.092	0.056	9.733	0.542	0.420	0.000	0.695	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	411	50	0	113	88
N.S.	1	1.00	0.76	0.92	10.82	1.32	0.00	2.97	2.32
time (sec)	N/A	0.087	0.026	1.954	0.539	0.387	0.000	0.778	18.687

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	41	67	1146	75	0	151	393
N.S.	1	1.00	0.53	0.87	14.88	0.97	0.00	1.96	5.10
time (sec)	N/A	0.098	0.067	6.493	0.689	0.419	0.000	0.933	20.606

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	51	57	2187	100	0	184	589
N.S.	1	1.00	0.44	0.50	19.02	0.87	0.00	1.60	5.12
time (sec)	N/A	0.102	0.090	4.515	0.591	0.420	0.000	0.960	33.014

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	143	721	237	354	0	2790	-1
N.S.	1	1.00	0.81	4.07	1.34	2.00	0.00	15.76	-0.01
time (sec)	N/A	0.144	0.387	35.185	0.513	0.930	0.000	2.055	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	84	403	132	234	0	1011	-1
N.S.	1	1.00	0.71	3.42	1.12	1.98	0.00	8.57	-0.01
time (sec)	N/A	0.072	0.280	31.089	0.546	0.628	0.000	0.833	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	129	129	145	0	329	-1
N.S.	1	1.00	1.03	2.22	2.22	2.50	0.00	5.67	-0.02
time (sec)	N/A	0.038	0.040	28.333	0.524	0.502	0.000	0.582	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	58	45	135	0	49	-1
N.S.	1	1.00	0.98	1.07	0.83	2.50	0.00	0.91	-0.02
time (sec)	N/A	0.043	0.033	8.725	0.303	0.776	0.000	0.513	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	77	122	119	239	0	0	-1
N.S.	1	1.00	0.70	1.11	1.08	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.138	9.696	0.326	0.963	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	103	212	227	415	0	0	-1
N.S.	1	1.00	0.62	1.28	1.38	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.391	10.737	0.284	1.244	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	198	380	0	27	0	0	-1
N.S.	1	1.00	0.85	1.62	0.00	0.12	0.00	0.00	-0.00
time (sec)	N/A	0.192	1.396	17.988	0.000	0.138	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	140	294	0	27	0	0	-1
N.S.	1	1.00	0.82	1.72	0.00	0.16	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.336	16.095	0.000	0.098	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	18	0	0	-1
N.S.	1	1.00	1.20	1.39	0.00	0.35	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.065	5.293	0.000	0.098	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	143	156	0	27	0	0	-1
N.S.	1	1.00	0.82	0.90	0.00	0.16	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.390	9.195	0.000	0.129	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	197	351	0	27	0	0	-1
N.S.	1	1.00	0.85	1.51	0.00	0.12	0.00	0.00	-0.00
time (sec)	N/A	0.176	2.198	10.210	0.000	0.152	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	160	711	246	385	0	0	-1
N.S.	1	1.00	0.73	3.23	1.12	1.75	0.00	0.00	-0.00
time (sec)	N/A	0.167	1.338	25.310	0.506	0.785	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	116	567	175	265	0	2185	-1
N.S.	1	1.00	0.78	3.83	1.18	1.79	0.00	14.76	-0.01
time (sec)	N/A	0.089	0.342	24.247	0.579	0.624	0.000	1.450	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	393	166	186	0	1338	-1
N.S.	1	1.00	0.94	4.68	1.98	2.21	0.00	15.93	-0.01
time (sec)	N/A	0.051	0.100	20.985	0.536	0.626	0.000	0.796	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	86	64	175	0	71	-1
N.S.	1	1.00	0.88	1.10	0.82	2.24	0.00	0.91	-0.01
time (sec)	N/A	0.053	0.093	9.758	0.315	0.865	0.000	0.429	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	90	165	156	282	0	0	-1
N.S.	1	1.00	0.64	1.18	1.11	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.312	8.410	0.303	1.061	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	123	257	287	442	0	0	-1
N.S.	1	1.00	0.59	1.24	1.38	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.129	0.559	12.801	0.286	1.355	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	211	419	0	45	0	0	-1
N.S.	1	1.00	0.77	1.52	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.233	1.913	16.838	0.000	0.136	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	174	515	0	45	0	0	-1
N.S.	1	1.00	0.78	2.32	0.00	0.20	0.00	0.00	-0.00
time (sec)	N/A	0.147	1.969	18.767	0.000	0.128	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	156	266	0	18	0	0	-1
N.S.	1	1.00	1.01	1.73	0.00	0.12	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.533	11.441	0.000	0.101	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	173	204	0	45	0	0	-1
N.S.	1	1.00	0.78	0.91	0.00	0.20	0.00	0.00	-0.00
time (sec)	N/A	0.165	1.499	9.170	0.000	0.142	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	218	419	0	45	0	0	-1
N.S.	1	1.00	0.79	1.52	0.00	0.16	0.00	0.00	-0.00
time (sec)	N/A	0.227	3.284	10.507	0.000	0.192	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	108	644	254	328	0	2580	-1
N.S.	1	1.00	0.81	4.81	1.90	2.45	0.00	19.25	-0.01
time (sec)	N/A	0.115	0.330	27.454	0.568	0.515	0.000	1.906	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	353	128	220	0	793	-1
N.S.	1	1.00	0.95	4.36	1.58	2.72	0.00	9.79	-0.01
time (sec)	N/A	0.065	0.157	22.770	0.546	0.478	0.000	0.909	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	103	112	112	0	98	-1
N.S.	1	1.00	1.06	2.86	3.11	3.11	0.00	2.72	-0.03
time (sec)	N/A	0.034	0.031	19.824	0.545	0.455	0.000	0.606	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	26	100	0	31	-1
N.S.	1	1.00	1.00	1.27	0.79	3.03	0.00	0.94	-0.03
time (sec)	N/A	0.040	0.028	5.682	0.293	0.441	0.000	0.449	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	109	81	220	0	0	-1
N.S.	1	1.00	0.95	1.45	1.08	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.122	9.036	0.276	0.471	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	101	205	167	388	0	898	-1
N.S.	1	1.00	0.80	1.63	1.33	3.08	0.00	7.13	-0.01
time (sec)	N/A	0.088	0.255	11.559	0.323	0.528	0.000	0.833	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	188	377	0	845	0	0	-1
N.S.	1	1.00	0.76	1.53	0.00	3.43	0.00	0.00	-0.00
time (sec)	N/A	0.161	1.495	18.121	0.000	0.199	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	100	222	0	730	0	0	-1
N.S.	1	1.00	0.92	2.04	0.00	6.70	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.269	17.383	0.000	0.161	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	60	0	305	0	0	-1
N.S.	1	1.00	1.18	1.18	0.00	5.98	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.057	0.243	0.000	0.109	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	101	120	0	723	0	0	-1
N.S.	1	1.00	0.95	1.13	0.00	6.82	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.259	7.063	0.000	0.155	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	186	351	0	1045	0	0	-1
N.S.	1	1.00	0.78	1.46	0.00	4.35	0.00	0.00	-0.00
time (sec)	N/A	0.176	2.702	10.261	0.000	0.171	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	107	3763	341	593	0	2776	-1
N.S.	1	1.00	0.60	21.26	1.93	3.35	0.00	15.68	-0.01
time (sec)	N/A	0.151	0.335	90.773	0.522	0.561	0.000	2.134	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	75	2199	198	445	0	1011	-1
N.S.	1	1.00	0.64	18.64	1.68	3.77	0.00	8.57	-0.01
time (sec)	N/A	0.081	0.084	71.232	0.517	0.510	0.000	1.086	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	1317	151	281	0	250	-1
N.S.	1	1.00	0.86	20.90	2.40	4.46	0.00	3.97	-0.02
time (sec)	N/A	0.045	0.047	59.158	0.518	0.454	0.000	0.739	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	62	48	225	0	57	-1
N.S.	1	1.00	0.81	1.09	0.84	3.95	0.00	1.00	-0.02
time (sec)	N/A	0.050	0.037	8.878	0.272	0.465	0.000	0.511	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	70	148	123	406	0	525	-1
N.S.	1	1.00	0.64	1.35	1.12	3.69	0.00	4.77	-0.01
time (sec)	N/A	0.078	0.070	10.357	0.324	0.466	0.000	0.808	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	94	265	231	652	0	1150	-1
N.S.	1	1.00	0.56	1.59	1.38	3.90	0.00	6.89	-0.01
time (sec)	N/A	0.112	0.224	10.391	0.293	0.531	0.000	1.054	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	197	368	0	1228	0	0	-1
N.S.	1	1.00	0.67	1.26	0.00	4.21	0.00	0.00	-0.00
time (sec)	N/A	0.211	1.599	18.771	0.000	0.226	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	145	278	0	1048	0	0	-1
N.S.	1	1.00	0.65	1.24	0.00	4.68	0.00	0.00	-0.00
time (sec)	N/A	0.148	0.623	18.806	0.000	0.198	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	938	0	0	-1
N.S.	1	1.00	0.89	1.02	0.00	9.29	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.104	10.734	0.000	0.167	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	142	141	0	996	0	0	-1
N.S.	1	1.00	0.68	0.67	0.00	4.77	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.530	9.345	0.000	0.162	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	199	353	0	1465	0	0	-1
N.S.	1	1.00	0.67	1.19	0.00	4.93	0.00	0.00	-0.00
time (sec)	N/A	0.240	2.616	9.993	0.000	0.200	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	107	1909	432	995	0	3826	-1
N.S.	1	1.00	0.49	8.76	1.98	4.56	0.00	17.55	-0.00
time (sec)	N/A	0.188	0.344	63.339	0.563	0.636	0.000	3.086	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	76	1065	268	769	0	1792	-1
N.S.	1	1.00	0.50	6.96	1.75	5.03	0.00	11.71	-0.01
time (sec)	N/A	0.097	0.087	47.820	0.591	0.522	0.000	1.554	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	895	214	521	0	842	-1
N.S.	1	1.00	0.62	9.84	2.35	5.73	0.00	9.25	-0.01
time (sec)	N/A	0.054	0.055	38.530	0.565	0.536	0.000	0.996	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	49	260	69	382	0	74	-1
N.S.	1	1.00	0.59	3.13	0.83	4.60	0.00	0.89	-0.01
time (sec)	N/A	0.057	0.040	20.122	0.341	0.501	0.000	0.455	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	69	1038	164	666	0	751	-1
N.S.	1	1.00	0.48	7.26	1.15	4.66	0.00	5.25	-0.01
time (sec)	N/A	0.092	0.178	30.439	0.355	0.498	0.000	0.990	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	117	988	295	984	0	1411	-1
N.S.	1	1.00	0.56	4.75	1.42	4.73	0.00	6.78	-0.00
time (sec)	N/A	0.131	0.574	41.831	0.377	0.564	0.000	1.253	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	235	667	0	1730	0	0	-1
N.S.	1	1.00	0.68	1.92	0.00	4.97	0.00	0.00	-0.00
time (sec)	N/A	0.285	2.333	26.053	0.000	0.358	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	199	851	0	1632	0	0	-1
N.S.	1	1.00	0.68	2.91	0.00	5.59	0.00	0.00	-0.00
time (sec)	N/A	0.204	1.830	20.928	0.000	0.279	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	172	547	0	1531	0	0	-1
N.S.	1	1.00	0.77	2.45	0.00	6.87	0.00	0.00	-0.00
time (sec)	N/A	0.173	0.916	10.520	0.000	0.220	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	209	411	0	1595	0	0	-1
N.S.	1	1.00	0.73	1.43	0.00	5.56	0.00	0.00	-0.00
time (sec)	N/A	0.232	1.820	14.218	0.000	0.253	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	226	633	0	1971	0	0	-1
N.S.	1	1.00	0.65	1.82	0.00	5.66	0.00	0.00	-0.00
time (sec)	N/A	0.327	2.089	12.108	0.000	0.266	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	121	0	0	29	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.366	2.007	0.000	0.635	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	83	0	0	27	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.174	0.921	0.000	0.404	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	25	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.42	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.048	0.517	0.000	0.390	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	25	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.46	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.043	0.516	0.000	0.413	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	73	0	0	27	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.301	0.516	0.000	0.487	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	27	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.086	5.325	0.626	0.000	0.452	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	27	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.220	0.497	0.000	0.391	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	0	0	27	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.145	0.556	0.000	0.414	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	27	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.071	4.990	0.556	0.000	0.435	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	143	132	152	1764	0	144	2003
N.S.	1	1.00	0.93	0.86	0.99	11.53	0.00	0.94	13.09
time (sec)	N/A	0.125	0.228	0.766	0.526	38.729	0.000	0.499	17.542

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	52	0	0	38	-1
N.S.	1	1.00	1.00	0.76	1.16	0.00	0.00	0.84	-0.02
time (sec)	N/A	0.047	0.020	6.684	0.502	0.000	0.000	0.435	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	39	0	0	24	-1
N.S.	1	1.00	1.00	0.75	1.39	0.00	0.00	0.86	-0.04
time (sec)	N/A	0.044	0.012	2.297	0.519	0.000	0.000	0.427	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	0	68	195	0	0	-1
N.S.	1	1.00	0.93	0.00	1.15	3.31	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.038	2.602	0.519	0.551	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	0	0	361	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	4.06	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.132	1.026	0.000	0.565	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	65	72	0	240	0	0	-1
N.S.	1	1.00	1.27	1.41	0.00	4.71	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.063	0.493	0.000	0.536	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	50	140	0	0	-1
N.S.	1	1.00	1.00	0.00	1.43	4.00	0.00	0.00	-0.03
time (sec)	N/A	0.048	0.019	1.809	0.523	0.523	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	0	79	247	0	0	-1
N.S.	1	1.00	0.94	0.00	1.13	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.055	2.703	0.559	0.554	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	141	0	166	371	0	0	-1
N.S.	1	1.00	1.31	0.00	1.54	3.44	0.00	0.00	-0.01
time (sec)	N/A	0.116	1.992	1.033	0.498	0.518	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	291	0	0	37	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.09	0.00	0.00	-0.00
time (sec)	N/A	0.267	54.377	2.437	0.000	0.114	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	304	396	0	28	0	0	-1
N.S.	1	1.00	1.88	2.44	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.056	1.903	58.507	0.000	0.094	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	372	0	0	37	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.08	0.00	0.00	-0.00
time (sec)	N/A	0.249	58.311	1.862	0.000	0.120	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.027	6.753	1.738	0.000	1.794	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	875	0	0	37	0	0	-1
N.S.	1	1.00	3.14	0.00	0.00	0.13	0.00	0.00	-0.00
time (sec)	N/A	0.200	17.373	0.758	0.000	0.482	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	463	0	0	35	0	0	-1
N.S.	1	1.00	3.28	0.00	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.084	7.691	1.685	0.000	0.437	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	35	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.65	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.068	1.615	0.000	0.421	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	119	0	0	37	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.463	1.632	0.000	0.483	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.026	45.620	0.767	0.000	0.488	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.025	2.317	0.763	0.000	0.483	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	28	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.65	0.00	0.00	-0.06
time (sec)	N/A	0.008	0.163	1.674	0.000	0.423	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.025	2.015	0.799	0.000	0.441	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	37	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.42	0.00	0.00	-0.04
time (sec)	N/A	0.025	52.175	0.727	0.000	0.461	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	3544	0	0	59	0	0	-1
N.S.	1	1.00	11.58	0.00	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.288	17.742	2.289	0.000	0.427	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	2368	0	0	41	0	0	-1
N.S.	1	1.00	11.01	0.00	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.183	14.740	0.875	0.000	0.403	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	1065	0	0	23	0	0	-1
N.S.	1	1.00	8.59	0.00	0.00	0.19	0.00	0.00	-0.01
time (sec)	N/A	0.101	12.462	0.727	0.000	0.434	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.034	5.994	0.429	0.000	0.397	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	43	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	1.65	0.00	0.00	-0.04
time (sec)	N/A	0.035	54.860	1.185	0.000	0.407	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	57	97	0	46	-1
N.S.	1	1.00	0.96	0.81	1.21	2.06	0.00	0.98	-0.02
time (sec)	N/A	0.051	0.020	3.086	0.513	0.413	0.000	0.520	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	41	74	0	27	-1
N.S.	1	1.00	1.00	0.83	1.41	2.55	0.00	0.93	-0.03
time (sec)	N/A	0.048	0.012	0.518	0.520	0.421	0.000	0.516	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.032	6.876	1.300	0.000	0.407	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.032	51.652	0.419	0.000	0.426	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	23	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.713	0.358	0.000	0.408	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	0	23	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.42	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.040	0.221	0.000	0.445	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	129	0	0	25	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.18	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.734	0.386	0.000	0.414	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.031	61.883	0.372	0.000	0.458	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.031	5.630	0.320	0.000	0.421	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	16	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.94	0.00	0.00	-0.06
time (sec)	N/A	0.009	0.478	0.180	0.000	0.416	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.031	4.731	0.301	0.000	0.444	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	25	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.96	0.00	0.00	-0.04
time (sec)	N/A	0.031	80.629	0.447	0.000	0.445	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	102	324	0	125	0	0	-1
N.S.	1	1.07	0.95	3.03	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.140	183.300	0.000	0.111	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	135	0	0	39	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.642	1.006	0.000	1.215	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	325	4067171	0	44	0	0	-1
N.S.	1	1.00	4.11	51483.18	0.00	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.199	4.481	0.000	0.464	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	529	258013	0	0	0	0	-1
N.S.	1	1.00	6.70	3265.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	1.129	5.211	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [591] had the largest ratio of [37]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	10	0.300
2	A	3	3	1.00	10	0.300
3	A	2	2	1.00	10	0.200
4	A	2	2	1.00	10	0.200
5	A	3	3	1.00	10	0.300
6	A	4	3	1.00	10	0.300
7	A	6	3	1.00	10	0.300
8	A	4	3	1.00	10	0.300
9	A	3	3	1.00	10	0.300
10	A	3	3	1.00	10	0.300
11	A	4	3	1.00	10	0.300
12	A	6	3	1.00	10	0.300
13	A	7	3	1.00	10	0.300
14	A	5	3	1.00	10	0.300
15	A	3	3	1.00	10	0.300
16	A	3	3	1.00	10	0.300
17	A	3	2	1.00	10	0.200
18	A	3	2	1.00	10	0.200
19	A	2	2	1.00	14	0.143
20	A	2	2	1.00	14	0.143
21	A	2	2	1.00	14	0.143
22	A	2	2	1.00	14	0.143
23	A	2	2	1.00	14	0.143
24	A	2	2	1.00	14	0.143
25	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	12	0.167
27	A	2	2	1.00	12	0.167
28	A	2	2	1.00	12	0.167
29	A	2	2	1.00	14	0.143
30	A	2	2	1.00	14	0.143
31	A	3	2	1.00	9	0.222
32	A	4	3	1.00	11	0.273
33	A	5	3	1.00	11	0.273
34	A	6	3	1.00	11	0.273
35	A	4	3	1.00	24	0.125
36	A	4	3	1.00	24	0.125
37	A	4	3	1.00	24	0.125
38	A	3	3	1.00	22	0.136
39	A	4	4	1.00	22	0.182
40	A	5	5	1.00	24	0.208
41	A	6	5	1.00	24	0.208
42	A	6	5	1.00	24	0.208
43	A	5	5	1.00	24	0.208
44	A	4	4	1.00	24	0.167
45	A	3	3	1.00	15	0.200
46	A	4	3	1.00	24	0.125
47	A	4	3	1.00	24	0.125
48	A	4	3	1.00	24	0.125
49	A	4	3	1.00	24	0.125
50	A	4	3	1.00	24	0.125
51	A	3	2	1.00	24	0.083
52	A	3	3	1.00	22	0.136
53	A	5	4	1.00	22	0.182
54	A	6	5	1.00	24	0.208
55	A	6	5	1.00	24	0.208
56	A	4	3	1.00	24	0.125
57	A	3	3	1.00	24	0.125
58	A	3	2	1.00	15	0.133
59	A	4	3	1.00	24	0.125
60	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	11	0.182
62	A	3	2	1.00	11	0.182
63	A	3	2	1.00	11	0.182
64	A	3	2	1.00	21	0.095
65	A	2	1	1.00	19	0.053
66	A	2	2	1.00	19	0.105
67	A	2	2	1.00	21	0.095
68	A	4	3	1.00	21	0.143
69	A	3	3	1.00	21	0.143
70	A	3	2	1.00	12	0.167
71	A	2	2	1.00	21	0.095
72	A	3	3	1.00	21	0.143
73	A	3	2	1.00	21	0.095
74	A	3	2	1.00	8	0.250
75	A	1	1	1.00	10	0.100
76	A	2	2	1.00	10	0.200
77	A	3	3	1.00	10	0.300
78	A	4	3	1.00	23	0.130
79	A	4	3	1.00	23	0.130
80	A	3	3	1.00	23	0.130
81	A	2	2	1.00	21	0.095
82	A	4	4	1.00	21	0.190
83	A	5	5	1.00	23	0.217
84	A	6	6	1.00	23	0.261
85	A	7	6	1.00	23	0.261
86	A	6	6	1.00	23	0.261
87	A	5	5	1.00	23	0.217
88	A	3	3	1.00	23	0.130
89	A	2	2	1.00	14	0.143
90	A	3	3	1.00	23	0.130
91	A	4	3	1.00	23	0.130
92	A	4	3	1.00	23	0.130
93	A	4	3	1.00	23	0.130
94	A	5	4	1.00	23	0.174
95	A	5	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	23	0.130
97	A	3	3	1.00	21	0.143
98	A	5	5	1.00	21	0.238
99	A	6	6	1.00	23	0.261
100	A	6	6	1.00	23	0.261
101	A	5	5	1.00	23	0.217
102	A	4	4	1.00	23	0.174
103	A	4	4	1.00	14	0.286
104	A	4	4	1.02	23	0.174
105	A	5	4	1.00	23	0.174
106	A	6	6	1.00	23	0.261
107	A	4	3	1.00	23	0.130
108	A	5	4	1.00	23	0.174
109	A	5	5	1.00	14	0.357
110	A	5	5	1.00	23	0.217
111	A	6	5	1.00	14	0.357
112	A	7	5	1.00	14	0.357
113	A	2	2	1.00	13	0.154
114	A	3	3	1.00	13	0.231
115	A	2	2	1.00	21	0.095
116	A	5	4	1.00	13	0.308
117	A	4	4	1.00	13	0.308
118	A	3	3	1.00	13	0.231
119	A	3	3	1.00	13	0.231
120	A	4	4	1.00	13	0.308
121	A	5	4	1.00	13	0.308
122	A	5	5	1.00	25	0.200
123	A	4	4	1.00	23	0.174
124	A	6	6	1.00	23	0.261
125	A	4	4	1.00	25	0.160
126	A	5	5	1.00	25	0.200
127	A	8	8	1.00	25	0.320
128	A	6	6	1.00	25	0.240
129	A	2	2	1.00	16	0.125
130	A	8	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	8	8	1.00	25	0.320
132	A	6	5	1.00	25	0.200
133	A	5	4	1.00	23	0.174
134	A	7	7	1.00	23	0.304
135	A	7	7	1.00	25	0.280
136	A	5	4	1.00	25	0.160
137	A	6	5	1.00	25	0.200
138	A	9	8	1.00	25	0.320
139	A	7	6	1.00	25	0.240
140	A	6	6	1.00	16	0.375
141	A	7	7	1.00	25	0.280
142	A	8	8	1.00	25	0.320
143	A	7	7	1.00	16	0.438
144	A	4	4	1.00	25	0.160
145	A	3	3	1.00	23	0.130
146	A	3	3	1.00	23	0.130
147	A	4	4	1.00	25	0.160
148	A	7	7	1.00	25	0.280
149	A	5	5	1.00	25	0.200
150	A	2	2	1.00	16	0.125
151	A	8	8	1.00	25	0.320
152	A	8	8	1.00	25	0.320
153	A	4	4	1.00	25	0.160
154	A	2	2	1.00	23	0.087
155	A	4	4	1.00	23	0.174
156	A	6	6	1.00	25	0.240
157	A	8	8	1.00	25	0.320
158	A	7	7	1.00	25	0.280
159	A	6	6	1.00	25	0.240
160	A	4	4	1.00	16	0.250
161	A	8	8	1.00	25	0.320
162	A	5	5	1.00	25	0.200
163	A	3	3	1.00	25	0.120
164	A	3	3	1.00	23	0.130
165	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	8	8	1.00	25	0.320
167	A	8	8	1.00	25	0.320
168	A	7	6	1.00	25	0.240
169	A	7	7	1.00	16	0.438
170	A	9	9	1.00	25	0.360
171	A	3	3	1.00	25	0.120
172	A	5	5	1.00	23	0.217
173	A	4	4	1.00	23	0.174
174	A	3	3	1.00	21	0.143
175	A	3	3	1.00	21	0.143
176	A	3	3	1.00	23	0.130
177	A	3	3	1.00	23	0.130
178	A	3	3	1.00	23	0.130
179	A	3	3	1.00	23	0.130
180	A	3	3	1.00	23	0.130
181	A	3	3	1.00	23	0.130
182	A	17	7	1.00	23	0.304
183	A	15	7	1.00	23	0.304
184	A	13	5	1.00	23	0.217
185	A	11	4	1.00	21	0.190
186	A	14	6	1.00	21	0.286
187	A	15	7	1.00	23	0.304
188	A	18	8	1.00	23	0.348
189	A	15	6	1.00	23	0.261
190	A	14	5	1.00	23	0.217
191	A	11	5	1.00	23	0.217
192	A	11	4	1.00	14	0.286
193	A	15	6	1.00	23	0.261
194	A	16	7	1.00	23	0.304
195	A	6	5	1.00	24	0.208
196	A	6	5	1.00	24	0.208
197	A	6	5	1.00	24	0.208
198	A	4	4	1.00	24	0.167
199	A	4	4	1.00	22	0.182
200	A	7	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	7	6	1.00	24	0.250
202	A	7	6	1.00	24	0.250
203	A	12	6	1.00	24	0.250
204	A	9	6	1.00	24	0.250
205	A	7	5	1.00	24	0.208
206	A	4	3	1.00	24	0.125
207	A	4	3	1.00	15	0.200
208	A	6	4	1.00	24	0.167
209	A	6	4	1.00	24	0.167
210	A	6	4	1.00	24	0.167
211	A	6	4	1.00	24	0.167
212	A	7	6	1.00	24	0.250
213	A	5	5	1.00	24	0.208
214	A	5	5	1.00	24	0.208
215	A	5	5	1.00	24	0.208
216	A	5	5	1.00	22	0.227
217	A	11	7	1.00	22	0.318
218	A	14	9	1.00	24	0.375
219	A	6	5	1.00	24	0.208
220	A	7	6	1.00	24	0.250
221	A	5	4	1.00	24	0.167
222	A	5	4	1.00	15	0.267
223	A	7	5	1.00	24	0.208
224	A	6	6	1.00	24	0.250
225	A	6	6	1.00	24	0.250
226	A	6	6	1.00	24	0.250
227	A	6	5	1.00	24	0.208
228	A	6	6	1.00	22	0.273
229	A	16	7	1.00	22	0.318
230	A	9	7	1.00	24	0.292
231	A	6	5	1.00	24	0.208
232	A	6	5	1.00	24	0.208
233	A	6	5	1.00	24	0.208
234	A	6	5	1.00	15	0.333
235	A	8	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	3	1.80	10	0.300
237	A	10	6	1.00	10	0.600
238	A	10	6	1.00	8	0.750
239	A	5	5	1.00	23	0.217
240	A	8	7	1.00	23	0.304
241	A	5	5	1.00	25	0.200
242	A	4	4	1.00	25	0.160
243	A	2	2	1.00	23	0.087
244	A	4	4	1.00	23	0.174
245	A	7	7	1.00	25	0.280
246	A	4	4	1.00	25	0.160
247	A	2	2	1.00	16	0.125
248	A	5	5	1.00	25	0.200
249	A	17	4	1.00	10	0.400
250	A	7	3	1.00	10	0.300
251	A	9	3	1.00	10	0.300
252	A	17	4	1.00	11	0.364
253	A	7	3	1.00	11	0.273
254	A	9	3	1.00	11	0.273
255	A	15	5	1.00	8	0.625
256	A	7	3	1.00	8	0.375
257	A	9	3	0.59	8	0.375
258	A	15	5	1.00	10	0.500
259	A	8	6	1.00	10	0.600
260	A	10	6	1.00	10	0.600
261	A	3	2	1.00	16	0.125
262	A	3	2	1.00	16	0.125
263	A	3	2	1.00	16	0.125
264	A	2	2	1.00	16	0.125
265	A	2	2	1.00	14	0.143
266	A	4	3	1.00	16	0.188
267	A	4	3	1.00	16	0.188
268	A	3	3	1.00	16	0.188
269	A	2	2	1.00	16	0.125
270	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	2	1.00	16	0.125
272	A	3	2	1.00	16	0.125
273	A	3	2	1.00	16	0.125
274	A	3	2	1.00	16	0.125
275	A	2	2	1.00	16	0.125
276	A	2	2	1.00	16	0.125
277	A	3	3	1.00	14	0.214
278	A	4	3	1.00	14	0.214
279	A	4	3	1.00	16	0.188
280	A	3	3	1.00	16	0.188
281	A	2	2	1.00	16	0.125
282	A	3	3	1.00	16	0.188
283	A	3	2	1.00	16	0.125
284	A	3	2	1.00	16	0.125
285	A	6	4	1.00	21	0.190
286	A	5	4	1.00	21	0.190
287	A	4	4	1.00	21	0.190
288	A	3	2	1.00	12	0.167
289	A	3	3	1.00	21	0.143
290	A	2	1	1.00	21	0.048
291	A	3	2	1.00	21	0.095
292	A	3	2	1.00	21	0.095
293	A	6	5	1.00	23	0.217
294	A	5	5	1.00	23	0.217
295	A	1	1	1.00	14	0.071
296	A	5	4	1.00	23	0.174
297	A	4	3	1.00	23	0.130
298	A	3	2	1.00	23	0.087
299	A	3	2	1.00	23	0.087
300	A	3	2	1.00	23	0.087
301	A	4	3	1.00	15	0.200
302	A	6	6	1.00	15	0.400
303	A	4	3	1.00	15	0.200
304	A	5	5	1.00	15	0.333
305	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
306	A	4	4	1.00	15	0.267
307	A	2	2	1.00	13	0.154
308	A	4	4	1.00	13	0.308
309	A	3	3	1.00	15	0.200
310	A	5	5	1.00	15	0.333
311	A	4	3	1.00	15	0.200
312	A	6	6	1.00	15	0.400
313	A	4	3	1.00	15	0.200
314	A	6	6	1.00	15	0.400
315	A	5	4	1.00	15	0.267
316	A	5	5	1.00	15	0.333
317	A	3	3	1.00	15	0.200
318	A	3	3	1.00	15	0.200
319	A	3	3	1.00	13	0.231
320	A	5	5	1.00	13	0.385
321	A	5	4	1.00	15	0.267
322	A	6	6	1.00	15	0.400
323	A	5	4	1.00	15	0.267
324	A	5	5	1.00	25	0.200
325	A	4	4	1.00	23	0.174
326	A	6	5	1.00	23	0.217
327	A	4	4	1.00	25	0.160
328	A	5	5	1.00	25	0.200
329	A	8	8	1.18	25	0.320
330	A	7	7	1.25	25	0.280
331	A	2	2	1.00	16	0.125
332	A	8	8	1.31	25	0.320
333	A	8	8	1.20	25	0.320
334	A	6	5	1.00	25	0.200
335	A	5	4	1.00	23	0.174
336	A	7	6	1.00	23	0.261
337	A	7	6	1.00	25	0.240
338	A	5	4	1.00	25	0.160
339	A	6	5	1.00	25	0.200
340	A	9	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	8	8	1.00	25	0.320
342	A	6	6	1.00	16	0.375
343	A	7	7	1.00	25	0.280
344	A	8	8	1.00	25	0.320
345	A	4	4	1.00	25	0.160
346	A	3	3	1.00	23	0.130
347	A	3	3	1.00	23	0.130
348	A	4	4	1.00	25	0.160
349	A	7	7	1.24	25	0.280
350	A	6	6	1.34	25	0.240
351	A	2	2	1.00	16	0.125
352	A	8	8	1.29	25	0.320
353	A	8	8	1.19	25	0.320
354	A	4	4	1.00	25	0.160
355	A	2	2	1.00	23	0.087
356	A	4	4	1.00	23	0.174
357	A	6	6	1.00	25	0.240
358	A	8	8	1.00	25	0.320
359	A	7	7	1.00	25	0.280
360	A	7	7	1.00	25	0.280
361	A	4	4	1.00	16	0.250
362	A	8	8	1.00	25	0.320
363	A	5	5	1.00	25	0.200
364	A	3	3	1.00	25	0.120
365	A	3	3	1.00	23	0.130
366	A	6	6	1.00	23	0.261
367	A	8	8	1.16	25	0.320
368	A	8	8	1.18	25	0.320
369	A	8	8	1.18	25	0.320
370	A	7	7	1.00	16	0.438
371	A	9	8	1.14	25	0.320
372	A	3	3	1.00	25	0.120
373	A	5	5	1.00	23	0.217
374	A	4	4	0.96	23	0.174
375	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
376	A	3	3	1.00	21	0.143
377	A	3	3	1.00	23	0.130
378	A	3	3	1.00	23	0.130
379	A	3	3	1.00	23	0.130
380	A	3	3	1.00	14	0.214
381	A	3	3	1.00	23	0.130
382	A	3	3	1.00	23	0.130
383	A	11	10	1.00	23	0.435
384	A	9	9	1.00	23	0.391
385	A	7	7	1.00	21	0.333
386	A	11	10	1.00	21	0.476
387	A	11	10	1.00	23	0.435
388	A	38	8	1.00	23	0.348
389	A	24	7	1.00	23	0.304
390	A	11	4	1.00	14	0.286
391	F	0	0	N/A	0.	N/A
392	F	0	0	N/A	0.	N/A
393	A	10	9	1.00	23	0.391
394	A	8	8	1.00	23	0.348
395	A	9	9	1.00	23	0.391
396	A	8	8	1.00	21	0.381
397	A	18	11	1.00	21	0.524
398	A	18	11	1.00	23	0.478
399	A	0	0	0.00	0	0.000
400	A	0	0	0.00	0	0.000
401	A	0	0	0.00	0	0.000
402	A	0	0	0.00	0	0.000
403	A	0	0	0.00	0	0.000
404	A	6	5	1.00	24	0.208
405	A	6	5	1.00	24	0.208
406	A	4	4	1.00	24	0.167
407	A	4	4	1.00	22	0.182
408	A	7	6	1.00	22	0.273
409	A	7	6	1.00	24	0.250
410	A	7	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	16	6	1.00	24	0.250
412	A	12	6	1.00	24	0.250
413	A	9	6	1.00	24	0.250
414	A	7	5	1.00	24	0.208
415	A	4	3	1.00	24	0.125
416	A	6	4	1.00	24	0.167
417	A	6	4	1.00	24	0.167
418	A	6	4	1.00	24	0.167
419	A	0	0	0.00	0	0.000
420	A	8	7	0.97	23	0.304
421	A	7	6	1.00	23	0.261
422	A	3	3	1.00	21	0.143
423	A	7	6	1.00	21	0.286
424	A	9	6	1.00	23	0.261
425	A	0	0	0.00	0	0.000
426	A	0	0	0.00	0	0.000
427	A	0	0	0.00	0	0.000
428	A	0	0	0.00	0	0.000
429	A	0	0	0.00	0	0.000
430	A	0	0	0.00	0	0.000
431	A	9	6	1.00	23	0.261
432	A	7	6	1.00	23	0.261
433	A	3	3	1.00	21	0.143
434	A	0	0	0.00	0	0.000
435	A	0	0	0.00	0	0.000
436	A	0	0	0.00	0	0.000
437	A	0	0	0.00	0	0.000
438	A	0	0	0.00	0	0.000
439	A	0	0	0.00	0	0.000
440	A	0	0	0.00	0	0.000
441	A	3	2	1.00	23	0.087
442	A	3	2	1.00	23	0.087
443	A	3	2	1.00	23	0.087
444	A	4	3	1.00	21	0.143
445	A	4	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	3	2	1.00	23	0.087
447	A	3	2	1.00	23	0.087
448	A	3	2	1.00	23	0.087
449	A	4	3	1.00	23	0.130
450	A	4	3	1.00	23	0.130
451	A	4	3	1.00	23	0.130
452	A	3	3	1.00	23	0.130
453	A	3	3	1.00	23	0.130
454	A	4	3	1.00	23	0.130
455	A	5	3	1.00	23	0.130
456	A	6	3	1.00	23	0.130
457	A	5	4	1.00	26	0.154
458	A	5	4	1.00	26	0.154
459	A	4	4	1.00	24	0.167
460	A	5	5	1.00	24	0.208
461	A	7	7	1.00	26	0.269
462	A	7	6	1.00	26	0.231
463	A	6	6	1.00	26	0.231
464	A	5	5	1.00	26	0.192
465	A	5	4	1.00	26	0.154
466	A	5	4	1.00	26	0.154
467	A	5	4	1.00	26	0.154
468	A	5	4	1.00	26	0.154
469	A	5	4	1.00	26	0.154
470	A	4	4	1.00	24	0.167
471	A	4	4	1.00	24	0.167
472	A	6	6	1.00	26	0.231
473	A	5	4	1.00	26	0.154
474	A	4	4	1.00	26	0.154
475	A	4	4	1.00	26	0.154
476	A	4	3	1.00	26	0.115
477	A	5	4	1.00	26	0.154
478	A	5	4	1.00	26	0.154
479	A	5	4	1.00	26	0.154
480	A	4	4	1.00	24	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	5	5	1.00	24	0.208
482	A	6	6	1.00	26	0.231
483	A	5	5	1.00	26	0.192
484	A	5	5	1.00	26	0.192
485	A	4	4	1.00	26	0.154
486	A	5	4	1.00	26	0.154
487	A	5	4	1.00	26	0.154
488	A	6	6	1.00	25	0.240
489	A	5	5	1.00	25	0.200
490	A	4	4	1.00	23	0.174
491	A	4	4	1.00	23	0.174
492	A	5	5	1.00	25	0.200
493	A	6	6	1.00	25	0.240
494	A	8	8	1.00	25	0.320
495	A	7	7	1.00	25	0.280
496	A	2	2	1.00	16	0.125
497	A	7	7	1.00	25	0.280
498	A	8	8	1.00	25	0.320
499	A	7	6	1.00	25	0.240
500	A	6	5	1.00	25	0.200
501	A	5	4	1.00	23	0.174
502	A	5	4	1.00	23	0.174
503	A	6	5	1.00	25	0.200
504	A	7	6	1.00	25	0.240
505	A	9	9	1.00	25	0.360
506	A	8	8	1.00	25	0.320
507	A	6	6	1.00	16	0.375
508	A	8	8	1.00	25	0.320
509	A	9	9	1.00	25	0.360
510	A	5	5	1.00	25	0.200
511	A	4	4	1.00	25	0.160
512	A	3	3	1.00	23	0.130
513	A	3	3	1.00	23	0.130
514	A	4	4	1.00	25	0.160
515	A	5	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	8	8	1.00	25	0.320
517	A	4	4	1.00	25	0.160
518	A	2	2	1.00	16	0.125
519	A	5	5	1.00	25	0.200
520	A	8	8	1.00	25	0.320
521	A	6	6	1.00	25	0.240
522	A	5	5	1.00	25	0.200
523	A	4	4	1.00	23	0.174
524	A	4	4	1.00	23	0.174
525	A	5	5	1.00	25	0.200
526	A	6	6	1.00	25	0.240
527	A	9	8	1.00	25	0.320
528	A	8	8	1.00	25	0.320
529	A	4	4	1.00	16	0.250
530	A	8	8	1.00	25	0.320
531	A	9	8	1.00	25	0.320
532	A	7	6	1.00	25	0.240
533	A	6	5	1.00	25	0.200
534	A	5	4	1.00	23	0.174
535	A	5	4	1.00	23	0.174
536	A	6	5	1.00	25	0.200
537	A	7	6	1.00	25	0.240
538	A	10	8	1.00	25	0.320
539	A	9	8	1.00	25	0.320
540	A	7	7	1.00	16	0.438
541	A	9	9	1.00	25	0.360
542	A	10	9	1.00	25	0.360
543	A	3	3	1.00	25	0.120
544	A	3	3	1.00	23	0.130
545	A	2	2	1.00	21	0.095
546	A	2	2	1.00	21	0.095
547	A	3	3	1.00	23	0.130
548	A	3	3	1.00	23	0.130
549	A	3	3	1.00	23	0.130
550	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	3	3	1.00	23	0.130
552	A	11	10	1.00	15	0.667
553	A	5	5	1.00	15	0.333
554	A	4	4	1.00	15	0.267
555	A	5	5	1.00	23	0.217
556	A	4	4	1.00	25	0.160
557	A	3	3	1.00	23	0.130
558	A	4	4	1.00	23	0.174
559	A	5	5	1.00	25	0.200
560	A	6	6	1.00	25	0.240
561	A	4	4	1.00	25	0.160
562	A	2	2	1.00	16	0.125
563	A	6	6	1.00	25	0.240
564	A	0	0	0.00	0	0.000
565	A	11	10	1.00	23	0.435
566	A	7	6	1.00	21	0.286
567	A	3	3	1.00	21	0.143
568	A	6	6	1.00	23	0.261
569	A	0	0	0.00	0	0.000
570	A	0	0	0.00	0	0.000
571	A	0	0	0.00	0	0.000
572	A	0	0	0.00	0	0.000
573	A	0	0	0.00	0	0.000
574	A	10	5	1.00	23	0.217
575	A	8	5	1.00	23	0.217
576	A	6	5	1.00	21	0.238
577	A	0	0	0.00	0	0.000
578	A	0	0	0.00	0	0.000
579	A	5	5	1.00	15	0.333
580	A	4	4	1.00	15	0.267
581	A	0	0	0.00	0	0.000
582	A	0	0	0.00	0	0.000
583	A	0	0	0.00	0	0.000
584	A	3	3	1.00	21	0.143
585	A	7	6	1.00	23	0.261

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	0	0	0.00	0	0.000
587	A	0	0	0.00	0	0.000
588	A	0	0	0.00	0	0.000
589	A	0	0	0.00	0	0.000
590	A	0	0	0.00	0	0.000
591	A	7	7	1.07	37	0.189
592	A	3	3	1.00	35	0.086
593	A	3	3	1.00	25	0.120
594	A	3	3	1.00	25	0.120

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(a\sin^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=53

$$-\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

```
[Out] -4/15*a*cot(x)*(a*sin(x)^2)^(3/2)-1/5*cot(x)*(a*sin(x)^2)^(5/2)-8/15*a^2*co
t(x)*(a*sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3282, 3286, 2718}

$$-\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(5/2),x]
```

```
[Out] (-8*a^2*Cot[x]*Sqrt[a*Sin[x]^2])/15 - (4*a*Cot[x]*(a*Sin[x]^2)^(3/2))/15 -
(Cot[x]*(a*Sin[x]^2)^(5/2))/5
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sin^2(x)\right)^{5/2} dx = -\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2} + \frac{1}{5}(4a)\int \left(a\sin^2(x)\right)^{3/2} dx$$
$$= -\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2} - \frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2} + \frac{1}{15}(8a^2)\int \sqrt{a\sin^2(x)}\, dx$$
$$= -\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2} - \frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2} + \frac{1}{15}\left(8a^2\csc(x)\sqrt{a\sin^2(x)}\right)$$
$$= -\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)} - \frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2} - \frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

Mathematica [**A**]
time = 0.03, size = 36, normalized size = 0.68

$$-\frac{1}{240}a^2(150\cos(x) - 25\cos(3x) + 3\cos(5x))\csc(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(5/2),x]
[Out] -1/240*(a^2*(150*Cos[x] - 25*Cos[3*x] + 3*Cos[5*x])*Csc[x]*Sqrt[a*Sin[x]^2]
)
```

Maple [**A**]
time = 3.80, size = 32, normalized size = 0.60

method	result
default	$-\frac{a^3\sin(x)\cos(x)\left(3\left(\sin^4(x)\right)+4\left(\sin^2(x)\right)+8\right)}{15\sqrt{a\left(\sin^2(x)\right)}}$
risch	$-\frac{ia^2 e^{6ix}\sqrt{-a\left(e^{2ix}-1\right)^2 e^{-2ix}}}{160(e^{2ix}-1)} - \frac{5ia^2 e^{2ix}\sqrt{-a\left(e^{2ix}-1\right)^2 e^{-2ix}}}{16(e^{2ix}-1)} - \frac{5i\sqrt{-a\left(e^{2ix}-1\right)^2 e^{-2ix}}\, a^2}{16(e^{2ix}-1)} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/15*a^3*sin(x)*cos(x)*(3*sin(x)^4+4*sin(x)^2+8)/(a*sin(x)^2)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^2)^(5/2), x)
```

Fricas [A]
time = 0.39, size = 43, normalized size = 0.81

$$-\frac{\left(3\,a^{2}\cos\left(x\right)^{5}-10\,a^{2}\cos\left(x\right)^{3}+15\,a^{2}\cos\left(x\right)\right)\sqrt{-a\cos\left(x\right)^{2}+a}}{15\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x))*sqrt(-a*cos(x)^2 +
 a)/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin^{2}\left(x\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**2)**(5/2),x)
```

```
[Out] Integral((a*sin(x)**2)**(5/2), x)
```

Giac [A]
time = 0.51, size = 45, normalized size = 0.85

$$\frac{1}{15}\left(8\,a^{2}\mathrm{sgn}(\sin\left(x\right))-\left(3\,a^{2}\cos\left(x\right)^{5}-10\,a^{2}\cos\left(x\right)^{3}+15\,a^{2}\cos\left(x\right)\right)\mathrm{sgn}(\sin\left(x\right))\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*(8*a^2*sgn(sin(x)) - (3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x)
)*sgn(sin(x)))*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int{\left(a\,{\sin\left(x\right)}^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(5/2),x)
```

```
[Out] int((a*sin(x)^2)^(5/2), x)
```

3.2 $\int \left(a\sin^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

```
[Out] -1/3*cot(x)*(a*sin(x)^2)^(3/2)-2/3*a*cot(x)*(a*sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {3282, 3286, 2718}

$$-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(3/2),x]
```

```
[Out] (-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3 - (Cot[x]*(a*Sin[x]^2)^(3/2))/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(a\sin^2(x)\right)^{3/2}\,dx &= -\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}+\frac{1}{3}(2a)\int\sqrt{a\sin^2(x)}\,dx\\ &= -\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}+\frac{1}{3}\left(2a\csc(x)\sqrt{a\sin^2(x)}\right)\int\sin(x)\,dx\\ &= -\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 0.76

$$\frac{1}{12}a(-9\cos(x)+\cos(3x))\csc(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(3/2),x]
[Out] (a*(-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[a*Sin[x]^2])/12
```

Maple [A]
time = 3.98, size = 24, normalized size = 0.71

method	result
default	$-\frac{a^2\sin(x)\cos(x)(\sin^2(x)+2)}{3\sqrt{a\left(\sin^2(x)\right)}}$
risch	$\frac{ia\,e^{4ix}\sqrt{-a\left(e^{2ix}-1\right)^2e^{-2ix}}}{24\,e^{2ix}-24}-\frac{3ia\,e^{2ix}\sqrt{-a\left(e^{2ix}-1\right)^2e^{-2ix}}}{8(e^{2ix}-1)}-\frac{3i\sqrt{-a\left(e^{2ix}-1\right)^2e^{-2ix}}\,a}{8(e^{2ix}-1)}+\frac{ia\,e^{-2ix}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3*a^2*sin(x)*cos(x)*(sin(x)^2+2)/(a*sin(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(x)^2)^(3/2), x)
```

Fricas [A]
time = 0.39, size = 29, normalized size = 0.85

$$\frac{\left(a\cos\left(x\right)^{3}-3\,a\cos\left(x\right)\right)\sqrt{-a\cos\left(x\right)^{2}+a}}{3\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(a*cos(x)^3 - 3*a*cos(x))*sqrt(-a*cos(x)^2 + a)/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**2)**(3/2),x)
[Out] Integral((a*sin(x)**2)**(3/2), x)
```

Giac [A]
time = 0.42, size = 24, normalized size = 0.71

$$\frac{1}{3}\left(\left(\cos\left(x\right)^{3}-3\,\cos\left(x\right)\right)\operatorname{sgn}(\sin\left(x\right))+2\operatorname{sgn}(\sin\left(x\right))\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(3/2),x, algorithm="giac")
[Out] 1/3*((cos(x)^3 - 3*cos(x))*sgn(sin(x)) + 2*sgn(sin(x)))*a^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int{\left(a\,{\sin\left(x\right)}^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(3/2),x)
[Out] int((a*sin(x)^2)^(3/2), x)
```

3.3 $\int \sqrt{a\sin^2(x)}\,dx$

Optimal. Leaf size=14

$$-\cot(x)\sqrt{a\sin^2(x)}$$

```
[Out] -cot(x)*(a*sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 2718}

$$-\cot(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[x]^2],x]
[Out] -(Cot[x]*Sqrt[a*Sin[x]^2])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\sin^2(x)}\,dx &= \left(\csc(x)\sqrt{a\sin^2(x)}\right)\int \sin(x)\,dx\\ &= -\cot(x)\sqrt{a\sin^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\cot(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[a*Sin[x]^2])
```

Maple [A]
time = 2.30, size = 16, normalized size = 1.14

method	result	size
default	$-\frac{a\cos(x)\sin(x)}{\sqrt{a\left(\sin^2(x)\right)}}$	16
risch	$-\frac{i\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}\,\mathrm{e}^{2ix}}{2(\mathrm{e}^{2ix}-1)}-\frac{i\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}}{2(\mathrm{e}^{2ix}-1)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*sin(x)^2)^(1/2)*a*cos(x)*sin(x)
```

Maxima [A]
time = 0.50, size = 13, normalized size = 0.93

$$-\frac{\sqrt{a}}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a)/sqrt(tan(x)^2 + 1)
```

Fricas [A]
time = 0.40, size = 19, normalized size = 1.36

$$-\frac{\sqrt{-a\cos(x)^2+a}\,\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-a*cos(x)^2 + a)*cos(x)/sin(x)
```

Sympy [A]
time = 0.11, size = 17, normalized size = 1.21

$$-\frac{\sqrt{a\sin^2(x)}\,\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**2)**(1/2),x)
```

```
[Out] -sqrt(a*sin(x)**2)*cos(x)/sin(x)
```

Giac [A]
time = 0.45, size = 17, normalized size = 1.21

$$-(\cos(x)\,\mathrm{sgn}(\sin(x)) - \mathrm{sgn}(\sin(x)))\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(cos(x)*sgn(sin(x)) - sgn(sin(x)))*sqrt(a)
```

Mupad [B]
time = 13.64, size = 40, normalized size = 2.86

$$-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{2\sin(x)^2}\,\left(-\sin(x)^2 + \frac{\sin(2\,x)\,1\mathrm{i}}{2} + 1\right)}{\sin(x)^2\,2\mathrm{i} + \sin(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(1/2),x)
```

```
[Out] -(2^(1/2)*a^(1/2)*(2*sin(x)^2)^(1/2)*((sin(2*x)*1i)/2 - sin(x)^2 + 1))/(sin
(2*x) + sin(x)^2*2i)
```

3.4 $\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx$

Optimal. Leaf size=17

$$-\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{a\sin^2(x)}}$$

```
[Out] -arctanh(cos(x))*sin(x)/(a*sin(x)^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3855}

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{a\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sin[x]^2],x]
```

```
[Out] -((ArcTanh[Cos[x]]*Sin[x])/Sqrt[a*Sin[x]^2])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx &= \frac{\sin(x)\int \csc(x)\,dx}{\sqrt{a\sin^2(x)}}\\ &= -\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{a\sin^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.76

$$\frac{\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)}{\sqrt{a\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sin[x]^2],x]
```

```
[Out] ((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[a*Sin[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 4.02, size = 49, normalized size = 2.88

method	result	size
default	$-\frac{\sin(x)\sqrt{a\left(\cos^2(x)\right)}\ \ln\left(\frac{2\sqrt{a}\ \sqrt{a\left(\cos^2(x)\right)}+2a}{\sin(x)}\right)}{\sqrt{a}\ \cos(x)\sqrt{a\left(\sin^2(x)\right)}}$	49
risch	$\frac{2\ln\left(e^{ix}-1\right)\sin(x)}{\sqrt{-a\left(e^{2ix}-1\right)^2 e^{-2ix}}}-\frac{2\ln\left(e^{ix}+1\right)\sin(x)}{\sqrt{-a\left(e^{2ix}-1\right)^2 e^{-2ix}}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -sin(x)*(a*cos(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a)/sin(
x))/cos(x)/(a*sin(x)^2)^(1/2)
```

Maxima [A]
time = 0.55, size = 26, normalized size = 1.53

$$\frac{\sqrt{-a}\left(\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a)*(arctan2(sin(x), cos(x) + 1) - arctan2(sin(x), cos(x) - 1))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

time = 0.39, size = 70, normalized size = 4.12

$$\left[\frac{\sqrt{-a\cos\left(x\right)^{2}+a}\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{2\,a\sin\left(x\right)},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a\cos\left(x\right)^{2}+a}\,\sqrt{-a}\,\cos(x)}{a\sin(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*sqrt(-a*cos(x)^2 + a)*log(-(cos(x) - 1)/(cos(x) + 1))/(a*sin(x)), sqrt
(-a)*arctan(sqrt(-a*cos(x)^2 + a)*sqrt(-a)*cos(x)/(a*sin(x)))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(a*sin(x)**2), x)
```

Giac [**A**]
time = 0.54, size = 15, normalized size = 0.88

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="giac")
[Out] log(abs(tan(1/2*x)))/(sqrt(a)*sgn(sin(x)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{a\,{\sin\left(x\right)}^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(1/2),x)
[Out] int(1/(a*sin(x)^2)^(1/2), x)
```

3.5 $\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}-\frac{\tanh^{-1}(\cos(x))\sin(x)}{2a\sqrt{a\sin^2(x)}}$$

```
[Out] -1/2*cot(x)/a/(a*sin(x)^2)^(1/2)-1/2*arctanh(cos(x))*sin(x)/a/(a*sin(x)^2)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3283, 3286, 3855}

$$-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}-\frac{\sin(x)\tanh^{-1}(\cos(x))}{2a\sqrt{a\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(-3/2),x]
```

```
[Out] -1/2*Cot[x]/(a*Sqrt[a*Sin[x]^2]) - (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Sin
[x]^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx &= -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}+\frac{\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx}{2a}\\
&= -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}+\frac{\sin(x)\int \csc(x)\,dx}{2a\sqrt{a\sin^2(x)}}\\
&= -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}-\frac{\tanh^{-1}(\cos(x))\sin(x)}{2a\sqrt{a\sin^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 1.31

$$
-\frac{\left(\csc^2\left(\frac{x}{2}\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)-\sec^2\left(\frac{x}{2}\right)\right)\sin^3(x)}{8\left(a\sin^2(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(-3/2),x]
```

```
[Out] -1/8*((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x]^
3)/(a*Sin[x]^2)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(34) = 68.
time = 6.58, size = 70, normalized size = 1.67

method	result	size
default	$-\frac{\sqrt{a\left(\cos^2\left(x\right)\right)}\,\left(\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2\left(x\right)\right)}+2a}{\sin\left(x\right)}\right)a\left(\sin^2\left(x\right)\right)+\sqrt{a}\,\sqrt{a\left(\cos^2\left(x\right)\right)}\right)}{2a^{\frac{5}{2}}\sin\left(x\right)\cos\left(x\right)\sqrt{a\left(\sin^2\left(x\right)\right)}}$	70
risch	$-\frac{i\left(e^{2ix}+1\right)}{a\left(e^{2ix}-1\right)\sqrt{-a\left(e^{2ix}-1\right)^2 e^{-2ix}}}-\frac{\ln\left(e^{ix}+1\right)\sin\left(x\right)}{a\sqrt{-a\left(e^{2ix}-1\right)^2 e^{-2ix}}}+\frac{\ln\left(e^{ix}-1\right)\sin\left(x\right)}{a\sqrt{-a\left(e^{2ix}-1\right)^2 e^{-2ix}}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^(5/2)/sin(x)*(a*cos(x)^2)^(1/2)*(ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a)
/sin(x))*a*sin(x)^2+a^(1/2)*(a*cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(34) = 68$.
time = 0.57, size = 314, normalized size = 7.48

$$-\frac{\left(\left(2\left(2\cos(2x)-1\right)\cos(4x)-\cos(4x)^2-4\cos(2x)^2-\sin(4x)^2+4\sin(4x)\sin(2x)-4\sin(2x)^2+4\cos(2x)-1\right)\arctan\left(\sin(x),\cos(x)+1\right)-\left(2\left(2\cos(2x)-1\right)\cos(4x)-\cos(4x)^2-4\cos(2x)^2-\sin(4x)^2+4\sin(4x)\sin(2x)-4\sin(2x)^2+4\cos(2x)-1\right)\arctan\left(\sin(x),\cos(x)-1\right)+2\left(\sin(3x)+\sin(x)\right)\cos(4x)-2\left(\cos(3x)+\cos(x)\right)\sin(4x)-2\left(2\cos(2x)-1\right)\sin(3x)+4\cos(3x)\sin(2x)+4\cos(x)\sin(2x)-4\cos(2x)\sin(x)+2\sin(x)\right)\sqrt{-a}}{2\left(a^2\cos(4x)^2+4a^2\cos(2x)^2+a^2\sin(4x)^2-4a^2\sin(4x)\sin(2x)+4a^2\sin(2x)^2-4a^2\cos(2x)+a^2-2\left(2a^2\cos(2x)-a^2\right)\cos(4x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2
 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), cos
(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4
*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x)
, cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))*sin(
4*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*sin(2*x
) - 4*cos(2*x)*sin(x) + 2*sin(x))*sqrt(-a)/(a^2*cos(4*x)^2 + 4*a^2*cos(2*x)
^2 + a^2*sin(4*x)^2 - 4*a^2*sin(4*x)*sin(2*x) + 4*a^2*sin(2*x)^2 - 4*a^2*co
s(2*x) + a^2 - 2*(2*a^2*cos(2*x) - a^2)*cos(4*x))
```

Fricas [A]
time = 0.38, size = 58, normalized size = 1.38

$$\frac{\sqrt{-a\cos\left(x\right)^2+a}\left(\left(\cos\left(x\right)^2-1\right)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)+2\cos\left(x\right)\right)}{4\left(a^2\cos\left(x\right)^2-a^2\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-a*cos(x)^2 + a)*((cos(x)^2 - 1)*log(-(cos(x) - 1)/(cos(x) + 1)) +
 2*cos(x))/((a^2*cos(x)^2 - a^2)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\sin^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**2)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(34) = 68$.
time = 0.57, size = 87, normalized size = 2.07

$$\frac{\frac{2\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{\sqrt{a}\,\mathrm{sgn}(\sin(x))}+\frac{\left(\sqrt{a}-\frac{2\sqrt{a}\,(\cos(x)-1)}{\cos(x)+1}\right)(\cos(x)+1)}{a(\cos(x)-1)\mathrm{sgn}(\sin(x))}-\frac{\cos(x)-1}{\sqrt{a}\,(\cos(x)+1)\mathrm{sgn}(\sin(x))}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*log(-(cos(x) - 1)/(cos(x) + 1))/(sqrt(a)*sgn(sin(x))) + (sqrt(a) - 2
*sqrt(a)*(cos(x) - 1)/(cos(x) + 1))*(cos(x) + 1)/(a*(cos(x) - 1)*sgn(sin(x)
)) - (cos(x) - 1)/(sqrt(a)*(cos(x) + 1)*sgn(sin(x))))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\sin\left(x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(3/2),x)
```

```
[Out] int(1/(a*sin(x)^2)^(3/2), x)
```

3.6 $\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\tanh^{-1}(\cos(x))\sin(x)}{8a^2\sqrt{a\sin^2(x)}}$$

```
[Out] -1/4*cot(x)/a/(a*sin(x)^2)^(3/2)-3/8*cot(x)/a^2/(a*sin(x)^2)^(1/2)-3/8*arct
anh(cos(x))*sin(x)/a^2/(a*sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3283, 3286, 3855}

$$-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\sin(x)\tanh^{-1}(\cos(x))}{8a^2\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(-5/2),x]
```

```
[Out] -1/4*Cot[x]/(a*(a*Sin[x]^2)^(3/2)) - (3*Cot[x])/(8*a^2*Sqrt[a*Sin[x]^2]) -
(3*ArcTanh[Cos[x]]*Sin[x])/(8*a^2*Sqrt[a*Sin[x]^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx &= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}+\frac{3\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx}{4a}\\
&= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}+\frac{3\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx}{8a^2}\\
&= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}+\frac{(3\sin(x))\int \csc(x)\,dx}{8a^2\sqrt{a\sin^2(x)}}\\
&= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\tanh^{-1}(\cos(x))\sin(x)}{8a^2\sqrt{a\sin^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 1.26

$$
-\frac{\csc(x)\left(6\csc^2\left(\frac{x}{2}\right)+\csc^4\left(\frac{x}{2}\right)+24\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-6\sec^2\left(\frac{x}{2}\right)-\sec^4\left(\frac{x}{2}\right)\right)\sqrt{a\sin^2(x)}}{64a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(-5/2),x]
```

```
[Out] -1/64*(Csc[x]*(6*Csc[x/2]^2 + Csc[x/2]^4 + 24*(Log[Cos[x/2]] - Log[Sin[x/2]
]) - 6*Sec[x/2]^2 - Sec[x/2]^4)*Sqrt[a*Sin[x]^2])/a^3
```

Maple [A]
time = 7.38, size = 89, normalized size = 1.46

method	result
default	$-\frac{\sqrt{a\left(\cos^2(x)\right)}\left(3\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\cos^2(x)\right)}+2a}{\sin(x)}\right)a\left(\sin^4(x)\right)+3\sqrt{a\left(\cos^2(x)\right)}\left(\sin^2(x)\right)\sqrt{a}+2\sqrt{a}\sqrt{a(\mathrm{co}}{8a^{\frac{7}{2}}\sin(x)^3\cos(x)\sqrt{a\left(\sin^2(x)\right)}}$
risch	$-\frac{i\left(3\,\mathrm{e}^{6ix}-11\,\mathrm{e}^{4ix}-11\,\mathrm{e}^{2ix}+3\right)}{4a^2\left(\mathrm{e}^{2ix}-1\right)^3\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}}+\frac{3\ln\left(\mathrm{e}^{ix}-1\right)\sin(x)}{4a^2\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}}-\frac{3\ln\left(\mathrm{e}^{ix}+1\right)\sin(x)}{4a^2\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a^(7/2)/sin(x)^3*(a*cos(x)^2)^(1/2)*(3*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2
)+a)/sin(x))*a*sin(x)^4+3*(a*cos(x)^2)^(1/2)*sin(x)^2*a^(1/2)+2*a^(1/2)*(a*
cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(49) = 98$.
time = 0.88, size = 931, normalized size = 15.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*(2*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2
 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x
) - 1)*cos(4*x) - 36*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*x
) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(6*
x) - 16*sin(6*x)^2 - 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^2 +
 8*cos(2*x) - 1)*arctan2(sin(x), cos(x) + 1) - 3*(2*(4*cos(6*x) - 6*cos(4*x
) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)
*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x) - 1)*cos(4*x) - 36*cos(4*x)^2 -
16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x
)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(6*x) - 16*sin(6*x)^2 - 36*sin(4*x)^2
 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^2 + 8*cos(2*x) - 1)*arctan2(sin(x), c
os(x) - 1) + 2*(3*sin(7*x) - 11*sin(5*x) - 11*sin(3*x) + 3*sin(x))*cos(8*x)
 + 12*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 8*(11*sin(5*x) + 11
*sin(3*x) - 3*sin(x))*cos(6*x) + 44*(3*sin(4*x) - 2*sin(2*x))*cos(5*x) - 12
*(11*sin(3*x) - 3*sin(x))*cos(4*x) - 2*(3*cos(7*x) - 11*cos(5*x) - 11*cos(3
*x) + 3*cos(x))*sin(8*x) - 6*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*sin
(7*x) - 8*(11*cos(5*x) + 11*cos(3*x) - 3*cos(x))*sin(6*x) - 22*(6*cos(4*x)
- 4*cos(2*x) + 1)*sin(5*x) + 12*(11*cos(3*x) - 3*cos(x))*sin(4*x) + 22*(4*c
os(2*x) - 1)*sin(3*x) - 88*cos(3*x)*sin(2*x) + 24*cos(x)*sin(2*x) - 24*cos(
2*x)*sin(x) + 6*sin(x))*sqrt(-a)/(a^3*cos(8*x)^2 + 16*a^3*cos(6*x)^2 + 36*a
^3*cos(4*x)^2 + 16*a^3*cos(2*x)^2 + a^3*sin(8*x)^2 + 16*a^3*sin(6*x)^2 + 36
*a^3*sin(4*x)^2 - 48*a^3*sin(4*x)*sin(2*x) + 16*a^3*sin(2*x)^2 - 8*a^3*cos(
2*x) + a^3 - 2*(4*a^3*cos(6*x) - 6*a^3*cos(4*x) + 4*a^3*cos(2*x) - a^3)*cos
(8*x) - 8*(6*a^3*cos(4*x) - 4*a^3*cos(2*x) + a^3)*cos(6*x) - 12*(4*a^3*cos(
2*x) - a^3)*cos(4*x) - 4*(2*a^3*sin(6*x) - 3*a^3*sin(4*x) + 2*a^3*sin(2*x))
*sin(8*x) - 16*(3*a^3*sin(4*x) - 2*a^3*sin(2*x))*sin(6*x))
```

Fricas [A]
time = 0.42, size = 78, normalized size = 1.28

$$\frac{\sqrt{-a\cos\left(x\right)^{2}+a}\,\left(6\,\cos\left(x\right)^{3}+3\,\left(\cos\left(x\right)^{4}-2\,\cos\left(x\right)^{2}+1\right)\log\left(-\frac{\cos\left(x\right)-1}{\cos\left(x\right)+1}\right)-10\,\cos\left(x\right)\right)}{16\,\left(a^{3}\cos\left(x\right)^{4}-2\,a^{3}\cos\left(x\right)^{2}+a^{3}\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(-a*cos(x)^2 + a)*(6*cos(x)^3 + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(
-(cos(x) - 1)/(cos(x) + 1)) - 10*cos(x))/((a^3*cos(x)^4 - 2*a^3*cos(x)^2 +
a^3)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin^2(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**2)**(5/2),x)
```

```
[Out] Integral((a*sin(x)**2)**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(49) = 98.
time = 0.46, size = 126, normalized size = 2.07

$$\frac{3\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}(\sin(x))} - \frac{\left(\sqrt{a} - \frac{8\sqrt{a}\,(\cos(x)-1)}{\cos(x)+1} + \frac{18\sqrt{a}\,(\cos(x)-1)^2}{(\cos(x)+1)^2}\right)(\cos(x)+1)^2}{64\,a^3(\cos(x)-1)^2\mathrm{sgn}(\sin(x))} - \frac{\frac{8\,a^{\frac{7}{2}}(\cos(x)-1)\mathrm{sgn}(\sin(x))}{\cos(x)+1} - \frac{a^{\frac{7}{2}}(\cos(x)-1)^2\mathrm{sgn}(\sin(x))}{(\cos(x)+1)^2}}{64\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/16*log(-(cos(x) - 1)/(cos(x) + 1))/(a^(5/2)*sgn(sin(x))) - 1/64*(sqrt(a)
- 8*sqrt(a)*(cos(x) - 1)/(cos(x) + 1) + 18*sqrt(a)*(cos(x) - 1)^2/(cos(x) +
 1)^2)*(cos(x) + 1)^2/(a^3*(cos(x) - 1)^2*sgn(sin(x))) - 1/64*(8*a^(7/2)*(c
os(x) - 1)*sgn(sin(x))/(cos(x) + 1) - a^(7/2)*(cos(x) - 1)^2*sgn(sin(x))/(c
os(x) + 1)^2)/a^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a\sin(x)^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(5/2),x)
```

```
[Out] int(1/(a*sin(x)^2)^(5/2), x)
```

3.7 $\int \left(a\sin^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=123

$$-\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)}-\frac{26a^2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{77\sin^{\frac{3}{2}}(x)}-\frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)}-\frac{26}{165}a^2\cos(x)\sin$$

```
[Out] -26/77*a^2*cot(x)*(a*sin(x)^3)^(1/2)-26/77*a^2*(sin(1/4*Pi+1/2*x)^2)^(1/2)/
sin(1/4*Pi+1/2*x)*EllipticF(cos(1/4*Pi+1/2*x),2^(1/2))*(a*sin(x)^3)^(1/2)/s
in(x)^(3/2)-78/385*a^2*cos(x)*sin(x)*(a*sin(x)^3)^(1/2)-26/165*a^2*cos(x)*s
in(x)^3*(a*sin(x)^3)^(1/2)-2/15*a^2*cos(x)*sin(x)^5*(a*sin(x)^3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 2720}

$$-\frac{26}{165}a^2\sin^3(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{78}{385}a^2\sin(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{2}{15}a^2\sin^5(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)}-\frac{26a^2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{77\sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(5/2),x]
```

```
[Out] (-26*a^2*Cot[x]*Sqrt[a*Sin[x]^3])/77 - (26*a^2*EllipticF[Pi/4 - x/2, 2]*Sqr
t[a*Sin[x]^3])/(77*Sin[x]^(3/2)) - (78*a^2*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^3])/
385 - (26*a^2*Cos[x]*Sin[x]^3*Sqrt[a*Sin[x]^3])/165 - (2*a^2*Cos[x]*Sin[x]^
5*Sqrt[a*Sin[x]^3])/15
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sin^3(x)\right)^{5/2}\,dx = \frac{\left(a^2\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{15}{2}}(x)\,dx}{\sin^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{15}a^2\cos(x)\sin^5(x)\sqrt{a\sin^3(x)} + \frac{\left(13a^2\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{11}{2}}(x)\,dx}{15\sin^{\frac{3}{2}}(x)}$$

$$= -\frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\sin^3(x)} - \frac{2}{15}a^2\cos(x)\sin^5(x)\sqrt{a\sin^3(x)} + \frac{\left(39a^2\sqrt{a\sin\right.}{5}$$

$$= -\frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)} - \frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\sin^3(x)} - \frac{2}{15}a^2\cos(x)\,\mathrm{s}$$

$$= -\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)} - \frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)} - \frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{}$$

$$= -\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)} - \frac{26a^2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{77\sin^{\frac{3}{2}}(x)} - \frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a}$$

Mathematica [A]
time = 0.13, size = 65, normalized size = 0.53

$$\frac{a\left(-12480F\left(\frac{1}{4}(\pi-2x)\middle|2\right)+\left(-15465\cos(x)+3657\cos(3x)-749\cos(5x)+77\cos(7x)\right)\sqrt{\sin(x)}\right)\left(a\sin^3(x)\right)^{3/2}}{36960\sin^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(5/2),x]
```

```
[Out] (a*(-12480*EllipticF[(Pi - 2*x)/4, 2] + (-15465*Cos[x] + 3657*Cos[3*x] - 74
9*Cos[5*x] + 77*Cos[7*x])*Sqrt[Sin[x]])*(a*Sin[x]^3)^(3/2))/(36960*Sin[x]^(
9/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.54, size = 152, normalized size = 1.24

method	result
default	$-\frac{\left(-154\left(\cos^{8}(x)\right)+195i\sqrt{2}\,\sin(x)\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\,\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\,\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\right.\right.}{1155}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1155*(-154*cos(x)^8+195*I*2^(1/2)*sin(x)*((I*cos(x)+sin(x)-I)/sin(x))^(1
/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*Ellip
ticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+154*cos(x)^7+644*cos(x
)^6-644*cos(x)^5-1060*cos(x)^4+1060*cos(x)^3+960*cos(x)^2-960*cos(x))*(a*(1
-cos(x)^2)*sin(x))^(5/2)/sin(x)^7/(-1+cos(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 110, normalized size = 0.89

$$\frac{195\sqrt{2}\sqrt{-i\,a}\,a^{2}\sin(x)\,\mathrm{weierstrassPInverse}(4,0,\cos(x)+i\sin(x))+195\sqrt{2}\sqrt{i\,a}\,a^{2}\sin(x)\,\mathrm{weierstrassPInverse}(4,0,\cos(x)-i\sin(x))+2\left(77a^{2}\cos(x)^{7}-322a^{2}\cos(x)^{5}+530a^{2}\cos(x)^{3}-480a^{2}\cos(x)\right)\sqrt{-\left(a\cos(x)^{2}-a\right)\sin(x)}}{1155\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(195*sqrt(2)*sqrt(-I*a)*a^2*sin(x)*weierstrassPInverse(4, 0, cos(x)
+ I*sin(x)) + 195*sqrt(2)*sqrt(I*a)*a^2*sin(x)*weierstrassPInverse(4, 0, co
s(x) - I*sin(x)) + 2*(77*a^2*cos(x)^7 - 322*a^2*cos(x)^5 + 530*a^2*cos(x)^3
 - 480*a^2*cos(x))*sqrt(-(a*cos(x)^2 - a)*sin(x)))/sin(x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(x)^3)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sin\left(x\right)^3\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(5/2),x)
[Out] int((a*sin(x)^3)^(5/2), x)
```

3.8 $\int \left(a\sin^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=73

$$-\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)}-\frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)}-\frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)}$$

```
[Out] -14/45*a*cos(x)*(a*sin(x)^3)^(1/2)-14/15*a*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(
1/4*Pi+1/2*x)*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*(a*sin(x)^3)^(1/2)/sin(x
)^(3/2)-2/9*a*cos(x)*sin(x)^2*(a*sin(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 2719}

$$-\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)}-\frac{2}{9}a\sin^2(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(3/2),x]
```

```
[Out] (-14*a*Cos[x]*Sqrt[a*Sin[x]^3])/45 - (14*a*EllipticE[Pi/4 - x/2, 2]*Sqrt[a*
Sin[x]^3])/(15*Sin[x]^(3/2)) - (2*a*Cos[x]*Sin[x]^2*Sqrt[a*Sin[x]^3])/9
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{9}{2}}(x)\,dx}{\sin^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)} + \frac{\left(7a\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{5}{2}}(x)\,dx}{9\sin^{\frac{3}{2}}(x)}$$

$$= -\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)} - \frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)} + \frac{\left(7a\sqrt{a\sin^3(x)}\right)\int \sqrt{\mathrm{si}}}{15\sin^{\frac{3}{2}}(x)}$$

$$= -\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)} - \frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)} - \frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.74

$$\frac{\left(a\sin^3(x)\right)^{3/2}\left(-168E\left(\frac{1}{4}(\pi-2x)\middle|2\right)+\sqrt{\sin(x)}\,(-38\sin(2x)+5\sin(4x))\right)}{180\sin^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(3/2),x]
```

```
[Out] ((a*Sin[x]^3)^(3/2)*(-168*EllipticE[(Pi - 2*x)/4, 2] + Sqrt[Sin[x]]*(-38*Si
n[2*x] + 5*Sin[4*x])))/(180*Sin[x]^(9/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 337, normalized size = 4.62

method	result
default	$-\frac{\left(42\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\,\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\,\sqrt{2}\,\cos(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/45*(42*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^
(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*(-I*(-1+cos
(x))/sin(x))^(1/2)*2^(1/2)*cos(x)-21*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-
I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((
I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)+42*((I*cos(x)+
sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*c
os(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(x))/sin(x))^(1/2)*2^
(1/2)-21*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(
1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(
1/2),1/2*2^(1/2))*2^(1/2)+10*cos(x)^5-34*cos(x)^3+66*cos(x)-42)*(a*sin(x)^3
)^(3/2)/sin(x)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 80, normalized size = 1.10

$$\frac{7}{15} i \sqrt{2} \sqrt{-i a} a \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(x) + i \sin(x))) - \frac{7}{15} i \sqrt{2} \sqrt{i a} a \text{weierstrassZeta}(4, 0, \text{weierstrassPInverse}(4, 0, \cos(x) - i \sin(x))) + \frac{2}{45} (5 a \cos(x)^3 - 12 a \cos(x)) \sqrt{-(a \cos(x)^2 - a) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 7/15*I*sqrt(2)*sqrt(-I*a)*a*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
 cos(x) + I*sin(x))) - 7/15*I*sqrt(2)*sqrt(I*a)*a*weierstrassZeta(4, 0, wei
erstrassPInverse(4, 0, cos(x) - I*sin(x))) + 2/45*(5*a*cos(x)^3 - 12*a*cos(
x))*sqrt(-(a*cos(x)^2 - a)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sin^3(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**3)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(x)^3)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sin\left(x\right)^3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(3/2),x)
[Out] int((a*sin(x)^3)^(3/2), x)
```

3.9 $\int \sqrt{a \sin^3(x)}\, dx$

Optimal. Leaf size=50

$$-\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)}-\frac{2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}$$

```
[Out] -2/3*cot(x)*(a*sin(x)^3)^(1/2)-2/3*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*Pi+1
/2*x)*EllipticF(cos(1/4*Pi+1/2*x),2^(1/2))*(a*sin(x)^3)^(1/2)/sin(x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 2720}

$$-\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)}-\frac{2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[x]^3],x]
```

```
[Out] (-2*Cot[x]*Sqrt[a*Sin[x]^3])/3 - (2*EllipticF[Pi/4 - x/2, 2]*Sqrt[a*Sin[x]^
3])/(3*Sin[x]^(3/2))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a\sin^3(x)}\,dx = \frac{\sqrt{a\sin^3(x)}\int \sin^{\frac{3}{2}}(x)\,dx}{\sin^{\frac{3}{2}}(x)}$$
$$= -\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)} + \frac{\sqrt{a\sin^3(x)}\int \frac{1}{\sqrt{\sin(x)}}\,dx}{3\sin^{\frac{3}{2}}(x)}$$
$$= -\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)} - \frac{2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.82

$$-\frac{2\left(F\left(\frac{1}{4}(\pi-2x)\middle|2\right)+\cos(x)\sqrt{\sin(x)}\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[x]^3],x]
[Out] (-2*(EllipticF[(Pi - 2*x)/4, 2] + Cos[x]*Sqrt[Sin[x]])*Sqrt[a*Sin[x]^3])/(3
*Sin[x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.54, size = 124, normalized size = 2.48

method	result
default	$-\frac{\left(i\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\,\sin(x)\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\,\sqrt{-\frac{i\cos(x)-\sin(x)-i}{\sin(x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{\sqrt{2}}{2}\right)+(\mathrm{co}\right.}{6\sin(x)(-1+\cos(x))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/6*(I*(-I*(-1+cos(x))/sin(x))^(1/2)*sin(x)*((I*cos(x)+sin(x)-I)/sin(x))^(
1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin
(x))^(1/2),1/2*2^(1/2))+cos(x)^2*2^(1/2)-cos(x)*2^(1/2))*(a*(1-cos(x)^2)*si
n(x))^(1/2)/sin(x)/(-1+cos(x))*8^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 69, normalized size = 1.38

$$\frac{\sqrt{2}\,\sqrt{-i\,a}\,\sin\left(x\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(x\right)+i\,\sin\left(x\right))+\sqrt{2}\,\sqrt{i\,a}\,\sin\left(x\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(x\right)-i\,\sin\left(x\right))-2\,\sqrt{-\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)}\,\cos\left(x\right)}{3\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*sqrt(-I*a)*sin(x)*weierstrassPInverse(4, 0, cos(x) + I*sin(x))
 + sqrt(2)*sqrt(I*a)*sin(x)*weierstrassPInverse(4, 0, cos(x) - I*sin(x)) -
2*sqrt(-(a*cos(x)^2 - a)*sin(x))*cos(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin^{3}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{a\sin\left(x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(1/2),x)
```

```
[Out] int((a*sin(x)^3)^(1/2), x)
```

3.10 $\int \frac{1}{\sqrt{a \sin^3(x)}} \, dx$

Optimal. Leaf size=48

$$-\frac{2\cos(x)\sin(x)}{\sqrt{a\sin^3(x)}}+\frac{2E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{\sqrt{a\sin^3(x)}}$$

```
[Out] -2*cos(x)*sin(x)/(a*sin(x)^3)^(1/2)+2*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*P
i+1/2*x)*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*sin(x)^(3/2)/(a*sin(x)^3)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2716, 2719}

$$\frac{2\sin^{\frac{3}{2}}(x)E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{\sqrt{a\sin^3(x)}}-\frac{2\sin(x)\cos(x)}{\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sin[x]^3],x]
```

```
[Out] (-2*Cos[x]*Sin[x])/Sqrt[a*Sin[x]^3] + (2*EllipticE[Pi/4 - x/2, 2]*Sin[x]^(3
/2))/Sqrt[a*Sin[x]^3]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx &= \frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\sin^3(x)}}\\ &= -\frac{2\cos(x)\sin(x)}{\sqrt{a\sin^3(x)}} - \frac{\sin^{\frac{3}{2}}(x)\int \sqrt{\sin(x)}\,dx}{\sqrt{a\sin^3(x)}}\\ &= -\frac{2\cos(x)\sin(x)}{\sqrt{a\sin^3(x)}} + \frac{2E\left(\frac{\pi}{4}-\frac{x}{2}\middle|\,2\right)\sin^{\frac{3}{2}}(x)}{\sqrt{a\sin^3(x)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.77

$$\frac{2E\left(\frac{1}{4}(\pi-2x)\middle|\,2\right)\sin^{\frac{3}{2}}(x)-\sin(2x)}{\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sin[x]^3],x]
[Out] (2*EllipticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2) - Sin[2*x])/Sqrt[a*Sin[x]^3]
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 318, normalized size = 6.62

method	result
default	$\frac{\left(2\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\,\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\,\sqrt{2}\,\cos(x)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] (2*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*E
llipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(x))/si
n(x))^(1/2)*2^(1/2)*cos(x)-((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+s
in(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+s
in(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)+2*((I*cos(x)+sin(x)-I)/s
```

```
in(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x
)-I)/sin(x))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(x))/sin(x))^(1/2)*2^(1/2)-((I*c
os(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+c
os(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/
2))*2^(1/2)-2)*sin(x)/(a*sin(x)^3)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sin(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 102, normalized size = 2.12

$$\frac{\left(-i\sqrt{2}\cos\left(x\right)^{2}+i\sqrt{2}\right)\sqrt{-i a}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(x\right)+i\sin\left(x\right)))+\left(i\sqrt{2}\cos\left(x\right)^{2}-i\sqrt{2}\right)\sqrt{i a}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(x\right)-i\sin\left(x\right)))+2\sqrt{-\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)}\cos\left(x\right)}{a\cos\left(x\right)^{2}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] ((-I*sqrt(2)*cos(x)^2 + I*sqrt(2))*sqrt(-I*a)*weierstrassZeta(4, 0, weierst
rassPInverse(4, 0, cos(x) + I*sin(x))) + (I*sqrt(2)*cos(x)^2 - I*sqrt(2))*s
qrt(I*a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(x) - I*sin(x))
) + 2*sqrt(-(a*cos(x)^2 - a)*sin(x))*cos(x))/(a*cos(x)^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin^{3}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sin(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a\sin\left(x\right)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*sin(x)^3)^(1/2), x)
```

3.11 $\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}-\frac{10F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{21a\sqrt{a\sin^3(x)}}$$

```
[Out] -10/21*cos(x)/a/(a*sin(x)^3)^(1/2)-2/7*cot(x)*csc(x)/a/(a*sin(x)^3)^(1/2)-1
0/21*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*Pi+1/2*x)*EllipticF(cos(1/4*Pi+1/2
*x),2^(1/2))*sin(x)^(3/2)/a/(a*sin(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2716, 2720}

$$-\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}}-\frac{10\sin^{\frac{3}{2}}(x)F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{21a\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(-3/2),x]
```

```
[Out] (-10*Cos[x])/(21*a*Sqrt[a*Sin[x]^3]) - (2*Cot[x]*Csc[x])/(7*a*Sqrt[a*Sin[x]
^3]) - (10*EllipticF[Pi/4 - x/2, 2]*Sin[x]^(3/2))/(21*a*Sqrt[a*Sin[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx &= \frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\sin^3(x)}}\\
&= -\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}+\frac{\left(5\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\sin^3(x)}}\\
&= -\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}+\frac{\left(5\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\sin(x)}}\,dx}{21a\sqrt{a\sin^3(x)}}\\
&= -\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}-\frac{10F\left(\frac{\pi}{4}-\frac{x}{2}\middle|\,2\right)\sin^{\frac{3}{2}}(x)}{21a\sqrt{a\sin^3(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.62

$$-\frac{2\sin^2(x)\left(3\cot(x)+5\cos(x)\sin(x)+5F\left(\frac{1}{4}(\pi-2x)\middle|\,2\right)\sin^{\frac{5}{2}}(x)\right)}{21\left(a\sin^3(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(-3/2),x]
[Out] (-2*Sin[x]^2*(3*Cot[x] + 5*Cos[x]*Sin[x] + 5*EllipticF[(Pi - 2*x)/4, 2]*Sin
[x]^(5/2)))/(21*(a*Sin[x]^3)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 372, normalized size = 4.83

method	result
default	$-\frac{(\cos(x)+1)^2(-1+\cos(x))^2\left(5i(\cos^3(x))\sin(x)\sqrt{2}\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{-\frac{i\cos(x)-\sin(x)-i}{\sin(x)}}\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\ \text{Ellipti}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*(cos(x)+1)^2*(-1+cos(x))^2*(5*I*cos(x)^3*sin(x)*2^(1/2)*((I*cos(x)+si
n(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/s
in(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+5*I*
cos(x)^2*sin(x)*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(
x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(
x)-I)/sin(x))^(1/2),1/2*2^(1/2))-5*I*cos(x)*sin(x)*2^(1/2)*((I*cos(x)+sin(x
)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(
x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-5*I*((I
*cos(x)+sin(x)-I)/sin(x))^(1/2)*2^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)
*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2)
,1/2*2^(1/2))*sin(x)-10*cos(x)^3+16*cos(x))/(a*sin(x)^3)^(3/2)/sin(x)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 139, normalized size = 1.81

$$\frac{5\left(\sqrt{2}\cos\left(x\right)^{4}-2\sqrt{2}\cos\left(x\right)^{2}+\sqrt{2}\right)\sqrt{-ia}\sin\left(x\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(x\right)+i\sin\left(x\right))+5\left(\sqrt{2}\cos\left(x\right)^{4}-2\sqrt{2}\cos\left(x\right)^{2}+\sqrt{2}\right)\sqrt{ia}\sin\left(x\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(x\right)-i\sin\left(x\right))+2\left(5\cos\left(x\right)^{3}-8\cos\left(x\right)\right)\sqrt{-\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)}}{21\left(a^{2}\cos\left(x\right)^{4}-2a^{2}\cos\left(x\right)^{2}+a^{2}\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(5*(sqrt(2)*cos(x)^4 - 2*sqrt(2)*cos(x)^2 + sqrt(2))*sqrt(-I*a)*sin(x)
*weierstrassPInverse(4, 0, cos(x) + I*sin(x)) + 5*(sqrt(2)*cos(x)^4 - 2*sqr
t(2)*cos(x)^2 + sqrt(2))*sqrt(I*a)*sin(x)*weierstrassPInverse(4, 0, cos(x)
- I*sin(x)) + 2*(5*cos(x)^3 - 8*cos(x))*sqrt(-(a*cos(x)^2 - a)*sin(x)))/((a
^2*cos(x)^4 - 2*a^2*cos(x)^2 + a^2)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin^{3}\left(x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**3)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(x)^3)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\sin\left(x\right)^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*sin(x)^3)^(3/2), x)
```

3.12 $\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}}-\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}}-\frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\sin^3(x)}}+\frac{154E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{195a^2\sqrt{a\sin^3(x)}}$$

```
[Out] -154/585*cot(x)/a^2/(a*sin(x)^3)^(1/2)-22/117*cot(x)*csc(x)^2/a^2/(a*sin(x)
^3)^(1/2)-2/13*cot(x)*csc(x)^4/a^2/(a*sin(x)^3)^(1/2)-154/195*cos(x)*sin(x)
/a^2/(a*sin(x)^3)^(1/2)+154/195*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*Pi+1/2*
x)*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*sin(x)^(3/2)/a^2/(a*sin(x)^3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2716, 2719}

$$-\frac{154\sin(x)\cos(x)}{195a^2\sqrt{a\sin^3(x)}}-\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}}+\frac{154\sin^{\frac{3}{2}}(x)E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}}-\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(-5/2),x]
```

```
[Out] (-154*Cot[x])/(585*a^2*Sqrt[a*Sin[x]^3]) - (22*Cot[x]*Csc[x]^2)/(117*a^2*Sq
rt[a*Sin[x]^3]) - (2*Cot[x]*Csc[x]^4)/(13*a^2*Sqrt[a*Sin[x]^3]) - (154*Cos[
x]*Sin[x])/(195*a^2*Sqrt[a*Sin[x]^3]) + (154*EllipticE[Pi/4 - x/2, 2]*Sin[x
]^(3/2))/(195*a^2*Sqrt[a*Sin[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx = \frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} + \frac{\left(11\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} + \frac{\left(77\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} + \frac{\left(77\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{3}{2}}(x)}}{195a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} - \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\sin^3(x)}} - $$

$$= -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} - \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\sin^3(x)}} + $$

Mathematica [A]
time = 0.16, size = 60, normalized size = 0.49

$$-\frac{2\left(\cot(x)\left(77+55\csc^2(x)+45\csc^4(x)\right)+231\cos(x)\sin(x)-231E\left(\frac{1}{4}(\pi-2x)\middle|2\right)\sin^{\frac{3}{2}}(x)\right)}{585a^2\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(-5/2),x]
```

```
[Out] (-2*(Cot[x]*(77 + 55*Csc[x]^2 + 45*Csc[x]^4) + 231*Cos[x]*Sin[x] - 231*Elli
pticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2)))/(585*a^2*Sqrt[a*Sin[x]^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 1349, normalized size = 10.97

method	result	size
default	Expression too large to display	1349

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/585*(462*cos(x)^7*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)
-sin(x)-I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2
^(1/2))*(-I*(-1+cos(x))/sin(x))^(1/2)-231*cos(x)^7*2^(1/2)*((I*cos(x)+sin(x
)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(
x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+462*cos
(x)^6*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(
x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*(-I*(-1
+cos(x))/sin(x))^(1/2)-231*cos(x)^6*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1
/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*Ellip
ticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-1386*cos(x)^5*2^(1/2)*
((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*Elli
pticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(x))/sin(x
))^(1/2)+693*cos(x)^5*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x
)-sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x
)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-1386*cos(x)^4*2^(1/2)*((I*cos(x)+sin
(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*EllipticE(((I*cos(
x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(x))/sin(x))^(1/2)+693*c
os(x)^4*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/si
n(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/si
n(x))^(1/2),1/2*2^(1/2))+1386*cos(x)^3*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))
^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/s
in(x))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(x))/sin(x))^(1/2)-693*cos(x)^3*2^(1/2
)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*(-
I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/
2*2^(1/2))+1386*cos(x)^2*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*co
s(x)-sin(x)-I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1
/2*2^(1/2))*(-I*(-1+cos(x))/sin(x))^(1/2)-693*cos(x)^2*2^(1/2)*((I*cos(x)+s
in(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/
sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-462
*cos(x)^6-462*cos(x)*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)
-sin(x)-I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2
^(1/2))*(-I*(-1+cos(x))/sin(x))^(1/2)+231*cos(x)*2^(1/2)*((I*cos(x)+sin(x)-
I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x)
)^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+154*cos(x
)^5-462*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/si
n(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*(-I*(
-1+cos(x))/sin(x))^(1/2)+231*2^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(
```

```
I*cos(x)-sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((
I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+1386*cos(x)^4-418*cos(x)^3-13
86*cos(x)^2+354*cos(x)+462)*sin(x)/(a*sin(x)^3)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 209, normalized size = 1.70

$$-\frac{231 \left(i \sqrt{2} \cos \left(x\right)^{8} - 4 i \sqrt{2} \cos \left(x\right)^{6} + 6 i \sqrt{2} \cos \left(x\right)^{4} - 4 i \sqrt{2} \cos \left(x\right)^{2} + i \sqrt{2}\right) \sqrt{-i a} \,\mathrm{weierstrassZeta}(4, 0, \mathrm{weierstrassPInverse}(4, 0, \cos \left(x\right) + i \sin \left(x\right))) + 231 \left(-i \sqrt{2} \cos \left(x\right)^{8} + 4 i \sqrt{2} \cos \left(x\right)^{6} - 6 i \sqrt{2} \cos \left(x\right)^{4} + 4 i \sqrt{2} \cos \left(x\right)^{2} - i \sqrt{2}\right) \sqrt{i a} \,\mathrm{weierstrassZeta}(4, 0, \mathrm{weierstrassPInverse}(4, 0, \cos \left(x\right) - i \sin \left(x\right))) - 2 \left(231 \cos \left(x\right)^{7} - 770 \cos \left(x\right)^{5} + 902 \cos \left(x\right)^{3} - 408 \cos \left(x\right)\right) \sqrt{-\left(a \cos \left(x\right)^{2} - a\right) \sin \left(x\right)}}{585 \left(a^{3} \cos \left(x\right)^{8} - 4 a^{3} \cos \left(x\right)^{6} + 6 a^{3} \cos \left(x\right)^{4} - 4 a^{3} \cos \left(x\right)^{2} + a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/585*(231*(I*sqrt(2)*cos(x)^8 - 4*I*sqrt(2)*cos(x)^6 + 6*I*sqrt(2)*cos(x)
^4 - 4*I*sqrt(2)*cos(x)^2 + I*sqrt(2))*sqrt(-I*a)*weierstrassZeta(4, 0, wei
erstrassPInverse(4, 0, cos(x) + I*sin(x))) + 231*(-I*sqrt(2)*cos(x)^8 + 4*I
*sqrt(2)*cos(x)^6 - 6*I*sqrt(2)*cos(x)^4 + 4*I*sqrt(2)*cos(x)^2 - I*sqrt(2)
)*sqrt(I*a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(x) - I*sin(
x))) - 2*(231*cos(x)^7 - 770*cos(x)^5 + 902*cos(x)^3 - 408*cos(x))*sqrt(-(a
*cos(x)^2 - a)*sin(x)))/(a^3*cos(x)^8 - 4*a^3*cos(x)^6 + 6*a^3*cos(x)^4 - 4
*a^3*cos(x)^2 + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \sin^{3}{\left(x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**3)**(5/2),x)
```

```
[Out] Integral((a*sin(x)**3)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(x)^3)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\sin\left(x\right)^3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(5/2),x)
[Out] int(1/(a*sin(x)^3)^(5/2), x)
```

3.13 $\int \left(a\sin^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=132

$$-\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)}+\frac{63}{256}a^2x\csc^2(x)\sqrt{a\sin^4(x)}-\frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\cos(x)\sin^3$$

```
[Out] -63/256*a^2*cot(x)*(a*sin(x)^4)^(1/2)+63/256*a^2*x*csc(x)^2*(a*sin(x)^4)^(1
/2)-21/128*a^2*cos(x)*sin(x)*(a*sin(x)^4)^(1/2)-21/160*a^2*cos(x)*sin(x)^3*
(a*sin(x)^4)^(1/2)-9/80*a^2*cos(x)*sin(x)^5*(a*sin(x)^4)^(1/2)-1/10*a^2*cos
(x)*sin(x)^7*(a*sin(x)^4)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$-\frac{21}{128}a^2\sin(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{1}{10}a^2\sin^7(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{9}{80}a^2\sin^5(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\sin^3(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)}+\frac{63}{256}a^2x\csc^2(x)\sqrt{a\sin^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(5/2),x]
```

```
[Out] (-63*a^2*Cot[x]*Sqrt[a*Sin[x]^4])/256 + (63*a^2*x*Csc[x]^2*Sqrt[a*Sin[x]^4]
)/256 - (21*a^2*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^4])/128 - (21*a^2*Cos[x]*Sin[x]
^3*Sqrt[a*Sin[x]^4])/160 - (9*a^2*Cos[x]*Sin[x]^5*Sqrt[a*Sin[x]^4])/80 - (a
^2*Cos[x]*Sin[x]^7*Sqrt[a*Sin[x]^4])/10
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \left(a\sin^4(x)\right)^{5/2}\,dx &= \left(a^2\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^{10}(x)\,dx \\
&= -\frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{a\sin^4(x)}+\frac{1}{10}\left(9a^2\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^8(x)\,dx \\
&= -\frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{a\sin^4(x)}-\frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{a\sin^4(x)}+\frac{1}{80}\Big(63a^2\csc^2(\\
&= -\frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4(x)}-\frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{a\sin^4(x)}-\frac{1}{10}a^2\cos(x)\,\mathrm{s} \\
&= -\frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4(x)}-\frac{9}{80}a^2\cos(x)\,\mathrm{s} \\
&= -\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)}-\frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\cos(x)\sin^3(x) \\
&= -\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)}+\frac{63}{256}a^2x\csc^2(x)\sqrt{a\sin^4(x)}-\frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\,\mathrm{s}}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 53, normalized size = 0.40

$$\frac{a\csc^6(x)\left(a\sin^4(x)\right)^{3/2}\left(2520x-2100\sin(2x)+600\sin(4x)-150\sin(6x)+25\sin(8x)-2\sin(10x)\right)}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(5/2),x]
```

```
[Out] (a*Csc[x]^6*(a*Sin[x]^4)^(3/2)*(2520*x - 2100*Sin[2*x] + 600*Sin[4*x] - 150
*Sin[6*x] + 25*Sin[8*x] - 2*Sin[10*x]))/10240
```

Maple [A]
time = 0.38, size = 63, normalized size = 0.48

method	result
default	$-\frac{\left(a\left(1-\left(\cos^2(x)\right)\right)^2\right)^{\frac{5}{2}}\left(128\left(\cos^9(x)\right)\sin(x)-656\left(\cos^7(x)\right)\sin(x)+1368\left(\cos^5(x)\right)\sin(x)-1490\left(\cos^3(x)\right)\sin(x)+965\sin(x)\cos(x)-\right.}{1280\sin(x)^{10}}$
risch	$-\frac{63a^2\mathrm{e}^{2ix}\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}\,x}{256\left(\mathrm{e}^{2ix}-1\right)^2}-\frac{ia^2\mathrm{e}^{12ix}\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}}{10240\left(\mathrm{e}^{2ix}-1\right)^2}+\frac{5ia^2\mathrm{e}^{10ix}\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}}{4096\left(\mathrm{e}^{2ix}-1\right)^2}-1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1280*(a*(1-cos(x)^2)^2)^(5/2)*(128*cos(x)^9*sin(x)-656*cos(x)^7*sin(x)+1
368*cos(x)^5*sin(x)-1490*cos(x)^3*sin(x)+965*sin(x)*cos(x)-315*x)/sin(x)^10
```

Maxima [A]
time = 0.57, size = 85, normalized size = 0.64

$$\frac{63}{256}\,a^{\frac{5}{2}}x-\frac{965\,a^{\frac{5}{2}}\tan\left(x\right)^{9}+2370\,a^{\frac{5}{2}}\tan\left(x\right)^{7}+2688\,a^{\frac{5}{2}}\tan\left(x\right)^{5}+1470\,a^{\frac{5}{2}}\tan\left(x\right)^{3}+315\,a^{\frac{5}{2}}\tan\left(x\right)}{1280\left(\tan\left(x\right)^{10}+5\,\tan\left(x\right)^{8}+10\,\tan\left(x\right)^{6}+10\,\tan\left(x\right)^{4}+5\,\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 63/256*a^(5/2)*x - 1/1280*(965*a^(5/2)*tan(x)^9 + 2370*a^(5/2)*tan(x)^7 + 2
688*a^(5/2)*tan(x)^5 + 1470*a^(5/2)*tan(x)^3 + 315*a^(5/2)*tan(x))/(tan(x)^
10 + 5*tan(x)^8 + 10*tan(x)^6 + 10*tan(x)^4 + 5*tan(x)^2 + 1)
```

Fricas [A]
time = 0.39, size = 82, normalized size = 0.62

$$-\frac{\sqrt{a\cos\left(x\right)^{4}-2\,a\cos\left(x\right)^{2}+a}\,\left(315\,a^{2}x-\left(128\,a^{2}\cos\left(x\right)^{9}-656\,a^{2}\cos\left(x\right)^{7}+1368\,a^{2}\cos\left(x\right)^{5}-1490\,a^{2}\cos\left(x\right)^{3}+965\,a^{2}\cos\left(x\right)\right)\sin\left(x\right)\right)}{1280\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/1280*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(315*a^2*x - (128*a^2*cos(x)^9
- 656*a^2*cos(x)^7 + 1368*a^2*cos(x)^5 - 1490*a^2*cos(x)^3 + 965*a^2*cos(x)
)*sin(x))/(cos(x)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin^{4}\left(x\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sin(x)**4)**(5/2), x)
```

Giac [A]
time = 0.45, size = 57, normalized size = 0.43

$$\frac{1}{10240}\left(2520\,a^{2}x-2\,a^{2}\sin\left(10\,x\right)+25\,a^{2}\sin\left(8\,x\right)-150\,a^{2}\sin\left(6\,x\right)+600\,a^{2}\sin\left(4\,x\right)-2100\,a^{2}\sin\left(2\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/10240*(2520*a^2*x - 2*a^2*sin(10*x) + 25*a^2*sin(8*x) - 150*a^2*sin(6*x)
+ 600*a^2*sin(4*x) - 2100*a^2*sin(2*x))*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sin\left(x\right)^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(5/2),x)
```

```
[Out] int((a*sin(x)^4)^(5/2), x)
```

3.14 $\int \left(a\sin^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)}+\frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)}-\frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)}$$

```
[Out] -5/16*a*cot(x)*(a*sin(x)^4)^(1/2)+5/16*a*x*csc(x)^2*(a*sin(x)^4)^(1/2)-5/24
*a*cos(x)*sin(x)*(a*sin(x)^4)^(1/2)-1/6*a*cos(x)*sin(x)^3*(a*sin(x)^4)^(1/2
)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$-\frac{5}{24}a\sin(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{1}{6}a\sin^3(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)}+\frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(3/2),x]
```

```
[Out] (-5*a*Cot[x]*Sqrt[a*Sin[x]^4])/16 + (5*a*x*Csc[x]^2*Sqrt[a*Sin[x]^4])/16 -
(5*a*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^4])/24 - (a*Cos[x]*Sin[x]^3*Sqrt[a*Sin[x]^
4])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(a\sin^4(x)\right)^{3/2}\,dx &= \left(a\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^6(x)\,dx\\ &= -\frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} + \frac{1}{6}\left(5a\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^4(x)\,dx\\ &= -\frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} + \frac{1}{8}\Big(5a\csc^2(x)\sqrt{a\,\mathrm{s}}\\ &= -\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)} - \frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin}\\ &= -\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)} + \frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)} - \frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 38, normalized size = 0.49

$$-\frac{1}{192}\csc^6(x)\left(a\sin^4(x)\right)^{3/2}(-60x+45\sin(2x)-9\sin(4x)+\sin(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(3/2),x]
```

```
[Out] -1/192*(Csc[x]^6*(a*Sin[x]^4)^(3/2)*(-60*x + 45*Sin[2*x] - 9*Sin[4*x] + Sin
[6*x]))
```

Maple [A]
time = 0.23, size = 47, normalized size = 0.60

method	result
default	$-\frac{\left(a\left(1-\left(\cos^2(x)\right)\right)^2\right)^{\frac{3}{2}}\left(8\left(\cos^5(x)\right)\sin(x)-26\left(\cos^3(x)\right)\sin(x)+33\sin(x)\cos(x)-15x\right)}{48\sin(x)^6}$
risch	$-\frac{5a\,\mathrm{e}^{2ix}\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}\,x}{16\left(\mathrm{e}^{2ix}-1\right)^2}-\frac{ia\,\mathrm{e}^{8ix}\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}}{384\left(\mathrm{e}^{2ix}-1\right)^2}+\frac{3ia\,\mathrm{e}^{6ix}\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}}{128\left(\mathrm{e}^{2ix}-1\right)^2}-\frac{15ia\,\mathrm{e}^{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(a*(1-cos(x)^2)^2)^(3/2)*(8*cos(x)^5*sin(x)-26*cos(x)^3*sin(x)+33*sin
(x)*cos(x)-15*x)/sin(x)^6
```

Maxima [A]
time = 0.51, size = 55, normalized size = 0.71

$$\frac{5}{16}\,a^{\frac{3}{2}}x-\frac{33\,a^{\frac{3}{2}}\tan\left(x\right)^{5}+40\,a^{\frac{3}{2}}\tan\left(x\right)^{3}+15\,a^{\frac{3}{2}}\tan\left(x\right)}{48\,\left(\tan\left(x\right)^{6}+3\,\tan\left(x\right)^{4}+3\,\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^(3/2)*x - 1/48*(33*a^(3/2)*tan(x)^5 + 40*a^(3/2)*tan(x)^3 + 15*a^(3/
2)*tan(x))/(tan(x)^6 + 3*tan(x)^4 + 3*tan(x)^2 + 1)
```

Fricas [A]
time = 0.38, size = 56, normalized size = 0.72

$$-\frac{\sqrt{a\cos\left(x\right)^{4}-2\,a\cos\left(x\right)^{2}+a}\,\left(15\,ax-\left(8\,a\cos\left(x\right)^{5}-26\,a\cos\left(x\right)^{3}+33\,a\cos\left(x\right)\right)\sin\left(x\right)\right)}{48\,\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(15*a*x - (8*a*cos(x)^5 - 26*a*co
s(x)^3 + 33*a*cos(x))*sin(x))/(cos(x)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin^{4}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**4)**(3/2), x)
```

Giac [A]
time = 0.42, size = 27, normalized size = 0.35

$$\frac{1}{192}\,a^{\frac{3}{2}}(60\,x-\sin\left(6\,x\right)+9\,\sin\left(4\,x\right)-45\,\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*a^(3/2)*(60*x - sin(6*x) + 9*sin(4*x) - 45*sin(2*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(a\,{\sin\left(x\right)}^4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(3/2),x)
```

```
[Out] int((a*sin(x)^4)^(3/2), x)
```

3.15 $\int \sqrt{a \sin^4(x)} \, dx$

Optimal. Leaf size=36

$$-\frac{1}{2} \cot(x) \sqrt{a \sin^4(x)} + \frac{1}{2} x \csc^2(x) \sqrt{a \sin^4(x)}$$

```
[Out] -1/2*cot(x)*(a*sin(x)^4)^(1/2)+1/2*x*csc(x)^2*(a*sin(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$\frac{1}{2} x \csc^2(x) \sqrt{a \sin^4(x)} - \frac{1}{2} \cot(x) \sqrt{a \sin^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[x]^4],x]
```

```
[Out] -1/2*(Cot[x]*Sqrt[a*Sin[x]^4]) + (x*Csc[x]^2*Sqrt[a*Sin[x]^4])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a \sin^4(x)}\, dx &= \left(\csc^2(x)\sqrt{a \sin^4(x)}\right)\int \sin^2(x)\, dx \\ &= -\frac{1}{2}\cot(x)\sqrt{a\sin^4(x)} + \frac{1}{2}\left(\csc^2(x)\sqrt{a\sin^4(x)}\right)\int 1\, dx \\ &= -\frac{1}{2}\cot(x)\sqrt{a\sin^4(x)} + \frac{1}{2}x\csc^2(x)\sqrt{a\sin^4(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.69

$$\frac{1}{2}\csc(x)(-\cos(x) + x\csc(x))\sqrt{a\sin^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[x]^4],x]
```

```
[Out] (Csc[x]*(-Cos[x] + x*Csc[x])*Sqrt[a*Sin[x]^4])/2
```

Maple [A]
time = 0.20, size = 33, normalized size = 0.92

method	result	size
default	$-\frac{\sqrt{a\left(1-\left(\cos^2\left(x\right)\right)\right)^2}\,\left(\sin(x)\cos(x)-x\right)\sqrt{16}}{8\sin(x)^2}$	33
risch	$-\frac{\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}\,\mathrm{e}^{2ix}x}{2\left(\mathrm{e}^{2ix}-1\right)^2} - \frac{i\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}\,\mathrm{e}^{4ix}}{8\left(\mathrm{e}^{2ix}-1\right)^2} + \frac{i\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}}{8\left(\mathrm{e}^{2ix}-1\right)^2}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(a*(1-cos(x)^2)^2)^(1/2)*(sin(x)*cos(x)-x)/sin(x)^2*16^(1/2)
```

Maxima [A]
time = 0.53, size = 22, normalized size = 0.61

$$\frac{1}{2}\sqrt{a}\,x - \frac{\sqrt{a}\,\tan\left(x\right)}{2\left(\tan\left(x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(1/2),x, algorithm="maxima")
```

[Out] 1/2*sqrt(a)*x - 1/2*sqrt(a)*tan(x)/(tan(x)^2 + 1)

Fricas [A]
time = 0.41, size = 36, normalized size = 1.00

$$\frac{\sqrt{a \cos{(x)}^{4} - 2\, a \cos{(x)}^{2} + a}\, (\cos{(x)} \sin{(x)} - x)}{2\, \left(\cos{(x)}^{2} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(x)^4)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(cos(x)*sin(x) - x)/(cos(x)^2 - 1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin^{4}{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(x)**4)**(1/2),x)

[Out] Integral(sqrt(a*sin(x)**4), x)

Giac [A]
time = 0.43, size = 15, normalized size = 0.42

$$\frac{1}{4}\, \sqrt{a}\, (2\, x - \sin{(2\, x)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(x)^4)^(1/2),x, algorithm="giac")

[Out] 1/4*sqrt(a)*(2*x - sin(2*x))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a \sin{(x)}^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sin(x)^4)^(1/2),x)

[Out] int((a*sin(x)^4)^(1/2), x)

3.16 $\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx$

Optimal. Leaf size=16

$$-\frac{\cos(x)\sin(x)}{\sqrt{a\sin^4(x)}}$$

```
[Out] -cos(x)*sin(x)/(a*sin(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 3852, 8}

$$-\frac{\sin(x)\cos(x)}{\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sin[x]^4],x]
```

```
[Out] -((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = \frac{\sin^2(x)\int \csc^2(x)\,dx}{\sqrt{a\sin^4(x)}}$$
$$= -\frac{\sin^2(x)\text{Subst}(\int 1\,dx, x, \cot(x))}{\sqrt{a\sin^4(x)}}$$
$$= -\frac{\cos(x)\sin(x)}{\sqrt{a\sin^4(x)}}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{\cos(x)\sin(x)}{\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sin[x]^4],x]
[Out] -((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

Maple [A]
time = 0.17, size = 15, normalized size = 0.94

method	result	size
default	$-\frac{\cos(x)\sin(x)}{\sqrt{a\left(\sin^4(x)\right)}}$	15
risch	$\frac{2i\left(1-e^{-2ix}\right)}{\sqrt{a\left(e^{2ix}-1\right)^4 e^{-4ix}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -cos(x)*sin(x)/(a*sin(x)^4)^(1/2)
```

Maxima [A]
time = 0.59, size = 9, normalized size = 0.56

$$-\frac{1}{\sqrt{a}\,\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(a)*tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.39, size = 36, normalized size = 2.25

$$\frac{\sqrt{a\cos\left(x\right)^{4}-2\,a\cos\left(x\right)^{2}+a}\,\cos\left(x\right)}{\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*cos(x)/((a*cos(x)^2 - a)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\sin^{4}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(x)**4), x)
```

Giac [A]
time = 0.52, size = 9, normalized size = 0.56

$$-\frac{1}{\sqrt{a}\,\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(a)*tan(x))
```

Mupad [B]
time = 13.71, size = 7, normalized size = 0.44

$$-\frac{\cot(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(1/2),x)
```

```
[Out] -cot(x)/a^(1/2)
```

3.17 $\int \frac{1}{(a\sin^4(x))^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}}-\frac{\cos(x)\sin(x)}{a\sqrt{a\sin^4(x)}}$$

```
[Out] -2/3*cos(x)^2*cot(x)/a/(a*sin(x)^4)^(1/2)-1/5*cos(x)^2*cot(x)^3/a/(a*sin(x)
^4)^(1/2)-cos(x)*sin(x)/a/(a*sin(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3286, 3852}

$$-\frac{\sin(x)\cos(x)}{a\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}}-\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(-3/2),x]
```

```
[Out] (-2*Cos[x]^2*Cot[x])/(3*a*Sqrt[a*Sin[x]^4]) - (Cos[x]^2*Cot[x]^3)/(5*a*Sqrt
[a*Sin[x]^4]) - (Cos[x]*Sin[x])/(a*Sqrt[a*Sin[x]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a\sin^4(x))^{3/2}}\,dx = \frac{\sin^2(x)\int \csc^6(x)\,dx}{a\sqrt{a\sin^4(x)}}$$
$$= -\frac{\sin^2(x)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, \cot(x)\right)}{a\sqrt{a\sin^4(x)}}$$
$$= -\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}} - \frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}} - \frac{\cos(x)\sin(x)}{a\sqrt{a\sin^4(x)}}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.50

$$-\frac{\cos(x)\left(8+4\csc^2(x)+3\csc^4(x)\right)\sin^5(x)}{15\left(a\sin^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(-3/2),x]
[Out] -1/15*(Cos[x]*(8 + 4*Csc[x]^2 + 3*Csc[x]^4)*Sin[x]^5)/(a*Sin[x]^4)^(3/2)
```

Maple [A]
time = 0.19, size = 29, normalized size = 0.43

method	result	size
default	$-\frac{\left(8\left(\cos^4(x)\right)-20\left(\cos^2(x)\right)+15\right)\cos(x)\sin(x)}{15\left(a\left(\sin^4(x)\right)\right)^{\frac{3}{2}}}$	29
risch	$\frac{16i(-5+11\cos(2x)+9i\sin(2x))}{15a(e^{2ix}-1)^3\sqrt{a\left(e^{2ix}-1\right)^4 e^{-4ix}}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(8*cos(x)^4-20*cos(x)^2+15)*cos(x)*sin(x)/(a*sin(x)^4)^(3/2)
```

Maxima [A]
time = 0.49, size = 23, normalized size = 0.34

$$-\frac{15\tan\left(x\right)^4+10\tan\left(x\right)^2+3}{15\,a^{\frac{3}{2}}\tan\left(x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="maxima")
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/(a^(3/2)*tan(x)^5)
```

Fricas [A]
time = 0.37, size = 74, normalized size = 1.09

$$\frac{\sqrt{a\cos(x)^4 - 2a\cos(x)^2 + a}\left(8\cos(x)^5 - 20\cos(x)^3 + 15\cos(x)\right)}{15\left(a^2\cos(x)^6 - 3a^2\cos(x)^4 + 3a^2\cos(x)^2 - a^2\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="fricas")
[Out] 1/15*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos
(x))/((a^2*cos(x)^6 - 3*a^2*cos(x)^4 + 3*a^2*cos(x)^2 - a^2)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**4)**(3/2),x)
[Out] Integral((a*sin(x)**4)**(-3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 14.28, size = 44, normalized size = 0.65

$$\frac{\frac{8\mathrm{i}}{15\,a^{3/2}} - \frac{4\left(2\sin(2x)^3 - 9\sin(2x) + 3\sin(4x) + 2\mathrm{i}\right)}{15\,a^{3/2}}}{\left(\cos(2x) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(3/2),x)
[Out] (8i/(15*a^(3/2)) - (4*(3*sin(4*x) - 9*sin(2*x) + 2*sin(2*x)^3 + 2i))/(15*a^
(3/2)))/(cos(2*x) - 1)^3
```

3.18 $\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}}-\frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}}-\frac{\cos(x)\sin(x)}{a^2\sqrt{a\sin^4(x)}}$$

```
[Out] -4/3*cos(x)^2*cot(x)/a^2/(a*sin(x)^4)^(1/2)-6/5*cos(x)^2*cot(x)^3/a^2/(a*si
n(x)^4)^(1/2)-4/7*cos(x)^2*cot(x)^5/a^2/(a*sin(x)^4)^(1/2)-1/9*cos(x)^2*cot
(x)^7/a^2/(a*sin(x)^4)^(1/2)-cos(x)*sin(x)/a^2/(a*sin(x)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3852}

$$-\frac{\sin(x)\cos(x)}{a^2\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}}-\frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(-5/2),x]
```

```
[Out] (-4*Cos[x]^2*Cot[x])/(3*a^2*Sqrt[a*Sin[x]^4]) - (6*Cos[x]^2*Cot[x]^3)/(5*a^
2*Sqrt[a*Sin[x]^4]) - (4*Cos[x]^2*Cot[x]^5)/(7*a^2*Sqrt[a*Sin[x]^4]) - (Cos
[x]^2*Cot[x]^7)/(9*a^2*Sqrt[a*Sin[x]^4]) - (Cos[x]*Sin[x])/(a^2*Sqrt[a*Sin[
x]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a\sin^4(x))^{5/2}}\,dx = \frac{\sin^2(x)\int \csc^{10}(x)\,dx}{a^2\sqrt{a\sin^4(x)}}$$
$$= -\frac{\sin^2(x)\text{Subst}\left(\int (1+4x^2+6x^4+4x^6+x^8)\,dx, x, \cot(x)\right)}{a^2\sqrt{a\sin^4(x)}}$$
$$= -\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}} - \frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}} - \frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}} - \frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}} - \frac{\cos(}{a^2\sqrt{}}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 0.40

$$-\frac{\cos(x)\,(128+64\csc^2(x)+48\csc^4(x)+40\csc^6(x)+35\csc^8(x))\sin(x)}{315a^2\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(-5/2),x]
[Out] -1/315*(Cos[x]*(128 + 64*Csc[x]^2 + 48*Csc[x]^4 + 40*Csc[x]^6 + 35*Csc[x]^8
)*Sin[x])/(a^2*Sqrt[a*Sin[x]^4])
```

Maple [A]
time = 0.24, size = 41, normalized size = 0.35

method	result	size
default	$-\frac{(128(\cos^8(x))-576(\cos^6(x))+1008(\cos^4(x))-840(\cos^2(x))+315)\cos(x)\sin(x)}{315(a(\sin^4(x)))^{\frac{5}{2}}}$	41
risch	$\frac{256i(126\,e^{6ix}-84\,e^{4ix}-9+37\cos(2x)+35i\sin(2x))}{315a^2(e^{2ix}-1)^7\sqrt{a\,(e^{2ix}-1)^4\,e^{-4ix}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/315*(128*cos(x)^8-576*cos(x)^6+1008*cos(x)^4-840*cos(x)^2+315)*cos(x)*si
n(x)/(a*sin(x)^4)^(5/2)
```

Maxima [A]
time = 0.51, size = 35, normalized size = 0.30

$$-\frac{315\,\tan(x)^8+420\,\tan(x)^6+378\,\tan(x)^4+180\,\tan(x)^2+35}{315\,a^{\frac{5}{2}}\tan(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/315*(315*tan(x)^8 + 420*tan(x)^6 + 378*tan(x)^4 + 180*tan(x)^2 + 35)/(a^
(5/2)*tan(x)^9)
```

Fricas [A]
time = 0.37, size = 104, normalized size = 0.88

$$\frac{\left(128 \cos\left(x\right)^{9} - 576 \cos\left(x\right)^{7} + 1008 \cos\left(x\right)^{5} - 840 \cos\left(x\right)^{3} + 315 \cos\left(x\right)\right) \sqrt{a \cos\left(x\right)^{4} - 2 \, a \cos\left(x\right)^{2} + a}}{315 \, \left(a^{3} \cos\left(x\right)^{10} - 5 \, a^{3} \cos\left(x\right)^{8} + 10 \, a^{3} \cos\left(x\right)^{6} - 10 \, a^{3} \cos\left(x\right)^{4} + 5 \, a^{3} \cos\left(x\right)^{2} - a^{3}\right) \sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(128*cos(x)^9 - 576*cos(x)^7 + 1008*cos(x)^5 - 840*cos(x)^3 + 315*cos
(x))*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)/((a^3*cos(x)^10 - 5*a^3*cos(x)^8 +
 10*a^3*cos(x)^6 - 10*a^3*cos(x)^4 + 5*a^3*cos(x)^2 - a^3)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \sin^{4}\left(x\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sin(x)**4)**(-5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 16.56, size = 117, normalized size = 0.99

$$\frac{256\,\left({\mathrm{e}}^{x\,46{}\mathrm{i}}\,1{}\mathrm{i}-{\mathrm{e}}^{x\,48{}\mathrm{i}}\,9{}\mathrm{i}+{\mathrm{e}}^{x\,50{}\mathrm{i}}\,36{}\mathrm{i}-{\mathrm{e}}^{x\,52{}\mathrm{i}}\,84{}\mathrm{i}+{\mathrm{e}}^{x\,54{}\mathrm{i}}\,126{}\mathrm{i}\right)}{315\,a^{5/2}\,\left({\mathrm{e}}^{x\,46{}\mathrm{i}}-9\,{\mathrm{e}}^{x\,48{}\mathrm{i}}+36\,{\mathrm{e}}^{x\,50{}\mathrm{i}}-84\,{\mathrm{e}}^{x\,52{}\mathrm{i}}+126\,{\mathrm{e}}^{x\,54{}\mathrm{i}}-126\,{\mathrm{e}}^{x\,56{}\mathrm{i}}+84\,{\mathrm{e}}^{x\,58{}\mathrm{i}}-36\,{\mathrm{e}}^{x\,60{}\mathrm{i}}+9\,{\mathrm{e}}^{x\,62{}\mathrm{i}}-{\mathrm{e}}^{x\,64{}\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(5/2),x)
```

```
[Out] (256*(exp(x*46i)*1i - exp(x*48i)*9i + exp(x*50i)*36i - exp(x*52i)*84i + exp
(x*54i)*126i))/(315*a^(5/2)*(exp(x*46i) - 9*exp(x*48i) + 36*exp(x*50i) - 84
*exp(x*52i) + 126*exp(x*54i) - 126*exp(x*56i) + 84*exp(x*58i) - 36*exp(x*60
i) + 9*exp(x*62i) - exp(x*64i)))
```

3.19 $\int (c \sin^m(a+bx))^{5/2}\, dx$

Optimal. Leaf size=89

$$\frac{2c^2 \cos(a+bx)\, {}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+5m);\frac{1}{4}(6+5m);\sin^2(a+bx)\right)\sin^{1+2m}(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+5m)\sqrt{\cos^2(a+bx)}}$$

```
[Out] 2*c^2*cos(b*x+a)*hypergeom([1/2, 1/2+5/4*m],[3/2+5/4*m],sin(b*x+a)^2)*sin(b
*x+a)^(1+2*m)*(c*sin(b*x+a)^m)^(1/2)/b/(2+5*m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$\frac{2c^2 \cos(a+bx)\sin^{2m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\, {}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\sin^2(a+bx)\right)}{b(5m+2)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(5/2),x]
```

```
[Out] (2*c^2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, Sin[a
+ b*x]^2]*Sin[a + b*x]^(1 + 2*m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 5*m)*Sqrt[
Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \left(c^2\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\right)\int \sin^{\frac{5m}{2}}(a+bx)\,dx$$
$$= \frac{2c^2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+5m);\frac{1}{4}(6+5m);\sin^2(a+bx)\right)\sin^{1+2m}(a+bx)\sqrt{c}}{b(2+5m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A]
time = 0.12, size = 74, normalized size = 0.83

$$\frac{2\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+5m);\frac{1}{4}(6+5m);\sin^2(a+bx)\right)(c\sin^m(a+bx))^{5/2}\tan(a+bx)}{b(2+5m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, Si
n[a + b*x]^2]*(c*Sin[a + b*x]^m)^(5/2)*Tan[a + b*x])/(b*(2 + 5*m))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (c(\sin^m(bx+a)))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^m)^(5/2),x)
```

```
[Out] int((c*sin(b*x+a)^m)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**m)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c \sin (a + b\,x)^m)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^m)^(5/2),x)
```

```
[Out] int((c*sin(a + b*x)^m)^(5/2), x)
```

3.20 $\int (c\sin^m(a+bx))^{3/2}\,dx$

Optimal. Leaf size=83

$$\frac{2c\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+3m);\frac{3(2+m)}{4};\sin^2(a+bx)\right)\sin^{1+m}(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+3m)\sqrt{\cos^2(a+bx)}}$$

```
[Out] 2*c*cos(b*x+a)*hypergeom([1/2, 1/2+3/4*m],[3/2+3/4*m],sin(b*x+a)^2)*sin(b*x
+a)^(1+m)*(c*sin(b*x+a)^m)^(1/2)/b/(2+3*m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$\frac{2c\cos(a+bx)\sin^{m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\sin^2(a+bx)\right)}{b(3m+2)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(3/2),x]
```

```
[Out] (2*c*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4, Sin[a
+ b*x]^2]*Sin[a + b*x]^(1 + m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 3*m)*Sqrt[Co
s[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \left(c\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\right)\int \sin^{\frac{3m}{2}}(a+bx)\,dx$$
$$= \frac{2c\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+3m);\frac{3(2+m)}{4};\sin^2(a+bx)\right)\sin^{1+m}(a+bx)\sqrt{c\sin^m(a}{b(2+3m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A]
time = 0.08, size = 72, normalized size = 0.87

$$\frac{2\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+3m);\frac{3(2+m)}{4};\sin^2(a+bx)\right)(c\sin^m(a+bx))^{3/2}\tan(a+bx)}{b(2+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(3/2),x]
[Out] (2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4,
Sin[a + b*x]^2]*(c*Sin[a + b*x]^m)^(3/2)*Tan[a + b*x])/(b*(2 + 3*m))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (c(\sin^m(bx+a)))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^m)^(3/2),x)
[Out] int((c*sin(b*x+a)^m)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a)^m)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin^m\left(a+bx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**m)**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x)**m)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\sin\left(a+b\,x\right)}^m\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^m)^(3/2),x)
```

```
[Out] int((c*sin(a + b*x)^m)^(3/2), x)
```

3.21 $\int \sqrt{c\sin^m(a+bx)}\,dx$

Optimal. Leaf size=74

$$\frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{4};\frac{6+m}{4};\sin^2(a+bx)\right)\sin(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+m)\sqrt{\cos^2(a+bx)}}$$

```
[Out] 2*cos(b*x+a)*hypergeom([1/2, 1/2+1/4*m],[3/2+1/4*m],sin(b*x+a)^2)*sin(b*x+a
)*(c*sin(b*x+a)^m)^(1/2)/b/(2+m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$\frac{2\sin(a+bx)\cos(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\sin^2(a+bx)\right)}{b(m+2)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a + b*x]^2
]*Sin[a + b*x]*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\sin^m(a+bx)}\,dx &= \left(\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\right)\int \sin^{\frac{m}{2}}(a+bx)\,dx\\ &= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{4};\frac{6+m}{4};\sin^2(a+bx)\right)\sin(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+m)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.92

$$\frac{2\sqrt{\cos^2(a+bx)}\ {}_2F_1\left(\frac{1}{2},\frac{2+m}{4};\frac{6+m}{4};\sin^2(a+bx)\right)\sqrt{c\sin^m(a+bx)}\ \tan(a+bx)}{b(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]^m],x]
[Out] (2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a
+ b*x]^2]*Sqrt[c*Sin[a + b*x]^m]*Tan[a + b*x])/(b*(2 + m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(\sin^m\left(bx+a\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^m)^(1/2),x)
[Out] int((c*sin(b*x+a)^m)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a)^m), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\sin^m\left(a+bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x)**m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a)^m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\sin\left(a+b\,x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^m)^(1/2),x)
```

```
[Out] int((c*sin(a + b*x)^m)^(1/2), x)
```

3.22 $\int \frac{1}{\sqrt{c \sin^m(a+bx)}} \, dx$

Optimal. Leaf size=80

$$\frac{2 \cos(a+bx) \, {}_2F_1\left(\frac{1}{2}, \frac{2-m}{4}; \frac{6-m}{4}; \sin^2(a+bx)\right) \sin(a+bx)}{b(2-m)\sqrt{\cos^2(a+bx)} \sqrt{c \sin^m(a+bx)}}$$

```
[Out] 2*cos(b*x+a)*hypergeom([1/2, 1/2-1/4*m],[3/2-1/4*m],sin(b*x+a)^2)*sin(b*x+a
)/b/(2-m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$\frac{2 \sin(a+bx) \cos(a+bx) \, {}_2F_1\left(\frac{1}{2}, \frac{2-m}{4}; \frac{6-m}{4}; \sin^2(a+bx)\right)}{b(2-m)\sqrt{\cos^2(a+bx)} \sqrt{c \sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[a + b*x]^2
]*Sin[a + b*x])/(b*(2 - m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]^m])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{c \sin^m(a+bx)}}\,dx = \frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{m}{2}}(a+bx)\,dx}{\sqrt{c \sin^m(a+bx)}}$$
$$= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right)\sin(a+bx)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c \sin^m(a+bx)}}$$

Mathematica [**A**]
time = 0.05, size = 72, normalized size = 0.90

$$-\frac{2\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right)\tan(a+bx)}{b(-2+m)\sqrt{c \sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Sin[a + b*x]^m],x]
[Out] (-2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[a
 + b*x]^2]*Tan[a + b*x])/(b*(-2 + m)*Sqrt[c*Sin[a + b*x]^m])
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(\sin^m(bx+a)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a)^m)^(1/2),x)
[Out] int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c \sin^m{(a + bx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*sin(a + b*x)**m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,{\sin\left(a+b\,x\right)}^m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x)^m)^(1/2),x)
```

```
[Out] int(1/(c*sin(a + b*x)^m)^(1/2), x)
```

3.23 $\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\sin^2(a+bx)\right)\sin^{1-m}(a+bx)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\,\sqrt{c\sin^m(a+bx)}}$$

```
[Out] 2*cos(b*x+a)*hypergeom([1/2, 1/2-3/4*m],[3/2-3/4*m],sin(b*x+a)^2)*sin(b*x+a
)^(1-m)/b/c/(2-3*m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$\frac{2\cos(a+bx)\sin^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\sin^2(a+bx)\right)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\,\sqrt{c\sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(-3/2),x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4, Sin[a +
b*x]^2]*Sin[a + b*x]^(1 - m))/(b*c*(2 - 3*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Si
n[a + b*x]^m])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{3m}{2}}(a+bx)\,dx}{c\sqrt{c\sin^m(a+bx)}}$$
$$= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\sin^2(a+bx)\right)\sin^{1-m}(a+bx)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

Mathematica [A]
time = 0.07, size = 71, normalized size = 0.80

$$\frac{\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);-\frac{3}{4}(-2+m);\sin^2(a+bx)\right)\tan(a+bx)}{\left(b-\frac{3bm}{2}\right)(c\sin^m(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(-3/2),x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (-3*(-2 + m))/4,
Sin[a + b*x]^2]*Tan[a + b*x])/((b - (3*b*m)/2)*(c*Sin[a + b*x]^m)^(3/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(c(\sin^m(bx+a)))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a)^m)^(3/2),x)
[Out] int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin^m{(a+bx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)**m)**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x)**m)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,{\sin\left(a+b\,x\right)}^m\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x)^m)^(3/2),x)
```

```
[Out] int(1/(c*sin(a + b*x)^m)^(3/2), x)
```

3.24 $\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\sin^2(a+bx)\right)\sin^{1-2m}(a+bx)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\,\sqrt{c\sin^m(a+bx)}}$$

```
[Out] 2*cos(b*x+a)*hypergeom([1/2, 1/2-5/4*m],[3/2-5/4*m],sin(b*x+a)^2)*sin(b*x+a
)^(1-2*m)/b/c^2/(2-5*m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$\frac{2\cos(a+bx)\sin^{1-2m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\sin^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\,\sqrt{c\sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(-5/2),x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin[a + b*
x]^2]*Sin[a + b*x]^(1 - 2*m))/(b*c^2*(2 - 5*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*
Sin[a + b*x]^m])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{5m}{2}}(a+bx)\,dx}{c^2\sqrt{c\sin^m(a+bx)}}$$
$$= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\sin^2(a+bx)\right)\sin^{1-2m}(a+bx)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\,\sqrt{c\sin^m(a+bx)}}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 0.82

$$\frac{\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\sin^2(a+bx)\right)\tan(a+bx)}{\left(b-\frac{5bm}{2}\right)(c\sin^m(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(-5/2),x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin[
a + b*x]^2]*Tan[a + b*x])/((b - (5*b*m)/2)*(c*Sin[a + b*x]^m)^(5/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(c(\sin^m(bx+a)))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a)^m)^(5/2),x)
[Out] int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sin^{m}\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)**m)**(5/2),x)
```

```
[Out] Integral((c*sin(a + b*x)**m)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,{\sin\left(a+b\,x\right)}^m\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x)^m)^(5/2),x)
```

```
[Out] int(1/(c*sin(a + b*x)^m)^(5/2), x)
```

3.25 $\int (b\sin^n(c+dx))^p\,dx$

Optimal. Leaf size=77

$$\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)\sin(c+dx)\,(b\sin^n(c+dx))^p}{d(1+np)\sqrt{\cos^2(c+dx)}}$$

```
[Out] cos(d*x+c)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(d*x+c)^2)*sin(d*x
+c)*(b*sin(d*x+c)^n)^p/d/(n*p+1)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3287, 2722}

$$\frac{\sin(c+dx)\cos(c+dx)\,(b\sin^n(c+dx))^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[c + d*x]^n)^p,x]
```

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(b*Sin[c + d*x]^n)^p)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (b\sin^n(c+dx))^p\,dx &= \left(\sin^{-np}(c+dx)\,(b\sin^n(c+dx))^p\right)\int \sin^{np}(c+dx)\,dx\\ &= \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)\sin(c+dx)\,(b\sin^n(c+dx))^p}{d(1+np)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.92

$$\frac{\sqrt{\cos^2(c+dx)}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)(b\sin^n(c+dx))^p\tan(c+dx)}{d(1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[c + d*x]^n)^p,x]
[Out] (Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[
c + d*x]^2]*(b*Sin[c + d*x]^n)^p*Tan[c + d*x])/(d*(1 + n*p))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (b(\sin^n(dx+c)))^p\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(d*x+c)^n)^p,x)
[Out] int((b*sin(d*x+c)^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n)^p, x)
```

Fricas [F]
time = 0.43, size = 14, normalized size = 0.18

$$\text{integral}((b\sin(dx+c)^n)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sin^n(c+dx))^p\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)**n)**p,x)
```

```
[Out] Integral((b*sin(c + d*x)**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\sin\left(c+d\,x\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(c + d*x)^n)^p,x)
```

```
[Out] int((b*sin(c + d*x)^n)^p, x)
```

3.26 $\int \left(c \sin^2(a+bx)\right)^p \, dx$

Optimal. Leaf size=77

$$\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2p);\frac{1}{2}(3+2p);\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^2(a+bx)\right)^p}{b(1+2p)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+p],[3/2+p],sin(b*x+a)^2)*sin(b*x+a)*(c*sin(b
*x+a)^2)^p/b/(1+2*p)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3286, 2722}

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^2(a+bx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2p+1);\frac{1}{2}(2p+3);\sin^2(a+bx)\right)}{b(2p+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^2)^p,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 2*p)/2, (3 + 2*p)/2, Sin[a + b*x]
^2]*Sin[a + b*x]*(c*Sin[a + b*x]^2)^p)/(b*(1 + 2*p)*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^2(a+bx)\right)^p\,dx &= \left(\sin^{-2p}(a+bx)\left(c\sin^2(a+bx)\right)^p\right)\int \sin^{2p}(a+bx)\,dx\\ &= \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2p);\frac{1}{2}(3+2p);\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^2(a+bx)\right)^p}{b(1+2p)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.79

$$\frac{\sqrt{\cos^2(a+bx)}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}+p;\frac{3}{2}+p;\sin^2(a+bx)\right)\left(c\sin^2(a+bx)\right)^p\tan(a+bx)}{b+2bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^2)^p,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + p, 3/2 + p, Sin[a + b*x]
^2]*(c*Sin[a + b*x]^2)^p*Tan[a + b*x])/(b + 2*b*p)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(c\left(\sin^2\left(bx+a\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^2)^p,x)
[Out] int((c*sin(b*x+a)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^2)^p,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a)^2)^p, x)
```

Fricas [F]
time = 0.41, size = 17, normalized size = 0.22

$$\mathrm{integral}\left(\left(-c\cos\left(bx+a\right)^2+c\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^2)^p,x, algorithm="fricas")
[Out] integral((-c*cos(b*x + a)^2 + c)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin^2\left(a+bx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**2)**p,x)
```

```
[Out] Integral((c*sin(a + b*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^2)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\sin\left(a+b\,x\right)}^2\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^2)^p,x)
```

```
[Out] int((c*sin(a + b*x)^2)^p, x)
```

3.27 $\int (c\sin^3(a+bx))^p\,dx$

Optimal. Leaf size=75

$$\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+3p);\frac{3(1+p)}{2};\sin^2(a+bx)\right)\sin(a+bx)\,(c\sin^3(a+bx))^p}{b(1+3p)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+3/2*p],[3/2+3/2*p],sin(b*x+a)^2)*sin(b*x+a)*
(c*sin(b*x+a)^3)^p/b/(1+3*p)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3286, 2722}

$$\frac{\sin(a+bx)\cos(a+bx)\,(c\sin^3(a+bx))^p\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}(3p+1);\frac{3(p+1)}{2};\sin^2(a+bx)\right)}{b(3p+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^p,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, Sin[a + b*
x]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^3)^p)/(b*(1 + 3*p)*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (c\sin^3(a+bx))^p\,dx = \left(\sin^{-3p}(a+bx)\left(c\sin^3(a+bx)\right)^p\right)\int \sin^{3p}(a+bx)\,dx$$
$$= \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+3p);\frac{3(1+p)}{2};\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^3(a+bx)\right)^p}{b(1+3p)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 0.89

$$\frac{\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+3p);\frac{3(1+p)}{2};\sin^2(a+bx)\right)\left(c\sin^3(a+bx)\right)^p\tan(a+bx)}{b+3bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^p,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, Si
n[a + b*x]^2]*(c*Sin[a + b*x]^3)^p*Tan[a + b*x])/(b + 3*b*p)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(c\left(\sin^3(bx+a)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^p,x)
[Out] int((c*sin(b*x+a)^3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^p,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a)^3)^p, x)
```

Fricas [F]
time = 0.39, size = 26, normalized size = 0.35

$$\text{integral}\left(\left(-\left(c\cos(bx+a)^2-c\right)\sin(bx+a)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^p,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin^3\left(a+bx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**p,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^p,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\sin\left(a+b\,x\right)}^3\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^p,x)
```

```
[Out] int((c*sin(a + b*x)^3)^p, x)
```

3.28 $\int \left(c\sin^4(a+bx)\right)^p\,dx$

Optimal. Leaf size=77

$$\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+4p);\frac{1}{2}(3+4p);\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^4(a+bx)\right)^p}{b(1+4p)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+2*p],[3/2+2*p],sin(b*x+a)^2)*sin(b*x+a)*(c*s
in(b*x+a)^4)^p/b/(1+4*p)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3286, 2722}

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^4(a+bx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(4p+1);\frac{1}{2}(4p+3);\sin^2(a+bx)\right)}{b(4p+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^4)^p,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 4*p)/2, (3 + 4*p)/2, Sin[a + b*x]
^2]*Sin[a + b*x]*(c*Sin[a + b*x]^4)^p)/(b*(1 + 4*p)*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^4(a+bx)\right)^p\,dx &= \left(\sin^{-4p}(a+bx)\left(c\sin^4(a+bx)\right)^p\right)\int \sin^{4p}(a+bx)\,dx\\ &= \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+4p);\frac{1}{2}(3+4p);\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^4(a+bx)\right)^p}{b(1+4p)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 0.84

$$\frac{\sqrt{\cos^2(a+bx)}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}+2p;\frac{3}{2}+2p;\sin^2(a+bx)\right)\left(c\sin^4(a+bx)\right)^p\tan(a+bx)}{b+4bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^4)^p,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + 2*p, 3/2 + 2*p, Sin[a + 
b*x]^2]*(c*Sin[a + b*x]^4)^p*Tan[a + b*x])/(b + 4*b*p)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(c\left(\sin^4\left(bx+a\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^4)^p,x)
[Out] int((c*sin(b*x+a)^4)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^4)^p,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a)^4)^p, x)
```

Fricas [F]
time = 0.40, size = 27, normalized size = 0.35

$$\mathrm{integral}\left(\left(c\cos\left(bx+a\right)^4-2\,c\cos\left(bx+a\right)^2+c\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^4)^p,x, algorithm="fricas")
[Out] integral((c*cos(b*x + a)^4 - 2*c*cos(b*x + a)^2 + c)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin^4\left(a+bx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**4)**p,x)
```

```
[Out] Integral((c*sin(a + b*x)**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^4)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\sin\left(a+b\,x\right)}^4\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^4)^p,x)
```

```
[Out] int((c*sin(a + b*x)^4)^p, x)
```

3.29 $\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx$

Optimal. Leaf size=25

$$-\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

```
[Out] -cot(b*x+a)*(c*sin(b*x+a)^n)^(1/n)/b
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2718}

$$-\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx &= \left(\csc(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}\right)\int \sin(a+bx)\,dx\\ &= -\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (c(\sin^n (bx+a)))^{\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^n)^(1/n),x)
```

```
[Out] int((c*sin(b*x+a)^n)^(1/n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^n)^(1/n), x)
```

Fricas [A]
time = 0.40, size = 16, normalized size = 0.64

$$-\frac{c^{\left(\frac{1}{n}\right)}\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="fricas")
```

```
[Out] -c^(1/n)*cos(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(22) = 44$.

time = 0.38, size = 58, normalized size = 2.32

$$\begin{cases} x(c\sin^n{(a)})^{\frac{1}{n}} & \text{for } b = 0 \\ x(0^n c)^{\frac{1}{n}} & \text{for } a = -bx \vee a = -bx + \pi \\ -\frac{(c\sin^n{(a+bx)})^{\frac{1}{n}}\cos{(a+bx)}}{b\sin{(a+bx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**n)**(1/n),x)
```

```
[Out] Piecewise((x*(c*sin(a)**n)**(1/n), Eq(b, 0)), (x*(0**n*c)**(1/n), Eq(a, -b*
x) | Eq(a, -b*x + pi)), (-(c*sin(a + b*x)**n)**(1/n)*cos(a + b*x)/(b*sin(a
+ b*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(25) = 50.
time = 1.78, size = 384, normalized size = 15.36

$$\frac{|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4n}-\frac{\pi}{4n}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4-2|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4n}-\frac{\pi}{4n}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2+4|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4n}-\frac{\pi}{4n}\right)\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^3-|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4n}-\frac{\pi}{4n}\right)^2-4|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4n}-\frac{\pi}{4n}\right)\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)+2|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2-|c|^{\left(\frac{1}{n}\right)}}{b\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4n}-\frac{\pi}{4n}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+2b\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4n}-\frac{\pi}{4n}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2+b\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+b\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4n}-\frac{\pi}{4n}\right)^2+2b\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="giac")
```

```
[Out] (abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b
*x + 1/2*a)^4 - 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*
pi/n)^2*tan(1/2*b*x + 1/2*a)^2 + 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*p
i*sgn(c)/n - 1/4*pi/n)*tan(1/2*b*x + 1/2*a)^3 - abs(c)^(1/n)*tan(1/2*b*x +
1/2*a)^4 + abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2
 - 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)*tan(1/2
*b*x + 1/2*a) + 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a)^2 - abs(c)^(1/n))/(b*ta
n(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*x + 1/2*a)^4 +
2*b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*x + 1/2*a
)^2 + b*tan(1/2*b*x + 1/2*a)^4 + b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n -
1/4*pi/n)^2 + 2*b*tan(1/2*b*x + 1/2*a)^2 + b)
```

Mupad [B]
time = 13.75, size = 36, normalized size = 1.44

$$-\frac{\sin\left(2\,a+2\,b\,x\right)\,{\left(c\,{\sin\left(a+b\,x\right)}^n\right)}^{1/n}}{2\,b\,{\sin\left(a+b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^n)^(1/n),x)
```

```
[Out] -(sin(2*a + 2*b*x)*(c*sin(a + b*x)^n)^(1/n))/(2*b*sin(a + b*x)^2)
```

3.30 $\int (a(b\sin(c+dx))^p)^n \, dx$

Optimal. Leaf size=79

$$\frac{\cos(c+dx)\, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)\sin(c+dx)\left(a(b\sin(c+dx))^p\right)^n}{d(1+np)\sqrt{\cos^2(c+dx)}}$$

```
[Out] cos(d*x+c)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(d*x+c)^2)*sin(d*x
+c)*(a*(b*sin(d*x+c))^p)^n/d/(n*p+1)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$\frac{\sin(c+dx)\cos(c+dx)\, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)\left(a(b\sin(c+dx))^p\right)^n}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Sin[c + d*x])^p)^n,x]
```

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(a*(b*Sin[c + d*x])^p)^n)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2
])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (a(b\sin(c+dx))^p)^n \, dx = \left((b\sin(c+dx))^{-np}\left(a(b\sin(c+dx))^p\right)^n\right)\int (b\sin(c+dx))^{np}\,dx$$
$$= \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)\sin(c+dx)\left(a(b\sin(c+d\right.}{d(1+np)\sqrt{\cos^2(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 0.92

$$\frac{\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)\left(a(b\sin(c+dx))^p\right)^n\tan(c+dx)}{d(1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*Sin[c + d*x])^p)^n,x]
[Out] (Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[
c + d*x]^2]*(a*(b*Sin[c + d*x])^p)^n*Tan[c + d*x])/(d*(1 + n*p))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (a(b\sin(dx+c))^p)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*sin(d*x+c))^p)^n,x)
[Out] int((a*(b*sin(d*x+c))^p)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="maxima")
[Out] integrate(((b*sin(d*x + c))^p*a)^n, x)
```

Fricas [F]
time = 0.41, size = 16, normalized size = 0.20

$$\text{integral}\left(\left((b\sin(dx+c))^p\, a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="fricas")
```

```
[Out] integral(((b*sin(d*x + c))^p*a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(b\sin{(c+dx)})^p\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))**p)**n,x)
```

```
[Out] Integral((a*(b*sin(c + d*x))**p)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*sin(d*x + c))^p*a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,{\left(b\,\sin\left(c+d\,x\right)\right)}^p\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*sin(c + d*x))^p)^n,x)
```

```
[Out] int((a*(b*sin(c + d*x))^p)^n, x)
```

3.31 $\int \left(a-a\sin^2(x)\right)\,dx$

Optimal. Leaf size=16

$$\frac{ax}{2}+\frac{1}{2}a\cos(x)\sin(x)$$

```
[Out] 1/2*a*x+1/2*a*cos(x)*sin(x)
```

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2715, 8}

$$\frac{ax}{2}+\frac{1}{2}a\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[a - a*Sin[x]^2,x]
```

```
[Out] (a*x)/2 + (a*Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sin^2(x)\right)\,dx &= ax-a\int \sin^2(x)\,dx\\ &= ax+\frac{1}{2}a\cos(x)\sin(x)-\frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2}+\frac{1}{2}a\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]

time = 0.00, size = 16, normalized size = 1.00

$$a\left(\frac{x}{2}+\frac{1}{4}\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a - a*Sin[x]^2,x]
```

```
[Out] a*(x/2 + Sin[2*x]/4)
```

Maple [A]
time = 0.05, size = 18, normalized size = 1.12

method	result	size
risch	$\frac{ax}{2}+\frac{a\sin(2x)}{4}$	13
default	$ax-a\left(-\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)$	18
norman	$\frac{a\tan\left(\frac{x}{2}\right)+ax\left(\tan^{2}\left(\frac{x}{2}\right)\right)+\frac{ax}{2}-a\left(\tan^{3}\left(\frac{x}{2}\right)\right)+\frac{ax\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a-a*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x-a*(-1/2*sin(x)*cos(x)+1/2*x)
```

Maxima [A]
time = 0.29, size = 17, normalized size = 1.06

$$-\frac{1}{4}\,a(2\,x-\sin\left(2\,x\right))+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*x - sin(2*x)) + a*x
```

Fricas [A]
time = 0.39, size = 12, normalized size = 0.75

$$\frac{1}{2}\,a\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*cos(x)*sin(x) + 1/2*a*x
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.94

$$ax-a\left(\frac{x}{2}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)**2,x)
```

```
[Out] a*x - a*(x/2 - sin(x)*cos(x)/2)
```

Giac [A]
time = 0.41, size = 17, normalized size = 1.06

$$-\frac{1}{4}\,a(2\,x - \sin\left(2\,x\right)) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/4*a*(2*x - sin(2*x)) + a*x
```

Mupad [B]
time = 13.58, size = 11, normalized size = 0.69

$$\frac{a\left(2\,x + \sin\left(2\,x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a - a*sin(x)^2,x)
```

```
[Out] (a*(2*x + sin(2*x)))/4
```

3.32 $\int \left(a-a\sin^2(x)\right)^2 \, dx$

Optimal. Leaf size=33

$$\frac{3a^2x}{8}+\frac{3}{8}a^2\cos(x)\sin(x)+\frac{1}{4}a^2\cos^3(x)\sin(x)$$

```
[Out] 3/8*a^2*x+3/8*a^2*cos(x)*sin(x)+1/4*a^2*cos(x)^3*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3254, 2715, 8}

$$\frac{3a^2x}{8}+\frac{1}{4}a^2\sin(x)\cos^3(x)+\frac{3}{8}a^2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^2,x]
```

```
[Out] (3*a^2*x)/8 + (3*a^2*Cos[x]*Sin[x])/8 + (a^2*Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(a - a\sin^2(x)\right)^2 \, dx = a^2 \int \cos^4(x)\, dx$$
$$= \frac{1}{4}a^2\cos^3(x)\sin(x) + \frac{1}{4}\left(3a^2\right)\int \cos^2(x)\, dx$$
$$= \frac{3}{8}a^2\cos(x)\sin(x) + \frac{1}{4}a^2\cos^3(x)\sin(x) + \frac{1}{8}\left(3a^2\right)\int 1\, dx$$
$$= \frac{3a^2x}{8} + \frac{3}{8}a^2\cos(x)\sin(x) + \frac{1}{4}a^2\cos^3(x)\sin(x)$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.79

$$a^2\left(\frac{3x}{8} + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^2,x]
```

```
[Out] a^2*((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)
```

Maple [A]
time = 0.11, size = 43, normalized size = 1.30

method	result
risch	$\frac{3a^2x}{8} + \frac{a^2\sin(4x)}{32} + \frac{a^2\sin(2x)}{4}$
default	$a^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4} + \frac{3x}{8}\right) - 2a^2\left(-\frac{\sin(x)\cos(x)}{2} + \frac{x}{2}\right) + a^2x$
norman	$\frac{\frac{3a^2x}{8}+\frac{5a^2\tan\left(\frac{x}{2}\right)}{4}-\frac{3a^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{3a^2\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{5a^2\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3a^2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9a^2x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3a^2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3a^2x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-2*a^2*(-1/2*sin(x)*cos(x)+1/2
*x)+a^2*x
```

Maxima [A]
time = 0.31, size = 40, normalized size = 1.21

$$\frac{1}{32}\,a^2(12\,x + \sin\left(4\,x\right) - 8\,\sin\left(2\,x\right)) - \frac{1}{2}\,a^2(2\,x - \sin\left(2\,x\right)) + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a^2*(12*x + sin(4*x) - 8*sin(2*x)) - 1/2*a^2*(2*x - sin(2*x)) + a^2*x
```

Fricas [A]
time = 0.39, size = 28, normalized size = 0.85

$$\frac{3}{8}\,a^2x+\frac{1}{8}\left(2\,a^2\cos\left(x\right)^3+3\,a^2\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 3/8*a^2*x + 1/8*(2*a^2*cos(x)^3 + 3*a^2*cos(x))*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(34) = 68$.
time = 0.14, size = 110, normalized size = 3.33

$$\frac{3a^2x\sin^4\left(x\right)}{8}+\frac{3a^2x\sin^2\left(x\right)\cos^2\left(x\right)}{4}-a^2x\sin^2\left(x\right)+\frac{3a^2x\cos^4\left(x\right)}{8}-a^2x\cos^2\left(x\right)+a^2x-\frac{5a^2\sin^3\left(x\right)\cos\left(x\right)}{8}-\frac{3a^2\sin\left(x\right)\cos^3\left(x\right)}{8}+a^2\sin\left(x\right)\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**2,x)
```

```
[Out] 3*a**2*x*sin(x)**4/8 + 3*a**2*x*sin(x)**2*cos(x)**2/4 - a**2*x*sin(x)**2 +
3*a**2*x*cos(x)**4/8 - a**2*x*cos(x)**2 + a**2*x - 5*a**2*sin(x)**3*cos(x)/
8 - 3*a**2*sin(x)*cos(x)**3/8 + a**2*sin(x)*cos(x)
```

Giac [A]
time = 0.44, size = 25, normalized size = 0.76

$$\frac{3}{8}\,a^2x+\frac{1}{32}\,a^2\sin\left(4\,x\right)+\frac{1}{4}\,a^2\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*x + 1/32*a^2*sin(4*x) + 1/4*a^2*sin(2*x)
```

Mupad [B]
time = 13.76, size = 33, normalized size = 1.00

$$\frac{\frac{3\,a^2\,\tan(x)^3}{8}+\frac{5\,a^2\,\tan(x)}{8}}{{\left(\tan\left(x\right)^2+1\right)}^2}+\frac{3\,a^2\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^2,x)
```

```
[Out] ((5*a^2*tan(x))/8 + (3*a^2*tan(x)^3)/8)/(tan(x)^2 + 1)^2 + (3*a^2*x)/8
```

3.33 $\int \left(a-a\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=46

$$\frac{5a^3x}{16}+\frac{5}{16}a^3\cos(x)\sin(x)+\frac{5}{24}a^3\cos^3(x)\sin(x)+\frac{1}{6}a^3\cos^5(x)\sin(x)$$

```
[Out] 5/16*a^3*x+5/16*a^3*cos(x)*sin(x)+5/24*a^3*cos(x)^3*sin(x)+1/6*a^3*cos(x)^5
*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {3254, 2715, 8}

$$\frac{5a^3x}{16}+\frac{1}{6}a^3\sin(x)\cos^5(x)+\frac{5}{24}a^3\sin(x)\cos^3(x)+\frac{5}{16}a^3\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^3,x]
```

```
[Out] (5*a^3*x)/16 + (5*a^3*Cos[x]*Sin[x])/16 + (5*a^3*Cos[x]^3*Sin[x])/24 + (a^3
*Cos[x]^5*Sin[x])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(a-a\sin^2(x)\right)^3\,dx &= a^3\int \cos^6(x)\,dx \\
&= \frac{1}{6}a^3\cos^5(x)\sin(x)+\frac{1}{6}\left(5a^3\right)\int \cos^4(x)\,dx \\
&= \frac{5}{24}a^3\cos^3(x)\sin(x)+\frac{1}{6}a^3\cos^5(x)\sin(x)+\frac{1}{8}\left(5a^3\right)\int \cos^2(x)\,dx \\
&= \frac{5}{16}a^3\cos(x)\sin(x)+\frac{5}{24}a^3\cos^3(x)\sin(x)+\frac{1}{6}a^3\cos^5(x)\sin(x)+\frac{1}{16}\left(5a^3\right)\int 1\,dx \\
&= \frac{5a^3x}{16}+\frac{5}{16}a^3\cos(x)\sin(x)+\frac{5}{24}a^3\cos^3(x)\sin(x)+\frac{1}{6}a^3\cos^5(x)\sin(x)
\end{aligned}
$$

Mathematica [**A**]
time = 0.00, size = 34, normalized size = 0.74

$$a^3\left(\frac{5x}{16}+\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)+\frac{1}{192}\sin(6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^3,x]
```

```
[Out] a^3*((5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192)
```

Maple [**A**]
time = 0.22, size = 72, normalized size = 1.57

method	result
risch	$\frac{5a^3x}{16}+\frac{a^3\sin(6x)}{192}+\frac{3a^3\sin(4x)}{64}+\frac{15a^3\sin(2x)}{64}$
default	$-a^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3a^3\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-3a^3\left(-\frac{\sin(x)\,c}{2}\right.$
norman	$\frac{\frac{5a^3x}{16}+\frac{11a^3\tan\left(\frac{x}{2}\right)}{8}-\frac{5a^3\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{15a^3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{15a^3\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{5a^3\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}-\frac{11a^3\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15a^3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{75a^3x(}{}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+3*a^3*(-1/4*(
sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-3*a^3*(-1/2*sin(x)*cos(x)+1/2*x)+a^3*x
```

Maxima [A]
time = 0.30, size = 69, normalized size = 1.50

$$-\frac{1}{192}\left(4\,\sin\left(2\,x\right)^{3}+60\,x+9\,\sin\left(4\,x\right)-48\,\sin\left(2\,x\right)\right)a^{3}+\frac{3}{32}\,a^{3}\left(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right)\right)-\frac{3}{4}\,a^{3}\left(2\,x-\sin\left(2\,x\right)\right)+a^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^3 + 3/32*a^3*(12*
x + sin(4*x) - 8*sin(2*x)) - 3/4*a^3*(2*x - sin(2*x)) + a^3*x
```

Fricas [A]
time = 0.41, size = 37, normalized size = 0.80

$$\frac{5}{16}\,a^{3}x+\frac{1}{48}\,\left(8\,a^{3}\cos\left(x\right)^{5}+10\,a^{3}\cos\left(x\right)^{3}+15\,a^{3}\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 5/16*a^3*x + 1/48*(8*a^3*cos(x)^5 + 10*a^3*cos(x)^3 + 15*a^3*cos(x))*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(49) = 98$.
time = 0.32, size = 233, normalized size = 5.07

$$-\frac{5a^{3}x\sin^{6}(x)}{16}-\frac{15a^{3}x\sin^{4}(x)\cos^{2}(x)}{16}+\frac{9a^{3}x\sin^{4}(x)}{8}-\frac{15a^{3}x\sin^{2}(x)\cos^{4}(x)}{16}+\frac{9a^{3}x\sin^{2}(x)\cos^{2}(x)}{4}-\frac{3a^{3}x\sin^{2}(x)}{2}-\frac{5a^{3}x\cos^{6}(x)}{16}+\frac{9a^{3}x\cos^{4}(x)}{8}-\frac{3a^{3}x\cos^{2}(x)}{2}+a^{3}x+\frac{11a^{3}\sin^{5}(x)\cos(x)}{16}+\frac{5a^{3}\sin^{3}(x)\cos^{3}(x)}{6}-\frac{15a^{3}\sin^{3}(x)\cos(x)}{8}+\frac{5a^{3}\sin(x)\cos^{5}(x)}{16}-\frac{9a^{3}\sin(x)\cos^{3}(x)}{8}+\frac{3a^{3}\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**3,x)
```

```
[Out] -5*a**3*x*sin(x)**6/16 - 15*a**3*x*sin(x)**4*cos(x)**2/16 + 9*a**3*x*sin(x)
**4/8 - 15*a**3*x*sin(x)**2*cos(x)**4/16 + 9*a**3*x*sin(x)**2*cos(x)**2/4 -
 3*a**3*x*sin(x)**2/2 - 5*a**3*x*cos(x)**6/16 + 9*a**3*x*cos(x)**4/8 - 3*a*
*3*x*cos(x)**2/2 + a**3*x + 11*a**3*sin(x)**5*cos(x)/16 + 5*a**3*sin(x)**3*
cos(x)**3/6 - 15*a**3*sin(x)**3*cos(x)/8 + 5*a**3*sin(x)*cos(x)**5/16 - 9*a
**3*sin(x)*cos(x)**3/8 + 3*a**3*sin(x)*cos(x)/2
```

Giac [A]
time = 0.44, size = 34, normalized size = 0.74

$$\frac{5}{16}\,a^{3}x+\frac{1}{192}\,a^{3}\sin\left(6\,x\right)+\frac{3}{64}\,a^{3}\sin\left(4\,x\right)+\frac{15}{64}\,a^{3}\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] 5/16*a^3*x + 1/192*a^3*sin(6*x) + 3/64*a^3*sin(4*x) + 15/64*a^3*sin(2*x)
```

Mupad [B]
time = 13.68, size = 42, normalized size = 0.91

$$\frac{11\,a^3\cos\left(x\right)^5\,\sin\left(x\right)}{16}+\frac{5\,a^3\cos\left(x\right)^3\sin\left(x\right)^3}{6}+\frac{5\,a^3\,\cos\left(x\right)\,\sin\left(x\right)^5}{16}+\frac{5\,x\,a^3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^3,x)
```

```
[Out] (5*a^3*x)/16 + (5*a^3*cos(x)*sin(x)^5)/16 + (11*a^3*cos(x)^5*sin(x))/16 + (
5*a^3*cos(x)^3*sin(x)^3)/6
```

3.34 $\int \left(a-a\sin^2(x)\right)^4\,dx$

Optimal. Leaf size=59

$$\frac{35a^4x}{128}+\frac{35}{128}a^4\cos(x)\sin(x)+\frac{35}{192}a^4\cos^3(x)\sin(x)+\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)$$

```
[Out] 35/128*a^4*x+35/128*a^4*cos(x)*sin(x)+35/192*a^4*cos(x)^3*sin(x)+7/48*a^4*c
os(x)^5*sin(x)+1/8*a^4*cos(x)^7*sin(x)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3254, 2715, 8}

$$\frac{35a^4x}{128}+\frac{1}{8}a^4\sin(x)\cos^7(x)+\frac{7}{48}a^4\sin(x)\cos^5(x)+\frac{35}{192}a^4\sin(x)\cos^3(x)+\frac{35}{128}a^4\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^4,x]
```

```
[Out] (35*a^4*x)/128 + (35*a^4*Cos[x]*Sin[x])/128 + (35*a^4*Cos[x]^3*Sin[x])/192
+ (7*a^4*Cos[x]^5*Sin[x])/48 + (a^4*Cos[x]^7*Sin[x])/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(a - a\sin^2(x)\right)^4 \, dx &= a^4 \int \cos^8(x) \, dx \\
&= \frac{1}{8}a^4 \cos^7(x)\sin(x) + \frac{1}{8}\left(7a^4\right) \int \cos^6(x) \, dx \\
&= \frac{7}{48}a^4 \cos^5(x)\sin(x) + \frac{1}{8}a^4 \cos^7(x)\sin(x) + \frac{1}{48}\left(35a^4\right) \int \cos^4(x) \, dx \\
&= \frac{35}{192}a^4 \cos^3(x)\sin(x) + \frac{7}{48}a^4 \cos^5(x)\sin(x) + \frac{1}{8}a^4 \cos^7(x)\sin(x) + \frac{1}{64}\left(35a^4\right) \int \cos \\
&= \frac{35}{128}a^4 \cos(x)\sin(x) + \frac{35}{192}a^4 \cos^3(x)\sin(x) + \frac{7}{48}a^4 \cos^5(x)\sin(x) + \frac{1}{8}a^4 \cos^7(x)\sin \\
&= \frac{35a^4x}{128} + \frac{35}{128}a^4 \cos(x)\sin(x) + \frac{35}{192}a^4 \cos^3(x)\sin(x) + \frac{7}{48}a^4 \cos^5(x)\sin(x) + \frac{1}{8}a^4 c
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 0.71

$$
a^4\left(\frac{35x}{128} + \frac{7}{32}\sin(2x) + \frac{7}{128}\sin(4x) + \frac{1}{96}\sin(6x) + \frac{\sin(8x)}{1024}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^4,x]
```

```
[Out] a^4*((35*x)/128 + (7*Sin[2*x])/32 + (7*Sin[4*x])/128 + Sin[6*x]/96 + Sin[8*
x]/1024)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(49) = 98.
time = 0.23, size = 105, normalized size = 1.78

method	result
risch	$\frac{35a^4x}{128} + \frac{a^4\sin(8x)}{1024} + \frac{a^4\sin(6x)}{96} + \frac{7a^4\sin(4x)}{128} + \frac{7a^4\sin(2x)}{32}$
default	$a^4\left(-\frac{\left(\sin^7(x)+\frac{7\left(\sin^5(x)\right)}{6}+\frac{35\left(\sin^3(x)\right)}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8} + \frac{35x}{128}\right) - 4a^4\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6} +\right.$
norman	$\frac{35a^4x}{128}+\frac{93a^4\tan\left(\frac{x}{2}\right)}{64}+\frac{91a^4\left(\tan^3\left(\frac{x}{2}\right)\right)}{192}+\frac{1799a^4\left(\tan^5\left(\frac{x}{2}\right)\right)}{192}-\frac{1085a^4\left(\tan^7\left(\frac{x}{2}\right)\right)}{192}+\frac{1085a^4\left(\tan^9\left(\frac{x}{2}\right)\right)}{192}-\frac{1799a^4\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{192}-\frac{91a^4\left(\tan^{13}\left(\frac{x}{2}\right.\right.}{192}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/128
*x)-4*a^4*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+6*a^4*(-
1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-4*a^4*(-1/2*sin(x)*cos(x)+1/2*x)+a^
4*x
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.
time = 0.30, size = 104, normalized size = 1.76

$$\frac{1}{3072}\left(128\sin\left(2x\right)^{3}+840x+3\sin\left(8x\right)+168\sin\left(4x\right)-768\sin\left(2x\right)\right)a^{4}-\frac{1}{48}\left(4\sin\left(2x\right)^{3}+60x+9\sin\left(4x\right)-48\sin\left(2x\right)\right)a^{4}+\frac{3}{16}a^{4}\left(12x+\sin\left(4x\right)-8\sin\left(2x\right)\right)-a^{4}\left(2x-\sin\left(2x\right)\right)+a^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/3072*(128*sin(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))*
a^4 - 1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^4 + 3/16*a^4*
(12*x + sin(4*x) - 8*sin(2*x)) - a^4*(2*x - sin(2*x)) + a^4*x
```

Fricas [**A**]
time = 0.39, size = 46, normalized size = 0.78

$$\frac{35}{128}a^{4}x+\frac{1}{384}\left(48a^{4}\cos\left(x\right)^{7}+56a^{4}\cos\left(x\right)^{5}+70a^{4}\cos\left(x\right)^{3}+105a^{4}\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^4,x, algorithm="fricas")
```

```
[Out] 35/128*a^4*x + 1/384*(48*a^4*cos(x)^7 + 56*a^4*cos(x)^5 + 70*a^4*cos(x)^3 +
 105*a^4*cos(x))*sin(x)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(65) = 130$.
time = 0.71, size = 376, normalized size = 6.37

$$\frac{35a^{4}x\sin^{8}\left(x\right)}{128}+\frac{35a^{4}x\sin^{6}\left(x\right)\cos^{2}\left(x\right)}{32}-\frac{5a^{4}x\sin^{6}\left(x\right)}{4}+\frac{105a^{4}x\sin^{4}\left(x\right)\cos^{4}\left(x\right)}{64}-\frac{15a^{4}x\sin^{4}\left(x\right)\cos^{2}\left(x\right)}{4}+\frac{9a^{4}x\sin^{4}\left(x\right)}{4}+\frac{35a^{4}x\sin^{2}\left(x\right)\cos^{6}\left(x\right)}{32}-\frac{15a^{4}x\sin^{2}\left(x\right)\cos^{4}\left(x\right)}{4}+\frac{9a^{4}x\sin^{2}\left(x\right)\cos^{2}\left(x\right)}{2}-2a^{4}x\sin^{2}\left(x\right)+\frac{35a^{4}x\cos^{8}\left(x\right)}{128}-\frac{5a^{4}x\cos^{6}\left(x\right)}{4}+\frac{9a^{4}x\cos^{4}\left(x\right)}{4}-2a^{4}x\cos^{2}\left(x\right)+a^{4}x-\frac{93a^{4}\sin^{7}\left(x\right)\cos\left(x\right)}{128}-\frac{511a^{4}\sin^{5}\left(x\right)\cos^{3}\left(x\right)}{384}+\frac{11a^{4}\sin^{5}\left(x\right)\cos\left(x\right)}{4}-\frac{385a^{4}\sin^{3}\left(x\right)\cos^{5}\left(x\right)}{384}+\frac{10a^{4}\sin^{3}\left(x\right)\cos^{3}\left(x\right)}{3}-\frac{15a^{4}\sin^{3}\left(x\right)\cos\left(x\right)}{4}-\frac{35a^{4}\sin\left(x\right)\cos^{7}\left(x\right)}{128}+\frac{5a^{4}\sin\left(x\right)\cos^{5}\left(x\right)}{4}-\frac{9a^{4}\sin\left(x\right)\cos^{3}\left(x\right)}{4}+2a^{4}\sin\left(x\right)\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**4,x)
```

```
[Out] 35*a**4*x*sin(x)**8/128 + 35*a**4*x*sin(x)**6*cos(x)**2/32 - 5*a**4*x*sin(x
)**6/4 + 105*a**4*x*sin(x)**4*cos(x)**4/64 - 15*a**4*x*sin(x)**4*cos(x)**2/
4 + 9*a**4*x*sin(x)**4/4 + 35*a**4*x*sin(x)**2*cos(x)**6/32 - 15*a**4*x*sin
(x)**2*cos(x)**4/4 + 9*a**4*x*sin(x)**2*cos(x)**2/2 - 2*a**4*x*sin(x)**2 +
35*a**4*x*cos(x)**8/128 - 5*a**4*x*cos(x)**6/4 + 9*a**4*x*cos(x)**4/4 - 2*a
**4*x*cos(x)**2 + a**4*x - 93*a**4*sin(x)**7*cos(x)/128 - 511*a**4*sin(x)**
5*cos(x)**3/384 + 11*a**4*sin(x)**5*cos(x)/4 - 385*a**4*sin(x)**3*cos(x)**5
```

```
/384 + 10*a**4*sin(x)**3*cos(x)**3/3 - 15*a**4*sin(x)**3*cos(x)/4 - 35*a**4
*sin(x)*cos(x)**7/128 + 5*a**4*sin(x)*cos(x)**5/4 - 9*a**4*sin(x)*cos(x)**3
/4 + 2*a**4*sin(x)*cos(x)
```

Giac [A]
time = 0.44, size = 43, normalized size = 0.73

$$\frac{35}{128}\,a^4x + \frac{1}{1024}\,a^4\sin\left(8\,x\right) + \frac{1}{96}\,a^4\sin\left(6\,x\right) + \frac{7}{128}\,a^4\sin\left(4\,x\right) + \frac{7}{32}\,a^4\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^4,x, algorithm="giac")
```

```
[Out] 35/128*a^4*x + 1/1024*a^4*sin(8*x) + 1/96*a^4*sin(6*x) + 7/128*a^4*sin(4*x)
 + 7/32*a^4*sin(2*x)
```

Mupad [B]
time = 13.69, size = 51, normalized size = 0.86

$$\frac{\frac{35\,a^4\,\tan(x)^7}{128}+\frac{385\,a^4\,\tan(x)^5}{384}+\frac{511\,a^4\,\tan(x)^3}{384}+\frac{93\,a^4\,\tan(x)}{128}}{{\left({\tan\left(x\right)}^2+1\right)}^4}+\frac{35\,a^4\,x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^4,x)
```

```
[Out] ((93*a^4*tan(x))/128 + (511*a^4*tan(x)^3)/384 + (385*a^4*tan(x)^5)/384 + (3
5*a^4*tan(x)^7)/128)/(tan(x)^2 + 1)^4 + (35*a^4*x)/128
```

3.35 $\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{3\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{ad}+\frac{\cos^5(c+dx)}{5ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] 3*cos(d*x+c)/a/d-cos(d*x+c)^3/a/d+1/5*cos(d*x+c)^5/a/d+sec(d*x+c)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3254, 2670, 276}

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{ad}+\frac{3\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (3*Cos[c + d*x])/(a*d) - Cos[c + d*x]^3/(a*d) + Cos[c + d*x]^5/(5*a*d) + Se
c[c + d*x]/(a*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \sin^5(c+dx)\tan^2(c+dx)\,dx}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^2}\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= -\frac{\text{Subst}\left(\int \left(-3+\frac{1}{x^2}+3x^2-x^4\right)\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= \frac{3\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{ad} + \frac{\cos^5(c+dx)}{5ad} + \frac{\sec(c+dx)}{ad}$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 0.94

$$\frac{\frac{19\cos(c+dx)}{8d} - \frac{3\cos(3(c+dx))}{16d} + \frac{\cos(5(c+dx))}{80d} + \frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] ((19*Cos[c + d*x])/(8*d) - (3*Cos[3*(c + d*x)])/(16*d) + Cos[5*(c + d*x)]/(
80*d) + Sec[c + d*x]/d)/a
```

Maple [A]
time = 0.24, size = 45, normalized size = 0.73

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^5(dx+c)\right)}{5}-\left(\cos^3(dx+c)\right)+3\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	45
default	$\frac{\frac{\left(\cos^5(dx+c)\right)}{5}-\left(\cos^3(dx+c)\right)+3\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	45
risch	$\frac{19\,e^{i(dx+c)}}{16ad}+\frac{19\,e^{-i(dx+c)}}{16ad}+\frac{2\,e^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}+\frac{\cos(5dx+5c)}{80ad}-\frac{3\cos(3dx+3c)}{16ad}$	100
norman	$\frac{-\frac{32\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{32}{5ad}-\frac{192\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{448\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{448\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/5*cos(d*x+c)^5-cos(d*x+c)^3+3*cos(d*x+c)+1/cos(d*x+c))
```

Maxima [A]
time = 0.36, size = 50, normalized size = 0.81

$$\frac{\frac{\cos(dx+c)^5-5\cos(dx+c)^3+15\cos(dx+c)}{a}+\frac{5}{a\cos(dx+c)}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/5*((cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 15*cos(d*x + c))/a + 5/(a*cos(d*x
 + c)))/d
```

Fricas [A]
time = 0.38, size = 46, normalized size = 0.74

$$\frac{\cos(dx+c)^6-5\cos(dx+c)^4+15\cos(dx+c)^2+5}{5\,ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/5*(cos(d*x + c)^6 - 5*cos(d*x + c)^4 + 15*cos(d*x + c)^2 + 5)/(a*d*cos(d*
x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(46) = 92$.
time = 12.16, size = 314, normalized size = 5.06

$$\begin{cases} -\frac{160\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{5ad\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+20ad\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+25ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-25ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-20ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-5ad}-\frac{128\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{5ad\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+20ad\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+25ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-25ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-20ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-5ad}-\frac{32}{5ad\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+20ad\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+25ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-25ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-20ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-5ad} & \text{for}\ d\neq 0 \\ \frac{x\sin^7(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-160*tan(c/2 + d*x/2)**4/(5*a*d*tan(c/2 + d*x/2)**12 + 20*a*d*ta
n(c/2 + d*x/2)**10 + 25*a*d*tan(c/2 + d*x/2)**8 - 25*a*d*tan(c/2 + d*x/2)**
4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a*d) - 128*tan(c/2 + d*x/2)**2/(5*a*d*ta
n(c/2 + d*x/2)**12 + 20*a*d*tan(c/2 + d*x/2)**10 + 25*a*d*tan(c/2 + d*x/2)*
*8 - 25*a*d*tan(c/2 + d*x/2)**4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a*d) - 32/
(5*a*d*tan(c/2 + d*x/2)**12 + 20*a*d*tan(c/2 + d*x/2)**10 + 25*a*d*tan(c/2
+ d*x/2)**8 - 25*a*d*tan(c/2 + d*x/2)**4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a
*d), Ne(d, 0)), (x*sin(c)**7/(-a*sin(c)**2 + a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(60) = 120$.
time = 0.44, size = 149, normalized size = 2.40

$$\frac{2\left(\frac{5}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}+\frac{\frac{50\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{80\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{30\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{5\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-11}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2/5*(5/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + (50*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 80*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 30
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 - 11)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

Mupad [B]
time = 13.65, size = 54, normalized size = 0.87

$$\frac{\frac{3\cos(c+d\,x)}{a}+\frac{1}{a\cos(c+d\,x)}-\frac{\cos(c+d\,x)^3}{a}+\frac{\cos(c+d\,x)^5}{5\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - a*sin(c + d*x)^2),x)
```

```
[Out] ((3*cos(c + d*x))/a + 1/(a*cos(c + d*x)) - cos(c + d*x)^3/a + cos(c + d*x)^
5/(5*a))/d
```

3.36 $\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{2\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] 2*cos(d*x+c)/a/d-1/3*cos(d*x+c)^3/a/d+sec(d*x+c)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2670, 276}

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{2\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (2*Cos[c + d*x])/(a*d) - Cos[c + d*x]^3/(3*a*d) + Sec[c + d*x]/(a*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin^3(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, \cos(c+dx)\right)}{ad}\\ &= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, \cos(c+dx)\right)}{ad}\\ &= \frac{2\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.93

$$\frac{\frac{7\cos(c+dx)}{4d}-\frac{\cos(3(c+dx))}{12d}+\frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
[Out] ((7*Cos[c + d*x])/(4*d) - Cos[3*(c + d*x)]/(12*d) + Sec[c + d*x]/d)/a
```

Maple [A]
time = 0.20, size = 35, normalized size = 0.76

method	result	size
derivativedivides	$\frac{-\frac{\left(\cos^3(dx+c)\right)}{3}+2\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	35
default	$\frac{-\frac{\left(\cos^3(dx+c)\right)}{3}+2\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	35
risch	$\frac{7\,e^{i(dx+c)}}{8ad}+\frac{7\,e^{-i(dx+c)}}{8ad}+\frac{2\,e^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}-\frac{\cos(3dx+3c)}{12ad}$	83
norman	$\frac{-\frac{32\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{16}{3ad}-\frac{64\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{80\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/3*cos(d*x+c)^3+2*cos(d*x+c)+1/cos(d*x+c))
```

Maxima [A]
time = 0.32, size = 40, normalized size = 0.87

$$-\frac{\frac{\cos(dx+c)^3-6\,\cos(dx+c)}{a}-\frac{3}{a\cos(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/3*((cos(d*x + c)^3 - 6*cos(d*x + c))/a - 3/(a*cos(d*x + c)))/d
```

Fricas [A]
time = 0.39, size = 36, normalized size = 0.78

$$-\frac{\cos(dx+c)^4 - 6\cos(dx+c)^2 - 3}{3\,ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^4 - 6*cos(d*x + c)^2 - 3)/(a*d*cos(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(34) = 68$.
time = 4.72, size = 143, normalized size = 3.11

$$\begin{cases} -\frac{32\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-6ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3ad} - \frac{16}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-6ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3ad} & \text{for } d \neq 0 \\ \frac{x\sin^5(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-32*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**8 + 6*a*d*tan(c
/2 + d*x/2)**6 - 6*a*d*tan(c/2 + d*x/2)**2 - 3*a*d) - 16/(3*a*d*tan(c/2 + d
*x/2)**8 + 6*a*d*tan(c/2 + d*x/2)**6 - 6*a*d*tan(c/2 + d*x/2)**2 - 3*a*d),
Ne(d, 0)), (x*sin(c)**5/(-a*sin(c)**2 + a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(44) = 88$.
time = 0.46, size = 105, normalized size = 2.28

$$\frac{2\left(\frac{3}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)} + \frac{\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - 5}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2/3*(3/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + (12*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5)/
(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^3))/d
```

Mupad [B]
time = 0.06, size = 38, normalized size = 0.83

$$\frac{-\cos\left(c+d\,x\right)^4+6\cos\left(c+d\,x\right)^2+3}{3\,a\,d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - a*sin(c + d*x)^2),x)
[Out] (6*cos(c + d*x)^2 - cos(c + d*x)^4 + 3)/(3*a*d*cos(c + d*x))
```

3.37 $\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] cos(d*x+c)/a/d+sec(d*x+c)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3254, 2670, 14}

$$\frac{\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] Cos[c + d*x]/(a*d) + Sec[c + d*x]/(a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \sin(c+dx)\tan^2(c+dx)\,dx}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= \frac{\cos(c+dx)}{ad} + \frac{\sec(c+dx)}{ad}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.93

$$\frac{\frac{\cos(c+dx)}{d} + \frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
[Out] (Cos[c + d*x]/d + Sec[c + d*x]/d)/a
```

Maple [A]
time = 0.18, size = 23, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	23
default	$\frac{\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	23
risch	$\frac{e^{i(dx+c)}}{2ad}+\frac{e^{-i(dx+c)}}{2ad}+\frac{2\,e^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}$	66
norman	$\frac{-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{4}{ad}-\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(cos(d*x+c)+1/cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 27, normalized size = 1.00

$$\frac{\frac{\cos(dx+c)}{a} + \frac{1}{a\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
[Out] (cos(d*x + c)/a + 1/(a*cos(d*x + c)))/d
```

Fricas [A]
time = 0.39, size = 25, normalized size = 0.93

$$\frac{\cos(dx+c)^2+1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
[Out] (cos(d*x + c)^2 + 1)/(a*d*cos(d*x + c))
```

Sympy [A]
time = 1.68, size = 36, normalized size = 1.33

$$\begin{cases} -\frac{4}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x\sin^3(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-a*sin(d*x+c)**2),x)
[Out] Piecewise((-4/(a*d*tan(c/2 + d*x/2)**4 - a*d), Ne(d, 0)), (x*sin(c)**3/(-a*
sin(c)**2 + a), True))
```

Giac [A]
time = 0.54, size = 29, normalized size = 1.07

$$\frac{\cos(dx+c)}{ad}+\frac{1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="giac")
[Out] cos(d*x + c)/(a*d) + 1/(a*d*cos(d*x + c))
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.93

$$\frac{\cos(c+d\,x)^2+1}{a\,d\,\cos(c+d\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - a*sin(c + d*x)^2),x)
[Out] (cos(c + d*x)^2 + 1)/(a*d*cos(c + d*x))
```

3.38 $\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=13

$$\frac{\sec(c+dx)}{ad}$$

```
[Out] sec(d*x+c)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3254, 2686, 8}

$$\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] Sec[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sec(c+dx)\tan(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{ad}\\ &= \frac{\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] Sec[c + d*x]/(a*d)
```

Maple [A]
time = 0.12, size = 16, normalized size = 1.23

method	result	size
derivativedivides	$\frac{1}{da\cos(dx+c)}$	16
default	$\frac{1}{da\cos(dx+c)}$	16
risch	$\frac{2\,\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}$	31
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a/cos(d*x+c)
```

Maxima [A]
time = 0.30, size = 15, normalized size = 1.15

$$\frac{1}{ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/(a*d*cos(d*x + c))
```

Fricas [A]
time = 0.38, size = 15, normalized size = 1.15

$$\frac{1}{ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/(a*d*cos(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(8) = 16$.
time = 0.63, size = 34, normalized size = 2.62

$$\begin{cases} -\frac{2}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d \neq 0 \\ \frac{x\sin(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-2/(a*d*tan(c/2 + d*x/2)**2 - a*d), Ne(d, 0)), (x*sin(c)/(-a*sin
(c)**2 + a), True))
```

Giac [A]
time = 0.46, size = 15, normalized size = 1.15

$$\frac{1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/(a*d*cos(d*x + c))
```

Mupad [B]
time = 13.60, size = 15, normalized size = 1.15

$$\frac{1}{a\,d\,\cos(c+d\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - a*sin(c + d*x)^2),x)
```

```
[Out] 1/(a*d*cos(c + d*x))
```

3.39 $\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=29

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3254, 2702, 327, 213}

$$\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
```

```
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc(c+dx)\sec^2(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad}\\ &= \frac{\sec(c+dx)}{ad}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.59

$$\frac{-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a - a*Sin[c + d*x]^2),x]
[Out] (-(Log[Cos[(c + d*x)/2]]/d) + Log[Sin[(c + d*x)/2]]/d + Sec[c + d*x]/d)/a
```

Maple [A]
time = 0.22, size = 39, normalized size = 1.34

method	result	size
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{2}-\frac{\ln(1+\cos(dx+c))}{2}+\frac{1}{\cos(dx+c)}}{da}$	39
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{2}-\frac{\ln(1+\cos(dx+c))}{2}+\frac{1}{\cos(dx+c)}}{da}$	39
norman	$-\frac{2}{ad\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	42
risch	$\frac{2\,e^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{ad}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/2*ln(cos(d*x+c)-1)-1/2*ln(1+cos(d*x+c))+1/cos(d*x+c))
```

Maxima [A]
time = 0.30, size = 46, normalized size = 1.59

$$-\frac{\frac{\log(\cos(dx+c)+1)}{a}-\frac{\log(\cos(dx+c)-1)}{a}-\frac{2}{a\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(cos(d*x + c) + 1)/a - log(cos(d*x + c) - 1)/a - 2/(a*cos(d*x + c)
))/d
```

Fricas [A]
time = 0.39, size = 55, normalized size = 1.90

$$-\frac{\cos\left(dx+c\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-2}{2\,ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - cos(d*x + c)*log(-1/2*cos(
d*x + c) + 1/2) - 2)/(a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\csc\left(c+dx\right)}{\sin^{2}\left(c+dx\right)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)/(sin(c + d*x)**2 - 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.
time = 0.44, size = 62, normalized size = 2.14

$$\frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{4}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + 4/(a*((cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B]
time = 0.08, size = 31, normalized size = 1.07

$$\frac{1}{a\,d\,\cos\left(c+d\,x\right)}-\frac{\mathrm{atanh}(\cos\left(c+d\,x\right))}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - a*sin(c + d*x)^2)),x)
[Out] 1/(a*d*cos(c + d*x)) - atanh(cos(c + d*x))/(a*d)
```

3.40 $\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3\sec(c+dx)}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] -3/2*arctanh(cos(d*x+c))/a/d+3/2*sec(d*x+c)/a/d-1/2*csc(d*x+c)^2*sec(d*x+c)
/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3254, 2702, 294, 327, 213}

$$\frac{3\sec(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (3*Sec[c + d*x])/(2*a*d) - (Csc[c + d*
x]^2*Sec[c + d*x])/(2*a*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^3(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= -\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}+\frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2ad}\\
&= \frac{3\sec(c+dx)}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}+\frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2ad}\\
&= -\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3\sec(c+dx)}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(58) = 116.
time = 0.18, size = 146, normalized size = 2.52

$$\frac{\csc^4(c+dx)\left(2-6\cos(2(c+dx))+2\cos(3(c+dx))+3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\cos(c+dx)\left(-2-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{2ad\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (Csc[c + d*x]^4*(2 - 6*Cos[2*(c + d*x)] + 2*Cos[3*(c + d*x)] + 3*Cos[3*(c +
 d*x)]*Log[Cos[(c + d*x)/2]] - 3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + C
os[c + d*x]*(-2 - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]])))/(2*a
*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

Maple [**A**]
time = 0.28, size = 63, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\frac{1}{4+4\cos(dx+c)}-\frac{3\ln(1+\cos(dx+c))}{4}+\frac{1}{4\cos(dx+c)-4}+\frac{3\ln(\cos(dx+c)-1)}{4}+\frac{1}{\cos(dx+c)}}{da}$	63
default	$\frac{\frac{1}{4+4\cos(dx+c)}-\frac{3\ln(1+\cos(dx+c))}{4}+\frac{1}{4\cos(dx+c)-4}+\frac{3\ln(\cos(dx+c)-1)}{4}+\frac{1}{\cos(dx+c)}}{da}$	63
norman	$\frac{\frac{1}{8ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{9\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$	94
risch	$\frac{3\,e^{5i(dx+c)}-2\,e^{3i(dx+c)}+3\,e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}-\frac{3\ln\left(e^{i(dx+c)}+1\right)}{2ad}+\frac{3\ln\left(e^{i(dx+c)}-1\right)}{2ad}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/4/(1+cos(d*x+c))-3/4*ln(1+cos(d*x+c))+1/4/(cos(d*x+c)-1)+3/4*ln(co
s(d*x+c)-1)+1/cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.21

$$\frac{\frac{2\left(3\cos(dx+c)^2-2\right)}{a\cos(dx+c)^3-a\cos(dx+c)}-\frac{3\log(\cos(dx+c)+1)}{a}+\frac{3\log(\cos(dx+c)-1)}{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*cos(d*x + c)^2 - 2)/(a*cos(d*x + c)^3 - a*cos(d*x + c)) - 3*log(c
os(d*x + c) + 1)/a + 3*log(cos(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.43, size = 98, normalized size = 1.69

$$\frac{6\cos(dx+c)^2-3\left(\cos(dx+c)^3-\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\left(\cos(dx+c)^3-\cos(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-4}{4\left(ad\cos(dx+c)^3-ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) + 3*(cos(d*x + c)^3 - cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2)
 - 4)/(a*d*cos(d*x + c)^3 - a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\csc^3(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**3/(sin(c + d*x)**2 - 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(52) = 104$.
time = 0.50, size = 149, normalized size = 2.57

$$\frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{\frac{14(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}-\frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (14*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 1)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2)) - (cos(d*x + c) - 1)/(a*(cos(d*x + c) + 1)))/d
```

Mupad [B]
time = 0.09, size = 55, normalized size = 0.95

$$-\frac{\frac{3\cos(c+d\,x)^2}{2}-1}{d\,\left(a\,\cos\left(c+d\,x\right)-a\cos\left(c+d\,x\right)^3\right)}-\frac{3\,\mathrm{atanh}(\cos\left(c+d\,x\right))}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] - ((3*cos(c + d*x)^2)/2 - 1)/(d*(a*cos(c + d*x) - a*cos(c + d*x)^3)) - (3*a
tanh(cos(c + d*x)))/(2*a*d)
```

3.41 $\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{15\sec(c+dx)}{8ad}-\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad}-\frac{\csc^4(c+dx)\sec(c+dx)}{4ad}$$

```
[Out] -15/8*arctanh(cos(d*x+c))/a/d+15/8*sec(d*x+c)/a/d-5/8*csc(d*x+c)^2*sec(d*x+
c)/a/d-1/4*csc(d*x+c)^4*sec(d*x+c)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3254, 2702, 294, 327, 213}

$$\frac{15\sec(c+dx)}{8ad}-\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\csc^4(c+dx)\sec(c+dx)}{4ad}-\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-15*ArcTanh[Cos[c + d*x]])/(8*a*d) + (15*Sec[c + d*x])/(8*a*d) - (5*Csc[c
+ d*x]^2*Sec[c + d*x])/(8*a*d) - (Csc[c + d*x]^4*Sec[c + d*x])/(4*a*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^5(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= -\frac{\csc^4(c+dx)\sec(c+dx)}{4ad} + \frac{5\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{4ad}\\
&= -\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad} - \frac{\csc^4(c+dx)\sec(c+dx)}{4ad} + \frac{15\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,\right.}{8ad}\\
&= \frac{15\sec(c+dx)}{8ad} - \frac{5\csc^2(c+dx)\sec(c+dx)}{8ad} - \frac{\csc^4(c+dx)\sec(c+dx)}{4ad} + \frac{15\text{Sub}}{}\\
&= -\frac{15\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{15\sec(c+dx)}{8ad} - \frac{5\csc^2(c+dx)\sec(c+dx)}{8ad} - \frac{\csc^4(}{}
\end{aligned}$$

Mathematica [A]
time = 2.78, size = 132, normalized size = 1.61

$$-\frac{14\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(78+\cos(c+dx)\left(-8\left(8+15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)\right)-14\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{-1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}{64ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -1/64*(14*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4 + (Sec[(c + d*x)/2]^2*(78
 + Cos[c + d*x]*(-8*(8 + 15*Log[Cos[(c + d*x)/2]] - 15*Log[Sin[(c + d*x)/2]
]) + Sec[(c + d*x)/2]^4) - 14*Tan[(c + d*x)/2]^2))/(-1 + Tan[(c + d*x)/2]^2
))/(a*d)
```

Maple [A]
time = 0.29, size = 87, normalized size = 1.06

method	result
derivativedivides	$\frac{\frac{1}{\cos(dx+c)}+\frac{1}{16(1+\cos(dx+c))^2}+\frac{7}{16(1+\cos(dx+c))}-\frac{15\ln(1+\cos(dx+c))}{16}-\frac{1}{16(\cos(dx+c)-1)^2}+\frac{7}{16(\cos(dx+c)-1)}+\frac{15\ln(\cos(dx+c)-1)}{16}}{da}$
default	$\frac{\frac{1}{\cos(dx+c)}+\frac{1}{16(1+\cos(dx+c))^2}+\frac{7}{16(1+\cos(dx+c))}-\frac{15\ln(1+\cos(dx+c))}{16}-\frac{1}{16(\cos(dx+c)-1)^2}+\frac{7}{16(\cos(dx+c)-1)}+\frac{15\ln(\cos(dx+c)-1)}{16}}{da}$
norman	$\frac{\frac{1}{64ad}+\frac{15\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{15\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}$
risch	$\frac{15\,e^{9i(dx+c)}-40\,e^{7i(dx+c)}+18\,e^{5i(dx+c)}-40\,e^{3i(dx+c)}+15\,e^{i(dx+c)}}{4da\left(e^{2i(dx+c)}-1\right)^4\left(e^{2i(dx+c)}+1\right)}-\frac{15\ln\left(e^{i(dx+c)}+1\right)}{8ad}+\frac{15\ln\left(e^{i(dx+c)}-1\right)}{8ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/cos(d*x+c)+1/16/(1+cos(d*x+c))^2+7/16/(1+cos(d*x+c))-15/16*ln(1+co
s(d*x+c))-1/16/(cos(d*x+c)-1)^2+7/16/(cos(d*x+c)-1)+15/16*ln(cos(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 90, normalized size = 1.10

$$\frac{\frac{2\left(15\cos(dx+c)^4-25\cos(dx+c)^2+8\right)}{a\cos(dx+c)^5-2\,a\cos(dx+c)^3+a\cos(dx+c)}-\frac{15\log(\cos(dx+c)+1)}{a}+\frac{15\log(\cos(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(15*cos(d*x + c)^4 - 25*cos(d*x + c)^2 + 8)/(a*cos(d*x + c)^5 - 2*a
*cos(d*x + c)^3 + a*cos(d*x + c)) - 15*log(cos(d*x + c) + 1)/a + 15*log(cos
(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.40, size = 135, normalized size = 1.65

$$\frac{30\cos(dx+c)^4-50\cos(dx+c)^2-15\left(\cos(dx+c)^5-2\cos(dx+c)^3+\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15\left(\cos(dx+c)^5-2\cos(dx+c)^3+\cos(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+16}{16\left(ad\cos(dx+c)^5-2\,ad\cos(dx+c)^3+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(30*cos(d*x + c)^4 - 50*cos(d*x + c)^2 - 15*(cos(d*x + c)^5 - 2*cos(d*
x + c)^3 + cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^5 -
 2*cos(d*x + c)^3 + cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 16)/(a*d*c
os(d*x + c)^5 - 2*a*d*cos(d*x + c)^3 + a*d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc^5(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**5/(sin(c + d*x)**2 - 1), x)/a
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(74) = 148.
time = 0.44, size = 181, normalized size = 2.21

$$\frac{\frac{\left(\frac{16(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{90(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2}+\frac{60\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}-\frac{\frac{16a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{128}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*((16*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 90*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2) + 60
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a - (16*a*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^2
+ 128/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [**B**]
time = 0.10, size = 74, normalized size = 0.90

$$\frac{\frac{15\cos(c+d\,x)^4}{8}-\frac{25\cos(c+d\,x)^2}{8}+1}{d\,\left(a\cos\left(c+d\,x\right)^5-2\,a\cos\left(c+d\,x\right)^3+a\,\cos\left(c+d\,x\right)\right)}-\frac{15\,\mathrm{atanh}(\cos\left(c+d\,x\right))}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] ((15*cos(c + d*x)^4)/8 - (25*cos(c + d*x)^2)/8 + 1)/(d*(a*cos(c + d*x) - 2*
a*cos(c + d*x)^3 + a*cos(c + d*x)^5)) - (15*atanh(cos(c + d*x)))/(8*a*d)
```

3.42 $\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{15x}{8a}+\frac{15\tan(c+dx)}{8ad}-\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad}-\frac{\sin^4(c+dx)\tan(c+dx)}{4ad}$$

```
[Out] -15/8*x/a+15/8*tan(d*x+c)/a/d-5/8*sin(d*x+c)^2*tan(d*x+c)/a/d-1/4*sin(d*x+c
)^4*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3254, 2671, 294, 327, 209}

$$\frac{15\tan(c+dx)}{8ad}-\frac{\sin^4(c+dx)\tan(c+dx)}{4ad}-\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad}-\frac{15x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-15*x)/(8*a) + (15*Tan[c + d*x])/(8*a*d) - (5*Sin[c + d*x]^2*Tan[c + d*x])
/(8*a*d) - (Sin[c + d*x]^4*Tan[c + d*x])/(4*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin^4(c+dx)\tan^2(c+dx)\,dx}{a} \\
&= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{ad} \\
&= -\frac{\sin^4(c+dx)\tan(c+dx)}{4ad} + \frac{5\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{4ad} \\
&= -\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} - \frac{\sin^4(c+dx)\tan(c+dx)}{4ad} + \frac{15\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x\right.}{8ad} \\
&= \frac{15\tan(c+dx)}{8ad} - \frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} - \frac{\sin^4(c+dx)\tan(c+dx)}{4ad} - \frac{15\text{Su}}{} \\
&= -\frac{15x}{8a} + \frac{15\tan(c+dx)}{8ad} - \frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} - \frac{\sin^4(c+dx)\tan(c+dx}{4ad}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 44, normalized size = 0.60

$$-\frac{60c+60dx-16\sin(2(c+dx))+\sin(4(c+dx))-32\tan(c+dx)}{32ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
[Out] -1/32*(60*c + 60*d*x - 16*Sin[2*(c + d*x)] + Sin[4*(c + d*x)] - 32*Tan[c +
d*x])/(a*d)
```

Maple [A]
time = 0.21, size = 57, normalized size = 0.78

method	result
derivativedivides	$\frac{\tan(dx+c)-\frac{-\frac{9\left(\tan^3(dx+c)\right)}{8}-\frac{7\tan(dx+c)}{8}}{\left(\tan^2(dx+c)+1\right)^2}-\frac{15\arctan(\tan(dx+c))}{8}}{da}$
default	$\frac{\tan(dx+c)-\frac{-\frac{9\left(\tan^3(dx+c)\right)}{8}-\frac{7\tan(dx+c)}{8}}{\left(\tan^2(dx+c)+1\right)^2}-\frac{15\arctan(\tan(dx+c))}{8}}{da}$
risch	$-\frac{15x}{8a}-\frac{i\mathrm{e}^{2i(dx+c)}}{4ad}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4ad}+\frac{2i}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{\sin(4dx+4c)}{32ad}$
norman	$\frac{15x}{8a}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{35\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{113\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{29\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{113\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{35\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(tan(d*x+c)-(-9/8*tan(d*x+c)^3-7/8*tan(d*x+c))/(tan(d*x+c)^2+1)^2-15/
8*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 72, normalized size = 0.99

$$\frac{\frac{9\,\tan(dx+c)^3+7\,\tan(dx+c)}{a\tan(dx+c)^4+2\,a\tan(dx+c)^2+a}-\frac{15\,(dx+c)}{a}+\frac{8\,\tan(dx+c)}{a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((9*tan(d*x + c)^3 + 7*tan(d*x + c))/(a*tan(d*x + c)^4 + 2*a*tan(d*x +
c)^2 + a) - 15*(d*x + c)/a + 8*tan(d*x + c)/a)/d
```

Fricas [A]
time = 0.38, size = 56, normalized size = 0.77

$$-\frac{15\,dx\cos\left(dx+c\right)+\left(2\,\cos\left(dx+c\right)^4-9\,\cos\left(dx+c\right)^2-8\right)\sin\left(dx+c\right)}{8\,ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(15*d*x*cos(d*x + c) + (2*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 8)*sin(d
*x + c))/(a*d*cos(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(61) = 122$.
time = 7.85, size = 1161, normalized size = 15.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**10/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*
d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2
)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 45*d*x*tan(c/2 + d*x/2)**8/(8*
a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*
x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d)
- 30*d*x*tan(c/2 + d*x/2)**6/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 +
 d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a
*d*tan(c/2 + d*x/2)**2 - 8*a*d) + 30*d*x*tan(c/2 + d*x/2)**4/(8*a*d*tan(c/2
 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 1
6*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) + 45*d*x*ta
n(c/2 + d*x/2)**2/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8
+ 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2
+ d*x/2)**2 - 8*a*d) + 15*d*x/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2
+ d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*
a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 30*tan(c/2 + d*x/2)**9/(8*a*d*tan(c/2 +
d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a
*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 80*tan(c/2 +
 d*x/2)**7/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*
d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2
)**2 - 8*a*d) - 36*tan(c/2 + d*x/2)**5/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d
*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)
**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 80*tan(c/2 + d*x/2)**3/(8*a*d*t
an(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)*
*6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 30*
tan(c/2 + d*x/2)/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 +
 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 +
 d*x/2)**2 - 8*a*d), Ne(d, 0)), (x*sin(c)**6/(-a*sin(c)**2 + a), True))
```

Giac [A]
time = 0.43, size = 63, normalized size = 0.86

$$-\frac{\frac{15\,(dx+c)}{a}-\frac{8\,\tan(dx+c)}{a}-\frac{9\,\tan(dx+c)^3+7\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2 a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(15*(d*x + c)/a - 8*tan(d*x + c)/a - (9*tan(d*x + c)^3 + 7*tan(d*x + c
))/((tan(d*x + c)^2 + 1)^2*a))/d
```

Mupad [B]

time = 13.72, size = 68, normalized size = 0.93

$$\frac{\tan(c+d\,x)}{a\,d}-\frac{15\,x}{8\,a}+\frac{\frac{9\tan(c+d\,x)^3}{8}+\frac{7\tan(c+d\,x)}{8}}{d\,\left(a\tan\left(c+d\,x\right)^4+2\,a\tan\left(c+d\,x\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a - a*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)/(a*d) - (15*x)/(8*a) + ((7*tan(c + d*x))/8 + (9*tan(c + d*x)^3
)/8)/(d*(a + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4))
```

3.43 $\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=49

$$-\frac{3x}{2a}+\frac{3\tan(c+dx)}{2ad}-\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}$$

```
[Out] -3/2*x/a+3/2*tan(d*x+c)/a/d-1/2*sin(d*x+c)^2*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3254, 2671, 294, 327, 209}

$$\frac{3\tan(c+dx)}{2ad}-\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-3*x)/(2*a) + (3*Tan[c + d*x])/(2*a*d) - (Sin[c + d*x]^2*Tan[c + d*x])/(2*
a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin^2(c+dx)\tan^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{ad}\\
&= -\frac{\sin^2(c+dx)\tan(c+dx)}{2ad} + \frac{3\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2ad}\\
&= \frac{3\tan(c+dx)}{2ad} - \frac{\sin^2(c+dx)\tan(c+dx)}{2ad} - \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2ad}\\
&= -\frac{3x}{2a} + \frac{3\tan(c+dx)}{2ad} - \frac{\sin^2(c+dx)\tan(c+dx)}{2ad}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 34, normalized size = 0.69

$$\frac{-6(c+dx)+\sin(2(c+dx))+4\tan(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
[Out] (-6*(c + d*x) + Sin[2*(c + d*x)] + 4*Tan[c + d*x])/(4*a*d)
```

Maple [A]
time = 0.18, size = 44, normalized size = 0.90

method	result
derivativedivides	$\frac{\tan(dx+c)+\frac{\tan(dx+c)}{2\left(\tan^2(dx+c)\right)+2}-\frac{3\arctan(\tan(dx+c))}{2}}{da}$

default	$\frac{\tan(dx+c)+\frac{\tan(dx+c)}{2\left(\tan^2(dx+c)\right)+2}-\frac{3\arctan(\tan(dx+c))}{2}}{da}$
risch	$-\frac{3x}{2a}-\frac{i\mathrm{e}^{2i(dx+c)}}{8ad}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8ad}+\frac{2i}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}$
norman	$\frac{\frac{3x}{2a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{10\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{9x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{3x(\mathrm{ta}}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(tan(d*x+c)+1/2*tan(d*x+c)/(tan(d*x+c)^2+1)-3/2*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 49, normalized size = 1.00

$$-\frac{\frac{3\,(dx+c)}{a}-\frac{\tan(dx+c)}{a\tan(dx+c)^2+a}-\frac{2\,\tan(dx+c)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*(d*x + c)/a - tan(d*x + c)/(a*tan(d*x + c)^2 + a) - 2*tan(d*x + c)/
a)/d
```

Fricas [A]
time = 0.39, size = 45, normalized size = 0.92

$$-\frac{3\,dx\cos\left(dx+c\right)-\left(\cos\left(dx+c\right)^2+2\right)\sin\left(dx+c\right)}{2\,ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*d*x*cos(d*x + c) - (cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d*cos(d*x
+ c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(39) = 78$.
time = 2.89, size = 502, normalized size = 10.24

$$\begin{cases} -\frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad}-\frac{3dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad}+\frac{3dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad}+\frac{3dx}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad}-\frac{6\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad}-\frac{4\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad}-\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad} & \text{for}\: d \neq 0 \\ \frac{x\sin^4\left(c\right)}{-a\sin^2\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**6/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*ta
n(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) - 3*d*x*tan(c/2 + d*
x/2)**4/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(
c/2 + d*x/2)**2 - 2*a*d) + 3*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2
)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) + 3*d
*x/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 +
 d*x/2)**2 - 2*a*d) - 6*tan(c/2 + d*x/2)**5/(2*a*d*tan(c/2 + d*x/2)**6 + 2*
a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) - 4*tan(c/2 +
d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*ta
n(c/2 + d*x/2)**2 - 2*a*d) - 6*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**6
+ 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d), Ne(d, 0))
, (x*sin(c)**4/(-a*sin(c)**2 + a), True))
```

Giac [A]
time = 0.44, size = 50, normalized size = 1.02

$$-\frac{\frac{3\,(dx+c)}{a}-\frac{2\,\tan(dx+c)}{a}-\frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*(d*x + c)/a - 2*tan(d*x + c)/a - tan(d*x + c)/((tan(d*x + c)^2 + 1)
*a))/d
```

Mupad [B]
time = 13.53, size = 45, normalized size = 0.92

$$\frac{\tan(c+d\,x)}{2\,d\,\left(a\tan\left(c+d\,x\right)^2+a\right)}-\frac{3\,x}{2\,a}+\frac{\tan(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - a*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)/(2*d*(a + a*tan(c + d*x)^2)) - (3*x)/(2*a) + tan(c + d*x)/(a*d
)
```

3.44 $\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=20

$$-\frac{x}{a}+\frac{\tan(c+dx)}{ad}$$

[Out] -x/a+tan(d*x+c)/a/d

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3250, 3254, 3852, 8}

$$\frac{\tan(c+dx)}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]

[Out] -(x/a) + Tan[c + d*x]/(a*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx &= -\frac{x}{a}+\int \frac{1}{a-a\sin^2(c+dx)}\,dx\\ &= -\frac{x}{a}+\frac{\int \sec^2(c+dx)\,dx}{a}\\ &= -\frac{x}{a}-\frac{\text{Subst}(\int 1\,dx,x,-\tan(c+dx))}{ad}\\ &= -\frac{x}{a}+\frac{\tan(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.35

$$\frac{-\frac{\tan^{-1}(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
[Out] (-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d)/a
```

Maple [A]
time = 0.15, size = 24, normalized size = 1.20

method	result	size
derivativedivides	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{da}$	24
default	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{da}$	24
risch	$-\frac{x}{a}+\frac{2i}{da\left(e^{2i(dx+c)}+1\right)}$	30
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(tan(d*x+c)-arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.53, size = 26, normalized size = 1.30

$$-\frac{\frac{dx+c}{a}-\frac{\tan(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -((d*x + c)/a - tan(d*x + c)/a)/d
```

Fricas [A]
time = 0.39, size = 34, normalized size = 1.70

$$-\frac{dx\cos(dx+c)-\sin(dx+c)}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) - sin(d*x + c))/(a*d*cos(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(12) = 24$.
time = 1.01, size = 100, normalized size = 5.00

$$\begin{cases} -\frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad}+\frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x\sin^2(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 - a*d) + d*x/(
a*d*tan(c/2 + d*x/2)**2 - a*d) - 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**
2 - a*d), Ne(d, 0)), (x*sin(c)**2/(-a*sin(c)**2 + a), True))
```

Giac [A]
time = 0.50, size = 26, normalized size = 1.30

$$-\frac{\frac{dx+c}{a}-\frac{\tan(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - tan(d*x + c)/a)/d
```

Mupad [B]
time = 13.74, size = 20, normalized size = 1.00

$$\frac{\tan(c+d\,x)}{a\,d}-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - a*sin(c + d*x)^2),x)
[Out] tan(c + d*x)/(a*d) - x/a
```

3.45 $\int \frac{1}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=13

$$\frac{\tan(c+dx)}{ad}$$

[Out] tan(d*x+c)/a/d

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3254, 3852, 8}

$$\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[(a - a*Sin[c + d*x]^2)^(-1),x]

[Out] Tan[c + d*x]/(a*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{ad}\\ &= \frac{\tan(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[c + d*x]^2)^(-1),x]
```

```
[Out] Tan[c + d*x]/(a*d)
```

Maple [A]
time = 0.16, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\tan(dx+c)}{ad}$	14
default	$\frac{\tan(dx+c)}{ad}$	14
risch	$\frac{2i}{da\left(e^{2i(dx+c)}+1\right)}$	23
norman	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] tan(d*x+c)/a/d
```

Maxima [A]
time = 0.32, size = 13, normalized size = 1.00

$$\frac{\tan\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] tan(d*x + c)/(a*d)
```

Fricas [A]
time = 0.39, size = 21, normalized size = 1.62

$$\frac{\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] sin(d*x + c)/(a*d*cos(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(8) = 16$.
time = 0.47, size = 41, normalized size = 3.15

$$\begin{cases} -\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d \neq 0 \\ \frac{x}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 - a*d), Ne(d, 0)),
(x/(-a*sin(c)**2 + a), True))
```

Giac [A]
time = 0.46, size = 13, normalized size = 1.00

$$\frac{\tan(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] tan(d*x + c)/(a*d)
```

Mupad [B]
time = 13.59, size = 13, normalized size = 1.00

$$\frac{\tan(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)/(a*d)
```

3.46 $\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=28

$$-\frac{\cot(c+dx)}{ad}+\frac{\tan(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d+tan(d*x+c)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3254, 2700, 14}

$$\frac{\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(Cot[c + d*x]/(a*d)) + Tan[c + d*x]/(a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \csc^2(c+dx)\sec^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{\cot(c+dx)}{ad} + \frac{\tan(c+dx)}{ad}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 0.57

$$-\frac{2\cot(2(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
[Out] (-2*Cot[2*(c + d*x)])/(a*d)
```

Maple [A]
time = 0.22, size = 25, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\tan(dx+c)-\frac{1}{\tan(dx+c)}}{da}$	25
default	$\frac{\tan(dx+c)-\frac{1}{\tan(dx+c)}}{da}$	25
risch	$-\frac{4i}{da\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)}$	36
norman	$\frac{\frac{1}{2ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(tan(d*x+c)-1/tan(d*x+c))
```

Maxima [A]
time = 0.30, size = 28, normalized size = 1.00

$$\frac{\frac{\tan(dx+c)}{a} - \frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (tan(d*x + c)/a - 1/(a*tan(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 36, normalized size = 1.29

$$-\frac{2\cos(dx+c)^2-1}{ad\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(2*cos(d*x + c)^2 - 1)/(a*d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\csc^2(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**2/(sin(c + d*x)**2 - 1), x)/a
```

Giac [A]
time = 0.45, size = 19, normalized size = 0.68

$$-\frac{2}{ad\tan(2\,dx+2\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/(a*d*tan(2*d*x + 2*c))
```

Mupad [B]
time = 13.50, size = 17, normalized size = 0.61

$$-\frac{2\cot(2\,c+2\,d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] -(2*cot(2*c + 2*d*x))/(a*d)
```

3.47 $\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$-\frac{2\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}$$

```
[Out] -2*cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+tan(d*x+c)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3254, 2700, 276}

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{2\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-2*Cot[c + d*x])/(a*d) - Cot[c + d*x]^3/(3*a*d) + Tan[c + d*x]/(a*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \csc^4(c+dx)\sec^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{2\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\tan(c+dx)}{ad}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 1.07

$$\frac{-\frac{5\cot(c+dx)}{3d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d} + \frac{\tan(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
[Out] ((-5*Cot[c + d*x])/(3*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) + Tan[c + d*
x]/d)/a
```

Maple [A]
time = 0.24, size = 35, normalized size = 0.76

method	result	size
derivativedivides	$\frac{\tan(dx+c)-\frac{1}{3\tan(dx+c)^3}-\frac{2}{\tan(dx+c)}}{da}$	35
default	$\frac{\tan(dx+c)-\frac{1}{3\tan(dx+c)^3}-\frac{2}{\tan(dx+c)}}{da}$	35
risch	$\frac{16i\left(2\,e^{2i(dx+c)}-1\right)}{3da\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)}$	49
norman	$\frac{\frac{1}{24ad}+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{15\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(tan(d*x+c)-1/3/tan(d*x+c)^3-2/tan(d*x+c))
```

Maxima [A]
time = 0.32, size = 42, normalized size = 0.91

$$\frac{\frac{3\,\tan(dx+c)}{a} - \frac{6\,\tan(dx+c)^2+1}{a\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*tan(d*x + c)/a - (6*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

Fricas [A]
time = 0.38, size = 56, normalized size = 1.22

$$-\frac{8\cos\left(dx+c\right)^{4}-12\cos\left(dx+c\right)^{2}+3}{3\left(ad\cos\left(dx+c\right)^{3}-ad\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 3)/((a*d*cos(d*x + c)^3 - a*d*
cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\csc^{4}\left(c+dx\right)}{\sin^{2}\left(c+dx\right)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**4/(sin(c + d*x)**2 - 1), x)/a
```

Giac [A]
time = 0.44, size = 42, normalized size = 0.91

$$\frac{\frac{3\tan(dx+c)}{a}-\frac{6\tan(dx+c)^{2}+1}{a\tan(dx+c)^{3}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*tan(d*x + c)/a - (6*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

Mupad [B]
time = 13.73, size = 38, normalized size = 0.83

$$-\frac{-\tan(c+d\,x)^{4}+2\tan(c+d\,x)^{2}+\frac{1}{3}}{a\,d\tan\left(c+d\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] -(2*tan(c + d*x)^2 - tan(c + d*x)^4 + 1/3)/(a*d*tan(c + d*x)^3)
```

3.48 $\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=62

$$-\frac{3\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\tan(c+dx)}{ad}$$

```
[Out] -3*cot(d*x+c)/a/d-cot(d*x+c)^3/a/d-1/5*cot(d*x+c)^5/a/d+tan(d*x+c)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2700, 276}

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{ad}-\frac{3\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-3*Cot[c + d*x])/(a*d) - Cot[c + d*x]^3/(a*d) - Cot[c + d*x]^5/(5*a*d) + T
an[c + d*x]/(a*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \csc^6(c+dx)\sec^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^6}+\frac{3}{x^4}+\frac{3}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{3\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{\tan(c+dx)}{ad}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 1.13

$$\frac{-\frac{11\cot(c+dx)}{5d} - \frac{3\cot(c+dx)\csc^2(c+dx)}{5d} - \frac{\cot(c+dx)\csc^4(c+dx)}{5d} + \frac{\tan(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
[Out] ((-11*Cot[c + d*x])/(5*d) - (3*Cot[c + d*x]*Csc[c + d*x]^2)/(5*d) - (Cot[c
+ d*x]*Csc[c + d*x]^4)/(5*d) + Tan[c + d*x]/d)/a
```

Maple [A]
time = 0.25, size = 45, normalized size = 0.73

method	result
derivativedivides	$\frac{\tan(dx+c)-\frac{1}{5\tan(dx+c)^5}-\frac{3}{\tan(dx+c)}-\frac{1}{\tan(dx+c)^3}}{da}$
default	$\frac{\tan(dx+c)-\frac{1}{5\tan(dx+c)^5}-\frac{3}{\tan(dx+c)}-\frac{1}{\tan(dx+c)^3}}{da}$
risch	$-\frac{32i\left(5\,e^{4i(dx+c)}-4\,e^{2i(dx+c)}+1\right)}{5da\left(e^{2i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)}$
norman	$\frac{\frac{1}{160ad}+\frac{7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80ad}+\frac{35\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-\frac{35\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{35\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{7\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80ad}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(tan(d*x+c)-1/5/tan(d*x+c)^5-3/tan(d*x+c)-1/tan(d*x+c)^3)
```

Maxima [A]
time = 0.30, size = 52, normalized size = 0.84

$$\frac{\frac{5\,\tan(dx+c)}{a} - \frac{15\,\tan(dx+c)^4+5\,\tan(dx+c)^2+1}{a\tan(dx+c)^5}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/5*(5*tan(d*x + c)/a - (15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/(a*tan(d
*x + c)^5))/d
```

Fricas [A]
time = 0.37, size = 77, normalized size = 1.24

$$-\frac{16\cos(dx+c)^6-40\cos(dx+c)^4+30\cos(dx+c)^2-5}{5\left(ad\cos(dx+c)^5-2\,ad\cos(dx+c)^3+ad\cos(dx+c)\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/5*(16*cos(d*x + c)^6 - 40*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 5)/((a*d*
cos(d*x + c)^5 - 2*a*d*cos(d*x + c)^3 + a*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\csc^6(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**6/(sin(c + d*x)**2 - 1), x)/a
```

Giac [A]
time = 0.41, size = 52, normalized size = 0.84

$$\frac{\frac{5\tan(dx+c)}{a}-\frac{15\tan(dx+c)^4+5\tan(dx+c)^2+1}{a\tan(dx+c)^5}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/5*(5*tan(d*x + c)/a - (15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/(a*tan(d
*x + c)^5))/d
```

Mupad [B]
time = 13.96, size = 50, normalized size = 0.81

$$\frac{\tan(c+d\,x)}{a\,d}-\frac{3\tan(c+d\,x)^4+\tan(c+d\,x)^2+\frac{1}{5}}{a\,d\tan\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] tan(c + d*x)/(a*d) - (tan(c + d*x)^2 + 3*tan(c + d*x)^4 + 1/5)/(a*d*tan(c +
 d*x)^5)
```

3.49 $\int \frac{\sin^7(c+dx)}{(a-a\sin^2(c+dx))^2} dx$

Optimal. Leaf size=65

$$-\frac{3\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{3\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

```
[Out] -3*cos(d*x+c)/a^2/d+1/3*cos(d*x+c)^3/a^2/d-3*sec(d*x+c)/a^2/d+1/3*sec(d*x+c
)^3/a^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2670, 276}

$$\frac{\cos^3(c+dx)}{3a^2d}-\frac{3\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{3\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-3*Cos[c + d*x])/(a^2*d) + Cos[c + d*x]^3/(3*a^2*d) - (3*Sec[c + d*x])/(a^
2*d) + Sec[c + d*x]^3/(3*a^2*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = \frac{\int \sin^3(c+dx)\tan^4(c+dx)\,dx}{a^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{a^2 d}$$
$$= -\frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)\,dx, x, \cos(c+dx)\right)}{a^2 d}$$
$$= -\frac{3\cos(c+dx)}{a^2 d}+\frac{\cos^3(c+dx)}{3a^2 d}-\frac{3\sec(c+dx)}{a^2 d}+\frac{\sec^3(c+dx)}{3a^2 d}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.91

$$\frac{-\frac{11\cos(c+dx)}{4d}+\frac{\cos(3(c+dx))}{12d}-\frac{3\sec(c+dx)}{d}+\frac{\sec^3(c+dx)}{3d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-11*Cos[c + d*x])/(4*d) + Cos[3*(c + d*x)]/(12*d) - (3*Sec[c + d*x])/d +
Sec[c + d*x]^3/(3*d))/a^2
```

Maple [A]
time = 0.16, size = 47, normalized size = 0.72

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^3(dx+c)\right)}{3}-3\cos(dx+c)-\frac{3}{\cos(dx+c)}+\frac{1}{3\cos(dx+c)^3}}{d\,a^2}$	47
default	$\frac{\frac{\left(\cos^3(dx+c)\right)}{3}-3\cos(dx+c)-\frac{3}{\cos(dx+c)}+\frac{1}{3\cos(dx+c)^3}}{d\,a^2}$	47
risch	$\frac{e^{3i(dx+c)}}{24d\,a^2}-\frac{11\,e^{i(dx+c)}}{8d\,a^2}-\frac{11\,e^{-i(dx+c)}}{8d\,a^2}+\frac{e^{-3i(dx+c)}}{24d\,a^2}-\frac{2\left(9\,e^{5i(dx+c)}+14\,e^{3i(dx+c)}+9\,e^{i(dx+c)}\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/3*cos(d*x+c)^3-3*cos(d*x+c)-3/cos(d*x+c)+1/3/cos(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.80

$$\frac{\frac{\cos(dx+c)^3-9\cos(dx+c)}{a^2}-\frac{9\cos(dx+c)^2-1}{a^2\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 9*cos(d*x + c))/a^2 - (9*cos(d*x + c)^2 - 1)/(a^2*co
s(d*x + c)^3))/d
```

Fricas [A]
time = 0.40, size = 46, normalized size = 0.71

$$\frac{\cos\left(dx+c\right)^{6}-9\,\cos\left(dx+c\right)^{4}-9\,\cos\left(dx+c\right)^{2}+1}{3\,a^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^6 - 9*cos(d*x + c)^4 - 9*cos(d*x + c)^2 + 1)/(a^2*d*cos(d
*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(56) = 112.
time = 29.63, size = 156, normalized size = 2.40

$$\begin{cases} -\frac{96\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d}+\frac{32}{3a^{2}d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d} & \text{for}\ d\neq 0 \\ \frac{x\sin^{7}\left(c\right)}{\left(-a\sin^{2}\left(c\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-96*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**12 - 9*a**2*
d*tan(c/2 + d*x/2)**8 + 9*a**2*d*tan(c/2 + d*x/2)**4 - 3*a**2*d) + 32/(3*a*
*2*d*tan(c/2 + d*x/2)**12 - 9*a**2*d*tan(c/2 + d*x/2)**8 + 9*a**2*d*tan(c/2
 + d*x/2)**4 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**7/(-a*sin(c)**2 + a)**2, Tr
ue))
```

Giac [A]
time = 0.45, size = 57, normalized size = 0.88

$$-\frac{32\left(\frac{3\left(\cos(dx+c)-1\right)^{2}}{\left(\cos(dx+c)+1\right)^{2}}-1\right)}{3\,a^{2}d\left(\frac{\left(\cos(dx+c)-1\right)^{2}}{\left(\cos(dx+c)+1\right)^{2}}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -32/3*(3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)/(a^2*d*((cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)^3)
```

Mupad [B]
time = 13.64, size = 48, normalized size = 0.74

$$-\frac{-\cos\left(c+d\,x\right)^6+9\cos\left(c+d\,x\right)^4+9\cos\left(c+d\,x\right)^2-1}{3\,a^2\,d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] -(9*cos(c + d*x)^2 + 9*cos(c + d*x)^4 - cos(c + d*x)^6 - 1)/(3*a^2*d*cos(c
+ d*x)^3)
```

3.50 $\int \frac{\sin^5(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2} dx$

Optimal. Leaf size=47

$$-\frac{\cos(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

[Out] -cos(d*x+c)/a^2/d-2*sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2670, 276}

$$-\frac{\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2)^2,x]

[Out] -(Cos[c + d*x]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) + Sec[c + d*x]^3/(3*a^2*d)

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\int \sin(c+dx)\tan^4(c+dx)\,dx}{a^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{a^2 d}$$
$$= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx, x, \cos(c+dx)\right)}{a^2 d}$$
$$= -\frac{\cos(c+dx)}{a^2 d} - \frac{2\sec(c+dx)}{a^2 d} + \frac{\sec^3(c+dx)}{3a^2 d}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.89

$$\frac{-\frac{\cos(c+dx)}{d} - \frac{2\sec(c+dx)}{d} + \frac{\sec^3(c+dx)}{3d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-(Cos[c + d*x]/d) - (2*Sec[c + d*x])/d + Sec[c + d*x]^3/(3*d))/a^2
```

Maple [A]
time = 0.20, size = 37, normalized size = 0.79

method	result	size
derivativedivides	$\frac{-\cos(dx+c)+\frac{1}{3\cos(dx+c)^3}-\frac{2}{\cos(dx+c)}}{d\,a^2}$	37
default	$\frac{-\cos(dx+c)+\frac{1}{3\cos(dx+c)^3}-\frac{2}{\cos(dx+c)}}{d\,a^2}$	37
risch	$-\frac{e^{i(dx+c)}}{2d\,a^2}-\frac{e^{-i(dx+c)}}{2d\,a^2}-\frac{4\left(3\,e^{5i(dx+c)}+4\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	91
norman	$\frac{-\frac{112\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{16}{3ad}+\frac{32\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{32\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{128\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{32\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-cos(d*x+c)+1/3/cos(d*x+c)^3-2/cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.87

$$-\frac{\frac{3\,\cos(dx+c)}{a^2}+\frac{6\,\cos(dx+c)^2-1}{a^2\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*cos(d*x + c)/a^2 + (6*cos(d*x + c)^2 - 1)/(a^2*cos(d*x + c)^3))/d
```

Fricas [A]
time = 0.42, size = 38, normalized size = 0.81

$$-\frac{3\cos(dx+c)^4+6\cos(dx+c)^2-1}{3a^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(39) = 78.
time = 12.87, size = 156, normalized size = 3.32

$$\begin{cases} -\frac{32\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d}+\frac{16}{3a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d\neq 0 \\ \frac{x\sin^5(c)}{(-a\sin^2(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-32*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 - 6*a**2*d
*tan(c/2 + d*x/2)**6 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 16/(3*a**
2*d*tan(c/2 + d*x/2)**8 - 6*a**2*d*tan(c/2 + d*x/2)**6 + 6*a**2*d*tan(c/2 +
 d*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**5/(-a*sin(c)**2 + a)**2, True
))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(45) = 90.
time = 0.44, size = 106, normalized size = 2.26

$$\frac{2\left(\frac{3}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)}-\frac{\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+5}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 2/3*(3/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (12*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5
)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

Mupad [B]
time = 0.05, size = 36, normalized size = 0.77

$$-\frac{\cos\left(c+d\,x\right)^4+2\cos\left(c+d\,x\right)^2-\frac{1}{3}}{a^2\,d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] -(2*cos(c + d*x)^2 + cos(c + d*x)^4 - 1/3)/(a^2*d*cos(c + d*x)^3)
```

3.51 $\int \frac{\sin^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

```
[Out] -sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3254, 2686}

$$\frac{\sec^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(Sec[c + d*x]/(a^2*d)) + Sec[c + d*x]^3/(3*a^2*d)
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a^2}\\ &= \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\sec(c+dx)\right)}{a^2d}\\ &= -\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 0.94

$$\frac{-\frac{\sec(c+dx)}{d}+\frac{\sec^3(c+dx)}{3d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
[Out] (-(Sec[c + d*x]/d) + Sec[c + d*x]^3/(3*d))/a^2
```

Maple [A]
time = 0.20, size = 29, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\frac{1}{3\cos(dx+c)^3}-\frac{1}{\cos(dx+c)}}{d\,a^2}$	29
default	$\frac{\frac{1}{3\cos(dx+c)^3}-\frac{1}{\cos(dx+c)}}{d\,a^2}$	29
risch	$-\frac{2\left(3\,e^{5i(dx+c)}+2\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	56
norman	$\frac{-\frac{8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{32\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{4}{3ad}-\frac{4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/3/cos(d*x+c)^3-1/cos(d*x+c))
```

Maxima [A]
time = 0.30, size = 28, normalized size = 0.85

$$-\frac{3\cos\left(dx+c\right)^2-1}{3\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

Fricas [A]
time = 0.39, size = 28, normalized size = 0.85

$$-\frac{3\cos\left(dx+c\right)^2-1}{3\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(26) = 52$.
time = 4.58, size = 156, normalized size = 4.73

$$\begin{cases} -\frac{12\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d}+\frac{4}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d \neq 0 \\ \frac{x\sin^3(c)}{\left(-a\sin^2(c)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-12*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d
*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 4/(3*a**2
*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 +
d*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**3/(-a*sin(c)**2 + a)**2, True)
)
```

Giac [A]
time = 0.41, size = 28, normalized size = 0.85

$$-\frac{3\cos\left(dx+c\right)^2-1}{3\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

Mupad [B]
time = 13.56, size = 26, normalized size = 0.79

$$-\frac{\cos\left(c+d\,x\right)^2-\frac{1}{3}}{a^2\,d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] -(cos(c + d*x)^2 - 1/3)/(a^2*d*cos(c + d*x)^3)
```

3.52 $\int \frac{\sin(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{\sec^3(c+dx)}{3a^2d}$$

```
[Out] 1/3*sec(d*x+c)^3/a^2/d
```

Rubi [A]

time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3254, 2686, 30}

$$\frac{\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = \frac{\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^2\,dx, x, \sec(c+dx)\right)}{a^2 d}$$
$$= \frac{\sec^3(c+dx)}{3a^2 d}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{\sec^3(c+dx)}{3a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^2*d)
```

Maple [A]
time = 0.15, size = 17, normalized size = 0.94

method	result	size
derivativedivides	$\frac{1}{3d\,a^2\cos(dx+c)^3}$	17
default	$\frac{1}{3d\,a^2\cos(dx+c)^3}$	17
risch	$\frac{8\,e^{3i(dx+c)}}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	31
norman	$\frac{-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2}{3ad}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d/a^2/cos(d*x+c)^3
```

Maxima [A]
time = 0.31, size = 16, normalized size = 0.89

$$\frac{1}{3\,a^2 d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3/(a^2*d*cos(d*x + c)^3)
```

Fricas [A]
time = 0.37, size = 16, normalized size = 0.89

$$\frac{1}{3\,a^2 d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3/(a^2*d*cos(d*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 2.15, size = 156, normalized size = 8.67

$$\begin{cases} -\frac{6\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} - \frac{2}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for}\ d\neq 0 \\ \frac{x\sin\left(c\right)}{\left(-a\sin^2\left(c\right)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-6*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 2/(3*a**2*
d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d
*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)/(-a*sin(c)**2 + a)**2, True))
```

Giac [A]
time = 0.52, size = 16, normalized size = 0.89

$$\frac{1}{3\,a^2 d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3/(a^2*d*cos(d*x + c)^3)
```

Mupad [B]
time = 13.59, size = 16, normalized size = 0.89

$$\frac{1}{3\,a^2\,d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] 1/(3*a^2*d*cos(c + d*x)^3)
```

3.53 $\int \frac{\csc(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3254, 2702, 308, 213}

$$\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
```

```
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a^2}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2 d}\\
&= \frac{\operatorname{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \sec(c+dx)\right)}{a^2 d}\\
&= \frac{\sec(c+dx)}{a^2 d}+\frac{\sec^3(c+dx)}{3a^2 d}+\frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2 d}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d}+\frac{\sec(c+dx)}{a^2 d}+\frac{\sec^3(c+dx)}{3a^2 d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 1.30

$$\frac{-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\sec(c+dx)}{d}+\frac{\sec^3(c+dx)}{3d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-(Log[Cos[(c + d*x)/2]]/d) + Log[Sin[(c + d*x)/2]]/d + Sec[c + d*x]/d + Se
c[c + d*x]^3/(3*d))/a^2
```

Maple [A]
time = 0.25, size = 49, normalized size = 1.04

method	result	size
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{2}-\frac{\ln(1+\cos(dx+c))}{2}+\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}}{d\,a^2}$	49
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{2}-\frac{\ln(1+\cos(dx+c))}{2}+\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}}{d\,a^2}$	49
norman	$\frac{\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{8}{3ad}-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$	85
risch	$\frac{2\,e^{5i(dx+c)}+\frac{20\,e^{3i(dx+c)}}{3}+2\,e^{i(dx+c)}}{d\,a^2\left(e^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{a^2 d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{a^2 d}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/2*ln(cos(d*x+c)-1)-1/2*ln(1+cos(d*x+c))+1/3/cos(d*x+c)^3+1/cos(d
*x+c))
```

Maxima [A]
time = 0.30, size = 59, normalized size = 1.26

$$-\frac{\frac{3\,\log\left(\cos\left(dx+c\right)+1\right)}{a^2}-\frac{3\,\log\left(\cos\left(dx+c\right)-1\right)}{a^2}-\frac{2\left(3\,\cos\left(dx+c\right)^2+1\right)}{a^2\cos\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*log(cos(d*x + c) + 1)/a^2 - 3*log(cos(d*x + c) - 1)/a^2 - 2*(3*cos(
d*x + c)^2 + 1)/(a^2*cos(d*x + c)^3))/d
```

Fricas [A]
time = 0.39, size = 70, normalized size = 1.49

$$-\frac{3\,\cos\left(dx+c\right)^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,\cos\left(dx+c\right)^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-6\,\cos\left(dx+c\right)^2-2}{6\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*cos(d*x + c)^3*log(-
1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c)^2 - 2)/(a^2*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc\left(c+dx\right)}{\sin^4\left(c+dx\right)-2\sin^2\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(45) = 90.

time = 0.43, size = 107, normalized size = 2.28

$$\frac{\frac{3 \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{8\left(\frac{3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + 2\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + 8*(3*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 2)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

Mupad [B]
time = 0.09, size = 41, normalized size = 0.87

$$\frac{\cos\left(c+d\,x\right)^2+\frac{1}{3}}{a^2\,d\,\cos\left(c+d\,x\right)^3}-\frac{\mathrm{atanh}(\cos\left(c+d\,x\right))}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - a*sin(c + d*x)^2)^2),x)
```

```
[Out] (cos(c + d*x)^2 + 1/3)/(a^2*d*cos(c + d*x)^3) - atanh(cos(c + d*x))/(a^2*d)
```

3.54 $\int \frac{\csc^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

Optimal. Leaf size=78

$$-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{5\sec(c+dx)}{2a^2d}+\frac{5\sec^3(c+dx)}{6a^2d}-\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d}$$

```
[Out] -5/2*arctanh(cos(d*x+c))/a^2/d+5/2*sec(d*x+c)/a^2/d+5/6*sec(d*x+c)^3/a^2/d-
1/2*csc(d*x+c)^2*sec(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3254, 2702, 294, 308, 213}

$$\frac{5\sec^3(c+dx)}{6a^2d}+\frac{5\sec(c+dx)}{2a^2d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (5*Sec[c + d*x])/(2*a^2*d) + (5*Sec[
c + d*x]^3)/(6*a^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*a^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx &= \frac{\int \csc^3(c+dx)\sec^4(c+dx)\,dx}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{a^2d} \\
&= -\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d} + \frac{5\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2a^2d} \\
&= -\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d} + \frac{5\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \sec(c+dx)\right)}{2a^2d} \\
&= \frac{5\sec(c+dx)}{2a^2d} + \frac{5\sec^3(c+dx)}{6a^2d} - \frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d} + \frac{5\text{Subst}\left(\int \frac{1}{-1+x}\right.}{} \\
&= -\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{5\sec(c+dx)}{2a^2d} + \frac{5\sec^3(c+dx)}{6a^2d} - \frac{\csc^2(c+dx)\sec}{2a^2d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(78) = 156.
time = 0.32, size = 208, normalized size = 2.67

$$\frac{2\csc^8(c+dx)\left(22-40\cos(2(c+dx))+13\cos(3(c+dx))-30\cos(4(c+dx))+13\cos(5(c+dx))+15\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+15\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-15\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-15\cos(5(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\cos(c+dx)\left(-26-30\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+30\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{3a^2d\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (2*Csc[c + d*x]^8*(22 - 40*Cos[2*(c + d*x)] + 13*Cos[3*(c + d*x)] - 30*Cos[
4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 15*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/
2]] + 15*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 15*Cos[3*(c + d*x)]*Log[S
in[(c + d*x)/2]] - 15*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]] + Cos[c + d*x]
*(-26 - 30*Log[Cos[(c + d*x)/2]] + 30*Log[Sin[(c + d*x)/2]])))/(3*a^2*d*(Cs
c[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)^3)
```

Maple [A]
time = 0.31, size = 75, normalized size = 0.96

method	result	size
derivativedivides	$\frac{\frac{1}{4+4\cos(dx+c)}-\frac{5\ln(1+\cos(dx+c))}{4}+\frac{1}{3\cos(dx+c)^{3}}+\frac{2}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)-4}+\frac{5\ln(\cos(dx+c)-1)}{4}}{d\,a^{2}}$	75
default	$\frac{\frac{1}{4+4\cos(dx+c)}-\frac{5\ln(1+\cos(dx+c))}{4}+\frac{1}{3\cos(dx+c)^{3}}+\frac{2}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)-4}+\frac{5\ln(\cos(dx+c)-1)}{4}}{d\,a^{2}}$	75
risch	$\frac{15\,e^{9i(dx+c)}+20\,e^{7i(dx+c)}-22\,e^{5i(dx+c)}+20\,e^{3i(dx+c)}+15\,e^{i(dx+c)}}{3d\,a^{2}\left(e^{2i(dx+c)}+1\right)^{3}\left(e^{2i(dx+c)}-1\right)^{2}}-\frac{5\ln\left(e^{i(dx+c)}+1\right)}{2a^{2}d}+\frac{5\ln\left(e^{i(dx+c)}-1\right)}{2a^{2}d}$	132
norman	$\frac{\frac{1}{8ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{75\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{65\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{55\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/4/(1+cos(d*x+c))-5/4*ln(1+cos(d*x+c))+1/3/cos(d*x+c)^3+2/cos(d*x
+c)+1/4/(cos(d*x+c)-1)+5/4*ln(cos(d*x+c)-1))
```

Maxima [A]
time = 0.34, size = 86, normalized size = 1.10

$$\frac{\frac{2\left(15\cos(dx+c)^{4}-10\cos(dx+c)^{2}-2\right)}{a^{2}\cos(dx+c)^{5}-a^{2}\cos(dx+c)^{3}}-\frac{15\log(\cos(dx+c)+1)}{a^{2}}+\frac{15\log(\cos(dx+c)-1)}{a^{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(a^2*cos(d*x + c)^5 - a
^2*cos(d*x + c)^3) - 15*log(cos(d*x + c) + 1)/a^2 + 15*log(cos(d*x + c) - 1
)/a^2)/d
```

Fricas [A]
time = 0.41, size = 118, normalized size = 1.51

$$\frac{30\cos(dx+c)^{4}-20\cos(dx+c)^{2}-15\left(\cos(dx+c)^{5}-\cos(dx+c)^{3}\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15\left(\cos(dx+c)^{5}-\cos(dx+c)^{3}\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-4}{12\left(a^{2}d\cos(dx+c)^{5}-a^{2}d\cos(dx+c)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(30*cos(d*x + c)^4 - 20*cos(d*x + c)^2 - 15*(cos(d*x + c)^5 - cos(d*x
+ c)^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^5 - cos(d*x + c)^3)*
log(-1/2*cos(d*x + c) + 1/2) - 4)/(a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c
)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.
time = 0.44, size = 175, normalized size = 2.24

$$-\frac{\frac{3\left(\frac{10(\cos(dx+c)-1)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}-\frac{30\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{3(\cos(dx+c)-1)}{a^2(\cos(dx+c)+1)}-\frac{16\left(\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+7\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*(10*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)/
(a^2*(cos(d*x + c) - 1)) - 30*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/a^2 + 3*(cos(d*x + c) - 1)/(a^2*(cos(d*x + c) + 1)) - 16*(12*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 7)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

Mupad [**B**]
time = 13.76, size = 70, normalized size = 0.90

$$\frac{-\frac{5\cos(c+d\,x)^4}{2}+\frac{5\cos(c+d\,x)^2}{3}+\frac{1}{3}}{d\,\left(a^2\cos\left(c+d\,x\right)^3-a^2\cos\left(c+d\,x\right)^5\right)}-\frac{5\,\mathrm{atanh}(\cos\left(c+d\,x\right))}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a - a*sin(c + d*x)^2)^2),x)
```

```
[Out] ((5*cos(c + d*x)^2)/3 - (5*cos(c + d*x)^4)/2 + 1/3)/(d*(a^2*cos(c + d*x)^3
- a^2*cos(c + d*x)^5)) - (5*atanh(cos(c + d*x)))/(2*a^2*d)
```

3.55 $\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

Optimal. Leaf size=69

$$\frac{5x}{2a^2}-\frac{5\tan(c+dx)}{2a^2d}+\frac{5\tan^3(c+dx)}{6a^2d}-\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}$$

```
[Out] 5/2*x/a^2-5/2*tan(d*x+c)/a^2/d+5/6*tan(d*x+c)^3/a^2/d-1/2*sin(d*x+c)^2*tan(
d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3254, 2671, 294, 308, 209}

$$\frac{5\tan^3(c+dx)}{6a^2d}-\frac{5\tan(c+dx)}{2a^2d}-\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{5x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (5*x)/(2*a^2) - (5*Tan[c + d*x])/(2*a^2*d) + (5*Tan[c + d*x]^3)/(6*a^2*d) -
 (Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\int \sin^2(c+dx)\tan^4(c+dx)\,dx}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{a^2 d}$$
$$= -\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d} + \frac{5\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2a^2d}$$
$$= -\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d} + \frac{5\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx, x, \tan(c+dx\right)}{2a^2d}$$
$$= -\frac{5\tan(c+dx)}{2a^2d} + \frac{5\tan^3(c+dx)}{6a^2d} - \frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d} + \frac{5\text{Subst}\left(\int \frac{}{1+}\right.}{}$$
$$= \frac{5x}{2a^2} - \frac{5\tan(c+dx)}{2a^2d} + \frac{5\tan^3(c+dx)}{6a^2d} - \frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}$$

Mathematica [A]
time = 0.15, size = 46, normalized size = 0.67

$$\frac{30(c+dx) - 3\sin(2(c+dx)) + 4(-7+\sec^2(c+dx))\tan(c+dx)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (30*(c + d*x) - 3*Sin[2*(c + d*x)] + 4*(-7 + Sec[c + d*x]^2)*Tan[c + d*x])/
(12*a^2*d)
```

Maple [A]
time = 0.14, size = 56, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-2\tan(dx+c)-\frac{\tan(dx+c)}{2\left(\tan^2(dx+c)+1\right)}+\frac{5\arctan(\tan(dx+c))}{2}}{d\,a^2}$	56
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-2\tan(dx+c)-\frac{\tan(dx+c)}{2\left(\tan^2(dx+c)+1\right)}+\frac{5\arctan(\tan(dx+c))}{2}}{d\,a^2}$	56
risch	$\frac{5x}{2a^2}+\frac{i\mathrm{e}^{2i(dx+c)}}{8d\,a^2}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8d\,a^2}-\frac{2i\left(9\,\mathrm{e}^{4i(dx+c)}+12\,\mathrm{e}^{2i(dx+c)}+7\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3-2*tan(d*x+c)-1/2*tan(d*x+c)/(tan(d*x+c)^2+1)+5/2*
arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.53, size = 64, normalized size = 0.93

$$-\frac{\frac{3\,\tan(dx+c)}{a^2\tan(dx+c)^2+a^2}-\frac{2\left(\tan(dx+c)^3-6\,\tan(dx+c)\right)}{a^2}-\frac{15\,(dx+c)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*tan(d*x + c)/(a^2*tan(d*x + c)^2 + a^2) - 2*(tan(d*x + c)^3 - 6*tan
(d*x + c))/a^2 - 15*(d*x + c)/a^2)/d
```

Fricas [A]
time = 0.39, size = 59, normalized size = 0.86

$$\frac{15\,dx\cos\left(dx+c\right)^3-\left(3\,\cos\left(dx+c\right)^4+14\,\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)}{6\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(15*d*x*cos(d*x + c)^3 - (3*cos(d*x + c)^4 + 14*cos(d*x + c)^2 - 2)*sin
(d*x + c))/(a^2*d*cos(d*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(63) = 126$.
time = 21.95, size = 1275, normalized size = 18.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**10/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a
**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c
/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 15*d*x*tan(c/2
+ d*x/2)**8/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 -
 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*t
an(c/2 + d*x/2)**2 - 6*a**2*d) - 30*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c
/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)
**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2
*d) + 30*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*
tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d
*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) + 15*d*x*tan(c/2 + d*x/
2)**2/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a*
*2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2
 + d*x/2)**2 - 6*a**2*d) - 15*d*x/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d
*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 +
d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) + 30*tan(c/2 + d*x/2)*
*9/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*
d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 +
d*x/2)**2 - 6*a**2*d) - 40*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**
10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2
*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 44*tan(
c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)*
*8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2
*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 40*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c
/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)
**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2
*d) + 30*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2
 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**
4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d), Ne(d, 0)), (x*sin(c)**6/(-a*s
in(c)**2 + a)**2, True))
```

Giac [A]
time = 0.45, size = 68, normalized size = 0.99

$$\frac{\frac{15\,(dx+c)}{a^2} - \frac{3\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)a^2} + \frac{2\left(a^4\tan(dx+c)^3 - 6\,a^4\tan(dx+c)\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)/a^2 - 3*tan(d*x + c)/((tan(d*x + c)^2 + 1)*a^2) + 2*(a^4*
tan(d*x + c)^3 - 6*a^4*tan(d*x + c))/a^6)/d
```

Mupad [B]
time = 13.81, size = 66, normalized size = 0.96

$$\frac{5\,x}{2\,a^2}-\frac{\tan(c+d\,x)}{2\,d\,\left(a^2\tan\left(c+d\,x\right)^2+a^2\right)}-\frac{2\tan(c+d\,x)}{a^2\,d}+\frac{\tan(c+d\,x)^3}{3\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] (5*x)/(2*a^2) - tan(c + d*x)/(2*d*(a^2 + a^2*tan(c + d*x)^2)) - (2*tan(c +
d*x))/(a^2*d) + tan(c + d*x)^3/(3*a^2*d)
```

3.56 $\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=38

$$\frac{x}{a^2}-\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}$$

```
[Out] x/a^2-tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 3554, 8}

$$\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - Tan[c + d*x]/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \tan^4(c+dx)\,dx}{a^2}\\ &= \frac{\tan^3(c+dx)}{3a^2d}-\frac{\int \tan^2(c+dx)\,dx}{a^2}\\ &= -\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}+\frac{\int 1\,dx}{a^2}\\ &= \frac{x}{a^2}-\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.11

$$\frac{\frac{\tan^{-1}(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
[Out] (ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))/a^2
```

Maple [A]
time = 0.20, size = 34, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d\,a^2}$
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d\,a^2}$
risch	$\frac{x}{a^2}-\frac{4i\left(3\,e^{4i(dx+c)}+3\,e^{2i(dx+c)}+2\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{\frac{x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x}{a}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{38\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{24\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{38\left(\tan\right.}{}}{\left(1+\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 37, normalized size = 0.97

$$\frac{\frac{\tan(dx+c)^3-3\,\tan(dx+c)}{a^2}+\frac{3\,(dx+c)}{a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 - 3*tan(d*x + c))/a^2 + 3*(d*x + c)/a^2)/d
```

Fricas [A]
time = 0.39, size = 49, normalized size = 1.29

$$\frac{3\,dx\cos\left(dx+c\right)^{3}-\left(4\,\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)}{3\,a^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c)^3 - (4*cos(d*x + c)^2 - 1)*sin(d*x + c))/(a^2*d*cos
(d*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(31) = 62$.
time = 8.06, size = 551, normalized size = 14.50

$$\begin{cases} \frac{3dx\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d}-\frac{9dx\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d}+\frac{9dx\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d}-\frac{3dx}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d}+\frac{6\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d}-\frac{20\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d}+\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d} & \text{for}\: d \neq 0 \\ \frac{x\sin^{4}\left(c\right)}{\left(-a\sin^{2}\left(c\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2
*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 9*d*x*t
an(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2
)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2)**2
/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*ta
n(c/2 + d*x/2)**2 - 3*a**2*d) - 3*d*x/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**
2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 6*tan(
c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**
4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 20*tan(c/2 + d*x/2)**3/(3*a*
*2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
+ d*x/2)**2 - 3*a**2*d) + 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6
- 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d),
Ne(d, 0)), (x*sin(c)**4/(-a*sin(c)**2 + a)**2, True))
```

Giac [A]
time = 0.44, size = 44, normalized size = 1.16

$$\frac{\frac{3\,(dx+c)}{a^{2}}+\frac{a^{4}\tan(dx+c)^{3}-3\,a^{4}\tan(dx+c)}{a^{6}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)/a^2 + (a^4*tan(d*x + c)^3 - 3*a^4*tan(d*x + c))/a^6)/d
```

Mupad [B]
time = 13.48, size = 31, normalized size = 0.82

$$\frac{x}{a^2} - \frac{\tan(c + d\,x) - \frac{\tan(c+d\,x)^3}{3}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] x/a^2 - (tan(c + d*x) - tan(c + d*x)^3/3)/(a^2*d)
```

3.57 $\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^3(c+dx)}{3a^2d}$$

```
[Out] 1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2687, 30}

$$\frac{\tan^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Tan[c + d*x]^3/(3*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{a^2d}$$
$$= \frac{\tan^3(c+dx)}{3a^2d}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{\tan^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Tan[c + d*x]^3/(3*a^2*d)
```

Maple [A]
time = 0.17, size = 17, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\tan^3(dx+c)}{3a^2d}$	17
default	$\frac{\tan^3(dx+c)}{3a^2d}$	17
risch	$-\frac{2i\left(3\,\mathrm{e}^{4i(dx+c)}+1\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	36
norman	$\frac{-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*tan(d*x+c)^3/a^2/d
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.89

$$\frac{\tan\left(dx+c\right)^3}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*tan(d*x + c)^3/(a^2*d)
```

Fricas [A]
time = 0.38, size = 32, normalized size = 1.78

$$-\frac{\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)}{3\,a^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(14) = 28$.
time = 3.27, size = 94, normalized size = 5.22

$$\begin{cases} -\frac{8\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d} & \text{for}\ d\neq 0 \\ \frac{x\sin^{2}\left(c\right)}{\left(-a\sin^{2}\left(c\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-8*tan(c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d), Ne(d, 0)),
(x*sin(c)**2/(-a*sin(c)**2 + a)**2, True))
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.89

$$\frac{\tan\left(dx+c\right)^{3}}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*tan(d*x + c)^3/(a^2*d)
```

Mupad [B]
time = 13.38, size = 16, normalized size = 0.89

$$\frac{{\tan\left(c+d\,x\right)}^{3}}{3\,a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] tan(c + d*x)^3/(3*a^2*d)
```

3.58 $\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=32

$$\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}$$

```
[Out] tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {3254, 3852}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] Tan[c + d*x]/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sec^4(c+dx)\,dx}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{a^2d}\\
&= \frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 0.81

$$\frac{\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[c + d*x]^2)^(-2),x]
[Out] (Tan[c + d*x] + Tan[c + d*x]^3/3)/(a^2*d)
```

Maple [A]
time = 0.16, size = 25, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)}{d\,a^2}$	25
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)}{d\,a^2}$	25
risch	$\frac{4i\left(3\,e^{2i(dx+c)}+1\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	36
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3+tan(d*x+c))
```

Maxima [A]
time = 0.31, size = 25, normalized size = 0.78

$$\frac{\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d)
```

Fricas [A]
time = 0.37, size = 34, normalized size = 1.06

$$\frac{\left(2\,\cos\left(dx+c\right)^2+1\right)\sin\left(dx+c\right)}{3\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(26) = 52$.
time = 1.33, size = 238, normalized size = 7.44

$$\begin{cases} -\frac{6\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} + \frac{4\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} - \frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d \neq 0 \\ \frac{x}{\left(-a\sin^2(c)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-6*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 4*tan(c/2
+ d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 +
9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 6*tan(c/2 + d*x/2)/(3*a**2*d*tan
(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)
**2 - 3*a**2*d), Ne(d, 0)), (x/(-a*sin(c)**2 + a)**2, True))
```

Giac [A]
time = 0.51, size = 25, normalized size = 0.78

$$\frac{\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)}{3\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d)
```

Mupad [B]
time = 13.45, size = 24, normalized size = 0.75

$$\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2+3\right)}{3\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] (tan(c + d*x)*(tan(c + d*x)^2 + 3))/(3*a^2*d)
```

3.59 $\int \frac{\csc^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}$$

```
[Out] -cot(d*x+c)/a^2/d+2*tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3254, 2700, 276}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(Cot[c + d*x]/(a^2*d)) + (2*Tan[c + d*x])/(a^2*d) + Tan[c + d*x]^3/(3*a^2*
d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x, \tan(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)\,dx, x, \tan(c+dx)\right)}{a^2 d}$$
$$= -\frac{\cot(c+dx)}{a^2 d} + \frac{2\tan(c+dx)}{a^2 d} + \frac{\tan^3(c+dx)}{3a^2 d}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 1.06

$$\frac{-\frac{\cot(c+dx)}{d} + \frac{5\tan(c+dx)}{3d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
[Out] (-(Cot[c + d*x]/d) + (5*Tan[c + d*x])/(3*d) + (Sec[c + d*x]^2*Tan[c + d*x])
/(3*d))/a^2
```

Maple [A]
time = 0.26, size = 37, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+2\tan(dx+c)-\frac{1}{\tan(dx+c)}}{d\,a^2}$	37
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+2\tan(dx+c)-\frac{1}{\tan(dx+c)}}{d\,a^2}$	37
risch	$-\frac{16i\left(2\,e^{2i(dx+c)}+1\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}-1\right)}$	49
norman	$\frac{\frac{1}{2ad}-\frac{6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{25\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{6\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3+2*tan(d*x+c)-1/tan(d*x+c))
```

Maxima [A]
time = 0.30, size = 40, normalized size = 0.85

$$\frac{\frac{\tan(dx+c)^3+6\,\tan(dx+c)}{a^2}-\frac{3}{a^2\tan(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 6*tan(d*x + c))/a^2 - 3/(a^2*tan(d*x + c)))/d
```

Fricas [A]
time = 0.36, size = 46, normalized size = 0.98

$$-\frac{8\,\cos\left(dx+c\right)^4-4\,\cos\left(dx+c\right)^2-1}{3\,a^2d\cos\left(dx+c\right)^3\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3*sin(d*
x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

Giac [A]
time = 0.46, size = 48, normalized size = 1.02

$$-\frac{\frac{3}{a^2\tan(dx+c)}-\frac{a^4\tan(dx+c)^3+6\,a^4\tan(dx+c)}{a^6}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3/(a^2*tan(d*x + c)) - (a^4*tan(d*x + c)^3 + 6*a^4*tan(d*x + c))/a^6)
/d
```

Mupad [B]
time = 13.58, size = 36, normalized size = 0.77

$$\frac{\tan(c+d\,x)^4+6\tan(c+d\,x)^2-3}{3\,a^2\,d\tan\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - a*sin(c + d*x)^2)^2),x)
[Out] (6*tan(c + d*x)^2 + tan(c + d*x)^4 - 3)/(3*a^2*d*tan(c + d*x))
```

3.60 $\int \frac{\csc^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{3\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{3\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}$$

```
[Out] -3*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+3*tan(d*x+c)/a^2/d+1/3*tan(d*x+c
)^3/a^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2700, 276}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{3\tan(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-3*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d) + (3*Tan[c + d*x])/(a^
2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \csc^4(c+dx)\sec^4(c+dx)\,dx}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^4}\,dx, x, \tan(c+dx)\right)}{a^2 d} \\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)\,dx, x, \tan(c+dx)\right)}{a^2 d} \\
&= -\frac{3\cot(c+dx)}{a^2 d} - \frac{\cot^3(c+dx)}{3a^2 d} + \frac{3\tan(c+dx)}{a^2 d} + \frac{\tan^3(c+dx)}{3a^2 d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.71

$$\frac{16\left(-\frac{\cot(2(c+dx))}{3d} - \frac{\cot(2(c+dx))\csc^2(2(c+dx))}{6d}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (16*(-1/3*Cot[2*(c + d*x)]/d - (Cot[2*(c + d*x)]*Csc[2*(c + d*x)]^2)/(6*d))
)/a^2
```

Maple [A]
time = 0.30, size = 47, normalized size = 0.72

method	result	s
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+3\tan(dx+c)-\frac{1}{3\tan(dx+c)^3}-\frac{3}{\tan(dx+c)}}{d\,a^2}$	4
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+3\tan(dx+c)-\frac{1}{3\tan(dx+c)^3}-\frac{3}{\tan(dx+c)}}{d\,a^2}$	4
risch	$\frac{32i\left(3\,e^{4i(dx+c)}-1\right)}{3d\,a^2\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)^3}$	4
norman	$\frac{\frac{1}{24ad}+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{91\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{35\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{91\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{5\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3+3*tan(d*x+c)-1/3/tan(d*x+c)^3-3/tan(d*x+c))
```

Maxima [A]
time = 0.31, size = 52, normalized size = 0.80

$$\frac{\frac{\tan(dx+c)^3+9\tan(dx+c)}{a^2}-\frac{9\tan(dx+c)^2+1}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 9*tan(d*x + c))/a^2 - (9*tan(d*x + c)^2 + 1)/(a^2*ta
n(d*x + c)^3))/d
```

Fricas [A]
time = 0.37, size = 72, normalized size = 1.11

$$-\frac{16\,\cos\left(dx+c\right)^6-24\,\cos\left(dx+c\right)^4+6\,\cos\left(dx+c\right)^2+1}{3\,\left(a^2d\cos\left(dx+c\right)^5-a^2d\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(16*cos(d*x + c)^6 - 24*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 1)/((a^2*d
*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^4\left(c+dx\right)}{\sin^4\left(c+dx\right)-2\sin^2\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

Giac [A]
time = 0.45, size = 34, normalized size = 0.52

$$-\frac{8\left(3\,\tan\left(2\,dx+2\,c\right)^2+1\right)}{3\,a^2d\tan\left(2\,dx+2\,c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -8/3*(3*tan(2*d*x + 2*c)^2 + 1)/(a^2*d*tan(2*d*x + 2*c)^3)
```

Mupad [B]
time = 13.68, size = 48, normalized size = 0.74

$$-\frac{-\tan(c+d\,x)^6-9\tan(c+d\,x)^4+9\tan(c+d\,x)^2+1}{3\,a^2\,d\tan\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a - a*sin(c + d*x)^2)^2),x)
```

```
[Out] -(9*tan(c + d*x)^2 - 9*tan(c + d*x)^4 - tan(c + d*x)^6 + 1)/(3*a^2*d*tan(c
+ d*x)^3)
```

3.61 $\int \frac{1}{\left(a-a\sin^2(x)\right)^3}\,dx$

Optimal. Leaf size=29

$$\frac{\tan(x)}{a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan^5(x)}{5a^3}$$

```
[Out] tan(x)/a^3+2/3*tan(x)^3/a^3+1/5*tan(x)^5/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {3254, 3852}

$$\frac{\tan^5(x)}{5a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-3),x]
```

```
[Out] Tan[x]/a^3 + (2*Tan[x]^3)/(3*a^3) + Tan[x]^5/(5*a^3)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-a\sin^2(x)\right)^3}\,dx&=\frac{\int \sec^6(x)\,dx}{a^3}\\&=-\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(x)\right)}{a^3}\\&=\frac{\tan(x)}{a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan^5(x)}{5a^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.07

$$\frac{\frac{8\tan(x)}{15}+\frac{4}{15}\sec^2(x)\tan(x)+\frac{1}{5}\sec^4(x)\tan(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-3),x]
[Out] ((8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5)/a^3
```

Maple [A]
time = 0.13, size = 20, normalized size = 0.69

method	result	size
default	$\frac{\frac{\left(\tan^5(x)\right)}{5}+\frac{2\left(\tan^3(x)\right)}{3}+\tan(x)}{a^3}$	20
risch	$\frac{16i\left(10\,e^{4ix}+5\,e^{2ix}+1\right)}{15\left(e^{2ix}+1\right)^5 a^3}$	32
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{116\left(\tan^5\left(\frac{x}{2}\right)\right)}{15a}+\frac{8\left(\tan^7\left(\frac{x}{2}\right)\right)}{3a}-\frac{2\left(\tan^9\left(\frac{x}{2}\right)\right)}{a}}{a^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)^5}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^3,x,method=_RETURNVERBOSE)
[Out] 1/a^3*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

Maxima [A]
time = 0.29, size = 22, normalized size = 0.76

$$\frac{3\,\tan\left(x\right)^5+10\,\tan\left(x\right)^3+15\,\tan\left(x\right)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^3,x, algorithm="maxima")
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

Fricas [A]
time = 0.37, size = 25, normalized size = 0.86

$$\frac{\left(8\,\cos\left(x\right)^4+4\,\cos\left(x\right)^2+3\right)\sin\left(x\right)}{15\,a^3\cos\left(x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^3*cos(x)^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(27) = 54$.
time = 2.31, size = 362, normalized size = 12.48

$$-\frac{30\tan^{9}\left(\frac{x}{2}\right)}{15a^{3}\tan^{10}\left(\frac{x}{2}\right)-75a^{3}\tan^{8}\left(\frac{x}{2}\right)+150a^{3}\tan^{6}\left(\frac{x}{2}\right)-150a^{3}\tan^{4}\left(\frac{x}{2}\right)+75a^{3}\tan^{2}\left(\frac{x}{2}\right)-15a^{3}}+\frac{40\tan^{7}\left(\frac{x}{2}\right)}{15a^{3}\tan^{10}\left(\frac{x}{2}\right)-75a^{3}\tan^{8}\left(\frac{x}{2}\right)+150a^{3}\tan^{6}\left(\frac{x}{2}\right)-150a^{3}\tan^{4}\left(\frac{x}{2}\right)+75a^{3}\tan^{2}\left(\frac{x}{2}\right)-15a^{3}}-\frac{116\tan^{5}\left(\frac{x}{2}\right)}{15a^{3}\tan^{10}\left(\frac{x}{2}\right)-75a^{3}\tan^{8}\left(\frac{x}{2}\right)+150a^{3}\tan^{6}\left(\frac{x}{2}\right)-150a^{3}\tan^{4}\left(\frac{x}{2}\right)+75a^{3}\tan^{2}\left(\frac{x}{2}\right)-15a^{3}}+\frac{40\tan^{3}\left(\frac{x}{2}\right)}{15a^{3}\tan^{10}\left(\frac{x}{2}\right)-75a^{3}\tan^{8}\left(\frac{x}{2}\right)+150a^{3}\tan^{6}\left(\frac{x}{2}\right)-150a^{3}\tan^{4}\left(\frac{x}{2}\right)+75a^{3}\tan^{2}\left(\frac{x}{2}\right)-15a^{3}}-\frac{30\tan\left(\frac{x}{2}\right)}{15a^{3}\tan^{10}\left(\frac{x}{2}\right)-75a^{3}\tan^{8}\left(\frac{x}{2}\right)+150a^{3}\tan^{6}\left(\frac{x}{2}\right)-150a^{3}\tan^{4}\left(\frac{x}{2}\right)+75a^{3}\tan^{2}\left(\frac{x}{2}\right)-15a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**3,x)
```

```
[Out] -30*tan(x/2)**9/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(
x/2)**6 - 150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(x/
2)**7/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 -
150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 116*tan(x/2)**5/(15
*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*
tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(x/2)**3/(15*a**3*tan(
x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan(x/2)**
4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 30*tan(x/2)/(15*a**3*tan(x/2)**10 - 75
*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan(x/2)**4 + 75*a**3*t
an(x/2)**2 - 15*a**3)
```

Giac [A]
time = 0.52, size = 22, normalized size = 0.76

$$\frac{3\tan\left(x\right)^{5}+10\tan\left(x\right)^{3}+15\tan\left(x\right)}{15\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

Mupad [B]
time = 13.41, size = 21, normalized size = 0.72

$$\frac{\tan(x)\,\left(3\tan(x)^{4}+10\tan(x)^{2}+15\right)}{15\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^3,x)
```

```
[Out] (tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a^3)
```

3.62 $\int \frac{1}{(a-a\sin^2(x))^4}\,dx$

Optimal. Leaf size=37

$$\frac{\tan(x)}{a^4}+\frac{\tan^3(x)}{a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^7(x)}{7a^4}$$

```
[Out] tan(x)/a^4+tan(x)^3/a^4+3/5*tan(x)^5/a^4+1/7*tan(x)^7/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3254, 3852}

$$\frac{\tan^7(x)}{7a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^3(x)}{a^4}+\frac{\tan(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-4),x]
[Out] Tan[x]/a^4 + Tan[x]^3/a^4 + (3*Tan[x]^5)/(5*a^4) + Tan[x]^7/(7*a^4)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-a\sin^2(x))^4}\,dx &= \frac{\int \sec^8(x)\,dx}{a^4}\\ &= -\frac{\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx,x,-\tan(x)\right)}{a^4}\\ &= \frac{\tan(x)}{a^4}+\frac{\tan^3(x)}{a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^7(x)}{7a^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.11

$$\frac{\frac{16\tan(x)}{35}+\frac{8}{35}\sec^2(x)\tan(x)+\frac{6}{35}\sec^4(x)\tan(x)+\frac{1}{7}\sec^6(x)\tan(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-4),x]
[Out] ((16*Tan[x])/35 + (8*Sec[x]^2*Tan[x])/35 + (6*Sec[x]^4*Tan[x])/35 + (Sec[x]
^6*Tan[x])/7)/a^4
```

Maple [A]
time = 0.18, size = 24, normalized size = 0.65

method	result	size
default	$\frac{\frac{\left(\tan^7(x)\right)}{7}+\frac{3\left(\tan^5(x)\right)}{5}+\tan^3(x)+\tan(x)}{a^4}$	24
risch	$\frac{32i(35\,e^{6ix}+21\,e^{4ix}+7\,e^{2ix}+1)}{35(e^{2ix}+1)^7a^4}$	39
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{86\left(\tan^5\left(\frac{x}{2}\right)\right)}{5a}+\frac{424\left(\tan^7\left(\frac{x}{2}\right)\right)}{35a}-\frac{86\left(\tan^9\left(\frac{x}{2}\right)\right)}{5a}+\frac{4\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{a}}{a^3\left(\tan^2\left(\frac{x}{2}\right)-1\right)^7}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^4,x,method=_RETURNVERBOSE)
[Out] 1/a^4*(1/7*tan(x)^7+3/5*tan(x)^5+tan(x)^3+tan(x))
```

Maxima [A]
time = 0.30, size = 28, normalized size = 0.76

$$\frac{5\,\tan(x)^7+21\,\tan(x)^5+35\,\tan(x)^3+35\,\tan(x)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^4,x, algorithm="maxima")
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

Fricas [A]
time = 0.38, size = 31, normalized size = 0.84

$$\frac{\left(16\,\cos(x)^6+8\,\cos(x)^4+6\,\cos(x)^2+5\right)\sin(x)}{35\,a^4\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(16*cos(x)^6 + 8*cos(x)^4 + 6*cos(x)^2 + 5)*sin(x)/(a^4*cos(x)^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(36) = 72$.
time = 7.55, size = 675, normalized size = 18.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**4,x)
```

```
[Out] -70*tan(x/2)**13/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*t
an(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(
x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 140*tan(x/2)**11/(35*a**4*tan(x
/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2
)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2
- 35*a**4) - 602*tan(x/2)**9/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12
+ 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 7
35*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 424*tan(x/2)**7/(35
*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a
**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*t
an(x/2)**2 - 35*a**4) - 602*tan(x/2)**5/(35*a**4*tan(x/2)**14 - 245*a**4*ta
n(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(
x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 140*tan(
x/2)**3/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**
10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 +
 245*a**4*tan(x/2)**2 - 35*a**4) - 70*tan(x/2)/(35*a**4*tan(x/2)**14 - 245*
a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a*
*4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4)
```

Giac [A]
time = 0.48, size = 28, normalized size = 0.76

$$\frac{5\,\tan\left(x\right)^{7} + 21\,\tan\left(x\right)^{5} + 35\,\tan\left(x\right)^{3} + 35\,\tan\left(x\right)}{35\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^4,x, algorithm="giac")
```

```
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

Mupad [B]
time = 13.41, size = 33, normalized size = 0.89

$$\frac{\tan(x)}{a^4}+\frac{\tan(x)^3}{a^4}+\frac{3\tan(x)^5}{5\,a^4}+\frac{\tan(x)^7}{7\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^4,x)
[Out] tan(x)/a^4 + tan(x)^3/a^4 + (3*tan(x)^5)/(5*a^4) + tan(x)^7/(7*a^4)
```

3.63 $\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx$

Optimal. Leaf size=51

$$\frac{\tan(x)}{a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{\tan^9(x)}{9a^5}$$

```
[Out] tan(x)/a^5+4/3*tan(x)^3/a^5+6/5*tan(x)^5/a^5+4/7*tan(x)^7/a^5+1/9*tan(x)^9/
a^5
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3254, 3852}

$$\frac{\tan^9(x)}{9a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{\tan(x)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-5),x]
```

```
[Out] Tan[x]/a^5 + (4*Tan[x]^3)/(3*a^5) + (6*Tan[x]^5)/(5*a^5) + (4*Tan[x]^7)/(7*
a^5) + Tan[x]^9/(9*a^5)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx &= \frac{\int \sec^{10}(x)\,dx}{a^5}\\
&= -\frac{\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -\tan(x)\right)}{a^5}\\
&= \frac{\tan(x)}{a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{\tan^9(x)}{9a^5}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.00

$$\frac{\frac{128\tan(x)}{315}+\frac{64}{315}\sec^2(x)\tan(x)+\frac{16}{105}\sec^4(x)\tan(x)+\frac{8}{63}\sec^6(x)\tan(x)+\frac{1}{9}\sec^8(x)\tan(x)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-5),x]
```

```
[Out] ((128*Tan[x])/315 + (64*Sec[x]^2*Tan[x])/315 + (16*Sec[x]^4*Tan[x])/105 + (
8*Sec[x]^6*Tan[x])/63 + (Sec[x]^8*Tan[x])/9)/a^5
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.63

method	result	size
default	$\frac{\frac{\left(\tan^9(x)\right)}{9}+\frac{4\left(\tan^7(x)\right)}{7}+\frac{6\left(\tan^5(x)\right)}{5}+\frac{4\left(\tan^3(x)\right)}{3}+\tan(x)}{a^5}$	32
risch	$\frac{256i\left(126\,e^{8ix}+84\,e^{6ix}+36\,e^{4ix}+9\,e^{2ix}+1\right)}{315\left(e^{2ix}+1\right)^9 a^5}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/9*tan(x)^9+4/7*tan(x)^7+6/5*tan(x)^5+4/3*tan(x)^3+tan(x))
```

Maxima [A]
time = 0.32, size = 34, normalized size = 0.67

$$\frac{35\tan(x)^9+180\tan(x)^7+378\tan(x)^5+420\tan(x)^3+315\tan(x)}{315\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^5
```

Fricas [A]
time = 0.37, size = 37, normalized size = 0.73

$$\frac{\left(128\cos(x)^8+64\cos(x)^6+48\cos(x)^4+40\cos(x)^2+35\right)\sin(x)}{315\,a^5\cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^5,x, algorithm="fricas")
```

```
[Out] 1/315*(128*cos(x)^8 + 64*cos(x)^6 + 48*cos(x)^4 + 40*cos(x)^2 + 35)*sin(x)/
(a^5*cos(x)^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. $2(51) = 102$.
time = 22.90, size = 1083, normalized size = 21.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**5,x)
```

```
[Out] -630*tan(x/2)**17/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a
**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 3969
0*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835
*a**5*tan(x/2)**2 - 315*a**5) + 1680*tan(x/2)**15/(315*a**5*tan(x/2)**18 -
2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12
+ 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6
 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 9576*tan(x/
2)**13/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2
)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(
x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x
/2)**2 - 315*a**5) + 10224*tan(x/2)**11/(315*a**5*tan(x/2)**18 - 2835*a**5*
tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a*
*5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a
**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 21316*tan(x/2)**9/(31
5*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26
460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 +
26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 3
15*a**5) + 10224*tan(x/2)**7/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**1
6 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)
**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2
)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 9576*tan(x/2)**5/(315*a**5*tan(x
/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan
(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*t
an(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) + 1
680*tan(x/2)**3/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**
5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*
a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a
**5*tan(x/2)**2 - 315*a**5) - 630*tan(x/2)/(315*a**5*tan(x/2)**18 - 2835*a*
*5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690
*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 1134
0*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5)
```

Giac [A]

time = 0.43, size = 34, normalized size = 0.67

$$\frac{35\,\tan\left(x\right)^{9} + 180\,\tan\left(x\right)^{7} + 378\,\tan\left(x\right)^{5} + 420\,\tan\left(x\right)^{3} + 315\,\tan\left(x\right)}{315\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^5,x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^5
```

Mupad [B]
time = 13.34, size = 43, normalized size = 0.84

$$\frac{\tan(x)}{a^5} + \frac{4\tan(x)^3}{3\,a^5} + \frac{6\tan(x)^5}{5\,a^5} + \frac{4\tan(x)^7}{7\,a^5} + \frac{\tan(x)^9}{9\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^5,x)
```

```
[Out] tan(x)/a^5 + (4*tan(x)^3)/(3*a^5) + (6*tan(x)^5)/(5*a^5) + (4*tan(x)^7)/(7*
a^5) + tan(x)^9/(9*a^5)
```

3.64 $\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+b)\cos(c+dx)}{d}+\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{b\cos^5(c+dx)}{5d}$$

```
[Out] -(a+b)*cos(d*x+c)/d+1/3*(a+2*b)*cos(d*x+c)^3/d-1/5*b*cos(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3092, 380}

$$\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((a + b)*Cos[c + d*x])/d) + ((a + 2*b)*Cos[c + d*x]^3)/(3*d) - (b*Cos[c +
 d*x]^5)/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-bx^2\right)\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(a\left(1+\frac{b}{a}\right)-(a+2b)x^2+bx^4\right)\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{(a+b)\cos(c+dx)}{d}+\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{b\cos^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 1.51

$$-\frac{3a\cos(c+dx)}{4d}-\frac{5b\cos(c+dx)}{8d}+\frac{a\cos(3(c+dx))}{12d}+\frac{5b\cos(3(c+dx))}{48d}-\frac{b\cos(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-3*a*Cos[c + d*x])/(4*d) - (5*b*Cos[c + d*x])/(8*d) + (a*Cos[3*(c + d*x)])
/(12*d) + (5*b*Cos[3*(c + d*x)])/(48*d) - (b*Cos[5*(c + d*x)])/(80*d)
```

Maple [A]
time = 0.24, size = 54, normalized size = 1.06

method	result	size
derivativedivides	$\frac{-\frac{b\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$	54
default	$\frac{-\frac{b\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$	54
risch	$-\frac{3\cos(dx+c)a}{4d}-\frac{5b\cos(dx+c)}{8d}-\frac{b\cos(5dx+5c)}{80d}+\frac{a\cos(3dx+3c)}{12d}+\frac{5\cos(3dx+3c)b}{48d}$	71
norman	$\frac{-\frac{20a+16b}{15d}-\frac{4a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(14a+16b)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{(20a+16b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*b*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-1/3*a*(2+sin(d*x
+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.84

$$-\frac{3\,b\cos\left(dx+c\right)^5-5\left(a+2\,b\right)\cos\left(dx+c\right)^3+15\left(a+b\right)\cos\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - 5*(a + 2*b)*cos(d*x + c)^3 + 15*(a + b)*cos(d*x
 + c))/d
```

Fricas [A]
time = 0.39, size = 43, normalized size = 0.84

$$-\frac{3\,b\cos\left(dx+c\right)^{5}-5\,(a+2\,b)\cos\left(dx+c\right)^{3}+15\,(a+b)\cos\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - 5*(a + 2*b)*cos(d*x + c)^3 + 15*(a + b)*cos(d*x
 + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(44) = 88$.
time = 0.26, size = 107, normalized size = 2.10

$$\begin{cases} -\frac{a\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{d}-\frac{2a\cos^{3}\left(c+dx\right)}{3d}-\frac{b\sin^{4}\left(c+dx\right)\cos\left(c+dx\right)}{d}-\frac{4b\sin^{2}\left(c+dx\right)\cos^{3}\left(c+dx\right)}{3d}-\frac{8b\cos^{5}\left(c+dx\right)}{15d} & \text{for}\ d\neq 0\\ x(a+b\sin^{2}\left(c\right))\sin^{3}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)/d - 2*a*cos(c + d*x)**3/(3*d) -
b*sin(c + d*x)**4*cos(c + d*x)/d - 4*b*sin(c + d*x)**2*cos(c + d*x)**3/(3*d
) - 8*b*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c)**3,
True))
```

Giac [A]
time = 0.43, size = 67, normalized size = 1.31

$$-\frac{b\cos\left(dx+c\right)^{5}}{5\,d}+\frac{a\cos\left(dx+c\right)^{3}}{3\,d}+\frac{2\,b\cos\left(dx+c\right)^{3}}{3\,d}-\frac{a\cos\left(dx+c\right)}{d}-\frac{b\cos\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/5*b*cos(d*x + c)^5/d + 1/3*a*cos(d*x + c)^3/d + 2/3*b*cos(d*x + c)^3/d -
 a*cos(d*x + c)/d - b*cos(d*x + c)/d
```

Mupad [B]
time = 13.37, size = 44, normalized size = 0.86

$$-\frac{\frac{b\cos(c+d\,x)^5}{5}+\left(-\frac{a}{3}-\frac{2\,b}{3}\right)\,\cos\left(c+d\,x\right)^3+\left(a+b\right)\,\cos\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*sin(c + d*x)^2),x)
```

```
[Out] -((b*cos(c + d*x)^5)/5 - cos(c + d*x)^3*(a/3 + (2*b)/3) + cos(c + d*x)*(a +
 b))/d
```

3.65 $\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$-\frac{(a+b)\cos(c+dx)}{d}+\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] -(a+b)*cos(d*x+c)/d+1/3*b*cos(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3092}

$$\frac{b\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((a + b)*Cos[c + d*x])/d) + (b*Cos[c + d*x]^3)/(3*d)
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx&=-\frac{\text{Subst}\left(\int\left(a+b-bx^2\right)\,dx,x,\cos(c+dx)\right)}{d}\\&=-\frac{(a+b)\cos(c+dx)}{d}+\frac{b\cos^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.74

$$-\frac{a\cos(c)\cos(dx)}{d}-\frac{3b\cos(c+dx)}{4d}+\frac{b\cos(3(c+dx))}{12d}+\frac{a\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a*Cos[c]*Cos[d*x])/d) - (3*b*Cos[c + d*x])/(4*d) + (b*Cos[3*(c + d*x)])/
(12*d) + (a*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.18, size = 34, normalized size = 1.10

method	result	size
derivativedivides	$\frac{-\frac{b\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3}-a\cos(dx+c)}{d}$	34
default	$\frac{-\frac{b\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3}-a\cos(dx+c)}{d}$	34
risch	$-\frac{\cos(dx+c)a}{d}-\frac{3b\cos(dx+c)}{4d}+\frac{\cos(3dx+3c)b}{12d}$	41
norman	$\frac{-\frac{6a+4b}{3d}-\frac{2a\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(4a+4b)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/3*b*(2+sin(d*x+c)^2)*cos(d*x+c)-a*cos(d*x+c))
```

Maxima [A]
time = 0.30, size = 34, normalized size = 1.10

$$\frac{\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)b-3\,a\cos\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*b - 3*a*cos(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 27, normalized size = 0.87

$$\frac{b\cos\left(dx+c\right)^{3}-3\left(a+b\right)\cos\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
[Out] 1/3*(b*cos(d*x + c)^3 - 3*(a + b)*cos(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(26) = 52$.

time = 0.11, size = 58, normalized size = 1.87

$$\begin{cases} -\frac{a\cos(c+dx)}{d} - \frac{b\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2b\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a+b\sin^2(c))\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)/d - b*sin(c + d*x)**2*cos(c + d*x)/d - 2*b*cos(c
 + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c), True))
```

Giac [A]
time = 0.57, size = 40, normalized size = 1.29

$$\frac{1}{3}\left(\frac{\cos(dx+c)^3}{d} - \frac{3\cos(dx+c)}{d}\right)b - \frac{a\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(cos(d*x + c)^3/d - 3*cos(d*x + c)/d)*b - a*cos(d*x + c)/d
```

Mupad [B]
time = 13.31, size = 27, normalized size = 0.87

$$\frac{\frac{b\cos(c+d\,x)^3}{3} - \cos(c+d\,x)\,(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*sin(c + d*x)^2),x)
```

```
[Out] ((b*cos(c + d*x)^3)/3 - cos(c + d*x)*(a + b))/d
```

3.66 $\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=26

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d-b*cos(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3093, 3855}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (b*Cos[c + d*x])/d
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{b\cos(c+dx)}{d}+a\int \csc(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.
time = 0.02, size = 63, normalized size = 2.42

$$-\frac{b\cos(c)\cos(dx)}{d}-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) - (a*Log[Cos[c/2 + (d*x)/2]])/d + (a*Log[Sin[c/2 +
 (d*x)/2]])/d + (b*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.19, size = 33, normalized size = 1.27

method	result	size
derivativedivides	$\frac{a\ln(\csc(dx+c)-\cot(dx+c))-b\cos(dx+c)}{d}$	33
default	$\frac{a\ln(\csc(dx+c)-\cot(dx+c))-b\cos(dx+c)}{d}$	33
risch	$-\frac{b\,e^{i(dx+c)}}{2d}-\frac{b\,e^{-i(dx+c)}}{2d}+\frac{a\ln(e^{i(dx+c)}-1)}{d}-\frac{a\ln(e^{i(dx+c)}+1)}{d}$	67
norman	$\frac{\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*ln(csc(d*x+c)-cot(d*x+c))-b*cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 38, normalized size = 1.46

$$-\frac{2\,b\cos\left(dx+c\right)+a\log\left(\cos\left(dx+c\right)+1\right)-a\log\left(\cos\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*cos(d*x + c) + a*log(cos(d*x + c) + 1) - a*log(cos(d*x + c) - 1))
/d
```

Fricas [A]
time = 0.43, size = 42, normalized size = 1.62

$$-\frac{2\,b\cos\left(dx+c\right)+a\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*cos(d*x + c) + a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*x
 + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^2\left(c + dx\right)\right)\csc\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.
time = 0.42, size = 58, normalized size = 2.23

$$\frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) + \frac{4b}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + 4*b/((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 1))/d
```

Mupad [B]
time = 13.37, size = 23, normalized size = 0.88

$$-\frac{b\cos\left(c + d\,x\right) + a\,\mathrm{atanh}\left(\cos\left(c + d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x),x)
```

```
[Out] -(b*cos(c + d*x) + a*atanh(cos(c + d*x)))/d
```

3.67 $\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$-\frac{(a+2b)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -1/2*(a+2*b)*arctanh(cos(d*x+c))/d-1/2*a*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3091, 3855}

$$-\frac{(a+2b)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/2*((a + 2*b)*ArcTanh[Cos[c + d*x]])/d - (a*Cot[c + d*x]*Csc[c + d*x])/(2
*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx&=-\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}(a+2b)\int \csc(c+dx)\,dx\\&=-\frac{(a+2b)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(40) = 80.
time = 0.03, size = 118, normalized size = 2.95

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Log[Cos[c/2 + (d*x)/2]])/d - (a*Log[Cos[
(c + d*x)/2]])/(2*d) + (b*Log[Sin[c/2 + (d*x)/2]])/d + (a*Log[Sin[(c + d*x)
/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A]
time = 0.25, size = 59, normalized size = 1.48

method	result	size
derivativedivides	$\frac{a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\ln(\csc(dx+c)-\cot(dx+c))}{d}$	59
default	$\frac{a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\ln(\csc(dx+c)-\cot(dx+c))}{d}$	59
norman	$\frac{-\frac{a}{8d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(a+2b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$	107
risch	$\frac{a\left(e^{3i(dx+c)}+e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}+\frac{b\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}-\frac{b\ln\left(e^{i(dx+c)}+1\right)}{d}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c)))+b*ln(csc(
d*x+c)-cot(d*x+c)))
```

Maxima [A]
time = 0.29, size = 58, normalized size = 1.45

$$-\frac{(a+2b)\log\left(\cos\left(dx+c\right)+1\right)-(a+2b)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2a\cos(dx+c)}{\cos(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*((a + 2*b)*log(cos(d*x + c) + 1) - (a + 2*b)*log(cos(d*x + c) - 1) - 2
*a*cos(d*x + c)/(cos(d*x + c)^2 - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(36) = 72.
time = 0.40, size = 95, normalized size = 2.38

$$\frac{2a\cos\left(dx+c\right)-\left((a+2b)\cos\left(dx+c\right)^2-a-2b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left((a+2b)\cos\left(dx+c\right)^2-a-2b\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - ((a + 2*b)*cos(d*x + c)^2 - a - 2*b)*log(1/2*cos(d*
x + c) + 1/2) + ((a + 2*b)*cos(d*x + c)^2 - a - 2*b)*log(-1/2*cos(d*x + c)
+ 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^2\left(c+dx\right)\right)\csc^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(36) = 72.
time = 0.44, size = 121, normalized size = 3.02

$$\frac{2\left(a+2\,b\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+\frac{\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a - 2
*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/d
```

Mupad [**B**]
time = 13.39, size = 42, normalized size = 1.05

$$\frac{a\,\cos\left(c+d\,x\right)}{2\,d\,\left({\cos\left(c+d\,x\right)}^2-1\right)}-\frac{\mathrm{atanh}(\cos\left(c+d\,x\right))\,\left(\frac{a}{2}+b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x)^3,x)
```

```
[Out] (a*cos(c + d*x))/(2*d*(cos(c + d*x)^2 - 1)) - (atanh(cos(c + d*x))*(a/2 + b
))/d
```

3.68 $\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{1}{16}(6a+5b)x-\frac{(6a+5b)\cos(c+dx)\sin(c+dx)}{16d}-\frac{(6a+5b)\cos(c+dx)\sin^3(c+dx)}{24d}-\frac{b\cos(c+dx)\sin^5(c+d}{6d}$$

```
[Out] 1/16*(6*a+5*b)*x-1/16*(6*a+5*b)*cos(d*x+c)*sin(d*x+c)/d-1/24*(6*a+5*b)*cos(
d*x+c)*sin(d*x+c)^3/d-1/6*b*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3093, 2715, 8}

$$-\frac{(6a+5b)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{(6a+5b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6a+5b)-\frac{b\sin^5(c+dx)\cos(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((6*a + 5*b)*x)/16 - ((6*a + 5*b)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - ((6*a
 + 5*b)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) - (b*Cos[c + d*x]*Sin[c + d*x]^
5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{b\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(6a+5b)\int \sin^4(c+dx)\,dx$$
$$= -\frac{(6a+5b)\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{b\cos(c+dx)\sin^5(c+dx)}{6d}$$
$$= -\frac{(6a+5b)\cos(c+dx)\sin(c+dx)}{16d} - \frac{(6a+5b)\cos(c+dx)\sin^3(c}{24d}$$
$$= \frac{1}{16}(6a+5b)x - \frac{(6a+5b)\cos(c+dx)\sin(c+dx)}{16d} - \frac{(6a+5b)\cos}{}$$

Mathematica [A]
time = 0.08, size = 70, normalized size = 0.79

$$\frac{72ac+60bc+72adx+60bdx-3(16a+15b)\sin(2(c+dx))+(6a+9b)\sin(4(c+dx))-b\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (72*a*c + 60*b*c + 72*a*d*x + 60*b*d*x - 3*(16*a + 15*b)*Sin[2*(c + d*x)] +
 (6*a + 9*b)*Sin[4*(c + d*x)] - b*Sin[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 0.28, size = 86, normalized size = 0.97

method	result
risch	$\frac{3ax}{8}+\frac{5bx}{16}-\frac{b\sin(6dx+6c)}{192d}+\frac{\sin(4dx+4c)a}{32d}+\frac{3b\sin(4dx+4c)}{64d}-\frac{\sin(2dx+2c)a}{4d}-\frac{15b\sin(2dx+2c)}{64d}$
derivativedivides	$\frac{b\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\right.}{d}$
default	$\frac{b\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\right.}{d}$
norman	$\frac{\left(\frac{3a}{8}+\frac{5b}{16}\right)x+\left(\frac{3a}{8}+\frac{5b}{16}\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9a}{4}+\frac{15b}{8}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9a}{4}+\frac{15b}{8}\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15a}{2}+\frac{25b}{4}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/1
6*d*x+5/16*c)+a*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*
c))
```

Maxima [A]
time = 0.53, size = 104, normalized size = 1.17

$$\frac{3\,(dx+c)(6\,a+5\,b)-\frac{3\,(10\,a+11\,b)\tan(dx+c)^5+8\,(6\,a+5\,b)\tan(dx+c)^3+3\,(6\,a+5\,b)\tan(dx+c)}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(d*x + c)*(6*a + 5*b) - (3*(10*a + 11*b)*tan(d*x + c)^5 + 8*(6*a +
5*b)*tan(d*x + c)^3 + 3*(6*a + 5*b)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d
*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.40, size = 69, normalized size = 0.78

$$\frac{3\,(6\,a+5\,b)dx-\left(8\,b\cos\left(dx+c\right)^5-2\,(6\,a+13\,b)\cos\left(dx+c\right)^3+3\,(10\,a+11\,b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(6*a + 5*b)*d*x - (8*b*cos(d*x + c)^5 - 2*(6*a + 13*b)*cos(d*x + c)
^3 + 3*(10*a + 11*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(82) = 164$.
time = 0.43, size = 258, normalized size = 2.90

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8}+\frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3ax\cos^4(c+dx)}{8}-\frac{5a\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{3a\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{5bx\sin^6(c+dx)}{16}+\frac{15bx\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15bx\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5bx\cos^6(c+dx)}{16}-\frac{11b\sin^5(c+dx)\cos(c+dx)}{16d}-\frac{5b\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{5b\sin(c+dx)\cos^5(c+dx)}{16d} & \text{for}\ d\neq 0 \\ x(a+b\sin^2(c))\sin^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 - 5*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + 5*b*x*sin(c + d*x)**6/16 + 15*b*x*sin(
c + d*x)**4*cos(c + d*x)**2/16 + 15*b*x*sin(c + d*x)**2*cos(c + d*x)**4/16
+ 5*b*x*cos(c + d*x)**6/16 - 11*b*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*b
*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 5*b*sin(c + d*x)*cos(c + d*x)**5/(
16*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c)**4, True))
```

Giac [A]
time = 0.48, size = 68, normalized size = 0.76

$$\frac{1}{16}\,(6\,a+5\,b)x-\frac{b\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{(2\,a+3\,b)\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{(16\,a+15\,b)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a + 5*b)*x - 1/192*b*sin(6*d*x + 6*c)/d + 1/64*(2*a + 3*b)*sin(4*d*
x + 4*c)/d - 1/64*(16*a + 15*b)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 13.94, size = 92, normalized size = 1.03

$$x\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)-\frac{\left(\frac{5\,a}{8}+\frac{11\,b}{16}\right)\,\tan(c+d\,x)^5+\left(a+\frac{5\,b}{6}\right)\,\tan(c+d\,x)^3+\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^6+3\tan\left(c+d\,x\right)^4+3\tan\left(c+d\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b*sin(c + d*x)^2),x)
```

```
[Out] x*((3*a)/8 + (5*b)/16) - (tan(c + d*x)^5*((5*a)/8 + (11*b)/16) + tan(c + d*
x)*((3*a)/8 + (5*b)/16) + tan(c + d*x)^3*(a + (5*b)/6))/(d*(3*tan(c + d*x)^
2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

3.69 $\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{1}{8}(4a+3b)x-\frac{(4a+3b)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}$$

```
[Out] 1/8*(4*a+3*b)*x-1/8*(4*a+3*b)*cos(d*x+c)*sin(d*x+c)/d-1/4*b*cos(d*x+c)*sin(
d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3093, 2715, 8}

$$-\frac{(4a+3b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3b)-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((4*a + 3*b)*x)/8 - ((4*a + 3*b)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (b*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{b\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{4}(4a+3b)\int \sin^2(c+dx)\,dx$$
$$= -\frac{(4a+3b)\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\cos(c+dx)\sin^3(c+dx)}{4d} -$$
$$= \frac{1}{8}(4a+3b)x - \frac{(4a+3b)\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\cos(c+dx)\,\mathrm{s}}{4d}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 0.74

$$\frac{4(4a+3b)(c+dx)-8(a+b)\sin(2(c+dx))+b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (4*(4*a + 3*b)*(c + d*x) - 8*(a + b)*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])
/(32*d)
```

Maple [A]
time = 0.22, size = 65, normalized size = 1.07

method	result
risch	$\frac{ax}{2}+\frac{3bx}{8}+\frac{b\sin(4dx+4c)}{32d}-\frac{\sin(2dx+2c)a}{4d}-\frac{b\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{b\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{b\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\left(\frac{a}{2}+\frac{3b}{8}\right)x+\left(2a+\frac{3b}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2a+\frac{3b}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a+\frac{9b}{4}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{a}{2}+\frac{3b}{8}\right)x\left(\tan^8\left(\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+a*(-1/
2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.54, size = 74, normalized size = 1.21

$$\frac{(dx+c)(4\,a+3\,b)-\frac{(4\,a+5\,b)\tan(dx+c)^3+(4\,a+3\,b)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((d*x + c)*(4*a + 3*b) - ((4*a + 5*b)*tan(d*x + c)^3 + (4*a + 3*b)*tan(
d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.40, size = 50, normalized size = 0.82

$$\frac{(4\,a+3\,b)dx+\left(2\,b\cos\left(dx+c\right)^{3}-(4\,a+5\,b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((4*a + 3*b)*d*x + (2*b*cos(d*x + c)^3 - (4*a + 5*b)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(53) = 106$.
time = 0.18, size = 158, normalized size = 2.59

$$\begin{cases} \frac{ax\sin^{2}\left(c+dx\right)}{2}+\frac{ax\cos^{2}\left(c+dx\right)}{2}-\frac{a\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{3bx\sin^{4}\left(c+dx\right)}{8}+\frac{3bx\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{3bx\cos^{4}\left(c+dx\right)}{8}-\frac{5b\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{3b\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d} & \text{for}\ d\neq 0 \\ x(a+b\sin^{2}\left(c\right))\sin^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 - a*sin(c + d*x)*c
os(c + d*x)/(2*d) + 3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c +
 d*x)**2/4 + 3*b*x*cos(c + d*x)**4/8 - 5*b*sin(c + d*x)**3*cos(c + d*x)/(8*
d) - 3*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sin(c)**2
)*sin(c)**2, True))
```

Giac [A]
time = 0.46, size = 43, normalized size = 0.70

$$\frac{1}{8}\,(4\,a+3\,b)x+\frac{b\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{(a+b)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*a + 3*b)*x + 1/32*b*sin(4*d*x + 4*c)/d - 1/4*(a + b)*sin(2*d*x + 2*c
)/d
```

Mupad [B]
time = 13.55, size = 68, normalized size = 1.11

$$x\left(\frac{a}{2}+\frac{3\,b}{8}\right)-\frac{\left(\frac{a}{2}+\frac{5\,b}{8}\right)\,\tan(c+d\,x)^3+\left(\frac{a}{2}+\frac{3\,b}{8}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*sin(c + d*x)^2),x)
```

```
[Out] x*(a/2 + (3*b)/8) - (tan(c + d*x)^3*(a/2 + (5*b)/8) + tan(c + d*x)*(a/2 + (
3*b)/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.70 $\int \left(a+b\sin^2(c+dx)\right) \, dx$

Optimal. Leaf size=30

$$ax+\frac{bx}{2}-\frac{b\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] a*x+1/2*b*x-1/2*b*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2715, 8}

$$ax-\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d*x]^2,x]
```

```
[Out] a*x + (b*x)/2 - (b*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(c+dx)\right) \, dx &= ax+b\int \sin^2(c+dx) \, dx\\ &= ax-\frac{b\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}b\int 1 \, dx\\ &= ax+\frac{bx}{2}-\frac{b\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.10

$$ax+\frac{b(c+dx)}{2d}-\frac{b\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d*x]^2,x]
```

```
[Out] a*x + (b*(c + d*x))/(2*d) - (b*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.14, size = 32, normalized size = 1.07

method	result	size
risch	$ax+\frac{bx}{2}-\frac{b\sin(2dx+2c)}{4d}$	24
default	$ax+\frac{b\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	32
derivativedivides	$\frac{(dx+c)a+b\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	37
norman	$\frac{\left(a+\frac{b}{2}\right)x+\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\left(a+\frac{b}{2}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2a+b)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+sin(d*x+c)^2*b,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b/d*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)
```

Maxima [A]
time = 0.28, size = 29, normalized size = 0.97

$$ax+\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))b}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a*x + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*b/d
```

Fricas [A]
time = 0.40, size = 29, normalized size = 0.97

$$\frac{(2\,a+b)dx-b\cos(dx+c)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a + b)*d*x - b*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [A]
time = 0.07, size = 51, normalized size = 1.70

$$ax + b\left(\begin{cases} \frac{x\sin^2(c+dx)}{2} + \frac{x\cos^2(c+dx)}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x\sin^2(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)**2,x)
[Out] a*x + b*Piecewise((x*sin(c + d*x)**2/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)
*cos(c + d*x)/(2*d), Ne(d, 0)), (x*sin(c)**2, True))
```

Giac [A]
time = 0.41, size = 25, normalized size = 0.83

$$\frac{1}{4}b\left(2x - \frac{\sin(2dx + 2c)}{d}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)^2,x, algorithm="giac")
[Out] 1/4*b*(2*x - sin(2*d*x + 2*c)/d) + a*x
```

Mupad [B]
time = 13.40, size = 27, normalized size = 0.90

$$-\frac{\frac{b\sin(2c+2dx)}{4} - dx\left(a + \frac{b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(c + d*x)^2,x)
[Out] -((b*sin(2*c + 2*d*x))/4 - d*x*(a + b/2))/d
```

3.71 $\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=16

$$bx-\frac{a\cot(c+dx)}{d}$$

```
[Out] b*x-a*cot(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3091, 8}

$$bx-\frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] b*x - (a*Cot[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx&=-\frac{a\cot(c+dx)}{d}+b\int 1\,dx\\&=bx-\frac{a\cot(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$bx-\frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] b*x - (a*Cot[c + d*x])/d
```

Maple [A]
time = 0.20, size = 22, normalized size = 1.38

method	result
derivativedivides	$\frac{-\cot(dx+c)a+b(dx+c)}{d}$
default	$\frac{-\cot(dx+c)a+b(dx+c)}{d}$
risch	$bx-\frac{2ia}{d\left(e^{2i(dx+c)}-1\right)}$
norman	$\frac{bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+bx\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{2d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+2bx\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-cot(d*x+c)*a+b*(d*x+c))
```

Maxima [A]
time = 0.51, size = 23, normalized size = 1.44

$$\frac{(dx+c)b-\frac{a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*b - a/tan(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 32, normalized size = 2.00

$$\frac{bdx\sin(dx+c)-a\cos(dx+c)}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (b*d*x*sin(d*x + c) - a*cos(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^2\left(c+dx\right)\right)\csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sin(d*x+c)**2),x)
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.
time = 0.48, size = 39, normalized size = 2.44

$$\frac{2\left(dx+c\right)b+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*(2*(d*x + c)*b + a*tan(1/2*d*x + 1/2*c) - a/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 13.36, size = 16, normalized size = 1.00

$$b\,x-\frac{a\cot(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x)^2,x)
[Out] b*x - (a*cot(c + d*x))/d
```

3.72 $\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=43

$$-\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

```
[Out] -1/3*(2*a+3*b)*cot(d*x+c)/d-1/3*a*cot(d*x+c)*csc(d*x+c)^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3091, 3852, 8}

$$-\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/3*((2*a + 3*b)*Cot[c + d*x])/d - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}+\frac{1}{3}(2a+3b)\int \csc^2(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{(2a+3b)\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{3d}\\ &= -\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.14

$$-\frac{2a\cot(c+dx)}{3d}-\frac{b\cot(c+dx)}{d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
[Out] (-2*a*Cot[c + d*x])/(3*d) - (b*Cot[c + d*x])/d - (a*Cot[c + d*x]*Csc[c + d*
x]^2)/(3*d)
```

Maple [A]
time = 0.26, size = 35, normalized size = 0.81

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)-\cot(dx+c)b}{d}$
default	$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)-\cot(dx+c)b}{d}$
risch	$-\frac{2i\left(3b\,e^{4i(dx+c)}-6a\,e^{2i(dx+c)}-6b\,e^{2i(dx+c)}+2a+3b\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$
norman	$\frac{-\frac{a}{24d}+\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{(5a+6b)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{(5a+6b)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{(11a+12b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{(11a+12b)\left(\tan^8\left(\right.\right.}{24d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)-cot(d*x+c)*b)
```

Maxima [A]
time = 0.29, size = 28, normalized size = 0.65

$$-\frac{3\,(a+b)\tan\left(dx+c\right)^2+a}{3\,d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
[Out] -1/3*(3*(a + b)*tan(d*x + c)^2 + a)/(d*tan(d*x + c)^3)
```

Fricas [A]
time = 0.39, size = 54, normalized size = 1.26

$$-\frac{(2\,a+3\,b)\cos\left(dx+c\right)^3-3\,(a+b)\cos\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*((2*a + 3*b)*cos(d*x + c)^3 - 3*(a + b)*cos(d*x + c))/((d*cos(d*x + c)
^2 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^2\left(c+dx\right)\right)\csc^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**4, x)
```

Giac [A]
time = 0.46, size = 37, normalized size = 0.86

$$-\frac{3\,a\tan\left(dx+c\right)^2+3\,b\tan\left(dx+c\right)^2+a}{3\,d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*tan(d*x + c)^2 + 3*b*tan(d*x + c)^2 + a)/(d*tan(d*x + c)^3)
```

Mupad [B]
time = 13.37, size = 29, normalized size = 0.67

$$-\frac{a\cot(c+d\,x)^3}{3\,d}-\frac{\cot(c+d\,x)\,\left(a+b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x)^4,x)
```

```
[Out] - (a*cot(c + d*x)^3)/(3*d) - (cot(c + d*x)*(a + b))/d
```

3.73 $\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=65

$$-\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

```
[Out] -1/5*(4*a+5*b)*cot(d*x+c)/d-1/15*(4*a+5*b)*cot(d*x+c)^3/d-1/5*a*cot(d*x+c)*
csc(d*x+c)^4/d
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3091, 3852}

$$-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/5*((4*a + 5*b)*Cot[c + d*x])/d - ((4*a + 5*b)*Cot[c + d*x]^3)/(15*d) - (
a*Cot[c + d*x]*Csc[c + d*x]^4)/(5*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}+\frac{1}{5}(4a+5b)\int \csc^4(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{(4a+5b)\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,c\right.}{5d}\\ &= -\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{a\cot(c+dx)}{5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 95, normalized size = 1.46

$$-\frac{8a\cot(c+dx)}{15d}-\frac{2b\cot(c+dx)}{3d}-\frac{4a\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{b\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (2*b*Cot[c + d*x])/(3*d) - (4*a*Cot[c + d*x]*C
sc[c + d*x]^2)/(15*d) - (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (a*Cot[c +
d*x]*Csc[c + d*x]^4)/(5*d)
```

Maple [A]
time = 0.29, size = 56, normalized size = 0.86

method	result
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+b\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+b\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
risch	$\frac{4i\left(15b\,e^{6i(dx+c)}-40a\,e^{4i(dx+c)}-35b\,e^{4i(dx+c)}+20a\,e^{2i(dx+c)}+25b\,e^{2i(dx+c)}-4a-5b\right)}{15d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{a}{160d}+\frac{a\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}-\frac{5(7a+8b)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{5(7a+8b)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{(31a+20b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}+\frac{(31a+20b)\left(\tan^{12}\right.}{480d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+b*(-2/3-1/3*cs
c(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.30, size = 45, normalized size = 0.69

$$-\frac{15\,(a+b)\tan\left(dx+c\right)^4+5\,(2\,a+b)\tan\left(dx+c\right)^2+3\,a}{15\,d\tan\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a + b)*tan(d*x + c)^4 + 5*(2*a + b)*tan(d*x + c)^2 + 3*a)/(d*tan
(d*x + c)^5)
```

Fricas [A]
time = 0.36, size = 81, normalized size = 1.25

$$-\frac{2\,(4\,a+5\,b)\cos\left(dx+c\right)^{5}-5\,(4\,a+5\,b)\cos\left(dx+c\right)^{3}+15\,(a+b)\cos\left(dx+c\right)}{15\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(4*a + 5*b)*cos(d*x + c)^5 - 5*(4*a + 5*b)*cos(d*x + c)^3 + 15*(a
+ b)*cos(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^{2}\left(c+dx\right)\right)\csc^{6}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**6, x)
```

Giac [A]
time = 0.58, size = 61, normalized size = 0.94

$$-\frac{15\,a\tan\left(dx+c\right)^{4}+15\,b\tan\left(dx+c\right)^{4}+10\,a\tan\left(dx+c\right)^{2}+5\,b\tan\left(dx+c\right)^{2}+3\,a}{15\,d\tan\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*a*tan(d*x + c)^4 + 15*b*tan(d*x + c)^4 + 10*a*tan(d*x + c)^2 + 5*
b*tan(d*x + c)^2 + 3*a)/(d*tan(d*x + c)^5)
```

Mupad [B]
time = 13.38, size = 49, normalized size = 0.75

$$-\frac{a\cot(c+d\,x)^{5}}{5\,d}-\frac{\cot(c+d\,x)\,\left(a+b\right)}{d}-\frac{\cot(c+d\,x)^{3}\,\left(\frac{2\,a}{3}+\frac{b}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x)^6,x)
```

```
[Out] - (a*cot(c + d*x)^5)/(5*d) - (cot(c + d*x)*(a + b))/d - (cot(c + d*x)^3*((2
*a)/3 + b/3))/d
```

3.74 $\int (a + b\sin^2(x))\, dx$

Optimal. Leaf size=19

$$ax + \frac{bx}{2} - \frac{1}{2}b\cos(x)\sin(x)$$

```
[Out] a*x+1/2*b*x-1/2*b*cos(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$ax + \frac{bx}{2} - \frac{1}{2}b\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[x]^2,x]
```

```
[Out] a*x + (b*x)/2 - (b*Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int (a + b\sin^2(x))\, dx &= ax + b\int \sin^2(x)\, dx\\ &= ax - \frac{1}{2}b\cos(x)\sin(x) + \frac{1}{2}b\int 1\, dx\\ &= ax + \frac{bx}{2} - \frac{1}{2}b\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$ax + \frac{bx}{2} - \frac{1}{4}b\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[x]^2,x]
```

```
[Out] a*x + (b*x)/2 - (b*Sin[2*x])/4
```

Maple [A]
time = 0.08, size = 17, normalized size = 0.89

method	result	size
risch	$ax+\frac{bx}{2}-\frac{b\sin(2x)}{4}$	16
default	$ax+b\left(-\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)$	17
norman	$\frac{b\left(\tan^{3}\left(\frac{x}{2}\right)\right)+\left(a+\frac{b}{2}\right)x+\left(a+\frac{b}{2}\right)x\left(\tan^{4}\left(\frac{x}{2}\right)\right)+(2a+b)x\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*(-1/2*sin(x)*cos(x)+1/2*x)
```

Maxima [A]
time = 0.29, size = 17, normalized size = 0.89

$$\frac{1}{4}\,b(2\,x-\sin\left(2\,x\right))+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*x - sin(2*x)) + a*x
```

Fricas [A]
time = 0.39, size = 16, normalized size = 0.84

$$-\frac{1}{2}\,b\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,(2\,a+b)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*b*cos(x)*sin(x) + 1/2*(2*a + b)*x
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.79

$$ax+b\left(\frac{x}{2}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)**2,x)
```

```
[Out] a*x + b*(x/2 - sin(x)*cos(x)/2)
```

Giac [A]
time = 0.53, size = 17, normalized size = 0.89

$$\frac{1}{4}\,b(2\,x - \sin{(2\,x)}) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*(2*x - sin(2*x)) + a*x
```

Mupad [B]
time = 13.31, size = 15, normalized size = 0.79

$$x\left(a + \frac{b}{2}\right) - \frac{b\,\sin{(2\,x)}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(x)^2,x)
```

```
[Out] x*(a + b/2) - (b*sin(2*x))/4
```

3.75 $\int (a+b\sin^2(x))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{1}{8}b(8a+3b)\cos(x)\sin(x)-\frac{1}{4}b^2\cos(x)\sin^3(x)$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*x-1/8*b*(8*a+3*b)*cos(x)*sin(x)-1/4*b^2*cos(x)*sin(
x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {3258}

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{1}{8}b(8a+3b)\sin(x)\cos(x)-\frac{1}{4}b^2\sin^3(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[x]*Sin[x])/8 - (b^2*Cos[
x]*Sin[x]^3)/4
```

Rule 3258

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sin^2(x)\right)^2\,dx=\frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{1}{8}b(8a+3b)\cos(x)\sin(x)-\frac{1}{4}b^2\cos(x)\sin^3(x)$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 0.86

$$\frac{1}{32}\left(4\left(8a^2+8ab+3b^2\right)x-8b(2a+b)\sin(2x)+b^2\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)^2,x]
```

```
[Out] (4*(8*a^2 + 8*a*b + 3*b^2)*x - 8*b*(2*a + b)*Sin[2*x] + b^2*Sin[4*x])/32
```

Maple [A]
time = 0.19, size = 42, normalized size = 0.84

method	result
default	$b^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+2ab\left(-\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)+a^2x$
risch	$a^2x+abx+\frac{3b^2x}{8}+\frac{b^2\sin(4x)}{32}-\frac{\sin(2x)ab}{2}-\frac{\sin(2x)b^2}{4}$
norman	$\frac{\left(-2ab-\frac{11}{4}b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(-2ab-\frac{3}{4}b^2\right)\tan\left(\frac{x}{2}\right)+\left(2ab+\frac{3}{4}b^2\right)\left(\tan^7\left(\frac{x}{2}\right)\right)+\left(2ab+\frac{11}{4}b^2\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(a^2+ab+\frac{3}{8}b^2\right)x+\left(a^2+ab+\frac{3}{8}b^2\right)x(\mathrm{ta}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+2*a*b*(-1/2*sin(x)*cos(x)+1/2
*x)+a^2*x
```

Maxima [A]
time = 0.29, size = 39, normalized size = 0.78

$$\frac{1}{32}\,b^2(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))+\frac{1}{2}\,ab(2\,x-\sin\left(2\,x\right))+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/32*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + 1/2*a*b*(2*x - sin(2*x)) + a^2*x
```

Fricas [A]
time = 0.39, size = 47, normalized size = 0.94

$$\frac{1}{8}\left(8\,a^2+8\,ab+3\,b^2\right)x+\frac{1}{8}\left(2\,b^2\cos\left(x\right)^3-\left(8\,ab+5\,b^2\right)\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/8*(2*b^2*cos(x)^3 - (8*a*b + 5*b^2)*cos(x
))*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(44) = 88$.
time = 0.14, size = 110, normalized size = 2.20

$$a^2x+abx\sin^2\left(x\right)+abx\cos^2\left(x\right)-ab\sin\left(x\right)\cos\left(x\right)+\frac{3b^2x\sin^4\left(x\right)}{8}+\frac{3b^2x\sin^2\left(x\right)\cos^2\left(x\right)}{4}+\frac{3b^2x\cos^4\left(x\right)}{8}-\frac{5b^2\sin^3\left(x\right)\cos\left(x\right)}{8}-\frac{3b^2\sin\left(x\right)\cos^3\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)**2,x)
```

```
[Out] a**2*x + a*b*x*sin(x)**2 + a*b*x*cos(x)**2 - a*b*sin(x)*cos(x) + 3*b**2*x*s
in(x)**4/8 + 3*b**2*x*sin(x)**2*cos(x)**2/4 + 3*b**2*x*cos(x)**4/8 - 5*b**2
*sin(x)**3*cos(x)/8 - 3*b**2*sin(x)*cos(x)**3/8
```

Giac [A]
time = 0.41, size = 42, normalized size = 0.84

$$\frac{1}{32}\,b^2\sin\left(4\,x\right)+\frac{1}{8}\left(8\,a^2+8\,ab+3\,b^2\right)x-\frac{1}{4}\left(2\,ab+b^2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/32*b^2*sin(4*x) + 1/8*(8*a^2 + 8*a*b + 3*b^2)*x - 1/4*(2*a*b + b^2)*sin(2
*x)
```

Mupad [B]
time = 13.52, size = 44, normalized size = 0.88

$$x\,a^2-\sin\left(x\right)\,a\,b\,\cos\left(x\right)+x\,a\,b+\frac{\sin\left(x\right)\,b^2\cos\left(x\right)^3}{4}-\frac{5\,\sin\left(x\right)\,b^2\,\cos\left(x\right)}{8}+\frac{3\,x\,b^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)^2,x)
```

```
[Out] a^2*x + (3*b^2*x)/8 + (b^2*cos(x)^3*sin(x))/4 + a*b*x - (5*b^2*cos(x)*sin(x
))/8 - a*b*cos(x)*sin(x)
```

3.76 $\int \left(a+b\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{1}{16}(2a+b)\left(8a^2+8ab+5b^2\right)x-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\cos(x)\sin(x)-\frac{5}{24}b^2(2a+b)\cos(x)\sin^3(x)-\frac{1}{6}b\cos(x$$

```
[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*x-1/48*b*(64*a^2+54*a*b+15*b^2)*cos(x)*sin
(x)-5/24*b^2*(2*a+b)*cos(x)*sin(x)^3-1/6*b*cos(x)*sin(x)*(a+b*sin(x)^2)^2
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3259, 3248}

$$\frac{1}{16}x(2a+b)\left(8a^2+8ab+5b^2\right)-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\sin(x)\cos(x)-\frac{5}{24}b^2(2a+b)\sin^3(x)\cos(x)-\frac{1}{6}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)^3,x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x)/16 - (b*(64*a^2 + 54*a*b + 15*b^2)*Co
s[x]*Sin[x])/48 - (5*b^2*(2*a + b)*Cos[x]*Sin[x]^3)/24 - (b*Cos[x]*Sin[x]*(
a + b*Sin[x]^2)^2)/6
```

Rule 3248

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rubi steps

$$\int \left(a+b\sin^2(x)\right)^3\,dx=-\frac{1}{6}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2+\frac{1}{6}\int \left(a+b\sin^2(x)\right)\left(a(6a+b)+5b(2a+b)\sin\right.$$

$$=\frac{1}{16}(2a+b)\left(8a^2+8ab+5b^2\right)x-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\cos(x)\sin(x)-\frac{5}{24}b^2(2a$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 80, normalized size = 0.92

$$\frac{1}{192}(12(2a+b)\left(8a^2+8ab+5b^2\right)x+9ib(4ia+(1+2i)b)(4a+(2+i)b)\sin(2x)+9b^2(2a+b)\sin(4x)-b^3\sin(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)^3,x]
```

```
[Out] (12*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x + (9*I)*b*((4*I)*a + (1 + 2*I)*b)*(
4*a + (2 + I)*b)*Sin[2*x] + 9*b^2*(2*a + b)*Sin[4*x] - b^3*Sin[6*x])/192
```

Maple [A]
time = 0.21, size = 73, normalized size = 0.84

method	result
default	$b^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3a\,b^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+3a^2b\left(-\frac{\sin(x)}{2}\right.$
risch	$a^3x+\frac{3x\,a^2b}{2}+\frac{9a\,b^2x}{8}+\frac{5b^3x}{16}-\frac{b^3\sin(6x)}{192}+\frac{3\sin(4x)a\,b^2}{32}+\frac{3\sin(4x)b^3}{64}-\frac{3\sin(2x)a^2b}{4}-\frac{3\sin(2x)a\,b^2}{4}-\frac{15\sin(2x)}{64}$
norman	$\frac{\left(-9a^2b-\frac{51}{4}a\,b^2-\frac{85}{24}b^3\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(-6a^2b-\frac{21}{2}a\,b^2-\frac{33}{4}b^3\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(-3a^2b-\frac{9}{4}a\,b^2-\frac{5}{8}b^3\right)\tan\left(\frac{x}{2}\right)+\left(3a^2b+\frac{9}{4}a\,b^2+\frac{5}{8}b^3\right)\left(\tan^{11}\left(\frac{x}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+3*a*b^2*(-1/4*
(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+3*a^2*b*(-1/2*sin(x)*cos(x)+1/2*x)+a^3*
x
```

Maxima [A]
time = 0.29, size = 71, normalized size = 0.82

$$\frac{1}{192}\left(4\sin(2x)^3+60x+9\sin(4x)-48\sin(2x)\right)b^3+\frac{3}{32}ab^2(12x+\sin(4x)-8\sin(2x))+\frac{3}{4}a^2b(2x-\sin(2x))+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*b^3 + 3/32*a*b^2*(12
*x + sin(4*x) - 8*sin(2*x)) + 3/4*a^2*b*(2*x - sin(2*x)) + a^3*x
```

Fricas [A]
time = 0.40, size = 81, normalized size = 0.93

$$\frac{1}{16}\left(16a^3+24a^2b+18ab^2+5b^3\right)x-\frac{1}{48}\left(8b^3\cos(x)^5-2\left(18ab^2+13b^3\right)\cos(x)^3+3\left(24a^2b+30ab^2+11b^3\right)\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x - 1/48*(8*b^3*cos(x)^5 - 2*(1
8*a*b^2 + 13*b^3)*cos(x)^3 + 3*(24*a^2*b + 30*a*b^2 + 11*b^3)*cos(x))*sin(x
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(88) = 176$.
time = 0.32, size = 246, normalized size = 2.83

$$a^3x + \frac{3a^2bx\sin^2(x)}{2} + \frac{3a^2bx\cos^2(x)}{2} - \frac{3a^2b\sin(x)\cos(x)}{2} + \frac{9ab^2x\sin^4(x)}{8} + \frac{9ab^2x\sin^2(x)\cos^2(x)}{4} + \frac{9ab^2x\cos^4(x)}{8} - \frac{15ab^2\sin^3(x)\cos(x)}{8} - \frac{9ab^2\sin(x)\cos^3(x)}{8} + \frac{5b^3x\sin^6(x)}{16} + \frac{15b^3x\sin^4(x)\cos^2(x)}{16} + \frac{15b^3x\sin^2(x)\cos^4(x)}{16} + \frac{5b^3x\cos^6(x)}{16} - \frac{11b^3\sin^5(x)\cos(x)}{16} - \frac{5b^3\sin^3(x)\cos^3(x)}{6} - \frac{5b^3\sin(x)\cos^5(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x*sin(x)**2/2 + 3*a**2*b*x*cos(x)**2/2 - 3*a**2*b*sin(x)*
cos(x)/2 + 9*a*b**2*x*sin(x)**4/8 + 9*a*b**2*x*sin(x)**2*cos(x)**2/4 + 9*a*
b**2*x*cos(x)**4/8 - 15*a*b**2*sin(x)**3*cos(x)/8 - 9*a*b**2*sin(x)*cos(x)*
*3/8 + 5*b**3*x*sin(x)**6/16 + 15*b**3*x*sin(x)**4*cos(x)**2/16 + 15*b**3*x
*sin(x)**2*cos(x)**4/16 + 5*b**3*x*cos(x)**6/16 - 11*b**3*sin(x)**5*cos(x)/
16 - 5*b**3*sin(x)**3*cos(x)**3/6 - 5*b**3*sin(x)*cos(x)**5/16
```

Giac [A]
time = 0.42, size = 76, normalized size = 0.87

$$-\frac{1}{192}\,b^3\sin\left(6\,x\right) + \frac{1}{16}\left(16\,a^3 + 24\,a^2b + 18\,ab^2 + 5\,b^3\right)x + \frac{3}{64}\left(2\,ab^2 + b^3\right)\sin\left(4\,x\right) - \frac{3}{64}\left(16\,a^2b + 16\,ab^2 + 5\,b^3\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/192*b^3*sin(6*x) + 1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x + 3/64*
(2*a*b^2 + b^3)*sin(4*x) - 3/64*(16*a^2*b + 16*a*b^2 + 5*b^3)*sin(2*x)
```

Mupad [B]
time = 14.13, size = 118, normalized size = 1.36

$$a^3\,x + \frac{5\,b^3\,x}{16} - \frac{\left(72\,a^2\,b + 90\,a\,b^2 + 33\,b^3\right)\,\tan(x)^5 + \left(144\,a^2\,b + 144\,a\,b^2 + 40\,b^3\right)\,\tan(x)^3 + \left(72\,a^2\,b + 54\,a\,b^2 + 15\,b^3\right)\,\tan(x)}{48\tan\left(x\right)^6 + 144\tan\left(x\right)^4 + 144\tan\left(x\right)^2 + 48} + \frac{9\,a\,b^2\,x}{8} + \frac{3\,a^2\,b\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)^3,x)
```

```
[Out] a^3*x + (5*b^3*x)/16 - (tan(x)^5*(90*a*b^2 + 72*a^2*b + 33*b^3) + tan(x)^3*
(144*a*b^2 + 144*a^2*b + 40*b^3) + tan(x)*(54*a*b^2 + 72*a^2*b + 15*b^3))/(
144*tan(x)^2 + 144*tan(x)^4 + 48*tan(x)^6 + 48) + (9*a*b^2*x)/8 + (3*a^2*b*
x)/2
```

3.77 $\int \left(a+b\sin^2(x)\right)^4 \, dx$

Optimal. Leaf size=140

$$\frac{1}{128}\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)x-\frac{1}{384}b\left(608a^3+808a^2b+480ab^2+105b^3\right)\cos(x)\sin(x)-$$

```
[Out] 1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)*x-1/384*b*(608*a^3+8
08*a^2*b+480*a*b^2+105*b^3)*cos(x)*sin(x)-1/192*b^2*(104*a^2+104*a*b+35*b^2
)*cos(x)*sin(x)^3-7/48*b*(2*a+b)*cos(x)*sin(x)*(a+b*sin(x)^2)^2-1/8*b*cos(x
)*sin(x)*(a+b*sin(x)^2)^3
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3259, 3249, 3248}

$$-\frac{1}{192}b^2\left(104a^2+104ab+35b^2\right)\sin^3(x)\cos(x)-\frac{1}{384}b\left(608a^3+808a^2b+480ab^2+105b^3\right)\sin(x)\cos(x)+\frac{1}{128}x\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)-\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3-\frac{7}{48}b(2a+b)\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)^4,x]
```

```
[Out] ((128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x)/128 - (b*(608*
a^3 + 808*a^2*b + 480*a*b^2 + 105*b^3)*Cos[x]*Sin[x])/384 - (b^2*(104*a^2 +
 104*a*b + 35*b^2)*Cos[x]*Sin[x]^3)/192 - (7*b*(2*a + b)*Cos[x]*Sin[x]*(a +
 b*Sin[x]^2)^2)/48 - (b*Cos[x]*Sin[x]*(a + b*Sin[x]^2)^3)/8
```

Rule 3248

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3249

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e
 + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p
 + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Gt
Q[p, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
```

```
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(x)\right)^4\,dx &= -\frac{1}{8}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^3+\frac{1}{8}\int \left(a+b\sin^2(x)\right)^2\left(a(8a+b)+7b(2a+b)\,\mathrm{si}\right.\\ &= -\frac{7}{48}b(2a+b)\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2-\frac{1}{8}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^3+\frac{1}{48}\\ &= \frac{1}{128}\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)x-\frac{1}{384}b\left(608a^3+808a^2b+480ab\right.\end{aligned}$$

Mathematica [A]
time = 0.11, size = 113, normalized size = 0.81

$$\frac{24(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4)\,x-96b(2a+b)\left(16a^2+16ab+7b^2\right)\sin(2x)+24b^2\left(24a^2+24ab+7b^2\right)\sin(4x)-32b^3(2a+b)\sin(6x)+3b^4\sin(8x)}{3072}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)^4,x]
```

```
[Out] (24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x - 96*b*(2*a
+ b)*(16*a^2 + 16*a*b + 7*b^2)*Sin[2*x] + 24*b^2*(24*a^2 + 24*a*b + 7*b^2)*
Sin[4*x] - 32*b^3*(2*a + b)*Sin[6*x] + 3*b^4*Sin[8*x])/3072
```

Maple [A]
time = 0.27, size = 110, normalized size = 0.79

method	result
default	$b^4\left(-\frac{\left(\sin^7(x)+\frac{7\left(\sin^5(x)\right)}{6}+\frac{35\left(\sin^3(x)\right)}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)+4a\,b^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\right.$
risch	$a^4x+2x\,a^3b+\frac{9x\,a^2b^2}{4}+\frac{5xa\,b^3}{4}+\frac{35x\,b^4}{128}+\frac{b^4\sin(8x)}{1024}-\frac{\sin(6x)a\,b^3}{48}-\frac{\sin(6x)b^4}{96}+\frac{3\sin(4x)a^2b^2}{16}+\frac{3\sin(4x)a\,b^3}{16}+$
norman	$\frac{\left(-36a^3b-\frac{153}{2}a^2b^2-\frac{383}{6}a\,b^3-\frac{2681}{192}b^4\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(-20a^3b-\frac{93}{2}a^2b^2-\frac{283}{6}a\,b^3-\frac{5053}{192}b^4\right)\left(\tan^7\left(\frac{x}{2}\right)\right)+\left(-20a^3b-\frac{69}{2}a^2b^2-\frac{115}{6}a\,b^3-\frac{805}{192}b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^4*(-1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/128
*x)+4*a*b^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+6*a^2*
```

```
b^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+4*a^3*b*(-1/2*sin(x)*cos(x)+1
/2*x)+a^4*x
```

Maxima [A]
time = 0.29, size = 108, normalized size = 0.77

$$\frac{1}{48}\left(4 \sin\left(2 x\right)^{3}+60 x+9 \sin\left(4 x\right)-48 \sin\left(2 x\right)\right) a b^{3}+\frac{1}{3072}\left(128 \sin\left(2 x\right)^{3}+840 x+3 \sin\left(8 x\right)+168 \sin\left(4 x\right)-768 \sin\left(2 x\right)\right) b^{4}+\frac{3}{16} a^{2} b^{2}\left(12 x+\sin\left(4 x\right)-8 \sin\left(2 x\right)\right)+a^{3} b\left(2 x-\sin\left(2 x\right)\right)+a^{4} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a*b^3 + 1/3072*(128*s
in(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))*b^4 + 3/16*a^
2*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + a^3*b*(2*x - sin(2*x)) + a^4*x
```

Fricas [A]
time = 0.40, size = 123, normalized size = 0.88

$$\frac{1}{128}\left(128 a^{4}+256 a^{3} b+288 a^{2} b^{2}+160 a b^{3}+35 b^{4}\right) x+\frac{1}{384}\left(48 b^{4} \cos\left(x\right)^{7}-8\left(32 a b^{3}+25 b^{4}\right) \cos\left(x\right)^{5}+2\left(288 a^{2} b^{2}+416 a b^{3}+163 b^{4}\right) \cos\left(x\right)^{3}-3\left(256 a^{3} b+480 a^{2} b^{2}+352 a b^{3}+93 b^{4}\right) \cos\left(x\right)\right) \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x + 1/384*(4
8*b^4*cos(x)^7 - 8*(32*a*b^3 + 25*b^4)*cos(x)^5 + 2*(288*a^2*b^2 + 416*a*b^
3 + 163*b^4)*cos(x)^3 - 3*(256*a^3*b + 480*a^2*b^2 + 352*a*b^3 + 93*b^4)*co
s(x))*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(146) = 292$.
time = 0.72, size = 410, normalized size = 2.93

$$a^{4} x + 2 a^{3} b x \sin^{2}{\left(x \right)} + 2 a^{3} b x \cos^{2}{\left(x \right)} - 2 a^{3} b \sin{\left(x \right)} \cos{\left(x \right)} + \frac{9 a^{2} b^{2} x \sin^{4}{\left(x \right)}}{4} + \frac{9 a^{2} b^{2} x \sin^{2}{\left(x \right)} \cos^{2}{\left(x \right)}}{2} + \frac{9 a^{2} b^{2} x \cos^{4}{\left(x \right)}}{4} - \frac{15 a^{2} b^{2} \sin^{3}{\left(x \right)} \cos{\left(x \right)}}{4} - \frac{9 a^{2} b^{2} \sin{\left(x \right)} \cos^{3}{\left(x \right)}}{4} + \frac{5 a b^{3} x \sin^{6}{\left(x \right)}}{4} + \frac{15 a b^{3} x \sin^{4}{\left(x \right)} \cos^{2}{\left(x \right)}}{4} + \frac{15 a b^{3} x \sin^{2}{\left(x \right)} \cos^{4}{\left(x \right)}}{4} + \frac{5 a b^{3} x \cos^{6}{\left(x \right)}}{4} - \frac{11 a b^{3} \sin^{5}{\left(x \right)} \cos{\left(x \right)}}{4} - \frac{10 a b^{3} \sin^{3}{\left(x \right)} \cos^{3}{\left(x \right)}}{3} - \frac{5 a b^{3} \sin{\left(x \right)} \cos^{5}{\left(x \right)}}{4} + \frac{35 b^{4} x \sin^{8}{\left(x \right)}}{128} + \frac{35 b^{4} x \sin^{6}{\left(x \right)} \cos^{2}{\left(x \right)}}{32} + \frac{105 b^{4} x \sin^{4}{\left(x \right)} \cos^{4}{\left(x \right)}}{64} + \frac{35 b^{4} x \sin^{2}{\left(x \right)} \cos^{6}{\left(x \right)}}{32} + \frac{35 b^{4} x \cos^{8}{\left(x \right)}}{128} - \frac{93 b^{4} \sin^{7}{\left(x \right)} \cos{\left(x \right)}}{128} - \frac{511 b^{4} \sin^{5}{\left(x \right)} \cos^{3}{\left(x \right)}}{384} - \frac{385 b^{4} \sin^{3}{\left(x \right)} \cos^{5}{\left(x \right)}}{384} - \frac{35 b^{4} \sin{\left(x \right)} \cos^{7}{\left(x \right)}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)**4,x)
```

```
[Out] a**4*x + 2*a**3*b*x*sin(x)**2 + 2*a**3*b*x*cos(x)**2 - 2*a**3*b*sin(x)*cos(
x) + 9*a**2*b**2*x*sin(x)**4/4 + 9*a**2*b**2*x*sin(x)**2*cos(x)**2/2 + 9*a*
*2*b**2*x*cos(x)**4/4 - 15*a**2*b**2*sin(x)**3*cos(x)/4 - 9*a**2*b**2*sin(x
)*cos(x)**3/4 + 5*a*b**3*x*sin(x)**6/4 + 15*a*b**3*x*sin(x)**4*cos(x)**2/4
+ 15*a*b**3*x*sin(x)**2*cos(x)**4/4 + 5*a*b**3*x*cos(x)**6/4 - 11*a*b**3*si
n(x)**5*cos(x)/4 - 10*a*b**3*sin(x)**3*cos(x)**3/3 - 5*a*b**3*sin(x)*cos(x)
**5/4 + 35*b**4*x*sin(x)**8/128 + 35*b**4*x*sin(x)**6*cos(x)**2/32 + 105*b*
*4*x*sin(x)**4*cos(x)**4/64 + 35*b**4*x*sin(x)**2*cos(x)**6/32 + 35*b**4*x*
```

```
cos(x)**8/128 - 93*b**4*sin(x)**7*cos(x)/128 - 511*b**4*sin(x)**5*cos(x)**3
/384 - 385*b**4*sin(x)**3*cos(x)**5/384 - 35*b**4*sin(x)*cos(x)**7/128
```

Giac [A]
time = 0.46, size = 118, normalized size = 0.84

$$\frac{1}{1024}\,b^4\sin\left(8\,x\right)+\frac{1}{128}\,\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)x-\frac{1}{96}\,\left(2\,ab^3+b^4\right)\sin\left(6\,x\right)+\frac{1}{128}\,\left(24\,a^2b^2+24\,ab^3+7\,b^4\right)\sin\left(4\,x\right)-\frac{1}{32}\,\left(32\,a^3b+48\,a^2b^2+30\,ab^3+7\,b^4\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^4,x, algorithm="giac")
```

```
[Out] 1/1024*b^4*sin(8*x) + 1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3
+ 35*b^4)*x - 1/96*(2*a*b^3 + b^4)*sin(6*x) + 1/128*(24*a^2*b^2 + 24*a*b^3
+ 7*b^4)*sin(4*x) - 1/32*(32*a^3*b + 48*a^2*b^2 + 30*a*b^3 + 7*b^4)*sin(2*x
)
```

Mupad [B]
time = 13.63, size = 147, normalized size = 1.05

$$x\,a^4-2\,\sin\left(x\right)\,a^3\,b\,\cos\left(x\right)+2\,x\,a^3\,b+\frac{3\,\sin\left(x\right)\,a^2\,b^2\cos\left(x\right)^3}{2}-\frac{15\,\sin\left(x\right)\,a^2\,b^2\,\cos\left(x\right)}{4}+\frac{9\,x\,a^2\,b^2}{4}-\frac{2\,\sin\left(x\right)\,a\,b^3\cos\left(x\right)^5}{3}+\frac{13\,\sin\left(x\right)\,a\,b^3\cos\left(x\right)^3}{6}-\frac{11\,\sin\left(x\right)\,a\,b^3\,\cos\left(x\right)}{4}+\frac{5\,x\,a\,b^3}{4}+\frac{\sin\left(x\right)\,b^4\cos\left(x\right)^7}{8}-\frac{25\,\sin\left(x\right)\,b^4\cos\left(x\right)^5}{48}+\frac{163\,\sin\left(x\right)\,b^4\cos\left(x\right)^3}{192}-\frac{93\,\sin\left(x\right)\,b^4\,\cos\left(x\right)}{128}+\frac{35\,x\,b^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)^4,x)
```

```
[Out] a^4*x + (35*b^4*x)/128 + (163*b^4*cos(x)^3*sin(x))/192 - (25*b^4*cos(x)^5*s
in(x))/48 + (b^4*cos(x)^7*sin(x))/8 + (9*a^2*b^2*x)/4 - (93*b^4*cos(x)*sin(
x))/128 + (5*a*b^3*x)/4 + 2*a^3*b*x + (3*a^2*b^2*cos(x)^3*sin(x))/2 - (11*a
*b^3*cos(x)*sin(x))/4 - 2*a^3*b*cos(x)*sin(x) - (15*a^2*b^2*cos(x)*sin(x))/
4 + (13*a*b^3*cos(x)^3*sin(x))/6 - (2*a*b^3*cos(x)^5*sin(x))/3
```

3.78 $\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}\,d}-\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}$$

```
[Out] -(a^2-a*b+b^2)*cos(d*x+c)/b^3/d-1/3*(a-2*b)*cos(d*x+c)^3/b^2/d-1/5*cos(d*x+
c)^5/b/d+a^3*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3265, 398, 214}

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}d\sqrt{a+b}}-\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^3*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b]*d) -
 ((a^2 - a*b + b^2)*Cos[c + d*x])/(b^3*d) - ((a - 2*b)*Cos[c + d*x]^3)/(3*b
^2*d) - Cos[c + d*x]^5/(5*b*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{b^3}+\frac{(a-2b)x^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3(a+b-bx^2)}\right)dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}+\frac{a^3\text{Subst}\left(\int\right.}{}\\
&= \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}\,d}-\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.96, size = 180, normalized size = 1.70

$$\frac{-240a^3\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)-240a^3\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)-2\sqrt{-a-b}\sqrt{b}\cos(c+dx)\left(120a^2-100ab+89b^2+4(5a-7b)b\cos(2(c+dx))+3b^2\cos(4(c+dx))\right)}{240\sqrt{-a-b}\,b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-240*a^3*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 240
*a^3*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 2*Sqrt[-
a - b]*Sqrt[b]*Cos[c + d*x]*(120*a^2 - 100*a*b + 89*b^2 + 4*(5*a - 7*b)*b*C
os[2*(c + d*x)] + 3*b^2*Cos[4*(c + d*x)]))/(240*Sqrt[-a - b]*b^(7/2)*d)
```

Maple [A]
time = 0.36, size = 110, normalized size = 1.04

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\cos^5(dx+c)\right)b^2}{5}+\frac{ab\left(\cos^3(dx+c)\right)}{3}-\frac{2b^2\left(\cos^3(dx+c)\right)}{3}+a^2\cos(dx+c)-ab\cos(dx+c)+b^2\cos(dx+c)}{b^3}+\frac{a^3\,\text{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right.}{b^3\sqrt{(a+b)b}}}{d}$
default	$\frac{-\frac{\frac{\left(\cos^5(dx+c)\right)b^2}{5}+\frac{ab\left(\cos^3(dx+c)\right)}{3}-\frac{2b^2\left(\cos^3(dx+c)\right)}{3}+a^2\cos(dx+c)-ab\cos(dx+c)+b^2\cos(dx+c)}{b^3}+\frac{a^3\,\text{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right.}{b^3\sqrt{(a+b)b}}}{d}$

risch	$-\frac{e^{i(dx+c)}a^2}{2b^3d}+\frac{3a\,e^{i(dx+c)}}{8b^2d}-\frac{5\,e^{i(dx+c)}}{16bd}-\frac{e^{-i(dx+c)}a^2}{2b^3d}+\frac{3a\,e^{-i(dx+c)}}{8b^2d}-\frac{5\,e^{-i(dx+c)}}{16bd}-\frac{ia^3\ln\left(e^{2i(dx+c)}-\ldots\right.}{2\sqrt{-ab\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^3*(1/5*cos(d*x+c)^5*b^2+1/3*a*b*cos(d*x+c)^3-2/3*b^2*cos(d*x+c)^3
+a^2*cos(d*x+c)-a*b*cos(d*x+c)+b^2*cos(d*x+c))+a^3/b^3/((a+b)*b)^(1/2)*arct
anh(b*cos(d*x+c)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 116, normalized size = 1.09

$$-\frac{\frac{15\,a^3\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,b^3}+\frac{2\left(3\,b^2\cos(dx+c)^5+5\,(ab-2\,b^2)\cos(dx+c)^3+15\,(a^2-ab+b^2)\cos(dx+c)\right)}{b^3}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/30*(15*a^3*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt
((a + b)*b)))/(sqrt((a + b)*b)*b^3) + 2*(3*b^2*cos(d*x + c)^5 + 5*(a*b - 2*
b^2)*cos(d*x + c)^3 + 15*(a^2 - a*b + b^2)*cos(d*x + c))/b^3)/d
```

Fricas [A]
time = 0.42, size = 272, normalized size = 2.57

$$\left[-\frac{6\,(ab^3+b^4)\cos(dx+c)^5-15\,\sqrt{ab+b^2}\,a^3\log\left(\frac{b\cos(dx+c)^2+2\,\sqrt{ab+b^2}\,\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+10\,(a^2b^2-ab^3-2\,b^4)\cos(dx+c)^3+30\,(a^3b+b^4)\cos(dx+c)}{30\,(ab^4+b^5)d},\ -\frac{3\,(ab^3+b^4)\cos(dx+c)^5+15\,\sqrt{-ab-b^2}\,a^3\arctan\left(\frac{\sqrt{-ab-b^2}\,\cos(dx+c)}{a+b}\right)+5\,(a^2b^2-ab^3-2\,b^4)\cos(dx+c)^3+15\,(a^3b+b^4)\cos(dx+c)}{15\,(ab^4+b^5)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*(a*b^3 + b^4)*cos(d*x + c)^5 - 15*sqrt(a*b + b^2)*a^3*log((b*cos(
d*x + c)^2 + 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a
- b)) + 10*(a^2*b^2 - a*b^3 - 2*b^4)*cos(d*x + c)^3 + 30*(a^3*b + b^4)*cos(
d*x + c))/((a*b^4 + b^5)*d), -1/15*(3*(a*b^3 + b^4)*cos(d*x + c)^5 + 15*sqr
t(-a*b - b^2)*a^3*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a + b)) + 5*(a^2*b^
2 - a*b^3 - 2*b^4)*cos(d*x + c)^3 + 15*(a^3*b + b^4)*cos(d*x + c))/((a*b^4
+ b^5)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(94) = 188.
time = 0.50, size = 332, normalized size = 3.13

$$-\frac{\frac{15\,a^3\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\,\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b^3}-\frac{2\left(15\,a^2-10\,ab+8\,b^2-\frac{60\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{50\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{40\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{90\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{70\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{80\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{60\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{30\,ab(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{15\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}\right)}{b^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*a^3*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c
) + sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*b^3) - 2*(15*a^2 - 10*a*b + 8*b^2
- 60*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 50*a*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) - 40*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 90*a^2*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 70*a*b*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 80*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 60*a
^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 30*a*b*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 15*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/(b
^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

Mupad [B]
time = 0.16, size = 112, normalized size = 1.06

$$\frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{b^{7/2}\,d\,\sqrt{a+b}}-\frac{\cos\left(c+d\,x\right)^5}{5\,b\,d}-\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{a+b}{3\,b^2}-\frac{1}{b}\right)}{d}-\frac{\cos\left(c+d\,x\right)\,\left(\frac{3}{b}+\frac{(a+b)\,\left(\frac{a+b}{b^2}-\frac{3}{b}\right)}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (a^3*atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2)))/(b^(7/2)*d*(a + b)^(1/2))
 - cos(c + d*x)^5/(5*b*d) - (cos(c + d*x)^3*((a + b)/(3*b^2) - 1/b))/d - (c
os(c + d*x)*(3/b + ((a + b)*((a + b)/b^2 - 3/b))/b))/d
```

3.79 $\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,d}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

[Out] (a-b)*cos(d*x+c)/b^2/d+1/3*cos(d*x+c)^3/b/d-a^2*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(5/2)/d/(a+b)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 398, 214}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]

[Out] -((a^2*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*d)) + ((a - b)*Cos[c + d*x])/(b^2*d) + Cos[c + d*x]^3/(3*b*d)

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{a-b}{b^2}-\frac{x^2}{b}+\frac{a^2}{b^2(a+b-bx^2)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}-\frac{a^2\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{b^2d}$$
$$= -\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,d}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 150, normalized size = 1.95

$$\frac{6a^2\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+6a^2\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\sqrt{-a-b}\sqrt{b}\cos(c+dx)(6a-5b+b\cos(2(c+dx)))}{6\sqrt{-a-b}\,b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (6*a^2*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + 6*a^2*
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*
Sqrt[b]*Cos[c + d*x]*(6*a - 5*b + b*Cos[2*(c + d*x)]))/(6*Sqrt[-a - b]*b^(5
/2)*d)
```

Maple [A]
time = 0.27, size = 70, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{\frac{\left(\cos^3(dx+c)\right)b}{3}+a\cos(dx+c)-b\cos(dx+c)}{b^2}-\frac{a^2\,\text{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{b^2\sqrt{(a+b)\,b}}}{d}$
default	$\frac{\frac{\frac{\left(\cos^3(dx+c)\right)b}{3}+a\cos(dx+c)-b\cos(dx+c)}{b^2}-\frac{a^2\,\text{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{b^2\sqrt{(a+b)\,b}}}{d}$
risch	$\frac{a\,e^{i(dx+c)}}{2b^2d}-\frac{3\,e^{i(dx+c)}}{8bd}+\frac{a\,e^{-i(dx+c)}}{2b^2d}-\frac{3\,e^{-i(dx+c)}}{8bd}+\frac{ia^2\ln\left(e^{2i(dx+c)}-\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{2\sqrt{-ab-b^2}\,d\,b^2}-\frac{ia^2\ln\left(e^{2i(dx+c)}\right.}{2\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*(1/3*cos(d*x+c)^3*b+a*cos(d*x+c)-b*cos(d*x+c))-a^2/b^2/((a+b)*b)
^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 88, normalized size = 1.14

$$\frac{\frac{3\,a^2\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,b^2}+\frac{2\left(b\cos(dx+c)^3+3\,(a-b)\cos(dx+c)\right)}{b^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a
 + b)*b)))/(sqrt((a + b)*b)*b^2) + 2*(b*cos(d*x + c)^3 + 3*(a - b)*cos(d*x
+ c))/b^2)/d
```

Fricas [A]
time = 0.41, size = 218, normalized size = 2.83

$$\left[\frac{2\,(ab^2+b^3)\cos(dx+c)^3+3\,\sqrt{ab+b^2}\,a^2\log\left(-\frac{b\cos(dx+c)^2-2\,\sqrt{ab+b^2}\,\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+6\,(a^2b-b^3)\cos(dx+c)}{6\,(ab^3+b^4)d},\ \frac{(ab^2+b^3)\cos(dx+c)^3+3\,\sqrt{-ab-b^2}\,a^2\arctan\left(\frac{\sqrt{-ab-b^2}\,\cos(dx+c)}{a+b}\right)+3\,(a^2b-b^3)\cos(dx+c)}{3\,(ab^3+b^4)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a*b^2 + b^3)*cos(d*x + c)^3 + 3*sqrt(a*b + b^2)*a^2*log(-(b*cos(d*
x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a -
b)) + 6*(a^2*b - b^3)*cos(d*x + c))/((a*b^3 + b^4)*d), 1/3*((a*b^2 + b^3)*c
os(d*x + c)^3 + 3*sqrt(-a*b - b^2)*a^2*arctan(sqrt(-a*b - b^2)*cos(d*x + c)
/(a + b)) + 3*(a^2*b - b^3)*cos(d*x + c))/((a*b^3 + b^4)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(67) = 134.
time = 0.44, size = 173, normalized size = 2.25

$$\frac{\frac{3\,a^2\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b^2}-\frac{2\left(3\,a-2\,b-\frac{6\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
 sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*b^2) - 2*(3*a - 2*b - 6*a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*
a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(b^2*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^3))/d
```

Mupad [B]
time = 0.11, size = 72, normalized size = 0.94

$$\frac{\cos\left(c+d\,x\right)\,\left(\frac{a+b}{b^2}-\frac{2}{b}\right)}{d}+\frac{{\cos\left(c+d\,x\right)}^3}{3\,b\,d}-\frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos\left(c+d\,x\right)}{\sqrt{a+b}}\right)}{b^{5/2}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (cos(c + d*x)*((a + b)/b^2 - 2/b))/d + cos(c + d*x)^3/(3*b*d) - (a^2*atanh(
(b^(1/2)*cos(c + d*x))/(a + b)^(1/2)))/(b^(5/2)*d*(a + b)^(1/2))
```

3.80 $\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}\,d}-\frac{\cos(c+dx)}{bd}$$

```
[Out] -cos(d*x+c)/b/d+a*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/d/(a+b)^(
1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 396, 214}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*d) - C
os[c + d*x]/(b*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\cos(c+dx)}{bd} + \frac{a\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{bd}$$
$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}\,d} - \frac{\cos(c+dx)}{bd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 125, normalized size = 2.40

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right) + a\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right) + \sqrt{-a-b}\sqrt{b}\cos(c+dx)}{\sqrt{-a-b}\,b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + a*ArcTan
[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*Sqrt[b
]*Cos[c + d*x])/(Sqrt[-a - b]*b^(3/2)*d))
```

Maple [A]
time = 0.24, size = 45, normalized size = 0.87

method	result	siz
derivativedivides	$\frac{-\frac{\cos(dx+c)}{b}+\frac{a\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{b\sqrt{(a+b)\,b}}}{d}$	45
default	$\frac{-\frac{\cos(dx+c)}{b}+\frac{a\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{b\sqrt{(a+b)\,b}}}{d}$	45
risch	$-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}-\frac{ia\ln\left(e^{2i(dx+c)}-\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{2\sqrt{-ab-b^2}\,db}+\frac{ia\ln\left(e^{2i(dx+c)}+\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{2\sqrt{-ab-b^2}\,db}$	158

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*cos(d*x+c)+1/b*a/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(
1/2)))
```

Maxima [A]
time = 0.59, size = 67, normalized size = 1.29

$$-\frac{\frac{a\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,b}+\frac{2\cos(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a +
b)*b)))/(sqrt((a + b)*b)*b) + 2*cos(d*x + c)/b)/d
```

Fricas [A]
time = 0.41, size = 165, normalized size = 3.17

$$\left[\frac{\sqrt{ab+b^2}\,a\log\left(\frac{b\cos(dx+c)^2+2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-2\,(ab+b^2)\cos(dx+c)}{2\,(ab^2+b^3)d},\ -\frac{\sqrt{-ab-b^2}\,a\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)+(ab+b^2)\cos(dx+c)}{(ab^2+b^3)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*b + b^2)*a*log((b*cos(d*x + c)^2 + 2*sqrt(a*b + b^2)*cos(d*x +
 c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(a*b + b^2)*cos(d*x + c))/((a*
b^2 + b^3)*d), -(sqrt(-a*b - b^2)*a*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a
 + b)) + (a*b + b^2)*cos(d*x + c))/((a*b^2 + b^3)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 57, normalized size = 1.10

$$-\frac{a\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,bd}-\frac{\cos(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b*d) - cos(d*x
 + c)/(b*d)
```

Mupad [B]
time = 0.10, size = 44, normalized size = 0.85

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{b^{3/2}\,d\,\sqrt{a+b}}-\frac{\cos\left(c+d\,x\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (a*atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2)))/(b^(3/2)*d*(a + b)^(1/2)) -
 cos(c + d*x)/(b*d)
```

3.81 $\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=37

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,d}$$

```
[Out] -arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/d/b^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3265, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 97, normalized size = 2.62

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + ArcTan[(Sqrt
[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/(Sqrt[-a - b]*Sqrt[b]*d)
```

Maple [A]
time = 0.17, size = 29, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{d\sqrt{(a+b)\,b}}$	29
default	$-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{d\sqrt{(a+b)\,b}}$	29
risch	$\frac{i\ln\left(e^{2i(dx+c)}-\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{2\sqrt{-ab-b^2}\,d}-\frac{i\ln\left(e^{2i(dx+c)}+\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{2\sqrt{-ab-b^2}\,d}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 50, normalized size = 1.35

$$\frac{\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a + b)*b
)))/(sqrt((a + b)*b)*d)
```

Fricas [A]
time = 0.41, size = 117, normalized size = 3.16

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)}{2\sqrt{ab+b^2}\,d}, \frac{\sqrt{-ab-b^2}\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)}{(ab+b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*co
s(d*x + c)^2 - a - b))/(sqrt(a*b + b^2)*d), sqrt(-a*b - b^2)*arctan(sqrt(-a
*b - b^2)*cos(d*x + c)/(a + b))/((a*b + b^2)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 367693 vs. $2(34) = 68$.
time = 86.31, size = 367693, normalized size = 9937.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2 + d*
x/2))/(b*d), Eq(a, 0)), (2/(b*d*tan(c/2 + d*x/2)**2 - b*d), Eq(a, -b)), (-c
os(c + d*x)/(a*d), Eq(b, 0)), (x*sin(c)/(a + b*sin(c)**2), Eq(d, 0)), (74*a
**37*b*log(-sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*
a**38*b*d + 5478*a**37*b**2*d - 148*a**37*b*d*sqrt(a*b + b**2) + 2502532*a*
*36*b**3*d - 135124*a**36*b**2*d*sqrt(a*b + b**2) + 456961248*a**35*b**4*d
- 36983424*a**35*b**3*d*sqrt(a*b + b**2) + 44602414272*a**34*b**5*d - 48095
99808*a**34*b**4*d*sqrt(a*b + b**2) + 2698911348224*a**33*b**6*d - 36352456
1920*a**33*b**5*d*sqrt(a*b + b**2) + 110776036340736*a**32*b**7*d - 1789193
1206656*a**32*b**6*d*sqrt(a*b + b**2) + 3275718126403584*a**31*b**8*d - 616
808259780608*a**31*b**7*d*sqrt(a*b + b**2) + 72854727629602816*a**30*b**9*d
 - 15666762815766528*a**30*b**8*d*sqrt(a*b + b**2) + 1258467596957384704*a*
*29*b**10*d - 304230303833522176*a**29*b**9*d*sqrt(a*b + b**2) + 1730614089
1880620032*a**28*b**11*d - 4645206174395269120*a**28*b**10*d*sqrt(a*b + b**
2) + 193199008739227598848*a**27*b**12*d - 57001938802859573248*a**27*b**11
*d*sqrt(a*b + b**2) + 1778515685235870400512*a**26*b**13*d - 57202990741937
6123904*a**26*b**12*d*sqrt(a*b + b**2) + 13673782930644613988352*a**25*b**1
4*d - 4761020109769125396480*a**25*b**13*d*sqrt(a*b + b**2) + 8872218313957
7965838336*a**24*b**15*d - 33244276082712682430464*a**24*b**14*d*sqrt(a*b +
 b**2) + 490030319626953299066880*a**23*b**16*d - 196589323247525507891200*
a**23*b**15*d*sqrt(a*b + b**2) + 2320264659880999460536320*a**22*b**17*d -
```

```
992185245208510642257920*a**22*b**16*d*sqrt(a*b + b**2) + 94732374230503145
65550080*a**21*b**18*d - 4301031135604201236725760*a**21*b**17*d*sqrt(a*b +
 b**2) + 33508135815008970573086720*a**20*b**19*d - 16096759227611109665013
760*a**20*b**18*d*sqrt(a*b + b**2) + 103066001007281297455841280*a**19*b**2
0*d - 52224655042483407940485120*a**19*b**19*d*sqrt(a*b + b**2) + 276460659
949410743463444480*a**18*b**21*d - 147353756231010598099353600*a**18*b**20*
d*sqrt(a*b + b**2) + 648017072918162395858206720*a**17*b**22*d - 3624060024
26292494841937920*a**17*b**21*d*sqrt(a*b + b**2) + 132896736702920448874119
1680*a**16*b**23*d - 778070309276565523251855360*a**16*b**22*d*sqrt(a*b + b
**2) + 2385704631316968523085905920*a**15*b**24*d - 14592140482343417820060
05760*a**15*b**23*d*sqrt(a*b + b**2) + 3747529655246565869805895680*a**14*b
**25*d - 2390143076492438985108357120*a**14*b**24*d*sqrt(a*b + b**2) + 5144
960127422757831513735168*a**13*b**26*d - 3415726795297830225523507200*a**13
*b**25*d*sqrt(a*b + b**2) + 6160343368926179873261617152*a**12*b**27*d - 42
50434573627220170723295232*a**12*b**26*d*sqrt(a*b + b**2) + 641255305238666
2194178162688*a**11*b**28*d - 4591390200361817020475375616*a**11*b**27*d*sq
rt(a*b + b**2) + 5777443964131114181336236032*a**10*b**29*d - 4286847917414
518496444809216*a**10*b**28*d*sqrt(a*b + b**2) + 44785211323812567795084820
48*a**9*b**30*d - 3439316943952955273874767872*a**9*b**29*d*sqrt(a*b + b**2
) + 2963524367539447964941418496*a**8*b**31*d - 235269177855673739570577408
0*a**8*b**30*d*sqrt(a*b + b**2) + 1656691644402709111026745344*a**7*b**32*d
 - 1358109508879153777946394624*a**7*b**31*d*sqrt(a*b + b**2) + 77163922119
9942160467623936*a**6*b**33*d - 652517637350019802209452032*a**6*b**32*d*sq
rt(a*b + b**2) + 293857629218373137558667264*a**5*b**34*d - 256080136201453
725194125312*a**5*b**33*d*sqrt(a*b + b**2) + 89102865177381478141526016*a**
4*b**35*d - 79945311123381698013691904*a**4*b**34*d*sqrt(a*b + b**2) + 2068
3374899158687330336768*a**3*b**36*d - 19090166507000540776366080*a**3*b**35
*d*sqrt(a*b + b**2) + 3450869307356993239908352*a**2*b**37*d - 327378056424
9381544394752*a**2*b**36*d*sqrt(a*b + b**2) + 368344585663832326668288*a*b*
*38*d - 358899852698093036240896*a*b**37*d*sqrt(a*b + b**2) + 1888946593147
8580854784*b**39*d - 18889465931478580854784*b**38*d*sqrt(a*b + b**2)) + 74
*a**37*b*log(sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2
*a**38*b*d + 5478*a**37*b**2*d - 148*a**37*b*d*sqrt(a*b + b**2) + 2502532*a
**36*b**3*d - 135124*a**36*b**2*d*sqrt(a*b + b**2) + 456961248*a**35*b**4*d
 - 36983424*a**35*b**3*d*sqrt(a*b + b**2) + 44602414272*a**34*b**5*d - 4809
599808*a**34*b**4*d*sqrt(a*b + b**2) + 2698911348224*a**33*b**6*d - 3635245
61920*a**33*b**5*d*sqrt(a*b + b**2) + 110776036340736*a**32*b**7*d - 178919
31206656*a**32*b**6*d*sqrt(a*b + b**2) + 3275718126403584*a**31*b**8*d - 61
6808259780608*a**31*b**7*d*sqrt(a*b + b**2) + 72854727629602816*a**30*b**9*
d - 15666762815766528*a**30*b**8*d*sqrt(a*b + b**2) + 1258467596957384704*a
**29*b**10*d - 304230303833522176*a**29*b**9*d*sqrt(a*b + b**2) + 173061408
91880620032*a**28*b**11*d - 4645206174395269120*a**28*b**10*d*sqrt(a*b + b*
*2) + 193199008739227598848*a**27*b**12*d - 57001938802859573248*a**27*b**1
1*d*sqrt(a*b + b**2) + 1778515685235870400512*a**26*b**13*d - 5720299074193
76123904*a**26*b**12*d*sqrt(a*b + b**2) + 13673782930644613988352*a**25*b**
```

```
14*d - 4761020109769125396480*a**25*b**13*d*sqr...
```

Giac [A]
time = 0.44, size = 37, normalized size = 1.00

$$\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*d)
```

Mupad [B]
time = 0.09, size = 29, normalized size = 0.78

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*sin(c + d*x)^2),x)
```

```
[Out] -atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2))/(b^(1/2)*d*(a + b)^(1/2))
```

3.82 $\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a/
d/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3265, 400, 212, 214}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\cos(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sq
rt[a + b]])/(a*Sqrt[a + b]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad}+\frac{b\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 143, normalized size = 2.60

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/Sq
rt[-a - b] + (Sqrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a
 - b]])/Sqrt[-a - b] + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])/(a*d)
)
```

Maple [A]
time = 0.28, size = 62, normalized size = 1.13

method	result
derivativedivides	$\frac{\frac{b\,\text{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{a\sqrt{(a+b)\,b}}+\frac{\ln(\cos(dx+c)-1)}{2a}-\frac{\ln(1+\cos(dx+c))}{2a}}{d}$
default	$\frac{\frac{b\,\text{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{a\sqrt{(a+b)\,b}}+\frac{\ln(\cos(dx+c)-1)}{2a}-\frac{\ln(1+\cos(dx+c))}{2a}}{d}$

risch	$\frac{\ln(e^{i(dx+c)}-1)}{ad} - \frac{\ln(e^{i(dx+c)}+1)}{ad} + \frac{i\sqrt{-(a+b)\,b}\,\ln\left(e^{2i(dx+c)} + \frac{2i\sqrt{-(a+b)\,b}\,e^{i(dx+c)}}{b} + 1\right)}{2(a+b)da} - \frac{i\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*b/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))+1/2/a*ln(c
os(d*x+c)-1)-1/2/a*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.55, size = 83, normalized size = 1.51

$$-\frac{\frac{b\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a} + \frac{\log(\cos(dx+c)+1)}{a} - \frac{\log(\cos(dx+c)-1)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a +
b)*b)))/(sqrt((a + b)*b)*a) + log(cos(d*x + c) + 1)/a - log(cos(d*x + c) -
1)/a)/d
```

Fricas [A]
time = 0.42, size = 161, normalized size = 2.93

$$\left[\frac{\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^2+2\,(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right) - \log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,ad}, -\frac{2\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\cos(dx+c)\right) + \log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - \log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b/(a + b))*cos
(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - log(1/2*cos(d*x + c) + 1/2
) + log(-1/2*cos(d*x + c) + 1/2))/(a*d), -1/2*(2*sqrt(-b/(a + b))*arctan(sq
rt(-b/(a + b))*cos(d*x + c)) + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d
*x + c) + 1/2))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(47) = 94.
time = 0.55, size = 100, normalized size = 1.82

$$-\frac{\frac{2\,b\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a) - log(abs(-cos(d*x + c) + 1)/abs(co
s(d*x + c) + 1))/a)/d
```

Mupad [B]
time = 13.73, size = 457, normalized size = 8.31

$$-\frac{\operatorname{atanh}(\cos(c+d\,x))}{a\,d}-\frac{\operatorname{atan}\left(\frac{\frac{\left(2\,b^3\cos(c+d\,x)+\frac{\left(2\,a^2\,b^2-\frac{\cos(c+d\,x)\,(8\,a^3\,b^2+16\,a^2\,b^3)\,\sqrt{b\,(a+b)}}{4\,(a^2+b\,a)}\right)\sqrt{b\,(a+b)}}{2\,(a^2+b\,a)}\right)\sqrt{b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\cos(c+d\,x)-\frac{\left(2\,a^2\,b^2+\frac{\cos(c+d\,x)\,(8\,a^3\,b^2+16\,a^2\,b^3)\,\sqrt{b\,(a+b)}}{4\,(a^2+b\,a)}\right)\sqrt{b\,(a+b)}}{2\,(a^2+b\,a)}\right)\sqrt{b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}}{\frac{\left(2\,b^3\cos(c+d\,x)+\frac{\left(2\,a^2\,b^2-\frac{\cos(c+d\,x)\,(8\,a^3\,b^2+16\,a^2\,b^3)\,\sqrt{b\,(a+b)}}{4\,(a^2+b\,a)}\right)\sqrt{b\,(a+b)}}{2\,(a^2+b\,a)}\right)\sqrt{b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\cos(c+d\,x)-\frac{\left(2\,a^2\,b^2+\frac{\cos(c+d\,x)\,(8\,a^3\,b^2+16\,a^2\,b^3)\,\sqrt{b\,(a+b)}}{4\,(a^2+b\,a)}\right)\sqrt{b\,(a+b)}}{2\,(a^2+b\,a)}\right)\sqrt{b\,(a+b)}}{a^2+b\,a}}\right)\sqrt{b\,(a+b)}\,1\mathrm{i}}{d\,(a^2+b\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] - atanh(cos(c + d*x))/(a*d) - (atan((((2*b^3*cos(c + d*x) + ((2*a^2*b^2 - (
cos(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(
b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*
b^3*cos(c + d*x) - ((2*a^2*b^2 + (cos(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*
(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a
+ b))^(1/2)*1i)/(a*b + a^2))/(((2*b^3*cos(c + d*x) + ((2*a^2*b^2 - (cos(c +
 d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a +
b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*cos(c
+ d*x) - ((2*a^2*b^2 + (cos(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(
1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2
))/(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(d*(a*b + a^2))
```

3.83 $\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{(a-2b)\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*(a-2*b)*arctanh(cos(d*x+c))/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d-b^(3/2
)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/a^2/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3265, 425, 536, 212, 214}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}-\frac{(a-2b)\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/2*((a - 2*b)*ArcTanh[Cos[c + d*x]])/(a^2*d) - (b^(3/2)*ArcTanh[(Sqrt[b]*
Cos[c + d*x])/Sqrt[a + b]])/(a^2*Sqrt[a + b]*d) - (Cot[c + d*x]*Csc[c + d*x
])/(2*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\text{Subst}\left(\int \frac{a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{2ad}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{2a^2d} - \frac{b^2\text{Subst}$$

$$= -\frac{(a-2b)\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,d} - \frac{\cot(c+dx)\csc}{2ad}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.46, size = 224, normalized size = 2.64

$$-\frac{(2a+b-b\cos(2(c+dx)))\csc^2(c+dx)\left(-8b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan(\frac{1}{2}(c+dx))}{\sqrt{-a-b}}\right)-8b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan(\frac{1}{2}(c+dx))}{\sqrt{-a-b}}\right)+\sqrt{-a-b}\left(a\csc^2\left(\frac{1}{2}(c+dx)\right)+4(a-2b)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-a\sec^2\left(\frac{1}{2}(c+dx)\right)\right)\right)}{16a^2\sqrt{-a-b}\,d\,(b+a\csc^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/16*((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(-8*b^(3/2)*ArcTan[(Sq
rt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 8*b^(3/2)*ArcTan[(Sqrt[
b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*(a*Csc[(c + d
*x)/2]^2 + 4*(a - 2*b)*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) - a*
Sec[(c + d*x)/2]^2)))/(a^2*Sqrt[-a - b]*d*(b + a*Csc[c + d*x]^2))
```

Maple [A]

time = 0.37, size = 107, normalized size = 1.26

method	result
derivativedivides	$\dfrac{\frac{1}{4a(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4a^2}-\frac{b^2\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{a^2\sqrt{(a+b)\,b}}+\frac{1}{4a(1+\cos(dx+c))}+\frac{(-a+2b)\ln(1+\cos(dx+c))}{4a^2}}{d}$
default	$\dfrac{\frac{1}{4a(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4a^2}-\frac{b^2\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{a^2\sqrt{(a+b)\,b}}+\frac{1}{4a(1+\cos(dx+c))}+\frac{(-a+2b)\ln(1+\cos(dx+c))}{4a^2}}{d}$
risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2ad}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}-\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{i\sqrt{-(a+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a/(cos(d*x+c)-1)+1/4*(a-2*b)/a^2*ln(cos(d*x+c)-1)-b^2/a^2/((a+b)*b
)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))+1/4/a/(1+cos(d*x+c))+1/4/a^2*
(-a+2*b)*ln(1+cos(d*x+c)))
```

Maxima [A]

time = 0.54, size = 120, normalized size = 1.41

$$\frac{\frac{2\,b^2\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2}+\frac{2\cos(dx+c)}{a\cos(dx+c)^2-a}-\frac{(a-2\,b)\log(\cos(dx+c)+1)}{a^2}+\frac{(a-2\,b)\log(\cos(dx+c)-1)}{a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a
 + b)*b)))/(sqrt((a + b)*b)*a^2) + 2*cos(d*x + c)/(a*cos(d*x + c)^2 - a) -
(a - 2*b)*log(cos(d*x + c) + 1)/a^2 + (a - 2*b)*log(cos(d*x + c) - 1)/a^2)/
d
```

Fricas [A]

time = 0.42, size = 327, normalized size = 3.85

$$\left[\frac{2\left(b\cos(dx+c)^2-b\right)\sqrt{\frac{b}{a+b}}\log\left(-\frac{b\cos(dx+c)^2-2(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+2\,a\cos(dx+c)-\left((a-2\,b)\cos(dx+c)^2-a+2\,b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left((a-2\,b)\cos(dx+c)^2-a+2\,b\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{4\left(a^2d\cos(dx+c)^2-a^2d\right)},\ \frac{4\left(b\cos(dx+c)^2-b\right)\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\cos(dx+c)\right)+2\,a\cos(dx+c)-\left((a-2\,b)\cos(dx+c)^2-a+2\,b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left((a-2\,b)\cos(dx+c)^2-a+2\,b\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{4\left(a^2d\cos(dx+c)^2-a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*cos(d*x + c)^2 - b)*sqrt(b/(a + b))*log(-(b*cos(d*x + c)^2 - 2*(
a + b)*sqrt(b/(a + b))*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) +
2*a*cos(d*x + c) - ((a - 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c
) + 1/2) + ((a - 2*b)*cos(d*x + c)^2 - a + 2*b)*log(-1/2*cos(d*x + c) + 1/2
))/(a^2*d*cos(d*x + c)^2 - a^2*d), 1/4*(4*(b*cos(d*x + c)^2 - b)*sqrt(-b/(a
 + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) + 2*a*cos(d*x + c) - ((a - 2*b
)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c) + 1/2) + ((a - 2*b)*cos(d*
x + c)^2 - a + 2*b)*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d*cos(d*x + c)^2 - a
^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(73) = 146.
time = 0.43, size = 196, normalized size = 2.31

$$\frac{\frac{8\,b^2\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2}+\frac{2\,(a-2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
 sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a^2) + 2*(a - 2*b)*log(abs(-cos(d*x +
 c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (a - 2*a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(
a^2*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/(a*(cos(d*x + c) + 1)))/d
```

Mupad [B]
time = 13.91, size = 592, normalized size = 6.96

$$-\frac{a\left(b\cos(c+d\,x)-b\,\mathrm{atanh}(\cos(c+d\,x))+b\cos(c+d\,x)^2\,\mathrm{atanh}(\cos(c+d\,x))\right)+a^2\left(\cos(c+d\,x)+\mathrm{atanh}(\cos(c+d\,x))-\cos(c+d\,x)^2\,\mathrm{atanh}(\cos(c+d\,x))\right)-2\,b^2\,\mathrm{atanh}(\cos(c+d\,x))+\mathrm{atan}\left(\text{[illegible]}\right)\sqrt{b^4+a\,b^3}\,2{}\mathrm{i}+2\,b^2\cos(c+d\,x)^2\,\mathrm{atanh}(\cos(c+d\,x))-\cos(c+d\,x)^2\,\mathrm{atan}\left(\text{[illegible]}\right)\sqrt{b^4+a\,b^3}\,2{}\mathrm{i}}{d\left(-2\,a^3\cos(c+d\,x)^2+2\,a^3-2\,b\,a^2\cos(c+d\,x)^2+2\,b\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] -(a*(b*cos(c + d*x) - b*atanh(cos(c + d*x)) + b*cos(c + d*x)^2*atanh(cos(c
+ d*x))) + a^2*(cos(c + d*x) + atanh(cos(c + d*x)) - cos(c + d*x)^2*atanh(c
os(c + d*x))) - 2*b^2*atanh(cos(c + d*x)) + atan((b^5*cos(c + d*x)*(a*b^3 +
 b^4)^(1/2)*8i - b*cos(c + d*x)*(a*b^3 + b^4)^(3/2)*8i - a*cos(c + d*x)*(a*
b^3 + b^4)^(3/2)*4i + a^2*b^3*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2
*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*2i + a*b^4*cos(c + d*x)*(a*b^3 + b^4)^(1/
2)*12i + a^4*b*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i)/(3*a^2*b^5 + 5*a^3*b^4
+ a^4*b^3 - a^5*b^2))*(a*b^3 + b^4)^(1/2)*2i + 2*b^2*cos(c + d*x)^2*atanh(c
os(c + d*x)) - cos(c + d*x)^2*atan((b^5*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*8i
 - b*cos(c + d*x)*(a*b^3 + b^4)^(3/2)*8i - a*cos(c + d*x)*(a*b^3 + b^4)^(3/
2)*4i + a^2*b^3*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2*cos(c + d*x)*
(a*b^3 + b^4)^(1/2)*2i + a*b^4*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*12i + a^4*b
*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i)/(3*a^2*b^5 + 5*a^3*b^4 + a^4*b^3 - a^
5*b^2))*(a*b^3 + b^4)^(1/2)*2i)/(d*(2*a^2*b + 2*a^3 - 2*a^3*cos(c + d*x)^2
- 2*a^2*b*cos(c + d*x)^2))
```

3.84 $\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{(3a^2-4ab+8b^2)\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,d}-\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d}-$$

```
[Out] -1/8*(3*a^2-4*a*b+8*b^2)*arctanh(cos(d*x+c))/a^3/d-1/8*(3*a-4*b)*cot(d*x+c)
*csc(d*x+c)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d+b^(5/2)*arctanh(cos(d*x+c
)*b^(1/2)/(a+b)^(1/2))/a^3/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3265, 425, 541, 536, 212, 214}

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}-\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{(3a^2-4ab+8b^2)\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/8*((3*a^2 - 4*a*b + 8*b^2)*ArcTanh[Cos[c + d*x]])/(a^3*d) + (b^(5/2)*Arc
Tanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]*d) - ((3*a - 4*b
)*Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a
*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+b-bx^2)}\,dx,x,\cos(c+dx)\right)}{d}\\
&= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\text{Subst}\left(\int \frac{3a-b-3bx^2}{(1-x^2)^2(a+b-bx^2)}\,dx,x,\cos(c+dx)\right)}{4ad}\\
&= -\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\text{Subst}\left(\int \frac{3a^2-ab-\cdots}{(1-\cdots}\right.}{\cdots}\\
&= -\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{b^3\text{Subst}\left(\int \frac{1}{a+b-\cdots}\right.}{\cdots}\\
&= -\frac{(3a^2-4ab+8b^2)\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,d}-\frac{(3a-4\cdots}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.24, size = 657, normalized size = 5.26

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{b}\cos\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-a-b}}\right)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)}{2a^3\sqrt{-a-b}\,d\,(b+a\csc^2(c+dx))}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{b}\cos\left(\frac{1}{2}(c+dx)\right)+i\sqrt{a}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-a-b}}\right)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)}{2a^3\sqrt{-a-b}\,d\,(b+a\csc^2(c+dx))}+\frac{(3a-4b)(-2a-b+b\cos(2(c+dx)))\csc^2\left(\frac{1}{2}(c+dx)\right)\csc^2(c+dx)}{64a^2d\,(b+a\csc^2(c+dx))}+\frac{(-2a-b+b\cos(2(c+dx)))\csc^4\left(\frac{1}{2}(c+dx)\right)\csc^2(c+dx)}{128ad\,(b+a\csc^2(c+dx))}+\frac{(3a^2-4ab+8b^2)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16a^3d\,(b+a\csc^2(c+dx))}+\frac{(-3a^2+4ab-8b^2)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16a^3d\,(b+a\csc^2(c+dx))}+\frac{(-3a+4b)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{64a^2d\,(b+a\csc^2(c+dx))}-\frac{(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}{128ad\,(b+a\csc^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(Sqrt[b]*Cos[(c + d*x)/2] - I*Sqrt[a]*Sin
[(c + d*x)/2]))/Sqrt[-a - b]]*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^
2)/(2*a^3*Sqrt[-a - b]*d*(b + a*Csc[c + d*x]^2)) + (b^(5/2)*ArcTan[(Sec[(c
+ d*x)/2]*(Sqrt[b]*Cos[(c + d*x)/2] + I*Sqrt[a]*Sin[(c + d*x)/2]))/Sqrt[-a
- b]]*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2)/(2*a^3*Sqrt[-a - b]*d
*(b + a*Csc[c + d*x]^2)) + ((3*a - 4*b)*(-2*a - b + b*Cos[2*(c + d*x)])*Csc
[(c + d*x)/2]^2*Csc[c + d*x]^2)/(64*a^2*d*(b + a*Csc[c + d*x]^2)) + ((-2*a
- b + b*Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*Csc[c + d*x]^2)/(128*a*d*(b +
a*Csc[c + d*x]^2)) + ((3*a^2 - 4*a*b + 8*b^2)*(-2*a - b + b*Cos[2*(c + d*x)
])*Csc[c + d*x]^2*Log[Cos[(c + d*x)/2]])/(16*a^3*d*(b + a*Csc[c + d*x]^2))
+ ((-3*a^2 + 4*a*b - 8*b^2)*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*
Log[Sin[(c + d*x)/2]])/(16*a^3*d*(b + a*Csc[c + d*x]^2)) + ((-3*a + 4*b)*(-
2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*Sec[(c + d*x)/2]^2)/(64*a^2*d*
(b + a*Csc[c + d*x]^2)) - ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*S
ec[(c + d*x)/2]^4)/(128*a*d*(b + a*Csc[c + d*x]^2))
```

Maple [A]
time = 0.40, size = 168, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{1}{16a(1+\cos(dx+c))^2}-\frac{-3a+4b}{16a^2(1+\cos(dx+c))}+\frac{\left(-3a^2+4ab-8b^2\right)\ln(1+\cos(dx+c))}{16a^3}-\frac{1}{16a(\cos(dx+c)-1)^2}-\frac{-3a+4b}{16a^2(\cos(dx+c)-1)}+\frac{\left(3a^2\right.}{}}{d}$
default	$\frac{\frac{1}{16a(1+\cos(dx+c))^2}-\frac{-3a+4b}{16a^2(1+\cos(dx+c))}+\frac{\left(-3a^2+4ab-8b^2\right)\ln(1+\cos(dx+c))}{16a^3}-\frac{1}{16a(\cos(dx+c)-1)^2}-\frac{-3a+4b}{16a^2(\cos(dx+c)-1)}+\frac{\left(3a^2\right.}{}}{d}$
risch	$\frac{3a\,e^{7i(dx+c)}-4b\,e^{7i(dx+c)}-11a\,e^{5i(dx+c)}+4b\,e^{5i(dx+c)}-11a\,e^{3i(dx+c)}+4b\,e^{3i(dx+c)}+3a\,e^{i(dx+c)}-4b\,e^{i(dx+c)}}{4d\,a^2\left(e^{2i(dx+c)}-1\right)^4}-\frac{3\ln(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/a/(1+cos(d*x+c))^2-1/16*(-3*a+4*b)/a^2/(1+cos(d*x+c))+1/16/a^3*(-
3*a^2+4*a*b-8*b^2)*ln(1+cos(d*x+c))-1/16/a/(cos(d*x+c)-1)^2-1/16*(-3*a+4*b)
```

/a^2/(cos(d*x+c)-1)+1/16*(3*a^2-4*a*b+8*b^2)/a^3*ln(cos(d*x+c)-1)+b^3/a^3/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2)))

Maxima [A]
time = 0.53, size = 181, normalized size = 1.45

$$-\frac{\frac{8\,b^{3}\log\left(\frac{b\cos\left(dx+c\right)-\sqrt{(a+b)b}}{b\cos\left(dx+c\right)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^{3}}-\frac{2\left((3\,a-4\,b)\cos\left(dx+c\right)^{3}-(5\,a-4\,b)\cos\left(dx+c\right)\right)}{a^{2}\cos\left(dx+c\right)^{4}-2\,a^{2}\cos\left(dx+c\right)^{2}+a^{2}}+\frac{(3\,a^{2}-4\,ab+8\,b^{2})\log\left(\cos\left(dx+c\right)+1\right)}{a^{3}}-\frac{(3\,a^{2}-4\,ab+8\,b^{2})\log\left(\cos\left(dx+c\right)-1\right)}{a^{3}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*b^3*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt(
(a + b)*b)))/(sqrt((a + b)*b)*a^3) - 2*((3*a - 4*b)*cos(d*x + c)^3 - (5*a -
 4*b)*cos(d*x + c))/(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2) + (3*
a^2 - 4*a*b + 8*b^2)*log(cos(d*x + c) + 1)/a^3 - (3*a^2 - 4*a*b + 8*b^2)*lo
g(cos(d*x + c) - 1)/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(111) = 222$.
time = 0.44, size = 612, normalized size = 4.90

$$\left[\frac{2\,(3\,a^{2}-4\,ab)\cos\left(dx+c\right)^{3}+8\,\left(b^{2}\cos\left(dx+c\right)^{4}-2\,b^{2}\cos\left(dx+c\right)^{2}+b^{2}\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos\left(dx+c\right)^{2}+2\,(a+b)\sqrt{\frac{b}{a+b}}\cos\left(dx+c\right)+a+b}{b\cos\left(dx+c\right)^{2}-a-b}\right)-2\,(5\,a^{2}-4\,ab)\cos\left(dx+c\right)-\left((3\,a^{2}-4\,ab+8\,b^{2})\cos\left(dx+c\right)^{4}-2\,(3\,a^{2}-4\,ab+8\,b^{2})\cos\left(dx+c\right)^{2}+3\,a^{2}-4\,ab+8\,b^{2}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left((3\,a^{2}-4\,ab+8\,b^{2})\cos\left(dx+c\right)^{4}-2\,(3\,a^{2}-4\,ab+8\,b^{2})\cos\left(dx+c\right)^{2}+3\,a^{2}-4\,ab+8\,b^{2}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{16\,\left(a^{3}d\cos\left(dx+c\right)^{4}-2\,a^{3}d\cos\left(dx+c\right)^{2}+a^{3}d\right)},\ \frac{2\,(3\,a^{2}-4\,ab)\cos\left(dx+c\right)^{3}-16\,\left(b^{2}\cos\left(dx+c\right)^{4}-2\,b^{2}\cos\left(dx+c\right)^{2}+b^{2}\right)\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\cos\left(dx+c\right)\right)-2\,(5\,a^{2}-4\,ab)\cos\left(dx+c\right)-\left((3\,a^{2}-4\,ab+8\,b^{2})\cos\left(dx+c\right)^{4}-2\,(3\,a^{2}-4\,ab+8\,b^{2})\cos\left(dx+c\right)^{2}+3\,a^{2}-4\,ab+8\,b^{2}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left((3\,a^{2}-4\,ab+8\,b^{2})\cos\left(dx+c\right)^{4}-2\,(3\,a^{2}-4\,ab+8\,b^{2})\cos\left(dx+c\right)^{2}+3\,a^{2}-4\,ab+8\,b^{2}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{16\,\left(a^{3}d\cos\left(dx+c\right)^{4}-2\,a^{3}d\cos\left(dx+c\right)^{2}+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 - 4*a*b)*cos(d*x + c)^3 + 8*(b^2*cos(d*x + c)^4 - 2*b^2*cos
(d*x + c)^2 + b^2)*sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b
/(a + b))*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(5*a^2 - 4*
a*b)*cos(d*x + c) - ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*
a*b + 8*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(1/2*cos(d*x + c) +
 1/2) + ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8*b^2)
*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3
*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d), 1/16*(2*(3*a^2 - 4*a*b
)*cos(d*x + c)^3 - 16*(b^2*cos(d*x + c)^4 - 2*b^2*cos(d*x + c)^2 + b^2)*sqr
t(-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) - 2*(5*a^2 - 4*a*b)*cos
(d*x + c) - ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8*
b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(1/2*cos(d*x + c) + 1/2) +
((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8*b^2)*cos(d*x
 + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*d*cos(d
*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}\left(c+dx\right)}{a+b\sin^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**5/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(111) = 222.

time = 0.44, size = 334, normalized size = 2.67

$$-\frac{\frac{64\,b^3\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3}+\frac{\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}-\frac{4\,(3\,a^2-4\,ab+8\,b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\left(a^2-\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{18\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{24\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{48\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{a^3(\cos(dx+c)-1)^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*b^3*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c
) + sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a^3) + (8*a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 8*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)/a^2 - 4*(3*a^2 - 4*a*b + 8*b^2)*log(abs(-co
s(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (a^2 - 8*a^2*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 8*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^2
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 24*a*b*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 48*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^2))/d
```

Mupad [B]

time = 13.93, size = 1105, normalized size = 8.84

$$-\frac{\frac{\cos(c+d\,x)\,(5\,a-4\,b)}{8\,a^2}-\frac{\cos(c+d\,x)^3\,(3\,a-4\,b)}{8\,a^2}}{d\,\left(\cos(c+d\,x)^4-\cos(c+d\,x)^2+\sin(c+d\,x)^2\right)}-\frac{\mathrm{atanh}\left(\frac{63\,b^4\cos(c+d\,x)}{64\,\left(\frac{63\,b^4}{64}-\frac{81\,a\,b^3}{256}+\frac{27\,a^2\,b^2}{256}-\frac{35\,b^5}{32\,a}+\frac{5\,b^6}{4\,a^2}\right)}-\frac{81\,b^3\cos(c+d\,x)}{256\,\left(\frac{27\,a\,b^2}{256}-\frac{81\,b^3}{256}+\frac{63\,b^4}{64\,a}-\frac{35\,b^5}{32\,a^2}+\frac{5\,b^6}{4\,a^3}\right)}-\frac{35\,b^5\cos(c+d\,x)}{32\,\left(\frac{63\,a\,b^4}{64}-\frac{35\,b^5}{32}-\frac{81\,a^2\,b^3}{256}+\frac{27\,a^3\,b^2}{256}+\frac{5\,b^6}{4\,a}\right)}+\frac{5\,b^6\cos(c+d\,x)}{4\,\left(\frac{5\,b^6}{4}-\frac{35\,a\,b^5}{32}+\frac{63\,a^2\,b^4}{64}-\frac{81\,a^3\,b^3}{256}+\frac{27\,a^4\,b^2}{256}\right)}+\frac{27\,b^2\cos(c+d\,x)}{256\,\left(\frac{27\,b^2}{256}-\frac{81\,b^3}{256\,a}+\frac{63\,b^4}{64\,a^2}-\frac{35\,b^5}{32\,a^3}+\frac{5\,b^6}{4\,a^4}\right)}\right)\,(3\,a^2-4\,a\,b+8\,b^2)}{8\,a^3\,d}-\frac{\mathrm{atan}\left(\frac{[illegible]}{[illegible]}\right)\sqrt{b^5\,(a+b)}\,1{}\mathrm{i}}{d\,(a^4+b\,a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] - ((cos(c + d*x)*(5*a - 4*b))/(8*a^2) - (cos(c + d*x)^3*(3*a - 4*b))/(8*a^2
))/(d*(cos(c + d*x)^4 - cos(c + d*x)^2 + sin(c + d*x)^2)) - (atanh((63*b^4*
cos(c + d*x))/(64*((63*b^4)/64 - (81*a*b^3)/256 + (27*a^2*b^2)/256 - (35*b^
5)/(32*a) + (5*b^6)/(4*a^2))) - (81*b^3*cos(c + d*x))/(256*((27*a*b^2)/256
- (81*b^3)/256 + (63*b^4)/(64*a) - (35*b^5)/(32*a^2) + (5*b^6)/(4*a^3))) -
(35*b^5*cos(c + d*x))/(32*((63*a*b^4)/64 - (35*b^5)/32 - (81*a^2*b^3)/256 +
 (27*a^3*b^2)/256 + (5*b^6)/(4*a))) + (5*b^6*cos(c + d*x))/(4*((5*b^6)/4 -
(35*a*b^5)/32 + (63*a^2*b^4)/64 - (81*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (
27*b^2*cos(c + d*x))/(256*((27*b^2)/256 - (81*b^3)/(256*a) + (63*b^4)/(64*a
^2) - (35*b^5)/(32*a^3) + (5*b^6)/(4*a^4))))*(3*a^2 - 4*a*b + 8*b^2))/(8*a^
3*d) - (atan((((b^5*(a + b))^(1/2)*((cos(c + d*x)*(128*b^7 - 64*a*b^6 + 64*
a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) + ((b^5*(a + b))^(1/2)*((2*a^6*
```

```
b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) - (cos(c + d*x)*(512*a^6*b^3 + 2
56*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)
))*1i)/(a^3*b + a^4) + ((b^5*(a + b))^(1/2)*((cos(c + d*x)*(128*b^7 - 64*a*
b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) - ((b^5*(a + b))^(1/2)
*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) + (cos(c + d*x)*(512*a^
6*b^3 + 256*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3
*b + a^4)))*1i)/(a^3*b + a^4))/((((5*a*b^7)/4 - b^8 - (3*a^2*b^6)/4 + (9*a^3
*b^5)/32)/a^6 + ((b^5*(a + b))^(1/2)*((cos(c + d*x)*(128*b^7 - 64*a*b^6 + 6
4*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) + ((b^5*(a + b))^(1/2)*((2*a^
6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) - (cos(c + d*x)*(512*a^6*b^3 +
 256*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^
4))))/(a^3*b + a^4) - ((b^5*(a + b))^(1/2)*((cos(c + d*x)*(128*b^7 - 64*a*b
^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) - ((b^5*(a + b))^(1/2)*
((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) + (cos(c + d*x)*(512*a^6
*b^3 + 256*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*
b + a^4))))/(a^3*b + a^4)))*(b^5*(a + b))^(1/2)*1i)/(d*(a^3*b + a^4))
```

3.85 $\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=163

$$-\frac{(16a^3-8a^2b+6ab^2-5b^3)\,x}{16b^4}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{b^4\sqrt{a+b}\,d}-\frac{(8a^2-6ab+5b^2)\cos(c+dx)\sin(c+dx)}{16b^3d}$$

```
[Out] -1/16*(16*a^3-8*a^2*b+6*a*b^2-5*b^3)*x/b^4-1/16*(8*a^2-6*a*b+5*b^2)*cos(d*x
+c)*sin(d*x+c)/b^3/d+1/24*(6*a-5*b)*cos(d*x+c)*sin(d*x+c)^3/b^2/d-1/6*cos(d
*x+c)*sin(d*x+c)^5/b/d+a^(7/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/b^4/d
/(a+b)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3266, 481, 592, 536, 209, 211}

$$\frac{a^{7/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{b^4d\sqrt{a+b}}-\frac{(8a^2-6ab+5b^2)\sin(c+dx)\cos(c+dx)}{16b^3d}-\frac{x(16a^3-8a^2b+6ab^2-5b^3)}{16b^4}+\frac{(6a-5b)\sin^3(c+dx)\cos(c+dx)}{24b^2d}-\frac{\sin^5(c+dx)\cos(c+dx)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/16*((16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*x)/b^4 + (a^(7/2)*ArcTan[(Sqrt[
a + b]*Tan[c + d*x])/Sqrt[a]])/(b^4*Sqrt[a + b]*d) - ((8*a^2 - 6*a*b + 5*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(16*b^3*d) + ((6*a - 5*b)*Cos[c + d*x]*Sin[c
+ d*x]^3)/(24*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5)/(6*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)^4(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{\cos(c+dx)\sin^5(c+dx)}{6bd} + \frac{\text{Subst}\left(\int \frac{x^4(5a+(-a+5b)x^2)}{(1+x^2)^3(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{6bd} \\
&= \frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d} - \frac{\cos(c+dx)\sin^5(c+dx)}{6bd} - \frac{\text{Subst}\left(\int \frac{x^2(3a(6}{(}\right.}{} \\
&= -\frac{(8a^2-6ab+5b^2)\cos(c+dx)\sin(c+dx)}{16b^3d} + \frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d} \\
&= -\frac{(8a^2-6ab+5b^2)\cos(c+dx)\sin(c+dx)}{16b^3d} + \frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d} \\
&= -\frac{(16a^3-8a^2b+6ab^2-5b^3)\,x}{16b^4} + \frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{b^4\sqrt{a+b}\,d} - \frac{(8a^2-6ab}{}
\end{aligned}
$$

Mathematica [A]
time = 0.82, size = 133, normalized size = 0.82

$$
-\frac{12(16a^3-8a^2b+6ab^2-5b^3)(c+dx) - \frac{192a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 3b(16a^2-16ab+15b^2)\sin(2(c+dx)) + 3(2a-3b)b^2\sin(4(c+dx)) + b^3\sin(6(c+dx))}{192b^4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/192*(12*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*(c + d*x) - (192*a^(7/2)*Ar
cTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b] + 3*b*(16*a^2 - 16*a*
b + 15*b^2)*Sin[2*(c + d*x)] + 3*(2*a - 3*b)*b^2*Sin[4*(c + d*x)] + b^3*Sin
[6*(c + d*x)])/(b^4*d)
```

Maple [A]
time = 0.27, size = 168, normalized size = 1.03

method	result
derivativedivides	$-\frac{\frac{\frac{\left(\frac{1}{2}a^2b-\frac{5}{8}ab^2+\frac{11}{16}b^3\right)\left(\tan^5(dx+c)\right)+\left(a^2b-ab^2+\frac{5}{6}b^3\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}a^2b-\frac{3}{8}ab^2+\frac{5}{16}b^3\right)\tan(dx+c)}{\left(\tan^2(dx+c)+1\right)^3}+\frac{(16a^3-8a^2b+6ab^2-5}{1}}{b^4}}{d}$

default	$-\frac{-\frac{\left(\frac{1}{2}a^2b-\frac{5}{8}a\,b^2+\frac{11}{16}b^3\right)\left(\tan^5(dx+c)\right)+\left(a^2b-a\,b^2+\frac{5}{6}b^3\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}a^2b-\frac{3}{8}a\,b^2+\frac{5}{16}b^3\right)\tan(dx+c)}{\left(\tan^2(dx+c)+1\right)^3}+\frac{\left(16a^3-8a^2b+6a\,b^2-5b^3\right.}{16}}{b^4}}{d}$
risch	$-\frac{x\,a^3}{b^4}+\frac{x\,a^2}{2b^3}-\frac{3ax}{8b^2}+\frac{5x}{16b}+\frac{i\mathrm{e}^{2i(dx+c)}a^2}{8b^3d}-\frac{i\mathrm{e}^{2i(dx+c)}a}{8b^2d}+\frac{15i\mathrm{e}^{2i(dx+c)}}{128bd}-\frac{i\mathrm{e}^{-2i(dx+c)}a^2}{8b^3d}+\frac{i\mathrm{e}^{-2i(dx+c)}a}{8b^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(((1/2*a^2*b-5/8*a*b^2+11/16*b^3)*tan(d*x+c)^5+(a^2*b-a*b^2+5/6
*b^3)*tan(d*x+c)^3+(1/2*a^2*b-3/8*a*b^2+5/16*b^3)*tan(d*x+c))/(tan(d*x+c)^2
+1)^3+1/16*(16*a^3-8*a^2*b+6*a*b^2-5*b^3)*arctan(tan(d*x+c)))+a^4/b^4/(a*(a
+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

Maxima [A]

time = 0.53, size = 192, normalized size = 1.18

$$\frac{\frac{48\,a^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^4}-\frac{3\left(8\,a^2-10\,ab+11\,b^2\right)\tan(dx+c)^5+8\left(6\,a^2-6\,ab+5\,b^2\right)\tan(dx+c)^3+3\left(8\,a^2-6\,ab+5\,b^2\right)\tan(dx+c)}{b^3\tan(dx+c)^6+3\,b^3\tan(dx+c)^4+3\,b^3\tan(dx+c)^2+b^3}-\frac{3\left(16\,a^3-8\,a^2b+6\,ab^2-5\,b^3\right)(dx+c)}{b^4}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(48*a^4*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*
b^4) - (3*(8*a^2 - 10*a*b + 11*b^2)*tan(d*x + c)^5 + 8*(6*a^2 - 6*a*b + 5*b
^2)*tan(d*x + c)^3 + 3*(8*a^2 - 6*a*b + 5*b^2)*tan(d*x + c))/(b^3*tan(d*x +
 c)^6 + 3*b^3*tan(d*x + c)^4 + 3*b^3*tan(d*x + c)^2 + b^3) - 3*(16*a^3 - 8*
a^2*b + 6*a*b^2 - 5*b^3)*(d*x + c)/b^4)/d
```

Fricas [A]

time = 0.45, size = 453, normalized size = 2.78

$$\left[\frac{12\,a^3\sqrt{-\frac{a}{a+b}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2-4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(dx+c)^3-\left(a^2+2\,ab+b^2\right)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)-3\left(16\,a^3-8\,a^2b+6\,ab^2-5\,b^3\right)dx-\left(8\,b^3\cos(dx+c)^5+2\left(6\,ab^2-13\,b^3\right)\cos(dx+c)^3+3\left(8\,a^2b-10\,ab^2+11\,b^3\right)\cos(dx+c)\right)\sin(dx+c)}{48\,b^4d},-\frac{24\,a^3\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)+3\left(16\,a^3-8\,a^2b+6\,ab^2-5\,b^3\right)dx+\left(8\,b^3\cos(dx+c)^5+2\left(6\,ab^2-13\,b^3\right)\cos(dx+c)^3+3\left(8\,a^2b-10\,ab^2+11\,b^3\right)\cos(dx+c)\right)\sin(dx+c)}{48\,b^4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*a^3*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 -
2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
 c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a
^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
```

```
+ 2*a*b + b^2)) - 3*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*d*x - (8*b^3*cos(d
*x + c)^5 + 2*(6*a*b^2 - 13*b^3)*cos(d*x + c)^3 + 3*(8*a^2*b - 10*a*b^2 + 1
1*b^3)*cos(d*x + c))*sin(d*x + c))/(b^4*d), -1/48*(24*a^3*sqrt(a/(a + b))*a
rctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c
)*sin(d*x + c))) + 3*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*d*x + (8*b^3*cos(
d*x + c)^5 + 2*(6*a*b^2 - 13*b^3)*cos(d*x + c)^3 + 3*(8*a^2*b - 10*a*b^2 +
11*b^3)*cos(d*x + c))*sin(d*x + c))/(b^4*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 233, normalized size = 1.43

$$\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^4}{\sqrt{a^2+ab}\,b^4}-\frac{3\left(16a^3-8a^2b+6ab^2-5b^3\right)(dx+c)}{b^4}-\frac{24a^2\tan(dx+c)^5-30ab\tan(dx+c)^5+33b^2\tan(dx+c)^5+48a^2\tan(dx+c)^3-48ab\tan(dx+c)^3+40b^2\tan(dx+c)^3+24a^2\tan(dx+c)-18ab\tan(dx+c)+15b^2\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^3b^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/48*(48*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
 c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^4/(sqrt(a^2 + a*b)*b^4) - 3*(16*a
^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*(d*x + c)/b^4 - (24*a^2*tan(d*x + c)^5 - 30
*a*b*tan(d*x + c)^5 + 33*b^2*tan(d*x + c)^5 + 48*a^2*tan(d*x + c)^3 - 48*a*
b*tan(d*x + c)^3 + 40*b^2*tan(d*x + c)^3 + 24*a^2*tan(d*x + c) - 18*a*b*tan
(d*x + c) + 15*b^2*tan(d*x + c))/((tan(d*x + c)^2 + 1)^3*b^3))/d
```

Mupad [**B**]
time = 15.32, size = 2244, normalized size = 13.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (atan(((((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^
7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((((
5*a*b^12)/4 + a^2*b^11 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 - (t
an(c + d*x)*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 1024*b^
```

11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*1i)/(32*b^4) + (((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) + ((((5*a*b^12)/4 + a^2*b^11 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 + (tan(c + d*x)*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*1i)/(32*b^4))/(((a^10*b)/4 + a^11 + (25*a^4*b^7)/128 - (5*a^5*b^6)/64 + (21*a^6*b^5)/128 - (21*a^7*b^4)/32 - (15*a^8*b^3)/32 - (a^9*b^2)/8)/b^9 - (((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((((5*a*b^12)/4 + a^2*b^11 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 - (tan(c + d*x)*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^4) + (((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) + ((((5*a*b^12)/4 + a^2*b^11 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 + (tan(c + d*x)*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^4)))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*1i)/(16*b^4*d) - ((tan(c + d*x)*(8*a^2 - 6*a*b + 5*b^2))/(16*b^3) + (tan(c + d*x)^3*(6*a^2 - 6*a*b + 5*b^2))/(6*b^3) + (tan(c + d*x)^5*(8*a^2 - 10*a*b + 11*b^2))/(16*b^3))/(d*(3*tan(c + d*x)^2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1)) + (atan((((-a^7*(a + b))^(1/2)*((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((-a^7*(a + b))^(1/2)*((320*a*b^12 + 256*a^2*b^11 + 64*a^3*b^10 + 640*a^4*b^9 + 512*a^5*b^8)/(256*b^9) - (tan(c + d*x)*(-a^7*(a + b))^(1/2)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(256*b^6*(a*b^4 + b^5))))/(2*(a*b^4 + b^5)))*1i)/(2*(a*b^4 + b^5)) + ((-a^7*(a + b))^(1/2)*((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) + ((-a^7*(a + b))^(1/2)*((320*a*b^12 + 256*a^2*b^11 + 64*a^3*b^10 + 640*a^4*b^9 + 512*a^5*b^8)/(256*b^9) + (tan(c + d*x)*(-a^7*(a + b))^(1/2)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(256*b^6*(a*b^4 + b^5))))/(2*(a*b^4 + b^5)))*1i)/(2*(a*b^4 + b^5)))/((32*a^10*b + 128*a^11 + 25*a^4*b^7 - 10*a^5*b^6 + 21*a^6*b^5 - 84*a^7*b^4 - 60*a^8*b^3 - 16*a^9*b^2)/(128*b^9) - ((-a^7*(a + b))^(1/2)*((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((-a^7*(a + b))^(1/2)*((320*a*b^12 + 256*a^2*b^11 + 64*a^3*b^10 + 640*a^4*b^9 + 512*a^5*b^8)/(256*b^9) - (tan(c + d*x)*(-a^7*(a + b))^(1/2)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(256*b^6*(a*b^4 + b^5))))/(2*(a*b^4 + b^5))))/(2*(a*b^4 + b^5)) + ((-a^7*(a + b))^(1/2)*((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*

```
b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(
128*b^6) + ((-a^7*(a + b))^(1/2)*((320*a*b^12 + 256*a^2*b^11 + 64*a^3*b^10
+ 640*a^4*b^9 + 512*a^5*b^8)/(256*b^9) + (tan(c + d*x)*(-a^7*(a + b))^(1/2)
*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(256*b^6*(a*b^4 +
 b^5))))/(2*(a*b^4 + b^5))))/(2*(a*b^4 + b^5))))*(-a^7*(a + b))^(1/2)*1i)/(
d*(a*b^4 + b^5))
```

3.86 $\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{(8a^2-4ab+3b^2)\,x}{8b^3}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}\,d}+\frac{(4a-3b)\cos(c+dx)\sin(c+dx)}{8b^2d}-\frac{\cos(c+dx)\sin^3(c+dx)}{4bd}$$

```
[Out] 1/8*(8*a^2-4*a*b+3*b^2)*x/b^3+1/8*(4*a-3*b)*cos(d*x+c)*sin(d*x+c)/b^2/d-1/4
*cos(d*x+c)*sin(d*x+c)^3/b/d-a^(5/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))
/b^3/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3266, 481, 592, 536, 209, 211}

$$-\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{b^3d\sqrt{a+b}}+\frac{x(8a^2-4ab+3b^2)}{8b^3}+\frac{(4a-3b)\sin(c+dx)\cos(c+dx)}{8b^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((8*a^2 - 4*a*b + 3*b^2)*x)/(8*b^3) - (a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c +
d*x])/Sqrt[a]])/(b^3*Sqrt[a + b]*d) + ((4*a - 3*b)*Cos[c + d*x]*Sin[c + d*x
])/(8*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(-a+3b)x^2)}{(1+x^2)^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{4bd}$$
$$= \frac{(4a-3b)\cos(c+dx)\sin(c+dx)}{8b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} - \frac{\text{Subst}\left(\int \frac{a(4a-3b)+}{(1+x^2}\right.}{}$$
$$= \frac{(4a-3b)\cos(c+dx)\sin(c+dx)}{8b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} - \frac{a^3\text{Subst}\left(\int \frac{1}{a+(a+b}\right.}{}$$
$$= \frac{(8a^2-4ab+3b^2)\,x}{8b^3} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}\,d} + \frac{(4a-3b)\cos(c+dx)\sin}{8b^2d}$$

Mathematica [A]

time = 0.32, size = 95, normalized size = 0.81

$$\frac{4(8a^2-4ab+3b^2)(c+dx) - \frac{32a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 8(a-b)b\sin(2(c+dx)) + b^2\sin(4(c+dx))}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (4*(8*a^2 - 4*a*b + 3*b^2)*(c + d*x) - (32*a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[
c + d*x])/Sqrt[a]])/Sqrt[a + b] + 8*(a - b)*b*Sin[2*(c + d*x)] + b^2*Sin[4*
(c + d*x)])/(32*b^3*d)
```

Maple [A]

time = 0.30, size = 118, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{1}{2}ab-\frac{5}{8}b^2\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}ab-\frac{3}{8}b^2\right)\tan(dx+c)}{\left(\tan^2(dx+c)+1\right)^2}+\frac{\left(8a^2-4ab+3b^2\right)\arctan(\tan(dx+c))}{8}}{b^3}-\frac{a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^3\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{\frac{\left(\frac{1}{2}ab-\frac{5}{8}b^2\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}ab-\frac{3}{8}b^2\right)\tan(dx+c)}{\left(\tan^2(dx+c)+1\right)^2}+\frac{\left(8a^2-4ab+3b^2\right)\arctan(\tan(dx+c))}{8}}{b^3}-\frac{a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^3\sqrt{a(a+b)}}}{d}$

risch	$\frac{x\,a^2}{b^3}-\frac{ax}{2b^2}+\frac{3x}{8b}-\frac{i\mathrm{e}^{2i(dx+c)}a}{8b^2d}+\frac{i\mathrm{e}^{2i(dx+c)}}{8bd}+\frac{i\mathrm{e}^{-2i(dx+c)}a}{8b^2d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8bd}-\frac{\sqrt{-a\,(a+b)}\,a^2\ln\left(\mathrm{e}^{2i(\cdots}\right.}{2(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*(((1/2*a*b-5/8*b^2)*tan(d*x+c)^3+(1/2*a*b-3/8*b^2)*tan(d*x+c))/(
tan(d*x+c)^2+1)^2+1/8*(8*a^2-4*a*b+3*b^2)*arctan(tan(d*x+c)))-a^3/b^3/(a*(a
+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.54, size = 128, normalized size = 1.09

$$-\frac{\frac{8\,a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^3}-\frac{(4\,a-5\,b)\tan(dx+c)^3+(4\,a-3\,b)\tan(dx+c)}{b^2\tan(dx+c)^4+2\,b^2\tan(dx+c)^2+b^2}-\frac{(8\,a^2-4\,ab+3\,b^2)(dx+c)}{b^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^3*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b
^3) - ((4*a - 5*b)*tan(d*x + c)^3 + (4*a - 3*b)*tan(d*x + c))/(b^2*tan(d*x
+ c)^4 + 2*b^2*tan(d*x + c)^2 + b^2) - (8*a^2 - 4*a*b + 3*b^2)*(d*x + c)/b^
3)/d
```

Fricas [A]
time = 0.47, size = 372, normalized size = 3.18

$$\left[\frac{2\,a^2\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\,\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+2\,ab+b^2)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)+(8\,a^2-4\,ab+3\,b^2)dx+\left(2\,b^2\cos(dx+c)^3+(4\,ab-5\,b^2)\cos(dx+c)\right)\sin(dx+c)}{8\,b^3d},\ \frac{4\,a^2\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)+(8\,a^2-4\,ab+3\,b^2)dx+\left(2\,b^2\cos(dx+c)^3+(4\,ab-5\,b^2)\cos(dx+c)\right)\sin(dx+c)}{8\,b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^2*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*
(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c
)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2
 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 +
2*a*b + b^2)) + (8*a^2 - 4*a*b + 3*b^2)*d*x + (2*b^2*cos(d*x + c)^3 + (4*a*
b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/(b^3*d), 1/8*(4*a^2*sqrt(a/(a + b))*
arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x +
c)*sin(d*x + c))) + (8*a^2 - 4*a*b + 3*b^2)*d*x + (2*b^2*cos(d*x + c)^3 + (
4*a*b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/(b^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 157, normalized size = 1.34

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^3}{\sqrt{a^2+ab}\,b^3}-\frac{(8a^2-4ab+3b^2)(dx+c)}{b^3}-\frac{4a\tan(dx+c)^3-5b\tan(dx+c)^3+4a\tan(dx+c)-3b\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2b^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^3/(sqrt(a^2 + a*b)*b^3) - (8*a^2 -
 4*a*b + 3*b^2)*(d*x + c)/b^3 - (4*a*tan(d*x + c)^3 - 5*b*tan(d*x + c)^3 +
4*a*tan(d*x + c) - 3*b*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*b^2))/d
```

Mupad [B]
time = 14.82, size = 1892, normalized size = 16.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*sin(c + d*x)^2),x)
```

```
[Out] ((tan(c + d*x)*(4*a - 3*b))/(8*b^2) + (tan(c + d*x)^3*(4*a - 5*b))/(8*b^2))
/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) - (atan(((((((((3*a*b^9)/2 + a^
2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) - (tan(c + d*x)*(-a^5*(a + b))^(
1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*b^4*(a*b^3 +
 b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(3*a*b^6 +
192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5
*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2)*1i)/(a*b^3 + b^4) - ((((((3*a*b^9)/2
+ a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) + (tan(c + d*x)*(-a^5*(a + b
))^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*b^4*(a*b
^3 + b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) + (tan(c + d*x)*(3*a*b^
6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64
*a^5*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2)*1i)/(a*b^3 + b^4))/(((((((3*a*b^9
)/2 + a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) - (tan(c + d*x)*(-a^5*(a
```

```
 + b))^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*b^4*
(a*b^3 + b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(3*
a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3
+ 64*a^5*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2))/(a*b^3 + b^4) - ((a^7*b)/4 +
 a^8 + (9*a^3*b^5)/32 - (3*a^4*b^4)/16 + (25*a^5*b^3)/32 + (a^6*b^2)/2)/b^6
 + ((((((3*a*b^9)/2 + a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) + (tan(c
 + d*x)*(-a^5*(a + b))^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3
*b^6))/(128*b^4*(a*b^3 + b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) + (
tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b
^4 + 40*a^4*b^3 + 64*a^5*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2))/(a*b^3 + b^4
)))*(-a^5*(a + b))^(1/2)*1i)/(d*(a*b^3 + b^4)) - (atan(((((((3*a*b^9)/2 +
a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/b^6 - (tan(c + d*x)*(a^2*8i - a*b*4i +
 b^2*3i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a
^2*8i - a*b*4i + b^2*3i))/(16*b^3) - (tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 1
28*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5*b^2))/(32*b^
4))*(a^2*8i - a*b*4i + b^2*3i)*1i)/(16*b^3) - ((((((3*a*b^9)/2 + a^2*b^8 -
(5*a^3*b^7)/2 - 2*a^4*b^6)/b^6 + (tan(c + d*x)*(a^2*8i - a*b*4i + b^2*3i)*(
1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a^2*8i - a*
b*4i + b^2*3i))/(16*b^3) + (tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9
*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5*b^2))/(32*b^4))*(a^2*8
i - a*b*4i + b^2*3i)*1i)/(16*b^3))/(((((((3*a*b^9)/2 + a^2*b^8 - (5*a^3*b^7
)/2 - 2*a^4*b^6)/b^6 - (tan(c + d*x)*(a^2*8i - a*b*4i + b^2*3i)*(1024*a*b^8
 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a^2*8i - a*b*4i + b^2
*3i))/(16*b^3) - (tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*
a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5*b^2))/(32*b^4))*(a^2*8i - a*b*4i
 + b^2*3i))/(16*b^3) - ((a^7*b)/4 + a^8 + (9*a^3*b^5)/32 - (3*a^4*b^4)/16 +
 (25*a^5*b^3)/32 + (a^6*b^2)/2)/b^6 + ((((((3*a*b^9)/2 + a^2*b^8 - (5*a^3*b
^7)/2 - 2*a^4*b^6)/b^6 + (tan(c + d*x)*(a^2*8i - a*b*4i + b^2*3i)*(1024*a*b
^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a^2*8i - a*b*4i + b
^2*3i))/(16*b^3) + (tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 1
9*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5*b^2))/(32*b^4))*(a^2*8i - a*b*
4i + b^2*3i))/(16*b^3)))*(a^2*8i - a*b*4i + b^2*3i)*1i)/(8*b^3*d)
```

3.87 $\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{(2a-b)x}{2b^2}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}\,d}-\frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a-b)*x/b^2-1/2*cos(d*x+c)*sin(d*x+c)/b/d+a^(3/2)*arctan((a+b)^(1/2)
*tan(d*x+c)/a^(1/2))/b^2/d/(a+b)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3266, 481, 536, 209, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/2*((2*a - b)*x)/b^2 + (a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]
])/(b^2*Sqrt[a + b]*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
```

```
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{2bd}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{b^2d} - \frac{(2a-b)}{}$$

$$= -\frac{(2a-b)x}{2b^2} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}\,d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

Mathematica [A]
time = 0.22, size = 69, normalized size = 0.90

$$-\frac{2(2a-b)(c+dx) - \frac{4a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + b\sin(2(c+dx))}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/4*(2*(2*a - b)*(c + d*x) - (4*a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/
Sqrt[a]])/Sqrt[a + b] + b*Sin[2*(c + d*x)])/(b^2*d)
```

Maple [A]
time = 0.24, size = 81, normalized size = 1.05

method	result
derivativedivides	$\frac{\frac{a^2 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}-\frac{\frac{b\tan(dx+c)}{2\left(\tan^2(dx+c)\right)+2}+\frac{(2a-b)\arctan(\tan(dx+c))}{2}}{b^2}}{d}$
default	$\frac{\frac{a^2 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}-\frac{\frac{b\tan(dx+c)}{2\left(\tan^2(dx+c)\right)+2}+\frac{(2a-b)\arctan(\tan(dx+c))}{2}}{b^2}}{d}$
risch	$-\frac{ax}{b^2}+\frac{x}{2b}+\frac{i e^{2i(dx+c)}}{8bd}-\frac{i e^{-2i(dx+c)}}{8bd}+\frac{\sqrt{-a(a+b)}\, a\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2(a+b)d\,b^2}-\ \sqrt{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/b^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/b^2
*(1/2*b*tan(d*x+c)/(tan(d*x+c)^2+1)+1/2*(2*a-b)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.51, size = 78, normalized size = 1.01

$$\frac{\frac{2\,a^2 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^2}-\frac{(dx+c)(2\,a-b)}{b^2}-\frac{\tan(dx+c)}{b\tan(dx+c)^2+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^
2) - (d*x + c)*(2*a - b)/b^2 - tan(d*x + c)/(b*tan(d*x + c)^2 + b))/d
```

Fricas [A]
time = 0.42, size = 305, normalized size = 3.96

$$\left[-\frac{2\,(2\,a-b)dx+2\,b\cos(dx+c)\sin(dx+c)-a\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2-4\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+2\,ab+b^2)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)}{4\,b^2d},\ -\frac{(2\,a-b)dx+b\cos(dx+c)\sin(dx+c)+a\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)}{2\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(2*a - b)*d*x + 2*b*cos(d*x + c)*sin(d*x + c) - a*sqrt(-a/(a + b))
*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*
x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*co
s(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x
 + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)))/(b^2*d), -1/2
*((2*a - b)*d*x + b*cos(d*x + c)*sin(d*x + c) + a*sqrt(a/(a + b))*arctan(1/
2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*
x + c))))/(b^2*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 114, normalized size = 1.48

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^2}{\sqrt{a^2+ab}\,b^2}-\frac{(dx+c)(2a-b)}{b^2}-\frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^2/(sqrt(a^2 + a*b)*b^2) - (d*x + c)
*(2*a - b)/b^2 - tan(d*x + c)/((tan(d*x + c)^2 + 1)*b))/d
```

Mupad [B]
time = 13.97, size = 481, normalized size = 6.25

$$\frac{b^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{d\,\left(2\,b^3+2\,a\,b^2\right)}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{d\,\left(2\,b^3+2\,a\,b^2\right)}-\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\left(2\,b^3+2\,a\,b^2\right)}-\frac{a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\left(2\,b^3+2\,a\,b^2\right)}-\frac{a\,b\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{d\,\left(2\,b^3+2\,a\,b^2\right)}-\frac{\mathrm{atan}\left(\frac{a\,\sin(c+d\,x)\,{\left(-a^4-b\,a^3\right)}^{3/2}\,8{}\mathrm{i}+b\,\sin(c+d\,x)\,{\left(-a^4-b\,a^3\right)}^{3/2}\,4{}\mathrm{i}+a^5\,\sin(c+d\,x)\,\sqrt{-a^4-b\,a^3}\,8{}\mathrm{i}+b^5\,\sin(c+d\,x)\,\sqrt{-a^4-b\,a^3}\,1{}\mathrm{i}-a\,b^4\,\sin(c+d\,x)\,\sqrt{-a^4-b\,a^3}\,1{}\mathrm{i}+a^4\,b\,\sin(c+d\,x)\,\sqrt{-a^4-b\,a^3}\,12{}\mathrm{i}-a^2\,b^3\,\sin(c+d\,x)\,\sqrt{-a^4-b\,a^3}\,5{}\mathrm{i}+a^3\,b^2\,\sin(c+d\,x)\,\sqrt{-a^4-b\,a^3}\,1{}\mathrm{i}}{3\,\cos(c+d\,x)\,a^5\,b^2+5\,\cos(c+d\,x)\,a^4\,b^3+\cos(c+d\,x)\,a^3\,b^4-\cos(c+d\,x)\,a^2\,b^5}\right)\,\sqrt{-a^4-b\,a^3}\,2{}\mathrm{i}}{d\,\left(2\,b^3+2\,a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (b^2*atan(sin(c + d*x)/cos(c + d*x)))/(d*(2*a*b^2 + 2*b^3)) - (2*a^2*atan(s
in(c + d*x)/cos(c + d*x)))/(d*(2*a*b^2 + 2*b^3)) - (b^2*sin(2*c + 2*d*x))/(
2*d*(2*a*b^2 + 2*b^3)) - (atan((a*sin(c + d*x)*(- a^3*b - a^4)^(3/2)*8i + b
```

```
*sin(c + d*x)*(- a^3*b - a^4)^(3/2)*4i + a^5*sin(c + d*x)*(- a^3*b - a^4)^(
1/2)*8i + b^5*sin(c + d*x)*(- a^3*b - a^4)^(1/2)*1i - a*b^4*sin(c + d*x)*(-
 a^3*b - a^4)^(1/2)*1i + a^4*b*sin(c + d*x)*(- a^3*b - a^4)^(1/2)*12i - a^2
*b^3*sin(c + d*x)*(- a^3*b - a^4)^(1/2)*5i + a^3*b^2*sin(c + d*x)*(- a^3*b
- a^4)^(1/2)*1i)/(a^3*b^4*cos(c + d*x) - a^2*b^5*cos(c + d*x) + 5*a^4*b^3*c
os(c + d*x) + 3*a^5*b^2*cos(c + d*x)))*(- a^3*b - a^4)^(1/2)*2i)/(d*(2*a*b^
2 + 2*b^3)) - (a*b*sin(2*c + 2*d*x))/(2*d*(2*a*b^2 + 2*b^3)) - (a*b*atan(si
n(c + d*x)/cos(c + d*x)))/(d*(2*a*b^2 + 2*b^3))
```

3.88 $\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{b}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}\,d}$$

```
[Out] x/b-arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1/2)/b/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3250, 3260, 211}

$$\frac{x}{b}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]*d
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{x}{b} - \frac{a\int \frac{1}{a+b\sin^2(c+dx)}\,dx}{b}$$
$$= \frac{x}{b} - \frac{a\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{bd}$$
$$= \frac{x}{b} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.11, size = 46, normalized size = 1.00

$$\frac{c+dx-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (c + d*x - (Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b]
)/(b*d)
```

Maple [A]
time = 0.21, size = 48, normalized size = 1.04

method	result
derivativedivides	$\frac{-\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}+\frac{\arctan(\tan(dx+c))}{b}}{d}$
default	$\frac{-\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}+\frac{\arctan(\tan(dx+c))}{b}}{d}$
risch	$\frac{x}{b}-\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2(a+b)db}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{-a}\ldots}{}\right)}{2(a+b)db}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*a/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/b*ar
ctan(tan(d*x+c)))
```

Maxima [A]
time = 0.57, size = 46, normalized size = 1.00

$$-\frac{\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b}-\frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(a*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) - (d*x
 + c)/b)/d
```

Fricas [A]
time = 0.43, size = 260, normalized size = 5.65

$$\left[\frac{4\,dx+\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+2\,ab+b^2)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)}{4\,bd},\ \frac{2\,dx+\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)}{2\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*d*x + sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 -
2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
 c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a
^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
+ 2*a*b + b^2)))/(b*d), 1/2*(2*d*x + sqrt(a/(a + b))*arctan(1/2*((2*a + b)*
cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))))/(b*
d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.
time = 0.42, size = 81, normalized size = 1.76

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}\,b}-\frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*a/(sqrt(a^2 + a*b)*b) - (d*x + c)/b)/d
```

Mupad [B]
time = 13.48, size = 104, normalized size = 2.26

$$\frac{\operatorname{atan}\left(\frac{2\,a\,b^2\,\tan(c+d\,x)}{2\,a^2\,b+2\,a\,b^2}+\frac{2\,a^2\,b\tan(c+d\,x)}{2\,a^2\,b+2\,a\,b^2}\right)}{b\,d}+\frac{\operatorname{atanh}\left(\frac{\tan(c+d\,x)\,\sqrt{-a\,(a+b)}}{a}\right)\,\sqrt{-a\,(a+b)}}{d\,(b^2+a\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^2),x)
```

```
[Out] atan((2*a*b^2*tan(c + d*x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(c + d*x))/(2
*a*b^2 + 2*a^2*b))/(b*d) + (atanh((tan(c + d*x)*(-a*(a + b))^(1/2))/a)*(-a*
(a + b))^(1/2))/(d*(a*b + b^2))
```

3.89 $\int \frac{1}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}\,d}$$

```
[Out] arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/d/a^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3260, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 36, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Maple [A]
time = 0.22, size = 30, normalized size = 0.83

method	result
derivativedivides	$\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\sqrt{a(a+b)}}$
default	$\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\sqrt{a(a+b)}}$
risch	$-\frac{\ln\left(e^{2i(dx+c)}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia^2+2iab-2a\sqrt{-a^2-ab}-b}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

Maxima [A]
time = 0.55, size = 29, normalized size = 0.81

$$\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(28) = 56$.
time = 0.41, size = 236, normalized size = 6.56

$$\left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2(4a^2+5ab+b^2)\cos(dx+c)^2+4\left((2a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\sin(dx+c)+a^2+2ab+b^2}{b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right)}{4(a^2+ab)d}, -\frac{\arctan\left(\frac{(2a+b)\cos(dx+c)^2-a-b}{2\sqrt{a^2+ab}\cos(dx+c)\sin(dx+c)}\right)}{2\sqrt{a^2+ab}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2
 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(
d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x +
 c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))/((a^2 + a*b)*d),
 -1/2*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*
x + c)*sin(d*x + c)))/(sqrt(a^2 + a*b)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 16298 vs. $2(32) = 64$.
time = 14.55, size = 16298, normalized size = 452.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((tan(c/2 + d*
x/2)/(2*d) - 1/(2*d*tan(c/2 + d*x/2)))/b, Eq(a, 0)), (2*tan(c/2 + d*x/2)/(b
*d*tan(c/2 + d*x/2)**2 - b*d), Eq(a, -b)), (x/a, Eq(b, 0)), (x/(a + b*sin(c
)**2), Eq(d, 0)), (6*a**3*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(-sq
rt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(10*a**4*b*d*sqrt
(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)
 - 2*a**4*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-
1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*b**2*d*sqrt(-1 - 2*b/a - 2*sqrt
(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 26*a**3*b*d*sqrt(
a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqr
t(a*b + b**2)/a) + 72*a**2*b**3*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*s
qrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 56*a**2*b**2*d*sqrt(a*b + b**2)*sq
rt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/
a) + 32*a*b**4*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a +
2*sqrt(a*b + b**2)/a) - 32*a*b**3*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sq
rt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)) - 6*a**3*b*sqrt(
-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)
/a) + tan(c/2 + d*x/2))/(10*a**4*b*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a
```

)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 2*a**4*d*sqrt(a*b + b**2)*sqrt(
-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)
+ 50*a**3*b**2*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a +
2*sqrt(a*b + b**2)/a) - 26*a**3*b*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sq
rt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 72*a**2*b**3*d*
sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2
)/a) - 56*a**2*b**2*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)
/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 32*a*b**4*d*sqrt(-1 - 2*b/a -
2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 32*a*b**3*
d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a
+ 2*sqrt(a*b + b**2)/a)) + 4*a**3*b*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)
*log(-sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(10*a**4*
b*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b +
b**2)/a) - 2*a**4*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a
)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*b**2*d*sqrt(-1 - 2*b/a
- 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 26*a**3*b
*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a
+ 2*sqrt(a*b + b**2)/a) + 72*a**2*b**3*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b*
*2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 56*a**2*b**2*d*sqrt(a*b +
b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b
+ b**2)/a) + 32*a*b**4*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 -
2*b/a + 2*sqrt(a*b + b**2)/a) - 32*a*b**3*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/
a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)) - 4*a**3
*b*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a - 2*sqrt(a*b
+ b**2)/a) + tan(c/2 + d*x/2))/(10*a**4*b*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b +
b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 2*a**4*d*sqrt(a*b + b**
2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b
2)/a) + 50*a3*b**2*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 -
2*b/a + 2*sqrt(a*b + b**2)/a) - 26*a**3*b*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/
a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 72*a**2
*b**3*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*
b + b**2)/a) - 56*a**2*b**2*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b
+ b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 32*a*b**4*d*sqrt(-1 -
2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 32
*a*b**3*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1
- 2*b/a + 2*sqrt(a*b + b**2)/a)) - a**3*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a -
2*sqrt(a*b + b**2)/a)*log(-sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(c/
2 + d*x/2))/(10*a**4*b*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 -
2*b/a + 2*sqrt(a*b + b**2)/a) - 2*a**4*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a -
2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*b*
*2*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b +
b**2)/a) - 26*a**3*b*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**
2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 72*a**2*b**3*d*sqrt(-1 - 2*
b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 56*a*
*2*b**2*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1

```
- 2*b/a + 2*sqrt(a*b + b**2)/a) + 32*a*b**4*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b
+ b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 32*a*b**3*d*sqrt(a*b +
 b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b
 + b**2)/a)) + a**3*sqrt(a*b + b**2)*sqrt(-1 - ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(28) = 56$.
time = 0.47, size = 64, normalized size = 1.78

$$\frac{\pi \left\lfloor \frac{dx+c}{\pi} + \frac{1}{2} \right\rfloor \mathrm{sgn}(2\,a + 2\,b) + \arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*t
an(d*x + c))/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*b)*d)
```

Mupad [B]
time = 13.53, size = 33, normalized size = 0.92

$$\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)}{\sqrt{a^2+b\,a}}\right)}{d\,\sqrt{a^2+b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2),x)
```

```
[Out] atan((tan(c + d*x)*(a + b))/(a*b + a^2)^(1/2))/(d*(a*b + a^2)^(1/2))
```

3.90 $\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}\,d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d-b*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/d/(a+b)^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3266, 464, 211}

$$-\frac{b\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a+b}}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]*d)) -
 Cot[c + d*x]/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
```

IntegerQ[p]

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{b\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}\,d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A]
time = 0.22, size = 53, normalized size = 1.00

$$\frac{-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} - \sqrt{a}\,\cot(c+dx)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-((b*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b]) - Sqrt[a]*Co
t[c + d*x])/(a^(3/2)*d)
```

Maple [A]
time = 0.30, size = 50, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{a\tan(dx+c)} - \frac{b\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{1}{a\tan(dx+c)} - \frac{b\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{d}$

risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,da}-\frac{b\ln\left(e^{2i(dx+c)}+\frac{2ia^2+2i\cdots}{\cdots}\right)}{2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a/tan(d*x+c)-1/a*b/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b)
)^(1/2)))
```

Maxima [A]
time = 0.54, size = 48, normalized size = 0.91

$$-\frac{\frac{b\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a) + 1/(a
*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(45) = 90$.
time = 0.42, size = 313, normalized size = 5.91

$$\left[-\frac{\sqrt{-a^2-ab}\,b\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2(4a^2+5ab+b^2)\cos(dx+c)^2-4\left((2a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\sin(dx+c)+a^2+2ab+b^2}{b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right)\sin(dx+c)+4(a^2+ab)\cos(dx+c)}{4(a^3+a^2b)d\sin(dx+c)}, \frac{\sqrt{a^2+ab}\,b\arctan\left(\frac{(2a+b)\cos(dx+c)^2-a-b}{2\sqrt{a^2+ab}\cos(dx+c)\sin(dx+c)}\right)\sin(dx+c)-2(a^2+ab)\cos(dx+c)}{2(a^3+a^2b)d\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*
a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*c
os(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*
x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c)
+ 4*(a^2 + a*b)*cos(d*x + c))/((a^3 + a^2*b)*d*sin(d*x + c)), 1/2*(sqrt(a^2
 + a*b)*b*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*co
s(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 2*(a^2 + a*b)*cos(d*x + c))/((a^3
+ a^2*b)*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

Giac [A]
time = 0.53, size = 83, normalized size = 1.57

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b}{\sqrt{a^2+ab}\,a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*b/(sqrt(a^2 + a*b)*a) + 1/(a*tan(d*x + c))
)/d
```

Mupad [B]
time = 13.48, size = 45, normalized size = 0.85

$$-\frac{\cot(c+d\,x)}{a\,d}-\frac{b\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{3/2}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] - cot(c + d*x)/(a*d) - (b*atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2)))/(a^(3
/2)*d*(a + b)^(1/2))
```

3.91 $\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}\,d}-\frac{(a-b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

```
[Out] -(a-b)*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a/d+b^2*arctan((a+b)^(1/2)*tan(d*x
+c)/a^(1/2))/a^(5/2)/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3266, 472, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a+b}}-\frac{(a-b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]*d) -
((a - b)*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{a-b}{a^2x^2}+\frac{b^2}{a^2(a+(a+b)x^2)}\right)\,dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{(a-b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}+\frac{b^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{a^2d}\\
&= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}\,d}-\frac{(a-b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 119, normalized size = 1.55

$$-\frac{(2a+b-b\cos(2(c+dx)))\csc^2(c+dx)\left(-3b^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\,\sqrt{a+b}\,\cot(c+dx)\,(2a-3b+a\csc^2(c+dx))\right)}{6a^{5/2}\sqrt{a+b}\,d\,(b+a\csc^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/6*((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(-3*b^2*ArcTan[(Sqrt[a
+ b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a + b]*Cot[c + d*x]*(2*a - 3*b +
 a*Csc[c + d*x]^2)))/(a^(5/2)*Sqrt[a + b]*d*(b + a*Csc[c + d*x]^2))
```

Maple [A]
time = 0.33, size = 69, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{1}{3a\tan(dx+c)^3}-\frac{a-b}{a^2\tan(dx+c)}+\frac{b^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{1}{3a\tan(dx+c)^3}-\frac{a-b}{a^2\tan(dx+c)}+\frac{b^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{d}$

risch	$\frac{2i\left(3b\,\mathrm{e}^{4i(dx+c)}+6a\,\mathrm{e}^{2i(dx+c)}-6b\,\mathrm{e}^{2i(dx+c)}-2a+3b\right)}{3d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}-1\right)^{3}}-\frac{b^{2}\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{2ia^{2}+2iab+2a\sqrt{-a^{2}-ab}+b\sqrt{-a^{2}-a}}{b\sqrt{-a^{2}-ab}}\right.}{2\sqrt{-a^{2}-ab}\,d\,a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/a/tan(d*x+c)^3-(a-b)/a^2/tan(d*x+c)+b^2/a^2/(a*(a+b))^(1/2)*arcta
n((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.53, size = 69, normalized size = 0.90

$$\frac{\frac{3\,b^{2}\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^{2}}-\frac{3\,(a-b)\tan(dx+c)^{2}+a}{a^{2}\tan(dx+c)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b^2*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^
2) - (3*(a - b)*tan(d*x + c)^2 + a)/(a^2*tan(d*x + c)^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(67) = 134.
time = 0.43, size = 451, normalized size = 5.86

$$\left[-\frac{4\,(2\,a^{3}-a^{2}b-3\,ab^{2})\cos(dx+c)^{3}+3\,(b^{2}\cos(dx+c)^{2}-b^{2})\sqrt{-a^{2}-ab}\,\log\left(\frac{(8\,a^{2}+8\,ab+b^{2})\cos(dx+c)^{4}-2\,(4\,a^{2}+5\,ab+b^{2})\cos(dx+c)^{2}+4\,\left((2\,a+b)\cos(dx+c)^{3}-(a+b)\cos(dx+c)\right)\sqrt{-a^{2}-ab}\,\sin(dx+c)+a^{2}+2\,ab+b^{2}}{b^{2}\cos(dx+c)^{4}-2\,(ab+b^{2})\cos(dx+c)^{2}+a^{2}+2\,ab+b^{2}}\right)\sin(dx+c)-12\,(a^{3}-ab^{2})\cos(dx+c)}{12\,\left((a^{4}+a^{3}b)d\cos(dx+c)^{2}-(a^{4}+a^{3}b)d\right)\sin(dx+c)},\ -\frac{2\,(2\,a^{3}-a^{2}b-3\,ab^{2})\cos(dx+c)^{3}+3\,(b^{2}\cos(dx+c)^{2}-b^{2})\sqrt{a^{2}+ab}\,\arctan\left(\frac{(2\,a+b)\cos(dx+c)^{2}-a-b}{2\sqrt{a^{2}+ab}\cos(dx+c)\sin(dx+c)}\right)\sin(dx+c)-6\,(a^{3}-ab^{2})\cos(dx+c)}{6\,\left((a^{4}+a^{3}b)d\cos(dx+c)^{2}-(a^{4}+a^{3}b)d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a^3 - a^2*b - 3*a*b^2)*cos(d*x + c)^3 + 3*(b^2*cos(d*x + c)^2
- b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^
2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos
(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x
+ c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) -
12*(a^3 - a*b^2)*cos(d*x + c))/(((a^4 + a^3*b)*d*cos(d*x + c)^2 - (a^4 + a^
3*b)*d)*sin(d*x + c)), -1/6*(2*(2*a^3 - a^2*b - 3*a*b^2)*cos(d*x + c)^3 + 3
*(b^2*cos(d*x + c)^2 - b^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x +
 c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) -
6*(a^3 - a*b^2)*cos(d*x + c))/(((a^4 + a^3*b)*d*cos(d*x + c)^2 - (a^4 + a^3
*b)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

Giac [A]
time = 0.46, size = 111, normalized size = 1.44

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^2}{\sqrt{a^2+ab}\,a^2}-\frac{3\,a\tan(dx+c)^2-3\,b\tan(dx+c)^2+a}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^2/(sqrt(a^2 + a*b)*a^2) - (3*a*tan(
d*x + c)^2 - 3*b*tan(d*x + c)^2 + a)/(a^2*tan(d*x + c)^3))/d
```

Mupad [B]
time = 13.43, size = 68, normalized size = 0.88

$$\frac{b^2\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{5/2}\,d\,\sqrt{a+b}}-\frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(a-b)}{a^2}}{d\,\tan\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] (b^2*atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2)))/(a^(5/2)*d*(a + b)^(1/2))
- (1/(3*a) + (tan(c + d*x)^2*(a - b))/a^2)/(d*tan(c + d*x)^3)
```

3.92 $\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}\,d}-\frac{(a^2-ab+b^2)\cot(c+dx)}{a^3d}-\frac{(2a-b)\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5ad}$$

```
[Out] -(a^2-a*b+b^2)*cot(d*x+c)/a^3/d-1/3*(2*a-b)*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+
c)^5/a/d-b^3*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3266, 472, 211}

$$-\frac{b^3\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a+b}}-\frac{(2a-b)\cot^3(c+dx)}{3a^2d}-\frac{(a^2-ab+b^2)\cot(c+dx)}{a^3d}-\frac{\cot^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a + b]*d))
 - ((a^2 - a*b + b^2)*Cot[c + d*x])/(a^3*d) - ((2*a - b)*Cot[c + d*x]^3)/(3
*a^2*d) - Cot[c + d*x]^5/(5*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6}+\frac{2a-b}{a^2x^4}+\frac{a^2-ab+b^2}{a^3x^2}+\frac{b^3}{a^3(-a-(a+b)x^2)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{(a^2-ab+b^2)\cot(c+dx)}{a^3d}-\frac{(2a-b)\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5ad}+\frac{b^3\text{Subst}\left(\right.}{}$$
$$= -\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}\,d}-\frac{(a^2-ab+b^2)\cot(c+dx)}{a^3d}-\frac{(2a-b)\cot^3(c+}{3a^2d}$$

Mathematica [A]

time = 1.02, size = 147, normalized size = 1.35

$$-\frac{(2a+b-b\cos(2(c+dx)))\csc^2(c+dx)\left(15b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a+b}\cot(c+dx)\left(8a^2-10ab+15b^2+a(4a-5b)\csc^2(c+dx)+3a^2\csc^4(c+dx)\right)\right)}{30a^{7/2}\sqrt{a+b}\,d\left(b+a\csc^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/30*((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(15*b^3*ArcTan[(Sqrt[a
 + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a + b]*Cot[c + d*x]*(8*a^2 - 10
*a*b + 15*b^2 + a*(4*a - 5*b)*Csc[c + d*x]^2 + 3*a^2*Csc[c + d*x]^4)))/(a^(
7/2)*Sqrt[a + b]*d*(b + a*Csc[c + d*x]^2))
```

Maple [A]

time = 0.36, size = 96, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{2a-b}{3a^2\tan(dx+c)^3}-\frac{a^2-ab+b^2}{a^3\tan(dx+c)}-\frac{b^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{2a-b}{3a^2\tan(dx+c)^3}-\frac{a^2-ab+b^2}{a^3\tan(dx+c)}-\frac{b^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{d}$

risch	$-\frac{2i(15b^2 e^{8i(dx+c)}+30ab\, e^{6i(dx+c)}-60b^2 e^{6i(dx+c)}+80a^2 e^{4i(dx+c)}-70ab\, e^{4i(dx+c)}+90b^2 e^{4i(dx+c)}-40a^2 e^{2i(dx+c)}+50b\, e}{15d\, a^3 (e^{2i(dx+c)}-1)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5/a/tan(d*x+c)^5-1/3*(2*a-b)/a^2/tan(d*x+c)^3-(a^2-a*b+b^2)/a^3/tan
(d*x+c)-b^3/a^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.53, size = 98, normalized size = 0.90

$$-\frac{\frac{15\,b^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^3}+\frac{15\,(a^2-ab+b^2)\tan(dx+c)^4+5\,(2\,a^2-ab)\tan(dx+c)^2+3\,a^2}{a^3\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^3*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)
*a^3) + (15*(a^2 - a*b + b^2)*tan(d*x + c)^4 + 5*(2*a^2 - a*b)*tan(d*x + c)
^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(97) = 194.
time = 0.44, size = 595, normalized size = 5.46

$$\left[-\frac{4\,(8\,a^4-2\,a^3b+5\,a^2b^2+15\,ab^3)\cos(dx+c)^5-20\,(4\,a^4-a^3b+a^2b^2+6\,ab^3)\cos(dx+c)^3+15\,(b^3\cos(dx+c)^4-2\,b^3\cos(dx+c)^2+b^3)\sqrt{-a^2-ab}\,\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2-4\,((2\,a+b)\cos(dx+c)^3-(a+b)\cos(dx+c))\sqrt{-a^2-ab}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)\sin(dx+c)+60\,(a^4+ab^3)\cos(dx+c)}{60\,((a^5+a^4b)d\cos(dx+c)^4-2\,(a^5+a^4b)d\cos(dx+c)^2+(a^5+a^4b)d)\sin(dx+c)},\ -\frac{2\,(8\,a^4-2\,a^3b+5\,a^2b^2+15\,ab^3)\cos(dx+c)^5-10\,(4\,a^4-a^3b+a^2b^2+6\,ab^3)\cos(dx+c)^3-15\,(b^3\cos(dx+c)^4-2\,b^3\cos(dx+c)^2+b^3)\sqrt{a^2+ab}\,\arctan\left(\frac{(2\,a+b)\cos(dx+c)^2-a-b}{2\,\sqrt{a^2+ab}\cos(dx+c)\sin(dx+c)}\right)\sin(dx+c)+30\,(a^4+ab^3)\cos(dx+c)}{30\,((a^5+a^4b)d\cos(dx+c)^4-2\,(a^5+a^4b)d\cos(dx+c)^2+(a^5+a^4b)d)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(d*x + c)^5 - 20*(4*a
^4 - a^3*b + a^2*b^2 + 6*a*b^3)*cos(d*x + c)^3 + 15*(b^3*cos(d*x + c)^4 - 2
*b^3*cos(d*x + c)^2 + b^3)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(
d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x
+ c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b
+ b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b
^2))*sin(d*x + c) + 60*(a^4 + a*b^3)*cos(d*x + c))/(((a^5 + a^4*b)*d*cos(d*
x + c)^4 - 2*(a^5 + a^4*b)*d*cos(d*x + c)^2 + (a^5 + a^4*b)*d)*sin(d*x + c)
), -1/30*(2*(8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(d*x + c)^5 - 10*(4
*a^4 - a^3*b + a^2*b^2 + 6*a*b^3)*cos(d*x + c)^3 - 15*(b^3*cos(d*x + c)^4 -
 2*b^3*cos(d*x + c)^2 + b^3)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x
```

```
+ c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) +
 30*(a^4 + a*b^3)*cos(d*x + c))/(((a^5 + a^4*b)*d*cos(d*x + c)^4 - 2*(a^5 +
 a^4*b)*d*cos(d*x + c)^2 + (a^5 + a^4*b)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**6/(a + b*sin(c + d*x)**2), x)
```

Giac [A]
time = 0.48, size = 155, normalized size = 1.42

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^3}{\sqrt{a^2+ab}\,a^3}+\frac{15a^2\tan(dx+c)^4-15ab\tan(dx+c)^4+15b^2\tan(dx+c)^4+10a^2\tan(dx+c)^2-5ab\tan(dx+c)^2+3a^2}{a^3\tan(dx+c)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
+ c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^3/(sqrt(a^2 + a*b)*a^3) + (15*a^
2*tan(d*x + c)^4 - 15*a*b*tan(d*x + c)^4 + 15*b^2*tan(d*x + c)^4 + 10*a^2*t
an(d*x + c)^2 - 5*a*b*tan(d*x + c)^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

Mupad [B]
time = 13.80, size = 95, normalized size = 0.87

$$-\frac{\tan(c+d\,x)^4\,(a^2-a\,b+b^2)+\frac{a^2}{5}-\tan(c+d\,x)^2\,\left(\frac{a\,b}{3}-\frac{2\,a^2}{3}\right)}{a^3\,d\tan\left(c+d\,x\right)^5}-\frac{b^3\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{7/2}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] - (tan(c + d*x)^4*(a^2 - a*b + b^2) + a^2/5 - tan(c + d*x)^2*((a*b)/3 - (2*
a^2)/3))/(a^3*d*tan(c + d*x)^5) - (b^3*atan((tan(c + d*x)*(a + b)^(1/2))/a^
(1/2)))/(a^(7/2)*d*(a + b)^(1/2))
```

3.93 $\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}\sqrt{a+b}\,d}-\frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4d}-\frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3d}-\frac{(3a-b)\cot^5(c}{5a^2d}$$

```
[Out] -(a-b)*(a^2+b^2)*cot(d*x+c)/a^4/d-1/3*(3*a^2-2*a*b+b^2)*cot(d*x+c)^3/a^3/d-
1/5*(3*a-b)*cot(d*x+c)^5/a^2/d-1/7*cot(d*x+c)^7/a/d+b^4*arctan((a+b)^(1/2)*
tan(d*x+c)/a^(1/2))/a^(9/2)/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$,
Rules used = {3266, 472, 211}

$$\frac{b^4\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d\sqrt{a+b}}-\frac{(3a-b)\cot^5(c+dx)}{5a^2d}-\frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4d}-\frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3d}-\frac{\cot^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^4*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*Sqrt[a + b]*d) -
((a - b)*(a^2 + b^2)*Cot[c + d*x])/(a^4*d) - ((3*a^2 - 2*a*b + b^2)*Cot[c +
 d*x]^3)/(3*a^3*d) - ((3*a - b)*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*x]^7/
(7*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
```

```
IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^8(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^8}+\frac{3a-b}{a^2x^6}+\frac{3a^2-2ab+b^2}{a^3x^4}+\frac{(a-b)(a^2+b^2)}{a^4x^2}+\frac{b^4}{a^4(a+(a+b)x^2)}\right) dx, x, \tan(c+d\right.}{d}$$

$$= -\frac{(a-b)(a^2+b^2)\cot(c+dx)}{a^4d}-\frac{(3a^2-2ab+b^2)\cot^3(c+dx)}{3a^3d}-\frac{(3a-b)\cot^5(}{5a^2d}$$

$$= \frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}\sqrt{a+b}\,d}-\frac{(a-b)(a^2+b^2)\cot(c+dx)}{a^4d}-\frac{(3a^2-2ab+b^2)}{3a^3c}$$

Mathematica [A]
time = 1.13, size = 137, normalized size = 0.98

$$\frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}\sqrt{a+b}\,d}-\frac{\cot(c+dx)\left(48a^3-56a^2b+70ab^2-105b^3+a(24a^2-28ab+35b^2)\csc^2(c+dx)+3a^2(6a-7b)\csc^4(c+dx)+15a^3\csc^6(c+dx)\right)}{105a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^4*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*Sqrt[a + b]*d) -
(Cot[c + d*x]*(48*a^3 - 56*a^2*b + 70*a*b^2 - 105*b^3 + a*(24*a^2 - 28*a*b
+ 35*b^2)*Csc[c + d*x]^2 + 3*a^2*(6*a - 7*b)*Csc[c + d*x]^4 + 15*a^3*Csc[c
+ d*x]^6))/(105*a^4*d)
```

Maple [A]
time = 0.36, size = 130, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{1}{7a\tan(dx+c)^7}-\frac{3a-b}{5a^2\tan(dx+c)^5}-\frac{3a^2-2ab+b^2}{3a^3\tan(dx+c)^3}-\frac{a^3-a^2b+a\,b^2-b^3}{a^4\tan(dx+c)}+\frac{b^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^4\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{1}{7a\tan(dx+c)^7}-\frac{3a-b}{5a^2\tan(dx+c)^5}-\frac{3a^2-2ab+b^2}{3a^3\tan(dx+c)^3}-\frac{a^3-a^2b+a\,b^2-b^3}{a^4\tan(dx+c)}+\frac{b^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^4\sqrt{a(a+b)}}}{d}$

risch	$\frac{2i(105b^3e^{12i(dx+c)}+210a\,b^2e^{10i(dx+c)}-630b^3e^{10i(dx+c)}+560a^2b\,e^{8i(dx+c)}-910a\,b^2e^{8i(dx+c)}+1575b^3e^{8i(dx+c)}+1680a^3e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7/a/tan(d*x+c)^7-1/5*(3*a-b)/a^2/tan(d*x+c)^5-1/3*(3*a^2-2*a*b+b^2)
/a^3/tan(d*x+c)^3-(a^3-a^2*b+a*b^2-b^3)/a^4/tan(d*x+c)+b^4/a^4/(a*(a+b))^(1
/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.55, size = 137, normalized size = 0.98

$$\frac{\frac{105\,b^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^4}-\frac{105\,(a^3-a^2b+ab^2-b^3)\tan(dx+c)^6+35\,(3\,a^3-2\,a^2b+ab^2)\tan(dx+c)^4+15\,a^3+21\,(3\,a^3-a^2b)\tan(dx+c)^2}{a^4\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*b^4*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a
)*a^4) - (105*(a^3 - a^2*b + a*b^2 - b^3)*tan(d*x + c)^6 + 35*(3*a^3 - 2*a^
2*b + a*b^2)*tan(d*x + c)^4 + 15*a^3 + 21*(3*a^3 - a^2*b)*tan(d*x + c)^2)/(
a^4*tan(d*x + c)^7))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(126) = 252$.
time = 0.45, size = 789, normalized size = 5.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/420*(4*(48*a^5 - 8*a^4*b + 14*a^3*b^2 - 35*a^2*b^3 - 105*a*b^4)*cos(d*x
 + c)^7 - 28*(24*a^5 - 4*a^4*b + 7*a^3*b^2 - 10*a^2*b^3 - 45*a*b^4)*cos(d*x
 + c)^5 + 140*(6*a^5 - a^4*b + a^3*b^2 - a^2*b^3 - 9*a*b^4)*cos(d*x + c)^3
+ 105*(b^4*cos(d*x + c)^6 - 3*b^4*cos(d*x + c)^4 + 3*b^4*cos(d*x + c)^2 - b
^4)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 +
 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*
x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c
)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 420
*(a^5 - a*b^4)*cos(d*x + c))/(((a^6 + a^5*b)*d*cos(d*x + c)^6 - 3*(a^6 + a^
```

```
5*b)*d*cos(d*x + c)^4 + 3*(a^6 + a^5*b)*d*cos(d*x + c)^2 - (a^6 + a^5*b)*d)
*sin(d*x + c)), -1/210*(2*(48*a^5 - 8*a^4*b + 14*a^3*b^2 - 35*a^2*b^3 - 105
*a*b^4)*cos(d*x + c)^7 - 14*(24*a^5 - 4*a^4*b + 7*a^3*b^2 - 10*a^2*b^3 - 45
*a*b^4)*cos(d*x + c)^5 + 70*(6*a^5 - a^4*b + a^3*b^2 - a^2*b^3 - 9*a*b^4)*c
os(d*x + c)^3 + 105*(b^4*cos(d*x + c)^6 - 3*b^4*cos(d*x + c)^4 + 3*b^4*cos(
d*x + c)^2 - b^4)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a
- b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 210*(a^5 -
 a*b^4)*cos(d*x + c))/(((a^6 + a^5*b)*d*cos(d*x + c)^6 - 3*(a^6 + a^5*b)*d*
cos(d*x + c)^4 + 3*(a^6 + a^5*b)*d*cos(d*x + c)^2 - (a^6 + a^5*b)*d)*sin(d*
x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
[Out] Timed out
```

Giac [A]
time = 0.44, size = 215, normalized size = 1.54

$$\frac{\frac{105\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^4}{\sqrt{a^2+ab}\,a^4}-\frac{105\,a^3\tan(dx+c)^6-105\,a^2b\tan(dx+c)^6+105\,ab^2\tan(dx+c)^6-105\,b^3\tan(dx+c)^6+105\,a^3\tan(dx+c)^4-70\,a^2b\tan(dx+c)^4+35\,ab^2\tan(dx+c)^4+63\,a^3\tan(dx+c)^2-21\,a^2b\tan(dx+c)^2+15\,a^3}{a^4\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
[Out] 1/105*(105*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
 + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^4/(sqrt(a^2 + a*b)*a^4) - (105*
a^3*tan(d*x + c)^6 - 105*a^2*b*tan(d*x + c)^6 + 105*a*b^2*tan(d*x + c)^6 -
105*b^3*tan(d*x + c)^6 + 105*a^3*tan(d*x + c)^4 - 70*a^2*b*tan(d*x + c)^4 +
 35*a*b^2*tan(d*x + c)^4 + 63*a^3*tan(d*x + c)^2 - 21*a^2*b*tan(d*x + c)^2
+ 15*a^3)/(a^4*tan(d*x + c)^7))/d
```

Mupad [B]
time = 15.07, size = 130, normalized size = 0.93

$$\frac{b^4\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{9/2}\,d\,\sqrt{a+b}}-\frac{\tan(c+d\,x)^4\,\left(a^3-\frac{2\,a^2\,b}{3}+\frac{a\,b^2}{3}\right)-\tan(c+d\,x)^2\,\left(\frac{a^2\,b}{5}-\frac{3\,a^3}{5}\right)+\tan(c+d\,x)^6\,\left(a^3-a^2\,b+a\,b^2-b^3\right)+\frac{a^3}{7}}{a^4\,d\,\tan\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^8*(a + b*sin(c + d*x)^2)),x)
[Out] (b^4*atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2)))/(a^(9/2)*d*(a + b)^(1/2))
- (tan(c + d*x)^4*((a*b^2)/3 - (2*a^2*b)/3 + a^3) - tan(c + d*x)^2*((a^2*b)
/5 - (3*a^3)/5) + tan(c + d*x)^6*(a*b^2 - a^2*b + a^3 - b^3) + a^3/7)/(a^4*
d*tan(c + d*x)^7)
```

3.94 $\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=128

$$-\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}(a+b)^{3/2}d}+\frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}+\frac{a^3\cos(c+dx)}{2b^3(a+b)d\,(a+b-b\cos^2(c+d$$

```
[Out] -1/2*a^2*(5*a+6*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/(a+b)^(3
/2)/d+(2*a-b)*cos(d*x+c)/b^3/d+1/3*cos(d*x+c)^3/b^2/d+1/2*a^3*cos(d*x+c)/b^
3/(a+b)/d/(a+b-b*cos(d*x+c)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 398, 393, 214}

$$\frac{a^3\cos(c+dx)}{2b^3d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}d(a+b)^{3/2}}+\frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(a^2*(5*a + 6*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(7/2)
*(a + b)^(3/2)*d) + ((2*a - b)*Cos[c + d*x])/(b^3*d) + Cos[c + d*x]^3/(3*b^
2*d) + (a^3*Cos[c + d*x])/(2*b^3*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{2a-b}{b^3}-\frac{x^2}{b^2}+\frac{a^2(2a+3b)-3a^2bx^2}{b^3\left(a+b-bx^2\right)^2}\right)\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}-\frac{\text{Subst}\left(\int \frac{a^2(2a+3b)-3a^2bx^2}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+\right.}{b^3d}$$

$$= \frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}+\frac{a^3\cos(c+dx)}{2b^3(a+b)d\left(a+b-b\cos^2(c+dx)\right)}-$$

$$= -\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}(a+b)^{3/2}d}+\frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.07, size = 194, normalized size = 1.52

$$\frac{-\frac{6a^2(5a+6b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}-\frac{6a^2(5a+6b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}+\sqrt{b}\left(\cos(c+dx)\left(24a-9b+\frac{12a^3}{(a+b)(2a+b-b\cos(2(c+dx)))}\right)+b\cos(3(c+dx))\right)}{12b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-6*a^2*(5*a + 6*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a
 - b]])/(-a - b)^(3/2) - (6*a^2*(5*a + 6*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[
(c + d*x)/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + Sqrt[b]*(Cos[c + d*x]*(24*a -
 9*b + (12*a^3)/((a + b)*(2*a + b - b*Cos[2*(c + d*x)]))) + b*Cos[3*(c + d*
x)]))/(12*b^(7/2)*d)
```

Maple [A]
time = 0.31, size = 116, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{\left(\cos^3(dx+c)\right)b}{3}+2a\cos(dx+c)-b\cos(dx+c)}{b^3}-\frac{a^2\left(-\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(5a+6b)\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{b^3}}{d}$
default	$\frac{\frac{\left(\cos^3(dx+c)\right)b}{3}+2a\cos(dx+c)-b\cos(dx+c)}{b^3}-\frac{a^2\left(-\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(5a+6b)\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{b^3}}{d}$
risch	$\frac{e^{3i(dx+c)}}{24b^2d}+\frac{a\,e^{i(dx+c)}}{b^3d}-\frac{3\,e^{i(dx+c)}}{8b^2d}+\frac{e^{-i(dx+c)}a}{d\,b^3}-\frac{3\,e^{-i(dx+c)}}{8d\,b^2}+\frac{e^{-3i(dx+c)}}{24b^2d}-\frac{a^3\left(e^{3i(dx+c)}+e^{i}\right.}{b^3(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i(d}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*(1/3*b*cos(d*x+c)^3+2*a*cos(d*x+c)-b*cos(d*x+c))-1/b^3*a^2*(-1/2
*a/(a+b)*cos(d*x+c)/(a+b-b*cos(d*x+c)^2)+1/2*(5*a+6*b)/(a+b)/((a+b)*b)^(1/2
)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.51, size = 154, normalized size = 1.20

$$\frac{\frac{6\,a^3\cos(dx+c)}{a^2b^3+2\,ab^4+b^5-(ab^4+b^5)\cos(dx+c)^2}+\frac{3\,(5\,a+6\,b)a^2\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{(ab^3+b^4)\sqrt{(a+b)b}}+\frac{4\left(b\cos(dx+c)^3+3\,(2\,a-b)\cos(dx+c)\right)}{b^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*a^3*cos(d*x + c)/(a^2*b^3 + 2*a*b^4 + b^5 - (a*b^4 + b^5)*cos(d*x +
 c)^2) + 3*(5*a + 6*b)*a^2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*
x + c) + sqrt((a + b)*b)))/((a*b^3 + b^4)*sqrt((a + b)*b)) + 4*(b*cos(d*x +
 c)^3 + 3*(2*a - b)*cos(d*x + c))/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(117) = 234$.
time = 0.45, size = 529, normalized size = 4.13

$$\left[\frac{4\,(a^2b^3+2\,ab^4+b^5)\cos(dx+c)^5+4\,(5\,a^3b^2+6\,a^2b^3-3\,ab^4-4\,b^5)\cos(dx+c)^3-3\left(5\,a^4+11\,a^3b+6\,a^2b^2-(5\,a^3b+6\,a^2b^2)\cos(dx+c)^2\right)\sqrt{ab+b^2}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-6\,(5\,a^4b+11\,a^3b^2+6\,a^2b^3-2\,ab^4-2\,b^5)\cos(dx+c)}{12\left((a^2b^5+2\,ab^6+b^7)d\cos(dx+c)^2-(a^3b^4+3\,a^2b^5+3\,ab^6+b^7)d\right)},\ \frac{2\,(a^2b^3+2\,ab^4+b^5)\cos(dx+c)^5+2\,(5\,a^3b^2+6\,a^2b^3-3\,ab^4-4\,b^5)\cos(dx+c)^3-3\left(5\,a^4+11\,a^3b+6\,a^2b^2-(5\,a^3b+6\,a^2b^2)\cos(dx+c)^2\right)\sqrt{-ab-b^2}\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)-3\,(5\,a^4b+11\,a^3b^2+6\,a^2b^3-2\,ab^4-2\,b^5)\cos(dx+c)}{6\left((a^2b^5+2\,ab^6+b^7)d\cos(dx+c)^2-(a^3b^4+3\,a^2b^5+3\,ab^6+b^7)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*(a^2*b^3 + 2*a*b^4 + b^5)*cos(d*x + c)^5 + 4*(5*a^3*b^2 + 6*a^2*b^
3 - 3*a*b^4 - 4*b^5)*cos(d*x + c)^3 - 3*(5*a^4 + 11*a^3*b + 6*a^2*b^2 - (5*
a^3*b + 6*a^2*b^2)*cos(d*x + c)^2)*sqrt(a*b + b^2)*log(-(b*cos(d*x + c)^2 -
 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 6*(5
*a^4*b + 11*a^3*b^2 + 6*a^2*b^3 - 2*a*b^4 - 2*b^5)*cos(d*x + c))/((a^2*b^5
+ 2*a*b^6 + b^7)*d*cos(d*x + c)^2 - (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*d
), 1/6*(2*(a^2*b^3 + 2*a*b^4 + b^5)*cos(d*x + c)^5 + 2*(5*a^3*b^2 + 6*a^2*b
^3 - 3*a*b^4 - 4*b^5)*cos(d*x + c)^3 - 3*(5*a^4 + 11*a^3*b + 6*a^2*b^2 - (5
*a^3*b + 6*a^2*b^2)*cos(d*x + c)^2)*sqrt(-a*b - b^2)*arctan(sqrt(-a*b - b^2
)*cos(d*x + c)/(a + b)) - 3*(5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3 - 2*a*b^4 - 2
*b^5)*cos(d*x + c))/((a^2*b^5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^2 - (a^3*b^4
+ 3*a^2*b^5 + 3*a*b^6 + b^7)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(117) = 234.
time = 0.55, size = 322, normalized size = 2.52

$$\frac{\frac{3\left(5a^3+6a^2b\right)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{(ab^3+b^4)\sqrt{-ab-b^2}}+\frac{6\left(a^3-\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(ab^3+b^4)\left(a-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}-\frac{8\left(3a-b-\frac{6a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{b^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(5*a^3 + 6*a^2*b)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*
cos(d*x + c) + sqrt(-a*b - b^2)))/((a*b^3 + b^4)*sqrt(-a*b - b^2)) + 6*(a^3
 - a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^2*b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))/((a*b^3 + b^4)*(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2)) - 8*(3*a - b - 6*a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + 3*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2)/(b^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^3))
/d
```

Mupad [B]
time = 0.22, size = 123, normalized size = 0.96

$$\frac{\cos\left(c+d\,x\right)\,\left(\frac{2\,\left(a+b\right)}{b^3}-\frac{3}{b^2}\right)}{d}+\frac{{\cos\left(c+d\,x\right)}^3}{3\,b^2\,d}+\frac{a^3\,\cos\left(c+d\,x\right)}{2\,d\,\left(a+b\right)\,\left(-b^4\,{\cos\left(c+d\,x\right)}^2+b^4+a\,b^3\right)}-\frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos\left(c+d\,x\right)}{\sqrt{a+b}}\right)\,\left(5\,a+6\,b\right)}{2\,b^{7/2}\,d\,{\left(a+b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (cos(c + d*x)*((2*(a + b))/b^3 - 3/b^2))/d + cos(c + d*x)^3/(3*b^2*d) + (a^
3*cos(c + d*x))/(2*d*(a + b)*(a*b^3 + b^4 - b^4*cos(c + d*x)^2)) - (a^2*ata
nh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2))*(5*a + 6*b))/(2*b^(7/2)*d*(a + b)^
(3/2))
```

3.95 $\int \frac{\sin^5(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=102

$$\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d}-\frac{\cos(c+dx)}{b^2d}-\frac{a^2\cos(c+dx)}{2b^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

```
[Out] 1/2*a*(3*a+4*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/2)
/d-cos(d*x+c)/b^2/d-1/2*a^2*cos(d*x+c)/b^2/(a+b)/d/(a+b-b*cos(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 398, 393, 214}

$$-\frac{a^2\cos(c+dx)}{2b^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}+\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}-\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(5/2)*(a +
 b)^(3/2)*d) - Cos[c + d*x]/(b^2*d) - (a^2*Cos[c + d*x])/(2*b^2*(a + b)*d*(
a + b - b*Cos[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a(a+2b)-2abx^2}{b^2\left(a+b-bx^2\right)^2}\right)dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{\cos(c+dx)}{b^2d}+\frac{\text{Subst}\left(\int \frac{a(a+2b)-2abx^2}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{b^2d} \\
&= -\frac{\cos(c+dx)}{b^2d}-\frac{a^2\cos(c+dx)}{2b^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)}+\frac{(a(3a+4b))\text{Subst}\left(\int \frac{}{a-}\right.}{2b^2(a} \\
&= \frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d}-\frac{\cos(c+dx)}{b^2d}-\frac{a^2\cos(c+dx)}{2b^2(a+b)d\left(a+b-b\cos^2\right.}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 172, normalized size = 1.69

$$\frac{\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}+\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}+2\sqrt{b}\cos(c+dx)\left(-1-\frac{a^2}{(a+b)(2a+b-b\cos(2(c+dx)))}\right)}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((a*(3*a + 4*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]
)/(-a - b)^(3/2) + (a*(3*a + 4*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)
/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + 2*Sqrt[b]*Cos[c + d*x]*(-1 - a^2/((a +
 b)*(2*a + b - b*Cos[2*(c + d*x)]))))/(2*b^(5/2)*d)
```

Maple [A]
time = 0.38, size = 90, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)}{b^2}+\frac{a\left(-\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(3a+4b)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{b^2}}{d}$
default	$\frac{-\frac{\cos(dx+c)}{b^2}+\frac{a\left(-\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(3a+4b)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{b^2}}{d}$
risch	$-\frac{e^{i(dx+c)}}{2b^2d}-\frac{e^{-i(dx+c)}}{2d\,b^2}+\frac{a^2\left(e^{3i(dx+c)}+e^{i(dx+c)}\right)}{b^2(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+b\right)}+\frac{3ia^2\ln\left(e^{2i(dx+c)}+\frac{2i(a+b)e^{i(dx}}{\sqrt{-ab-}}\right.}{4\sqrt{-ab-b^2}\,(a+b)c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2*cos(d*x+c)+1/b^2*a*(-1/2*a/(a+b)*cos(d*x+c)/(a+b-b*cos(d*x+c)^2
)+1/2*(3*a+4*b)/(a+b)/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))
))
```

Maxima [A]
time = 0.57, size = 131, normalized size = 1.28

$$-\frac{\frac{2\,a^2\cos(dx+c)}{a^2b^2+2\,ab^3+b^4-(ab^3+b^4)\cos(dx+c)^2}+\frac{(3\,a+4\,b)a\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{(ab^2+b^3)\sqrt{(a+b)b}}+\frac{4\cos(dx+c)}{b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*cos(d*x + c)/(a^2*b^2 + 2*a*b^3 + b^4 - (a*b^3 + b^4)*cos(d*x +
 c)^2) + (3*a + 4*b)*a*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x +
c) + sqrt((a + b)*b)))/((a*b^2 + b^3)*sqrt((a + b)*b)) + 4*cos(d*x + c)/b^2
)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(93) = 186.
time = 0.45, size = 427, normalized size = 4.19

$$\left[-\frac{4\,(a^2b^2+2\,ab^3+b^4)\cos(dx+c)^3+\left(3\,a^3+7\,a^2b+4\,ab^2-(3\,a^2b+4\,ab^2)\cos(dx+c)^2\right)\sqrt{ab+b^2}\,\log\left(\frac{b\cos(dx+c)^2+2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-2\,(3\,a^3b+7\,a^2b^2+6\,ab^3+2\,b^4)\cos(dx+c)}{4\left((a^2b^4+2\,ab^5+b^6)d\cos(dx+c)^2-(a^3b^3+3\,a^2b^4+3\,ab^5+b^6)d\right)},\ -\frac{2\,(a^2b^2+2\,ab^3+b^4)\cos(dx+c)^3-\left(3\,a^3+7\,a^2b+4\,ab^2-(3\,a^2b+4\,ab^2)\cos(dx+c)^2\right)\sqrt{-ab-b^2}\,\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)-(3\,a^3b+7\,a^2b^2+6\,ab^3+2\,b^4)\cos(dx+c)}{2\left((a^2b^4+2\,ab^5+b^6)d\cos(dx+c)^2-(a^3b^3+3\,a^2b^4+3\,ab^5+b^6)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b^2 + 2*a*b^3 + b^4)*cos(d*x + c)^3 + (3*a^3 + 7*a^2*b + 4*a*
b^2 - (3*a^2*b + 4*a*b^2)*cos(d*x + c)^2)*sqrt(a*b + b^2)*log((b*cos(d*x +
c)^2 + 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b))
- 2*(3*a^3*b + 7*a^2*b^2 + 6*a*b^3 + 2*b^4)*cos(d*x + c))/((a^2*b^4 + 2*a*b
^5 + b^6)*d*cos(d*x + c)^2 - (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*d), -1/2
*(2*(a^2*b^2 + 2*a*b^3 + b^4)*cos(d*x + c)^3 - (3*a^3 + 7*a^2*b + 4*a*b^2 -
 (3*a^2*b + 4*a*b^2)*cos(d*x + c)^2)*sqrt(-a*b - b^2)*arctan(sqrt(-a*b - b^
2)*cos(d*x + c)/(a + b)) - (3*a^3*b + 7*a^2*b^2 + 6*a*b^3 + 2*b^4)*cos(d*x
+ c))/((a^2*b^4 + 2*a*b^5 + b^6)*d*cos(d*x + c)^2 - (a^3*b^3 + 3*a^2*b^4 +
3*a*b^5 + b^6)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(93) = 186.
time = 0.44, size = 342, normalized size = 3.35

$$-\frac{\frac{(3\,a^2+4\,ab)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{(ab^2+b^3)\sqrt{-ab-b^2}}+\frac{2\left(3\,a^2+2\,ab-\frac{6\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{14\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{(ab^2+b^3)\left(a-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a^2 + 4*a*b)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos
(d*x + c) + sqrt(-a*b - b^2)))/((a*b^2 + b^3)*sqrt(-a*b - b^2)) + 2*(3*a^2
+ 2*a*b - 6*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 14*a*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 8*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3
*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2)/((a*b^2 + b^3)*(a - 3*a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
- a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)))/d
```

Mupad [B]
time = 0.16, size = 95, normalized size = 0.93

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)\,(3\,a+4\,b)}{2\,b^{5/2}\,d\,(a+b)^{3/2}}-\frac{a^2\,\cos\left(c+d\,x\right)}{2\,d\,\left(a+b\right)\,\left(-b^3\cos\left(c+d\,x\right)^2+b^3+a\,b^2\right)}-\frac{\cos\left(c+d\,x\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (a*atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2))*(3*a + 4*b))/(2*b^(5/2)*d*(a
 + b)^(3/2)) - (a^2*cos(c + d*x))/(2*d*(a + b)*(a*b^2 + b^3 - b^3*cos(c + d
*x)^2)) - cos(c + d*x)/(b^2*d)
```

3.96 $\int \frac{\sin^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d}+\frac{a\cos(c+dx)}{2b(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

```
[Out] -1/2*(a+2*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/d+
1/2*a*cos(d*x+c)/b/(a+b)/d/(a+b-b*cos(d*x+c)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 393, 214}

$$\frac{a\cos(c+dx)}{2bd(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a + 2*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(3/2)*(a +
b)^(3/2)*d) + (a*Cos[c + d*x])/(2*b*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a+b-bx^2)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{a\cos(c+dx)}{2b(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{(a+2b)\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2b(a+b)d}$$
$$= -\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d} + \frac{a\cos(c+dx)}{2b(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 160, normalized size = 1.93

$$\frac{\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} + \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} + \frac{2a\sqrt{b}\cos(c+dx)}{2a+b-b\cos(2(c+dx))}}{2b^{3/2}(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (((a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/Sq
rt[-a - b] + ((a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[
-a - b]])/Sqrt[-a - b] + (2*a*Sqrt[b]*Cos[c + d*x])/(2*a + b - b*Cos[2*(c +
 d*x)]))/(2*b^(3/2)*(a + b)*d)
```

Maple [A]
time = 0.31, size = 77, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{a\cos(dx+c)}{2(a+b)b\left(a+b-b\left(\cos^2(dx+c)\right)\right)} - \frac{(a+2b)\,\text{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)b\sqrt{(a+b)\,b}}}{d}$
default	$\frac{\frac{a\cos(dx+c)}{2(a+b)b\left(a+b-b\left(\cos^2(dx+c)\right)\right)} - \frac{(a+2b)\,\text{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)b\sqrt{(a+b)\,b}}}{d}$

risch	$-\frac{a\left(e^{3i(dx+c)}+e^{i(dx+c)}\right)}{b(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+b\right)}-\frac{ia\ln\left(e^{2i(dx+c)}+\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{4\sqrt{-ab-b^2}\,(a+b)db}-\frac{i\ln\left(e^{2i(dx+c)}+\frac{2}{\sqrt{}}\right.}{2\sqrt{-ab-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*a/(a+b)/b*cos(d*x+c)/(a+b-b*cos(d*x+c)^2)-1/2*(a+2*b)/(a+b)/b/((a+
b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.53, size = 111, normalized size = 1.34

$$\frac{\frac{2\,a\cos(dx+c)}{a^2b+2\,ab^2+b^3-(ab^2+b^3)\cos(dx+c)^2}+\frac{(a+2\,b)\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,(ab+b^2)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*cos(d*x + c)/(a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2
) + (a + 2*b)*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt
((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(74) = 148.
time = 0.42, size = 327, normalized size = 3.94

$$\left[\frac{\left((ab+2\,b^2)\cos(dx+c)^2-a^2-3\,ab-2\,b^2\right)\sqrt{ab+b^2}\log\left(-\frac{b\cos(dx+c)^2-2\,\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-2\,(a^2b+ab^2)\cos(dx+c)}{4\,\left((a^2b^3+2\,ab^4+b^5)d\cos(dx+c)^2-(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)d\right)},\ \frac{\left((ab+2\,b^2)\cos(dx+c)^2-a^2-3\,ab-2\,b^2\right)\sqrt{-ab-b^2}\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)-(a^2b+ab^2)\cos(dx+c)}{2\,\left((a^2b^3+2\,ab^4+b^5)d\cos(dx+c)^2-(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b + 2*b^2)*cos(d*x + c)^2 - a^2 - 3*a*b - 2*b^2)*sqrt(a*b + b^2)*
log(-(b*cos(d*x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x
 + c)^2 - a - b)) - 2*(a^2*b + a*b^2)*cos(d*x + c))/((a^2*b^3 + 2*a*b^4 + b
^5)*d*cos(d*x + c)^2 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d), 1/2*(((a*b
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 3*a*b - 2*b^2)*sqrt(-a*b - b^2)*arctan(sqr
t(-a*b - b^2)*cos(d*x + c)/(a + b)) - (a^2*b + a*b^2)*cos(d*x + c))/((a^2*b
^3 + 2*a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 93, normalized size = 1.12

$$\frac{(a+2b)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{2(ab+b^2)\sqrt{-ab-b^2}\,d}-\frac{a\cos(dx+c)}{2\left(b\cos(dx+c)^2-a-b\right)(ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a + 2*b)*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/((a*b + b^2)*sqrt(-a*
b - b^2)*d) - 1/2*a*cos(d*x + c)/((b*cos(d*x + c)^2 - a - b)*(a*b + b^2)*d)
```

Mupad [B]
time = 13.49, size = 71, normalized size = 0.86

$$\frac{a\cos(c+d\,x)}{2\,b\,d\,(a+b)\,\left(-b\cos(c+d\,x)^2+a+b\right)}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)\,(a+2\,b)}{2\,b^{3/2}\,d\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (a*cos(c + d*x))/(2*b*d*(a + b)*(a + b - b*cos(c + d*x)^2)) - (atanh((b^(1/
2)*cos(c + d*x))/(a + b)^(1/2))*(a + 2*b))/(2*b^(3/2)*d*(a + b)^(3/2))
```

3.97 $\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,(a+b)^{3/2}d}-\frac{\cos(c+dx)}{2(a+b)d\,(a+b-b\cos^2(c+dx))}$$

```
[Out] -1/2*cos(d*x+c)/(a+b)/d/(a+b-b*cos(d*x+c)^2)-1/2*arctanh(cos(d*x+c)*b^(1/2)
/(a+b)^(1/2))/(a+b)^(3/2)/d/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3265, 205, 214}

$$-\frac{\cos(c+dx)}{2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*(a + b)^(3/2)*d)
- Cos[c + d*x]/(2*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(a+b-bx^2)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\cos(c+dx)}{2(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2(a+b)d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,(a+b)^{3/2}d} - \frac{\cos(c+dx)}{2(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 149, normalized size = 2.01

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\sqrt{b}} - \frac{2\cos(c+dx)}{2a+b-b\cos(2(c+dx))}}{2(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]/(Sqrt[-a - b]*
Sqrt[b]) + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]/(Sqr
t[-a - b]*Sqrt[b]) - (2*Cos[c + d*x])/(2*a + b - b*Cos[2*(c + d*x)]))/(2*(a
 + b)*d)
```

Maple [A]
time = 0.23, size = 65, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}}{d}$
default	$\frac{-\frac{\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}}{d}$

risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}}{(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+b\right)}+\frac{i\ln\left(e^{2i(dx+c)}-\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{4\sqrt{-ab-b^2}\,(a+b)d}-\frac{i\ln\left(e^{2i(dx+c)}+\frac{2i(a+b}{\sqrt{-a}}\right.}{4\sqrt{-ab-b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*cos(d*x+c)/(a+b)/(a+b-b*cos(d*x+c)^2)-1/2/(a+b)/((a+b)*b)^(1/2)*a
rctanh(b*cos(d*x+c)/((a+b)*b)^(1/2)))
```

Maxima [A]
time = 0.53, size = 98, normalized size = 1.32

$$\frac{\frac{2\cos(dx+c)}{(ab+b^2)\cos(dx+c)^2-a^2-2\,ab-b^2}+\frac{\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,(a+b)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(d*x + c)/((a*b + b^2)*cos(d*x + c)^2 - a^2 - 2*a*b - b^2) + log(
(b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a + b)*b)))/(sqr
t((a + b)*b)*(a + b)))/d
```

Fricas [A]
time = 0.42, size = 282, normalized size = 3.81

$$\left[\frac{(b\cos(dx+c)^2-a-b)\sqrt{ab+b^2}\,\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+2\,(ab+b^2)\cos(dx+c)}{4\left((a^2b^2+2\,ab^3+b^4)d\cos(dx+c)^2-(a^3b+3\,a^2b^2+3\,ab^3+b^4)d\right)},\ \frac{(b\cos(dx+c)^2-a-b)\sqrt{-ab-b^2}\,\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)+(ab+b^2)\cos(dx+c)}{2\left((a^2b^2+2\,ab^3+b^4)d\cos(dx+c)^2-(a^3b+3\,a^2b^2+3\,ab^3+b^4)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*cos(d*x + c)^2 - a - b)*sqrt(a*b + b^2)*log(-(b*cos(d*x + c)^2 - 2
*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) + 2*(a*b
 + b^2)*cos(d*x + c))/((a^2*b^2 + 2*a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*d), 1/2*((b*cos(d*x + c)^2 - a - b)*sqrt(-a*b
- b^2)*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a + b)) + (a*b + b^2)*cos(d*x
+ c))/((a^2*b^2 + 2*a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 79, normalized size = 1.07

$$\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,(a+b)d}+\frac{\cos\left(dx+c\right)}{2\left(b\cos\left(dx+c\right)^2-a-b\right)(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*(a + b)*d) +
1/2*cos(d*x + c)/((b*cos(d*x + c)^2 - a - b)*(a + b)*d)
```

Mupad [B]
time = 0.11, size = 62, normalized size = 0.84

$$-\frac{\cos\left(c+d\,x\right)}{2\,d\,\left(a+b\right)\,\left(-b\cos\left(c+d\,x\right)^2+a+b\right)}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{2\,\sqrt{b}\,d\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] - cos(c + d*x)/(2*d*(a + b)*(a + b - b*cos(c + d*x)^2)) - atanh((b^(1/2)*co
s(c + d*x))/(a + b)^(1/2))/(2*b^(1/2)*d*(a + b)^(3/2))
```

3.98 $\int \frac{\csc(c+dx)}{(a+b\sin^2(c+dx))^2} \, dx$

Optimal. Leaf size=103

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d}+\frac{b\cos(c+dx)}{2a(a+b)d\,(a+b-b\cos^2(c+dx))}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+1/2*b*cos(d*x+c)/a/(a+b)/d/(a+b-b*cos(d*x+c)^2)+
1/2*(3*a+2*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b)^(3/
2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3265, 425, 536, 212, 214}

$$\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{b\cos(c+dx)}{2ad(a+b)\,(a-b\cos^2(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Co
s[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(3/2)*d) + (b*Cos[c + d*x])/(2*a*(
a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{b\cos(c+dx)}{2a(a+b)d\left(a+b-b\cos^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \cos(c+d\right.}{2a(a+b)d}$$

$$= \frac{b\cos(c+dx)}{2a(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{a^2d} + \frac{(b(3}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} + \frac{b}{2a(a+b)d\,(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.54, size = 194, normalized size = 1.88

$$\frac{\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + \frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + 2\left(\frac{ab\cos(c+dx)}{(a+b)(2a+b-b\cos(2(c+dx)))} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a
 - b]])/(-a - b)^(3/2) + (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*T
```

```
an[(c + d*x)/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + 2*((a*b*Cos[c + d*x])/((a
+ b)*(2*a + b - b*Cos[2*(c + d*x)])) - Log[Cos[(c + d*x)/2]] + Log[Sin[(c +
 d*x)/2]]))/(2*a^2*d)
```

Maple [A]
time = 0.41, size = 107, normalized size = 1.04

method	result
derivativedivides	$\frac{-\frac{\ln(1+\cos(dx+c))}{2a^2}+\frac{\ln(\cos(dx+c)-1)}{2a^2}+\frac{b\left(\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(3a+2b)\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{a^2}}{d}$
default	$\frac{-\frac{\ln(1+\cos(dx+c))}{2a^2}+\frac{\ln(\cos(dx+c)-1)}{2a^2}+\frac{b\left(\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(3a+2b)\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)\,b}}\right)}{2(a+b)\sqrt{(a+b)\,b}}\right)}{a^2}}{d}$
risch	$-\frac{b\left(e^{3i(dx+c)}+e^{i(dx+c)}\right)}{a(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+b\right)}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{3i\sqrt{-(a+b)\,b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2*ln(1+cos(d*x+c))+1/2/a^2*ln(cos(d*x+c)-1)+1/a^2*b*(1/2*a/(a+b
)*cos(d*x+c)/(a+b-b*cos(d*x+c)^2)+1/2*(3*a+2*b)/(a+b)/((a+b)*b)^(1/2)*arcta
nh(b*cos(d*x+c)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.50, size = 149, normalized size = 1.45

$$\frac{\frac{2\,b\cos(dx+c)}{a^3+2\,a^2b+ab^2-(a^2b+ab^2)\cos(dx+c)^2}-\frac{(3\,ab+2\,b^2)\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}}-\frac{2\log(\cos(dx+c)+1)}{a^2}+\frac{2\log(\cos(dx+c)-1)}{a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*cos(d*x + c)/(a^3 + 2*a^2*b + a*b^2 - (a^2*b + a*b^2)*cos(d*x + c)
^2) - (3*a*b + 2*b^2)*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c
) + sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)) - 2*log(cos(d*x + c)
+ 1)/a^2 + 2*log(cos(d*x + c) - 1)/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(94) = 188$.
time = 0.46, size = 455, normalized size = 4.42

$$\left[-\frac{2\,ab\cos(dx+c) - \left((3\,ab+2\,b^2)\cos(dx+c)^2 - 3\,a^2 - 5\,ab - 2\,b^2\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^2 + 2\,(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c) + a + b}{b\cos(dx+c)^2 - a - b}\right) + 2\left((ab+b^2)\cos(dx+c)^2 - a^2 - 2\,ab - b^2\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - 2\left((ab+b^2)\cos(dx+c)^2 - a^2 - 2\,ab - b^2\right)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{4\left((a^3b+a^2b^2)d\cos(dx+c)^2 - (a^4+2\,a^3b+a^2b^2)d\right)}, -\frac{ab\cos(dx+c) + \left((3\,ab+2\,b^2)\cos(dx+c)^2 - 3\,a^2 - 5\,ab - 2\,b^2\right)\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\cos(dx+c)\right) + \left((ab+b^2)\cos(dx+c)^2 - a^2 - 2\,ab - b^2\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - \left((ab+b^2)\cos(dx+c)^2 - a^2 - 2\,ab - b^2\right)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{2\left((a^3b+a^2b^2)d\cos(dx+c)^2 - (a^4+2\,a^3b+a^2b^2)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*cos(d*x + c) - ((3*a*b + 2*b^2)*cos(d*x + c)^2 - 3*a^2 - 5*a*b
 - 2*b^2)*sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b/(a + b))
*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) + 2*((a*b + b^2)*cos(d*x
 + c)^2 - a^2 - 2*a*b - b^2)*log(1/2*cos(d*x + c) + 1/2) - 2*((a*b + b^2)*c
os(d*x + c)^2 - a^2 - 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3*b +
a^2*b^2)*d*cos(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d), -1/2*(a*b*cos(d*x
 + c) + ((3*a*b + 2*b^2)*cos(d*x + c)^2 - 3*a^2 - 5*a*b - 2*b^2)*sqrt(-b/(a
 + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) + ((a*b + b^2)*cos(d*x + c)^2
- a^2 - 2*a*b - b^2)*log(1/2*cos(d*x + c) + 1/2) - ((a*b + b^2)*cos(d*x + c
)^2 - a^2 - 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3*b + a^2*b^2)*d
*cos(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sin(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(94) = 188$.
time = 0.49, size = 246, normalized size = 2.39

$$-\frac{\frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{(a^3+a^2b)\sqrt{-ab-b^2}} - \frac{2\left(ab-\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(a^3+a^2b)\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)} - \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b + 2*b^2)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos
(d*x + c) + sqrt(-a*b - b^2)))/((a^3 + a^2*b)*sqrt(-a*b - b^2)) - 2*(a*b -
```

```
a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b^2*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1))/((a^3 + a^2*b)*(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2)) - log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2)/
d
```

Mupad [B]
time = 14.63, size = 2039, normalized size = 19.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^2)^2),x)
```

```
[Out] (atan(((((cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 +
 a^2*b^2)) + ((b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a
^4*b + a^5 + a^3*b^2) - (cos(c + d*x)*(b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a
^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2
)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 +
a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^
5 + a^2*b^3 + 3*a^3*b^2)) + (((cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)
)/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*
b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) + (cos(c + d*x)*(b*(a + b)^3)^(1
/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*
a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))
/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)*(3*a + 2*b)
*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))/(((3*a*b^3)/2 + b^4)/(2*a^4
*b + a^5 + a^3*b^2) - (((cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(
2*a^3*b + a^4 + a^2*b^2)) + ((b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 +
4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (cos(c + d*x)*(b*(a + b)^3)^(1/2)*(3
*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b
+ a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3
*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(
3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((cos(c + d*x)*(20*a*b^4 + 8*b^5 +
 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((b*(a + b)^3)^(1/2)*((2*a^4*
b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) + (cos(c + d*x)*(b*(
a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*
b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*
(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)
*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(b*(a + b)^3)^(1/
2)*(3*a + 2*b)*1i)/(2*d*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (atan(((((
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*(2*a^4*b + a^5 + a^3*b^2)) - (cos(c
+ d*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b
 + a^4 + a^2*b^2)))*1i)/(2*a^2) + (cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*
```

```
b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b^3 + 4
*a^6*b^2)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (cos(c + d*x)*(32*a^4*b^5 + 80*a^
5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2
*a^2) - (cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4
 + a^2*b^2)))/a^2)/((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*(2*a^4*b + a^5
 + a^3*b^2)) - (cos(c + d*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7
*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (cos(c + d*x)*(20*a*b^4
 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + (((2*a^4*b^4 +
 6*a^5*b^3 + 4*a^6*b^2)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (cos(c + d*x)*(32*a
^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2
*b^2)))/(2*a^2) - (cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*
b + a^4 + a^2*b^2)))/a^2 - ((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2)))*
1i)/(a^2*d) + (b*cos(c + d*x))/(2*a*d*(a + b)*(a + b - b*cos(c + d*x)^2))
```

3.99 $\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=153

$$-\frac{(a-4b)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d}-\frac{b(a+2b)\cos(c+dx)}{2a^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)}-$$

```
[Out] -1/2*(a-4*b)*arctanh(cos(d*x+c))/a^3/d-1/2*b^(3/2)*(5*a+4*b)*arctanh(cos(d*
x+c)*b^(1/2)/(a+b)^(1/2))/a^3/(a+b)^(3/2)/d-1/2*b*(a+2*b)*cos(d*x+c)/a^2/(a
+b)/d/(a+b-b*cos(d*x+c)^2)-1/2*cot(d*x+c)*csc(d*x+c)/a/d/(a+b-b*cos(d*x+c)^
2)
```

Rubi [A]
time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3265, 425, 541, 536, 212, 214}

$$-\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}-\frac{(a-4b)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{b(a+2b)\cos(c+dx)}{2a^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\left(a-b\cos^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a - 4*b)*ArcTanh[Cos[c + d*x]])/(a^3*d) - (b^(3/2)*(5*a + 4*b)*ArcTa
nh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*d) - (b*(a + 2
*b)*Cos[c + d*x])/(2*a^2*(a + b)*d*(a + b - b*Cos[c + d*x]^2)) - (Cot[c + d
*x]*Csc[c + d*x])/(2*a*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad\,(a+b-b\cos^2(c+dx))} - \frac{\text{Subst}\left(\int \frac{a-b-3bx^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \cos(c+dx)\right)}{2ad}$$

$$= -\frac{b(a+2b)\cos(c+dx)}{2a^2(a+b)d\,(a+b-b\cos^2(c+dx))} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\,(a+b-b\cos^2(c+dx))} + \frac{\text{Subs}}{}$$

$$= -\frac{b(a+2b)\cos(c+dx)}{2a^2(a+b)d\,(a+b-b\cos^2(c+dx))} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\,(a+b-b\cos^2(c+dx))} - \frac{(a-}{}$$

$$= -\frac{(a-4b)\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d} - \frac{}{2a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.09, size = 390, normalized size = 2.55

$$\frac{(-2a-b+b\cos(2(c+dx)))\csc^3(c+dx)\left(\frac{8ab^2\cot(c+dx)}{a+b}+\frac{4b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)(2a+b-b\cos(2(c+dx)))\csc(c+dx)}{(-a-b)^{3/2}}+\frac{4b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)(2a+b-b\cos(2(c+dx)))\csc(c+dx)}{(-a-b)^{3/2}}+a(2a+b-b\cos(2(c+dx)))\csc^2\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)+4(a-4b)(2a+b-b\cos(2(c+dx)))\csc(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4(a-4b)(2a+b-b\cos(2(c+dx)))\csc(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-a(2a+b-b\cos(2(c+dx)))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)}{32a^3d\left(b+a\csc^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^3*((8*a*b^2*Cot[c + d*x])/(a
+ b) + (4*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])
/Sqrt[-a - b]]*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x])/(-a - b)^(3/2)
+ (4*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt
[-a - b]]*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x])/(-a - b)^(3/2) + a*(
2*a + b - b*Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^2*Csc[c + d*x] + 4*(a - 4*b)
*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Log[Cos[(c + d*x)/2]] - 4*(a -
 4*b)*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Log[Sin[(c + d*x)/2]] - a
*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Sec[(c + d*x)/2]^2))/(32*a^3*d
*(b + a*Csc[c + d*x]^2)^2)
```

Maple [A]
time = 0.48, size = 152, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{1}{4a^2(\cos(dx+c)-1)}+\frac{(a-4b)\ln(\cos(dx+c)-1)}{4a^3}+\frac{1}{4a^2(1+\cos(dx+c))}+\frac{(-a+4b)\ln(1+\cos(dx+c))}{4a^3}-\frac{b^2\left(\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\right.}{}}{d}$
default	$\frac{\frac{1}{4a^2(\cos(dx+c)-1)}+\frac{(a-4b)\ln(\cos(dx+c)-1)}{4a^3}+\frac{1}{4a^2(1+\cos(dx+c))}+\frac{(-a+4b)\ln(1+\cos(dx+c))}{4a^3}-\frac{b^2\left(\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\right.}{}}{d}$
risch	$\frac{ab\,e^{7i(dx+c)}+2b^2e^{7i(dx+c)}-4a^2e^{5i(dx+c)}-5b\,e^{5i(dx+c)}a-2b^2e^{5i(dx+c)}-4\,e^{3i(dx+c)}a^2-5ab\,e^{3i(dx+c)}-2b^2e^{3i(dx+c)}+b\,e^{i(}}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2(a+b)\left(b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+b\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2/(cos(d*x+c)-1)+1/4*(a-4*b)/a^3*ln(cos(d*x+c)-1)+1/4/a^2/(1+cos
(d*x+c))+1/4/a^3*(-a+4*b)*ln(1+cos(d*x+c))-b^2/a^3*(1/2*a/(a+b)*cos(d*x+c)/
(a+b-b*cos(d*x+c)^2)+1/2*(5*a+4*b)/(a+b)/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+
c)/((a+b)*b)^(1/2))))
```

Maxima [A]
time = 0.51, size = 223, normalized size = 1.46

$$\frac{\frac{(5\,ab^2+4\,b^3)\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}}+\frac{2\left((ab+2\,b^2)\cos(dx+c)^3-(a^2+2\,ab+2\,b^2)\cos(dx+c)\right)}{(a^3b+a^2b^2)\cos(dx+c)^4+a^4+2\,a^3b+a^2b^2-(a^4+3\,a^3b+2\,a^2b^2)\cos(dx+c)^2}-\frac{(a-4\,b)\log(\cos(dx+c)+1)}{a^3}+\frac{(a-4\,b)\log(\cos(dx+c)-1)}{a^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((5*a*b^2 + 4*b^3)*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x +
c) + sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)) + 2*((a*b + 2*b^2)*c
os(d*x + c)^3 - (a^2 + 2*a*b + 2*b^2)*cos(d*x + c))/((a^3*b + a^2*b^2)*cos(
d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 - (a^4 + 3*a^3*b + 2*a^2*b^2)*cos(d*x
+ c)^2) - (a - 4*b)*log(cos(d*x + c) + 1)/a^3 + (a - 4*b)*log(cos(d*x + c)
- 1)/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(143) = 286.
time = 0.52, size = 838, normalized size = 5.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*b + 2*a*b^2)*cos(d*x + c)^3 + ((5*a*b^2 + 4*b^3)*cos(d*x + c)^
4 + 5*a^2*b + 9*a*b^2 + 4*b^3 - (5*a^2*b + 14*a*b^2 + 8*b^3)*cos(d*x + c)^2
)*sqrt(b/(a + b))*log(-(b*cos(d*x + c)^2 - 2*(a + b)*sqrt(b/(a + b))*cos(d*
x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(a^3 + 2*a^2*b + 2*a*b^2)*c
os(d*x + c) - ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^2*b - 7
*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*log(1/2*c
os(d*x + c) + 1/2) + ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^
2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*lo
g(-1/2*cos(d*x + c) + 1/2))/((a^4*b + a^3*b^2)*d*cos(d*x + c)^4 - (a^5 + 3*
a^4*b + 2*a^3*b^2)*d*cos(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d), 1/4*(2*
(a^2*b + 2*a*b^2)*cos(d*x + c)^3 + 2*((5*a*b^2 + 4*b^3)*cos(d*x + c)^4 + 5*
a^2*b + 9*a*b^2 + 4*b^3 - (5*a^2*b + 14*a*b^2 + 8*b^3)*cos(d*x + c)^2)*sqrt
(-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) - 2*(a^3 + 2*a^2*b + 2*a
*b^2)*cos(d*x + c) - ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^
2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*lo
g(1/2*cos(d*x + c) + 1/2) + ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3
 - 2*a^2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c
)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^4*b + a^3*b^2)*d*cos(d*x + c)^4 - (a
^5 + 3*a^4*b + 2*a^3*b^2)*d*cos(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sin(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(143) = 286.
time = 0.46, size = 512, normalized size = 3.35

$$\frac{\frac{12\,(5\,ab^2+4\,b^3)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{(a^4+a^3b)\sqrt{-ab-b^2}}+\frac{3\,a^3+3\,a^2b-\frac{8\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{28\,ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{7\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{a^2b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{16\,ab^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{16\,b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{2\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{6\,a^2b(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{8\,ab^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{(a^4+a^3b)\left(\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{4\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)}+\frac{6\,(a-4\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}-\frac{3\,(\cos(dx+c)-1)}{a^2(\cos(dx+c)+1)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(12*(5*a*b^2 + 4*b^3)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2
)*cos(d*x + c) + sqrt(-a*b - b^2)))/((a^4 + a^3*b)*sqrt(-a*b - b^2)) + (3*a
^3 + 3*a^2*b - 8*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 28*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 7*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a^2*b*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 - 16*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 16*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a^3*(cos(d
*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 6*a^2*b*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 + 8*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((a^4 + a
^3*b)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + a*(co
s(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)) + 6*(a - 4*b)*log(abs(-cos(d*x + c
) + 1)/abs(cos(d*x + c) + 1))/a^3 - 3*(cos(d*x + c) - 1)/(a^2*(cos(d*x + c)
 + 1)))/d
```

Mupad [B]
time = 14.95, size = 2338, normalized size = 15.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^2)^2),x)
```

```
[Out] - ((cos(c + d*x)*(2*a*b + a^2 + 2*b^2))/(2*a^2*(a + b)) - (b*cos(c + d*x)^3
*(a + 2*b))/(2*a^2*(a + b)))/(d*(a + b + b*cos(c + d*x)^4 - cos(c + d*x)^2*
```

(a + 2*b))) - (atan((((a - 4*b)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (cos(c + d*x)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a - 4*b))/(4*a^3))*1i)/(4*a^3) + ((a - 4*b)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (cos(c + d*x)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a - 4*b))/(4*a^3))*1i)/(4*a^3))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^6*b^2) - ((a - 4*b)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (cos(c + d*x)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a - 4*b))/(4*a^3)))/(4*a^3) + ((a - 4*b)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (cos(c + d*x)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a - 4*b))/(4*a^3)))/(4*a^3)))*(a - 4*b)*1i)/(2*a^3*d) - (atan(((((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (cos(c + d*x)*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2) + (((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (cos(c + d*x)*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^6*b^2) - (((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (cos(c + d*x)*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2) + (((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (cos(c + d*x)*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b

```
^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6
+ a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^
6 + a^3*b^3 + 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4
 + b)*(b^3*(a + b)^3)^(1/2)*2i)/(d*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))
```

3.100 $\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=148

$$-\frac{(4a-b)x}{2b^3}+\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2b^3(a+b)^{3/2}d}-\frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)}-\frac{\sin^2(c+}{2bd\left(a+(a\right.}$$

```
[Out] -1/2*(4*a-b)*x/b^3+1/2*a^(3/2)*(4*a+5*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1
/2))/b^3/(a+b)^(3/2)/d-1/2*a*(2*a+b)*tan(d*x+c)/b^2/(a+b)/d/(a+(a+b)*tan(d*
x+c)^2)-1/2*sin(d*x+c)^2*tan(d*x+c)/b/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A]

time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3266, 481, 592, 536, 209, 211}

$$\frac{a^{3/2}(4a+5b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2b^3d(a+b)^{3/2}}-\frac{x(4a-b)}{2b^3}-\frac{a(2a+b)\tan(c+dx)}{2b^2d(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left((a+b)\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((4*a - b)*x)/b^3 + (a^(3/2)*(4*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c + d
*x])/Sqrt[a]])/(2*b^3*(a + b)^(3/2)*d) - (a*(2*a + b)*Tan[c + d*x])/(2*b^2*
(a + b)*d*(a + (a + b)*Tan[c + d*x]^2)) - (Sin[c + d*x]^2*Tan[c + d*x])/(2*
b*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2(a+(a+b)x^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\sin^2(c+dx)\tan(c+dx)}{2bd\,(a+(a+b)\tan^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(-a+b)x^2)}{(1+x^2)(a+(a+b)x^2)^2}\,dx, x, \tan(c+d\right.}{2bd}$$

$$= -\frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\,(a+(a+b)\tan^2(c+dx))} - \frac{\sin^2(c+dx)\tan(c+dx)}{2bd\,(a+(a+b)\tan^2(c+dx))} +$$

$$= -\frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\,(a+(a+b)\tan^2(c+dx))} - \frac{\sin^2(c+dx)\tan(c+dx)}{2bd\,(a+(a+b)\tan^2(c+dx))} -$$

$$= -\frac{(4a-b)x}{2b^3} + \frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^3(a+b)^{3/2}d} - \frac{a(2a+b)\tan}{2b^2(a+b)d\,(a+(a}$$

Mathematica [A]
time = 1.04, size = 106, normalized size = 0.72

$$\frac{-2(4a-b)(c+dx) + \frac{2a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + b\left(-1-\frac{2a^2}{(a+b)(2a+b-b\cos(2(c+dx)))}\right)\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-2*(4*a - b)*(c + d*x) + (2*a^(3/2)*(4*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c
+ d*x])/Sqrt[a]])/(a + b)^(3/2) + b*(-1 - (2*a^2)/((a + b)*(2*a + b - b*Cos
[2*(c + d*x)])))*Sin[2*(c + d*x)])/(4*b^3*d)
```

Maple [A]
time = 0.31, size = 134, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{a^2\left(-\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(4a+5b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^3}-\frac{\frac{b\tan(dx+c)}{2\left(\tan^2(dx+c)\right)+2}+\frac{(4a-b)\arctan(}{2}}{b^3}}{d}$

default	$\frac{\frac{a^2\left(-\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(4a+5b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^3}-\frac{\frac{b\tan(dx+c)}{2\left(\tan^2(dx+c)\right)+2}+\frac{(4a-b)\arctan(\tan\cdots}{2}}{b^3}}{d}$
risch	$-\frac{2ax}{b^3}+\frac{x}{2b^2}+\frac{i\mathrm{e}^{2i(dx+c)}}{8b^2d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8b^2d}+\frac{ia^2\left(2a\,\mathrm{e}^{2i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}-b\right)}{b^3(a+b)d\left(-b\,\mathrm{e}^{4i(dx+c)}+4a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}-b\right)}-\frac{\sqrt{-a(a\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/b^3*(-1/2*b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a)+1/2
*(4*a+5*b)/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))-
1/b^3*(1/2*b*tan(d*x+c)/(tan(d*x+c)^2+1)+1/2*(4*a-b)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.56, size = 181, normalized size = 1.22

$$\frac{\frac{\left(4\,a^3+5\,a^2b\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(ab^3+b^4)\sqrt{(a+b)a}}-\frac{(2\,a^2+2\,ab+b^2)\tan(dx+c)^3+(2\,a^2+ab)\tan(dx+c)}{(a^2b^2+2\,ab^3+b^4)\tan(dx+c)^4+a^2b^2+ab^3+(2\,a^2b^2+3\,ab^3+b^4)\tan(dx+c)^2}-\frac{(dx+c)(4\,a-b)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a^3 + 5*a^2*b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a*b^3
 + b^4)*sqrt((a + b)*a)) - ((2*a^2 + 2*a*b + b^2)*tan(d*x + c)^3 + (2*a^2 +
 a*b)*tan(d*x + c))/((a^2*b^2 + 2*a*b^3 + b^4)*tan(d*x + c)^4 + a^2*b^2 + a
*b^3 + (2*a^2*b^2 + 3*a*b^3 + b^4)*tan(d*x + c)^2) - (d*x + c)*(4*a - b)/b^
3)/d
```

Fricas [A]
time = 0.42, size = 623, normalized size = 4.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(4*a^2*b + 3*a*b^2 - b^3)*d*x*cos(d*x + c)^2 - 4*(4*a^3 + 7*a^2*b
+ 2*a*b^2 - b^3)*d*x + (4*a^3 + 9*a^2*b + 5*a*b^2 - (4*a^2*b + 5*a*b^2)*cos
(d*x + c)^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2
```

```
*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^
2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 +
 2*a*b + b^2)) + 4*((a*b^2 + b^3)*cos(d*x + c)^3 - (2*a^2*b + 2*a*b^2 + b^3
)*cos(d*x + c))*sin(d*x + c))/((a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^2*b^3 +
2*a*b^4 + b^5)*d), -1/4*(2*(4*a^2*b + 3*a*b^2 - b^3)*d*x*cos(d*x + c)^2 - 2
*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*d*x - (4*a^3 + 9*a^2*b + 5*a*b^2 - (4*a^
2*b + 5*a*b^2)*cos(d*x + c)^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*
x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))) + 2*((a*b^
2 + b^3)*cos(d*x + c)^3 - (2*a^2*b + 2*a*b^2 + b^3)*cos(d*x + c))*sin(d*x +
 c))/((a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^2*b^3 + 2*a*b^4 + b^5)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 223, normalized size = 1.51

$$\frac{\frac{\left(4\,a^3+5\,a^2 b\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(ab^3+b^4\right)\sqrt{a^2+ab}}-\frac{2\,a^2\tan(dx+c)^3+2\,ab\tan(dx+c)^3+b^2\tan(dx+c)^3+2\,a^2\tan(dx+c)+ab\tan(dx+c)}{\left(a\tan(dx+c)^4+b\tan(dx+c)^4+2\,a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)\left(ab^2+b^3\right)}-\frac{(dx+c)(4\,a-b)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((4*a^3 + 5*a^2*b)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arcta
n((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a*b^3 + b^4)*sqrt(a
^2 + a*b)) - (2*a^2*tan(d*x + c)^3 + 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c
)^3 + 2*a^2*tan(d*x + c) + a*b*tan(d*x + c))/((a*tan(d*x + c)^4 + b*tan(d*x
 + c)^4 + 2*a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a*b^2 + b^3)) - (d*x
+ c)*(4*a - b)/b^3)/d
```

Mupad [**B**]
time = 16.06, size = 2295, normalized size = 15.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (atan((((a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*((tan(c + d*x)*(96*a^5*b - 4*a
*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^
5)) + (((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) + (tan
(c + d*x)*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*(80*a*b^9 + 16*b^10 + 144*a^
2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*(a*b^4 + b^5)*(3*a*b^5 + b^6 + 3*a^2*
b^4 + a^3*b^3)))*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2))/(3*a*b^5 + b^6 + 3*a
^2*b^4 + a^3*b^3))*1i)/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3) + ((a + (5*b)/
4)*(-a^3*(a + b)^3)^(1/2)*((tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6
 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)) - (((2*a*b^9 +
8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) - (tan(c + d*x)*(a + (5*b
)/4)*(-a^3*(a + b)^3)^(1/2)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7
 + 32*a^4*b^6))/(2*(a*b^4 + b^5)*(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))*(a
 + (5*b)/4)*(-a^3*(a + b)^3)^(1/2))/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3))*
1i)/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3))/((16*a^5*b + 8*a^6 + (5*a^2*b^4)
/4 - (13*a^3*b^3)/2 + (3*a^4*b^2)/2)/(a*b^6 + b^7) + ((a + (5*b)/4)*(-a^3*(
a + b)^3)^(1/2)*((tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*
b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)) + (((2*a*b^9 + 8*a^2*b^8
+ 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) + (tan(c + d*x)*(a + (5*b)/4)*(-a^3
*(a + b)^3)^(1/2)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7 + 32*a^4*
b^6))/(2*(a*b^4 + b^5)*(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))*(a + (5*b)/4
)*(-a^3*(a + b)^3)^(1/2))/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))/(3*a*b^5
+ b^6 + 3*a^2*b^4 + a^3*b^3) - ((a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*((tan(
c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*
a^4*b^2))/(2*(a*b^4 + b^5)) - (((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b
^6)/(a*b^6 + b^7) - (tan(c + d*x)*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*(80*
a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*(a*b^4 + b^5)
*(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/
2))/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^
3*b^3)))*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*2i)/(d*(3*a*b^5 + b^6 + 3*a^2
*b^4 + a^3*b^3)) - (atan((((((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6
)/(a*b^6 + b^7) - (tan(c + d*x)*(a*1i - (b*1i)/4)*(80*a*b^9 + 16*b^10 + 144
*a^2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*b^3*(a*b^4 + b^5)))*(a*1i - (b*1i)
/4))/b^3 - (tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 +
20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)))*(a*1i - (b*1i)/4)*1i)/b^3 - ((
(((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) + (tan(c + d
*x)*(a*1i - (b*1i)/4)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7 + 32*
a^4*b^6))/(2*b^3*(a*b^4 + b^5)))*(a*1i - (b*1i)/4))/b^3 + (tan(c + d*x)*(96
*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2
*(a*b^4 + b^5)))*(a*1i - (b*1i)/4)*1i)/b^3)/((16*a^5*b + 8*a^6 + (5*a^2*b^4
)/4 - (13*a^3*b^3)/2 + (3*a^4*b^2)/2)/(a*b^6 + b^7) + (((((2*a*b^9 + 8*a^2*
b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) - (tan(c + d*x)*(a*1i - (b*1i)/
4)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*b^3*(a
*b^4 + b^5)))*(a*1i - (b*1i)/4))/b^3 - (tan(c + d*x)*(96*a^5*b - 4*a*b^5 +
32*a^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)))*(a
*1i - (b*1i)/4))/b^3 + (((((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(
```

```
a*b^6 + b^7) + (tan(c + d*x)*(a*1i - (b*1i)/4)*(80*a*b^9 + 16*b^10 + 144*a^
2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*b^3*(a*b^4 + b^5)))*(a*1i - (b*1i)/4)
)/b^3 + (tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 + 20*
a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)))*(a*1i - (b*1i)/4))/b^3))*(a*1i -
(b*1i)/4)*2i)/(b^3*d) - ((tan(c + d*x)^3*(2*a*b + 2*a^2 + b^2))/(2*b^2*(a +
 b)) + (a*tan(c + d*x)*(2*a + b))/(2*b^2*(a + b)))/(d*(a + tan(c + d*x)^4*(
a + b) + tan(c + d*x)^2*(2*a + b)))
```

3.101 $\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=93

$$\frac{x}{b^2}-\frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2b^2(a+b)^{3/2}d}+\frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)}$$

```
[Out] x/b^2-1/2*(2*a+3*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1/2)/b^2/(a+b
)^(3/2)/d+1/2*a*tan(d*x+c)/b/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3266, 481, 536, 209, 211}

$$-\frac{\sqrt{a}\,(2a+3b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2b^2d(a+b)^{3/2}}+\frac{a\tan(c+dx)}{2bd(a+b)\left((a+b)\tan^2(c+dx)+a\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] x/b^2 - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2
*b^2*(a + b)^(3/2)*d) + (a*Tan[c + d*x])/(2*b*(a + b)*d*(a + (a + b)*Tan[c
+ d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+(a+b)x^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(-a-2b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \tan(c\right.}{2b(a+b)d}$$

$$= \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{b^2d} - $$

$$= \frac{x}{b^2} - \frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2b^2(a+b)^{3/2}d} + \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(\right.}$$

Mathematica [A]
time = 0.59, size = 93, normalized size = 1.00

$$\frac{2(c+dx) - \frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + \frac{ab\sin(2(c+dx))}{(a+b)(2a+b-b\cos(2(c+dx)))}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[
a]])/(a + b)^(3/2) + (a*b*Sin[2*(c + d*x)])/((a + b)*(2*a + b - b*Cos[2*(c
+ d*x)])))/(2*b^2*d)
```

Maple [A]
time = 0.35, size = 101, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{b^2}-\frac{a\left(-\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(2a+3b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}}{d}$
default	$\frac{\frac{\arctan(\tan(dx+c))}{b^2}-\frac{a\left(-\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(2a+3b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}}{d}$
risch	$\frac{x}{b^2}-\frac{ia\left(2a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-b\right)}{b^2(a+b)d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)}+\frac{\sqrt{-a(a+b)}\,a\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{-a(a+b}}{b}\right.}{2(a+b)^2 d\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*arctan(tan(d*x+c))-a/b^2*(-1/2*b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^
2+b*tan(d*x+c)^2+a)+1/2*(2*a+3*b)/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*
x+c)/(a*(a+b))^(1/2))))
```

Maxima [A]
time = 0.53, size = 109, normalized size = 1.17

$$\frac{\frac{a\tan(dx+c)}{a^2b+ab^2+(a^2b+2\,ab^2+b^3)\tan(dx+c)^2}-\frac{(2\,a^2+3\,ab)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(ab^2+b^3)\sqrt{(a+b)a}}+\frac{2\,(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*tan(d*x + c)/(a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*tan(d*x + c)^2
) - (2*a^2 + 3*a*b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a*b^2 +
b^3)*sqrt((a + b)*a)) + 2*(d*x + c)/b^2)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(81) = 162$.
time = 0.46, size = 492, normalized size = 5.29

$$\left[\frac{8\,(ab+b^2)dx\cos(dx+c)^2-4\,ab\cos(dx+c)\sin(dx+c)-8\,(a^2+2\,ab+b^2)dx+\left((2\,ab+3\,b^2)\cos(dx+c)^2-2\,a^2-5\,ab-3\,b^2\right)\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+2\,ab+b^2)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)}{8\left((ab^3+b^4)d\cos(dx+c)^2-(a^2b^2+2\,ab^3+b^4)d\right)}, \frac{4\,(ab+b^2)dx\cos(dx+c)^2-2\,ab\cos(dx+c)\sin(dx+c)-4\,(a^2+2\,ab+b^2)dx+\left((2\,ab+3\,b^2)\cos(dx+c)^2-2\,a^2-5\,ab-3\,b^2\right)\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)}{4\left((ab^3+b^4)d\cos(dx+c)^2-(a^2b^2+2\,ab^3+b^4)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a*b + b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)*sin(d*x + c) -
8*(a^2 + 2*a*b + b^2)*d*x + ((2*a*b + 3*b^2)*cos(d*x + c)^2 - 2*a^2 - 5*a*b
 - 3*b^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4
*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c)^
3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2 +
 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*
a*b + b^2)))/((a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^2*b^2 + 2*a*b^3 + b^4)*d)
, 1/4*(4*(a*b + b^2)*d*x*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) -
 4*(a^2 + 2*a*b + b^2)*d*x + ((2*a*b + 3*b^2)*cos(d*x + c)^2 - 2*a^2 - 5*a*
b - 3*b^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sq
rt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))))/((a*b^3 + b^4)*d*cos(d*x + c)
^2 - (a^2*b^2 + 2*a*b^3 + b^4)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 140, normalized size = 1.51

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(2\,a^2+3\,ab)}{(ab^2+b^3)\sqrt{a^2+ab}}-\frac{a\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(ab+b^2)}-\frac{2\,(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(2*a^2 + 3*a*b)/((a*b^2 + b^3)*sqrt(a^
```

```
2 + a*b)) - a*tan(d*x + c)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a*b
+ b^2)) - 2*(d*x + c)/b^2)/d
```

Mupad [B]
time = 15.23, size = 1959, normalized size = 21.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (a*tan(c + d*x))/(2*d*(a + tan(c + d*x)^2*(a + b))*(a*b + b^2)) - atan(((((
(2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)*1i)/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(80
*a*b^7 + 16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*b^2*(a*b^2 +
b^3)))/(2*b^2) + (tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^
2*b^2))/(4*(a*b^2 + b^3)))/b^2 - ((((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)*1i)/(
2*(a*b^3 + b^4)) + (tan(c + d*x)*(80*a*b^7 + 16*b^8 + 144*a^2*b^6 + 112*a^3
*b^5 + 32*a^4*b^4))/(8*b^2*(a*b^2 + b^3)))/(2*b^2) - (tan(c + d*x)*(16*a*b^
3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^2*b^2))/(4*(a*b^2 + b^3)))/b^2)/((3*a*b
^2 + (7*a^2*b)/2 + a^3)/(a*b^3 + b^4) + (((((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^
4)*1i)/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(80*a*b^7 + 16*b^8 + 144*a^2*b^6 +
 112*a^3*b^5 + 32*a^4*b^4))/(8*b^2*(a*b^2 + b^3)))*1i)/(2*b^2) + (tan(c + d
*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^2*b^2)*1i)/(4*(a*b^2 + b^3)
))/b^2 + (((((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)*1i)/(2*(a*b^3 + b^4)) + (tan
(c + d*x)*(80*a*b^7 + 16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*
b^2*(a*b^2 + b^3)))*1i)/(2*b^2) - (tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^
4 + 4*b^4 + 33*a^2*b^2)*1i)/(4*(a*b^2 + b^3)))/b^2))/(b^2*d) - (atan((((-a*
(a + b)^3)^(1/2)*((tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a
^2*b^2))/(2*(a*b^2 + b^3)) - (((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)/(a*b^3 + b
^4) - (tan(c + d*x)*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*(80*a*b^7 + 16*b^8 + 1
44*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*(a*b^2 + b^3)*(3*a*b^4 + b^5 + 3
*a^2*b^3 + a^3*b^2)))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 +
 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b)*1i)/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3
*b^2)) + ((-a*(a + b)^3)^(1/2)*((tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4
+ 4*b^4 + 33*a^2*b^2))/(2*(a*b^2 + b^3)) + (((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b
^4)/(a*b^3 + b^4) + (tan(c + d*x)*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*(80*a*b^
7 + 16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*(a*b^2 + b^3)*(3*a
*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3
*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b)*1i)/(4*(3*a*b^4 + b^5 + 3
*a^2*b^3 + a^3*b^2)))/((3*a*b^2 + (7*a^2*b)/2 + a^3)/(a*b^3 + b^4) - ((-a*(
a + b)^3)^(1/2)*((tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^
2*b^2))/(2*(a*b^2 + b^3)) - (((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)/(a*b^3 + b^
4) - (tan(c + d*x)*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*(80*a*b^7 + 16*b^8 + 14
4*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*(a*b^2 + b^3)*(3*a*b^4 + b^5 + 3*
```

```
a^2*b^3 + a^3*b^2)))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 +
3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2
)) + ((-a*(a + b)^3)^(1/2)*((tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*
b^4 + 33*a^2*b^2))/(2*(a*b^2 + b^3)) + (((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)/
(a*b^3 + b^4) + (tan(c + d*x)*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*(80*a*b^7 +
16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*(a*b^2 + b^3)*(3*a*b^4
 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3*a*b
^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^
3 + a^3*b^2))))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*1i)/(2*d*(3*a*b^4 + b^5 +
3*a^2*b^3 + a^3*b^2))
```

3.102 $\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

Optimal. Leaf size=78

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a+b)^{3/2}d}-\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\,(a+b\sin^2(c+dx))}$$

```
[Out] -1/2*cos(d*x+c)*sin(d*x+c)/(a+b)/d/(a+b*sin(d*x+c)^2)+1/2*arctan((a+b)^(1/2
)*tan(d*x+c)/a^(1/2))/(a+b)^(3/2)/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3252, 12, 3260, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,d(a+b)^{3/2}}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\,(a+b\sin^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a + b)^(3/2)*d) - (C
os[c + d*x]*Sin[c + d*x])/(2*(a + b)*d*(a + b*Sin[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx &= -\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\,(a+b\sin^2(c+dx))}+\frac{\int \frac{a}{a+b\sin^2(c+dx)}\,dx}{2a(a+b)}\\ &= -\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\,(a+b\sin^2(c+dx))}+\frac{\int \frac{1}{a+b\sin^2(c+dx)}\,dx}{2(a+b)}\\ &= -\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\,(a+b\sin^2(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{2(a+b)d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a+b)^{3/2}d}-\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\,(a+b\sin^2(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 74, normalized size = 0.95

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}-\frac{\sin(2(c+dx))}{(a+b)(2a+b-b\cos(2(c+dx)))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a + b)^(3/2)) - Sin[2
*(c + d*x)]/((a + b)*(2*a + b - b*Cos[2*(c + d*x)])))/(2*d)
```

Maple [A]
time = 0.28, size = 75, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{d}$

default	$\frac{-\frac{\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{d}$
risch	$\frac{i\left(2a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-b\right)}{b(a+b)d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia^2+2iab-2a\sqrt{-a^2-ab}-b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right.}{4\sqrt{-a^2-ab}\,(a+b)d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^2/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a)+1/2/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))

Maxima [A]
time = 0.51, size = 74, normalized size = 0.95

$$-\frac{\frac{\tan(dx+c)}{(a^2+2ab+b^2)\tan(dx+c)^2+a^2+ab}-\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a+b)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/2*(tan(d*x + c)/((a^2 + 2*a*b + b^2)*tan(d*x + c)^2 + a^2 + a*b) - arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b)))/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(66) = 132$.
time = 0.42, size = 419, normalized size = 5.37

$$\left[\frac{4\left(a^2+ab\right)\cos(dx+c)\sin(dx+c)-\left(b\cos(dx+c)^2-a-b\right)\sqrt{-a^2-ab}\,\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2(4a^2+5ab+b^2)\cos(dx+c)^2+4\left((2a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\,\sin(dx+c)+a^2+2ab+b^2}{b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right)}{8\left((a^3b+2a^2b^2+ab^3)d\cos(dx+c)^2-(a^4+3a^3b+3a^2b^2+ab^3)d\right)},\ \frac{2\left(a^2+ab\right)\cos(dx+c)\sin(dx+c)-\left(b\cos(dx+c)^2-a-b\right)\sqrt{a^2+ab}\,\arctan\left(\frac{(2a+b)\cos(dx+c)^2-a-b}{2\sqrt{a^2+ab}\,\cos(dx+c)\sin(dx+c)}\right)}{4\left((a^3b+2a^2b^2+ab^3)d\cos(dx+c)^2-(a^4+3a^3b+3a^2b^2+ab^3)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/8*(4*(a^2 + a*b)*cos(d*x + c)*sin(d*x + c) - (b*cos(d*x + c)^2 - a - b)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)))/((a^3*b + 2*a^2*b^2 + a*b^3)*d*cos(d*x + c)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d), 1/4*(2*(a^2 + a*b)*cos(d*x + c)*sin(d*x + c) - (b*cos(d*x + c)^2 - a - b)*sqrt(a^2

```
+ a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d
*x + c)*sin(d*x + c))))/((a^3*b + 2*a^2*b^2 + a*b^3)*d*cos(d*x + c)^2 - (a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2)**2,x)
[Out] Timed out
```

Giac [A]
time = 0.47, size = 109, normalized size = 1.40

$$\frac{\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}\,(a+b)}-\frac{\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(a+b)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*b)*(a + b)) - tan(d*x + c
)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a + b)))/d
```

Mupad [B]
time = 13.43, size = 72, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)^2}{4\,\sqrt{a}\,(a+b)^{3/2}}\right)}{2\,\sqrt{a}\,d\,(a+b)^{3/2}}-\frac{\tan(c+d\,x)}{2\,d\,\left((a+b)\,\tan\left(c+d\,x\right)^2+a\right)\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^2)^2,x)
[Out] atan((tan(c + d*x)*(2*a + 2*b)^2)/(4*a^(1/2)*(a + b)^(3/2)))/(2*a^(1/2)*d*(
a + b)^(3/2)) - tan(c + d*x)/(2*d*(a + tan(c + d*x)^2*(a + b))*(a + b))
```

3.103 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=87

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d}+\frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)}$$

```
[Out] 1/2*(2*a+b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(3/2)/d+1/
2*b*cos(d*x+c)*sin(d*x+c)/a/(a+b)/d/(a+b*sin(d*x+c)^2)
```

Rubi [A]

time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3263, 12, 3260, 211}

$$\frac{(2a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(
3/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a + b)*d*(a + b*Sin[c + d*x]^
2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
```

```
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx &= \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)}-\frac{\int \frac{-2a-b}{a+b\sin^2(c+dx)}\,dx}{2a(a+b)}\\ &= \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)}+\frac{(2a+b)\int \frac{1}{a+b\sin^2(c+dx)}\,dx}{2a(a+b)}\\ &= \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)}+\frac{(2a+b)\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(c+dx\right.}{2a(a+b)d}\\ &= \frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d}+\frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 84, normalized size = 0.97

$$\frac{\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}}+\frac{\sqrt{a}\,b\sin(2(c+dx))}{(a+b)(2a+b-b\cos(2(c+dx)))}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-2),x]
[Out] (((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(3/2) + (Sq
rt[a]*b*Sin[2*(c + d*x)])/((a + b)*(2*a + b - b*Cos[2*(c + d*x)])))/(2*a^(3
/2)*d)
```

Maple [A]
time = 0.29, size = 87, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{b\tan(dx+c)}{2a(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(2a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{d}$

default	$\frac{\frac{b\tan(dx+c)}{2a(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(2a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{d}$
risch	$-\frac{i(2a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-b)}{a(a+b)d(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b)}-\frac{\ln\left(e^{2i(dx+c)}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-}}{b\sqrt{-a^2-ab}}\right.}{2\sqrt{-a^2-ab}\,(a+b)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b/a/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a)+1/2*(2*a+b)
/a/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.52, size = 89, normalized size = 1.02

$$\frac{\frac{b\tan(dx+c)}{a^3+a^2b+(a^3+2\,a^2b+ab^2)\tan(dx+c)^2}+\frac{(2\,a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+ab)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*tan(d*x + c)/(a^3 + a^2*b + (a^3 + 2*a^2*b + a*b^2)*tan(d*x + c)^2)
+ (2*a + b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(
a^2 + a*b)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(75) = 150.
time = 0.43, size = 463, normalized size = 5.32

$$\left[-\frac{4\,(a^2b+ab^2)\cos(dx+c)\sin(dx+c)+\left((2\,ab+b^2)\cos(dx+c)^2-2\,a^2-3\,ab-b^2\right)\sqrt{-a^2-ab}\,\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\left((2\,a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\,\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)}{8\left((a^4b+2\,a^3b^2+a^2b^3)d\cos(dx+c)^2-(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)d\right)},\ -\frac{2\,(a^2b+ab^2)\cos(dx+c)\sin(dx+c)+\left((2\,ab+b^2)\cos(dx+c)^2-2\,a^2-3\,ab-b^2\right)\sqrt{a^2+ab}\,\arctan\left(\frac{(2\,a+b)\cos(dx+c)^2-a-b}{2\sqrt{a^2+ab}\,\cos(dx+c)\sin(dx+c)}\right)}{4\left((a^4b+2\,a^3b^2+a^2b^3)d\cos(dx+c)^2-(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b + a*b^2)*cos(d*x + c)*sin(d*x + c) + ((2*a*b + b^2)*cos(d*x
 + c)^2 - 2*a^2 - 3*a*b - b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*
cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(
d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*
a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b
 + b^2)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cos(d*x + c)^2 - (a^5 + 3*a^4*b
```

```
+ 3*a^3*b^2 + a^2*b^3)*d), -1/4*(2*(a^2*b + a*b^2)*cos(d*x + c)*sin(d*x + c
) + ((2*a*b + b^2)*cos(d*x + c)^2 - 2*a^2 - 3*a*b - b^2)*sqrt(a^2 + a*b)*ar
ctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*s
in(d*x + c))))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cos(d*x + c)^2 - (a^5 + 3*a
^4*b + 3*a^3*b^2 + a^2*b^3)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.40, size = 113, normalized size = 1.30

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(2a+b)}{(a^2+ab)^{\frac{3}{2}}}+\frac{b\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(a^2+ab)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))*(2*a + b)/(a^2 + a*b)^(3/2) + b*tan(d*x
 + c)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a^2 + a*b)))/d
```

Mupad [**B**]
time = 13.43, size = 79, normalized size = 0.91

$$\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)\,(2\,a+b)}{2\,a^{3/2}\,d\,{(a+b)}^{3/2}}+\frac{b\tan(c+d\,x)}{2\,a\,d\,\left((a+b)\,{\tan\left(c+d\,x\right)}^2+a\right)\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (atan((tan(c + d*x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2)))*(2*a + b))/(2*a
^(3/2)*d*(a + b)^(3/2)) + (b*tan(c + d*x))/(2*a*d*(a + tan(c + d*x)^2*(a +
b))*(a + b))
```

3.104 $\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=127

$$-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^{3/2}d}-\frac{\cot(c+dx)}{ad\left(a+b\sin^2(c+dx)\right)}-\frac{(2ab+3b^2)\cos(c+dx)\sin(c+dx)}{2a^2(a+b)d\left(a+b\sin^2(c+dx)\right)}$$

```
[Out] -1/2*b*(4*a+3*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^(3/2)
/d-cot(d*x+c)/a/d/(a+b*sin(d*x+c)^2)-1/2*(2*a*b+3*b^2)*cos(d*x+c)*sin(d*x+c
)/a^2/(a+b)/d/(a+b*sin(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3266, 473, 393, 211}

$$-\frac{b(4a+3b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^{3/2}}-\frac{(2a^2+4ab+3b^2)\tan(c+dx)}{2a^2d(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{\cot(c+dx)}{ad\left((a+b)\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(b*(4*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*(a
 + b)^(3/2)*d) - Cot[c + d*x]/(a*d*(a + (a + b)*Tan[c + d*x]^2)) - ((2*a^2
+ 4*a*b + 3*b^2)*Tan[c + d*x])/(2*a^2*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)}{ad\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-a-3b+ax^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= -\frac{\cot(c+dx)}{ad\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\left(2a^2+4ab+3b^2\right)\tan(c+dx)}{2a^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{(b$$

$$= -\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^{3/2}d} - \frac{\cot(c+dx)}{ad\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{}{2a}$$

Mathematica [A]
time = 0.81, size = 155, normalized size = 1.22

$$-\frac{(2a+b-b\cos(2(c+dx)))\left(b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)(2a+b-b\cos(2(c+dx)))+\sqrt{a}\,\sqrt{a+b}\,\left(4a^2+6ab+3b^2-b(2a+3b)\cos(2(c+dx))\right)\cot(c+dx)\right)\csc^4(c+dx)}{8a^{5/2}(a+b)^{3/2}d\left(b+a\csc^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -1/8*((2*a + b - b*Cos[2*(c + d*x)])*(b*(4*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan
[c + d*x])/Sqrt[a]]*(2*a + b - b*Cos[2*(c + d*x)]) + Sqrt[a]*Sqrt[a + b]*(4
*a^2 + 6*a*b + 3*b^2 - b*(2*a + 3*b)*Cos[2*(c + d*x)])*Cot[c + d*x])*Csc[c
+ d*x]^4)/(a^(5/2)*(a + b)^(3/2)*d*(b + a*Csc[c + d*x]^2)^2)
```

Maple [A]
time = 0.40, size = 103, normalized size = 0.81

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(4a+3b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}-\frac{1}{a^2\tan(dx+c)}}{d}$
default	$\frac{-\frac{b\left(\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(4a+3b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}-\frac{1}{a^2\tan(dx+c)}}{d}$
risch	$-\frac{i\left(-4ab\,e^{4i(dx+c)}-3b^2e^{4i(dx+c)}+8a^2e^{2i(dx+c)}+14b\,e^{2i(dx+c)}a+6b^2e^{2i(dx+c)}-2ab-3b^2\right)}{a^2(a+b)d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)\left(e^{2i(dx+c)}-1\right)}+\frac{\ln\left(e^{2i(dx+c)}-\frac{2ia^2+2iab+}{}\right.}{\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*b*(1/2*b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a)+1/2
*(4*a+3*b)/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))-
1/a^2/tan(d*x+c))
```

Maxima [A]
time = 0.55, size = 133, normalized size = 1.05

$$-\frac{\frac{\left(4\,ab+3\,b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^3+a^2b\right)\sqrt{(a+b)a}}+\frac{\left(2\,a^2+4\,ab+3\,b^2\right)\tan(dx+c)^2+2\,a^2+2\,ab}{\left(a^4+2\,a^3b+a^2b^2\right)\tan(dx+c)^3+\left(a^4+a^3b\right)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*b + 3*b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^3 +
a^2*b)*sqrt((a + b)*a)) + ((2*a^2 + 4*a*b + 3*b^2)*tan(d*x + c)^2 + 2*a^2 +
 2*a*b)/((a^4 + 2*a^3*b + a^2*b^2)*tan(d*x + c)^3 + (a^4 + a^3*b)*tan(d*x +
 c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(115) = 230$.
time = 0.47, size = 588, normalized size = 4.63

$$\left[-\frac{4\left(2\,a^3b+5\,a^2b^2+3\,ab^3\right)\cos(dx+c)^3-\left(4\,a^2b+7\,ab^2+3\,b^3-\left(4\,ab^2+3\,b^3\right)\cos(dx+c)^2\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2-4\left((2\,a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\,\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)\sin(dx+c)-4\left(2\,a^4+6\,a^3b+7\,a^2b^2+3\,ab^3\right)\cos(dx+c)}{8\left(\left(a^5b+2\,a^4b^2+a^3b^3\right)d\cos(dx+c)^2-\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)d\right)\sin(dx+c)},\ -\frac{2\left(2\,a^3b+5\,a^2b^2+3\,ab^3\right)\cos(dx+c)^3+\left(4\,a^2b+7\,ab^2+3\,b^3-\left(4\,ab^2+3\,b^3\right)\cos(dx+c)^2\right)\sqrt{a^2+ab}\,\arctan\left(\frac{(2\,a+b)\cos(dx+c)^2-a-b}{2\,\sqrt{a^2+ab}\,\cos(dx+c)\sin(dx+c)}\right)\sin(dx+c)-2\left(2\,a^4+6\,a^3b+7\,a^2b^2+3\,ab^3\right)\cos(dx+c)}{4\left(\left(a^5b+2\,a^4b^2+a^3b^3\right)d\cos(dx+c)^2-\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^3 - (4*a^2*b + 7*a*b^
2 + 3*b^3 - (4*a*b^2 + 3*b^3)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2
+ 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*
((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x
+ c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^
2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 4*(2*a^4 + 6*a^3*b + 7*a^2*b^2 + 3*a
*b^3)*cos(d*x + c))/(((a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cos(d*x + c)^2 - (a^6
 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d)*sin(d*x + c)), -1/4*(2*(2*a^3*b + 5*a^
2*b^2 + 3*a*b^3)*cos(d*x + c)^3 + (4*a^2*b + 7*a*b^2 + 3*b^3 - (4*a*b^2 + 3
*b^3)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2
- a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 2*(2*a
^4 + 6*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cos(d*x + c))/(((a^5*b + 2*a^4*b^2 + a^
3*b^3)*d*cos(d*x + c)^2 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d)*sin(d*x
+ c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.44, size = 179, normalized size = 1.41

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(4ab+3b^2)}{(a^3+a^2b)\sqrt{a^2+ab}}+\frac{2a^2\tan(dx+c)^2+4ab\tan(dx+c)^2+3b^2\tan(dx+c)^2+2a^2+2ab}{\left(a\tan(dx+c)^3+b\tan(dx+c)^3+a\tan(dx+c)\right)(a^3+a^2b)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(4*a*b + 3*b^2)/((a^3 + a^2*b)*sqrt(a^
2 + a*b)) + (2*a^2*tan(d*x + c)^2 + 4*a*b*tan(d*x + c)^2 + 3*b^2*tan(d*x +
c)^2 + 2*a^2 + 2*a*b)/((a*tan(d*x + c)^3 + b*tan(d*x + c)^3 + a*tan(d*x + c
))*(a^3 + a^2*b)))/d
```

Mupad [B]
time = 13.60, size = 132, normalized size = 1.04

$$-\frac{\frac{1}{a}+\frac{\tan(c+d\,x)^2\,\left(2\,a^2+4\,a\,b+3\,b^2\right)}{2\,a^2\,(a+b)}}{d\,\left((a+b)\,\tan\left(c+d\,x\right)^3+a\tan\left(c+d\,x\right)\right)}-\frac{b\,\mathrm{atan}\left(\frac{b\tan(c+d\,x)\,\left(a^3+b\,a^2\right)\,(4\,a+3\,b)}{a^{5/2}\,\sqrt{a+b}\,\left(3\,b^2+4\,a\,b\right)}\right)\,\left(4\,a+3\,b\right)}{2\,a^{5/2}\,d\,{\left(a+b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^2)^2),x)
```

```
[Out] - (1/a + (tan(c + d*x)^2*(4*a*b + 2*a^2 + 3*b^2))/(2*a^2*(a + b)))/(d*(a*ta
n(c + d*x) + tan(c + d*x)^3*(a + b))) - (b*atan((b*tan(c + d*x)*(a^2*b + a^
3)*(4*a + 3*b))/(a^(5/2)*(a + b)^(1/2)*(4*a*b + 3*b^2))*(4*a + 3*b))/(2*a^
(5/2)*d*(a + b)^(3/2))
```

3.105 $\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^{3/2}d}-\frac{(2a^2-ab-5b^2)\cot(c+dx)}{2a^3(a+b)d}-\frac{(2a+5b)\cot^3(c+dx)}{6a^2(a+b)d}+\frac{b\csc^3(}{2a(a+b)d\,(}$$

```
[Out] 1/2*b^2*(6*a+5*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/(a+b)^(3/2
)/d-1/2*(2*a^2-a*b-5*b^2)*cot(d*x+c)/a^3/(a+b)/d-1/6*(2*a+5*b)*cot(d*x+c)^3
/a^2/(a+b)/d+1/2*b*csc(d*x+c)^3*sec(d*x+c)/a/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A]

time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3266, 479, 584, 211}

$$\frac{b^2(6a+5b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^{3/2}}-\frac{(2a+5b)\cot^3(c+dx)}{6a^2d(a+b)}-\frac{(2a^2-ab-5b^2)\cot(c+dx)}{2a^3d(a+b)}+\frac{b\csc^3(c+dx)\sec(c+dx)}{2ad(a+b)\left((a+b)\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(a
+ b)^(3/2)*d) - ((2*a^2 - a*b - 5*b^2)*Cot[c + d*x])/(2*a^3*(a + b)*d) - ((
2*a + 5*b)*Cot[c + d*x]^3)/(6*a^2*(a + b)*d) + (b*Csc[c + d*x]^3*Sec[c + d*
x])/(2*a*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^4\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{b\csc^3(c+dx)\sec(c+dx)}{2a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(-2a-5b+(-2a-b)x^2\right)}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \right.}{2a(a+b)d}$$

$$= \frac{b\csc^3(c+dx)\sec(c+dx)}{2a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(\frac{-2a-5b}{ax^4} + \frac{-2a^2+ab+5b^2}{a^2x^2} + \frac{(-6}{a^2(a-}\right.\right.}{2a(a+b)}$$

$$= -\frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\cot^3(c+dx)}{6a^2(a+b)d} + \frac{b\csc^3(c+dx)}{2a(a+b)d\left(a+(a\right.}$$

$$= \frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^{3/2}d} - \frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3(a+b)d} - \frac{(2a+}{}$$

Mathematica [A]
time = 0.88, size = 202, normalized size = 1.25

$$\frac{(-2a-b+b\cos(2(c+dx)))\csc^4(c+dx)\left(\frac{3b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)(-2a-b+b\cos(2(c+dx)))}{(a+b)^{3/2}} + 4\sqrt{a}\,(a-3b)(2a+b-b\cos(2(c+dx)))\cot(c+dx) + 2a^{3/2}(2a+b-b\cos(2(c+dx)))\cot(c+dx)\csc^2(c+dx) - \frac{3\sqrt{a}\,b^3\sin(2(c+dx))}{a+b}\right)}{24a^{7/2}d\left(b+a\csc^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^4*((3*b^2*(6*a + 5*b)*ArcTan[
(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]*(-2*a - b + b*Cos[2*(c + d*x)]))/(a + b
```

)^(3/2) + 4*Sqrt[a]*(a - 3*b)*(2*a + b - b*Cos[2*(c + d*x)])*Cot[c + d*x] + 2*a^(3/2)*(2*a + b - b*Cos[2*(c + d*x)])*Cot[c + d*x]*Csc[c + d*x]^2 - (3*Sqrt[a]*b^3*Sin[2*(c + d*x)])/(a + b)))/(24*a^(7/2)*d*(b + a*Csc[c + d*x]^2)^2)

Maple [A]
time = 0.43, size = 122, normalized size = 0.75

method	result
derivativedivides	$\frac{-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a-2b}{a^3\tan(dx+c)}+\frac{b^2\left(\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(6a+5b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}}{d}$
default	$\frac{-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a-2b}{a^3\tan(dx+c)}+\frac{b^2\left(\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)}+\frac{(6a+5b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}}{d}$
risch	$\frac{i(-18a\,b^2e^{8i(dx+c)}-15b^3e^{8i(dx+c)}+36b\,e^{6i(dx+c)}a^2+102a\,b^2e^{6i(dx+c)}+60b^3e^{6i(dx+c)}+48a^3e^{4i(dx+c)}-20b\,e^{4i(dx+c)}a^2}{3d\,a^3\left(e^{2i(dx+c)}-1\right)^3(a+b)\left(-b\,e^{4i(}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4/(a+sin(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/3/a^2/tan(d*x+c)^3-(a-2*b)/a^3/tan(d*x+c)+b^2/a^3*(1/2*b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a)+1/2*(6*a+5*b)/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))))

Maxima [A]
time = 0.51, size = 172, normalized size = 1.06

$$\frac{\frac{3\left(6\,ab^2+5\,b^3\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^4+a^3b)\sqrt{(a+b)a}}-\frac{3\left(2\,a^3-6\,ab^2-5\,b^3\right)\tan(dx+c)^4+2\,a^3+2\,a^2b+2\left(4\,a^3-a^2b-5\,ab^2\right)\tan(dx+c)^2}{(a^5+2\,a^4b+a^3b^2)\tan(dx+c)^5+(a^5+a^4b)\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/6*(3*(6*a*b^2 + 5*b^3)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^4 + a^3*b)*sqrt((a + b)*a)) - (3*(2*a^3 - 6*a*b^2 - 5*b^3)*tan(d*x + c)^4 + 2*a^3 + 2*a^2*b + 2*(4*a^3 - a^2*b - 5*a*b^2)*tan(d*x + c)^2)/((a^5 + 2*a^4*b + a^3*b^2)*tan(d*x + c)^5 + (a^5 + a^4*b)*tan(d*x + c)^3))/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(146) = 292$.
time = 0.47, size = 843, normalized size = 5.20

$$\left[-\frac{4\left(4a^4b-4a^3b^2-23a^2b^3-15ab^4\right)\cos\left(dx+c\right)^5-8\left(2a^5+3a^4b-12a^3b^2-28a^2b^3-15ab^4\right)\cos\left(dx+c\right)^3+3\left(\left(6ab^3+5b^4\right)\cos\left(dx+c\right)^4+6a^2b^2+11ab^3+5b^4-\left(6a^2b^2+17ab^3+10b^4\right)\cos\left(dx+c\right)^2\right)\sqrt{-a^2-ab}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos\left(dx+c\right)^4-2\left(4a^2+5ab+b^2\right)\cos\left(dx+c\right)^2+4\left(\left(2a+b\right)\cos\left(dx+c\right)^3-\left(a+b\right)\cos\left(dx+c\right)\right)\sqrt{-a^2-ab}\sin\left(dx+c\right)+a^2+2ab+b^2}{b^2\cos\left(dx+c\right)^4-2\left(ab+b^2\right)\cos\left(dx+c\right)^2+a^2+2ab+b^2}\right)\sin\left(dx+c\right)+12\left(2a^5+2a^4b-6a^3b^2-11a^2b^3-5ab^4\right)\cos\left(dx+c\right)}{24\left(\left(a^6b+2a^5b^2+a^4b^3\right)d\cos\left(dx+c\right)^4-\left(a^7+4a^6b+5a^5b^2+2a^4b^3\right)d\cos\left(dx+c\right)^2+\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)d\right)\sin\left(dx+c\right)}, -\frac{2\left(4a^4b-4a^3b^2-23a^2b^3-15ab^4\right)\cos\left(dx+c\right)^5-4\left(2a^5+3a^4b-12a^3b^2-28a^2b^3-15ab^4\right)\cos\left(dx+c\right)^3+3\left(\left(6ab^3+5b^4\right)\cos\left(dx+c\right)^4+6a^2b^2+11ab^3+5b^4-\left(6a^2b^2+17ab^3+10b^4\right)\cos\left(dx+c\right)^2\right)\sqrt{a^2+ab}\arctan\left(\frac{\left(2a+b\right)\cos\left(dx+c\right)^2-a-b}{2\sqrt{a^2+ab}\cos\left(dx+c\right)\sin\left(dx+c\right)}\right)\sin\left(dx+c\right)+6\left(2a^5+2a^4b-6a^3b^2-11a^2b^3-5ab^4\right)\cos\left(dx+c\right)}{12\left(\left(a^6b+2a^5b^2+a^4b^3\right)d\cos\left(dx+c\right)^4-\left(a^7+4a^6b+5a^5b^2+2a^4b^3\right)d\cos\left(dx+c\right)^2+\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)d\right)\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^5 - 8*
(2*a^5 + 3*a^4*b - 12*a^3*b^2 - 28*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^3 + 3*(
(6*a*b^3 + 5*b^4)*cos(d*x + c)^4 + 6*a^2*b^2 + 11*a*b^3 + 5*b^4 - (6*a^2*b^
2 + 17*a*b^3 + 10*b^4)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b
 + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a +
 b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) +
a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
 + 2*a*b + b^2))*sin(d*x + c) + 12*(2*a^5 + 2*a^4*b - 6*a^3*b^2 - 11*a^2*b^
3 - 5*a*b^4)*cos(d*x + c))/(((a^6*b + 2*a^5*b^2 + a^4*b^3)*d*cos(d*x + c)^4
 - (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cos(d*x + c)^2 + (a^7 + 3*a^6*
b + 3*a^5*b^2 + a^4*b^3)*d)*sin(d*x + c)), -1/12*(2*(4*a^4*b - 4*a^3*b^2 -
23*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^5 - 4*(2*a^5 + 3*a^4*b - 12*a^3*b^2 - 2
8*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^3 + 3*((6*a*b^3 + 5*b^4)*cos(d*x + c)^4
+ 6*a^2*b^2 + 11*a*b^3 + 5*b^4 - (6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cos(d*x +
c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a
^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) + 6*(2*a^5 + 2*a^4*b - 6
*a^3*b^2 - 11*a^2*b^3 - 5*a*b^4)*cos(d*x + c))/(((a^6*b + 2*a^5*b^2 + a^4*b
^3)*d*cos(d*x + c)^4 - (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cos(d*x +
c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(c+dx\right)}{\left(a+b\sin^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sin(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.45, size = 174, normalized size = 1.07

$$\frac{\frac{3\,b^3\tan(dx+c)}{\left(a^4+a^3b\right)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)}+\frac{3\left(6\,ab^2+5\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^4+a^3b\right)\sqrt{a^2+ab}}-\frac{2\left(3\,a\tan(dx+c)^2-6\,b\tan(dx+c)^2+a\right)}{a^3\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)/((a^4 + a^3*b)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2
 + a)) + 3*(6*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) +
 arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^4 + a^3*b)*
sqrt(a^2 + a*b)) - 2*(3*a*tan(d*x + c)^2 - 6*b*tan(d*x + c)^2 + a)/(a^3*tan
(d*x + c)^3))/d
```

Mupad [B]
time = 14.40, size = 164, normalized size = 1.01

$$\frac{b^2\,\mathrm{atan}\left(\frac{b^2\,\tan(c+d\,x)\,\left(a^4+b\,a^3\right)\,(6\,a+5\,b)}{a^{7/2}\,(5\,b^3+6\,a\,b^2)\,\sqrt{a+b}}\right)\,(6\,a+5\,b)}{2\,a^{7/2}\,d\,(a+b)^{3/2}}-\frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(4\,a-5\,b)}{3\,a^2}-\frac{\tan(c+d\,x)^4\,\left(-2\,a^3+6\,a\,b^2+5\,b^3\right)}{2\,a^3\,(a+b)}}{d\,\left((a+b)\,\tan\left(c+d\,x\right)^5+a\tan\left(c+d\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*sin(c + d*x)^2)^2),x)
```

```
[Out] (b^2*atan((b^2*tan(c + d*x)*(a^3*b + a^4)*(6*a + 5*b))/(a^(7/2)*(6*a*b^2 +
5*b^3)*(a + b)^(1/2)))*(6*a + 5*b))/(2*a^(7/2)*d*(a + b)^(3/2)) - (1/(3*a)
+ (tan(c + d*x)^2*(4*a - 5*b))/(3*a^2) - (tan(c + d*x)^4*(6*a*b^2 - 2*a^3 +
 5*b^3))/(2*a^3*(a + b)))/(d*(tan(c + d*x)^5*(a + b) + a*tan(c + d*x)^3))
```

3.106 $\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$

Optimal. Leaf size=148

$$\frac{x}{b^3}-\frac{\sqrt{a}\,(8a^2+20ab+15b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8b^3(a+b)^{5/2}d}+\frac{a\tan^3(c+dx)}{4b(a+b)d\,(a+(a+b)\tan^2(c+dx))^2}+\frac{a(4}{8b^2(a+b)^2}$$

```
[Out] x/b^3-1/8*(8*a^2+20*a*b+15*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1
/2)/b^3/(a+b)^(5/2)/d+1/4*a*tan(d*x+c)^3/b/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)^2
+1/8*a*(4*a+7*b)*tan(d*x+c)/b^2/(a+b)^2/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A]
time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {3266, 481, 592, 536, 209, 211}

$$-\frac{\sqrt{a}\,(8a^2+20ab+15b^2)\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8b^3d(a+b)^{5/2}}+\frac{a(4a+7b)\tan(c+dx)}{8b^2d(a+b)^2\,((a+b)\tan^2(c+dx)+a)}+\frac{a\tan^3(c+dx)}{4bd(a+b)\,((a+b)\tan^2(c+dx)+a)^2}+\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] x/b^3 - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x]
)/Sqrt[a]])/(8*b^3*(a + b)^(5/2)*d) + (a*Tan[c + d*x]^3)/(4*b*(a + b)*d*(a
+ (a + b)*Tan[c + d*x]^2)^2) + (a*(4*a + 7*b)*Tan[c + d*x])/(8*b^2*(a + b)^
2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^6}{(1+x^2)(a+(a+b)x^2)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{x^2(3a+(-a-4b)x^2)}{(1+x^2)(a+(a+b)x^2)^2}\,dx, x, \tan(c\right.}{4b(a+b)d}$$

$$= \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{a(4a+7b)\tan(c+dx)}{8b^2(a+b)^2d\left(a+(a+b)\tan^2(c+dx\right.}$$

$$= \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{a(4a+7b)\tan(c+dx)}{8b^2(a+b)^2d\left(a+(a+b)\tan^2(c+dx\right.}$$

$$= \frac{x}{b^3} - \frac{\sqrt{a}\,(8a^2+20ab+15b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8b^3(a+b)^{5/2}d} + \frac{a\tan^3(c}{4b(a+b)d\left(a+(a\right.}$$

Mathematica [A]
time = 1.81, size = 134, normalized size = 0.91

$$\frac{8(c+dx) - \frac{\sqrt{a}\,(8a^2+20ab+15b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}} + \frac{ab(8a^2+20ab+9b^2-3b(2a+3b)\cos(2(c+dx)))\sin(2(c+dx))}{(a+b)^2(2a+b-b\cos(2(c+dx)))^2}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Tan[c
 + d*x])/Sqrt[a]])/(a + b)^(5/2) + (a*b*(8*a^2 + 20*a*b + 9*b^2 - 3*b*(2*a
+ 3*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2*a + b - b*Cos[2*(c
 + d*x)])^2))/(8*b^3*d)
```

Maple [A]
time = 0.34, size = 158, normalized size = 1.07

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{b^3} - \frac{a\left(\frac{-\frac{(4a+9b)b\left(\tan^3(dx+c)\right)}{8(a+b)} - \frac{ab(4a+7b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2} + \frac{\left(8a^2+20ab+15b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{b^3}}{d}$

default	$\frac{\frac{\arctan(\tan(dx+c))}{b^3}-\frac{a\left(\frac{-\frac{(4a+9b)b\left(\tan^3(dx+c)\right)}{8(a+b)}-\frac{ab(4a+7b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{\left(8a^2+20ab+15b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{b^3}}{d}$
risch	$\frac{x}{b^3}-\frac{ia\left(-16b\,e^{6i(dx+c)}a^2-28a\,b^2e^{6i(dx+c)}-9b^3e^{6i(dx+c)}+48a^3e^{4i(dx+c)}+120b\,e^{4i(dx+c)}a^2+90a\,b^2e^{4i(dx+c)}+27b^3e^{4}\right.}{4b^3(a+b)^2d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx-}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+sin(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*arctan(tan(d*x+c))-a/b^3*((-1/8*(4*a+9*b)*b/(a+b)*tan(d*x+c)^3-1
/8*a*b*(4*a+7*b)/(a^2+2*a*b+b^2)*tan(d*x+c))/(a*tan(d*x+c)^2+b*tan(d*x+c)^2
+a)^2+1/8*(8*a^2+20*a*b+15*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b
)*tan(d*x+c)/(a*(a+b))^(1/2))))
```

Maxima [A]
time = 0.55, size = 234, normalized size = 1.58

$$-\frac{\frac{\left(8\,a^3+20\,a^2b+15\,ab^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^2b^3+2\,ab^4+b^5\right)\sqrt{(a+b)a}}-\frac{\left(4\,a^3+13\,a^2b+9\,ab^2\right)\tan(dx+c)^3+\left(4\,a^3+7\,a^2b\right)\tan(dx+c)}{a^4b^2+2\,a^3b^3+a^2b^4+\left(a^4b^2+4\,a^3b^3+6\,a^2b^4+4\,ab^5+b^6\right)\tan(dx+c)^4+2\left(a^4b^2+3\,a^3b^3+3\,a^2b^4+ab^5\right)\tan(dx+c)^2}-\frac{8\,(dx+c)}{b^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((8*a^3 + 20*a^2*b + 15*a*b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b
)*a))/((a^2*b^3 + 2*a*b^4 + b^5)*sqrt((a + b)*a)) - ((4*a^3 + 13*a^2*b + 9*
a*b^2)*tan(d*x + c)^3 + (4*a^3 + 7*a^2*b)*tan(d*x + c))/(a^4*b^2 + 2*a^3*b^
3 + a^2*b^4 + (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*tan(d*x + c
)^4 + 2*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*tan(d*x + c)^2) - 8*(d*x
+ c)/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(134) = 268$.
time = 0.47, size = 950, normalized size = 6.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cos(d*x + c)^4 - 64*(a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*d*x*cos(d*x + c)^2 + 32*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
```

```
*a*b^3 + b^4)*d*x + ((8*a^2*b^2 + 20*a*b^3 + 15*b^4)*cos(d*x + c)^4 + 8*a^4
 + 36*a^3*b + 63*a^2*b^2 + 50*a*b^3 + 15*b^4 - 2*(8*a^3*b + 28*a^2*b^2 + 35
*a*b^3 + 15*b^4)*cos(d*x + c)^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2
)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a
*b + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b
))*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*co
s(d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*(3*(2*a^2*b^2 + 3*a*b^3)*cos(d*x + c
)^3 - (4*a^3*b + 13*a^2*b^2 + 9*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^
5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^4 - 2*(a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^
7)*d*cos(d*x + c)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b^5 + 4*a*b^6 + b^7)*d),
 1/16*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cos(d*x + c)^4 - 32*(a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*d*x*cos(d*x + c)^2 + 16*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*x + ((8*a^2*b^2 + 20*a*b^3 + 15*b^4)*cos(d*x + c)^4 + 8*a^4
 + 36*a^3*b + 63*a^2*b^2 + 50*a*b^3 + 15*b^4 - 2*(8*a^3*b + 28*a^2*b^2 + 35
*a*b^3 + 15*b^4)*cos(d*x + c)^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(
d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))) - 2*(3*(
2*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^3 - (4*a^3*b + 13*a^2*b^2 + 9*a*b^3)*cos(
d*x + c))*sin(d*x + c))/((a^2*b^5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^4 - 2*(a^
3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*d*cos(d*x + c)^2 + (a^4*b^3 + 4*a^3*b^4
+ 6*a^2*b^5 + 4*a*b^6 + b^7)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 224, normalized size = 1.51

$$-\frac{\frac{\left(8\,a^3+20\,a^2b+15\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^2b^3+2\,ab^4+b^5\right)\sqrt{a^2+ab}}-\frac{4\,a^3\tan(dx+c)^3+13\,a^2b\tan(dx+c)^3+9\,ab^2\tan(dx+c)^3+4\,a^3\tan(dx+c)+7\,a^2b\tan(dx+c)}{\left(a^2b^2+2\,ab^3+b^4\right)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2}-\frac{8\,(dx+c)}{b^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*a^3 + 20*a^2*b + 15*a*b^2)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a +
 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^2*b^
3 + 2*a*b^4 + b^5)*sqrt(a^2 + a*b)) - (4*a^3*tan(d*x + c)^3 + 13*a^2*b*tan(
d*x + c)^3 + 9*a*b^2*tan(d*x + c)^3 + 4*a^3*tan(d*x + c) + 7*a^2*b*tan(d*x
+ c))/((a^2*b^2 + 2*a*b^3 + b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^
2) - 8*(d*x + c)/b^3)/d
```

Mupad [B]
time = 17.96, size = 2500, normalized size = 16.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*sin(c + d*x)^2)^3,x)
```

```
[Out] ((tan(c + d*x)^3*(9*a*b + 4*a^2))/(8*(a*b^2 + b^3)) + (a*tan(c + d*x)*(7*a*
b + 4*a^2))/(8*(2*a*b^3 + b^4 + a^2*b^2)))/(d*(tan(c + d*x)^4*(2*a*b + a^2
+ b^2) + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2))) - atan((((((7*a*b^10)/2 +
(25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*(3*a*b
^8 + b^9 + 3*a^2*b^7 + a^3*b^6)) - (tan(c + d*x)*(1792*a*b^11 + 256*b^12 +
5120*a^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/
(128*b^3*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/(2*b^3) + (tan(c + d*x)*(3
84*a*b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 160
0*a^4*b^2))/(64*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/b^3 - (((((7*a*b^10
)/2 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*
(3*a*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6)) + (tan(c + d*x)*(1792*a*b^11 + 256*b
^12 + 5120*a^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*
b^6))/(128*b^3*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/(2*b^3) - (tan(c + d
*x)*(384*a*b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3
 + 1600*a^4*b^2))/(64*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/b^3)/(((15*a*
b^4)/4 + (19*a^4*b)/4 + a^5 + (295*a^2*b^3)/32 + (19*a^3*b^2)/2)/(3*a*b^8 +
 b^9 + 3*a^2*b^7 + a^3*b^6) + ((((((7*a*b^10)/2 + (25*a^2*b^9)/2 + (33*a^3*
b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*(3*a*b^8 + b^9 + 3*a^2*b^7 + a^
3*b^6)) - (tan(c + d*x)*(1792*a*b^11 + 256*b^12 + 5120*a^2*b^10 + 7680*a^3*
b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/(128*b^3*(3*a*b^6 + b^7 +
 3*a^2*b^5 + a^3*b^4)))*1i)/(2*b^3) + (tan(c + d*x)*(384*a*b^5 + 704*a^5*b
+ 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4*b^2)*1i)/(64*(3
*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/b^3 + ((((((7*a*b^10)/2 + (25*a^2*b^9
)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*(3*a*b^8 + b^9 +
3*a^2*b^7 + a^3*b^6)) + (tan(c + d*x)*(1792*a*b^11 + 256*b^12 + 5120*a^2*b^
10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/(128*b^3*(3
*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))*1i)/(2*b^3) - (tan(c + d*x)*(384*a*b^
5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4*b
^2)*1i)/(64*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/b^3))/(b^3*d) - (atan((
((-a*(a + b)^5)^(1/2)*((tan(c + d*x)*(384*a*b^5 + 704*a^5*b + 128*a^6 + 64*
b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4*b^2))/(32*(3*a*b^6 + b^7 + 3*a
^2*b^5 + a^3*b^4)) - ((-a*(a + b)^5)^(1/2)*(((7*a*b^10)/2 + (25*a^2*b^9)/2
+ (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)/(3*a*b^8 + b^9 + 3*a^2*b^7 +
 a^3*b^6) - (tan(c + d*x)*(-a*(a + b)^5)^(1/2)*(20*a*b + 8*a^2 + 15*b^2)*(1
792*a*b^11 + 256*b^12 + 5120*a^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*
a^5*b^7 + 512*a^6*b^6))/(512*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)*(5*a*b^7
 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)))*(20*a*b + 8*a^2 +
```

```
15*b^2))/(16*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^
3)))*(20*a*b + 8*a^2 + 15*b^2)*1i)/(16*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3
*b^5 + 5*a^4*b^4 + a^5*b^3)) + ((-a*(a + b)^5)^(1/2)*((tan(c + d*x)*(384*a*
b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4
*b^2))/(32*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)) + ((-a*(a + b)^5)^(1/2)*(
((7*a*b^10)/2 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^
6)/(3*a*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6) + (tan(c + d*x)*(-a*(a + b)^5)^(1/
2)*(20*a*b + 8*a^2 + 15*b^2)*(1792*a*b^11 + 256*b^12 + 5120*a^2*b^10 + 7680
*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/(512*(3*a*b^6 + b^7
+ 3*a^2*b^5 + a^3*b^4)*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4
 + a^5*b^3)))*(20*a*b + 8*a^2 + 15*b^2))/(16*(5*a*b^7 + b^8 + 10*a^2*b^6 +
10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)))*(20*a*b + 8*a^2 + 15*b^2)*1i)/(16*(5*a*
b^7 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)))/(((15*a*b^4)/4
 + (19*a^4*b)/4 + a^5 + (295*a^2*b^3)/32 + (19*a^3*b^2)/2)/(3*a*b^8 + b^9 +
 3*a^2*b^7 + a^3*b^6) - ((-a*(a + b)^5)^(1/2)*((tan(c + d*x)*(384*a*b^5 + 7
04*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4*b^2))/
(32*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)) - ((-a*(a + b)^5)^(1/2)*(((7*a*b
^10)/2 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)/(3*a
*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6) - (tan(c + d*x)*(-a*(a + b)^5)^(1/2)*(20*
a*b + 8*a^2 + 15*b^2)*(1792*a*b^11 + 256*b^12 + 5120*a^2*b^10 + 7680*a^3*b^
9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/(512*(3*a*b^6 + b^7 + 3*a^2
*b^5 + a^3*b^4)*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*
b^3)))*(20*a*b + 8*a^2 + 15*b^2))/(16*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3*
b^5 + 5*a^4*b^4 + a^5*b^3)))*(20*a*b + 8*a^2 + 15*b^2))/(16*(5*a*b^7 + b^8
+ 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)) + ((-a*(a + b)^5)^(1/2)*(
(tan(c + d*x)*(384*a*b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 18
80*a^3*b^3 + 1600*a^4*b^2))/(32*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)) + ((
-a*(a + b)^5)^(1/2)*(((7*a*b^10)/2 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*
a^4*b^7)/2 + 2*a^5*b^6)/(3*a*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6) + (tan(c + d*
x)*(-a*(a + b)^5)^(1/2)*(20*a*b + 8*a^2 + 15*b^...
```

3.107 $\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^3} dx$

Optimal. Leaf size=110

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a+b)^{5/2}d}-\frac{\tan^3(c+dx)}{4(a+b)d(a+(a+b)\tan^2(c+dx))^2}-\frac{3\tan(c+dx)}{8(a+b)^2d(a+(a+b)\tan^2(c+dx))}$$

```
[Out] 3/8*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(5/2)/d/a^(1/2)-1/4*tan(d*
x+c)^3/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)^2-3/8*tan(d*x+c)/(a+b)^2/d/(a+(a+b)*t
an(d*x+c)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3266, 294, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}d(a+b)^{5/2}}-\frac{3\tan(c+dx)}{8d(a+b)^2((a+b)\tan^2(c+dx)+a)}-\frac{\tan^3(c+dx)}{4d(a+b)((a+b)\tan^2(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a + b)^(5/2)*d)
- Tan[c + d*x]^3/(4*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2)^2) - (3*Tan[c +
d*x])/(8*(a + b)^2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
```

```
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(a+(a+b)x^2)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\tan^3(c+dx)}{4(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{3\text{Subst}\left(\int \frac{x^2}{(a+(a+b)x^2)^2}\,dx, x, \tan(c+dx\right.}{4(a+b)d}$$

$$= -\frac{\tan^3(c+dx)}{4(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{3\tan(c+dx)}{8(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a+b)^{5/2}d} - \frac{\tan^3(c+dx)}{4(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{}{8(a+b}$$

Mathematica [A]
time = 0.89, size = 97, normalized size = 0.88

$$\frac{\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}} + \frac{(-8a-5b+(2a+5b)\cos(2(c+dx)))\sin(2(c+dx))}{(a+b)^2(2a+b-b\cos(2(c+dx)))^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + (
(-8*a - 5*b + (2*a + 5*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2
*a + b - b*Cos[2*(c + d*x)])^2))/(8*d)
```

Maple [A]
time = 0.24, size = 109, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{-\frac{5\left(\tan^3(dx+c)\right)}{8(a+b)}-\frac{3a\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}}{d}$

default	$\dfrac{\dfrac{-\frac{5\left(\tan^3(dx+c)\right)}{8(a+b)}-\frac{3a\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2}+\dfrac{3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}}{d}$
risch	$\dfrac{i\left(-8b\,e^{6i(dx+c)}a^2-16a\,b^2e^{6i(dx+c)}-5b^3e^{6i(dx+c)}+16a^3e^{4i(dx+c)}+56b\,e^{4i(dx+c)}a^2+46a\,b^2e^{4i(dx+c)}+15b^3e^{4i(dx+c)}-8b\right.}{4b^2(a+b)^2d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+sin(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-5/8/(a+b)*tan(d*x+c)^3-3/8*a/(a^2+2*a*b+b^2)*tan(d*x+c))/(a*tan(d*x+
c)^2+b*tan(d*x+c)^2+a)^2+3/8/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*t
an(d*x+c)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.53, size = 158, normalized size = 1.44

$$-\frac{\frac{5\,(a+b)\tan(dx+c)^3+3\,a\tan(dx+c)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\tan(dx+c)^4+a^4+2\,a^3b+a^2b^2+2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\tan(dx+c)^2}-\frac{3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+2\,ab+b^2)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*(a + b)*tan(d*x + c)^3 + 3*a*tan(d*x + c))/((a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 2*(a^4 + 3
*a^3*b + 3*a^2*b^2 + a*b^3)*tan(d*x + c)^2) - 3*arctan((a + b)*tan(d*x + c)
/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + 2*a*b + b^2)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(96) = 192.
time = 0.43, size = 683, normalized size = 6.21

$$\left[-\frac{3\,(b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2)\sqrt{-a^2-ab}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\,((2\,a+b)\cos(dx+c)^3-(a+b)\cos(dx+c))\sqrt{-a^2-ab}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)-4\,((2\,a^3+7\,a^2b+5\,ab^2)\cos(dx+c)^3-5\,(a^3+2\,a^2b+ab^2)\cos(dx+c))\sin(dx+c)}{32\,((a^4b^2+3\,a^3b^3+3\,a^2b^4+ab^5)d\cos(dx+c)^4-2\,(a^5b+4\,a^4b^2+6\,a^3b^3+4\,a^2b^4+ab^5)d\cos(dx+c)^2+(a^6+5\,a^5b+10\,a^4b^2+10\,a^3b^3+5\,a^2b^4+ab^5)d)},\ -\frac{3\,(b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2)\sqrt{a^2+ab}\arctan\left(\frac{(2\,a+b)\cos(dx+c)^2-a-b}{2\,\sqrt{a^2+ab}\cos(dx+c)\sin(dx+c)}\right)-2\,((2\,a^3+7\,a^2b+5\,ab^2)\cos(dx+c)^3-5\,(a^3+2\,a^2b+ab^2)\cos(dx+c))\sin(dx+c)}{16\,((a^4b^2+3\,a^3b^3+3\,a^2b^4+ab^5)d\cos(dx+c)^4-2\,(a^5b+4\,a^4b^2+6\,a^3b^3+4\,a^2b^4+ab^5)d\cos(dx+c)^2+(a^6+5\,a^5b+10\,a^4b^2+10\,a^3b^3+5\,a^2b^4+ab^5)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b
+ b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^
2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos
(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x
+ c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*((2*a^3 + 7
```

```
*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - 5*(a^3 + 2*a^2*b + a*b^2)*cos(d*x + c))*
sin(d*x + c))/((a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*cos(d*x + c)^4 -
 2*(a^5*b + 4*a^4*b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*d*cos(d*x + c)^2 + (
a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*d), -1/16*(3*(
b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt
(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*
cos(d*x + c)*sin(d*x + c))) - 2*((2*a^3 + 7*a^2*b + 5*a*b^2)*cos(d*x + c)^3
 - 5*(a^3 + 2*a^2*b + a*b^2)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2 + 3*a^3*
b^3 + 3*a^2*b^4 + a*b^5)*d*cos(d*x + c)^4 - 2*(a^5*b + 4*a^4*b^2 + 6*a^3*b^
3 + 4*a^2*b^4 + a*b^5)*d*cos(d*x + c)^2 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10*
a^3*b^3 + 5*a^2*b^4 + a*b^5)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 152, normalized size = 1.38

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^2+2ab+b^2)\sqrt{a^2+ab}}-\frac{5a\tan(dx+c)^3+5b\tan(dx+c)^3+3a\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2(a^2+2ab+b^2)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a*b))
 - (5*a*tan(d*x + c)^3 + 5*b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/((a*tan(d*x
 + c)^2 + b*tan(d*x + c)^2 + a)^2*(a^2 + 2*a*b + b^2)))/d
```

Mupad [B]
time = 13.69, size = 149, normalized size = 1.35

$$\frac{3\,\mathrm{atan}\left(\frac{3\tan(c+d\,x)\,(2\,a+2\,b)\,\left(\frac{8\,a^2}{3}+\frac{16\,a\,b}{3}+\frac{8\,b^2}{3}\right)}{16\,\sqrt{a}\,(a+b)^{5/2}}\right)}{8\,\sqrt{a}\,d\,(a+b)^{5/2}}-\frac{\frac{5\tan(c+d\,x)^3}{8\,(a+b)}+\frac{3\,a\tan(c+d\,x)}{8\,(a^2+2\,a\,b+b^2)}}{d\,\left(\tan\left(c+d\,x\right)^4\,\left(a^2+2\,a\,b+b^2\right)+a^2+\tan\left(c+d\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*sin(c + d*x)^2)^3,x)
```

```
[Out] (3*atan((3*tan(c + d*x)*(2*a + 2*b)*((16*a*b)/3 + (8*a^2)/3 + (8*b^2)/3))/(
16*a^(1/2)*(a + b)^(5/2))))/(8*a^(1/2)*d*(a + b)^(5/2)) - ((5*tan(c + d*x)^
3)/(8*(a + b)) + (3*a*tan(c + d*x))/(8*(2*a*b + a^2 + b^2)))/(d*(tan(c + d*
x)^4*(2*a*b + a^2 + b^2) + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2)))
```

3.108 $\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^{5/2}d}-\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\,(a+b\sin^2(c+dx))^2}-\frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\,(a+b\sin^2(c+dx))}$$

```
[Out] 1/8*(4*a+b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(5/2)/d-1/
4*cos(d*x+c)*sin(d*x+c)/(a+b)/d/(a+b*sin(d*x+c)^2)^2-1/8*(2*a-b)*cos(d*x+c)
*sin(d*x+c)/a/(a+b)^2/d/(a+b*sin(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3252, 12, 3260, 211}

$$\frac{(4a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a+b)^{5/2}}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{8ad(a+b)^2\,(a+b\sin^2(c+dx))}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\,(a+b\sin^2(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] ((4*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a + b)^(
5/2)*d) - (Cos[c + d*x]*Sin[c + d*x])/(4*(a + b)*d*(a + b*Sin[c + d*x]^2)^2
) - ((2*a - b)*Cos[c + d*x]*Sin[c + d*x])/(8*a*(a + b)^2*d*(a + b*Sin[c + d
*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\,(a+b\sin^2(c+dx))^2} + \frac{\int \frac{a+2a\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx}{4a(a+b)}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\,(a+b\sin^2(c+dx))^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\,(a+b\sin^2(c+dx))} + \frac{\int \frac{a(}{a+b\,\mathrm{si}}}{8a^2(}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\,(a+b\sin^2(c+dx))^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\,(a+b\sin^2(c+dx))} + \frac{(4a+}{}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\,(a+b\sin^2(c+dx))^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\,(a+b\sin^2(c+dx))} + \frac{(4a+}{}$$

$$= \frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^{5/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\,(a+b\sin^2(c+dx))^2} - \frac{(2a-}{8a(a}$$

Mathematica [A]
time = 0.96, size = 112, normalized size = 0.85

$$\frac{\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}} - \frac{(8a^2+4ab-b^2+b(-2a+b)\cos(2(c+dx)))\sin(2(c+dx))}{a(a+b)^2(2a+b-b\cos(2(c+dx)))^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (((4*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*(a + b)^(5
/2)) - ((8*a^2 + 4*a*b - b^2 + b*(-2*a + b)*Cos[2*(c + d*x)])*Sin[2*(c + d*
x)])/(a*(a + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2))/(8*d)
```

Maple [A]
time = 0.36, size = 131, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{-\frac{(4a-b)\left(\tan^3(dx+c)\right)}{8a(a+b)}-\frac{(4a+b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(4a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{-\frac{(4a-b)\left(\tan^3(dx+c)\right)}{8a(a+b)}-\frac{(4a+b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(4a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{a(a+b)}}}{d}$
risch	$\frac{i\left(4a\,b^2\mathrm{e}^{6i(dx+c)}+b^3\mathrm{e}^{6i(dx+c)}+16a^3\mathrm{e}^{4i(dx+c)}+8b\,\mathrm{e}^{4i(dx+c)}a^2-2a\,b^2\mathrm{e}^{4i(dx+c)}-3b^3\mathrm{e}^{4i(dx+c)}-16b\,\mathrm{e}^{2i(dx+c)}a^2-4a\,b^2\mathrm{e}^{2i(dx}\right.}{4ba(a+b)^2d\left(-b\,\mathrm{e}^{4i(dx+c)}+4a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}-b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+sin(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/8*(4*a-b)/a/(a+b)*tan(d*x+c)^3-1/8*(4*a+b)/(a^2+2*a*b+b^2)*tan(d*x
+c))/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a)^2+1/8*(4*a+b)/(a^2+2*a*b+b^2)/a/(a*(
a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.52, size = 191, normalized size = 1.46

$$\frac{\frac{(4\,a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{(a+b)a}}-\frac{(4\,a^2+3\,ab-b^2)\tan(dx+c)^3+(4\,a^2+ab)\tan(dx+c)}{a^5+2\,a^4b+a^3b^2+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\tan(dx+c)^4+2\,(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\tan(dx+c)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((4*a + b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^3 + 2*a^2*b
 + a*b^2)*sqrt((a + b)*a)) - ((4*a^2 + 3*a*b - b^2)*tan(d*x + c)^3 + (4*a^2
 + a*b)*tan(d*x + c))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 4*a^4*b + 6*a^3*b^2
 + 4*a^2*b^3 + a*b^4)*tan(d*x + c)^4 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b
^3)*tan(d*x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(117) = 234.
time = 0.46, size = 771, normalized size = 5.89

$$\left[-\frac{\left((4\,ab^2+b^3)\cos(dx+c)^4+4\,a^3+9\,a^2b+6\,ab^2+b^3-2\,(4\,a^2b+5\,ab^2+b^3)\cos(dx+c)^2\right)\sqrt{-a^2-ab}\,\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\,\left((2\,a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\,\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)-4\,\left((2\,a^3b+a^2b^2-ab^3)\cos(dx+c)^3-(4\,a^4+7\,a^3b+2\,a^2b^2-ab^3)\cos(dx+c)\right)\sin(dx+c)}{32\,\left((a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5)d\cos(dx+c)^4-2\,(a^6b+4\,a^5b^2+6\,a^4b^3+4\,a^3b^4+a^2b^5)d\cos(dx+c)^2+(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5)d\right)},\ -\frac{\left((4\,ab^2+b^3)\cos(dx+c)^4+4\,a^3+9\,a^2b+6\,ab^2+b^3-2\,(4\,a^2b+5\,ab^2+b^3)\cos(dx+c)^2\right)\sqrt{a^2+ab}\,\arctan\left(\frac{(2\,a+b)\cos(dx+c)^2-a-b}{2\,\sqrt{a^2+ab}\,\cos(dx+c)\sin(dx+c)}\right)-2\,\left((2\,a^3b+a^2b^2-ab^3)\cos(dx+c)^3-(4\,a^4+7\,a^3b+2\,a^2b^2-ab^3)\cos(dx+c)\right)\sin(dx+c)}{16\,\left((a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5)d\cos(dx+c)^4-2\,(a^6b+4\,a^5b^2+6\,a^4b^3+4\,a^3b^4+a^2b^5)d\cos(dx+c)^2+(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((4*a*b^2 + b^3)*cos(d*x + c)^4 + 4*a^3 + 9*a^2*b + 6*a*b^2 + b^3 -
 2*(4*a^2*b + 5*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 +
 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*(
(2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x +
 c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2
 + a^2 + 2*a*b + b^2)) - 4*((2*a^3*b + a^2*b^2 - a*b^3)*cos(d*x + c)^3 - (4
*a^4 + 7*a^3*b + 2*a^2*b^2 - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^4 - 2*(a^6*b + 4*a^5*b^2 +
 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^
5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d), -1/16*(((4*a*b^2 + b^3)*cos(d
*x + c)^4 + 4*a^3 + 9*a^2*b + 6*a*b^2 + b^3 - 2*(4*a^2*b + 5*a*b^2 + b^3)*c
os(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b
)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c))) - 2*((2*a^3*b + a^2*b^2 - a*
b^3)*cos(d*x + c)^3 - (4*a^4 + 7*a^3*b + 2*a^2*b^2 - a*b^3)*cos(d*x + c))*s
in(d*x + c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^4
- 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^2
+ (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 191, normalized size = 1.46

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(4a+b)}{(a^3+2a^2b+ab^2)\sqrt{a^2+ab}}-\frac{4a^2\tan(dx+c)^3+3ab\tan(dx+c)^3-b^2\tan(dx+c)^3+4a^2\tan(dx+c)+ab\tan(dx+c)}{(a^3+2a^2b+ab^2)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))*(4*a + b)/((a^3 + 2*a^2*b + a*b^2)*sqrt
(a^2 + a*b)) - (4*a^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^3 - b^2*tan(d*x +
 c)^3 + 4*a^2*tan(d*x + c) + a*b*tan(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*(a*
tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^2))/d
```

Mupad [B]
time = 13.85, size = 159, normalized size = 1.21

$$\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,(a^2+2\,a\,b+b^2)}{2\,\sqrt{a}\,(a+b)^{5/2}}\right)\,(4\,a+b)}{8\,a^{3/2}\,d\,(a+b)^{5/2}}-\frac{\frac{\tan(c+d\,x)\,(4\,a+b)}{8\,(a^2+2\,a\,b+b^2)}+\frac{\tan(c+d\,x)^3\,(4\,a-b)}{8\,a\,(a+b)}}{d\,\left(\tan\left(c+d\,x\right)^4\,\left(a^2+2\,a\,b+b^2\right)+a^2+\tan\left(c+d\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^2)^3,x)
```

```
[Out] (atan((tan(c + d*x)*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/
2)))*(4*a + b))/(8*a^(3/2)*d*(a + b)^(5/2)) - ((tan(c + d*x)*(4*a + b))/(8*
(2*a*b + a^2 + b^2)) + (tan(c + d*x)^3*(4*a - b))/(8*a*(a + b)))/(d*(tan(c
+ d*x)^4*(2*a*b + a^2 + b^2) + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2)))
```

3.109 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{(8a^2+8ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d}+\frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2}+\frac{3b(2a+b)\cos(c+dx)\sin(c}{8a^2(a+b)^2d\left(a+b\sin^2(c\right.}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(5/2)/(a+b
)^(5/2)/d+1/4*b*cos(d*x+c)*sin(d*x+c)/a/(a+b)/d/(a+b*sin(d*x+c)^2)^2+3/8*b*
(2*a+b)*cos(d*x+c)*sin(d*x+c)/a^2/(a+b)^2/d/(a+b*sin(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3263, 3252, 12, 3260, 211}

$$\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{8a^2d(a+b)^2\left(a+b\sin^2(c+dx)\right)}+\frac{(8a^2+8ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-3),x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*a^(
5/2)*(a + b)^(5/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(4*a*(a + b)*d*(a + b
*Sin[c + d*x]^2)^2) + (3*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*(a +
 b)^2*d*(a + b*Sin[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{\int \frac{-4a-3b+2b\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx}{4a(a+b)}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)} - \frac{\int \frac{-8a^2\cdots}{a+b\sin\cdots}}{8a^2\cdots}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{(8a^2+\cdots}{\cdots}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{(8a^2+\cdots}{\cdots}$$

$$= \frac{(8a^2+8ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2}$$

Mathematica [A]
time = 0.84, size = 125, normalized size = 0.87

$$\frac{\frac{(8a^2+8ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{a}\,b(16a^2+16ab+3b^2-3b(2a+b)\cos(2(c+dx)))\sin(2(c+dx))}{(a+b)^2(2a+b-b\cos(2(c+dx)))^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-3),x]
```

```
[Out] (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a +
b)^(5/2) + (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 - 3*b*(2*a + b)*Cos[2*(c + d
*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2))/(8*a^
(5/2)*d)
```

Maple [A]
time = 0.38, size = 148, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{\frac{(8a+3b)b\left(\tan^3(dx+c)\right)}{8a^2(a+b)}+\frac{b(8a+5b)\tan(dx+c)}{8a\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8a^2\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{\frac{(8a+3b)b\left(\tan^3(dx+c)\right)}{8a^2(a+b)}+\frac{b(8a+5b)\tan(dx+c)}{8a\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8a^2\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}}{d}$
risch	$-\frac{i\left(-8b\,e^{6i(dx+c)}a^2-8a\,b^2e^{6i(dx+c)}-3b^3e^{6i(dx+c)}+48a^3e^{4i(dx+c)}+72b\,e^{4i(dx+c)}a^2+42a\,b^2e^{4i(dx+c)}+9b^3e^{4i(dx+c)}-4\cdots\right.}{4a^2(a+b)^2d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((1/8*(8*a+3*b)/a^2*b/(a+b)*tan(d*x+c)^3+1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b
^2)*tan(d*x+c))/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a)^2+1/8*(8*a^2+8*a*b+3*b^2)
/a^2/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2
)))
```

Maxima [A]
time = 0.55, size = 211, normalized size = 1.47

$$\frac{\frac{(8\,a^2+8\,ab+3\,b^2)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^4+2\,a^3b+a^2b^2)\sqrt{(a+b)a}}+\frac{(8\,a^2b+11\,ab^2+3\,b^3)\tan(dx+c)^3+(8\,a^2b+5\,ab^2)\tan(dx+c)}{a^6+2\,a^5b+a^4b^2+(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\tan(dx+c)^4+2\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\tan(dx+c)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((8*a^2 + 8*a*b + 3*b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(
(a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*a)) + ((8*a^2*b + 11*a*b^2 + 3*b^3)*
tan(d*x + c)^3 + (8*a^2*b + 5*a*b^2)*tan(d*x + c))/(a^6 + 2*a^5*b + a^4*b^2
 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*tan(d*x + c)^4 + 2*(a^
6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*tan(d*x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(130) = 260$.
time = 0.43, size = 843, normalized size = 5.85

$$\left[-\frac{\left(\left(8\,a^2b^2+8\,ab^3+3\,b^4\right)\cos\left(dx+c\right)^4+8\,a^4+24\,a^3b+27\,a^2b^2+14\,ab^3+3\,b^4-2\,\left(8\,a^3b+16\,a^2b^2+11\,ab^3+3\,b^4\right)\cos\left(dx+c\right)^2\right)\sqrt{-a^2-ab}\,\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2\,(4a^2+5ab+b^2)\cos(dx+c)^2+4\left((2a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\,\sin(dx+c)+a^2+2ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right)+4\,\left(3\,\left(2\,a^3b^2+3\,a^2b^3+ab^4\right)\cos\left(dx+c\right)^3-\left(8\,a^4b+19\,a^3b^2+14\,a^2b^3+3\,ab^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{32\,\left(\left(a^6b^2+3\,a^5b^3+3\,a^4b^4+a^3b^5\right)d\cos\left(dx+c\right)^4-2\,\left(a^7b+4\,a^6b^2+6\,a^5b^3+4\,a^4b^4+a^3b^5\right)d\cos\left(dx+c\right)^2+\left(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5\right)d\right)},\ -\frac{\left(\left(8\,a^2b^2+8\,ab^3+3\,b^4\right)\cos\left(dx+c\right)^4+8\,a^4+24\,a^3b+27\,a^2b^2+14\,ab^3+3\,b^4-2\,\left(8\,a^3b+16\,a^2b^2+11\,ab^3+3\,b^4\right)\cos\left(dx+c\right)^2\right)\sqrt{a^2+ab}\,\arctan\left(\frac{(2a+b)\cos(dx+c)^2-a-b}{2\,\sqrt{a^2+ab}\,\cos(dx+c)\sin(dx+c)}\right)+2\,\left(3\,\left(2\,a^3b^2+3\,a^2b^3+ab^4\right)\cos\left(dx+c\right)^3-\left(8\,a^4b+19\,a^3b^2+14\,a^2b^3+3\,ab^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{16\,\left(\left(a^6b^2+3\,a^5b^3+3\,a^4b^4+a^3b^5\right)d\cos\left(dx+c\right)^4-2\,\left(a^7b+4\,a^6b^2+6\,a^5b^3+4\,a^4b^4+a^3b^5\right)d\cos\left(dx+c\right)^2+\left(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(d*x + c)^4 + 8*a^4 + 24*a^3*b +
27*a^2*b^2 + 14*a*b^3 + 3*b^4 - 2*(8*a^3*b + 16*a^2*b^2 + 11*a*b^3 + 3*b^4)
*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4
 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (
a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^
2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*(
3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (8*a^4*b + 19*a^3*b^2 +
14*a^2*b^3 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 + 3*a^5*b^3 + 3
*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^4 - 2*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4
*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5
*b^3 + 5*a^4*b^4 + a^3*b^5)*d), -1/16*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(d
*x + c)^4 + 8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4 - 2*(8*a^3*b +
 16*a^2*b^2 + 11*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*
((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x +
c))) + 2*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (8*a^4*b + 19*
a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 + 3*a
^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^4 - 2*(a^7*b + 4*a^6*b^2 + 6*a
^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^
2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 211, normalized size = 1.47

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan\left(dx+c\right)+b\tan\left(dx+c\right)}{\sqrt{a^2+ab}}\right)\right)\left(8\,a^2+8\,ab+3\,b^2\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{a^2+ab}}+\frac{8\,a^2b\tan\left(dx+c\right)^3+11\,ab^2\tan\left(dx+c\right)^3+3\,b^3\tan\left(dx+c\right)^3+8\,a^2b\tan\left(dx+c\right)+5\,ab^2\tan\left(dx+c\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\left(a\tan\left(dx+c\right)^2+b\tan\left(dx+c\right)^2+a\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))*(8*a^2 + 8*a*b + 3*b^2)/((a^4 + 2*a^3*b
 + a^2*b^2)*sqrt(a^2 + a*b)) + (8*a^2*b*tan(d*x + c)^3 + 11*a*b^2*tan(d*x +
 c)^3 + 3*b^3*tan(d*x + c)^3 + 8*a^2*b*tan(d*x + c) + 5*a*b^2*tan(d*x + c))
/((a^4 + 2*a^3*b + a^2*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^2))/d
```

Mupad [B]
time = 13.82, size = 176, normalized size = 1.22

$$\frac{\frac{\tan(c+d\,x)^3\,\left(3\,b^2+8\,a\,b\right)}{8\,a^2\,\left(a+b\right)}+\frac{\tan(c+d\,x)\,\left(5\,b^2+8\,a\,b\right)}{8\,a\,\left(a^2+2\,a\,b+b^2\right)}}{d\,\left({\tan\left(c+d\,x\right)}^4\,\left(a^2+2\,a\,b+b^2\right)+a^2+{\tan\left(c+d\,x\right)}^2\,\left(2\,a^2+2\,b\,a\right)\right)}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\left(2\,a+2\,b\right)\,\left(a^2+2\,a\,b+b^2\right)}{2\,\sqrt{a}\,\left(a+b\right)^{5/2}}\right)\,\left(8\,a^2+8\,a\,b+3\,b^2\right)}{8\,a^{5/2}\,d\,{\left(a+b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2)^3,x)
```

```
[Out] ((tan(c + d*x)^3*(8*a*b + 3*b^2))/(8*a^2*(a + b)) + (tan(c + d*x)*(8*a*b +
5*b^2))/(8*a*(2*a*b + a^2 + b^2)))/(d*(tan(c + d*x)^4*(2*a*b + a^2 + b^2) +
 a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2))) + (atan((tan(c + d*x)*(2*a + 2*b)*(
2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2)))*(8*a*b + 8*a^2 + 3*b^2))/(8*
a^(5/2)*d*(a + b)^(5/2))
```

3.110 $\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=196

$$-\frac{3b(8a^2+12ab+5b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^{5/2}d}-\frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3(a+b)^2d}+\frac{b\csc(c+dx)\sec^3(}{4a(a+b)d\,(a+(a+b)\,\mathrm{t}}$$

```
[Out] -3/8*b*(8*a^2+12*a*b+5*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/
(a+b)^(5/2)/d-1/8*(2*a+3*b)*(4*a+5*b)*cot(d*x+c)/a^3/(a+b)^2/d+1/4*b*csc(d*
x+c)*sec(d*x+c)^3/a/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)^2+1/8*b*cot(d*x+c)*(4*a+
5*b+(4*a+b)*tan(d*x+c)^2)/a^2/(a+b)^2/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A]
time = 0.18, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3266, 479, 591, 464, 211}

$$-\frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3d(a+b)^2}+\frac{b\cot(c+dx)\left((4a+b)\tan^2(c+dx)+4a+5b\right)}{8a^2d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}-\frac{3b(8a^2+12ab+5b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^{5/2}}+\frac{b\csc(c+dx)\sec^3(c+dx)}{4ad(a+b)\left((a+b)\tan^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/
(8*a^(7/2)*(a + b)^(5/2)*d) - ((2*a + 3*b)*(4*a + 5*b)*Cot[c + d*x])/(8*a^3
*(a + b)^2*d) + (b*Csc[c + d*x]*Sec[c + d*x]^3)/(4*a*(a + b)*d*(a + (a + b)
*Tan[c + d*x]^2)^2) + (b*Cot[c + d*x]*(4*a + 5*b + (4*a + b)*Tan[c + d*x]^2
))/(8*a^2*(a + b)^2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^2\left(a+(a+b)x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(-4a-5b+(-4a-b)x^2\right)}{x^2\left(a+(a+b)x^2\right)^2}\,dx, x\right.}{4a(a+b)d}$$

$$= \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{b\cot(c+dx)\left(4a+5b+(4a+b)\tan^2\right.}{8a^2(a+b)^2d\left(a+(a+b)\tan^2(c+\right.}$$

$$= -\frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3(a+b)^2d} + \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{b}{}$$

$$= -\frac{3b\left(8a^2+12ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^{5/2}d} - \frac{(2a+3b)(4a+5b)\cot(c+}{8a^3(a+b)^2d}$$

Mathematica [A]

time = 1.16, size = 214, normalized size = 1.09

$$\frac{(-2a-b+b\cos(2(c+dx)))\csc^6(c+dx)\left(\frac{3b\left(8a^2+12ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)(2a+b-b\cos(2(c+dx)))^2}{(a+b)^{5/2}} + 8\sqrt{a}\,(2a+b-b\cos(2(c+dx)))^2\cot(c+dx) + \frac{4a^{3/2}b^2\sin(2(c+dx))}{a+b} + \frac{\sqrt{a}\,b^2(10a+7b)(2a+b-b\cos(2(c+dx)))\sin(2(c+dx))}{(a+b)^2}\right)}{64a^{7/2}d\left(b+a\csc^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^6*((3*b*(8*a^2 + 12*a*b + 5*b
^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]*(2*a + b - b*Cos[2*(c + d*x)
])^2)/(a + b)^(5/2) + 8*Sqrt[a]*(2*a + b - b*Cos[2*(c + d*x)])^2*Cot[c + d*
x] + (4*a^(3/2)*b^2*Sin[2*(c + d*x)])/(a + b) + (Sqrt[a]*b^2*(10*a + 7*b)*(
2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(a + b)^2))/(64*a^(7/2)*d*(
b + a*Csc[c + d*x]^2)^3)
```

Maple [A]

time = 0.52, size = 160, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{(12a+7b)b\left(\tan^3(dx+c)\right)}{8a+8b}+\frac{3ab(4a+3b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{3\left(8a^2+12ab+5b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{a^3}-\frac{1}{a^3\tan(dx+c)}}{d}$

default	$-\frac{\frac{b\left(\frac{\frac{(12a+7b)b\left(\tan^3(dx+c)\right)}{8a+8b}+\frac{3ab(4a+3b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{3\left(8a^2+12ab+5b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{a^3}-\frac{1}{a^3\tan(dx+c)}}{d}$
risch	$-\frac{i\left(24a^2b^2\mathrm{e}^{8i(dx+c)}+36a\,b^3\mathrm{e}^{8i(dx+c)}+15b^4\mathrm{e}^{8i(dx+c)}-144a^3b\,\mathrm{e}^{6i(dx+c)}-312a^2b^2\mathrm{e}^{6i(dx+c)}-234a\,b^3\mathrm{e}^{6i(dx+c)}-60b^4\mathrm{e}^{6i\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+sin(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b/a^3*((1/8*(12*a+7*b)*b/(a+b)*tan(d*x+c)^3+3/8*a*b*(4*a+3*b)/(a^2+2*
a*b+b^2)*tan(d*x+c))/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a)^2+3/8*(8*a^2+12*a*b+
5*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1
/2)))-1/a^3/tan(d*x+c))
```

Maxima [A]
time = 0.50, size = 270, normalized size = 1.38

$$-\frac{\frac{3\,(8\,a^2b+12\,ab^2+5\,b^3)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)a}}+\frac{(8\,a^4+32\,a^3b+60\,a^2b^2+51\,ab^3+15\,b^4)\tan(dx+c)^4+8\,a^4+16\,a^3b+8\,a^2b^2+(16\,a^4+48\,a^3b+60\,a^2b^2+25\,ab^3)\tan(dx+c)^2}{(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\tan(dx+c)^5+2\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\tan(dx+c)^3+(a^7+2\,a^6b+a^5b^2)\tan(dx+c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*arctan((a + b)*tan(d*x + c)/sqrt((a +
b)*a))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*a)) + ((8*a^4 + 32*a^3*b + 6
0*a^2*b^2 + 51*a*b^3 + 15*b^4)*tan(d*x + c)^4 + 8*a^4 + 16*a^3*b + 8*a^2*b^
2 + (16*a^4 + 48*a^3*b + 60*a^2*b^2 + 25*a*b^3)*tan(d*x + c)^2)/((a^7 + 4*a
^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*tan(d*x + c)^5 + 2*(a^7 + 3*a^6*b +
 3*a^5*b^2 + a^4*b^3)*tan(d*x + c)^3 + (a^7 + 2*a^6*b + a^5*b^2)*tan(d*x +
c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(180) = 360.
time = 0.47, size = 1003, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^5 -
 4*(16*a^5*b + 76*a^4*b^2 + 137*a^3*b^3 + 107*a^2*b^4 + 30*a*b^5)*cos(d*x +
 c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 37*a^2*b^3 + 22*a*b^4 + 5*b^5 + (8*a^2*b^
3 + 12*a*b^4 + 5*b^5)*cos(d*x + c)^4 - 2*(8*a^3*b^2 + 20*a^2*b^3 + 17*a*b^4
 + 5*b^5)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d
*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x +
 c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b +
 b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^
2))*sin(d*x + c) + 4*(8*a^6 + 40*a^5*b + 92*a^4*b^2 + 111*a^3*b^3 + 66*a^2*
b^4 + 15*a*b^5)*cos(d*x + c))/(((a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)
*d*cos(d*x + c)^4 - 2*(a^8*b + 4*a^7*b^2 + 6*a^6*b^3 + 4*a^5*b^4 + a^4*b^5)
*d*cos(d*x + c)^2 + (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 +
a^4*b^5)*d)*sin(d*x + c)), -1/16*(2*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 +
15*a*b^5)*cos(d*x + c)^5 - 2*(16*a^5*b + 76*a^4*b^2 + 137*a^3*b^3 + 107*a^2
*b^4 + 30*a*b^5)*cos(d*x + c)^3 - 3*(8*a^4*b + 28*a^3*b^2 + 37*a^2*b^3 + 22
*a*b^4 + 5*b^5 + (8*a^2*b^3 + 12*a*b^4 + 5*b^5)*cos(d*x + c)^4 - 2*(8*a^3*b
^2 + 20*a^2*b^3 + 17*a*b^4 + 5*b^5)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(
1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*
x + c)))*sin(d*x + c) + 2*(8*a^6 + 40*a^5*b + 92*a^4*b^2 + 111*a^3*b^3 + 66
*a^2*b^4 + 15*a*b^5)*cos(d*x + c))/(((a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4
*b^5)*d*cos(d*x + c)^4 - 2*(a^8*b + 4*a^7*b^2 + 6*a^6*b^3 + 4*a^5*b^4 + a^4
*b^5)*d*cos(d*x + c)^2 + (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b
^4 + a^4*b^5)*d)*sin(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 232, normalized size = 1.18

$$-\frac{\frac{3\,\left(8\,a^{2}b+12\,ab^{2}+5\,b^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan\left(dx+c\right)+b\tan\left(dx+c\right)}{\sqrt{a^{2}+ab}}\right)\right)}{\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\sqrt{a^{2}+ab}}+\frac{12\,a^{2}b^{2}\tan\left(dx+c\right)^{3}+19\,ab^{3}\tan\left(dx+c\right)^{3}+7\,b^{4}\tan\left(dx+c\right)^{3}+12\,a^{2}b^{2}\tan\left(dx+c\right)+9\,ab^{3}\tan\left(dx+c\right)}{\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\left(a\tan\left(dx+c\right)^{2}+b\tan\left(dx+c\right)^{2}+a\right)^{2}}+\frac{8}{a^{3}\tan\left(dx+c\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a
+ 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^5 +
```

```
 2*a^4*b + a^3*b^2)*sqrt(a^2 + a*b)) + (12*a^2*b^2*tan(d*x + c)^3 + 19*a*b^
3*tan(d*x + c)^3 + 7*b^4*tan(d*x + c)^3 + 12*a^2*b^2*tan(d*x + c) + 9*a*b^3
*tan(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c
)^2 + a)^2) + 8/(a^3*tan(d*x + c)))/d
```

Mupad [B]
time = 15.24, size = 251, normalized size = 1.28

$$-\frac{\frac{1}{a}+\frac{\tan(c+d\,x)^4\,\left(8\,a^3+24\,a^2\,b+36\,a\,b^2+15\,b^3\right)}{8\,a^3\,\left(a+b\right)}+\frac{\tan(c+d\,x)^2\,\left(16\,a^3+48\,a^2\,b+60\,a\,b^2+25\,b^3\right)}{8\,a^2\,\left(a^2+2\,a\,b+b^2\right)}}{d\,\left({\tan\left(c+d\,x\right)}^5\,\left(a^2+2\,a\,b+b^2\right)+a^2\,\tan\left(c+d\,x\right)+{\tan\left(c+d\,x\right)}^3\,\left(2\,a^2+2\,b\,a\right)\right)}-\frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\,\tan(c+d\,x)\,\left(a^5+2\,a^4\,b+a^3\,b^2\right)\,\left(8\,a^2+12\,a\,b+5\,b^2\right)}{a^{7/2}\,{\left(a+b\right)}^{3/2}\,\left(24\,a^2\,b+36\,a\,b^2+15\,b^3\right)}\right)\,\left(8\,a^2+12\,a\,b+5\,b^2\right)}{8\,a^{7/2}\,d\,{\left(a+b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^2)^3),x)
```

```
[Out] - (1/a + (tan(c + d*x)^4*(36*a*b^2 + 24*a^2*b + 8*a^3 + 15*b^3))/(8*a^3*(a
+ b)) + (tan(c + d*x)^2*(60*a*b^2 + 48*a^2*b + 16*a^3 + 25*b^3))/(8*a^2*(2*
a*b + a^2 + b^2)))/(d*(tan(c + d*x)^5*(2*a*b + a^2 + b^2) + a^2*tan(c + d*x
) + tan(c + d*x)^3*(2*a*b + 2*a^2))) - (3*b*atan((3*b*tan(c + d*x)*(2*a^4*b
 + a^5 + a^3*b^2)*(12*a*b + 8*a^2 + 5*b^2))/(a^(7/2)*(a + b)^(3/2)*(36*a*b^
2 + 24*a^2*b + 15*b^3)))*(12*a*b + 8*a^2 + 5*b^2))/(8*a^(7/2)*d*(a + b)^(5/
2))
```

3.111 $\int \frac{1}{(a+b\sin^2(c+dx))^4} dx$

Optimal. Leaf size=206

$$\frac{(2a+b)(8a^2+8ab+5b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^{7/2}d}+\frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3}+\frac{5b(2a+b)\cos(c+\ldots}{24a^2(a+b)^2d\left(a+\ldots\right.}$$

```
[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(
7/2)/(a+b)^(7/2)/d+1/6*b*cos(d*x+c)*sin(d*x+c)/a/(a+b)/d/(a+b*sin(d*x+c)^2)
^3+5/24*b*(2*a+b)*cos(d*x+c)*sin(d*x+c)/a^2/(a+b)^2/d/(a+b*sin(d*x+c)^2)^2+
1/48*b*(44*a^2+44*a*b+15*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/(a+b)^3/d/(a+b*sin(
d*x+c)^2)
```

Rubi [A]

time = 0.20, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3263, 3252, 12, 3260, 211}

$$\frac{5b(2a+b)\sin(c+dx)\cos(c+dx)}{24a^2d(a+b)^2\left(a+b\sin^2(c+dx)\right)^2}+\frac{(2a+b)(8a^2+8ab+5b^2)\,\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^{7/2}}+\frac{b(44a^2+44ab+15b^2)\sin(c+dx)\cos(c+dx)}{48a^3d(a+b)^3\left(a+b\sin^2(c+dx)\right)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-4),x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a
]])/(16*a^(7/2)*(a + b)^(7/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(6*a*(a +
b)*d*(a + b*Sin[c + d*x]^2)^3) + (5*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/
(24*a^2*(a + b)^2*d*(a + b*Sin[c + d*x]^2)^2) + (b*(44*a^2 + 44*a*b + 15*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(48*a^3*(a + b)^3*d*(a + b*Sin[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
```

```
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx &= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3}-\frac{\int \frac{-6a-5b+4b\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx}{6a(a+b)}\\
&= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3}+\frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2}-\frac{\int \frac{-2}{}}{}\\
&= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3}+\frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2}+\frac{b(44}{}\\
&= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3}+\frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2}+\frac{b(44}{}\\
&= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3}+\frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2}+\frac{b(44}{}\\
&= \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^{7/2}d}+\frac{b\cos(c+dx)\sin(}{6a(a+b)d\left(a+b\sin^2\right.}
\end{aligned}$$

Mathematica [A]
time = 0.98, size = 201, normalized size = 0.98

$$\frac{\frac{3\left(16a^3+24a^2b+18ab^2+5b^3\right)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{7/2}}+\frac{32a^{5/2}b\sin(2(c+dx))}{(a+b)(2a+b-b\cos(2(c+dx)))^3}+\frac{20a^{3/2}b(2a+b)\sin(2(c+dx))}{(a+b)^2(2a+b-b\cos(2(c+dx)))^2}+\frac{\sqrt{a}\,b\left(44a^2+44ab+15b^2\right)\sin(2(c+dx))}{(a+b)^3(2a+b-b\cos(2(c+dx)))}}{48a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-4),x]
```

```
[Out] ((3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*ArcTan[(Sqrt[a + b]*Tan[c + d*x]
)/Sqrt[a]])/(a + b)^(7/2) + (32*a^(5/2)*b*Sin[2*(c + d*x)])/((a + b)*(2*a +
 b - b*Cos[2*(c + d*x)])^3) + (20*a^(3/2)*b*(2*a + b)*Sin[2*(c + d*x)])/((a
 + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2) + (Sqrt[a]*b*(44*a^2 + 44*a*b + 1
5*b^2)*Sin[2*(c + d*x)])/((a + b)^3*(2*a + b - b*Cos[2*(c + d*x)])))/(48*a^
(7/2)*d)
```

Maple [A]
time = 0.47, size = 230, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{\left(24a^2+18ab+5b^2\right)b\left(\tan^5(dx+c)\right)}{16a^3(a+b)}+\frac{\left(18a^2+18ab+5b^2\right)b\left(\tan^3(dx+c)\right)}{6a^2\left(a^2+2ab+b^2\right)}+\frac{b\left(24a^2+30ab+11b^2\right)\tan(dx+c)}{16a\left(a^3+3a^2b+3a\,b^2+b^3\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^3}+\frac{\left(16a^3+24a^2b+18a\,b^2+5b^3 \right.}{16a^3\left(a^3+3a^2b+3a\,b \right.}}{d}$
default	$\frac{\frac{\frac{\left(24a^2+18ab+5b^2\right)b\left(\tan^5(dx+c)\right)}{16a^3(a+b)}+\frac{\left(18a^2+18ab+5b^2\right)b\left(\tan^3(dx+c)\right)}{6a^2\left(a^2+2ab+b^2\right)}+\frac{b\left(24a^2+30ab+11b^2\right)\tan(dx+c)}{16a\left(a^3+3a^2b+3a\,b^2+b^3\right)}}{\left(a\left(\tan^2(dx+c)\right)+b\left(\tan^2(dx+c)\right)+a\right)^3}+\frac{\left(16a^3+24a^2b+18a\,b^2+5b^3 \right.}{16a^3\left(a^3+3a^2b+3a\,b \right.}}{d}$
risch	$-\frac{i\left(15b^5\mathrm{e}^{10i(dx+c)}+1408a^5\mathrm{e}^{6i(dx+c)}+150b^5\mathrm{e}^{6i(dx+c)}-75b^5\mathrm{e}^{8i(dx+c)}-150b^5\mathrm{e}^{4i(dx+c)}+75b^5\mathrm{e}^{2i(dx+c)}-15b^5-2592a^2b^3\mathrm{e}^{\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((1/16*(24*a^2+18*a*b+5*b^2)/a^3*b/(a+b)*tan(d*x+c)^5+1/6*(18*a^2+18*a*
b+5*b^2)/a^2*b/(a^2+2*a*b+b^2)*tan(d*x+c)^3+1/16*b*(24*a^2+30*a*b+11*b^2)/a
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c))/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a)^3+
1/16*(16*a^3+24*a^2*b+18*a*b^2+5*b^3)/a^3/(a^3+3*a^2*b+3*a*b^2+b^3)/(a*(a+b
))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.56, size = 378, normalized size = 1.83

$$\frac{\frac{3\,\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sqrt{(a+b)a}}+\frac{3\,\left(24\,a^4b+66\,a^3b^2+65\,a^2b^3+28\,ab^4+5\,b^5\right)\tan(dx+c)^5+8\,\left(18\,a^4b+36\,a^3b^2+23\,a^2b^3+5\,ab^4\right)\tan(dx+c)^3+3\,\left(24\,a^4b+30\,a^3b^2+11\,a^2b^3\right)\tan(dx+c)}{a^9+3\,a^8b+3\,a^7b^2+a^6b^3+\left(a^9+6\,a^8b+15\,a^7b^2+20\,a^6b^3+15\,a^5b^4+6\,a^4b^5+a^3b^6\right)\tan(dx+c)^6+3\,\left(a^9+5\,a^8b+10\,a^7b^2+10\,a^6b^3+5\,a^5b^4+a^4b^5\right)\tan(dx+c)^4+3\,\left(a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4\right)\tan(dx+c)^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*arctan((a + b)*tan(d*x + c)/
sqrt((a + b)*a))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*a)) +
(3*(24*a^4*b + 66*a^3*b^2 + 65*a^2*b^3 + 28*a*b^4 + 5*b^5)*tan(d*x + c)^5 +
 8*(18*a^4*b + 36*a^3*b^2 + 23*a^2*b^3 + 5*a*b^4)*tan(d*x + c)^3 + 3*(24*a^
4*b + 30*a^3*b^2 + 11*a^2*b^3)*tan(d*x + c))/(a^9 + 3*a^8*b + 3*a^7*b^2 + a
^6*b^3 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5
+ a^3*b^6)*tan(d*x + c)^6 + 3*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*
a^5*b^4 + a^4*b^5)*tan(d*x + c)^4 + 3*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^
3 + a^5*b^4)*tan(d*x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(190) = 380$.
time = 0.45, size = 1361, normalized size = 6.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6)*cos(d*x + c)^6 - 1
6*a^6 - 72*a^5*b - 138*a^4*b^2 - 147*a^3*b^3 - 93*a^2*b^4 - 33*a*b^5 - 5*b^
6 - 3*(16*a^4*b^2 + 40*a^3*b^3 + 42*a^2*b^4 + 23*a*b^5 + 5*b^6)*cos(d*x + c
)^4 + 3*(16*a^5*b + 56*a^4*b^2 + 82*a^3*b^3 + 65*a^2*b^4 + 28*a*b^5 + 5*b^6
)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^
4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 -
(a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b
^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*
((44*a^4*b^3 + 88*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^5 - 2*(54*a
^5*b^2 + 157*a^4*b^3 + 167*a^3*b^4 + 79*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^3
+ 3*(24*a^6*b + 90*a^5*b^2 + 131*a^4*b^3 + 93*a^3*b^4 + 33*a^2*b^5 + 5*a*b^
6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 + 4*a^7*b^4 + 6*a^6*b^5 + 4*a^5*b^
6 + a^4*b^7)*d*cos(d*x + c)^6 - 3*(a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 + 10*a^
6*b^5 + 5*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^4 + 3*(a^10*b + 6*a^9*b^2 + 15*
a^8*b^3 + 20*a^7*b^4 + 15*a^6*b^5 + 6*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^2 -
 (a^11 + 7*a^10*b + 21*a^9*b^2 + 35*a^8*b^3 + 35*a^7*b^4 + 21*a^6*b^5 + 7*a
^5*b^6 + a^4*b^7)*d), -1/96*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6
)*cos(d*x + c)^6 - 16*a^6 - 72*a^5*b - 138*a^4*b^2 - 147*a^3*b^3 - 93*a^2*b
^4 - 33*a*b^5 - 5*b^6 - 3*(16*a^4*b^2 + 40*a^3*b^3 + 42*a^2*b^4 + 23*a*b^5
+ 5*b^6)*cos(d*x + c)^4 + 3*(16*a^5*b + 56*a^4*b^2 + 82*a^3*b^3 + 65*a^2*b^
4 + 28*a*b^5 + 5*b^6)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)
*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c))) + 2*(
(44*a^4*b^3 + 88*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^5 - 2*(54*a^
5*b^2 + 157*a^4*b^3 + 167*a^3*b^4 + 79*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^3 +
 3*(24*a^6*b + 90*a^5*b^2 + 131*a^4*b^3 + 93*a^3*b^4 + 33*a^2*b^5 + 5*a*b^6
)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 + 4*a^7*b^4 + 6*a^6*b^5 + 4*a^5*b^6
```

```
 + a^4*b^7)*d*cos(d*x + c)^6 - 3*(a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6
*b^5 + 5*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^4 + 3*(a^10*b + 6*a^9*b^2 + 15*a
^8*b^3 + 20*a^7*b^4 + 15*a^6*b^5 + 6*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^2 -
(a^11 + 7*a^10*b + 21*a^9*b^2 + 35*a^8*b^3 + 35*a^7*b^4 + 21*a^6*b^5 + 7*a^
5*b^6 + a^4*b^7)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 344, normalized size = 1.67

$$\frac{\frac{3\,(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{a^2+ab}}+\frac{72\,a^4b\tan(dx+c)^5+198\,a^3b^2\tan(dx+c)^5+195\,a^2b^3\tan(dx+c)^5+84\,ab^4\tan(dx+c)^5+15\,b^5\tan(dx+c)^5+144\,a^4b\tan(dx+c)^3+288\,a^3b^2\tan(dx+c)^3+184\,a^2b^3\tan(dx+c)^3+40\,ab^4\tan(dx+c)^3+72\,a^4b\tan(dx+c)+90\,a^3b^2\tan(dx+c)+33\,a^2b^3\tan(dx+c)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2
)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)
))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a^2 + a*b)) + (72*a^4*b*tan(
d*x + c)^5 + 198*a^3*b^2*tan(d*x + c)^5 + 195*a^2*b^3*tan(d*x + c)^5 + 84*a
*b^4*tan(d*x + c)^5 + 15*b^5*tan(d*x + c)^5 + 144*a^4*b*tan(d*x + c)^3 + 28
8*a^3*b^2*tan(d*x + c)^3 + 184*a^2*b^3*tan(d*x + c)^3 + 40*a*b^4*tan(d*x +
c)^3 + 72*a^4*b*tan(d*x + c) + 90*a^3*b^2*tan(d*x + c) + 33*a^2*b^3*tan(d*x
 + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*(a*tan(d*x + c)^2 + b*tan(d*x
 + c)^2 + a)^3))/d
```

Mupad [B]
time = 15.44, size = 339, normalized size = 1.65

$$\frac{\frac{\tan(c+d\,x)\,(24\,a^2\,b+30\,a\,b^2+11\,b^3)}{16\,a\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}+\frac{\tan(c+d\,x)^3\,(18\,a^2\,b+18\,a\,b^2+5\,b^3)}{6\,a^2\,(a^2+2\,a\,b+b^2)}+\frac{\tan(c+d\,x)^5\,(24\,a^2\,b+18\,a\,b^2+5\,b^3)}{16\,a^3\,(a+b)}}{d\,\left(\tan\left(c+d\,x\right)^6\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)+\tan\left(c+d\,x\right)^2\,\left(3\,a^3+3\,b\,a^2\right)+\tan\left(c+d\,x\right)^4\,\left(3\,a^3+6\,a^2\,b+3\,a\,b^2\right)+a^3\right)}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+b)\,(2\,a+2\,b)\,(8\,a^2+8\,a\,b+5\,b^2)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{2\,\sqrt{a}\,(a+b)^{7/2}\,(16\,a^3+24\,a^2\,b+18\,a\,b^2+5\,b^3)}\right)\,\left(2\,a+b\right)\,\left(8\,a^2+8\,a\,b+5\,b^2\right)}{16\,a^{7/2}\,d\,{\left(a+b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2)^4,x)
```

```
[Out] ((tan(c + d*x)*(30*a*b^2 + 24*a^2*b + 11*b^3))/(16*a*(3*a*b^2 + 3*a^2*b + a
^3 + b^3)) + (tan(c + d*x)^3*(18*a*b^2 + 18*a^2*b + 5*b^3))/(6*a^2*(2*a*b +
 a^2 + b^2)) + (tan(c + d*x)^5*(18*a*b^2 + 24*a^2*b + 5*b^3))/(16*a^3*(a +
```

```
b)))/(d*(tan(c + d*x)^6*(3*a*b^2 + 3*a^2*b + a^3 + b^3) + tan(c + d*x)^2*(3
*a^2*b + 3*a^3) + tan(c + d*x)^4*(3*a*b^2 + 6*a^2*b + 3*a^3) + a^3)) + (ata
n((tan(c + d*x)*(2*a + b)*(2*a + 2*b)*(8*a*b + 8*a^2 + 5*b^2)*(3*a*b^2 + 3*
a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2)*(18*a*b^2 + 24*a^2*b + 16*a^3
+ 5*b^3)))*(2*a + b)*(8*a*b + 8*a^2 + 5*b^2))/(16*a^(7/2)*d*(a + b)^(7/2))
```

3.112 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx$

Optimal. Leaf size=279

$$\frac{\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{128a^{9/2}(a+b)^{9/2}d}+\frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4}+\frac{7}{4}$$

```
[Out] 1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)*arctan((a+b)^(1/2)*t
an(d*x+c)/a^(1/2))/a^(9/2)/(a+b)^(9/2)/d+1/8*b*cos(d*x+c)*sin(d*x+c)/a/(a+b
)/d/(a+b*sin(d*x+c)^2)^4+7/48*b*(2*a+b)*cos(d*x+c)*sin(d*x+c)/a^2/(a+b)^2/d
/(a+b*sin(d*x+c)^2)^3+1/192*b*(104*a^2+104*a*b+35*b^2)*cos(d*x+c)*sin(d*x+c
)/a^3/(a+b)^3/d/(a+b*sin(d*x+c)^2)^2+5/384*b*(2*a+b)*(40*a^2+40*a*b+21*b^2)
*cos(d*x+c)*sin(d*x+c)/a^4/(a+b)^4/d/(a+b*sin(d*x+c)^2)
```

Rubi [A]

time = 0.36, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3263, 3252, 12, 3260, 211}

$$\frac{7b(2a+b)\sin(c+dx)\cos(c+dx)}{48a^2d(a+b)^2\left(a+b\sin^2(c+dx)\right)^3}+\frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\sin(c+dx)\cos(c+dx)}{384a^4d(a+b)^4\left(a+b\sin^2(c+dx)\right)}+\frac{b\left(104a^2+104ab+35b^2\right)\sin(c+dx)\cos(c+dx)}{192a^3d(a+b)^3\left(a+b\sin^2(c+dx)\right)^2}+\frac{\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{128a^{9/2}d(a+b)^{9/2}}+\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-5),x]
```

```
[Out] ((128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*ArcTan[(Sqrt[a +
b]*Tan[c + d*x])/Sqrt[a]])/(128*a^(9/2)*(a + b)^(9/2)*d) + (b*Cos[c + d*x]*
Sin[c + d*x])/(8*a*(a + b)*d*(a + b*Sin[c + d*x]^2)^4) + (7*b*(2*a + b)*Cos
[c + d*x]*Sin[c + d*x])/(48*a^2*(a + b)^2*d*(a + b*Sin[c + d*x]^2)^3) + (b*
(104*a^2 + 104*a*b + 35*b^2)*Cos[c + d*x]*Sin[c + d*x])/(192*a^3*(a + b)^3*
d*(a + b*Sin[c + d*x]^2)^2) + (5*b*(2*a + b)*(40*a^2 + 40*a*b + 21*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(384*a^4*(a + b)^4*d*(a + b*Sin[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx = \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} - \frac{\int \frac{-8a-7b+6b\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^4}\,dx}{8a(a+b)}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} - \frac{\int \frac{-4}{}}{}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b(10}{}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b(10}{}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b(10}{}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b(10}{}$$

$$= \frac{\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{128a^{9/2}(a+b)^{9/2}d} + \frac{}{8a}$$

Mathematica [A]
time = 1.28, size = 312, normalized size = 1.12

$$\frac{\frac{24(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}}+\frac{2\sqrt{a}\,b(24576a^6+73728a^5b+97280a^4b^2+71680a^3b^3+32272a^2b^4+8720ab^5+1050b^6-b(27648a^5+69120a^4b+73616a^3b^2+41304a^2b^3+12310ab^4+1575b^5)\cos(2(c+dx))+2b^2(2816a^4+5632a^3b+4816a^2b^2+2000ab^3+315b^4)\cos(4(c+dx))-400a^3b^3\cos(6(c+dx))-600a^2b^4\cos(6(c+dx))-410ab^5\cos(6(c+dx))-105b^6\cos(6(c+dx)))\sin(2(c+dx))}{(a+b)^4(2a+b-b\cos(2(c+dx)))^4}}{3072a^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-5),x]
```

```
[Out] ((24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*ArcTan[(Sqrt[
a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(9/2) + (2*Sqrt[a]*b*(24576*a^6 + 73
728*a^5*b + 97280*a^4*b^2 + 71680*a^3*b^3 + 32272*a^2*b^4 + 8720*a*b^5 + 10
50*b^6 - b*(27648*a^5 + 69120*a^4*b + 73616*a^3*b^2 + 41304*a^2*b^3 + 12310
*a*b^4 + 1575*b^5)*Cos[2*(c + d*x)] + 2*b^2*(2816*a^4 + 5632*a^3*b + 4816*a
^2*b^2 + 2000*a*b^3 + 315*b^4)*Cos[4*(c + d*x)] - 400*a^3*b^3*Cos[6*(c + d*
x)] - 600*a^2*b^4*Cos[6*(c + d*x)] - 410*a*b^5*Cos[6*(c + d*x)] - 105*b^6*C
os[6*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^4*(2*a + b - b*Cos[2*(c + d*x)]
)^4))/(3072*a^(9/2)*d)
```

Maple [A]
time = 0.46, size = 336, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{b(256a^3+288a^2b+160a\,b^2+35b^3)(\tan^7(dx+c))}{128a^4(a+b)}+\frac{(2304a^3+3168a^2b+1760a\,b^2+385b^3)b(\tan^5(dx+c))}{384a^3(a^2+2ab+b^2)}+\frac{(2304a^3+3744a^2b+2336a\,b^2- \ldots}{384a^2(a^3+3a^2b+ \ldots}}{(a(\tan^2(dx+c))+b(\tan^2(dx+c))+a)^4}$
default	$\frac{\frac{b(256a^3+288a^2b+160a\,b^2+35b^3)(\tan^7(dx+c))}{128a^4(a+b)}+\frac{(2304a^3+3168a^2b+1760a\,b^2+385b^3)b(\tan^5(dx+c))}{384a^3(a^2+2ab+b^2)}+\frac{(2304a^3+3744a^2b+2336a\,b^2- \ldots}{384a^2(a^3+3a^2b+ \ldots}}{(a(\tan^2(dx+c))+b(\tan^2(dx+c))+a)^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((1/128*b*(256*a^3+288*a^2*b+160*a*b^2+35*b^3)/a^4/(a+b)*tan(d*x+c)^7+1
/384*(2304*a^3+3168*a^2*b+1760*a*b^2+385*b^3)/a^3*b/(a^2+2*a*b+b^2)*tan(d*x
+c)^5+1/384*(2304*a^3+3744*a^2*b+2336*a*b^2+511*b^3)/a^2*b/(a^3+3*a^2*b+3*a
*b^2+b^3)*tan(d*x+c)^3+1/128*b*(256*a^3+480*a^2*b+352*a*b^2+93*b^3)/a/(a^4+
4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*tan(d*x+c))/(a*tan(d*x+c)^2+b*tan(d*x+c)^2+a
)^4+1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)/a^4/(a^4+4*a^3*b
+6*a^2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(
1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(261) = 522$.

time = 0.54, size = 588, normalized size = 2.11

$$\frac{\frac{3\,(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4)\sqrt{(a+b)a}}+\frac{3\,(256\,a^6b+1056\,a^5b^2+1792\,a^4b^3+1635\,a^3b^4+873\,a^2b^5+265\,ab^6+35\,b^7)\tan(dx+c)^7+(2304\,a^6b+7776\,a^5b^2+10400\,a^4b^3+7073\,a^3b^4+2530\,a^2b^5+385\,ab^6)\tan(dx+c)^5+(2304\,a^6b+6048\,a^5b^2+6080\,a^4b^3+2847\,a^3b^4+511\,a^2b^5)\tan(dx+c)^3+3\,(256\,a^6b+480\,a^5b^2+352\,a^4b^3+93\,a^3b^4)\tan(dx+c)}{a^{12}+4\,a^{11}b+6\,a^{10}b^2+4\,a^9b^3+a^8b^4+(a^{12}+8\,a^{11}b+28\,a^{10}b^2+56\,a^9b^3+70\,a^8b^4+56\,a^7b^5+28\,a^6b^6+8\,a^5b^7+a^4b^8)\tan(dx+c)^8+4\,(a^{12}+7\,a^{11}b+21\,a^{10}b^2+35\,a^9b^3+35\,a^8b^4+21\,a^7b^5+7\,a^6b^6+a^5b^7)\tan(dx+c)^6+6\,(a^{12}+6\,a^{11}b+15\,a^{10}b^2+20\,a^9b^3+15\,a^8b^4+6\,a^7b^5+a^6b^6)\tan(dx+c)^4+4\,(a^{12}+5\,a^{11}b+10\,a^{10}b^2+10\,a^9b^3+5\,a^8b^4+a^7b^5)\tan(dx+c)^2}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*arctan((a
 + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3
 + a^4*b^4)*sqrt((a + b)*a)) + (3*(256*a^6*b + 1056*a^5*b^2 + 1792*a^4*b^3
+ 1635*a^3*b^4 + 873*a^2*b^5 + 265*a*b^6 + 35*b^7)*tan(d*x + c)^7 + (2304*a
^6*b + 7776*a^5*b^2 + 10400*a^4*b^3 + 7073*a^3*b^4 + 2530*a^2*b^5 + 385*a*b
^6)*tan(d*x + c)^5 + (2304*a^6*b + 6048*a^5*b^2 + 6080*a^4*b^3 + 2847*a^3*b
^4 + 511*a^2*b^5)*tan(d*x + c)^3 + 3*(256*a^6*b + 480*a^5*b^2 + 352*a^4*b^3
 + 93*a^3*b^4)*tan(d*x + c))/(a^12 + 4*a^11*b + 6*a^10*b^2 + 4*a^9*b^3 + a^
8*b^4 + (a^12 + 8*a^11*b + 28*a^10*b^2 + 56*a^9*b^3 + 70*a^8*b^4 + 56*a^7*b
^5 + 28*a^6*b^6 + 8*a^5*b^7 + a^4*b^8)*tan(d*x + c)^8 + 4*(a^12 + 7*a^11*b
+ 21*a^10*b^2 + 35*a^9*b^3 + 35*a^8*b^4 + 21*a^7*b^5 + 7*a^6*b^6 + a^5*b^7)
*tan(d*x + c)^6 + 6*(a^12 + 6*a^11*b + 15*a^10*b^2 + 20*a^9*b^3 + 15*a^8*b^
4 + 6*a^7*b^5 + a^6*b^6)*tan(d*x + c)^4 + 4*(a^12 + 5*a^11*b + 10*a^10*b^2
+ 10*a^9*b^3 + 5*a^8*b^4 + a^7*b^5)*tan(d*x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(261) = 522$.

time = 0.55, size = 2017, normalized size = 7.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="fricas")
```

```
[Out] [-1/1536*(3*((128*a^4*b^4 + 256*a^3*b^5 + 288*a^2*b^6 + 160*a*b^7 + 35*b^8)
*cos(d*x + c)^8 + 128*a^8 + 768*a^7*b + 2080*a^6*b^2 + 3360*a^5*b^3 + 3555*
a^4*b^4 + 2508*a^3*b^5 + 1138*a^2*b^6 + 300*a*b^7 + 35*b^8 - 4*(128*a^5*b^3
 + 384*a^4*b^4 + 544*a^3*b^5 + 448*a^2*b^6 + 195*a*b^7 + 35*b^8)*cos(d*x +
c)^6 + 6*(128*a^6*b^2 + 512*a^5*b^3 + 928*a^4*b^4 + 992*a^3*b^5 + 643*a^2*b
^6 + 230*a*b^7 + 35*b^8)*cos(d*x + c)^4 - 4*(128*a^7*b + 640*a^6*b^2 + 1440
*a^5*b^3 + 1920*a^4*b^4 + 1635*a^3*b^5 + 873*a^2*b^6 + 265*a*b^7 + 35*b^8)*
cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4
- 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a
 + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2
*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*(5
*(80*a^5*b^4 + 200*a^4*b^5 + 202*a^3*b^6 + 103*a^2*b^7 + 21*a*b^8)*cos(d*x
```

```
+ c)^7 - (1408*a^6*b^3 + 4824*a^5*b^4 + 6724*a^4*b^5 + 4923*a^3*b^6 + 1930*
a^2*b^7 + 315*a*b^8)*cos(d*x + c)^5 + (1728*a^7*b^2 + 7456*a^6*b^3 + 13370*
a^5*b^4 + 12969*a^4*b^5 + 7327*a^3*b^6 + 2315*a^2*b^7 + 315*a*b^8)*cos(d*x
+ c)^3 - 3*(256*a^8*b + 1312*a^7*b^2 + 2848*a^6*b^3 + 3427*a^5*b^4 + 2508*a
^4*b^5 + 1138*a^3*b^6 + 300*a^2*b^7 + 35*a*b^8)*cos(d*x + c))*sin(d*x + c))
/((a^10*b^4 + 5*a^9*b^5 + 10*a^8*b^6 + 10*a^7*b^7 + 5*a^6*b^8 + a^5*b^9)*d*
cos(d*x + c)^8 - 4*(a^11*b^3 + 6*a^10*b^4 + 15*a^9*b^5 + 20*a^8*b^6 + 15*a^
7*b^7 + 6*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^6 + 6*(a^12*b^2 + 7*a^11*b^3 +
21*a^10*b^4 + 35*a^9*b^5 + 35*a^8*b^6 + 21*a^7*b^7 + 7*a^6*b^8 + a^5*b^9)*d
*cos(d*x + c)^4 - 4*(a^13*b + 8*a^12*b^2 + 28*a^11*b^3 + 56*a^10*b^4 + 70*a
^9*b^5 + 56*a^8*b^6 + 28*a^7*b^7 + 8*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^2 +
(a^14 + 9*a^13*b + 36*a^12*b^2 + 84*a^11*b^3 + 126*a^10*b^4 + 126*a^9*b^5 +
 84*a^8*b^6 + 36*a^7*b^7 + 9*a^6*b^8 + a^5*b^9)*d), -1/768*(3*((128*a^4*b^4
 + 256*a^3*b^5 + 288*a^2*b^6 + 160*a*b^7 + 35*b^8)*cos(d*x + c)^8 + 128*a^8
 + 768*a^7*b + 2080*a^6*b^2 + 3360*a^5*b^3 + 3555*a^4*b^4 + 2508*a^3*b^5 +
1138*a^2*b^6 + 300*a*b^7 + 35*b^8 - 4*(128*a^5*b^3 + 384*a^4*b^4 + 544*a^3*
b^5 + 448*a^2*b^6 + 195*a*b^7 + 35*b^8)*cos(d*x + c)^6 + 6*(128*a^6*b^2 + 5
12*a^5*b^3 + 928*a^4*b^4 + 992*a^3*b^5 + 643*a^2*b^6 + 230*a*b^7 + 35*b^8)*
cos(d*x + c)^4 - 4*(128*a^7*b + 640*a^6*b^2 + 1440*a^5*b^3 + 1920*a^4*b^4 +
 1635*a^3*b^5 + 873*a^2*b^6 + 265*a*b^7 + 35*b^8)*cos(d*x + c)^2)*sqrt(a^2
+ a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d
*x + c)*sin(d*x + c))) + 2*(5*(80*a^5*b^4 + 200*a^4*b^5 + 202*a^3*b^6 + 103
*a^2*b^7 + 21*a*b^8)*cos(d*x + c)^7 - (1408*a^6*b^3 + 4824*a^5*b^4 + 6724*a
^4*b^5 + 4923*a^3*b^6 + 1930*a^2*b^7 + 315*a*b^8)*cos(d*x + c)^5 + (1728*a^
7*b^2 + 7456*a^6*b^3 + 13370*a^5*b^4 + 12969*a^4*b^5 + 7327*a^3*b^6 + 2315*
a^2*b^7 + 315*a*b^8)*cos(d*x + c)^3 - 3*(256*a^8*b + 1312*a^7*b^2 + 2848*a^
6*b^3 + 3427*a^5*b^4 + 2508*a^4*b^5 + 1138*a^3*b^6 + 300*a^2*b^7 + 35*a*b^8
)*cos(d*x + c))*sin(d*x + c))/((a^10*b^4 + 5*a^9*b^5 + 10*a^8*b^6 + 10*a^7*
b^7 + 5*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^8 - 4*(a^11*b^3 + 6*a^10*b^4 + 15
*a^9*b^5 + 20*a^8*b^6 + 15*a^7*b^7 + 6*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^6
+ 6*(a^12*b^2 + 7*a^11*b^3 + 21*a^10*b^4 + 35*a^9*b^5 + 35*a^8*b^6 + 21*a^7
*b^7 + 7*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^4 - 4*(a^13*b + 8*a^12*b^2 + 28*
a^11*b^3 + 56*a^10*b^4 + 70*a^9*b^5 + 56*a^8*b^6 + 28*a^7*b^7 + 8*a^6*b^8 +
 a^5*b^9)*d*cos(d*x + c)^2 + (a^14 + 9*a^13*b + 36*a^12*b^2 + 84*a^11*b^3 +
 126*a^10*b^4 + 126*a^9*b^5 + 84*a^8*b^6 + 36*a^7*b^7 + 9*a^6*b^8 + a^5*b^9
)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(261) = 522.
time = 0.43, size = 524, normalized size = 1.88

$$\frac{\frac{3\,(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4)\sqrt{a^2+ab}}+\frac{768\,a^6b\tan(dx+c)^7+3168\,a^5b^2\tan(dx+c)^7+5376\,a^4b^3\tan(dx+c)^7+4905\,a^3b^4\tan(dx+c)^7+2619\,a^2b^5\tan(dx+c)^7+795\,ab^6\tan(dx+c)^7+105\,b^7\tan(dx+c)^7+2304\,a^6b\tan(dx+c)^5+7776\,a^5b^2\tan(dx+c)^5+10400\,a^4b^3\tan(dx+c)^5+7073\,a^3b^4\tan(dx+c)^5+2530\,a^2b^5\tan(dx+c)^5+385\,ab^6\tan(dx+c)^5+2304\,a^6b\tan(dx+c)^3+6048\,a^5b^2\tan(dx+c)^3+6080\,a^4b^3\tan(dx+c)^3+2847\,a^3b^4\tan(dx+c)^3+511\,a^2b^5\tan(dx+c)^3+768\,a^6b\tan(dx+c)+1440\,a^5b^2\tan(dx+c)+1056\,a^4b^3\tan(dx+c)+279\,a^3b^4\tan(dx+c)}{(a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="giac")
```

```
[Out] 1/384*(3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*(pi*floor
((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x +
c))/sqrt(a^2 + a*b)))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*sq
rt(a^2 + a*b)) + (768*a^6*b*tan(d*x + c)^7 + 3168*a^5*b^2*tan(d*x + c)^7 +
5376*a^4*b^3*tan(d*x + c)^7 + 4905*a^3*b^4*tan(d*x + c)^7 + 2619*a^2*b^5*ta
n(d*x + c)^7 + 795*a*b^6*tan(d*x + c)^7 + 105*b^7*tan(d*x + c)^7 + 2304*a^6
*b*tan(d*x + c)^5 + 7776*a^5*b^2*tan(d*x + c)^5 + 10400*a^4*b^3*tan(d*x + c
)^5 + 7073*a^3*b^4*tan(d*x + c)^5 + 2530*a^2*b^5*tan(d*x + c)^5 + 385*a*b^6
*tan(d*x + c)^5 + 2304*a^6*b*tan(d*x + c)^3 + 6048*a^5*b^2*tan(d*x + c)^3 +
 6080*a^4*b^3*tan(d*x + c)^3 + 2847*a^3*b^4*tan(d*x + c)^3 + 511*a^2*b^5*ta
n(d*x + c)^3 + 768*a^6*b*tan(d*x + c) + 1440*a^5*b^2*tan(d*x + c) + 1056*a^
4*b^3*tan(d*x + c) + 279*a^3*b^4*tan(d*x + c))/((a^8 + 4*a^7*b + 6*a^6*b^2
+ 4*a^5*b^3 + a^4*b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^4))/d
```

Mupad [B]
time = 16.10, size = 450, normalized size = 1.61

$$\frac{\frac{\tan(c+dx)\,(256\,a^3b+480\,a^2b^2+352\,ab^3+93\,b^4)}{128\,a\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)}+\frac{\tan(c+dx)^3\,(2304\,a^3b+3744\,a^2b^2+2336\,ab^3+511\,b^4)}{384\,a^2\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{\tan(c+dx)^5\,(2304\,a^3b+3168\,a^2b^2+1760\,ab^3+385\,b^4)}{384\,a^3\,(a^2+2\,a\,b+b^2)}+\frac{\tan(c+dx)^7\,(256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4)}{128\,a^4\,(a+b)}}{d\,\left(\tan(c+dx)^4\,(6\,a^4+12\,a^3b+6\,a^2b^2)+\tan(c+dx)^2\,(4\,a^4+4\,b\,a^3)+\tan(c+dx)^6\,(4\,a^4+12\,a^3b+12\,a^2b^2+4\,ab^3)+a^4+\tan(c+dx)^8\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\right)}+\frac{\mathrm{atan}\left(\frac{\tan(c+dx)\,(2\,a+2\,b)\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)}{2\,\sqrt{a}\,(a+b)^{9/2}}\right)\,(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4)}{128\,a^{9/2}\,d\,(a+b)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2)^5,x)
```

```
[Out] ((tan(c + d*x)*(352*a*b^3 + 256*a^3*b + 93*b^4 + 480*a^2*b^2))/(128*a*(4*a*
b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (tan(c + d*x)^3*(2336*a*b^3 + 230
4*a^3*b + 511*b^4 + 3744*a^2*b^2))/(384*a^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)
) + (tan(c + d*x)^5*(1760*a*b^3 + 2304*a^3*b + 385*b^4 + 3168*a^2*b^2))/(38
4*a^3*(2*a*b + a^2 + b^2)) + (tan(c + d*x)^7*(160*a*b^3 + 256*a^3*b + 35*b^
4 + 288*a^2*b^2))/(128*a^4*(a + b)))/(d*(tan(c + d*x)^4*(12*a^3*b + 6*a^4 +
 6*a^2*b^2) + tan(c + d*x)^2*(4*a^3*b + 4*a^4) + tan(c + d*x)^6*(4*a*b^3 +
12*a^3*b + 4*a^4 + 12*a^2*b^2) + a^4 + tan(c + d*x)^8*(4*a*b^3 + 4*a^3*b +
a^4 + b^4 + 6*a^2*b^2))) + (atan((tan(c + d*x)*(2*a + 2*b)*(4*a*b^3 + 4*a^3
*b + a^4 + b^4 + 6*a^2*b^2))/(2*a^(1/2)*(a + b)^(9/2)))*(160*a*b^3 + 256*a^
3*b + 128*a^4 + 35*b^4 + 288*a^2*b^2))/(128*a^(9/2)*d*(a + b)^(9/2))
```

3.113 $\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}} \, dx$

Optimal. Leaf size=11

$$-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

```
[Out] -arcsin(1/2*cos(x)*2^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3265, 222}

$$-\text{ArcSin}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -ArcSin[Cos[x]/Sqrt[2]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}} \, dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}} \, dx, x, \cos(x)\right)\\ &= -\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 29, normalized size = 2.64

$$i \log\left(i\sqrt{2}\cos(x) + \sqrt{3-\cos(2x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] I*Log[I*Sqrt[2]*Cos[x] + Sqrt[3 - Cos[2*x]]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.
time = 3.62, size = 33, normalized size = 3.00

method	result	size
default	$\frac{\sqrt{(1+\sin^2(x))(\cos^2(x))}\ \arcsin(\sin^2(x))}{2\cos(x)\sqrt{1+\sin^2(x)}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((1+sin(x)^2)*cos(x)^2)^(1/2)*arcsin(sin(x)^2)/cos(x)/(1+sin(x)^2)^(1/2
)
```

Maxima [A]
time = 0.50, size = 10, normalized size = 0.91

$$-\arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/2*sqrt(2)*cos(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(10) = 20$.
time = 0.41, size = 57, normalized size = 5.18

$$\frac{1}{2}\arctan\left(-\frac{\cos(x)\sin(x)-\left(\cos(x)^3-\cos(x)\right)\sqrt{-\cos(x)^2+2}}{\cos(x)^4-3\cos(x)^2+1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(-(cos(x)*sin(x) - (cos(x)^3 - cos(x))*sqrt(-cos(x)^2 + 2))/(cos(
x)^4 - 3*cos(x)^2 + 1)) - 1/2*arctan(sin(x)/cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{(x)}}{\sqrt{\sin^2{(x)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(sin(x)/sqrt(sin(x)**2 + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.
time = 0.44, size = 25, normalized size = 2.27

$$-\frac{1}{2}\sqrt{-\cos{(x)}^2+2}\,\cos{(x)}-\arcsin\left(\frac{1}{2}\sqrt{2}\cos{(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

Mupad [B]
time = 13.38, size = 18, normalized size = 1.64

$$\ln\left(\sqrt{\sin{(x)}^2+1}+\cos{(x)}\ 1\mathrm{i}\right)\ 1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sin(x)^2 + 1)^(1/2),x)
```

```
[Out] log(cos(x)*1i + (sin(x)^2 + 1)^(1/2))*1i
```

3.114 $\int \sin(x)\sqrt{1+\sin^2(x)}\,dx$

Optimal. Leaf size=30

$$-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)-\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}$$

[Out] -arcsin(1/2*cos(x)*2^(1/2))-1/2*cos(x)*(2-cos(x)^2)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {3265, 201, 222}

$$-\text{ArcSin}\left(\frac{\cos(x)}{\sqrt{2}}\right)-\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sqrt[1 + Sin[x]^2],x]

[Out] -ArcSin[Cos[x]/Sqrt[2]] - (Cos[x]*Sqrt[2 - Cos[x]^2])/2

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin(x)\sqrt{1+\sin^2(x)}\,dx = -\text{Subst}\left(\int \sqrt{2-x^2}\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)} - \text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}}\,dx, x, \cos(x)\right)$$
$$= -\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right) - \frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 53, normalized size = 1.77

$$-\frac{\cos(x)\sqrt{3-\cos(2x)}}{2\sqrt{2}} + i\log\left(i\sqrt{2}\cos(x) + \sqrt{3-\cos(2x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -1/2*(Cos[x]*Sqrt[3 - Cos[2*x]])/Sqrt[2] + I*Log[I*Sqrt[2]*Cos[x] + Sqrt[3
- Cos[2*x]]]
```

Maple [A]
time = 5.52, size = 51, normalized size = 1.70

method	result	size
default	$-\frac{\sqrt{\left(1+\sin^2(x)\right)\left(\cos^2(x)\right)}\left(\sqrt{-\left(\cos^4(x)\right)+2\left(\cos^2(x)\right)}+\arcsin\left(\cos^2(x)-1\right)\right)}{2\cos(x)\sqrt{1+\sin^2(x)}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(1+sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((1+sin(x)^2)*cos(x)^2)^(1/2)*((-cos(x)^4+2*cos(x)^2)^(1/2)+arcsin(cos
(x)^2-1))/cos(x)/(1+sin(x)^2)^(1/2)
```

Maxima [A]
time = 0.56, size = 25, normalized size = 0.83

$$-\frac{1}{2}\sqrt{-\cos(x)^2+2}\cos(x) - \arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.
time = 0.41, size = 71, normalized size = 2.37

$$-\frac{1}{2}\sqrt{-\cos(x)^2+2}\cos(x)+\frac{1}{2}\arctan\left(-\frac{\cos(x)\sin(x)-(\cos(x)^3-\cos(x))\sqrt{-\cos(x)^2+2}}{\cos(x)^4-3\cos(x)^2+1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) + 1/2*arctan(-(cos(x)*sin(x) - (cos(x)^3 -
cos(x))*sqrt(-cos(x)^2 + 2))/(cos(x)^4 - 3*cos(x)^2 + 1)) - 1/2*arctan(sin(
x)/cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sin^2(x)+1}\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x)**2 + 1)*sin(x), x)
```

Giac [A]
time = 0.47, size = 25, normalized size = 0.83

$$-\frac{1}{2}\sqrt{-\cos(x)^2+2}\cos(x)-\arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\sin(x)\sqrt{\sin(x)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(sin(x)^2 + 1)^(1/2),x)
```

```
[Out] int(sin(x)*(sin(x)^2 + 1)^(1/2), x)
```

3.115 $\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\cos(7+3x)\right)$$

```
[Out] -1/3*arcsin(1/2*cos(7+3*x))
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3265, 222}

$$-\frac{1}{3}\text{ArcSin}\left(\frac{1}{2}\cos(3x+7)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[7 + 3*x]/Sqrt[3 + Sin[7 + 3*x]^2],x]
```

```
[Out] -1/3*ArcSin[Cos[7 + 3*x]/2]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \cos(7+3x)\right)\right)\\ &= -\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\cos(7+3x)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 39, normalized size = 2.60

$$\frac{1}{3}i\log\left(i\sqrt{2}\cos(7+3x)+\sqrt{7-\cos(2(7+3x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[7 + 3*x]/Sqrt[3 + Sin[7 + 3*x]^2],x]
```

```
[Out] (I/3)*Log[I*Sqrt[2]*Cos[7 + 3*x] + Sqrt[7 - Cos[2*(7 + 3*x)]]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.
time = 5.33, size = 57, normalized size = 3.80

method	result	size
default	$-\frac{\sqrt{\left(3+\sin^{2}\left(7+3x\right)\right)\left(\cos^{2}\left(7+3x\right)\right)}\,\arcsin\left(-1+\frac{\left(\cos^{2}\left(7+3x\right)\right)}{2}\right)}{6\cos\left(7+3x\right)\sqrt{3+\sin^{2}\left(7+3x\right)}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((3+sin(7+3*x)^2)*cos(7+3*x)^2)^(1/2)*arcsin(-1+1/2*cos(7+3*x)^2)/cos(
7+3*x)/(3+sin(7+3*x)^2)^(1/2)
```

Maxima [A]
time = 0.52, size = 11, normalized size = 0.73

$$-\frac{1}{3}\,\arcsin\left(\frac{1}{2}\,\cos\left(3\,x+7\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*arcsin(1/2*cos(3*x + 7))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(11) = 22$.
time = 0.38, size = 94, normalized size = 6.27

$$\frac{1}{6}\,\arctan\left(-\frac{4\,\cos\left(3\,x+7\right)\sin\left(3\,x+7\right)-\left(\cos\left(3\,x+7\right)^{3}-2\,\cos\left(3\,x+7\right)\right)\sqrt{-\cos\left(3\,x+7\right)^{2}+4}}{\cos\left(3\,x+7\right)^{4}-8\,\cos\left(3\,x+7\right)^{2}+4}\right)-\frac{1}{6}\,\arctan\left(\frac{\sin\left(3\,x+7\right)}{\cos\left(3\,x+7\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(-(4*cos(3*x + 7)*sin(3*x + 7) - (cos(3*x + 7)^3 - 2*cos(3*x + 7)
)*sqrt(-cos(3*x + 7)^2 + 4))/(cos(3*x + 7)^4 - 8*cos(3*x + 7)^2 + 4)) - 1/6
*arctan(sin(3*x + 7)/cos(3*x + 7))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(3x+7)}{\sqrt{\sin^2(3x+7)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sin(3*x + 7)/sqrt(sin(3*x + 7)**2 + 3), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(11) = 22$.
time = 0.78, size = 48, normalized size = 3.20

$$\frac{2}{3}\arctan\left(-\frac{1}{2}\sqrt{3}\tan\left(\frac{3}{2}x+\frac{7}{2}\right)^2-\frac{1}{2}\sqrt{3}+\frac{1}{2}\sqrt{3\tan\left(\frac{3}{2}x+\frac{7}{2}\right)^4+10\tan\left(\frac{3}{2}x+\frac{7}{2}\right)^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(-1/2*sqrt(3)*tan(3/2*x + 7/2)^2 - 1/2*sqrt(3) + 1/2*sqrt(3*tan(3
/2*x + 7/2)^4 + 10*tan(3/2*x + 7/2)^2 + 3))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sin(3\,x+7)}{\sqrt{\sin(3\,x+7)^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x + 7)/(sin(3*x + 7)^2 + 3)^(1/2),x)
```

```
[Out] int(sin(3*x + 7)/(sin(3*x + 7)^2 + 3)^(1/2), x)
```

3.116 $\int (a-a\sin^2(x))^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\sqrt{a\cos^2(x)}\,\tan(x)+\frac{4}{15}a\big(a\cos^2(x)\big)^{3/2}\tan(x)+\frac{1}{5}\big(a\cos^2(x)\big)^{5/2}\tan(x)$$

```
[Out] 4/15*a*(a*cos(x)^2)^(3/2)*tan(x)+1/5*(a*cos(x)^2)^(5/2)*tan(x)+8/15*a^2*(a*
cos(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3255, 3282, 3286, 2717}

$$\frac{8}{15}a^2\tan(x)\sqrt{a\cos^2(x)}+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{15}a\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[a*Cos[x]^2]*Tan[x])/15 + (4*a*(a*Cos[x]^2)^(3/2)*Tan[x])/15 + (
(a*Cos[x]^2)^(5/2)*Tan[x])/5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(a-a\sin^2(x)\right)^{5/2}\,dx &= \int \left(a\cos^2(x)\right)^{5/2}\,dx\\
&= \frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{5}(4a)\int \left(a\cos^2(x)\right)^{3/2}\,dx\\
&= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{15}\left(8a^2\right)\int \sqrt{a\cos^2(x)}\,dx\\
&= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{15}\left(8a^2\sqrt{a\cos^2(x)}\sec(x)\right)\\
&= \frac{8}{15}a^2\sqrt{a\cos^2(x)}\tan(x)+\frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2\sqrt{a\cos^2(x)}\sec(x)(150\sin(x)+25\sin(3x)+3\sin(5x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(5/2),x]
[Out] (a^2*Sqrt[a*Cos[x]^2]*Sec[x]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/240
```

Maple [A]
time = 2.80, size = 32, normalized size = 0.60

method	result
default	$\frac{a^3\cos(x)\sin(x)\left(3\left(\cos^4(x)\right)+4\left(\cos^2(x)\right)+8\right)}{15\sqrt{a\left(\cos^2(x)\right)}}$
risch	$-\frac{ia^2e^{6ix}\sqrt{a\left(e^{2ix}+1\right)^2e^{-2ix}}}{160(e^{2ix}+1)}-\frac{5ia^2e^{2ix}\sqrt{a\left(e^{2ix}+1\right)^2e^{-2ix}}}{16(e^{2ix}+1)}+\frac{5ia^2\sqrt{a\left(e^{2ix}+1\right)^2e^{-2ix}}}{16(e^{2ix}+1)}+\frac{5ia^2e^{-2ix}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/15*a^3*cos(x)*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/(a*cos(x)^2)^(1/2)
```

Maxima [A]
time = 0.56, size = 31, normalized size = 0.58

$$\frac{1}{240}\left(3a^2\sin(5x)+25a^2\sin(3x)+150a^2\sin(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*a^2*sin(5*x) + 25*a^2*sin(3*x) + 150*a^2*sin(x))*sqrt(a)
```

Fricas [A]
time = 0.38, size = 40, normalized size = 0.75

$$\frac{\left(3a^2\cos(x)^4+4a^2\cos(x)^2+8a^2\right)\sqrt{a\cos(x)^2}\sin(x)}{15\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^2*cos(x)^4 + 4*a^2*cos(x)^2 + 8*a^2)*sqrt(a*cos(x)^2)*sin(x)/cos(
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(41) = 82.
time = 0.48, size = 84, normalized size = 1.58

$$-\frac{2\left(15a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^4\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)-40a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)+48a^{\frac{5}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)\right)}{15\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^4*sgn(tan(1/2*x)^4 - 1) - 40*
a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) + 48*a^(5/2)*sg
n(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a - a\sin\left(x\right)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^(5/2),x)
[Out] int((a - a*sin(x)^2)^(5/2), x)
```

3.117 $\int \left(a-a\sin^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{2}{3}a\sqrt{a\cos^2(x)}\,\tan(x)+\frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)$$

```
[Out] 1/3*(a*cos(x)^2)^(3/2)*tan(x)+2/3*a*(a*cos(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3255, 3282, 3286, 2717}

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(a-a\sin^2(x)\right)^{3/2}\,dx &= \int \left(a\cos^2(x)\right)^{3/2}\,dx \\
&= \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{3}(2a)\int \sqrt{a\cos^2(x)}\,dx \\
&= \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{3}\left(2a\sqrt{a\cos^2(x)}\,\sec(x)\right)\int \cos(x)\,dx \\
&= \frac{2}{3}a\sqrt{a\cos^2(x)}\,\tan(x)+\frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.76

$$\frac{1}{12}a\sqrt{a\cos^2(x)}\,\sec(x)(9\sin(x)+\sin(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(3/2),x]
[Out] (a*Sqrt[a*Cos[x]^2]*Sec[x]*(9*Sin[x] + Sin[3*x]))/12
```

Maple [A]
time = 3.22, size = 24, normalized size = 0.71

method	result
default	$\frac{a^2\cos(x)\sin(x)\left(\cos^2(x)+2\right)}{3\sqrt{a\left(\cos^2(x)\right)}}$
risch	$-\frac{ia\,e^{4ix}\sqrt{a\left(e^{2ix}+1\right)^2e^{-2ix}}}{24\left(e^{2ix}+1\right)}-\frac{3ia\,e^{2ix}\sqrt{a\left(e^{2ix}+1\right)^2e^{-2ix}}}{8\left(e^{2ix}+1\right)}+\frac{3ia\sqrt{a\left(e^{2ix}+1\right)^2e^{-2ix}}}{8\left(e^{2ix}+1\right)}+\frac{ia\,e^{-2ix}\sqrt{a\,(}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*a^2*cos(x)*sin(x)*(cos(x)^2+2)/(a*cos(x)^2)^(1/2)
```

Maxima [A]
time = 0.57, size = 17, normalized size = 0.50

$$\frac{1}{12}\left(a\sin\left(3\,x\right)+9\,a\sin\left(x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(a*sin(3*x) + 9*a*sin(x))*sqrt(a)
```

Fricas [A]
time = 0.39, size = 26, normalized size = 0.76

$$\frac{\left(a\cos\left(x\right)^{2}+2\,a\right)\sqrt{a\cos\left(x\right)^{2}}\,\sin\left(x\right)}{3\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(x)^2 + 2*a)*sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\sin^{2}\left(x\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**(3/2),x)
```

```
[Out] Integral((-a*sin(x)**2 + a)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(26) = 52.
time = 0.43, size = 57, normalized size = 1.68

$$-\frac{2\left(3\,a^{\frac{3}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)-4\,a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)\right)}{3\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a^(3/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) - 4*a^(
3/2)*sgn(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int{\left(a-a\,{\sin\left(x\right)}^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^(3/2),x)
```

```
[Out] int((a - a*sin(x)^2)^(3/2), x)
```

3.118 $\int \sqrt{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=13

$$\sqrt{a\cos^2(x)}\,\tan(x)$$

[Out] (a*cos(x)^2)^(1/2)*tan(x)

Rubi [A]

time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3255, 3286, 2717}

$$\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - a*Sin[x]^2],x]

[Out] Sqrt[a*Cos[x]^2]*Tan[x]

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned} \int \sqrt{a-a\sin^2(x)}\,dx &= \int \sqrt{a\cos^2(x)}\,dx\\ &= \left(\sqrt{a\cos^2(x)}\,\sec(x)\right)\int \cos(x)\,dx\\ &= \sqrt{a\cos^2(x)}\,\tan(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\sqrt{a\cos^2(x)}\,\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[x]^2],x]
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Maple [A]
time = 1.24, size = 15, normalized size = 1.15

method	result	size
default	$\frac{a\cos(x)\sin(x)}{\sqrt{a\left(\cos^2(x)\right)}}$	15
risch	$-\frac{i\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}\,e^{2ix}}{2(e^{2ix}+1)}+\frac{i\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}{2\,e^{2ix}+2}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] a*cos(x)*sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [A]
time = 0.58, size = 6, normalized size = 0.46

$$\sqrt{a}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(a)*sin(x)
```

Fricas [A]
time = 0.40, size = 15, normalized size = 1.15

$$\frac{\sqrt{a\cos\left(x\right)^2}\,\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\sin^{2}(x) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sin(x)**2 + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.
time = 0.44, size = 27, normalized size = 2.08

$$-\frac{2\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^{4} - 1\right)}{\frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*sgn(tan(1/2*x)^4 - 1)/(1/tan(1/2*x) + tan(1/2*x))
```

Mupad [B]
time = 0.22, size = 46, normalized size = 3.54

$$\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{\cos(2x) + 1}\,\left(\cos(2x) - 1 + \sin(2x)\,1\mathrm{i}\right)}{2\,\left(\cos(2x)\,1\mathrm{i} - \sin(2x) + 1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*a^(1/2)*(cos(2*x) + 1)^(1/2)*(cos(2*x) + sin(2*x)*1i - 1))/(2*(cos
(2*x)*1i - sin(2*x) + 1i))
```

3.119 $\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}$$

[Out] arctanh(sin(x))*cos(x)/(a*cos(x)^2)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3255, 3286, 3855}

$$\frac{\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a - a*Sin[x]^2],x]

[Out] (ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cos^2(x)}}\,dx$$
$$= \frac{\cos(x)\int \sec(x)\,dx}{\sqrt{a\cos^2(x)}}$$
$$= \frac{\tanh^{-1}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.
time = 0.02, size = 46, normalized size = 2.88

$$\frac{\cos(x)\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sin[x]^2],x]
[Out] (Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]))/Sqrt[a*Cos[
x]^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 20, normalized size = 1.25

method	result	size
default	$\frac{\cos(x)\mathrm{am}^{-1}(x\mid 1)}{\sqrt{a\left(\cos^2(x)\right)}\,\mathrm{csgn}(\cos(x))}$	20
risch	$-\frac{2\ln\left(e^{ix}-i\right)\cos(x)}{\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}+\frac{2\ln\left(e^{ix}+i\right)\cos(x)}{\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(a*cos(x)^2)^(1/2)/csgn(cos(x))*cos(x)*InverseJacobiAM(x,1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(14) = 28.
time = 0.61, size = 38, normalized size = 2.38

$$\frac{\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1))/sqrt(a)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.
time = 0.42, size = 65, normalized size = 4.06

$$\left[-\frac{\sqrt{a\cos\left(x\right)^{2}}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)}{2\,a\cos\left(x\right)}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos\left(x\right)^{2}}\sqrt{-a}\sin(x)}{a\cos(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(x)^2)*log(-(sin(x) - 1)/(sin(x) + 1))/(a*cos(x)), -sqrt(-a
)*arctan(sqrt(a*cos(x)^2)*sqrt(-a)*sin(x)/(a*cos(x)))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-a\sin^{2}\left(x\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sin(x)**2 + a), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-a*sin(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{a-a\sin\left(x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^(1/2),x)
[Out] int(1/(a - a*sin(x)^2)^(1/2), x)
```

3.120 $\int \frac{1}{(a-a\sin^2(x))^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

```
[Out] 1/2*arctanh(sin(x))*cos(x)/a/(a*cos(x)^2)^(1/2)+1/2*tan(x)/a/(a*cos(x)^2)^(
1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3255, 3283, 3286, 3855}

$$\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\cos(x)\tanh^{-1}(\sin(x))}{2a\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-3/2),x]
```

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]
^2])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx &= \int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx\\ &= \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{2a}\\ &= \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\cos(x)\int \sec(x)\,dx}{2a\sqrt{a\cos^2(x)}}\\ &= \frac{\tanh^{-1}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.
time = 0.04, size = 91, normalized size = 2.17

$$-\frac{\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\cos(2x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-2\sin(x)\right)}{4\left(a\cos^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-3/2),x]
[Out] -1/4*(Cos[x]*(Log[Cos[x/2] - Sin[x/2]] + Cos[2*x]*(Log[Cos[x/2] - Sin[x/2]]
 - Log[Cos[x/2] + Sin[x/2]]) - Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x]))/(a*Cos
[x]^2)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 6.52, size = 70, normalized size = 1.67

method	result	size
default	$\frac{\sqrt{a\left(\sin^2(x)\right)}\left(\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\sin^2(x)\right)}+2a}{\cos(x)}\right)a\left(\cos^2(x)\right)+\sqrt{a}\,\sqrt{a\left(\sin^2(x)\right)}\right)}{2a^{\frac{5}{2}}\cos(x)\sin(x)\sqrt{a\left(\cos^2(x)\right)}}$	70

risch	$-\frac{i(e^{2ix}-1)}{a(e^{2ix}+1)\sqrt{a\left(e^{2ix}+1\right)^{2} e^{-2ix}}}+\frac{\ln(e^{ix}+i)\cos(x)}{a\sqrt{a\left(e^{2ix}+1\right)^{2} e^{-2ix}}}-\frac{\ln(e^{ix}-i)\cos(x)}{a\sqrt{a\left(e^{2ix}+1\right)^{2} e^{-2ix}}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^(5/2)/cos(x)*(a*sin(x)^2)^(1/2)*(ln(2/cos(x)*(a^(1/2)*(a*sin(x)^2)^(1
/2)+a))*a*cos(x)^2+a^(1/2)*(a*sin(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(34) = 68$.
time = 0.63, size = 304, normalized size = 7.24

$$\frac{4\,(\sin(3\,x)-\sin(x))\cos(4\,x)+(2\,(2\cos(2\,x)+1)\cos(4\,x)+\cos(4\,x)^2+4\cos(2\,x)^2+\sin(4\,x)^2+4\sin(4\,x)\sin(2\,x)+4\sin(2\,x)^2+4\cos(2\,x)+1)\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-(2\,(2\cos(2\,x)+1)\cos(4\,x)+\cos(4\,x)^2+4\cos(2\,x)^2+\sin(4\,x)^2+4\sin(4\,x)\sin(2\,x)+4\sin(2\,x)^2+4\cos(2\,x)+1)\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)-4\,(\cos(3\,x)-\cos(x))\sin(4\,x)+4\,(2\cos(2\,x)+1)\sin(3\,x)-8\cos(3\,x)\sin(2\,x)+8\cos(x)\sin(2\,x)-8\cos(2\,x)\sin(x)-4\sin(x)}{4\left(a\cos(4\,x)^2+4\,a\cos(2\,x)^2+a\sin(4\,x)^2+4\,a\sin(4\,x)\sin(2\,x)+4\,a\sin(2\,x)^2+2\,(2\,a\cos(2\,x)+a)\cos(4\,x)+4\,a\cos(2\,x)+a\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*x) - sin(x))*cos(4*x) + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*c
os(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (2*(2*cos(2*x) + 1)*
cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4
*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(
cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x)*sin(
2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))/((a*cos(4*x)^2 + 4
*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)^2 + 2*(
2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*sqrt(a))
```

Fricas [**A**]
time = 0.40, size = 40, normalized size = 0.95

$$-\frac{\sqrt{a\cos(x)^2}\left(\cos(x)^2\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\sin(x)\right)}{4\,a^2\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(x)^2)*(cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*sin(x))
/(a^2*cos(x)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a\sin^2(x)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**(3/2),x)
```

```
[Out] Integral((-a*sin(x)**2 + a)**(-3/2), x)
```

Giac [A]
time = 0.49, size = 47, normalized size = 1.12

$$-\frac{\frac{\log\left(\left|-\sqrt{a}\tan(x)+\sqrt{a\tan\left(x\right)^{2}+a}\right|\right)}{\sqrt{a}}-\frac{\sqrt{a\tan\left(x\right)^{2}+a}\tan(x)}{a}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a)))/sqrt(a) - sqrt(a*tan
(x)^2 + a)*tan(x)/a)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a-a\sin\left(x\right)^{2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^(3/2),x)
```

```
[Out] int(1/(a - a*sin(x)^2)^(3/2), x)
```

3.121 $\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\tanh^{-1}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

```
[Out] 3/8*arctanh(sin(x))*cos(x)/a^2/(a*cos(x)^2)^(1/2)+1/4*tan(x)/a/(a*cos(x)^2)
^(3/2)+3/8*tan(x)/a^2/(a*cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3255, 3283, 3286, 3855}

$$\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\cos(x)\tanh^{-1}(\sin(x))}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-5/2),x]
```

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x])/(8*a^2*Sqrt[a*Cos[x]^2]) + Tan[x]/(4*a*(a*Cos[x]
^2)^(3/2)) + (3*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^2])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx &= \int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx\\
&= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx}{4a}\\
&= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{8a^2}\\
&= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{(3\cos(x))\int \sec(x)\,dx}{8a^2\sqrt{a\cos^2(x)}}\\
&= \frac{3\tanh^{-1}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 72, normalized size = 1.18

$$
\frac{\cos^5(x)\left(-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\sec^4(x)(11\sin(x)+3\sin(3x))\right)}{16\left(a\cos^2(x)\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-5/2),x]
[Out] (Cos[x]^5*(-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[
x]^4*(11*Sin[x] + 3*Sin[3*x]))/2))/(16*(a*Cos[x]^2)^(5/2))
```

Maple [A]
time = 7.03, size = 89, normalized size = 1.46

method	result
default	$\frac{\sqrt{a\left(\sin^2(x)\right)}\left(3\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\sin^2(x)\right)}+2a}{\cos(x)}\right)a\left(\cos^4(x)\right)+3\sqrt{a\left(\sin^2(x)\right)}\left(\cos^2(x)\right)\sqrt{a}+2\sqrt{a}\sqrt{a\left(\sin^2(x\right.}\right.}{8a^{\frac{7}{2}}\cos(x)^3\sin(x)\sqrt{a\left(\cos^2(x)\right)}}$

risch	$-\frac{i(3\,\mathrm{e}^{6ix}+11\,\mathrm{e}^{4ix}-11\,\mathrm{e}^{2ix}-3)}{4a^2(\mathrm{e}^{2ix}+1)^3\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^2\mathrm{e}^{-2ix}}}-\frac{3\ln(\mathrm{e}^{ix}-i)\cos(x)}{4a^2\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^2\mathrm{e}^{-2ix}}}+\frac{3\ln(\mathrm{e}^{ix}+i)\cos(x)}{4a^2\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^2\mathrm{e}^{-2ix}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^(7/2)/cos(x)^3*(a*sin(x)^2)^(1/2)*(3*ln(2/cos(x)*(a^(1/2)*(a*sin(x)^2
)^(1/2)+a))*a*cos(x)^4+3*(a*sin(x)^2)^(1/2)*cos(x)^2*a^(1/2)+2*a^(1/2)*(a*s
in(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(49) = 98$.
time = 0.86, size = 933, normalized size = 15.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*sin(7*x) + 11*sin(5*x) - 11*sin(3*x) - 3*sin(x))*cos(8*x) - 24*(
2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 16*(11*sin(5*x) - 11*sin(3
*x) - 3*sin(x))*cos(6*x) - 88*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) - 24*(11*s
in(3*x) + 3*sin(x))*cos(4*x) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) +
 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*c
os(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 +
4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(
4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*s
in(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x
) + 1) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x
)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(
2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(
4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin
(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^
2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(3*cos(7*x)
 + 11*cos(5*x) - 11*cos(3*x) - 3*cos(x))*sin(8*x) + 12*(4*cos(6*x) + 6*cos(
4*x) + 4*cos(2*x) + 1)*sin(7*x) - 16*(11*cos(5*x) - 11*cos(3*x) - 3*cos(x))
*sin(6*x) + 44*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) + 24*(11*cos(3*x) + 3
*cos(x))*sin(4*x) - 44*(4*cos(2*x) + 1)*sin(3*x) + 176*cos(3*x)*sin(2*x) +
48*cos(x)*sin(2*x) - 48*cos(2*x)*sin(x) - 12*sin(x))/((a^2*cos(8*x)^2 + 16*
a^2*cos(6*x)^2 + 36*a^2*cos(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 1
6*a^2*sin(6*x)^2 + 36*a^2*sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*si
n(2*x)^2 + 8*a^2*cos(2*x) + a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^
2*cos(2*x) + a^2)*cos(8*x) + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(
6*x) + 12*(4*a^2*cos(2*x) + a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4
```

```
*x) + 2*a^2*sin(2*x))*sin(8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6
*x))*sqrt(a))
```

Fricas [A]
time = 0.43, size = 49, normalized size = 0.80

$$-\frac{\left(3\,\cos\left(x\right)^{4}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\,\left(3\,\cos\left(x\right)^{2}+2\right)\sin\left(x\right)\right)\sqrt{a\cos\left(x\right)^{2}}}{16\,a^{3}\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
[Out] -1/16*(3*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*cos(x)^2 + 2)*sin(
x))*sqrt(a*cos(x)^2)/(a^3*cos(x)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a\sin^{2}\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**(5/2),x)
[Out] Integral((-a*sin(x)**2 + a)**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(49) = 98.
time = 0.52, size = 129, normalized size = 2.11

$$-\frac{3\,\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)+2\right|\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)}+\frac{3\,\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)-2\right|\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)}-\frac{5\,\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{3}-12\,\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)}{4\left(\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{2}-4\right)^{2}a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="giac")
[Out] -3/16*log(abs(1/tan(1/2*x) + tan(1/2*x) + 2))/(a^(5/2)*sgn(tan(1/2*x)^4 - 1
)) + 3/16*log(abs(1/tan(1/2*x) + tan(1/2*x) - 2))/(a^(5/2)*sgn(tan(1/2*x)^4
 - 1)) - 1/4*(5*sqrt(a)*(1/tan(1/2*x) + tan(1/2*x))^3 - 12*sqrt(a)*(1/tan(1
/2*x) + tan(1/2*x)))/(((1/tan(1/2*x) + tan(1/2*x))^2 - 4)^2*a^3*sgn(tan(1/2
*x)^4 - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{{\left(a-a\sin\left(x\right)^{2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^(5/2),x)
```

```
[Out] int(1/(a - a*sin(x)^2)^(5/2), x)
```

3.122 $\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=125

$$\frac{(a-3b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{(a-3b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8bf}-\frac{\cos(e+$$

```
[Out] 1/8*(a-3*b)*(a+b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(
3/2)/f-1/4*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)/b/f+1/8*(a-3*b)*cos(f*x+e)
*(a+b-b*cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]
time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 396, 201, 223, 209}

$$\frac{(a-3b)(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4bf}+\frac{(a-3b)\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a - 3*b)*(a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x
]^2]])/(8*b^(3/2)*f) + ((a - 3*b)*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^
2])/(8*b*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int (1-x^2)\sqrt{a+b-bx^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{4bf} + \frac{(a-3b)\text{Subst}\left(\int \sqrt{a\right.}$$

$$= \frac{(a-3b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8bf} - \frac{\cos(e+fx)\,(a}{}$$

$$= \frac{(a-3b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8bf} - \frac{\cos(e+fx)\,(a}{}$$

$$= \frac{(a-3b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a-3b)\,\text{c}}{}$$

Mathematica [A]
time = 0.30, size = 119, normalized size = 0.95

$$\frac{\frac{\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}\,(-a-4b+b\cos(2(e+fx)))}{\sqrt{2}\,b}+\frac{(a+b)(-a+3b)\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{(-b)^{3/2}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(-a - 4*b + b*Cos[2*(e +
f*x)]))/(Sqrt[2]*b) + ((a + b)*(-a + 3*b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x]
 + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/(-b)^(3/2))/(8*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(109) = 218$.
time = 7.19, size = 311, normalized size = 2.49

method	result
default	$-\frac{\sqrt{\left(\cos^{2}\left(fx+e\right)\right)\left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)}\,\left(-4\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^{2}\left(fx+e\right)\right)}\,b^{\frac{5}{2}}\left(c\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(-4*(-b*cos(f*x+e)^4+(a+b)*co
s(f*x+e)^2)^(1/2)*b^(5/2)*cos(f*x+e)^2+10*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)
^2)^(1/2)*b^(5/2)+2*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(3/2)+ar
ctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2))*a^2*b-2*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+
e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*b^2-3*b^3*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b
)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/cos(f*x+e)/(
a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.54, size = 186, normalized size = 1.49

$$\frac{\frac{(a+b)a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{(a+b)\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{4a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-4\sqrt{b}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)-4\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cos\left(fx+e\right)-\frac{2\left(-b\cos(fx+e)^{2}+a+b\right)^{\frac{3}{2}}\cos(fx+e)}{b}+\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)\cos(fx+e)}{b}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*((a + b)*a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) + (a + b)*arc
sin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 4*a*arcsin(b*cos(f*x + e)/sqr
t((a + b)*b))/sqrt(b) - 4*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt((a + b)*b)) -
4*sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e) - 2*(-b*cos(f*x + e)^2 + a +
 b)^(3/2)*cos(f*x + e)/b + sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*cos(f*x
+ e)/b)/f
```

Fricas [A]
time = 0.66, size = 501, normalized size = 4.01

$$\left[\frac{(a^2-2ab-3b^2)\sqrt{-b}\log\left(128b^4\cos(fx+e)^8-256(ab^3+b^4)\cos(fx+e)^6+160(a^2b^2+2ab^3+b^4)\cos(fx+e)^4+a^4+4a^3b+6a^2b^2+4ab^3+b^4-32(a^3b+3a^2b^2+3ab^3+b^4)\cos(fx+e)^2+8\left(16b^3\cos(fx+e)^7-24(ab^2+b^3)\cos(fx+e)^5+10(a^2b+2ab^2+b^3)\cos(fx+e)^3-(a^3+3a^2b+3ab^2+b^3)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-b}\right)+8\left(2b^2\cos(fx+e)^3-(ab+5b^2)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}}{64b^2f},\ -\frac{(a^2-2ab-3b^2)\sqrt{b}\arctan\left(\frac{\left(8b^2\cos(fx+e)^4-8(ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{b}}{4\left(2b^3\cos(fx+e)^5-3(ab^2+b^3)\cos(fx+e)^3+(a^2b+2ab^2+b^3)\cos(fx+e)\right)}\right)-4\left(2b^2\cos(fx+e)^3-(ab+5b^2)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}}{32b^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b
^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x
 + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 8*(2*b
^2*cos(f*x + e)^3 - (a*b + 5*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
+ b))/(b^2*f), -1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f
*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x
+ e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*(2*b^2*cos(f*x + e)^3 -
 (a*b + 5*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b^2*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.76, size = 131, normalized size = 1.05

$$\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\left(2\,\cos\left(fx+e\right)^{2}-\frac{abf^{4}+5\,b^{2}f^{4}}{b^{2}f^{4}}\right)\cos\left(fx+e\right)}{8\,f}+\frac{\left(a^{2}-2\,ab-3\,b^{2}\right)\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}+\frac{\sqrt{-bf^{2}}\cos(fx+e)}{f}\right|\right)}{8\,\sqrt{-b}\,b|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-b*cos(f*x + e)^2 + a + b)*(2*cos(f*x + e)^2 - (a*b*f^4 + 5*b^2*f^
4)/(b^2*f^4))*cos(f*x + e)/f + 1/8*(a^2 - 2*a*b - 3*b^2)*log(abs(sqrt(-b*co
s(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*x + e)/f))/(sqrt(-b)*b*abs(f))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.123 $\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=78

$$-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{b}\,f}-\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f}$$

```
[Out] -1/2*(a+b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/b^(1/2)-
1/2*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 201, 223, 209}

$$-\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{b}\,f}-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/2*((a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/(Sqrt[b]*f) - (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \sqrt{a+b-bx^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{(a+b)\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.}{}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+bx^2}\right.}{}$$

$$= -\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{b}f} - \frac{\cos(e+fx)\sqrt{a+}}{}$$

Mathematica [A]
time = 0.18, size = 93, normalized size = 1.19

$$-\frac{\sqrt{2}\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))} + \frac{2(a+b)\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{\sqrt{-b}}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/4*(Sqrt[2]*Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]] + (2*(a + b)*
Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/Sq
rt[-b])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(66) = 132$.
time = 7.56, size = 182, normalized size = 2.33

method	result

default	$\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(b\arctan\left(\frac{-2b(\cos^2(fx+e))+a+b}{2\sqrt{b}\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+}\right.\right.}{4\sqrt{b}\ c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(b*arctan(1/2*(-2*b*cos(f*x+e)^
2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))+a*arctan(1/2*(-2
*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*
b^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))/b^(1/2)/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.54, size = 74, normalized size = 0.95

$$-\frac{\frac{a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+\sqrt{b}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)+\sqrt{-b\cos(fx+e)^2+a+b}\cos(fx+e)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) + sqrt(b)*arcsin(b*c
os(f*x + e)/sqrt((a + b)*b)) + sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)
)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(66) = 132.
time = 0.50, size = 433, normalized size = 5.55

$$\left[-\frac{8\sqrt{-b\cos(fx+e)^2+a+b}\,b\cos(fx+e)+(a+b)\sqrt{-b}\log\left(128b^4\cos(fx+e)^8-256(ab^3+b^4)\cos(fx+e)^6+160(a^2b^2+2ab^3+b^4)\cos(fx+e)^4+a^4+4a^3b+6a^2b^2+4ab^3+b^4-32(a^3b+3a^2b^2+3ab^3+b^4)\cos(fx+e)^2+8(16b^3\cos(fx+e)^7-24(ab^2+b^3)\cos(fx+e)^5+10(a^2b+2ab^2+b^3)\cos(fx+e)^3-(a^3+3a^2b+3ab^2+b^3)\cos(fx+e))\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}\right)}{16bf}, \frac{(a+b)\sqrt{b}\arctan\left(\frac{(8b^2\cos(fx+e)^4-8(ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{b}}{4(2b^3\cos(fx+e)^5-3(ab^2+b^3)\cos(fx+e)^3+(a^2b+2ab^2+b^3)\cos(fx+e))}\right)-4\sqrt{-b\cos(fx+e)^2+a+b}\,b\cos(fx+e)}{8bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (a + b)*sqrt(-b)
*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b
^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos
(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*
cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(
```

```
f*x + e)^2 + a + b)*sqrt(-b)))/(b*f), 1/8*((a + b)*sqrt(b)*arctan(1/4*(8*b^
2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*c
os(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-b*cos(f*x
+ e)^2 + a + b)*b*cos(f*x + e))/(b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sin(e + f*x), x)
```

Giac [A]
time = 0.88, size = 83, normalized size = 1.06

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cos\left(fx+e\right)}{2\,f}-\frac{(a+b)\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}+\frac{\sqrt{-bf^{2}}\cos(fx+e)}{f}\right|\right)}{2\sqrt{-b}\,|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/f - 1/2*(a + b)*log(abs(s
qrt(-b*cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*x + e)/f))/(sqrt(-b)*ab
s(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,\sqrt{b\sin\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.124 $\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*a^(1/2)/f-arctan(co
s(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3265, 399, 223, 209, 385, 212}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/f
) - (Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f} - \frac{b\text{Subst}\left(\right.}{}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{b\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{}{\sqrt{a+}}\right.}{}$$

Mathematica [A]
time = 0.08, size = 99, normalized size = 1.19

$$\frac{-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\sqrt{-b}\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

[Out] (-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]]) + Sqrt[-b]*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b *Cos[2*(e + f*x)]]])/f

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(71) = 142$.
time = 10.36, size = 174, normalized size = 2.10

method	result
default	$-\frac{\sqrt{\left(\cos^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)}\left(\sqrt{a}\,\ln\left(\frac{-(a-b)\left(\cos^2(fx+e)\right)-2\sqrt{a}\,\sqrt{-b\left(\cos^4(fx+e)\right)+(}{\cos^2(fx+e)-1}\right.\right.}{2\cos(fx+e)\sqrt{a+b}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(a^(1/2)*ln((-(a-b)*cos(f*x+e)^2-2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-a-b)/(cos(f*x+e)^2-1))-b^(1/2)*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [A]
time = 0.53, size = 139, normalized size = 1.67

$$-\frac{2\sqrt{b}\,\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{ab+b^2}}\right)+\sqrt{a}\,\log\left(b-\frac{\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a}}{\cos(fx+e)-1}-\frac{a}{\cos(fx+e)-1}\right)-\sqrt{a}\,\log\left(-b+\frac{\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a}}{\cos(fx+e)+1}+\frac{a}{\cos(fx+e)+1}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*(2*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt(a*b + b^2)) + sqrt(a)*log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) - 1) - a/(cos(f*x + e) - 1)) - sqrt(a)*log(-b + sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1)))/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(71) = 142$.
time = 0.60, size = 1158, normalized size = 13.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [1/8*(sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^

```
2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 -
 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2
*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)
)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 2*sqrt(a)*log(2*((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b
)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/8
*(4*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) +
sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 16
0*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16
*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2
 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/f, 1/4*(sqrt(b)*arctan(1/4*(8*b^2*co
s(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f
*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + sqrt(a)*log(2*((a^2 -
6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f
, 1/4*(2*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(
f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e)
)) + sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^
2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f
*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*
x + e))))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\,\csc\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

3.125 $\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f}$$

```
[Out] -1/2*(a+b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/a^(1/2)
-1/2*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3265, 386, 385, 212}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{a}\,f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/2*((a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]
])/(Sqrt[a]*f) - (Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e + f*x])
/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2f} - \frac{(a+b)\text{Sub}}{}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2f} - \frac{(a+b)\text{Sub}}{}$$

$$= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{a}\,f} - \frac{\sqrt{a+b-b\cos^2}}{}$$

Mathematica [A]
time = 0.18, size = 100, normalized size = 1.19

$$\frac{-2(a+b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right) - \sqrt{2}\sqrt{a}\sqrt{2a+b-b\cos(2(e+fx))}\,\cot(e+fx)\csc(e+fx)}{4\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*
x]*Csc[e + f*x])/(4*Sqrt[a]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(72) = 144.
time = 8.17, size = 227, normalized size = 2.70

method	result

default	$-\frac{\sqrt{\left(\cos^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)}\,\left(a\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\,\sqrt{-b\left(\cos^4(fx+e)\right)+(a+}{\sin(fx+e)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(a*ln(((a-b)*cos(f*x+e)^2+2*a^
(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x
+e)^2+b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^2+2*a^(1/2)*(cos(f*x+e)^2*(a+b*sin(f
*x+e)^2))^(1/2))/a^(1/2)/sin(f*x+e)^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

Fricas [A]
time = 0.45, size = 338, normalized size = 4.02

$$\left[\frac{4\sqrt{-b\cos(fx+e)^2+a+b}\,a\cos(fx+e)+\left((a+b)\cos(fx+e)^2-a-b\right)\sqrt{a}\,\log\left(\frac{2\left((a^2-6ab+b^2)\cos(fx+e)^4+2(3a^2+2ab-b^2)\cos(fx+e)^2-4\left((a-b)\cos(fx+e)^3+(a+b)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a}+a^2+2ab+b^2\right)}{\cos(fx+e)^4-2\cos(fx+e)^2+1}\right)}{8\left(af\cos(fx+e)^2-af\right)},\ \frac{\left((a+b)\cos(fx+e)^2-a-b\right)\sqrt{-a}\,\arctan\left(-\frac{\left((a-b)\cos(fx+e)^2+a+b\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-a}}{2\left(ab\cos(fx+e)^3-(a^2+ab)\cos(fx+e)\right)}\right)+2\sqrt{-b\cos(fx+e)^2+a+b}\,a\cos(fx+e)}{4\left(af\cos(fx+e)^2-af\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + ((a + b)*cos(f*x +
 e)^2 - a - b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2
 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*
x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f
*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(a*f*cos(f*x + e)^2 - a*f), 1/4*(((a +
b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a
 + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 +
 a*b)*cos(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e))/(a
*f*cos(f*x + e)^2 - a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2(e+fx)}\,\csc^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{16,[4,4]%%%}+%%%{%%%{32,[1]%%%},[4,3]%%%}+%%%{%%%{16,[2]%%%},
[4,2]%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

3.126 $\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=143

$$-\frac{(3a-b)(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8a^{3/2}f}-\frac{(3a-b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(}{8af}$$

```
[Out] -1/8*(3*a-b)*(a+b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a
^(3/2)/f-1/4*(a+b-b*cos(f*x+e)^2)^(3/2)*cot(f*x+e)*csc(f*x+e)^3/a/f-1/8*(3*
a-b)*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 390, 386, 385, 212}

$$-\frac{(3a-b)(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4af}-\frac{(3a-b)\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/8*((3*a - b)*(a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e
 + f*x]^2]])/(a^(3/2)*f) - ((3*a - b)*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e
+ f*x]*Csc[e + f*x])/(8*a*f) - ((a + b - b*Cos[e + f*x]^2)^(3/2)*Cot[e + f*
x]*Csc[e + f*x]^3)/(4*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
```

```
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{(1-x^2)^3}\,dx,x,\cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b-b\cos^2(e+fx))^{3/2}\cot(e+fx)\csc^3(e+fx)}{4af} - \frac{(3a-b)\text{S}}{}$$

$$= -\frac{(3a-b)\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{8af} - \frac{(a}{}$$

$$= -\frac{(3a-b)\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{8af} - \frac{(a}{}$$

$$= -\frac{(3a-b)(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8a^{3/2}f} - \frac{(3a-b)}{}$$

Mathematica [A]
time = 0.34, size = 127, normalized size = 0.89

$$\frac{(-6a^2-4ab+2b^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)-\sqrt{2}\sqrt{a}\sqrt{2a+b-b\cos(2(e+fx))}\cot(e+fx)\csc(e+fx)\left(3a+b+2a\csc^2(e+fx)\right)}{16a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((-6*a^2 - 4*a*b + 2*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x
)]]*Cot[e + f*x]*Csc[e + f*x]*(3*a + b + 2*a*Csc[e + f*x]^2))/(16*a^(3/2)*f
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(127) = 254$.
time = 10.09, size = 379, normalized size = 2.65

method	result
default	$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\left(3a^3\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\ \sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a}{\sin(fx+e)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(3*a^3*ln(((a-b)*cos(f*x+e)^2
+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*si
n(f*x+e)^4+2*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^4*a^2-ln(((a-b)*cos(f*x+e)^2+
2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*b^2
*sin(f*x+e)^4*a+6*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^2*a^(5
/2)+2*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^2*a^(3/2)+4*(cos
(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(5/2))/sin(f*x+e)^4/a^(5/2)/cos(f*x+e
)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

Fricas [A]
time = 0.57, size = 520, normalized size = 3.64

$$\left[-\frac{\left((3a^2+2ab-b^2)\cos(fx+e)^4-2(3a^2+2ab-b^2)\cos(fx+e)^2+3a^2+2ab-b^2\right)\sqrt{a}\log\left(\frac{2\left((a^2-6ab+b^2)\cos(fx+e)^4+2(3a^2+2ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+(a+b)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}+a^2+2ab+b^2\right)}{\cos(fx+e)^4-2\cos(fx+e)^2+1}\right)-4\left((3a^2+ab)\cos(fx+e)^3-(5a^2+ab)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}}{32\left(a^2f\cos(fx+e)^4-2a^2f\cos(fx+e)^2+a^2f\right)}, \frac{\left((3a^2+2ab-b^2)\cos(fx+e)^4-2(3a^2+2ab-b^2)\cos(fx+e)^2+3a^2+2ab-b^2\right)\sqrt{-a}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^2+a+b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a}}{2\left(ab\cos(fx+e)^3-(a^2+ab)\cos(fx+e)\right)}\right)+2\left((3a^2+ab)\cos(fx+e)^3-(5a^2+ab)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}}{16\left(a^2f\cos(fx+e)^4-2a^2f\cos(fx+e)^2+a^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*a^2 + 2*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 2*a*b - b^2)*cos
(f*x + e)^2 + 3*a^2 + 2*a*b - b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f
*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e
)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 +
 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((3*a^2 + a*b)*c
os(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)
)/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f), 1/16*(((3*a^2 +
2*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 3*a^
2 + 2*a*b - b^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(
f*x + e))) + 2*((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*
sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x +
e)^2 + a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2(e + fx)}\,\csc^5(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(135) = 270.
time = 0.64, size = 962, normalized size = 6.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a)*(tan(1/2*f*x + 1/2*e)^2 + (7*a + 2*b)/a) + 8*(3*a^2
 + 2*a*b - b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
)/sqrt(-a))/(sqrt(-a)*a) - 4*(3*a^(5/2) + 2*a^(3/2)*b - sqrt(a)*b^2)*log(ab
s(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(3/2) - 2*sqr
t(a)*b))/a^2 + 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*
```

```
a^2 + 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 2*(sqr
t(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f
*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(sqrt(a)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 4*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))*a^3 - 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a))*a^2*b + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b
^2 - 3*a^(7/2))/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 -
a)^2*a))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

3.127 $\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=259

$$-\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf}-\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f}-\frac{(2a$$

```
[Out] -1/15*(a+4*b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-1/5*cos(f*
x+e)*sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2)/f-1/15*(2*a^2-3*a*b-8*b^2)*Ellip
ticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+
e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+2/15*a*(a-2*b)*(a+b)*EllipticF
(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2
/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 489, 596, 538, 437, 435, 432, 430}

$$-\frac{(2a^2-3ab-8b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a-2b)(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{15b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{(a+4b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/15*((a + 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f
) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2])/(5*f) - ((2*a^
2 - 3*a*b - 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b
/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b^2*f*Sqrt[1 + (b*Sin[e +
 f*x]^2)/a]) + (2*a*(a - 2*b)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin
[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b^
2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e\right.}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f} + \frac{\left(\sqrt{\cos^2(e+f\right.}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+}{}$$

Mathematica [A]
time = 0.95, size = 199, normalized size = 0.77

$$\frac{-16a(2a^2-3ab-8b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+32a(a^2-ab-2b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b(8a^2+48ab+25b^2-4b(4a+7b)\cos(2(e+fx))+3b^2\cos(4(e+fx)))\sin(2(e+fx))}{240b^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-16*a*(2*a^2 - 3*a*b - 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] + 32*a*(a^2 - a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e
 + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(8*a^2 + 48*a*b + 25*b^
2 - 4*b*(4*a + 7*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e + f
*x)])/(240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 6.94, size = 413, normalized size = 1.59

method	result
default	$\frac{3b^3(\sin^7(fx+e))+4a\,b^2(\sin^5(fx+e))+b^3(\sin^5(fx+e))+2\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx}{a}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(3*b^3*sin(f*x+e)^7+4*a*b^2*sin(f*x+e)^5+b^3*sin(f*x+e)^5+2*EllipticF(
sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2
)*a^3-2*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin
(f*x+e),(-1/a*b)^(1/2))*b-4*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(
1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a^3+3*EllipticE
(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*a^2*b+8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*
sin(f*x+e)^2)/a)^(1/2)*a*b^2+a^2*b*sin(f*x+e)^3-4*b^3*sin(f*x+e)^3-a^2*b*si
n(f*x+e)-4*a*b^2*sin(f*x+e))/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Fricas [F]
time = 0.11, size = 39, normalized size = 0.15

$$\mathrm{integral}\left(\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 + a
 + b), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(e+f\,x)^4\,\sqrt{b\sin(e+f\,x)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.128 $\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=159

$$-\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{a(a+b)F}{}$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/3*(a+2*b)*(cos(f*x+
e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2
)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(
a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3249, 3251, 3257, 3256, 3262, 3261}

$$-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/3*(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + ((a + 2*b)*
EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])/(3*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3249

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e
 + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p
 + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Gt
Q[p, 0]
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
```

```
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{1}{3}\int \frac{a+(a+2\cdots}{\sqrt{a+b\,\mathrm{s}\cdots}}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a(a+b))\int \frac{\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left((a+2b)\sqrt{a+\cdots}\right.}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{(a+2b)E(e+\cdots}{3bf\sqrt{\cdots}}$$

Mathematica [A]
time = 0.56, size = 159, normalized size = 1.00

$$\frac{2\sqrt{2}\,a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}\,a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+b(-2a-b+b\cos(2(e+fx)))\sin(2(e+fx))}{6\sqrt{2}\,bf\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (2*Sqrt[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 5.97, size = 266, normalized size = 1.67

method	result
default	$-\frac{-b^2(\sin^5(fx+e))+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-b^2*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^2)^(1/2
)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-2*(
cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a*b-a*b*sin(f*x+e)^3+b^2*sin(f*x+e)^3+sin(f*x+e)*a*b)/b/cos(f*x+
e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Fricas [F]
time = 0.11, size = 30, normalized size = 0.19

$$\mathrm{integral}\left(-\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\left(\cos\left(fx+e\right)^2-1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2{(e + fx)}}\,\sin^2{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sin(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.129 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=51

$$\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3257, 3256}

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$
$$= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 1.20

$$\frac{a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 3.54, size = 71, normalized size = 1.39

method	result	size
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)}{\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.10, size = 18, normalized size = 0.35

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases}\frac{\sqrt{a}\ \mathrm{E}\left(e+f\,x\middle|-\frac{b}{a}\right)}{f} & \text{if}\ \ 0<a\\ \int\sqrt{b\sin\left(e+f\,x\right)^{2}+a}\ dx & \text{if}\ \ \neg 0<a\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2),x)
[Out] piecewise(0 < a, (a^(1/2)*ellipticE(e + f*x, -b/a))/f, ~0 < a, int((a + b*s
in(e + f*x)^2)^(1/2), x))
```

3.130 $\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=174

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)

Rubi [A]

time = 0.11, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 486, 21, 434, 437, 435, 432, 430}

$$\frac{(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]

[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - (Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1-x^2}}\,dx, x, \sin(e+\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+f\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(b\sqrt{\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+f\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+f\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin\right.}{f}$$

Mathematica [A]
time = 0.41, size = 137, normalized size = 0.79

$$\frac{-\sqrt{2}\,(2a+b-b\cos(2(e+fx)))\cot(e+fx) - 2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right) + 2(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*a*Sqrt[(2*a + b
 - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*(a + b)*Sqrt[(2*a
+ b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(2*f*Sqrt[2*a + b
- b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 6.20, size = 156, normalized size = 0.90

method	result
default	$\frac{\sin(fx+e)\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\left(\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a+\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\right.\right.}{\sin(fx+e)\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-Elli
pticE(sin(f*x+e),(-1/a*b)^(1/2))*a)+b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/sin
(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.13, size = 626, normalized size = 3.60

$$-\frac{2(-2ia-ib)\sqrt{-b}\sqrt{\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}F\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}(\cos(fx+e)+i\sin(fx+e))\right)\middle|\frac{8a^2+8ab+b^2-4(2ab+b^2)\sqrt{\frac{a^2+ab}{b^2}}}{b^2}\right)\sin(fx+e)+2(2ia+ib)\sqrt{-b}\sqrt{\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}F\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}(\cos(fx+e)-i\sin(fx+e))\right)\middle|\frac{8a^2+8ab+b^2-4(2ab+b^2)\sqrt{\frac{a^2+ab}{b^2}}}{b^2}\right)\sin(fx+e)+2\sqrt{-b\cos(fx+e)^2+a+b}\,b\cos(fx+e)+\left(2i\sqrt{-b}\,b\sqrt{\frac{a^2+ab}{b^2}}\sin(fx+e)\ldots\right.\text{[illegible]}}{2bf\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(-2*I*a - I*b)*sqrt(-b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/
b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x
 + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2
+ a*b)/b^2))/b^2)*sin(f*x + e) + 2*(2*I*a + I*b)*sqrt(-b)*sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)*sin(f*x + e) + 2*sqrt(-b*cos(f*
x + e)^2 + a + b)*b*cos(f*x + e) + (2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2)*si
n(f*x + e) + (2*I*a + I*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b
)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*
a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b
 + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2
)*sin(f*x + e) + (-2*I*a - I*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2
```

```
+ a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2)
 + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(
2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(b*f*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\,\csc^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

3.131 $\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=234

$$-\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(2a+b)\sqrt{\text{co}}}{}$$

```
[Out] -1/3*(2*a+b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-1/3*cot(f*x+e)*csc(f*x
+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*(2*a+b)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(
f*x+e)^2/a)^(1/2)+2/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(
cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 486, 597, 538, 437, 435, 432, 430}

$$\frac{2(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3af\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/3*((2*a + b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*f) - (Cot[e + f
*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((2*a + b)*Sqrt[Cos[
e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b
*Sin[e + f*x]^2])/(3*a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (2*(a + b)*Sqrt[
Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1
 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{1-x^2}}\,dx, x, \sin(e\right.}{f}$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cos^2(e+\right.}{}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(}{}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(}{}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(}{}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(}{}$$

Mathematica [A]
time = 2.13, size = 188, normalized size = 0.80

$$\frac{\frac{(4(2a^2+4ab+b^2)\cos(2(e+fx))-(2a+b)(8a+3b+b\cos(4(e+fx))))\cot(e+fx)\csc^2(e+fx)}{2\sqrt{2}} - 2a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right) + 4a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{6af\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((4*(2*a^2 + 4*a*b + b^2)*Cos[2*(e + f*x)] - (2*a + b)*(8*a + 3*b + b*Cos[
4*(e + f*x)]))*Cot[e + f*x]*Csc[e + f*x]^2)/(2*Sqrt[2]) - 2*a*(2*a + b)*Sqr
```

```
t[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 4*a*(a + b
)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6*a*f
*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 6.83, size = 342, normalized size = 1.46

method	result
default	$2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\operatorname{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2(\sin^3(fx+e))+2b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b}{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+2*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-2*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(
1/2))*a^2*sin(f*x+e)^3-(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+2*a*b*sin(f*x+e)^6+b^2*
sin(f*x+e)^6+2*a^2*sin(f*x+e)^4-b^2*sin(f*x+e)^4-a^2*sin(f*x+e)^2-2*a*b*sin
(f*x+e)^2-a^2)/a/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.17, size = 947, normalized size = 4.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*((-2*I*a*b - I*b^2)*cos(f*x + e)^2 + 2*I*a*b + I*b^2)*sqrt(-b)*sqrt
((a^2 + a*b)/b^2)*sin(f*x + e) - ((4*I*a^2 + 4*I*a*b + I*b^2)*cos(f*x + e)^
2 - 4*I*a^2 - 4*I*a*b - I*b^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 +
```

```
 a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2)
+ 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2
*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((2*I*a*b + I*b^2)*cos(f*x + e
)^2 - 2*I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((-4*I
*a^2 - 4*I*a*b - I*b^2)*cos(f*x + e)^2 + 4*I*a^2 + 4*I*a*b + I*b^2)*sqrt(-b
)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(ar
csin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*
x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2
) - 2*(2*((-I*a*b - I*b^2)*cos(f*x + e)^2 + I*a*b + I*b^2)*sqrt(-b)*sqrt((a
^2 + a*b)/b^2)*sin(f*x + e) + ((-2*I*a^2 - I*a*b)*cos(f*x + e)^2 + 2*I*a^2
+ I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/
b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x
 + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2
+ a*b)/b^2))/b^2) - 2*(2*((I*a*b + I*b^2)*cos(f*x + e)^2 - I*a*b - I*b^2)*s
qrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + ((2*I*a^2 + I*a*b)*cos(f*x + e
)^2 - 2*I*a^2 - I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^
2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b
^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*((2*a*b + b^2)*cos(f*x + e)^3 - (3*a*b
+ b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a*b*f*cos(f*x + e)^
2 - a*b*f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\csc^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

3.132 $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=169

$$\frac{(a-5b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{16bf}$$

```
[Out] 1/16*(a-5*b)*(a+b)^2*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/
b^(3/2)/f+1/24*(a-5*b)*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)/b/f-1/6*cos(f*
x+e)*(a+b-b*cos(f*x+e)^2)^(5/2)/b/f+1/16*(a-5*b)*(a+b)*cos(f*x+e)*(a+b-b*co
s(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]

time = 0.10, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 396, 201, 223, 209}

$$\frac{(a-5b)(a+b)^2\text{ArcTan}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6bf}+\frac{(a-5b)\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{24bf}+\frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{16bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 5*b)*(a + b)^2*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f
*x]^2]])/(16*b^(3/2)*f) + ((a - 5*b)*(a + b)*Cos[e + f*x]*Sqrt[a + b - b*Co
s[e + f*x]^2])/(16*b*f) + ((a - 5*b)*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2
)^(3/2))/(24*b*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(5/2))/(6*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int (1-x^2)\left(a+b-bx^2\right)^{3/2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{5/2}}{6bf} + \frac{(a-5b)\text{Subst}\left(\int (a\right.}{}$$

$$= \frac{(a-5b)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{24bf} - \frac{\cos(e+fx)\,(}{}$$

$$= \frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{16bf} + \frac{(a-5b}{}$$

$$= \frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{16bf} + \frac{(a-5b}{}$$

$$= \frac{(a-5b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\,\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{(a-5b)}{}$$

Mathematica [A]
time = 0.53, size = 152, normalized size = 0.90

$$\frac{-\frac{\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}\,\left(3a^2+29ab+23b^2-b(7a+9b)\cos(2(e+fx))+b^2\cos(4(e+fx))\right)}{3\sqrt{2}\,b}+\frac{(a+b)^2(-a+5b)\log\left(\sqrt{2}\,\sqrt{-b}\,\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{(-b)^{3/2}}}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-1/3*(Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(3*a^2 + 29*a*b + 23
*b^2 - b*(7*a + 9*b)*Cos[2*(e + f*x)] + b^2*Cos[4*(e + f*x)]))/(Sqrt[2]*b)
+ ((a + b)^2*(-a + 5*b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b -
b*Cos[2*(e + f*x)]]])/(-b)^(3/2))/(16*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(149) = 298$.
time = 9.45, size = 446, normalized size = 2.64

method	result
default	$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\,\left(16\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\,b^{\frac{7}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(16*(-b*cos(f*x+e)^4+(a+b)*co
s(f*x+e)^2)^(1/2)*b^(7/2)*cos(f*x+e)^4-4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)*b^(5/2)*(13*b+7*a)*cos(f*x+e)^2+66*b^(7/2)*(-b*cos(f*x+e)^4+(a+b)*
cos(f*x+e)^2)^(1/2)+72*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(5/2)
+6*a^2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(3/2)+3*arctan(1/2*(-2*
b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*a^3
*b-9*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2))*a^2*b^2-27*b^3*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2
)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-15*b^4*arctan(1/2*(-2*b*cos(f
*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/c
os(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.50, size = 262, normalized size = 1.55

$$\frac{\frac{3\,(a+b)^2 a \arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{3\,(a+b)^2 \arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{18\,(a+b)a \arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-18\,(a+b)\sqrt{b}\,\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)-12\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}\cos\left(fx+e\right)-18\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,(a+b)\cos\left(fx+e\right)-\frac{8\left(-b\cos(fx+e)^2+a+b\right)^{\frac{5}{2}}\cos(fx+e)}{b}+\frac{2\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\cos(fx+e)}{b}+\frac{3\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,(a+b)^2\cos(fx+e)}{b}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(a + b)^2*a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) + 3*(a +
 b)^2*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*a*arcsin(
b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*sqrt(b)*arcsin(b*cos(f
*x + e)/sqrt((a + b)*b)) - 12*(-b*cos(f*x + e)^2 + a + b)^(3/2)*cos(f*x + e
) - 18*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*cos(f*x + e) - 8*(-b*cos(f*x
 + e)^2 + a + b)^(5/2)*cos(f*x + e)/b + 2*(-b*cos(f*x + e)^2 + a + b)^(3/2)
```

```
*(a + b)*cos(f*x + e)/b + 3*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2*cos(f
*x + e)/b)/f
```

Fricas [A]
time = 1.43, size = 579, normalized size = 3.43

$$\left[\frac{3\,(a^3-3\,a^2b-9\,ab^2-5\,b^3)\sqrt{-b}\log\left(128\,b^4\cos(fx+e)^8-256\,(ab^3+b^4)\cos(fx+e)^6+160\,(a^2b^2+2\,ab^3+b^4)\cos(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4-32\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\cos(fx+e)^2+8\,(16\,b^3\cos(fx+e)^7-24\,(ab^2+b^3)\cos(fx+e)^5+10\,(a^2b+2\,ab^2+b^3)\cos(fx+e)^3-(a^3+3\,a^2b+3\,ab^2+b^3)\cos(fx+e))\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}\right)-8\,(8\,b^3\cos(fx+e)^5-2\,(7\,ab^2+13\,b^3)\cos(fx+e)^3+3\,(a^2b+12\,ab^2+11\,b^3)\cos(fx+e))\sqrt{-b\cos(fx+e)^2+a+b}}{384\,b^2f},\ -\frac{3\,(a^3-3\,a^2b-9\,ab^2-5\,b^3)\sqrt{b}\arctan\left(\frac{(8\,b^2\cos(fx+e)^4-8\,(ab+b^2)\cos(fx+e)^2+a^2+2\,ab+b^2)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{b}}{4\,(2\,b^3\cos(fx+e)^5-3\,(ab^2+b^3)\cos(fx+e)^3+(a^2b+2\,ab^2+b^3)\cos(fx+e))}\right)+4\,(8\,b^3\cos(fx+e)^5-2\,(7\,ab^2+13\,b^3)\cos(fx+e)^3+3\,(a^2b+12\,ab^2+11\,b^3)\cos(fx+e))\sqrt{-b\cos(fx+e)^2+a+b}}{192\,b^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*log(128*b^4*cos(f*x +
e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos
(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2
 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqr
t(-b)) - 8*(8*b^3*cos(f*x + e)^5 - 2*(7*a*b^2 + 13*b^3)*cos(f*x + e)^3 + 3*
(a^2*b + 12*a*b^2 + 11*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/
(b^2*f), -1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)
*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*(8*b^3*cos(f*x
 + e)^5 - 2*(7*a*b^2 + 13*b^3)*cos(f*x + e)^3 + 3*(a^2*b + 12*a*b^2 + 11*b^
3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b^2*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.86, size = 197, normalized size = 1.17

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\left(\frac{2\left(4\,bf^2\cos(fx+e)^2-\frac{7\,ab^4f^{10}+13\,b^5f^{10}}{b^4f^8}\right)\cos(fx+e)^2}{f^2}+\frac{3\,(a^2b^3f^8+12\,ab^4f^8+11\,b^5f^8)}{b^4f^8}\right)\cos\left(fx+e\right)}{48\,f}+\frac{(a^3-3\,a^2b-9\,ab^2-5\,b^3)\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^2+a+b}+\frac{\sqrt{-bf^2}\cos(fx+e)}{f}\right|\right)}{16\,\sqrt{-b}\,b|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(-b*cos(f*x + e)^2 + a + b)*(2*(4*b*f^2*cos(f*x + e)^2 - (7*a*b^4
*f^10 + 13*b^5*f^10)/(b^4*f^8))*cos(f*x + e)^2/f^2 + 3*(a^2*b^3*f^8 + 12*a*
```

```
b^4*f^8 + 11*b^5*f^8)/(b^4*f^8))*cos(f*x + e)/f + 1/16*(a^3 - 3*a^2*b - 9*a
*b^2 - 5*b^3)*log(abs(sqrt(-b*cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*
x + e)/f))/(sqrt(-b)*b*abs(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+fx)^3 \left(b\sin(e+fx)^2+a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.133 $\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=114

$$-\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{b}\,f}-\frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8f}-\frac{\cos(e+fx}{}$$

```
[Out] -1/4*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)/f-3/8*(a+b)^2*arctan(cos(f*x+e)*
b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/b^(1/2)-3/8*(a+b)*cos(f*x+e)*(a+b-b*c
os(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 201, 223, 209}

$$-\frac{3(a+b)^2\text{ArcTan}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8\sqrt{b}\,f}-\frac{3(a+b)\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a + b)^2*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/(8*Sqrt[b]*f) - (3*(a + b)*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(
8*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int \left(a+b-bx^2\right)^{3/2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{4f} - \frac{(3(a+b))\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8f} - \frac{\cos(e+fx)}{}$$

$$= -\frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8f} - \frac{\cos(e+fx)}{}$$

$$= -\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{b}\,f} - \frac{3(a+b)\cos(}{}$$

Mathematica [A]
time = 0.27, size = 113, normalized size = 0.99

$$-\frac{\frac{\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}\,(5a+4b-b\cos(2(e+fx)))}{\sqrt{2}}+\frac{3(a+b)^2\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{\sqrt{-b}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(5*a + 4*b - b*Cos[2
*(e + f*x)]))/Sqrt[2] + (3*(a + b)^2*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sq
rt[2*a + b - b*Cos[2*(e + f*x)]]])/Sqrt[-b])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(98) = 196$.
time = 8.80, size = 309, normalized size = 2.71

method	result
default	$\frac{\sqrt{(\cos^2(fx+e))\,(a+b(\sin^2(fx+e)))}\,\left(4\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,b^{\frac{3}{2}}(\cos^2(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(3/2)*cos(f*x+e)^2-10*b^(3/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+3*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*a^2+6*b*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))+3*b^2*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-10*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(1/2))/b^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [A]

time = 0.50, size = 112, normalized size = 0.98

$$-\frac{\frac{3\,(a+b)a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+3\,(a+b)\sqrt{b}\,\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)+2\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}\cos(fx+e)+3\sqrt{-b\cos(fx+e)^2+a+b}\,(a+b)\cos(fx+e)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] -1/8*(3*(a + b)*a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 3*(a + b)*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt((a + b)*b)) + 2*(-b*cos(f*x + e)^2 + a + b)^(3/2)*cos(f*x + e) + 3*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*cos(f*x + e))/f

Fricas [A]

time = 0.65, size = 495, normalized size = 4.34

$$\left[-\frac{3\,(a^2+2\,ab+b^2)\sqrt{-b}\,\log\left(128\,b^4\cos(fx+e)^8-256\,(ab^3+b^4)\cos(fx+e)^6+160\,(a^2b^2+2\,ab^3+b^4)\cos(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4-32\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\cos(fx+e)^2+8\left(16\,b^3\cos(fx+e)^7-24\,(ab^2+b^3)\cos(fx+e)^5+10\,(a^2b+2\,ab^2+b^3)\cos(fx+e)^3-(a^3+3\,a^2b+3\,ab^2+b^3)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-b}\right)-8\left(2\,b^2\cos(fx+e)^3-5\,(ab+b^2)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}}{64\,bf},\ \frac{3\,(a^2+2\,ab+b^2)\sqrt{b}\,\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\,(ab+b^2)\cos(fx+e)^2+a^2+2\,ab+b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{b}}{4\left(2\,b^3\cos(fx+e)^5-3\,(ab^2+b^3)\cos(fx+e)^3+(a^2b+2\,ab^2+b^3)\cos(fx+e)\right)}\right)+4\left(2\,b^2\cos(fx+e)^3-5\,(ab+b^2)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}}{32\,bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*

```
x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) - 8*(2*
b^2*cos(f*x + e)^3 - 5*(a*b + b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
 + b))/(b*f), 1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x
 + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x +
e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*(2*b^2*cos(f*x + e)^3 - 5
*(a*b + b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.79, size = 128, normalized size = 1.12

$$\frac{\left(2\,b\cos\left(fx+e\right)^{2}-\frac{5\,\left(ab^{2}f^{4}+b^{3}f^{4}\right)}{b^{2}f^{4}}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cos\left(fx+e\right)}{8\,f}-\frac{3\,\left(a^{2}+2\,ab+b^{2}\right)\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}+\frac{\sqrt{-bf^{2}}\cos(fx+e)}{f}\right|\right)}{8\,\sqrt{-b}\,|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*cos(f*x + e)^2 - 5*(a*b^2*f^4 + b^3*f^4)/(b^2*f^4))*sqrt(-b*cos(f*
x + e)^2 + a + b)*cos(f*x + e)/f - 3/8*(a^2 + 2*a*b + b^2)*log(abs(sqrt(-b*
cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*x + e)/f))/(sqrt(-b)*abs(f))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.134 $\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2f}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}-\frac{b\cos(e+fx}{}$$

```
[Out] -a^(3/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f-1/2*(3*a+
b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*b*co
s(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3265, 427, 537, 223, 209, 385, 212}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}-\frac{b\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e
+ f*x]^2]])/f - (a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[
e + f*x]^2]])/f - (b*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{1-x^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{-(a+b)(2a+b)}{(1-x^2)\sqrt{a+}}\right.}{}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{a^2\text{Subst}\left(\int \frac{}{(1-x^2)\sqrt{a}}\right.}{}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{a^2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx\right.}{}$$

$$= -\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2f} - \frac{a^{3/2}\tanh^{-1}}{}$$

Mathematica [A]
time = 0.63, size = 141, normalized size = 1.16

$$-\frac{4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\sqrt{2}\,b\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}-2\sqrt{-b}\,(3a+b)\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(4*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos
[2*(e + f*x)]]] + Sqrt[2]*b*Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
 - 2*Sqrt[-b]*(3*a + b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b -
b*Cos[2*(e + f*x)]]])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(104) = 208.
time = 10.34, size = 255, normalized size = 2.09

method	result
default	$\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(b^{\frac{3}{2}}\arctan\left(\frac{-2b\left(\cos^2(fx+e)\right)+a+b}{2\sqrt{b}\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx\right.}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(b^(3/2)*arctan(1/2*(-2*b*cos(f
*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*a^(3/2)*
ln((-(a-b)*cos(f*x+e)^2-2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2
)-a-b)/(cos(f*x+e)^2-1))+3*b^(1/2)*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(
1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*b*(-b*cos(f*x+e)^4+(a+b)
*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.54, size = 189, normalized size = 1.55

$$-\frac{3\,a\sqrt{b}\,\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{ab+b^2}}\right)+b^{\frac{3}{2}}\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{ab+b^2}}\right)+\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,b\cos\left(fx+e\right)+a^{\frac{3}{2}}\log\left(b-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)-1}-\frac{a}{\cos\left(fx+e\right)-1}\right)-a^{\frac{3}{2}}\log\left(-b+\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)+1}+\frac{a}{\cos\left(fx+e\right)+1}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*a*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt(a*b + b^2)) + b^(3/2)*arcsin(b
*cos(f*x + e)/sqrt(a*b + b^2)) + sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x
+ e) + a^(3/2)*log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e
) - 1) - a/(cos(f*x + e) - 1)) - a^(3/2)*log(-b + sqrt(-b*cos(f*x + e)^2 +
a + b)*sqrt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(104) = 208.
time = 0.70, size = 1282, normalized size = 10.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) - (3*a + b)*sqrt(-
b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2
*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
+ b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16*b^3*c
os(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3
)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)) - 4*a^(3/2)*log(2*((a^2 - 6*a*b + b^2)*cos(
f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x +
e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2
+ 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/16*(8*sqrt(-a
)*a*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) - 8*sqrt(-b
*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (3*a + b)*sqrt(-b)*log(128*b^4*co
s(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 +
```

```
b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^7 - 2
4*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3
- (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(-b)))/f, 1/8*((3*a + b)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4
- 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (
a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b
*cos(f*x + e) + 2*a^(3/2)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*
a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos
(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(co
s(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/8*(4*sqrt(-a)*a*arctan(-1/2*((a
 - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b
*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) + (3*a + b)*sqrt(b)*arctan(1/4
*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 +
b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-b*co
s(f*x + e)^2 + a + b)*b*cos(f*x + e))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}} \csc\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2)*csc(e + f*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}{\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x),x)
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

3.135 $\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=128

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2f}-\frac{a\sqrt{a+b-b}}{}$$

```
[Out] -b^(3/2)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f-1/2*(a+3*b
)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*a^(1/2)/f-1/2*a*co
t(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.280$, Rules used = {3265, 424, 537, 223, 209, 385, 212}

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}-\frac{a\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b^(3/2)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/f
) - (Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e
+ f*x]^2]])/(2*f) - (a*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e +
f*x])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^{3/2}}{(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{a\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2f} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{a\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2f} - \frac{b^2\text{Subs}}{}$$

$$= -\frac{a\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2f} - \frac{b^2\text{Subs}}{}$$

$$= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{\sqrt{a}\,(a+3b)\tanh^{-}}{}$$

Mathematica [A]
time = 0.78, size = 147, normalized size = 1.15

$$-\frac{2\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\sqrt{2}\,a\sqrt{2a+b-b\cos(2(e+fx))}\,\cot(e+fx)\csc(e+fx)+4(-b)^{3/2}\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(2*Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]]] + Sqrt[2]*a*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Co
t[e + f*x]*Csc[e + f*x] + 4*(-b)^(3/2)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] +
Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(110) = 220.
time = 11.00, size = 287, normalized size = 2.24

method	result
default	$\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(2b^{\frac{3}{2}}\arctan\left(\frac{2b\left(\sin^2(fx+e)\right)+a-b}{2\sqrt{b}\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(2*b^(3/2)*arctan(1/2/b^(1/2)*(
2*b*sin(f*x+e)^2+a-b)/(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))*sin(f*x+e)^2
-a^(3/2)*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)
^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^2-3*a^(1/2)*b*ln(((a-b)*cos(f*x+e)^
2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*s
in(f*x+e)^2-2*a*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/sin(f*x+e)^2/cos(f
*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(110) = 220.
time = 0.73, size = 1449, normalized size = 11.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + (b*cos(f*x + e)^2
- b)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6
 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 -
8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*
a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + ((a + 3*b)*cos(f*x + e)^2 - a
- 3*b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b
 - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^
4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x + e)^2 - f), 1/8*(2*((a + 3*b)*cos(f
*x + e)^2 - a - 3*b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*
cos(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + (b*cos(
f*x + e)^2 - b)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos
(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*
```

```
x + e)^2 - 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*
(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
s(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/(f*cos(f*x + e)^2 -
f), 1/8*(2*(b*cos(f*x + e)^2 - b)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4
- 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (
a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*a
*cos(f*x + e) + ((a + 3*b)*cos(f*x + e)^2 - a - 3*b)*sqrt(a)*log(2*((a^2 -
6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(
f*cos(f*x + e)^2 - f), 1/4*(((a + 3*b)*cos(f*x + e)^2 - a - 3*b)*sqrt(-a)*a
rctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) + (b*cos(f*x + e
)^2 - b)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x +
 e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*c
os(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*co
s(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e))/(f*cos(f*x
 + e)^2 - f)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}{{\sin\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

3.136 $\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=128

$$-\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{a}\,f}-\frac{3(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{8f}$$

```
[Out] -1/4*(a+b-b*cos(f*x+e)^2)^(3/2)*cot(f*x+e)*csc(f*x+e)^3/f-3/8*(a+b)^2*arcta
nh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/a^(1/2)-3/8*(a+b)*cot(f
*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3265, 386, 385, 212}

$$-\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8\sqrt{a}\,f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}-\frac{3(a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a + b)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]
])/(8*Sqrt[a]*f) - (3*(a + b)*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*C
sc[e + f*x])/(8*f) - ((a + b - b*Cos[e + f*x]^2)^(3/2)*Cot[e + f*x]*Csc[e +
 f*x]^3)/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
```

```
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{\left(1-x^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{4f} - \frac{(3(a}{}$$

$$= -\frac{3(a+b)\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{8f} -$$

$$= -\frac{3(a+b)\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{8f} -$$

$$= -\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{a}\,f} - \frac{3(a+b)\sqrt{}}{}$$

Mathematica [A]
time = 0.44, size = 114, normalized size = 0.89

$$-\frac{\frac{6(a+b)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{\sqrt{a}}+\sqrt{2}\sqrt{2a+b-b\cos(2(e+fx))}\,\cot(e+fx)\csc(e+fx)\left(3a+5b+2a\csc^2(e+fx)\right)}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/16*((6*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b
*Cos[2*(e + f*x)]]])/Sqrt[a] + Sqrt[2]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*C
ot[e + f*x]*Csc[e + f*x]*(3*a + 5*b + 2*a*Csc[e + f*x]^2))/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(112) = 224$.
time = 10.38, size = 376, normalized size = 2.94

method	result
default	$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\,\left(3a^2\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\,\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+}{\sin(fx+e)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(3*a^2*ln(((a-b)*cos(f*x+e)^2
+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*si
n(f*x+e)^4+6*a*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*co
s(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^4+3*b^2*ln(((a-b)*cos(f*x+e
)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)
*sin(f*x+e)^4+6*a^(3/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^
2+10*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(1/2)*sin(f*x+e)^2+4*a^(3/
2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/a^(1/2)/sin(f*x+e)^4/cos(f*x+e)
/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

Fricas [A]
time = 0.59, size = 484, normalized size = 3.78

$$\left[\frac{3\left((a^2+2ab+b^2)\cos(fx+e)^4-2(a^2+2ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{a}\log\left(\frac{2\left((a^2-6ab+b^2)\cos(fx+e)^4+2(3a^2+2ab-b^2)\cos(fx+e)^2-4\left((a-b)\cos(fx+e)^3+(a+b)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a}+a^2+2ab+b^2\right)}{\cos(fx+e)^4-2\cos(fx+e)^2+1}\right)+4\left((3a^2+5ab)\cos(fx+e)^3-5(a^2+ab)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}}{32\left(af\cos(fx+e)^4-2af\cos(fx+e)^2+af\right)}, \frac{3\left((a^2+2ab+b^2)\cos(fx+e)^4-2(a^2+2ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{-a}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^2+a+b\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-a}}{2\left(ab\cos(fx+e)^3-(a^2+ab)\cos(fx+e)\right)}\right)+2\left((3a^2+5ab)\cos(fx+e)^3-5(a^2+ab)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}}{16\left(af\cos(fx+e)^4-2af\cos(fx+e)^2+af\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x +
e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 +
 (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*
```

```
b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((3*a^2 + 5*a*b)*cos(
f*x + e)^3 - 5*(a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(
a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f), 1/16*(3*((a^2 + 2*a*b + b
^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b
^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) +
2*((3*a^2 + 5*a*b)*cos(f*x + e)^3 - 5*(a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos
(f*x + e)^2 + a + b))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

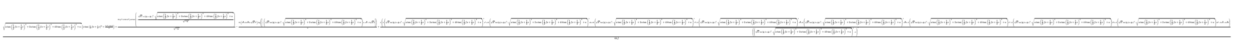
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 960 vs. 2(120) = 240.
time = 0.76, size = 960, normalized size = 7.50



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a)*(a*tan(1/2*f*x + 1/2*e)^2 + (7*a^2 + 10*a*b)/a) + 2
4*(a^2 + 2*a*b + b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))/sqrt(-a))/sqrt(-a) - 12*(a^(5/2) + 2*a^(3/2)*b + sqrt(a)*b^2)*log(a
bs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(3/2) - 2*sq
rt(a)*b))/a + 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a
^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 10*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 8*(sqrt(a)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
```

```
/2*f*x + 1/2*e)^2 + a))*a^3 - 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))*a^2*b - 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*
b^2 - 3*a^(7/2) - 4*a^(5/2)*b)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))^2 - a)^2)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

3.137 $\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=197

$$-\frac{(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16a^{3/2}f}-\frac{(5a-b)(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+$$

```
[Out] -1/16*(5*a-b)*(a+b)^2*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2)
)/a^(3/2)/f-1/24*(5*a-b)*(a+b-b*cos(f*x+e)^2)^(3/2)*cot(f*x+e)*csc(f*x+e)^3
/a/f-1/6*(a+b-b*cos(f*x+e)^2)^(5/2)*cot(f*x+e)*csc(f*x+e)^5/a/f-1/16*(5*a-b
)*(a+b)*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.11, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 390, 386, 385, 212}

$$-\frac{(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16a^{3/2}f}-\frac{\cot(e+fx)\csc^5(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6af}-\frac{(5a-b)\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{24af}-\frac{(5a-b)(a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{16af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/16*((5*a - b)*(a + b)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Co
s[e + f*x]^2]])/(a^(3/2)*f) - ((5*a - b)*(a + b)*Sqrt[a + b - b*Cos[e + f*x
]^2]*Cot[e + f*x]*Csc[e + f*x])/(16*a*f) - ((5*a - b)*(a + b - b*Cos[e + f*
x]^2)^(3/2)*Cot[e + f*x]*Csc[e + f*x]^3)/(24*a*f) - ((a + b - b*Cos[e + f*x
]^2)^(5/2)*Cot[e + f*x]*Csc[e + f*x]^5)/(6*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{\left(1-x^2\right)^4}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(a+b-b\cos^2(e+fx)\right)^{5/2}\cot(e+fx)\csc^5(e+fx)}{6af} - \frac{(5a-$$

$$= -\frac{(5a-b)\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{24af}$$

$$= -\frac{(5a-b)(a+b)\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+}{16af}$$

$$= -\frac{(5a-b)(a+b)\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+}{16af}$$

$$= -\frac{(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16a^{3/2}f} - \frac{(5a-}{}$$

Mathematica [A]

time = 0.80, size = 161, normalized size = 0.82

$$\frac{-6(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)-\sqrt{2}\sqrt{a}\sqrt{2a+b-b\cos(2(e+fx))}\csc^2(e+fx)\left((15a^2+22ab+3b^2)\cos(e+fx)+2a\cot(e+fx)\csc(e+fx)\left(5a+7b+4a\csc^2(e+fx)\right)\right)}{96a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-6*(5*a - b)*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b
 - b*Cos[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
]*Csc[e + f*x]^2*((15*a^2 + 22*a*b + 3*b^2)*Cos[e + f*x] + 2*a*Cot[e + f*x]
*Csc[e + f*x]*(5*a + 7*b + 4*a*Csc[e + f*x]^2)))/(96*a^(3/2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(177) = 354$.
time = 13.00, size = 565, normalized size = 2.87

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(15a^4\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\sqrt{-b(\cos^4(fx+e))+(a}}{\sin(fx+e)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(15*a^4*ln(((a-b)*cos(f*x+e)^
2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*s
in(f*x+e)^6+27*a^3*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b
)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^6+9*b^2*ln(((a-b)*cos(f
*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e
)^2)*sin(f*x+e)^6*a^2-3*b^3*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)
^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^6*a+30*a^(7/2)*(
cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^4+44*b*(cos(f*x+e)^2*(a+b
*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^4*a^(5/2)+6*b^2*(cos(f*x+e)^2*(a+b*sin(f*x
+e)^2))^(1/2)*sin(f*x+e)^4*a^(3/2)+20*a^(7/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)
^2))^(1/2)*sin(f*x+e)^2+28*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*
x+e)^2*a^(5/2)+16*a^(7/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/sin(f*x+
e)^6/a^(5/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^7, x)
```

Fricas [A]
time = 1.23, size = 752, normalized size = 3.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^6 - 3*(5*a^3 + 9
*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - 5*a^3 - 9*a^2*b - 3*a*b^2 + b^3 +
3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*((a^2 - 6
*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a
 - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b
)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4
*((15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)^5 - 2*(20*a^3 + 29*a^2*b + 3*a
*b^2)*cos(f*x + e)^3 + 3*(11*a^3 + 12*a^2*b + a*b^2)*cos(f*x + e))*sqrt(-b*
cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^6 - 3*a^2*f*cos(f*x + e)^4 + 3
*a^2*f*cos(f*x + e)^2 - a^2*f), 1/96*(3*((5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*
cos(f*x + e)^6 - 3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - 5*a^3
 - 9*a^2*b - 3*a*b^2 + b^3 + 3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x +
e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e)))
+ 2*((15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)^5 - 2*(20*a^3 + 29*a^2*b +
3*a*b^2)*cos(f*x + e)^3 + 3*(11*a^3 + 12*a^2*b + a*b^2)*cos(f*x + e))*sqrt(
-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^6 - 3*a^2*f*cos(f*x + e)^4
+ 3*a^2*f*cos(f*x + e)^2 - a^2*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**7*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1708 vs. 2(188) = 376.
time = 0.91, size = 1708, normalized size = 8.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a)*((a*tan(1/2*f*x + 1/2*e)^2 + (8*a^3 + 7*a^2*b)/a^2
)*tan(1/2*f*x + 1/2*e)^2 + (37*a^3 + 51*a^2*b + 6*a*b^2)/a^2) + 24*(5*a^3 +
 9*a^2*b + 3*a*b^2 - b^3)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a) - 12*(5*a^(7/2) + 9*a^(5/2)*b + 3*a^(3/2
)*b^2 - sqrt(a)*b^3)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a^2 + 2*(45*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))^5*a^3 + 132*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^5*a^2*b + 108*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))^5*a*b^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5
*b^3 + 63*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) +
120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b + 48*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 - 50*(sqr
t(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f
*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^4 - 156*(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3*b - 96*(sqrt(a)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b^2 + 32*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))^3*a*b^3 - 78*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))^2*a^(9/2) - 108*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))^2*a^(7/2)*b + 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))*a^5 + 72*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^4*b + 36
*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b^2 - 12*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b^3 + 31*a^(11/2) + 36*a^(
```

```
9/2)*b)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^3*a))
/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^7,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^7, x)
```

3.138 $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=325

$$-\frac{(a^2+11ab+8b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}-\frac{2(4a+3b)\cos(e+fx)\sin^3(e+fx)\sqrt{a}}{35f}$$

```
[Out] -1/35*(a^2+11*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f
-2/35*(4*a+3*b)*cos(f*x+e)*sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2)/f-1/7*b*co
s(f*x+e)*sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2)/f-2/35*(a+2*b)*(a^2-4*a*b-4*
b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+
b*sin(f*x+e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/35*a*(a+b)*(2*a^2-
5*a*b-8*b^2)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(
1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 488, 596, 538, 437, 435, 432, 430}

$$\frac{a(a+b)(2a^2-5ab-8b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{35b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)(a^2-4ab-4b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{35b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(a^2+11ab+8b^2)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}-\frac{b\sin^5(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f}-\frac{2(4a+3b)\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/35*((a^2 + 11*a*b + 8*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e +
f*x]^2])/(b*f) - (2*(4*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b*Sin[
e + f*x]^2])/(35*f) - (b*Cos[e + f*x]*Sin[e + f*x]^5*Sqrt[a + b*Sin[e + f*x
]^2])/(7*f) - (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt[Cos[e + f*x]^2]*Ellip
ticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])
/(35*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*(2*a^2 - 5*a*b - 8*
b^2)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f
*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(35*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{1-x^2}}\,dx,x,\sin(\right.}{f}$$

$$= -\frac{b\cos(e+fx)\sin^5(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f} - \left(\sqrt{\cos^2(\right.$$

$$= -\frac{2(4a+3b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f} -$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+f}}{35bf}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+f}}{35bf}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+f}}{35bf}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+f}}{35bf}$$

Mathematica [A]
time = 1.76, size = 249, normalized size = 0.77

$$\frac{-128a\left(a^3-2a^2b-12ab^2-8b^3\right)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+64a\left(2a^3-3a^2b-13ab^2-8b^3\right)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b\left(-32a^3-496a^2b-684ab^2-250b^3+b\left(144a^2+480ab+299b^2\right)\cos(2(e+fx))-2b^2(26a+27b)\cos(4(e+fx))+5b^3\cos(6(e+fx))\right)\sin(2(e+fx))}{2240b^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-128*a*(a^3 - 2*a^2*b - 12*a*b^2 - 8*b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x
)])/a]*EllipticE[e + f*x, -(b/a)] + 64*a*(2*a^3 - 3*a^2*b - 13*a*b^2 - 8*b^
3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt
[2]*b*(-32*a^3 - 496*a^2*b - 684*a*b^2 - 250*b^3 + b*(144*a^2 + 480*a*b + 2
99*b^2)*Cos[2*(e + f*x)] - 2*b^2*(26*a + 27*b)*Cos[4*(e + f*x)] + 5*b^3*Cos
[6*(e + f*x)])*Sin[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*
x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(299) = 598$.
time = 7.95, size = 602, normalized size = 1.85

method	result
default	$\frac{5b^4(\sin^9(fx+e))+13ab^3(\sin^7(fx+e))+b^4(\sin^7(fx+e))+9a^2b^2(\sin^5(fx+e))+4ab^3(\sin^5(fx+e))+2b^4(\sin^5(fx+e))+2\sqrt{\frac{\cos(2fx+2}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(5*b^4*sin(f*x+e)^9+13*a*b^3*sin(f*x+e)^7+b^4*sin(f*x+e)^7+9*a^2*b^2*s
in(f*x+e)^5+4*a*b^3*sin(f*x+e)^5+2*b^4*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*
((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4-3*(co
s(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b
)^(1/2))*a^3*b-13*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3-2*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^4
+4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^3*b+24*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+16*(cos(f*x+e)^2)^(1/2)*((a+b*s
in(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3+a^3*b*sin(
f*x+e)^3+2*a^2*b^2*sin(f*x+e)^3-9*a*b^3*sin(f*x+e)^3-8*b^4*sin(f*x+e)^3-a^3
*b*sin(f*x+e)-11*a^2*b^2*sin(f*x+e)-8*a*b^3*sin(f*x+e))/b^2/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)

Fricas [F]
time = 0.14, size = 68, normalized size = 0.21

$$\mathrm{integral}\left(-\left(b\cos\left(fx+e\right)^{6}-\left(a+3\,b\right)\cos\left(fx+e\right)^{4}+\left(2\,a+3\,b\right)\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(-(b*cos(f*x + e)^6 - (a + 3*b)*cos(f*x + e)^4 + (2*a + 3*b)*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b), x)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^4\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)

[Out] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)

3.139 $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=218

$$-\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}+$$

```
[Out] -1/5*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f-1/15*(3*a+4*b)*cos(f*
x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/15*(3*a^2+13*a*b+8*b^2)*(cos(f
*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e
)^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/15*a*(a+b)*(3*a+4*b)*(cos(f*x+e
)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/
a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3249, 3251, 3257, 3256, 3262, 3261}

$$\frac{(3a^2+13ab+8b^2)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}-\frac{(3a+4b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{a(a+b)(3a+4b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{15bf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*((3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f
- (Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(5*f) + ((3*a^2
+ 13*a*b + 8*b^2)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(1
5*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*(3*a + 4*b)*EllipticF[e
+ f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b*f*Sqrt[a + b*Sin[e + f
*x]^2])
```

Rule 3249

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e
 + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p
 + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Gt
Q[p, 0]
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
```

```
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}+\frac{1}{5}\int\sqrt{a+b\ldots}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{\cos\ldots}{\ldots}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{\cos\ldots}{\ldots}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{\cos\ldots}{\ldots}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{\cos\ldots}{\ldots}$$

Mathematica [A]
time = 0.94, size = 201, normalized size = 0.92

$$\frac{16a(3a^2+13ab+8b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-16a(3a^2+7ab+4b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b(48a^2+68ab+25b^2-4b(9a+7b)\cos(2(e+fx))+3b^2\cos(4(e+fx)))\sin(2(e+fx))}{240bf\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (16*a*(3*a^2 + 13*a*b + 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] - 16*a*(3*a^2 + 7*a*b + 4*b^2)*Sqrt[(2*a + b - b*Cos[
2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(48*a^2 + 68*a*b +
25*b^2 - 4*b*(9*a + 7*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(
e + f*x)])/(240*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 6.72, size = 429, normalized size = 1.97

method	result

default	$-\frac{-3b^3(\sin^7(fx+e))-9a\,b^2(\sin^5(fx+e))-b^3(\sin^5(fx+e))+3\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin}{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-3*b^3*sin(f*x+e)^7-9*a*b^2*sin(f*x+e)^5-b^3*sin(f*x+e)^5+3*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a^3+7*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(s
in(f*x+e),(-1/a*b)^(1/2))*b+4*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)
^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*(cos(f*x+e)^2)^(1/2)*((a+
b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-13*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(
1/2))*a^2*b-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))*a*b^2-6*a^2*b*sin(f*x+e)^3+5*a*b^2*sin(f*x+e)^3+4
*b^3*sin(f*x+e)^3+6*a^2*b*sin(f*x+e)+4*a*b^2*sin(f*x+e))/b/cos(f*x+e)/(a+b*
sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Fricas [F]
time = 0.11, size = 47, normalized size = 0.22

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-(a+2\,b)\cos\left(fx+e\right)^2+a+b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - (a + 2*b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(
f*x + e)^2 + a + b), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.140 $\int (a+b\sin^2(e+fx))^{3/2}\,dx$

Optimal. Leaf size=154

$$-\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{a(a+b)}{}$$

[Out] -1/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+2/3*(2*a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)

Rubi [A]

time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[e + f*x]^2)^(3/2),x]

[Out] -1/3*(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+}{\sqrt{a+b\sin^2(}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{1}{3}(a(a+b))\int \frac{}{\sqrt{a+b\,\mathrm{si}}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left(2(2a+b)\sqrt{a+b\sin^2(e}\right.}{3\sqrt{1}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{1+\frac{b\sin}{}}}$$

Mathematica [A]
time = 0.53, size = 156, normalized size = 1.01

$$\frac{4\sqrt{2}\,a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}\,a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+b(-2a-b+b\cos(2(e+fx)))\sin(2(e+fx))}{6\sqrt{2}\,f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 6.49, size = 266, normalized size = 1.73

method	result
default	$-\frac{\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}}{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3-1/3*s
in(f*x+e)*a*b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.11, size = 18, normalized size = 0.12

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2), x)
```

3.141 $\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=181

$$-\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] -a*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-(a-b)*EllipticE(sin(f*x+e),(-b/a)^
(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(
f*x+e)^2/a)^(1/2)+a*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(co
s(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 485, 538, 437, 435, 432, 430}

$$\frac{a(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - ((a - b)*Sqrt[Cos[e + f*
x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*Sqrt[Cos[e + f*
x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{1-x^2}}\,dx,x,\sin\right.}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(\right.}{}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left((-a+b)\sqrt{\cos^2(e+\right.}{}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(\left((-a+b)\sqrt{\cos^2(e+\right.\right.}{}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{(a-b)\sqrt{\cos^2(e+fx)}}{}$$

Mathematica [A]
time = 0.92, size = 141, normalized size = 0.78

$$-\frac{a\left(\sqrt{2}\,(2a+b-b\cos(2(e+fx)))\cot(e+fx)+2(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-2(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)\right)}{2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(a*(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x] + 2*(a - b)*Sq
rt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] - 2*(a + b)
*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)]))/(f*Sqr
t[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 6.46, size = 174, normalized size = 0.96

method	result

default	$\frac{a\left(\sin(fx+e)\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\left(\text{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a+\text{EllipticF}\left(\sin(fx+e),\sqrt{-\right.\right.\right.}{\sin(fx+e)\cos(fx+e)\sqrt{a+b(}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] a*(sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)+b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)

Fricas [F]
time = 0.13, size = 45, normalized size = 0.25

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^2, x)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

3.142 $\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=236

$$-\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{2(a+2b)\sqrt{}}{}$$

```
[Out] -2/3*(a+2*b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*a*cot(f*x+e)*csc(f*x
+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/f-2/3*(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*
x+e)^2/a)^(1/2)+1/3*(a+b)*(2*a+3*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(
f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)
^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 485, 597, 538, 437, 435, 432, 430}

$$\frac{(a+b)(2a+3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (a*Cot[e + f
*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (2*(a + 2*b)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*(2*a +
3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f
*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{1-x^2}}\,dx, x, \sin(e\right.}{f}$$

$$= -\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cos^2(e\right.}{}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\,\mathrm{cs}}{}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\,\mathrm{cs}}{}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\,\mathrm{cs}}{}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\,\mathrm{cs}}{}$$

Mathematica [**A**]
time = 2.97, size = 201, normalized size = 0.85

$$\frac{\frac{(-8a^2-13ab-6b^2+2(2a^2+7ab+4b^2)\cos(2(e+fx))-b(a+2b)\cos(4(e+fx)))\cot(e+fx)\csc^2(e+fx)}{\sqrt{2}} - 4a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right) + 2(2a^2+5ab+3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{6f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (((-8*a^2 - 13*a*b - 6*b^2 + 2*(2*a^2 + 7*a*b + 4*b^2)*Cos[2*(e + f*x)] - b
*(a + 2*b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] - 4*a*(a
```

+ 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] +
2*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[
e + f*x, -(b/a)])/(6*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])

Maple [A]
time = 8.54, size = 408, normalized size = 1.73

method	result
default	$\frac{2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2(\sin^3(fx+e))+5b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+5*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3+3*b^2*(c
os(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*
b)^(1/2))*sin(f*x+e)^3-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*
EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3-4*(cos(f*x+e)^2)^(1/2
)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin
(f*x+e)^3+2*a*b*sin(f*x+e)^6+4*b^2*sin(f*x+e)^6+2*a^2*sin(f*x+e)^4+3*a*b*si
n(f*x+e)^4-4*b^2*sin(f*x+e)^4-a^2*sin(f*x+e)^2-5*a*b*sin(f*x+e)^2-a^2)/sin(
f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)

Fricas [C] Result contains complex when optimal does not.
time = 0.16, size = 969, normalized size = 4.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] 1/3*((2*((-I*a*b - 2*I*b^2)*cos(f*x + e)^2 + I*a*b + 2*I*b^2)*sqrt(-b)*sqrt
((a^2 + a*b)/b^2)*sin(f*x + e) - ((2*I*a^2 + 5*I*a*b + 2*I*b^2)*cos(f*x + e

```
)^2 - 2*I*a^2 - 5*I*a*b - 2*I*b^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b + 2*I*b^2)*cos(f*x
 + e)^2 - I*a*b - 2*I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((
-2*I*a^2 - 5*I*a*b - 2*I*b^2)*cos(f*x + e)^2 + 2*I*a^2 + 5*I*a*b + 2*I*b^2)
*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellip
tic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) -
I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b
^2))/b^2) + (2*((I*a*b + I*b^2)*cos(f*x + e)^2 - I*a*b - I*b^2)*sqrt(-b)*sq
rt((a^2 + a*b)/b^2)*sin(f*x + e) - ((-2*I*a^2 - 7*I*a*b - 3*I*b^2)*cos(f*x
+ e)^2 + 2*I*a^2 + 7*I*a*b + 3*I*b^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt
((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b
)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2
 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((-I*a*b - I*b^2)*cos(f
*x + e)^2 + I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((
2*I*a^2 + 7*I*a*b + 3*I*b^2)*cos(f*x + e)^2 - 2*I*a^2 - 7*I*a*b - 3*I*b^2)*
sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellipt
ic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I
*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^
2))/b^2) - (2*(a*b + 2*b^2)*cos(f*x + e)^3 - (3*a*b + 4*b^2)*cos(f*x + e))*
sqrt(-b*cos(f*x + e)^2 + a + b))/((b*f*cos(f*x + e)^2 - b*f)*sin(f*x + e))
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8008 deep
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

3.143 $\int \left(a+b\sin^2(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=210

$$-\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d}-\frac{b\cos(c+dx)\sin(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}+$$

```
[Out] -1/5*b*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c)^2)^(3/2)/d-4/15*b*(2*a+b)*cos(
d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c)^2)^(1/2)/d+1/15*(23*a^2+23*a*b+8*b^2)*(co
s(d*x+c)^2)^(1/2)/cos(d*x+c)*EllipticE(sin(d*x+c),(-b/a)^(1/2))*(a+b*sin(d*
x+c)^2)^(1/2)/d/(1+b*sin(d*x+c)^2/a)^(1/2)-4/15*a*(a+b)*(2*a+b)*(cos(d*x+c)
^2)^(1/2)/cos(d*x+c)*EllipticF(sin(d*x+c),(-b/a)^(1/2))*(1+b*sin(d*x+c)^2/a
)^(1/2)/d/(a+b*sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3259, 3249, 3251, 3257, 3256, 3262, 3261}

$$\frac{(23a^2+23ab+8b^2)\sqrt{a+b\sin^2(c+dx)}\,E\left(c+dx\middle|-\frac{b}{a}\right)}{15d\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}-\frac{4b(2a+b)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\sin^2(c+dx)}{a}+1}\,F\left(c+dx\middle|-\frac{b}{a}\right)}{15d\sqrt{a+b\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(5/2),x]
```

```
[Out] (-4*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^2])/(15*d
) - (b*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x]^2)^(3/2))/(5*d) + ((23
*a^2 + 23*a*b + 8*b^2)*EllipticE[c + d*x, -(b/a)]*Sqrt[a + b*Sin[c + d*x]^2
])/(15*d*Sqrt[1 + (b*Sin[c + d*x]^2)/a]) - (4*a*(a + b)*(2*a + b)*EllipticF
[c + d*x, -(b/a)]*Sqrt[1 + (b*Sin[c + d*x]^2)/a])/(15*d*Sqrt[a + b*Sin[c +
d*x]^2])
```

Rule 3249

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e
 + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p
 + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Gt
Q[p, 0]
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
```

```
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx = -\frac{b\cos(c+dx)\sin(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d} + \frac{1}{5}\int \sqrt{a+b\sin^2(c+dx)}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d} - \frac{b\cos(c+dx)\sin}{}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d} - \frac{b\cos(c+dx)\sin}{}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d} - \frac{b\cos(c+dx)\sin}{}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d} - \frac{b\cos(c+dx)\sin}{}$$

Mathematica [A]
time = 0.97, size = 194, normalized size = 0.92

$$\frac{16a(23a^2+23ab+8b^2)\sqrt{\frac{2a+b-b\cos(2(c+dx))}{a}}\,E\left(c+dx\middle|-\frac{b}{a}\right) - 64a(2a^2+3ab+b^2)\sqrt{\frac{2a+b-b\cos(2(c+dx))}{a}}\,F\left(c+dx\middle|-\frac{b}{a}\right) - \sqrt{2}\,b(88a^2+88ab+25b^2-28b(2a+b)\cos(2(c+dx))+3b^2\cos(4(c+dx)))\sin(2(c+dx))}{240d\sqrt{2a+b-b\cos(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(5/2),x]
```

```
[Out] (16*a*(23*a^2 + 23*a*b + 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(c + d*x)])/a]*Elli
pticE[c + d*x, -(b/a)] - 64*a*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b - b*Cos[2
*(c + d*x)])/a]*EllipticF[c + d*x, -(b/a)] - Sqrt[2]*b*(88*a^2 + 88*a*b + 2
5*b^2 - 28*b*(2*a + b)*Cos[2*(c + d*x)] + 3*b^2*Cos[4*(c + d*x)])*Sin[2*(c
+ d*x)])/(240*d*Sqrt[2*a + b - b*Cos[2*(c + d*x)]])
```

Maple [A]
time = 7.26, size = 437, normalized size = 2.08

method	result

default	$\frac{-\frac{b^{3} \sin(dx+c)\left(\cos^{6}(dx+c)\right)}{5}+\frac{\left(14a\,b^{2}+10b^{3}\right)\left(\cos^{4}(dx+c)\right)\sin(dx+c)}{15}+\frac{\left(-11a^{2}b-18a\,b^{2}-7b^{3}\right)\left(\cos^{2}(dx+c)\right)\sin(dx+c)}{15}-\frac{8\sqrt{\frac{\cos(2dx+2c)}{2}+}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/5*b^3*sin(d*x+c)*cos(d*x+c)^6+1/15*(14*a*b^2+10*b^3)*cos(d*x+c)^4*sin(d
*x+c)+1/15*(-11*a^2*b-18*a*b^2-7*b^3)*cos(d*x+c)^2*sin(d*x+c)-8/15*(cos(d*x
+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticF(sin(d*x+c),(-1/a*b
)^(1/2))*a^3-4/5*a^2*(cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)
*EllipticF(sin(d*x+c),(-1/a*b)^(1/2))*b-4/15*a*(cos(d*x+c)^2)^(1/2)*(-b/a*c
os(d*x+c)^2+(a+b)/a)^(1/2)*EllipticF(sin(d*x+c),(-1/a*b)^(1/2))*b^2+23/15*(
cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticE(sin(d*x+c),
(-1/a*b)^(1/2))*a^3+23/15*(cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^
(1/2)*EllipticE(sin(d*x+c),(-1/a*b)^(1/2))*a^2*b+8/15*(cos(d*x+c)^2)^(1/2)*
(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticE(sin(d*x+c),(-1/a*b)^(1/2))*a*b^
2)/cos(d*x+c)/(a+sin(d*x+c)^2*b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^(5/2), x)
```

Fricas [F]
time = 0.12, size = 59, normalized size = 0.28

$$\text{integral}\left(\left(b^{2}\cos\left(dx+c\right)^{4}-2\left(ab+b^{2}\right)\cos\left(dx+c\right)^{2}+a^{2}+2\,ab+b^{2}\right)\sqrt{-b\cos\left(dx+c\right)^{2}+a+b}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b +
 b^2)*sqrt(-b*cos(d*x + c)^2 + a + b), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\sin\left(c+d\,x\right)}^2+a\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a + b*sin(c + d*x)^2)^(5/2), x)
```

3.144 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2b^{3/2}f}-\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf}$$

```
[Out] 1/2*(a-b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(3/2)/f-1
/2*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3265, 396, 223, 209}

$$\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2b^{3/2}f}-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2*
b^(3/2)*f) - (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\mathrm{Subst}\left(\int \frac{1-x^2}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf} + \frac{(a-b)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx,\right.}{2bf}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf} + \frac{(a-b)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{}{\sqrt{a+}}\right.}{2bf}$$

$$= \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2b^{3/2}f} - \frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e-}}{2bf}$$

Mathematica [A]
time = 0.21, size = 105, normalized size = 1.27

$$-\frac{\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}}{2\sqrt{2}\,bf} + \frac{(a-b)\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{2\sqrt{-b}\,bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/2*(Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])/(Sqrt[2]*b*f) + ((a
- b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
])/(2*Sqrt[-b]*b*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(71) = 142.
time = 7.69, size = 186, normalized size = 2.24

method	result

default	$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\,\left(2b^{\frac{3}{2}}\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}-\right.}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(2*b^(3/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-b^2*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))+b*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [A]
time = 0.57, size = 79, normalized size = 0.95

$$\frac{\frac{a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos(fx+e)}{b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*(a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) - arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/b)/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(71) = 142$.
time = 0.51, size = 438, normalized size = 5.28

$$\left[-\frac{8\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,b\cos\left(fx+e\right)-(a-b)\sqrt{-b}\log\left(128b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+160\left(a^2b^2+2ab^3+b^4\right)\cos\left(fx+e\right)^4+a^4+4a^3b+6a^2b^2+4ab^3+b^4-32\left(a^3b+3a^2b^2+3ab^3+b^4\right)\cos\left(fx+e\right)^2+8\left(16b^3\cos\left(fx+e\right)^7-24\left(ab^2+b^3\right)\cos\left(fx+e\right)^5+10\left(a^2b+2ab^2+b^3\right)\cos\left(fx+e\right)^3-\left(a^3+3a^2b+3ab^2+b^3\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-b}\right)}{16b^2f},\ -\frac{(a-b)\sqrt{b}\arctan\left(\frac{\left(8b^2\cos(fx+e)^4-8\left(ab+b^2\right)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{b}}{4\left(2b^3\cos(fx+e)^5-3\left(ab^2+b^3\right)\cos(fx+e)^3+\left(a^2b+2ab^2+b^3\right)\cos(fx+e)\right)}\right)+4\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,b\cos\left(fx+e\right)}{8b^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) - (a - b)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(

```
f*x + e)^2 + a + b)*sqrt(-b)))/(b^2*f), -1/8*((a - b)*sqrt(b)*arctan(1/4*(8
*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3
)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*sqrt(-b*cos(f
*x + e)^2 + a + b)*b*cos(f*x + e))/(b^2*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.91, size = 91, normalized size = 1.10

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)}{2\,bf}+\frac{(a-b)\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^2+a+b}+\frac{\sqrt{-bf^2}\cos(fx+e)}{f}\right|\right)}{2\,\sqrt{-b}\,b|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/(b*f) + 1/2*(a - b)*log(a
bs(sqrt(-b*cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*x + e)/f))/(sqrt(-b
)*b*abs(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.145 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

```
[Out] -arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[b]*f)
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Mathematica [A]
time = 0.09, size = 53, normalized size = 1.29

$$-\frac{\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{\sqrt{-b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]]/(
Sqrt[-b]*f))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(35) = 70$.
time = 4.66, size = 99, normalized size = 2.41

method	result
default	$\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\,\arctan\left(\frac{2b(\sin^2(fx+e))+a-b}{2\sqrt{b}\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}}\right)}{2\sqrt{b}\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)/b^(1/2)*arctan(1/2/b^(1/2)*(2*b
*sin(f*x+e)^2+a-b)/(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.55, size = 25, normalized size = 0.61

$$-\frac{\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/(sqrt(b)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(35) = 70.
time = 0.51, size = 370, normalized size = 9.02

$$\left[-\frac{\sqrt{-b}\log\left(128\,b^{4}\cos\left(fx+e\right)^{8}-256\,(ab^{3}+b^{4})\cos\left(fx+e\right)^{6}+160\,(a^{2}b^{2}+2\,ab^{3}+b^{4})\cos\left(fx+e\right)^{4}+a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}-32\,(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4})\cos\left(fx+e\right)^{2}+8\,\left(16\,b^{3}\cos\left(fx+e\right)^{7}-24\,(ab^{2}+b^{3})\cos\left(fx+e\right)^{5}+10\,(a^{2}b+2\,ab^{2}+b^{3})\cos\left(fx+e\right)^{3}-(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-b}\right)}{8\,bf},\ \frac{\arctan\left(\frac{\left(8\,b^{2}\cos(fx+e)^{4}-8\,(ab+b^{2})\cos(fx+e)^{2}+a^{2}+2\,ab+b^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{b}}{4\,\left(2\,b^{3}\cos(fx+e)^{5}-3\,(ab^{2}+b^{3})\cos(fx+e)^{3}+(a^{2}b+2\,ab^{2}+b^{3})\cos(fx+e)\right)}\right)}{4\,\sqrt{b}\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^
6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 +
 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2
*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)
)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b))/(b*f), 1/4*arctan(1/4*(8*b^2*co
s(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f
*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)))/(sqrt(b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.
time = 0.94, size = 83, normalized size = 2.02

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cos\left(fx+e\right)}{2\,f}-\frac{(a+b)\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}+\frac{\sqrt{-bf^{2}}\cos(fx+e)}{f}\right|\right)}{2\,\sqrt{-b}\,|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/f - 1/2*(a + b)*log(abs(s
qrt(-b*cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*x + e)/f))/(sqrt(-b)*ab
s(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(e+fx)}{\sqrt{b\sin(e+fx)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.146 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/a^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 385, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]

[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[a]*f
))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

Mathematica [A]
time = 0.11, size = 48, normalized size = 1.17

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
]/(Sqrt[a]*f))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(35) = 70.
time = 5.59, size = 112, normalized size = 2.73

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\,\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\,\sqrt{-b(\cos^4(fx+e))+(a+b)(}}{\sin(fx+e)^2}\right.}{2\sqrt{a}\,\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)/a^(1/2)*ln(((a-b)*cos(f*x+e)^2
+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)/co
s(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(37) = 74$.
time = 0.58, size = 115, normalized size = 2.80

$$-\frac{\frac{\log\left(b-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)-1}-\frac{a}{\cos\left(fx+e\right)-1}\right)}{\sqrt{a}}-\frac{\log\left(-b+\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)+1}+\frac{a}{\cos\left(fx+e\right)+1}\right)}{\sqrt{a}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) - 1) -
a/(cos(f*x + e) - 1))/sqrt(a) - log(-b + sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1))/sqrt(a))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(35) = 70$.
time = 0.48, size = 219, normalized size = 5.34

$$\left[\frac{\log\left(\frac{2\left(\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+2\left(3\,a^2+2\,ab-b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a-b\right)\cos\left(fx+e\right)^3+\left(a+b\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}+a^2+2\,ab+b^2\right)}{\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1}\right)}{4\,\sqrt{a}\,f},\ \frac{\sqrt{-a}\,\arctan\left(-\frac{\left(\left(a-b\right)\cos\left(fx+e\right)^2+a+b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a}}{2\left(ab\cos\left(fx+e\right)^3-\left(a^2+ab\right)\cos\left(fx+e\right)\right)}\right)}{2\,af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*co
s(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1))/(sqrt(a)*f), 1/2*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e
)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 -
 (a^2 + a*b)*cos(f*x + e)))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.147 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{3/2}f}-\frac{\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2af}$$

```
[Out] -1/2*(a-b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(3/2)/f
-1/2*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3265, 390, 385, 212}

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/2*((a - b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]
])/(a^(3/2)*f) - (Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e + f*x])
/(2*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2af} - \frac{(a-b)\text{Subst}\left(\int \frac{}{(1-x^2)}\right.}{}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2af} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-ax^2}\right.}{}$$

$$= -\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\sqrt{a+b-b\cos^2(e+fx)}\,\text{co}}{2af}$$

Mathematica [A]
time = 0.22, size = 102, normalized size = 1.15

$$\frac{-2(a-b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)-\sqrt{2}\sqrt{a}\sqrt{2a+b-b\cos(2(e+fx))}\cot(e+fx)\csc(e+fx)}{4a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a - b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*
x]*Csc[e + f*x])/(4*a^(3/2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(77) = 154.
time = 9.96, size = 231, normalized size = 2.60

method	result
default	$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\,\left(\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\,\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+}{\sin(fx+e)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(ln(((a-b)*cos(f*x+e)^2+2*a^(1
/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e
)^2*a^2-ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)+a+b)/sin(f*x+e)^2)*b*sin(f*x+e)^2*a+2*a^(3/2)*(cos(f*x+e)^2*(a+b*s
in(f*x+e)^2))^(1/2))/sin(f*x+e)^2/a^(5/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/
2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [A]
time = 0.49, size = 347, normalized size = 3.90

$$\left[\frac{4\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a\cos\left(fx+e\right)-\left((a-b)\cos\left(fx+e\right)^2-a+b\right)\sqrt{a}\,\log\left(\frac{2\left(\left(a^2-6ab+b^2\right)\cos(fx+e)^4+2\left(3a^2+2ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+(a+b)\cos(fx+e)\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}+a^2+2ab+b^2\right)}{\cos(fx+e)^4-2\cos(fx+e)^2+1}\right)}{8\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)}, \frac{\left((a-b)\cos\left(fx+e\right)^2-a+b\right)\sqrt{-a}\,\arctan\left(-\frac{\left((a-b)\cos(fx+e)^2+a+b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a}}{2\left(ab\cos(fx+e)^3-\left(a^2+ab\right)\cos(fx+e)\right)}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a\cos\left(fx+e\right)}{4\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) - ((a - b)*cos(f*x +
 e)^2 - a + b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2
 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*
x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f
*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(a^2*f*cos(f*x + e)^2 - a^2*f), 1/4*(((
a - b)*cos(f*x + e)^2 - a + b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2
 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a
^2 + a*b)*cos(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e)
)/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sin\left(e+f\,x\right)}^3\,\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.148 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=206

$$-\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+}{3b^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-2/3*(a-b)*EllipticE
(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2
)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(2*a-b)*EllipticF(sin(f*x+e)
,(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b
^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 490, 538, 437, 435, 432, 430}

$$\frac{a(2a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/3*(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f) - (2*(a -
 b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (
a*(2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b^2*f*Sqrt[a + b*Sin[e + f*x]
^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-x^2}\,\sqrt{a+bx^2}}\,dx, x, \sin(e+f\right.}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a-b)\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a-b)\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{2(a-b)\sqrt{\cos^2(e+fx)}\,E}{}$$

Mathematica [A]
time = 0.61, size = 163, normalized size = 0.79

$$\frac{-4\sqrt{2}\,a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}\,a(2a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+b(-2a-b+b\cos(2(e+fx)))\sin(2(e+fx))}{6\sqrt{2}\,b^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
[Out] (-4*Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
f*x, -(b/a)] + 2*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]
*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e +
f*x)])/(6*Sqrt[2]*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 6.52, size = 268, normalized size = 1.30

method	result

default	$\frac{b^2(\sin^5(fx+e))+2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2-a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b^2*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*(cos(f*x+e)^2)^(1
/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2
*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a*b+a*b*sin(f*x+e)^3-b^2*sin(f*x+e)^3-sin(f*x+e)*a*b)/b^2/cos(
f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.12, size = 59, normalized size = 0.29

$$\mathrm{integral}\left(-\frac{\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 +
a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.149 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=111

$$\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{aF\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)-a*(cos(f*x+e)^2)^(1/2)/cos(
f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a
+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3251, 3257, 3256, 3262, 3261}

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (b*Si
n[e + f*x]^2)/a]) - (a*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^
2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{b\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{\left(a\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\right.}{b\sqrt{a+b}}$$

$$= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{aF\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.17, size = 78, normalized size = 0.70

$$\frac{\sqrt{2a+b-b\cos(2(e+fx))}\left(E\left(e+fx\middle|-\frac{b}{a}\right)-F\left(e+fx\middle|-\frac{b}{a}\right)\right)}{bf\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(EllipticE[e + f*x, -(b/a)] - EllipticF
[e + f*x, -(b/a)]))/(b*f*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a])
```

Maple [A]
time = 4.92, size = 93, normalized size = 0.84

method	result	size
default	$-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\left(\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)\right)}{b\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)/b*(EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))-EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))/cos(f*x+e)/(a+b*sin
(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.10, size = 48, normalized size = 0.43

$$\mathrm{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\left(\cos\left(fx+e\right)^{2}-1\right)}{b\cos\left(fx+e\right)^{2}-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1)/(b*cos(f*x +
e)^2 - a - b), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^{2}\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(e+f\,x)^2}{\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.150 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin
(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3262, 3261}

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sin^2(e+fx)}}$$
$$= \frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [**A**]
time = 0.07, size = 60, normalized size = 1.18

$$\frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 60, normalized size = 1.18

method	result	size
default	$\frac{\sqrt{-\frac{b(\cos^2(fx+e))-a-b}{a}}\,\mathrm{am}^{-1}\left(fx+e\middle|\frac{i\sqrt{b}}{\sqrt{a}}\right)}{f\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/(a+b-b*cos(f*x+e)^2)^(1/2)*(-(b*cos(f*x+e)^2-a-b)/a)^(1/2)*InverseJacob
iAM(f*x+e,I/a^(1/2)*b^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.12, size = 305, normalized size = 5.98

$$-\frac{\left(2i\sqrt{-b}\,b\sqrt{\frac{a^2+ab}{b^2}}+(-2i\,a-i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,(\cos(fx+e)+i\,\sin(fx+e))\right)\mid\frac{8\,a^2+8\,ab+b^2-4\,(2\,ab+b^2)\sqrt{\frac{a^2+ab}{b^2}}}{b^2})+\left(-2i\sqrt{-b}\,b\sqrt{\frac{a^2+ab}{b^2}}+(2i\,a+i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,(\cos(fx+e)-i\,\sin(fx+e))\right)\mid\frac{8\,a^2+8\,ab+b^2-4\,(2\,ab+b^2)\sqrt{\frac{a^2+ab}{b^2}}}{b^2})}{b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -((2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2) + (-2*I*a - I*b)*sqrt(-b))*sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2
 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b
 + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b*sqr
t((a^2 + a*b)/b^2) + (2*I*a + I*b)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2
) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^
2)*sqrt((a^2 + a*b)/b^2))/b^2))/(b^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin^2{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.151 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=177

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}-\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-EllipticE(sin(f*x+e),(-b/a)^(1/2))
*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(f*x+
e)^2/a)^(1/2)+EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^
(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 491, 12, 507, 437, 435, 432, 430}

$$\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{af\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*f)) - (Sqrt[Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x
]^2])/(a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*Elliptic
F[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a]
)/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) && !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x, \sin(e+ \ldots}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\ldots}{\ldots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(b\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\ldots}{\ldots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\ldots}{\ldots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\,\mathrm{si}\ldots}{af\sqrt{\ldots}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|- \ldots}{af\sqrt{1+\frac{b\sin \ldots}{\ldots}}}$$

Mathematica [A]
time = 0.36, size = 138, normalized size = 0.78

$$\frac{-\sqrt{2}\,(2a+b-b\cos(2(e+fx)))\cot(e+fx)-2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{2af\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*a*Sqrt[(2*a + b
 - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*a*Sqrt[(2*a + b -
b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(2*a*f*Sqrt[2*a + b - b*
Cos[2*(e + f*x)]])
```

Maple [A]
time = 6.60, size = 140, normalized size = 0.79

method	result
default	$\frac{\sin(fx+e)\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,a\left(\text{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-\text{EllipticE}\left(\sin(fx+e),\sqrt{-\right.\right.}{a\sin(fx+e)\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))-EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))+b*cos
(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/a/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(
1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.13, size = 643, normalized size = 3.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(-4*I*sqrt(-b)*b*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((a
^2 + a*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2
)*sqrt((a^2 + a*b)/b^2))/b^2)*sin(f*x + e) + 4*I*sqrt(-b)*b*sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((a^2 + a*b)/b^2)*elliptic_f(arcsin(sqrt
((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))),
 (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)*sin(f*x
 + e) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (2*I*sqrt(-b)*b*
sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (2*I*a + I*b)*sqrt(-b)*sin(f*x + e))*s
qrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sq
rt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2
+ 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b
)*b*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (-2*I*a - I*b)*sqrt(-b)*sin(f*x +
```

```
e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8
*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(a*b*f*si
n(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.152 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=244

$$-\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{2(a-b)\sqrt{\mathrm{co}}$$

```
[Out] -2/3*(a-b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)*csc(f*x
+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a/f-2/3*(a-b)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f/(1+b*si
n(f*x+e)^2/a)^(1/2)+1/3*(2*a-b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+
e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a/f/(a+b*sin(f*x+e)^2)^(
1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 491, 597, 538, 437, 435, 432, 430}

$$-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\mathrm{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}+\frac{(2a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\mathrm{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3af\sqrt{a+b\sin^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a - b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*f) - (Cot[e + f
*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (2*(a - b)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*a^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((2*a - b)*S
qrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sq
rt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^2}\,\sqrt{a+bx^2}}\,dx, x, \sin(e+\right.}{f}$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a}}{3af}$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a}}{3af}$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a}}{3af}$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a}}{3af}$$

Mathematica [A]
time = 2.63, size = 195, normalized size = 0.80

$$\frac{\frac{(-8a^2-ab+3b^2+2(2a^2+ab-2b^2)\cos(2(e+fx))+b(-a+b)\cos(4(e+fx)))\cot(e+fx)\csc^2(e+fx)}{\sqrt{2}} - 4a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right) + 2a(2a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{6a^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((-8*a^2 - a*b + 3*b^2 + 2*(2*a^2 + a*b - 2*b^2)*Cos[2*(e + f*x)] + b*(-a
+ b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] - 4*a*(a - b)*S
qrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*a*(2*a
 - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6
*a^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 7.66, size = 354, normalized size = 1.45

method	result
default	$\frac{2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2(\sin^3(fx+e))-b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b(}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3-b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)
^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-2*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/
2))*a^2*sin(f*x+e)^3+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+2*a*b*sin(f*x+e)^6-2*b^
2*sin(f*x+e)^6+2*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4+2*b^2*sin(f*x+e)^4-a^2
*sin(f*x+e)^2+a*b*sin(f*x+e)^2-a^2)/a^2/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*
x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.15, size = 955, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*((-I*a*b + I*b^2)*cos(f*x + e)^2 + I*a*b - I*b^2)*sqrt(-b)*sqrt((a^
2 + a*b)/b^2)*sin(f*x + e) - ((2*I*a^2 - I*a*b - I*b^2)*cos(f*x + e)^2 - 2*
I*a^2 + I*a*b + I*b^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^
2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b
^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b - I*b^2)*cos(f*x + e)^2 - I*a*
b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((-2*I*a^2 + I*a*b
 + I*b^2)*cos(f*x + e)^2 + 2*I*a^2 - I*a*b - I*b^2)*sqrt(-b)*sin(f*x + e))*
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2
 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b -
 2*I*b^2)*cos(f*x + e)^2 - I*a*b + 2*I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*
sin(f*x + e) - ((-2*I*a^2 - I*a*b)*cos(f*x + e)^2 + 2*I*a^2 + I*a*b)*sqrt(-
b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(a
rcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f
*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^
2) + (2*((-I*a*b + 2*I*b^2)*cos(f*x + e)^2 + I*a*b - 2*I*b^2)*sqrt(-b)*sqrt
((a^2 + a*b)/b^2)*sin(f*x + e) - ((2*I*a^2 + I*a*b)*cos(f*x + e)^2 - 2*I*a^
2 - I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f
*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^
2 + a*b)/b^2))/b^2) - (2*(a*b - b^2)*cos(f*x + e)^3 - (3*a*b - 2*b^2)*cos(f
*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^2*b*f*cos(f*x + e)^2 - a^2*b*
f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin \left(e+f\,x\right)^4\,\sqrt{b\sin \left(e+f\,x\right)^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.153 $\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{3/2}f}+\frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

```
[Out] -arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(3/2)/f+a*cos(f*x+
e)/b/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3265, 393, 223, 209}

$$\frac{a\cos(e+fx)}{bf(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(b^(3/2)*f)
) + (a*Cos[e + f*x])/(b*(a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a+b-bx^2)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+\right.}{bf}$$

$$= \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(}}\right.}{bf}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{3/2}f} + \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

Mathematica [A]
time = 0.30, size = 96, normalized size = 1.22

$$\frac{\frac{\sqrt{2}\,ab\cos(e+fx)}{(a+b)\sqrt{2a+b-b\cos(2(e+fx))}} + \sqrt{-b}\,\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx) + \sqrt{2a+b-b\cos(2(e+fx))}\right)}{b^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
[Out] ((Sqrt[2]*a*b*Cos[e + f*x])/((a + b)*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]) +
Sqrt[-b]*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*
x)]]])/(b^2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(71) = 142.
time = 12.94, size = 156, normalized size = 1.97

method	result

default	$\dfrac{\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}\left(\dfrac{\arctan\left(\dfrac{\sqrt{b}\,\left(\sin^2(fx+e)-\frac{-a+b}{2b}\right)}{\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}}\right.}{2b^{\frac{3}{2}}}\right.}{\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/2/b^(3/2)*arctan(b^(1/2)*(sin(
f*x+e)^2-1/2*(-a+b)/b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))+1/b*a*cos
(f*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*
sin(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.55, size = 86, normalized size = 1.09

$$-\frac{\frac{\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{\cos(fx+e)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,(a+b)}-\frac{\cos(fx+e)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) + cos(f*x + e)/(sqrt(-b*co
s(f*x + e)^2 + a + b)*(a + b)) - cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a +
 b)*b))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(71) = 142.
time = 0.58, size = 564, normalized size = 7.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) + ((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256
*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^
4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*co
```

```
s(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/((
a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^2*b^2 + 2*a*b^3 + b^4)*f), -1/4*(4*sqrt(
-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*cos(f*x + e)^2 -
 a^2 - 2*a*b - b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2
)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^
2 + b^3)*cos(f*x + e))))/((a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^2*b^2 + 2*a*b
^3 + b^4)*f)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 1.00, size = 113, normalized size = 1.43

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a\cos\left(fx+e\right)}{\left(b\cos\left(fx+e\right)^2-a-b\right)\left(ab+b^2\right)f}-\frac{\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^2+a+b}+\frac{\sqrt{-bf^2}\cos(fx+e)}{f}\right|\right)}{\sqrt{-b}\,b|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e)/((b*cos(f*x + e)^2 - a - b)
*(a*b + b^2)*f) - log(abs(sqrt(-b*cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*co
s(f*x + e)/f))/(sqrt(-b)*b*abs(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.154 $\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{\cos(e+fx)}{(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

```
[Out] -cos(f*x+e)/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3265, 197}

$$-\frac{\cos(e+fx)}{f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]/((a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2]))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{(a+b-bx^2)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)}{(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 1.21

$$-\frac{\sqrt{2}\cos(e+fx)}{(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
[Out] -((Sqrt[2]*Cos[e + f*x])/((a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]))
```

Maple [A]
time = 4.88, size = 31, normalized size = 0.91

method	result	size
default	$-\frac{\cos(fx+e)}{(a+b)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\,f}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -cos(f*x+e)/(a+b)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.29, size = 34, normalized size = 1.00

$$-\frac{\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] -cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*f)
```

Fricas [A]
time = 0.45, size = 57, normalized size = 1.68

$$\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cos\left(fx+e\right)}{(ab+b^{2})f\cos\left(fx+e\right)^{2}-(a^{2}+2\,ab+b^{2})f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/((a*b + b^2)*f*cos(f*x + e)^2
- (a^2 + 2*a*b + b^2)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [**A**]
time = 0.63, size = 53, normalized size = 1.56

$$\frac{\sqrt{-b\cos(fx+e)^2+a+b}\,\cos(fx+e)}{\left(b\cos(fx+e)^2-a-b\right)(a+b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/((b*cos(f*x + e)^2 - a - b)*(a
 + b)*f)
```

Mupad [**B**]
time = 15.18, size = 119, normalized size = 3.50

$$-\frac{\sqrt{2}\,\sqrt{2\,a+b-b\,\cos(2\,e+2\,f\,x)}\,(4\,a\,\cos(e+f\,x)+b\,\cos(e+f\,x)-b\,\cos(3\,e+3\,f\,x))}{f\,(a+b)\,(8\,a\,b+8\,a^2+3\,b^2-4\,b^2\,\cos(2\,e+2\,f\,x)+b^2\,\cos(4\,e+4\,f\,x)-8\,a\,b\,\cos(2\,e+2\,f\,x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] -(2^(1/2)*(2*a + b - b*cos(2*e + 2*f*x))^(1/2)*(4*a*cos(e + f*x) + b*cos(e
+ f*x) - b*cos(3*e + 3*f*x)))/(f*(a + b)*(8*a*b + 8*a^2 + 3*b^2 - 4*b^2*cos
(2*e + 2*f*x) + b^2*cos(4*e + 4*f*x) - 8*a*b*cos(2*e + 2*f*x)))
```

3.155 $\int \frac{\csc(e+fx)}{(a+b\sin^2(e+fx))^{3/2}} \, dx$

Optimal. Leaf size=79

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{3/2}f}+\frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(3/2)/f+b*cos(f*x
+e)/a/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 390, 385, 212}

$$\frac{b\cos(e+fx)}{af(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(a^(3/2)*f
)) + (b*Cos[e + f*x])/(a*(a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \mathrm{c}\right.}{af}$$

$$= \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cos(e+f}{\sqrt{a+b-b\cos}}\right.}{af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{3/2}f} + \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx}}$$

Mathematica [A]
time = 0.23, size = 93, normalized size = 1.18

$$\frac{-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\frac{\sqrt{2}\sqrt{a}\,b\cos(e+fx)}{(a+b)\sqrt{2a+b-b\cos(2(e+fx))}}}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
] + (Sqrt[2]*Sqrt[a]*b*Cos[e + f*x])/((a + b)*Sqrt[2*a + b - b*Cos[2*(e + f
*x)]]))/(a^(3/2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(71) = 142$.
time = 12.07, size = 165, normalized size = 2.09

method	result
default	$\frac{\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(-\frac{\ln\left(\frac{2a+\left(-a+b\right)\left(\sin^{2}\left(fx+e\right)\right)+2\sqrt{a}\,\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)^{2}}\right.}{2a^{\frac{3}{2}}}\right.}{\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/a^(3/2)*ln((2*a+(-a+b)*sin(
f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2)
+1/a*b*cos(f*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*
x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(75) = 150$.
time = 0.54, size = 174, normalized size = 2.20

$$\frac{\frac{2\,b^{2}\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,a^{2}b+\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,ab^{2}}-\frac{\log\left(b-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)-1}-\frac{a}{\cos\left(fx+e\right)-1}\right)}{a^{\frac{3}{2}}}+\frac{\log\left(-b+\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)+1}+\frac{a}{\cos\left(fx+e\right)+1}\right)}{a^{\frac{3}{2}}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*a^2*b + sqrt(-b*co
s(f*x + e)^2 + a + b)*a*b^2) - log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt
(a)/(cos(f*x + e) - 1) - a/(cos(f*x + e) - 1))/a^(3/2) + log(-b + sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1))/a
^(3/2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(71) = 142$.
time = 0.50, size = 422, normalized size = 5.34

$$\left[-\frac{4\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,ab\cos\left(fx+e\right)-\left(\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}-a^{2}-2\,ab-b^{2}\right)\sqrt{a}\log\left(\frac{2\left(\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+2\left(3\,a^{2}+2\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}-4\left(\left(a-b\right)\cos\left(fx+e\right)^{3}+\left(a+b\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a}+a^{2}+2\,ab+b^{2}\right)}{\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1}\right)}{4\left(\left(a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)f\right)},-\frac{2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,ab\cos\left(fx+e\right)-\left(\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}-a^{2}-2\,ab-b^{2}\right)\sqrt{-a}\arctan\left(-\frac{\left(\left(a-b\right)\cos\left(fx+e\right)^{2}+a+b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-a}}{2\left(ab\cos\left(fx+e\right)^{3}-\left(a^{2}+ab\right)\cos\left(fx+e\right)\right)}\right)}{2\left(\left(a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*
```

```
x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)
^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 +
2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3*b + a^2*b^2)*f
*cos(f*x + e)^2 - (a^4 + 2*a^3*b + a^2*b^2)*f), -1/2*(2*sqrt(-b*cos(f*x + e
)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b -
 b^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))))
/((a^3*b + a^2*b^2)*f*cos(f*x + e)^2 - (a^4 + 2*a^3*b + a^2*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+f\,x)\,\left(b\,{\sin(e+f\,x)}^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.156 $\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}} dx$

Optimal. Leaf size=134

$$-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{5/2}f}-\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}}$$

```
[Out] -1/2*(a-3*b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(5/2)
/f-1/2*b*(a+3*b)*cos(f*x+e)/a^2/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)-1/2*cot(
f*x+e)*csc(f*x+e)/a/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3265, 425, 541, 12, 385, 212}

$$-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{b(a+3b)\cos(e+fx)}{2a^2f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((a - 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^
2]])/(a^(5/2)*f) - (b*(a + 3*b)*Cos[e + f*x])/(2*a^2*(a + b)*f*Sqrt[a + b -
 b*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*Sqrt[a + b - b*Cos
[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \cos(e+f\right.}{2af}$$

$$= -\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{S}}{}$$

$$= -\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{(}{}$$

$$= -\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{(}{}$$

$$= -\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{b(a+3b)\cos(e+}{2a^2(a+b)f\sqrt{a+b-b\cos}}$$

Mathematica [A]
time = 0.48, size = 134, normalized size = 1.00

$$\frac{-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{a^{5/2}} + \frac{(-2a^2-3ab-3b^2+b(a+3b)\cos(2(e+fx)))\cot(e+fx)\csc(e+fx)}{\sqrt{2}\,a^2(a+b)\sqrt{2a+b-b\cos(2(e+fx))}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(((a - 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2
*(e + f*x)]]])/a^(5/2)) + ((-2*a^2 - 3*a*b - 3*b^2 + b*(a + 3*b)*Cos[2*(e +
 f*x)])*Cot[e + f*x]*Csc[e + f*x])/(Sqrt[2]*a^2*(a + b)*Sqrt[2*a + b - b*Co
s[2*(e + f*x)]]))/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(118) = 236.
time = 16.32, size = 274, normalized size = 2.04

method	result
default	$\dfrac{\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(-\frac{\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}}{2a^{2}\sin\left(fx+e\right)^{2}}-\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/a^2/sin(f*x+e)^2*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)-1/4/a^(3/2)*ln((2*a+(-a+b)*sin(f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2)+3/4/a^(5/2)*b*ln((2*a+(-a+b)*sin(f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2)-b^2/a^2*cos(f*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(csc(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(118) = 236$.
time = 0.69, size = 634, normalized size = 4.73

$$\left[-\frac{\left(\left(a^{2}b-2ab^{2}-3b^{3}\right)\cos\left(fx+e\right)^{4}+a^{3}-a^{2}b-5ab^{2}-3b^{3}-\left(a^{3}-7ab^{2}-6b^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a}\log\left(\frac{2\left(\left(a^{2}-6ab+b^{2}\right)\cos\left(fx+e\right)^{4}+2\left(3a^{2}+2ab-b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a-b\right)\cos\left(fx+e\right)^{3}+\left(a+b\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{a}+a^{2}+2ab+b^{2}\right)}{\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1}\right)-4\left(\left(a^{2}b+3ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}+2a^{2}b+3ab^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{8\left(\left(a^{4}b+a^{3}b^{2}\right)f\cos\left(fx+e\right)^{4}-\left(a^{5}+3a^{4}b+2a^{3}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}+2a^{4}b+a^{3}b^{2}\right)f\right)},\ \frac{\left(\left(a^{2}b-2ab^{2}-3b^{3}\right)\cos\left(fx+e\right)^{4}+a^{3}-a^{2}b-5ab^{2}-3b^{3}-\left(a^{3}-7ab^{2}-6b^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a}\arctan\left(-\frac{\left(\left(a-b\right)\cos\left(fx+e\right)^{2}+a+b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{-a}}{2\left(ab\cos\left(fx+e\right)^{3}-\left(a^{2}+ab\right)\cos\left(fx+e\right)\right)}\right)+2\left(\left(a^{2}b+3ab^{2}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}+2a^{2}b+3ab^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{4\left(\left(a^{4}b+a^{3}b^{2}\right)f\cos\left(fx+e\right)^{4}-\left(a^{5}+3a^{4}b+2a^{3}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}+2a^{4}b+a^{3}b^{2}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/8*(((a^2*b - 2*a*b^2 - 3*b^3)*cos(f*x + e)^4 + a^3 - a^2*b - 5*a*b^2 - 3*b^3 - (a^3 - 7*a*b^2 - 6*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((a^2*b + 3*a*b^2)*cos(f*x + e)^3 - (a^3 + 2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^4 - (a^5 +

```
3*a^4*b + 2*a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f), 1/4*
(((a^2*b - 2*a*b^2 - 3*b^3)*cos(f*x + e)^4 + a^3 - a^2*b - 5*a*b^2 - 3*b^3
- (a^3 - 7*a*b^2 - 6*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos
(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x
+ e)^3 - (a^2 + a*b)*cos(f*x + e))) + 2*((a^2*b + 3*a*b^2)*cos(f*x + e)^3 -
 (a^3 + 2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(
(a^4*b + a^3*b^2)*f*cos(f*x + e)^4 - (a^5 + 3*a^4*b + 2*a^3*b^2)*f*cos(f*x
+ e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(125) = 250.
time = 0.72, size = 520, normalized size = 3.88

$$\frac{\frac{\left(\frac{(a^5b+a^4b^2)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^6b+a^5b^2}+\frac{2(a^5b+3a^4b^2+6a^3b^3)}{a^6b+a^5b^2}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\frac{a^5b+a^4b^2-8a^3b^3}{a^6b+a^5b^2}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}-\frac{2\left(a^{\frac{3}{2}}-3\sqrt{a}\,b\right)\log\left(\left|-\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)a-a^{\frac{3}{2}}-2\sqrt{a}\,b\right|\right)}{a^3}+\frac{2\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)a^{\frac{3}{2}}+2\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)\sqrt{a}\,b+a^2\right)}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\sqrt{a}-a^{\frac{3}{2}}\right)a^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((((a^5*b + a^4*b^2)*tan(1/2*f*x + 1/2*e)^2/(a^6*b + a^5*b^2) + 2*(a^5*
b + 3*a^4*b^2 + 6*a^3*b^3)/(a^6*b + a^5*b^2))*tan(1/2*f*x + 1/2*e)^2 + (a^5
*b + a^4*b^2 - 8*a^3*b^3)/(a^6*b + a^5*b^2))/sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - 2*(a^(3/2)
 - 3*sqrt(a)*b)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))*a - a^(3/2) - 2*sqrt(a)*b))/a^3 + 2*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*a^(3/2) + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*sqrt(a)*b + a^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a))^2*sqrt(a) - a^(3/2))*a^2))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+f\,x)^3\,\left(b\sin(e+f\,x)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.157 $\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=274

$$\frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}}-\frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^2(a+b)f}-\frac{(8a^2+3ab-2b^2)\sqrt{\ }}{}$$

```
[Out] a*cos(f*x+e)*sin(f*x+e)^3/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(4*a+b)*co
s(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b^2/(a+b)/f-1/3*(8*a^2+3*a*b-2
*b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a
+b*sin(f*x+e)^2)^(1/2)/b^3/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(8*a-b)
*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*si
n(f*x+e)^2/a)^(1/2)/b^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 481, 596, 538, 437, 435, 432, 430}

$$-\frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3b^3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{a(8a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3b^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(4a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^2f(a+b)}+\frac{a\sin^3(e+fx)\cos(e+fx)}{bf(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x]^3)/(b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) -
((4*a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*(a
+ b)*f) - ((8*a^2 + 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Si
n[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^3*(a + b
)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(8*a - b)*Sqrt[Cos[e + f*x]^2]*Ell
ipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^
2)/a])/(3*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1-x^2}\,(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx\right.}{f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \right.}{b(a+b}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin}}{3b^2(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin}}{3b^2(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin}}{3b^2(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin}}{3b^2(a+b)f}$$

Mathematica [A]
time = 0.89, size = 197, normalized size = 0.72

$$\frac{-2\sqrt{2}\,a(8a^2+3ab-2b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}\,a(8a^2+7ab-b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+b(-8a^2-3ab-b^2+b(a+b)\cos(2(e+fx)))\sin(2(e+fx))}{6\sqrt{2}\,b^3(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*a*(8*a^2 + 3*a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a
]*EllipticE[e + f*x, -(b/a)] + 2*Sqrt[2]*a*(8*a^2 + 7*a*b - b^2)*Sqrt[(2*a
+ b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + b*(-8*a^2 - 3*a*b
 - b^2 + b*(a + b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(6*Sqrt[2]*b^3*(a +
b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 7.31, size = 405, normalized size = 1.48

method	result
default	$\frac{a b^2 (\sin^5(fx+e))+b^3 (\sin^5(fx+e))+8\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,a^3+7a^2\sqrt{\mathrm{c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*b^2*sin(f*x+e)^5+b^3*sin(f*x+e)^5+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f
*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*a^2*(cos(f*x+e
)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*b^2-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+2*(cos(f*x+e)^2
)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a
*b^2+4*a^2*b*sin(f*x+e)^3-b^3*sin(f*x+e)^3-4*a^2*b*sin(f*x+e)-a*b^2*sin(f*x
+e))/b^3/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.14, size = 92, normalized size = 0.34

$$\mathrm{integral}\left(-\frac{\left(\cos\left(fx+e\right)^6-3\,\cos\left(fx+e\right)^4+3\,\cos\left(fx+e\right)^2-1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(-
b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^
2 + a^2 + 2*a*b + b^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^6}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.158 $\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$\frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin}}{b^2(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] a*cos(f*x+e)*sin(f*x+e)/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(2*a+b)*Elliptic
E(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^
2)^(1/2)/b^2/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-2*a*EllipticF(sin(f*x+e),(-
b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^2/
f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3267, 481, 538, 437, 435, 432, 430}

$$-\frac{2a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{a\sin(e+fx)\cos(e+fx)}{bf(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x])/(b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((
2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(b^2*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]
^2)/a]) - (2*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b^2*f*Sqrt[a + b*Sin[e + f*x
]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{1-x^2}\,(a+bx^2)^{3/2}}\,dx, x, \sin(e+f\ldots\right.}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \ldots\right.}{b(a+\ldots}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2a\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\ldots\right.}{b\ldots}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\left(-2a-b\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\ldots\right.}{b^2(a\ldots}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+\ldots\right.}{b^2(a+b)f\sqrt{1\ldots}}$$

Mathematica [A]
time = 0.47, size = 136, normalized size = 0.67

$$\frac{a\left(2(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-4(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b\sin(2(e+fx))\right)}{2b^2(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(2*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 4*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x
, -(b/a)] + Sqrt[2]*b*Sin[2*(e + f*x)]))/(2*b^2*(a + b)*f*Sqrt[2*a + b - b*
Cos[2*(e + f*x)]])
```

Maple [A]
time = 8.82, size = 241, normalized size = 1.19

method	result

default	$-\frac{a\left(2a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx\cdots}{a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(2*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ell
ipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))-(cos(f*x+e)^2)^(1/2)*((
a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b+sin(f*x+e
)^3*b-b*sin(f*x+e))/b^2/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.11, size = 81, normalized size = 0.40

$$\mathrm{integral}\left(\frac{\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 + a
 + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^
2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+f\,x)^4}{\left(b\sin(e+f\,x)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.159 $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}-\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{b(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -cos(f*x+e)*sin(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-(cos(f*x+e)^2)^(1/2
)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)/b/
(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*Elliptic
F(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2
)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$,
Rules used = {3252, 3251, 3257, 3256, 3262, 3261}

$$-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{bf(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Cos[e + f*x]*Sin[e + f*x])/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])) - (El
lipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*(a + b)*f*Sqrt[1 +
(b*Sin[e + f*x]^2)/a]) + (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*
x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
```

```
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \frac{a-a\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx}{a(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}dx}{b} - \frac{\int \sqrt{a+b\sin^2}}{b(}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\sqrt{a+b\sin^2(e+fx)}\int\sqrt{1+\frac{b\sin^2(e+}{a}}}{b(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{b(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{F}{}$$

Mathematica [A]
time = 0.29, size = 138, normalized size = 0.90

$$\frac{-\sqrt{2}\,a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-b\sin(2(e+fx))}{\sqrt{2}\,b(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/
a)]) + Sqrt[2]*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
 f*x, -(b/a)] - b*Sin[2*(e + f*x)])/(Sqrt[2]*b*(a + b)*f*Sqrt[2*a + b - b*C
os[2*(e + f*x)]])
```

Maple [A]
time = 8.41, size = 191, normalized size = 1.25

method	result
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{E}}{b(a+b)\cos(fx+e)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(
-1/a*b)^(1/2))+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(
sin(f*x+e),(-1/a*b)^(1/2))*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))+sin(f*x+e)^3*b-b*sin(f*x+e))/b/(
a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.16, size = 780, normalized size = 5.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^2*cos(f*x + e)*sin(f*x + e) - 2*((
-2*I*a*b - I*b^2)*cos(f*x + e)^2 + 2*I*a^2 + 3*I*a*b + I*b^2)*sqrt(-b)*sqrt
((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8
*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*((2*I*a*b + I*
b^2)*cos(f*x + e)^2 - 2*I*a^2 - 3*I*a*b - I*b^2)*sqrt(-b)*sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-I*b^2*cos(f*x + e)^2 + I
*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a*b + I*b^2)*cos(f*x +
 e)^2 - 2*I*a^2 - 3*I*a*b - I*b^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2
) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^
2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*b^2*cos(f*x + e)^2 - I*a*b - I*b^2)*
sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a*b - I*b^2)*cos(f*x + e)^2 + 2*I*a
^2 + 3*I*a*b + I*b^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/
```

```
b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x
 + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2
+ a*b)/b^2))/b^2))/((a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^2*b^2 + 2*a*b^3 + b
^4)*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.160 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(cos(f*x+e)^2)^(
1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)
/a/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3263, 21, 3257, 3256}

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
```

```
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{-a-b\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)}\\
&= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{a(a+b)}\\
&= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+f}{a}}}{a(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\
&= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 0.89

$$
\frac{2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b\sin(2(e+fx))}{2a(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sq
rt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)
```

Maple [A]
time = 6.24, size = 103, normalized size = 1.02

method	result	size
default	$\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,a\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+\sin(fx+e)(\cos^2(fx+e))b}{a(a+b)\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))+sin(f*x+e)*cos(f*x+e)^2*b)/a/(a+b)/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.18, size = 938, normalized size = 9.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^3*cos(f*x + e)*sin(f*x + e) - (2*
(I*b^3*cos(f*x + e)^2 - I*a*b^2 - I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (
2*I*a^2*b + 3*I*a*b^2 + I*b^3 + (-2*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqrt(-
b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8
*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(-I*b
^3*cos(f*x + e)^2 + I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-2*I
*a^2*b - 3*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b))*
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2
 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(2*(-I*a^2
*b - 2*I*a*b^2 - I*b^3 + (I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a
```

```
^2 + a*b)/b^2) + (2*I*a^3 + 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b - I*a*b^2)*co
s(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellip
tic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) +
I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b
^2))/b^2) + 2*(2*(I*a^2*b + 2*I*a*b^2 + I*b^3 + (-I*a*b^2 - I*b^3)*cos(f*x
+ e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + (-2*I*a^3 - 3*I*a^2*b - I*a*b^2 +
(2*I*a^2*b + I*a*b^2)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
+ b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
 b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/((a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 - (a
^3*b^2 + 2*a^2*b^3 + a*b^4)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sin(e + f*x)**2)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(1/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.161 $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=235

$$\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}-\frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f}-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\,E(\sin$$

```
[Out] b*cot(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-(a+2*b)*cot(f*x+e)*(a+b*sin
(f*x+e)^2)^(1/2)/a^2/(a+b)/f-(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec
(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/(a+b)/f/(1+b*sin(
f*x+e)^2/a)^(1/2)+EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)
^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 483, 597, 538, 437, 435, 432, 430}

$$-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{a^2f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f(a+b)}+\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{af\sqrt{a+b\sin^2(e+fx)}}+\frac{b\cot(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cot[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((a + 2*b)*Cot[
e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a^2*(a + b)*f) - ((a + 2*b)*Sqrt[Cos[
e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b
*Sin[e + f*x]^2])/(a^2*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^2}\,(a+bx^2)^{3/2}}\,dx, x, \sin(e+\right.}{f}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\right.}{a(a}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f}$$

Mathematica [A]
time = 0.89, size = 170, normalized size = 0.72

$$\frac{(-2a^2-3ab-2b^2+b(a+2b)\cos(2(e+fx)))\cot(e+fx)-\sqrt{2}\,a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{\sqrt{2}\,a^2(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

[Out] ((-2*a^2 - 3*a*b - 2*b^2 + b*(a + 2*b)*Cos[2*(e + f*x)])*Cot[e + f*x] - Sqr
t[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] + Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Elliptic
F[e + f*x, -(b/a)])/(Sqrt[2]*a^2*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)
]])

Maple [A]
time = 8.79, size = 199, normalized size = 0.85

method	result
default	$\frac{\sin(fx+e)\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\;a\left(\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a+\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\ldots}\right.\right.}{a^2\sin(fx+e)(a+b)\cos(fx+e)\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))
*b)+(a*b+2*b^2)*cos(f*x+e)^4+(-a^2-2*a*b-2*b^2)*cos(f*x+e)^2)/a^2/sin(f*x+e
)/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)

Fricas [C] Result contains complex when optimal does not.
time = 0.18, size = 1075, normalized size = 4.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] 1/2*((2*(I*a^2*b + 3*I*a*b^2 + 2*I*b^3 + (-I*a*b^2 - 2*I*b^3)*cos(f*x + e)^
2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (-2*I*a^3 - 7*I*a^2*b - 7*
I*a*b^2 - 2*I*b^3 + (2*I*a^2*b + 5*I*a*b^2 + 2*I*b^3)*cos(f*x + e)^2)*sqrt(
-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(

```
arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(
f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b
^2) + (2*(-I*a^2*b - 3*I*a*b^2 - 2*I*b^3 + (I*a*b^2 + 2*I*b^3)*cos(f*x + e)
^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (2*I*a^3 + 7*I*a^2*b + 7*
I*a*b^2 + 2*I*b^3 + (-2*I*a^2*b - 5*I*a*b^2 - 2*I*b^3)*cos(f*x + e)^2)*sqrt
(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e
(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin
(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/
b^2) - 2*(4*(I*a^2*b + 2*I*a*b^2 + I*b^3 + (-I*a*b^2 - I*b^3)*cos(f*x + e)^
2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (-2*I*a^3 - 3*I*a^2*b - I*
a*b^2 + (2*I*a^2*b + I*a*b^2)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((
2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a
*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(4*(-I*a^2*b - 2
*I*a*b^2 - I*b^3 + (I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a
*b)/b^2)*sin(f*x + e) + (2*I*a^3 + 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b - I*a*
b^2)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2)
 + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2
)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*((a*b^2 + 2*b^3)*cos(f*x + e)^3 - (a^2*b
+ 2*a*b^2 + 2*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(((a^3*b^
2 + a^2*b^3)*f*cos(f*x + e)^2 - (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x +
e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin \left(e+f\,x\right)^2\,{\left(b\,\sin \left(e+f\,x\right)^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)),x)
[Out] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.162 $\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{5/2}f}+\frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}}+\frac{a\cos(e+fx)\sin^2(e+}{3b(a+b)f\,(a+b-b\cos^2(e}$$

```
[Out] -arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(5/2)/f+1/3*a*cos(
f*x+e)*sin(f*x+e)^2/b/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)+1/3*a*(3*a+5*b)*co
s(f*x+e)/b^2/(a+b)^2/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 424, 393, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{5/2}f}+\frac{a(3a+5b)\cos(e+fx)}{3b^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}+\frac{a\sin^2(e+fx)\cos(e+fx)}{3bf(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(b^(5/2)*f)
) + (a*(3*a + 5*b)*Cos[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b - b*Cos[e +
f*x]^2]) + (a*Cos[e + f*x]*Sin[e + f*x]^2)/(3*b*(a + b)*f*(a + b - b*Cos[e
+ f*x]^2)^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a-3b+3(a+b)x^2}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e\right.}{3b(a+b)f}$$

$$= \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} + \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)}$$

$$= \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} + \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{5/2}f} + \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx}}$$

Mathematica [A]
time = 0.55, size = 133, normalized size = 0.97

$$\frac{\frac{2\sqrt{2}\,a\cos(e+fx)\left(3a^2+7ab+3b^2-b(2a+3b)\cos(2(e+fx))\right)}{(a+b)^2(2a+b-b\cos(2(e+fx)))^{3/2}}-\frac{3\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{\sqrt{-b}}}{3b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*Sqrt[2]*a*Cos[e + f*x]*(3*a^2 + 7*a*b + 3*b^2 - b*(2*a + 3*b)*Cos[2*(e
+ f*x)]))/((a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2)) - (3*Log[Sqrt[2]
*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/Sqrt[-b])/(3*
b^2*f)
```

Maple [A]
time = 18.34, size = 243, normalized size = 1.77

method	result
default	$\frac{\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}\left(\frac{\arctan\left(\frac{\sqrt{b}\left(\sin^2(fx+e)-\frac{-a+b}{2b}\right)}{\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right.}}\right.}{2b^{\frac{5}{2}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/2/b^(5/2)*arctan(b^(1/2)*(sin(
f*x+e)^2-1/2*(-a+b)/b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))-1/3*a^2/b
^2*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)^2/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2
)^(1/2)/(a+b*sin(f*x+e)^2)/(a^2+2*a*b+b^2)+2*a/b^2*cos(f*x+e)^2/(a+b)/(-(-b
*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(130) = 260$.
time = 0.49, size = 301, normalized size = 2.20

$$-\frac{\left(\frac{3\cos(fx+e)^2}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}b}-\frac{2a}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}b^2}-\frac{2}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}b}\right)\cos\left(fx+e\right)+\frac{3\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{2\cos(fx+e)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,(a+b)^2}+\frac{\cos(fx+e)}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)}-\frac{3\cos(fx+e)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,b^2}+\frac{2a\cos(fx+e)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,(a+b)b^2}-\frac{2\cos(fx+e)}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}b}+\frac{4\cos(fx+e)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,(a+b)b}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*cos(f*x + e)^2/((-b*cos(f*x + e)^2 + a + b)^(3/2)*b) - 2*a/((-b*co
s(f*x + e)^2 + a + b)^(3/2)*b^2) - 2/((-b*cos(f*x + e)^2 + a + b)^(3/2)*b))
*cos(f*x + e) + 3*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 2*cos(f*
x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x + e)/((-b*cos(
f*x + e)^2 + a + b)^(3/2)*(a + b)) - 3*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2
 + a + b)*b^2) + 2*a*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*
b^2) - 2*cos(f*x + e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*b) + 4*cos(f*x + e
)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*b))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(123) = 246$.
time = 0.87, size = 885, normalized size = 6.46

$$\left[-\frac{3\,\left(\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{4}+a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}-2\,\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-b}\log\left(128\,b^{4}\cos\left(fx+e\right)^{8}-256\,\left(ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{6}+160\,\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{4}+a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}-32\,\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{2}+8\,\left(16\,b^{3}\cos\left(fx+e\right)^{7}-24\,\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{5}+10\,\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{3}-\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{-b}\right)+8\,\left(2\,\left(2\,a^{2}b^{2}+3\,ab^{3}\right)\cos\left(fx+e\right)^{3}-3\,\left(a^{3}b+3\,a^{2}b^{2}+2\,ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{24\,\left(\left(a^{2}b^{5}+2\,ab^{6}+b^{7}\right)f\cos\left(fx+e\right)^{4}-2\,\left(a^{3}b^{4}+3\,a^{2}b^{5}+3\,ab^{6}+b^{7}\right)f\cos\left(fx+e\right)^{2}+\left(a^{4}b^{3}+4\,a^{3}b^{4}+6\,a^{2}b^{5}+4\,ab^{6}+b^{7}\right)f\right)},\ \frac{3\,\left(\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{4}+a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}-2\,\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{b}\arctan\left(\frac{\left(8\,b^{2}\cos\left(fx+e\right)^{4}-8\,\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{b}}{4\,\left(2\,b^{3}\cos\left(fx+e\right)^{5}-3\,\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{3}+\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)\right)}\right)-4\,\left(2\,\left(2\,a^{2}b^{2}+3\,ab^{3}\right)\cos\left(fx+e\right)^{3}-3\,\left(a^{3}b+3\,a^{2}b^{2}+2\,ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{12\,\left(\left(a^{2}b^{5}+2\,ab^{6}+b^{7}\right)f\cos\left(fx+e\right)^{4}-2\,\left(a^{3}b^{4}+3\,a^{2}b^{5}+3\,ab^{6}+b^{7}\right)f\cos\left(fx+e\right)^{2}+\left(a^{4}b^{3}+4\,a^{3}b^{4}+6\,a^{2}b^{5}+4\,ab^{6}+b^{7}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2
)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 +
160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(
16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b
^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 8*(2*(2*a^2*b^2 + 3*a*b^3)*cos(f*
x + e)^3 - 3*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x +
e)^2 + a + b))/((a^2*b^5 + 2*a*b^6 + b^7)*f*cos(f*x + e)^4 - 2*(a^3*b^4 + 3
*a^2*b^5 + 3*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b
^5 + 4*a*b^6 + b^7)*f), 1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 +
 a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*cos(f*x + e)^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b +
 b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*
a*b^2 + b^3)*cos(f*x + e))) - 4*(2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3
*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b
))/((a^2*b^5 + 2*a*b^6 + b^7)*f*cos(f*x + e)^4 - 2*(a^3*b^4 + 3*a^2*b^5 + 3
*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b^5 + 4*a*b^6
 + b^7)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(130) = 260$.
time = 0.78, size = 338, normalized size = 2.47

$$-\frac{\frac{\left(\left(\frac{\left(3\,a^{3}b^{8}+5\,a^{2}b^{9}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{a^{2}b^{10}+2\,ab^{11}+b^{12}}+\frac{3\,\left(a^{3}b^{8}+7\,a^{2}b^{9}+8\,ab^{10}\right)}{a^{2}b^{10}+2\,ab^{11}+b^{12}}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-\frac{3\,\left(a^{3}b^{8}+7\,a^{2}b^{9}+8\,ab^{10}\right)}{a^{2}b^{10}+2\,ab^{11}+b^{12}}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-\frac{3\,a^{3}b^{8}+5\,a^{2}b^{9}}{a^{2}b^{10}+2\,ab^{11}+b^{12}}}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+4\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a\right)^{\frac{3}{2}}}-\frac{6\,\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+4\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a}+\sqrt{a}}{2\,\sqrt{b}}\right)}{b^{\frac{5}{2}}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((((3*a^3*b^8 + 5*a^2*b^9)*tan(1/2*f*x + 1/2*e)^2/(a^2*b^10 + 2*a*b^1
1 + b^12) + 3*(a^3*b^8 + 7*a^2*b^9 + 8*a*b^10)/(a^2*b^10 + 2*a*b^11 + b^12)
)*tan(1/2*f*x + 1/2*e)^2 - 3*(a^3*b^8 + 7*a^2*b^9 + 8*a*b^10)/(a^2*b^10 + 2
*a*b^11 + b^12))*tan(1/2*f*x + 1/2*e)^2 - (3*a^3*b^8 + 5*a^2*b^9)/(a^2*b^10
 + 2*a*b^11 + b^12))/(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) - 6*arctan(-1/2*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(b))/b^(5/2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^5}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.163 $\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2\cos(e+fx)}{3(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}}-\frac{\cos(e+fx)\sin^2(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)^2/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)-2/3*cos(f*x
+e)/(a+b)^2/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3265, 386, 197}

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\sin^2(e+fx)\cos(e+fx)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*Cos[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2]) - (Cos[e +
 f*x]*Sin[e + f*x]^2)/(3*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a+b-bx^2)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)\sin^2(e+fx)}{3(a+b)f\,(a+b-b\cos^2(e+fx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{(a+b-bx^2)^{3/2}}\,dx, x, \cos(e$$
$$= -\frac{2\cos(e+fx)}{3(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cos(e+fx)\sin^2(e+fx)}{3(a+b)f\,(a+b-b\cos^2(e+fx))}$$

Mathematica [A]
time = 0.21, size = 64, normalized size = 0.79

$$\frac{\sqrt{2}\cos(e+fx)(-5a-3b+(a+3b)\cos(2(e+fx)))}{3(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
[Out] (Sqrt[2]*Cos[e + f*x]*(-5*a - 3*b + (a + 3*b)*Cos[2*(e + f*x)]))/(3*(a + b)
^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [A]
time = 5.91, size = 56, normalized size = 0.69

method	result	size
default	$-\frac{\left(a\left(\sin^2(fx+e)\right)+3b\left(\sin^2(fx+e)\right)+2a\right)\cos(fx+e)}{3(a+b)^2\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}f}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(a*sin(f*x+e)^2+3*b*sin(f*x+e)^2+2*a)*cos(f*x+e)/(a+b)^2/(a+b*sin(f*x+
e)^2)^(3/2)/f
```

Maxima [A]
time = 0.28, size = 129, normalized size = 1.59

$$-\frac{\frac{2\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}\,(a+b)^2} + \frac{\cos(fx+e)}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)} - \frac{\cos(fx+e)}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}b} + \frac{\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}\,(a+b)b}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] -1/3*(2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x
+ e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*(a + b)) - cos(f*x + e)/((-b*cos(f*
x + e)^2 + a + b)^(3/2)*b) + cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*
(a + b)*b))/f
```

Fricas [**A**]
time = 0.54, size = 137, normalized size = 1.69

$$\frac{\left((a+3\,b)\cos\left(fx+e\right)^{3}-3\,(a+b)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{3\,\left((a^{2}b^{2}+2\,ab^{3}+b^{4})f\cos\left(fx+e\right)^{4}-2\,(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4})f\cos\left(fx+e\right)^{2}+(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4})f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] 1/3*((a + 3*b)*cos(f*x + e)^3 - 3*(a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e
)^2 + a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*f*cos(f*x + e)^4 - 2*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [**A**]
time = 0.70, size = 149, normalized size = 1.84

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\left(\frac{3\,(abf^{2}+b^{2}f^{2})}{a^{2}bf^{2}+2\,ab^{2}f^{2}+b^{3}f^{2}}-\frac{(abf^{4}+3\,b^{2}f^{4})\cos(fx+e)^{2}}{(a^{2}bf^{2}+2\,ab^{2}f^{2}+b^{3}f^{2})f^{2}}\right)\cos\left(fx+e\right)}{3\left(b\cos\left(fx+e\right)^{2}-a-b\right)^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] -1/3*sqrt(-b*cos(f*x + e)^2 + a + b)*(3*(a*b*f^2 + b^2*f^2)/(a^2*b*f^2 + 2*
a*b^2*f^2 + b^3*f^2) - (a*b*f^4 + 3*b^2*f^4)*cos(f*x + e)^2/((a^2*b*f^2 + 2
*a*b^2*f^2 + b^3*f^2)*f^2))*cos(f*x + e)/((b*cos(f*x + e)^2 - a - b)^2*f)
```

Mupad [**B**]
time = 20.87, size = 176, normalized size = 2.17

$$\frac{2\,{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{a+b\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,\left(a+3\,b-10\,a\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-6\,b\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+3\,b\,{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)}{3\,f\,{\left(a+b\right)}^2\,{\left(b-4\,a\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-2\,b\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+b\,{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] (2*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b*((exp(- e*1i - f*x*1i
)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(a + 3*b - 10*a*exp(e*2i + f*
x*2i) + a*exp(e*4i + f*x*4i) - 6*b*exp(e*2i + f*x*2i) + 3*b*exp(e*4i + f*x*
4i)))/(3*f*(a + b)^2*(b - 4*a*exp(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*2i) +
 b*exp(e*4i + f*x*4i))^2)
```

3.164 $\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{\cos(e+fx)}{3(a+b)f\,(a+b-b\cos^2(e+fx))^{3/2}}-\frac{2\cos(e+fx)}{3(a+b)^2f\,\sqrt{a+b-b\cos^2(e+fx)}}$$

```
[Out] -1/3*cos(f*x+e)/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)-2/3*cos(f*x+e)/(a+b)^2/f
/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 198, 197}

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\cos(e+fx)}{3f(a+b)\,(a-b\cos^2(e+fx)+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*Cos[e + f*x]/((a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2)) - (2*Cos[e +
 f*x])/(3*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(a+b-bx^2)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{3(a+b)f\,(a+b-b\cos^2(e+fx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{(a+b-bx^2)^{3/2}}\,dx, x, \cos(e\right.}{3(a+b)f}$$

$$= -\frac{\cos(e+fx)}{3(a+b)f\,(a+b-b\cos^2(e+fx))^{3/2}} - \frac{2\cos(e+fx)}{3(a+b)^2f\sqrt{a+b-b\cos^2(e+f}}$$

Mathematica [A]
time = 0.13, size = 60, normalized size = 0.82

$$\frac{2\sqrt{2}\cos(e+fx)(-3a-2b+b\cos(2(e+fx)))}{3(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*Sqrt[2]*Cos[e + f*x]*(-3*a - 2*b + b*Cos[2*(e + f*x)]))/(3*(a + b)^2*f*(
2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [A]
time = 6.44, size = 55, normalized size = 0.75

method	result	size
default	$-\frac{\left(2b\left(\sin^2(fx+e)\right)+3a+b\right)\cos(fx+e)}{3\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}(a^2+2ab+b^2)f}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2)/(a^2+2*a*
b+b^2)/f
```

Maxima [A]
time = 0.28, size = 67, normalized size = 0.92

$$-\frac{\frac{2\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}\,(a+b)^2}+\frac{\cos(fx+e)}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x
+ e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(65) = 130$.

time = 0.49, size = 134, normalized size = 1.84

$$\frac{\left(2\,b\cos\left(fx+e\right)^{3}-3\,(a+b)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{3\,\left((a^{2}b^{2}+2\,ab^{3}+b^{4})f\cos\left(fx+e\right)^{4}-2\,(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4})f\cos\left(fx+e\right)^{2}+(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4})f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*cos(f*x + e)^3 - 3*(a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 +
a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*f*cos(f*x + e)^4 - 2*(a^3*b + 3*a^2*b^2 +
 3*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*f)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]

time = 0.62, size = 137, normalized size = 1.88

$$\frac{\left(\frac{2\,b^{2}f^{2}\cos(fx+e)^{2}}{a^{2}bf^{2}+2\,ab^{2}f^{2}+b^{3}f^{2}}-\frac{3\,(abf^{2}+b^{2}f^{2})}{a^{2}bf^{2}+2\,ab^{2}f^{2}+b^{3}f^{2}}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cos\left(fx+e\right)}{3\,\left(b\cos\left(fx+e\right)^{2}-a-b\right)^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*b^2*f^2*cos(f*x + e)^2/(a^2*b*f^2 + 2*a*b^2*f^2 + b^3*f^2) - 3*(a*b*
f^2 + b^2*f^2)/(a^2*b*f^2 + 2*a*b^2*f^2 + b^3*f^2))*sqrt(-b*cos(f*x + e)^2
+ a + b)*cos(f*x + e)/((b*cos(f*x + e)^2 - a - b)^2*f)
```

Mupad [B]

time = 20.80, size = 159, normalized size = 2.18

$$\frac{4\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{a+b\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^{2}}\left(b-6\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)}{3\,f\,(a+b)^{2}\left(b-4\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-2\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] (4*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b*((exp(- e*1i - f*x*1i
)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(b - 6*a*exp(e*2i + f*x*2i) -
 4*b*exp(e*2i + f*x*2i) + b*exp(e*4i + f*x*4i)))/(3*f*(a + b)^2*(b - 4*a*ex
p(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*2i) + b*exp(e*4i + f*x*4i))^2)
```

3.165 $\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{5/2}f}+\frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}+\frac{b(5a+3b)\cos(e+}{3a^2(a+b)^2f\sqrt{a+b-b\cos}}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*b*cos
(f*x+e)/a/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)+1/3*b*(5*a+3*b)*cos(f*x+e)/a^2
/(a+b)^2/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {3265, 425, 541, 12, 385, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{b(5a+3b)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}+\frac{b\cos(e+fx)}{3af(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(a^(5/2)*f
)) + (b*Cos[e + f*x])/(3*a*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2)) + (b
*(5*a + 3*b)*Cos[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3a-b-2bx^2}{(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \cos\right.}{3a(a+b)f}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+f}}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+f}}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+f}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{5/2}f} + \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)}$$

Mathematica [A]
time = 0.41, size = 127, normalized size = 0.98

$$\frac{-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{a^{5/2}} + \frac{\sqrt{2}\,b\cos(e+fx)\left(12a^2+13ab+3b^2-b(5a+3b)\cos(2(e+fx))\right)}{3a^2(a+b)^2(2a+b-b\cos(2(e+fx)))^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]
]]/a^(5/2)) + (Sqrt[2]*b*Cos[e + f*x]*(12*a^2 + 13*a*b + 3*b^2 - b*(5*a + 3
*b)*Cos[2*(e + f*x)]))/(3*a^2*(a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2
)))/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(115) = 230$.
time = 19.38, size = 249, normalized size = 1.93

method	result

default	$\frac{\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(-\frac{\ln\left(\frac{2a+(-a+b)\left(\sin^{2}(fx+e)\right)+2\sqrt{a}\,\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right.\right.\right.}}{\sin(fx+e)^{2}}\right.}{2a^{\frac{5}{2}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/a^(5/2)*ln((2*a+(-a+b)*sin(
f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2)
+1/3/a*b*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)^2/(-(-b*sin(f*x+e)^2-a)*cos(f*
x+e)^2)^(1/2)/(a+b*sin(f*x+e)^2)/(a^2+2*a*b+b^2)+1/a^2*b*cos(f*x+e)^2/(a+b)
/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(
1/2)/f
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(121) = 242$.
time = 0.52, size = 322, normalized size = 2.50

$$\frac{\frac{4b^{3}\cos(fx+e)}{\sqrt{-b\cos(fx+e)^{2}+a+b}\,a^{3}b^{2}+2\sqrt{-b\cos(fx+e)^{2}+a+b}\,a^{2}b^{3}+\sqrt{-b\cos(fx+e)^{2}+a+b}\,ab^{4}}+\frac{2b^{2}\cos(fx+e)}{\left(-b\cos(fx+e)^{2}+a+b\right)^{\frac{3}{2}}a^{2}b+\left(-b\cos(fx+e)^{2}+a+b\right)^{\frac{3}{2}}ab^{2}}+\frac{6b^{2}\cos(fx+e)}{\sqrt{-b\cos(fx+e)^{2}+a+b}\,a^{3}b+\sqrt{-b\cos(fx+e)^{2}+a+b}\,a^{2}b^{2}}-\frac{3\log\left(b-\frac{\sqrt{-b\cos(fx+e)^{2}+a+b}\,\sqrt{a}}{\cos(fx+e)-1}-\frac{a}{\cos(fx+e)-1}\right)}{a^{\frac{5}{2}}}+\frac{3\log\left(-b+\frac{\sqrt{-b\cos(fx+e)^{2}+a+b}\,\sqrt{a}}{\cos(fx+e)+1}+\frac{a}{\cos(fx+e)+1}\right)}{a^{\frac{5}{2}}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(4*b^3*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*a^3*b^2 + 2*sqrt(-
b*cos(f*x + e)^2 + a + b)*a^2*b^3 + sqrt(-b*cos(f*x + e)^2 + a + b)*a*b^4)
+ 2*b^2*cos(f*x + e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*a^2*b + (-b*cos(f*x
 + e)^2 + a + b)^(3/2)*a*b^2) + 6*b^2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2
+ a + b)*a^3*b + sqrt(-b*cos(f*x + e)^2 + a + b)*a^2*b^2) - 3*log(b - sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) - 1) - a/(cos(f*x + e) - 1
))/a^(5/2) + 3*log(-b + sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x +
e) + 1) + a/(cos(f*x + e) + 1))/a^(5/2))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(115) = 230$.
time = 0.67, size = 752, normalized size = 5.83

$$\left[\frac{3\left(\left(a^{2}b^{2}+2ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{4}+a^{4}+4a^{3}b+6a^{2}b^{2}+4ab^{3}+b^{4}-2\left(a^{3}b+3a^{2}b^{2}+3ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a}\log\left(\frac{2\left(\left(a^{2}-6ab+b^{2}\right)\cos(fx+e)^{4}+2\left(3a^{2}+2ab-b^{2}\right)\cos(fx+e)^{2}-4\left((a-b)\cos(fx+e)^{3}+(a+b)\cos(fx+e)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a}+a^{2}+2ab+b^{2}\right)}{\cos(fx+e)^{4}-2\cos(fx+e)^{2}+1}\right)-4\left(\left(5a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)^{3}-3\left(2a^{3}b+3a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{12\left(\left(a^{5}b^{2}+2a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{6}b+3a^{5}b^{2}+3a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{7}+4a^{6}b+6a^{5}b^{2}+4a^{4}b^{3}+a^{3}b^{4}\right)f\right)},\ \frac{3\left(\left(a^{2}b^{2}+2ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{4}+a^{4}+4a^{3}b+6a^{2}b^{2}+4ab^{3}+b^{4}-2\left(a^{3}b+3a^{2}b^{2}+3ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-a}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^{2}+a+b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-a}}{2\left(ab\cos(fx+e)^{3}-\left(a^{2}+ab\right)\cos(fx+e)\right)}\right)-2\left(\left(5a^{2}b^{2}+3ab^{3}\right)\cos\left(fx+e\right)^{3}-3\left(2a^{3}b+3a^{2}b^{2}+ab^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{6\left(\left(a^{5}b^{2}+2a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{6}b+3a^{5}b^{2}+3a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{7}+4a^{6}b+6a^{5}b^{2}+4a^{4}b^{3}+a^{3}b^{4}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2)
*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2
)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*
cos(f*x + e)^2 + 1)) - 4*((5*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3*(2*a^3*b
 + 3*a^2*b^2 + a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*
b^2 + 2*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 - 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
a^3*b^4)*f), 1/6*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(
f*x + e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x +
 e))) - 2*((5*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3*(2*a^3*b + 3*a^2*b^2 +
a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2 + 2*a^4*b^3
 + a^3*b^4)*f*cos(f*x + e)^4 - 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*
f*cos(f*x + e)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+f\,x)\,\left(b\,{\sin(e+f\,x)}^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

3.166 $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=285

$$\frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}}{3b^3(}$$

```
[Out] 1/3*a*cos(f*x+e)*sin(f*x+e)^3/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*a*(2*a
+3*b)*cos(f*x+e)*sin(f*x+e)/b^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(8*a
^2+13*a*b+3*b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^
2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/b^3/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-
1/3*a*(8*a+9*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2
)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^3/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 481, 592, 538, 437, 435, 432, 430}

$$\frac{(8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3b^3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(8a+9b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3b^3f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{2a(2a+3b)\sin(e+fx)\cos(e+fx)}{3b^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{a\sin^3(e+fx)\cos(e+fx)}{3bf(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x]^3)/(3*b*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)
) + (2*a*(2*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a +
 b*Sin[e + f*x]^2]) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[Cos[e + f*x]^2]*Ellipt
icE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/
(3*b^3*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(8*a + 9*b)*Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(3*b^3*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1-x^2}\,(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx\right.}{f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\right.}{3b(a-}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} -$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} -$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} -$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} +$$

Mathematica [A]
time = 1.42, size = 192, normalized size = 0.67

$$-\frac{a\left(-2a(8a^2+13ab+3b^2)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+2a(8a^2+17ab+9b^2)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b(-8a^2-17ab-7b^2+b(5a+7b)\cos(2(e+fx)))\sin(2(e+fx))\right)}{6b^3(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*(a*(-2*a*(8*a^2 + 13*a*b + 3*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(
3/2)*EllipticE[e + f*x, -(b/a)] + 2*a*(8*a^2 + 17*a*b + 9*b^2)*((2*a + b -
b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(-8*a^2
 - 17*a*b - 7*b^2 + b*(5*a + 7*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(b^3
*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(263) = 526$.

time = 9.64, size = 698, normalized size = 2.45

method	result
default	$-\frac{\left(\left(5a\,b^2+7b^3\right)\sin(fx+e)\left(\cos^4(fx+e)\right)+\left(-4a^2b-11a\,b^2-7b^3\right)\left(\cos^2(fx+e)\right)\sin(fx+e)-\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\sqrt{\cos(2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((5*a*b^2+7*b^3)*sin(f*x+e)*cos(f*x+e)^4+(-4*a^2*b-11*a*b^2-7*b^3)*cos
(f*x+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)
*b*(8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+17*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*a*b+9*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*EllipticE(sin(
f*x+e),(-1/a*b)^(1/2))*a^2-13*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b-3*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+8*(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3
+25*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*a^2*b+26*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)
/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2+9*(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3
-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a^3-21*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-16*(cos(f*x+e)^2)^(1/2)*(
-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2
-3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*b^3)*a/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/b^3/cos(f*x+e)/
f
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F]
time = 0.15, size = 130, normalized size = 0.46

$$\text{integral}\left(\frac{\left(\cos\left(fx+e\right)^{6}-3\,\cos\left(fx+e\right)^{4}+3\,\cos\left(fx+e\right)^{2}-1\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(-b
*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 + b^3)*cos(f*x + e)
^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)
^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^{6}}{{\left(b\sin\left(e+f\,x\right)^{2}+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.167 $\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=269

$$\frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\mathrm{s}\right.}{3b^2(a+b)}$$

```
[Out] 1/3*a*cos(f*x+e)*sin(f*x+e)/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)-2/3*(a+2*b)*
cos(f*x+e)*sin(f*x+e)/b/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(a+2*b)*Elli
pticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x
+e)^2)^(1/2)/b^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(2*a+3*b)*Ellipti
cF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)
^2/a)^(1/2)/b^2/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3267, 481, 541, 538, 437, 435, 432, 430}

$$\frac{(2a+3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\mathrm{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\mathrm{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2(a+2b)\sin(e+fx)\cos(e+fx)}{3bf(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{a\sin(e+fx)\cos(e+fx)}{3bf(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x])/(3*b*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
- (2*(a + 2*b)*Cos[e + f*x]*Sin[e + f*x])/(3*b*(a + b)^2*f*Sqrt[a + b*Sin[e
 + f*x]^2]) - (2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*(a + b)^2*f*Sq
rt[1 + (b*Sin[e + f*x]^2)/a]) + ((2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF
[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
/(3*b^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-x^2}\,(a+bx^2)^{5/2}}\,dx,x,\sin(e+f\cdots\right.}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\cdots\right.}{3b(a\cdots}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \cdots$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \cdots$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \cdots$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{2(\cdots}{\cdots}$$

Mathematica [A]
time = 1.13, size = 182, normalized size = 0.68

$$-\frac{2a^2(a+2b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-a(2a^2+5ab+3b^2)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b(-a^2-4ab-2b^2+b(a+2b)\cos(2(e+fx)))\sin(2(e+fx))}{3b^2(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*(2*a^2*(a + 2*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e
+ f*x, -(b/a)] - a*(2*a^2 + 5*a*b + 3*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/
a)^(3/2)*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(-a^2 - 4*a*b - 2*b^2 + b*(
a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b^2*(a + b)^2*f*(2*a + b - b*
Cos[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(247) = 494$.
time = 10.11, size = 623, normalized size = 2.32

method	result
default	$\frac{(2a\,b^2+4b^3)\sin(fx+e)(\cos^4(fx+e))+(-a^2b-5a\,b^2-4b^3)(\cos^2(fx+e))\sin(fx+e)-\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos(2fx+2e)}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((2*a*b^2+4*b^3)*sin(f*x+e)*cos(f*x+e)^4+(-a^2*b-5*a*b^2-4*b^3)*cos(f*x
+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*b*(
2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))*a*b+3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*EllipticE(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-4*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e
)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(
f*x+e),(-1/a*b)^(1/2))*a^3+7*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+8*(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^
2+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*b^3-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-6*(cos(f*x+e)^2)^(1/2)*(-b/
a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-4*
(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e)
,(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/b^2/cos(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.22, size = 1432, normalized size = 5.32

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")

```
[Out] 1/3*((2*((-I*a*b^3 - 2*I*b^4)*cos(f*x + e)^4 - I*a^3*b - 4*I*a^2*b^2 - 5*I*
a*b^3 - 2*I*b^4 - 2*(-I*a^2*b^2 - 3*I*a*b^3 - 2*I*b^4)*cos(f*x + e)^2)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a^2*b^2 + 5*I*a*b^3 + 2*I*b^4)*cos(f*x +
 e)^4 + 2*I*a^4 + 9*I*a^3*b + 14*I*a^2*b^2 + 9*I*a*b^3 + 2*I*b^4 + 2*(-2*I*
a^3*b - 7*I*a^2*b^2 - 7*I*a*b^3 - 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((
2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a
*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^3 + 2*I
*b^4)*cos(f*x + e)^4 + I*a^3*b + 4*I*a^2*b^2 + 5*I*a*b^3 + 2*I*b^4 - 2*(I*a
^2*b^2 + 3*I*a*b^3 + 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2
) - ((-2*I*a^2*b^2 - 5*I*a*b^3 - 2*I*b^4)*cos(f*x + e)^4 - 2*I*a^4 - 9*I*a^
3*b - 14*I*a^2*b^2 - 9*I*a*b^3 - 2*I*b^4 + 2*(2*I*a^3*b + 7*I*a^2*b^2 + 7*I
*a*b^3 + 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2)
 + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2
)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^3 + I*b^4)*cos(f*x + e)^4 + I*a^
3*b + 3*I*a^2*b^2 + 3*I*a*b^3 + I*b^4 - 2*(I*a^2*b^2 + 2*I*a*b^3 + I*b^4)*c
os(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a^2*b^2 - 7*I*a*b^3
 - 3*I*b^4)*cos(f*x + e)^4 - 2*I*a^4 - 11*I*a^3*b - 19*I*a^2*b^2 - 13*I*a*b^
3 - 3*I*b^4 + 2*(2*I*a^3*b + 9*I*a^2*b^2 + 10*I*a*b^3 + 3*I*b^4)*cos(f*x +
e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(ar
csin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*
x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2
) + (2*((-I*a*b^3 - I*b^4)*cos(f*x + e)^4 - I*a^3*b - 3*I*a^2*b^2 - 3*I*a*b
^3 - I*b^4 - 2*(-I*a^2*b^2 - 2*I*a*b^3 - I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sq
rt((a^2 + a*b)/b^2) - ((2*I*a^2*b^2 + 7*I*a*b^3 + 3*I*b^4)*cos(f*x + e)^4 +
 2*I*a^4 + 11*I*a^3*b + 19*I*a^2*b^2 + 13*I*a*b^3 + 3*I*b^4 + 2*(-2*I*a^3*b
 - 9*I*a^2*b^2 - 10*I*a*b^3 - 3*I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 +
 a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b +
 b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(a*b^3 + 2*b^4)*cos
(f*x + e)^3 - (a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sqrt(-b*cos(f*x + e
)^2 + a + b)*sin(f*x + e))/((a^2*b^5 + 2*a*b^6 + b^7)*f*cos(f*x + e)^4 - 2*
(a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^4*b^3 + 4*a^3*b
^4 + 6*a^2*b^5 + 4*a*b^6 + b^7)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8857 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.168 $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(}}{3ab(a+b)^2f\sqrt{1+\frac{b\sin^2(e+}{a}}}$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)-1/3*(a-b)*cos(f
*x+e)*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(a-b)*(cos(f*x+e)
^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^
(1/2)/a/b/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/(a+
b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3252, 3251, 3257, 3256, 3262, 3261}

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3abf(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*(Cos[e + f*x]*Sin[e + f*x])/((a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) -
 ((a - b)*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^2*f*Sqrt[a + b*Sin[e + f*
x]^2]) - ((a - b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3
*a*b*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (EllipticF[e + f*x, -(b/
a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^
2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
```

```
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\int \frac{a+a\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \cdots$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \cdots$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \cdots$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \cdots$$

Mathematica [A]
time = 1.00, size = 174, normalized size = 0.79

$$\frac{-2a^2(a-b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+2a^2(a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b\left(4a^2+ab-b^2+b(-a+b)\cos(2(e+fx))\right)\sin(2(e+fx))}{6ab(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x,
 -(b/a)] + 2*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF
[e + f*x, -(b/a)] - Sqrt[2]*b*(4*a^2 + a*b - b^2 + b*(-a + b)*Cos[2*(e + f*
x)])*Sin[2*(e + f*x)])/(6*a*b*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3
/2))
```

Maple [A]
time = 7.92, size = 483, normalized size = 2.19

method	result

default	$\frac{(a\,b^2-b^3)\sin(fx+e)(\cos^4(fx+e))+(-2a^2b-a\,b^2+b^3)(\cos^2(fx+e))\sin(fx+e)-\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos(2fx+2e)}{2}+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a*b^2-b^3)*sin(f*x+e)*cos(f*x+e)^4+(-2*a^2*b-a*b^2+b^3)*cos(f*x+e)^2*
sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*
b)*cos(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e
)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(
sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(
3/2)/(a+b)^2/a/b/cos(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.22, size = 1400, normalized size = 6.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*((-I*a*b^3 + I*b^4)*cos(f*x + e)^4 - I*a^3*b - I*a^2*b^2 + I*a*b^3
+ I*b^4 - 2*(-I*a^2*b^2 + I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/
b^2) - ((2*I*a^2*b^2 - I*a*b^3 - I*b^4)*cos(f*x + e)^4 + 2*I*a^4 + 3*I*a^3*
b - I*a^2*b^2 - 3*I*a*b^3 - I*b^4 + 2*(-2*I*a^3*b - I*a^2*b^2 + 2*I*a*b^3 +
 I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f
```

```
*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^
2 + a*b)/b^2))/b^2) + (2*((I*a*b^3 - I*b^4)*cos(f*x + e)^4 + I*a^3*b + I*a^
2*b^2 - I*a*b^3 - I*b^4 - 2*(I*a^2*b^2 - I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sq
rt((a^2 + a*b)/b^2) - ((-2*I*a^2*b^2 + I*a*b^3 + I*b^4)*cos(f*x + e)^4 - 2*
I*a^4 - 3*I*a^3*b + I*a^2*b^2 + 3*I*a*b^3 + I*b^4 + 2*(2*I*a^3*b + I*a^2*b^
2 - 2*I*a*b^3 - I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b
+ b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 4*(((I*a*b^3 + I*b^4)*cos(f*x + e)^4 +
 I*a^3*b + 3*I*a^2*b^2 + 3*I*a*b^3 + I*b^4 + 2*(-I*a^2*b^2 - 2*I*a*b^3 - I*
b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((-2*I*a^2*b^2 - I*a*
b^3)*cos(f*x + e)^4 - 2*I*a^4 - 5*I*a^3*b - 4*I*a^2*b^2 - I*a*b^3 + 2*(2*I*
a^3*b + 3*I*a^2*b^2 + I*a*b^3)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^
2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4
*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 4*(((-I*a*b^3 - I*b^4)*cos(f*x
 + e)^4 - I*a^3*b - 3*I*a^2*b^2 - 3*I*a*b^3 - I*b^4 + 2*(I*a^2*b^2 + 2*I*a*
b^3 + I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((2*I*a^2*b^2
 + I*a*b^3)*cos(f*x + e)^4 + 2*I*a^4 + 5*I*a^3*b + 4*I*a^2*b^2 + I*a*b^3 +
2*(-2*I*a^3*b - 3*I*a^2*b^2 - I*a*b^3)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 +
 a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b +
 b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*((a*b^3 - b^4)*cos(f
*x + e)^3 - (2*a^2*b^2 + a*b^3 - b^4)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2
+ a + b)*sin(f*x + e))/((a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*cos(f*x + e)^4 - 2*
(a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^5*b^2 + 4*a
^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.169 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3a^2(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] 1/3*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(2*a+b
)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(2*a+b)*
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/a^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(cos(f*x+e)^2
)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^
(1/2)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3263, 3252, 3251, 3257, 3256, 3262, 3261}

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
```

```
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a-2b+b\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} -$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} -$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} +$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} +$$

Mathematica [A]
time = 0.93, size = 172, normalized size = 0.77

$$\frac{2a^2(2a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-a^2(a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b(-5a^2-5ab-b^2+b(2a+b)\cos(2(e+fx)))\sin(2(e+fx))}{3a^2(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[
e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])
^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(245) = 490.
time = 9.72, size = 547, normalized size = 2.45

method	result

default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\;\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b(\sin^2(fx+e))+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{a+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*sin(f*x+
e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*b^3*sin(f*x+
e)^3-5*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a
+b)^2/cos(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.22, size = 1531, normalized size = 6.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*(2*I*a^3*b^2 + 5*I*a^2*b^3 + 4*I*a*b^4 + I*b^5 + (2*I*a*b^4 + I*b^5
)*cos(f*x + e)^4 - 2*(2*I*a^2*b^3 + 3*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) - (-4*I*a^4*b - 12*I*a^3*b^2 - 13*I*a^2*b^3 - 6*
I*a*b^4 - I*b^5 + (-4*I*a^2*b^3 - 4*I*a*b^4 - I*b^5)*cos(f*x + e)^4 + 2*(4*
```

```
I*a^3*b^2 + 8*I*a^2*b^3 + 5*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sqrt(-b))*sqrt
((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8
*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-2*I*a^3*b^2
 - 5*I*a^2*b^3 - 4*I*a*b^4 - I*b^5 + (-2*I*a*b^4 - I*b^5)*cos(f*x + e)^4 -
2*(-2*I*a^2*b^3 - 3*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a
*b)/b^2) - (4*I*a^4*b + 12*I*a^3*b^2 + 13*I*a^2*b^3 + 6*I*a*b^4 + I*b^5 + (
4*I*a^2*b^3 + 4*I*a*b^4 + I*b^5)*cos(f*x + e)^4 + 2*(-4*I*a^3*b^2 - 8*I*a^2
*b^3 - 5*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a
*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a
*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^4*b - 11*I*a^3*b^2 - 15*
I*a^2*b^3 - 9*I*a*b^4 - 2*I*b^5 + (-3*I*a^2*b^3 - 5*I*a*b^4 - 2*I*b^5)*cos(
f*x + e)^4 - 2*(-3*I*a^3*b^2 - 8*I*a^2*b^3 - 7*I*a*b^4 - 2*I*b^5)*cos(f*x +
 e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-6*I*a^5 - 17*I*a^4*b - 17*I*a^3*b
^2 - 7*I*a^2*b^3 - I*a*b^4 + (-6*I*a^3*b^2 - 5*I*a^2*b^3 - I*a*b^4)*cos(f*x
 + e)^4 + 2*(6*I*a^4*b + 11*I*a^3*b^2 + 6*I*a^2*b^3 + I*a*b^4)*cos(f*x + e)
^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcs
in(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x
+ e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)
+ (2*(3*I*a^4*b + 11*I*a^3*b^2 + 15*I*a^2*b^3 + 9*I*a*b^4 + 2*I*b^5 + (3*I*
a^2*b^3 + 5*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^4 - 2*(3*I*a^3*b^2 + 8*I*a^2*b^
3 + 7*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (
6*I*a^5 + 17*I*a^4*b + 17*I*a^3*b^2 + 7*I*a^2*b^3 + I*a*b^4 + (6*I*a^3*b^2
+ 5*I*a^2*b^3 + I*a*b^4)*cos(f*x + e)^4 + 2*(-6*I*a^4*b - 11*I*a^3*b^2 - 6*
I*a^2*b^3 - I*a*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
 b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(2*a*b^4 + b^5)*cos(f*x + e)^3 - (5*a
^2*b^3 + 7*a*b^4 + 2*b^5)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sin
(f*x + e))/((a^4*b^4 + 2*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^4 - 2*(a^5*b^3 +
 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^2 + (a^6*b^2 + 4*a^5*b^3 +
 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*f)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(-5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.170 $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=322

$$\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{(3a^2+13ab+8b^2)\cot(e+fx)\sqrt{a}}{3a^3(a+b)^2f}$$

```
[Out] 1/3*b*cot(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(3*a+2*b)*cot(f*x
+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(3*a^2+13*a*b+8*b^2)*cot(f*x
+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/(a+b)^2/f-1/3*(3*a^2+13*a*b+8*b^2)*Ellipti
cE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)
^2)^(1/2)/a^3/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(3*a+4*b)*EllipticF(
sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/
a)^(1/2)/a^2/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3267, 483, 593, 597, 538, 437, 435, 432, 430}

$$\frac{(3a+4b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{2b(3a+2b)\cot(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{(3a^2+13ab+8b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(3a^2+13ab+8b^2)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f(a+b)^2}+\frac{b\cot(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (b*Cot[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*b*(3*a +
 2*b)*Cot[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((3*a^
2 + 13*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)
^2*f) - ((3*a^2 + 13*a*b + 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin
[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)
^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((3*a + 4*b)*Sqrt[Cos[e + f*x]^2]*El
lipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]
^2)/a])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^2}\,(a+bx^2)^{5/2}}\,dx, x, \sin(e+f\right.}{f}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\right.}{3a(a}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{(}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(3}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(3}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(3}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(3}{}$$

Mathematica [A]
time = 1.57, size = 214, normalized size = 0.66

$$\frac{4a^2\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}\left(-\left((3a^2+13ab+8b^2)\,E\left(e+fx\middle|-\frac{b}{a}\right)\right)+(3a^2+7ab+4b^2)\,F\left(e+fx\middle|-\frac{b}{a}\right)\right)-2\sqrt{2}\left(3(a+b)^2(2a+b-b\cos(2(e+fx)))^2\cot(e+fx)+2ab^2(a+b)\sin(2(e+fx))+b^2(7a+5b)(2a+b-b\cos(2(e+fx)))\sin(2(e+fx))\right)}{12a^3(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (4*a^2*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(-((3*a^2 + 13*a*b + 8*b^2)
*EllipticE[e + f*x, -(b/a)]) + (3*a^2 + 7*a*b + 4*b^2)*EllipticF[e + f*x, -
(b/a)]) - 2*Sqrt[2]*(3*(a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^2*Cot[e + f
```

```
*x] + 2*a*b^2*(a + b)*Sin[2*(e + f*x)] + b^2*(7*a + 5*b)*(2*a + b - b*Cos[2
*(e + f*x)])*Sin[2*(e + f*x)]))/(12*a^3*(a + b)^2*f*(2*a + b - b*Cos[2*(e +
 f*x)])^(3/2))
```

Maple [A]
time = 10.81, size = 527, normalized size = 1.64

method	result
default	$\frac{\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,ab\left(3\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+13\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(3*Elliptic
E(sin(f*x+e),(-1/a*b)^(1/2))*a^2+13*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*
b+8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*EllipticF(sin(f*x+e),(-1/a*b
)^(1/2))*a^2-7*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b-4*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*b^2)*sin(f*x+e)*cos(f*x+e)^2-(-b/a*cos(f*x+e)^2+(a+b)/a
)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(3*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+
16*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+21*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a*b^2+8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^3-3*EllipticF(sin
(f*x+e),(-1/a*b)^(1/2))*a^3-10*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-1
1*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-4*EllipticF(sin(f*x+e),(-1/a*b
)^(1/2))*b^3)*sin(f*x+e)+(-3*a^2*b^2-13*a*b^3-8*b^4)*cos(f*x+e)^6+(6*a^3*b+
26*a^2*b^2+38*a*b^3+16*b^4)*cos(f*x+e)^4+(-3*a^4-12*a^3*b-26*a^2*b^2-25*a*b
^3-8*b^4)*cos(f*x+e)^2)/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/sin(f*x+e)/a^3/cos
(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.25, size = 1719, normalized size = 5.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] 1/6*((2*(-3*I*a^4*b - 19*I*a^3*b^2 - 37*I*a^2*b^3 - 29*I*a*b^4 - 8*I*b^5 +
(-3*I*a^2*b^3 - 13*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^4 - 2*(-3*I*a^3*b^2 - 16
*I*a^2*b^3 - 21*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b
)/b^2)*sin(f*x + e) - (6*I*a^5 + 41*I*a^4*b + 93*I*a^3*b^2 + 95*I*a^2*b^3 +
 45*I*a*b^4 + 8*I*b^5 + (6*I*a^3*b^2 + 29*I*a^2*b^3 + 29*I*a*b^4 + 8*I*b^5)
*cos(f*x + e)^4 + 2*(-6*I*a^4*b - 35*I*a^3*b^2 - 58*I*a^2*b^3 - 37*I*a*b^4
- 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b
)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*
a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b
 + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(3*I*a^4*b + 19*I*a^3*b^2 + 37*I*a
^2*b^3 + 29*I*a*b^4 + 8*I*b^5 + (3*I*a^2*b^3 + 13*I*a*b^4 + 8*I*b^5)*cos(f*
x + e)^4 - 2*(3*I*a^3*b^2 + 16*I*a^2*b^3 + 21*I*a*b^4 + 8*I*b^5)*cos(f*x +
e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (-6*I*a^5 - 41*I*a^4*b
- 93*I*a^3*b^2 - 95*I*a^2*b^3 - 45*I*a*b^4 - 8*I*b^5 + (-6*I*a^3*b^2 - 29*I
*a^2*b^3 - 29*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^4 + 2*(6*I*a^4*b + 35*I*a^3*b
^2 + 58*I*a^2*b^3 + 37*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x
+ e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))),
(8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 4*((-9
*I*a^4*b - 35*I*a^3*b^2 - 51*I*a^2*b^3 - 33*I*a*b^4 - 8*I*b^5 + (-9*I*a^2*b
^3 - 17*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^4 + 2*(9*I*a^3*b^2 + 26*I*a^2*b^3 +
 25*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f
*x + e) + (6*I*a^5 + 19*I*a^4*b + 22*I*a^3*b^2 + 11*I*a^2*b^3 + 2*I*a*b^4 +
 (6*I*a^3*b^2 + 7*I*a^2*b^3 + 2*I*a*b^4)*cos(f*x + e)^4 + 2*(-6*I*a^4*b - 1
3*I*a^3*b^2 - 9*I*a^2*b^3 - 2*I*a*b^4)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e
))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*
a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 4*((9*I*a
^4*b + 35*I*a^3*b^2 + 51*I*a^2*b^3 + 33*I*a*b^4 + 8*I*b^5 + (9*I*a^2*b^3 +
17*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^4 + 2*(-9*I*a^3*b^2 - 26*I*a^2*b^3 - 25*
I*a*b^4 - 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x +
 e) + (-6*I*a^5 - 19*I*a^4*b - 22*I*a^3*b^2 - 11*I*a^2*b^3 - 2*I*a*b^4 + (-
6*I*a^3*b^2 - 7*I*a^2*b^3 - 2*I*a*b^4)*cos(f*x + e)^4 + 2*(6*I*a^4*b + 13*I
*a^3*b^2 + 9*I*a^2*b^3 + 2*I*a*b^4)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2
 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*((3*a^2*b^
3 + 13*a*b^4 + 8*b^5)*cos(f*x + e)^5 - 2*(3*a^3*b^2 + 13*a^2*b^3 + 19*a*b^4
 + 8*b^5)*cos(f*x + e)^3 + (3*a^4*b + 12*a^3*b^2 + 26*a^2*b^3 + 25*a*b^4 +
8*b^5)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(((a^5*b^3 + 2*a^4*b^
4 + a^3*b^5)*f*cos(f*x + e)^4 - 2*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^
5)*f*cos(f*x + e)^2 + (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)
*f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral(csc(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e+f\,x)^2\,\left(b\sin(e+f\,x)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)),x)
[Out] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

3.171 $\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=122

$$-\frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}(d\sin\ldots}{f}$$

```
[Out] -d*AppellF1(1/2,1/2-1/2*m,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x
+e)*(a+b-b*cos(f*x+e)^2)^p*(d*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)
/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3268, 441, 440}

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(
a + b)]*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*(d*Sin[e + f*x])^(-1 + m)
*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3268

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m -
```

```
1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\left(d(d\sin(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\sin^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int (1-\right.}{f}$$

$$= -\frac{\left(d(a+b-b\cos^2(e+fx))^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}(d\sin(e+\right.}{}$$

$$= -\frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\,(}{}$$

Mathematica [A]
time = 0.33, size = 113, normalized size = 0.93

$$\frac{F_1\left(\frac{1+m}{2};\frac{1}{2},-p;\frac{3+m}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,(d\sin(e+fx))^m\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+fx)}{f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, 1/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^
2)/a)]*Sqrt[Cos[e + f*x]^2]*(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p*Tan
[e + f*x])/(f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^m \left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F]
time = 0.44, size = 29, normalized size = 0.24

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\left(d\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(e+fx\right)\right)^m\left(a+b\sin^2\left(e+fx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Integral((d*sin(e + f*x))**m*(a + b*sin(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(d\,\sin\left(e+f\,x\right)\right)^m\,\left(b\,\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)^2)^p, x)
```

3.172 $\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=220

$$\frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}-\frac{\left(3a^2-4ab(1+p)+4b^2(2+3p+p^2)\right)\cos(e+fx)}{}$$

[Out] (3*a-2*b*(2+p))*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+15)
-(3*a^2-4*a*b*(1+p)+4*b^2*(p^2+3*p+2))*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^p*hy
pergeom([1/2, -p],[3/2],b*cos(f*x+e)^2/(a+b))/b^2/f/(4*p^2+16*p+15)/((1-b*c
os(f*x+e)^2/(a+b))^p)-cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1+p)*sin(f*x+e)^2/b/
f/(5+2*p)

Rubi [A]
time = 0.15, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3265, 427, 396, 252, 251}

$$-\frac{\left(3a^2-4ab(p+1)+4b^2(p^2+3p+2)\right)\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{b^2f(2p+3)(2p+5)}+\frac{(3a-2b(p+2))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b^2f(2p+3)(2p+5)}-\frac{\sin^2(e+fx)\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{bf(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] ((3*a - 2*b*(2 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p))/(b^2*
f*(3 + 2*p)*(5 + 2*p)) - ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*C
os[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (b
*Cos[e + f*x]^2)/(a + b)])/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 - (b*Cos[e + f*x]^
2)/(a + b))^p) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p)*Sin[e + f
*x]^2)/(b*f*(5 + 2*p))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+b-bx^2\right)^p\,dx,x,\cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}\sin^2(e+fx)}{bf(5+2p)} + \frac{\text{Subst}(\int$$

$$= \frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos(e}{}$$

$$= \frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos(e}{}$$

$$= \frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{(3a^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.39, size = 98, normalized size = 0.45

$$\frac{F_1\left(3;\frac{1}{2},-p;4;\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3, 1/2, -p, 4, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[Cos[
e + f*x]^2]*Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(6*f*((a
+ b*Sin[e + f*x]^2)/a)^p)
```

Maple [**F**]
time = 1.12, size = 0, normalized size = 0.00

$$\int \left(\sin^{5}\left(f x+e\right)\right) \left(a+b \left(\sin^{2}\left(f x+e\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Fricas [**F**]
time = 0.44, size = 45, normalized size = 0.20

$$\mathrm{integral}\left(\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(-b*cos(f*x + e)^2 + a + b
)^p*sin(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^5\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^5*(a + b*sin(e + f*x)^2)^p, x)
```

3.173 $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=131

$$-\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)}+\frac{(a-2b(1+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(}{a-}\right.}{bf(3+2p)}$$

[Out] -cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1+p)/b/f/(3+2*p)+(a-2*b*(1+p))*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],b*cos(f*x+e)^2/(a+b))/b/f/(3+2*p)/((1-b*cos(f*x+e)^2/(a+b))^p)

Rubi [A]
time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3265, 396, 252, 251}

$$\frac{(a-2b(p+1))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{bf(2p+3)}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p))) + ((a
- 2*b*(1 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[
1/2, -p, 3/2, (b*Cos[e + f*x]^2)/(a + b)])/(b*f*(3 + 2*p)*(1 - (b*Cos[e + f
*x]^2)/(a + b))^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int (1-x^2)\left(a+b-bx^2\right)^p\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{(a-2b(1+p))\text{Subst}}{}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\Big((a-2b(1+p))\,(a}{}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{(a-2b(1+p))\cos(e}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.27, size = 98, normalized size = 0.75

$$\frac{F_1\left(2;\frac{1}{2},-p;3;\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[2, 1/2, -p, 3, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[Cos[
e + f*x]^2]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(4*f*((a
+ b*Sin[e + f*x]^2)/a)^p)
```

Maple [F]
time = 0.65, size = 0, normalized size = 0.00

$$\int \left(\sin^3(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.43, size = 36, normalized size = 0.27

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(-b*cos(f*x + e)^2 + a + b)^p*sin(f*x + e),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^p, x)
```

3.174 $\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=74

$$-\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],b*cos(f*x+e)^2
/(a+b))/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3265, 252, 251}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, (b*Cos[e + f*x]^2)/(a + b)])/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int (a+b-bx^2)^p\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\left((a+b-b\cos^2(e+fx))^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \left(1-\frac{b}{a}\right.\right.}{f}$$
$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},\right.}{f}$$

Mathematica [A]
time = 0.14, size = 74, normalized size = 1.00

$$-\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, (b*Cos[e + f*x]^2)/(a + b)])/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Fricas [F]
time = 0.41, size = 25, normalized size = 0.34

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sin(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^p, x)
```

3.175 $\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] -AppellF1(1/2,1,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x+e)*(a+b-b
*cos(f*x+e)^2)^p/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3265, 441, 440}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^p}{1-x^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left((a+b-b\cos^2(e+fx))^p \left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{b}{a}\right.}{1-}\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\,(a+b-b}{f}$$

Mathematica [F]
time = 15.14, size = 0, normalized size = 0.00

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \csc\left(fx+e\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Fricas [F]
time = 0.41, size = 25, normalized size = 0.30

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^2\left(e+fx\right)\right)^p\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**p*csc(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x), x)
```

3.176 $\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] -AppellF1(1/2,2,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x+e)*(a+b-b
*cos(f*x+e)^2)^p/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A]

time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 441, 440}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^p}{(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left((a+b-b\cos^2(e+fx))^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{bx}{a-}\right.}{(1-x}\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2}; 2, -p; \frac{3}{2}; \cos^2(e+fx), \frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\,(a+b-b}{f}$$

Mathematica [**F**]
time = 100.58, size = 0, normalized size = 0.00

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(\csc^3(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Fricas [F]
time = 0.43, size = 27, normalized size = 0.33

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

3.177 $\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}}{f}$$

```
[Out] -AppellF1(1/2,3,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x+e)*(a+b-b
*cos(f*x+e)^2)^p/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3265, 441, 440}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 3, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^p}{(1-x^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left((a+b-b\cos^2(e+fx))^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{(1-}{(1-}\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\,(a+b-}{f}$$

Mathematica [F]
time = 121.73, size = 0, normalized size = 0.00

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(\csc^5(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^5, x)
```

Fricas [F]
time = 0.45, size = 27, normalized size = 0.33

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\csc\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^5, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^5,x)
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^5, x)
```

3.178 $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right.}{5f}$$

```
[Out] 1/5*AppellF1(5/2,1/2,-p,7/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*sin(f*x+e)^4*(a
+b*sin(f*x+e)^2)^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^
p)
```

Rubi [A]

time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3267, 525, 524}

$$\frac{\sin^4(e+fx)\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(5*f*
(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4(a+bx^2)^p}{\sqrt{1-x^2}}\,dx, x, \sin(e+\right.}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+f}{a}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\sin^4(e}{5f}$$

Mathematica [A]
time = 0.39, size = 102, normalized size = 1.01

$$\frac{F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(5*f*
((a + b*Sin[e + f*x]^2)/a)^p)
```

Maple [F]
time = 1.38, size = 0, normalized size = 0.00

$$\int \left(\sin^4(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.43, size = 39, normalized size = 0.39

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\left(-b\cos\left(fx+e\right)^2+a+b\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(-b*cos(f*x + e)^2 + a + b
)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^4\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^p, x)
```

3.179 $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=99

$$\frac{F_1\left(\frac{3}{2};2+p,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\tan^3(e+fx)\left(1+\frac{(a$$
$$3f$$

```
[Out] 1/3*AppellF1(3/2,2+p,-p,5/2,-tan(f*x+e)^2,-(a+b)*tan(f*x+e)^2/a)*(sec(f*x+e
)^2)^p*(a+b*sin(f*x+e)^2)^p*tan(f*x+e)^3/f/((1+(a+b)*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3253, 525, 524}

$$\frac{\tan^3(e+fx)\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(\frac{(a+b)\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{(a+b)\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2 + p, -p, 5/2, -Tan[e + f*x]^2, -(((a + b)*Tan[e + f*x]^2)/
a)]*(Sec[e + f*x]^2)^p*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x]^3)/(3*f*(1 + (
(a + b)*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3253

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff*(a + b*Sin[e + f*x]^2)^p*((Sec[e + f*x]^2)^p/(f*(a + (a + b)*Tan[e +
```

```
f*x]^2)^p)), Subst[Int[(a + (a + b)*ff^2*x^2)^p*((A + (A + B)*ff^2*x^2)/(1
 + ff^2*x^2)^(p + 2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, A
, B}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(a+(a+b)\tan^2(e+fx)\right)^{-}\right.}{}$$

$$= \frac{\left(\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{(a+b)\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Su}}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2+p,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sec^2(e+f}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(99) = 198.
time = 0.50, size = 240, normalized size = 2.42

$$-\frac{2^{-2-p}\sqrt{\frac{b\cos^2(e+fx)}{a+b}}\,(2a+b-b\cos(2(e+fx)))^{1+p}\left(2a(2+p)F_1\left(1+p;\frac{1}{2},\frac{1}{2};2+p;\frac{2a+b-b\cos(2(e+fx))}{2(a+b)},\frac{2a+b-b\cos(2(e+fx))}{2a}\right)-(1+p)F_1\left(2+p;\frac{1}{2},\frac{1}{2};3+p;\frac{2a+b-b\cos(2(e+fx))}{2(a+b)},\frac{2a+b-b\cos(2(e+fx))}{2a}\right)(2a+b-b\cos(2(e+fx)))\right)\csc(2(e+fx))\sqrt{-\frac{b\sin^2(e+fx)}{a}}}{b^2f(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((2^(-2 - p)*Sqrt[(b*Cos[e + f*x]^2)/(a + b)]*(2*a + b - b*Cos[2*(e + f*x)
])^(1 + p)*(2*a*(2 + p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a + b - b*Cos[2
*(e + f*x)])/(2*(a + b)), (2*a + b - b*Cos[2*(e + f*x)])/(2*a)] - (1 + p)*A
ppellF1[2 + p, 1/2, 1/2, 3 + p, (2*a + b - b*Cos[2*(e + f*x)])/(2*(a + b)),
 (2*a + b - b*Cos[2*(e + f*x)])/(2*a)]*(2*a + b - b*Cos[2*(e + f*x)]))*Csc[
2*(e + f*x)]*Sqrt[-((b*Sin[e + f*x]^2)/a)])/(b^2*f*(1 + p)*(2 + p)))
```

Maple [F]
time = 1.03, size = 0, normalized size = 0.00

$$\int \left(\sin^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Fricas [F]
time = 0.40, size = 30, normalized size = 0.30

$$\mathrm{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(-b*cos(f*x + e)^2 + a + b)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^p, x)
```

3.180 $\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=97

$$-\frac{F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\csc(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p}{f}$$

```
[Out] -AppellF1(-1/2,1/2,-p,1/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*csc(f*x+e)*sec(f*
x+e)*(a+b*sin(f*x+e)^2)^p*(cos(f*x+e)^2)^(1/2)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3267, 525, 524}

$$-\frac{\sqrt{\cos^2(e+fx)}\,\csc(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sq
rt[Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(
1 + (b*Sin[e + f*x]^2)/a)^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
```

```
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\frac{(a+bx^2)^p}{x^2\sqrt{1-x^2}}\,dx,x,\sin(e\right.}{f}$$

$$=\frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+f}{a}\right.\right.}{f}$$

$$=-\frac{F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\csc}{f}$$

Mathematica [F]
time = 6.38, size = 0, normalized size = 0.00

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int\left(\csc^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.42, size = 27, normalized size = 0.28

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\sin\left(e+f\,x\right)}^2+a\right)}^p}{{\sin\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

3.181 $\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$-\frac{F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\csc^3(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right.}{3f}$$

```
[Out] -1/3*AppellF1(-3/2,1/2,-p,-1/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*csc(f*x+e)^3
*sec(f*x+e)*(a+b*sin(f*x+e)^2)^p*(cos(f*x+e)^2)^(1/2)/f/((1+b*sin(f*x+e)^2/
a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3267, 525, 524}

$$-\frac{\sqrt{\cos^2(e+fx)}\,\csc^3(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(AppellF1[-3/2, 1/2, -p, -1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)
]*Sqrt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p
)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3267

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\frac{(a+bx^2)^p}{x^4\sqrt{1-x^2}}\,dx,x,\sin(e$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+}{a}\right.\right.}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}}{}$$

Mathematica [F]
time = 11.37, size = 0, normalized size = 0.00

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(\csc^4\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.43, size = 27, normalized size = 0.27

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\csc\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^4, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{p}}{\sin\left(e+f\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^4,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

3.182 $\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=335

$$\frac{3x}{8b}+\frac{2(-1)^{2/3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{7/3}d}-\frac{2a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{7/3}d}+\frac{2\sqrt[3]{-1}\,a^{5/3}\,\mathrm{t}}{3}$$

[Out] 3/8*x/b+a*cos(d*x+c)/b^2/d-3/8*cos(d*x+c)*sin(d*x+c)/b/d-1/4*cos(d*x+c)*sin(d*x+c)^3/b/d-2/3*a^(5/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(7/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*(-1)^(1/3)*a^(5/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b^(7/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*(-1)^(2/3)*a^(5/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^(7/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)

Rubi [A]
time = 0.50, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3299, 2718, 2715, 8, 2739, 632, 210}

$$\frac{2(-1)^{2/3}a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\,a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4bd}-\frac{3\sin(c+dx)\cos(c+dx)}{8bd}+\frac{3x}{8b}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^3),x]

[Out] (3*x)/(8*b) + (2*(-1)^(2/3)*a^(5/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(7/3)*d) - (2*a^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(7/3)*d) + (2*(-1)^(1/3)*a^(5/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(7/3)*d) + (a*Cos[c + d*x])/(b^2*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{a\sin(c+dx)}{b^2} + \frac{\sin^4(c+dx)}{b} + \frac{a^2\sin(c+dx)}{b^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= -\frac{a\int \sin(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{b^2} + \frac{\int \sin^4(c+dx)\,dx}{b}$$

$$= \frac{a\cos(c+dx)}{b^2 d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{a^2\int\left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}\right.}{}$$

$$= \frac{a\cos(c+dx)}{b^2 d} - \frac{3\cos(c+dx)\sin(c+dx)}{8bd} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} - \frac{a^{5/3}\int -}{}$$

$$= \frac{3x}{8b} + \frac{a\cos(c+dx)}{b^2 d} - \frac{3\cos(c+dx)\sin(c+dx)}{8bd} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} - \frac{(2}{}$$

$$= \frac{3x}{8b} + \frac{a\cos(c+dx)}{b^2 d} - \frac{3\cos(c+dx)\sin(c+dx)}{8bd} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{(4}{}$$

$$= \frac{3x}{8b} + \frac{2(-1)^{2/3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{7/3}d} - \frac{2a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{}}{\sqrt{a^2}}\right.}{3\sqrt{a^{2/3}-b^{2/}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.37, size = 219, normalized size = 0.65

$$\frac{96a\cos(c+dx) - 32a^2\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right] + 3b(12(c+dx)-8\sin(2(c+dx))+\sin(4(c+dx)))}{96b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (96*a*Cos[c + d*x] - 32*a^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I
)*b*#1^4 + I*b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^2 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1
^2 + b*#1^4) & ] + 3*b*(12*(c + d*x) - 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)
]))/(96*b^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.68, size = 221, normalized size = 0.66

method	result
derivativedivides	$-\frac{4\left(\frac{-\frac{3b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{11b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{11b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{3a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}\right.}{b^2}$
default	$-\frac{4\left(\frac{-\frac{3b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{11b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{11b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{3a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}\right.}{b^2}$
risch	$\frac{3x}{8b}+\frac{a\,\mathrm{e}^{i(dx+c)}}{2b^2d}+\frac{a\,\mathrm{e}^{-i(dx+c)}}{2b^2d}+\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^{14}d^6-729b^{16}d^6\right)\_Z^6-3981312a^4b^{10}d^4\_Z^4-4398046511104a^{10}\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4/b^2*((-3/16*b*tan(1/2*d*x+1/2*c)^7-1/2*a*tan(1/2*d*x+1/2*c)^6-11/16
*b*tan(1/2*d*x+1/2*c)^5-3/2*a*tan(1/2*d*x+1/2*c)^4+11/16*b*tan(1/2*d*x+1/2*
c)^3-3/2*a*tan(1/2*d*x+1/2*c)^2+3/16*b*tan(1/2*d*x+1/2*c)-1/2*a)/(1+tan(1/2
*d*x+1/2*c)^2)^4-3/16*b*arctan(tan(1/2*d*x+1/2*c)))+2/3*a^2/b^2*sum((_R^3+_
R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6
*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/32*(32*b^2*d*integrate(-4*(3*a^2*b*cos(4*d*x + 4*c)^2 + 3*a^2*b*cos(2*d*
x + 2*c)^2 + 3*a^2*b*sin(4*d*x + 4*c)^2 + 8*a^3*cos(2*d*x + 2*c)*sin(3*d*x
+ 3*c) - 8*a^3*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 3*a^2*b*sin(2*d*x + 2*c)
^2 - a^2*b*cos(2*d*x + 2*c) - (a^2*b*cos(4*d*x + 4*c) - a^2*b*cos(2*d*x + 2
*c))*cos(6*d*x + 6*c) - (6*a^2*b*cos(2*d*x + 2*c) + 8*a^3*sin(3*d*x + 3*c)
- a^2*b)*cos(4*d*x + 4*c) - (a^2*b*sin(4*d*x + 4*c) - a^2*b*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 2*(4*a^3*cos(3*d*x + 3*c) - 3*a^2*b*sin(2*d*x + 2*c))
*sin(4*d*x + 4*c))/(b^4*cos(6*d*x + 6*c)^2 + 9*b^4*cos(4*d*x + 4*c)^2 + 64*
a^2*b^2*cos(3*d*x + 3*c)^2 + 9*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(6*d*x + 6*c
)^2 + 9*b^4*sin(4*d*x + 4*c)^2 + 64*a^2*b^2*sin(3*d*x + 3*c)^2 - 48*a*b^3*c
os(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*b^4*sin(2*d*x + 2*c)^2 - 6*b^4*cos(2*d
*x + 2*c) + b^4 - 2*(3*b^4*cos(4*d*x + 4*c) - 3*b^4*cos(2*d*x + 2*c) - 8*a*
b^3*sin(3*d*x + 3*c) + b^4)*cos(6*d*x + 6*c) - 6*(3*b^4*cos(2*d*x + 2*c) +
```

```
8*a*b^3*sin(3*d*x + 3*c) - b^4)*cos(4*d*x + 4*c) - 2*(8*a*b^3*cos(3*d*x + 3
*c) + 3*b^4*sin(4*d*x + 4*c) - 3*b^4*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6
*(8*a*b^3*cos(3*d*x + 3*c) - 3*b^4*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*
(3*a*b^3*cos(2*d*x + 2*c) - a*b^3)*sin(3*d*x + 3*c)), x) - 12*b*d*x - 32*a*
cos(d*x + c) - b*sin(4*d*x + 4*c) + 8*b*sin(2*d*x + 2*c))/(b^2*d)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.68, size = 21338, normalized size = 63.70

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(2/3)*sqrt(1/6)*b^2*d*sqrt(-(54*a^4 - (a^8*(-I*sqrt(3) + 1)/((-
1/1458*a^10/(a^2*b^14*d^6 - b^16*d^6) - 1/729*a^12/(a^2*b^4*d^2 - b^6*d^2)^
3 + 1/1458*(a^2 + b^2)*a^10/((a^2 - b^2)^2*b^14*d^6))^(1/3)*(a^2*b^4*d^2 -
b^6*d^2)^2) + 18*a^4/(a^2*b^4*d^2 - b^6*d^2) + 81*(-1/1458*a^10/(a^2*b^14*d
^6 - b^16*d^6) - 1/729*a^12/(a^2*b^4*d^2 - b^6*d^2)^3 + 1/1458*(a^2 + b^2)*
a^10/((a^2 - b^2)^2*b^14*d^6))^(1/3)*(I*sqrt(3) + 1))*(a^2*b^4 - b^6)*d^2 +
 3*sqrt(1/3)*(a^2*b^4 - b^6)*d^2*sqrt((972*a^8 - (a^4*b^8 - 2*a^2*b^10 + b^
12)*(a^8*(-I*sqrt(3) + 1)/((-1/1458*a^10/(a^2*b^14*d^6 - b^16*d^6) - 1/729*
a^12/(a^2*b^4*d^2 - b^6*d^2)^3 + 1/1458*(a^2 + b^2)*a^10/((a^2 - b^2)^2*b^1
4*d^6))^(1/3)*(a^2*b^4*d^2 - b^6*d^2)^2) + 18*a^4/(a^2*b^4*d^2 - b^6*d^2) +
 81*(-1/1458*a^10/(a^2*b^14*d^6 - b^16*d^6) - 1/729*a^12/(a^2*b^4*d^2 - b^6
*d^2)^3 + 1/1458*(a^2 + b^2)*a^10/((a^2 - b^2)^2*b^14*d^6))^(1/3)*(I*sqrt(3
) + 1))^2*d^4 + 36*(a^6*b^4 - a^4*b^6)*(a^8*(-I*sqrt(3) + 1)/((-1/1458*a^10
/(a^2*b^14*d^6 - b^16*d^6 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^7/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 15.24, size = 1978, normalized size = 5.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log((150994944*a^12*b^3*sin(c/2 + (d*x)/2) - 56623104*a^13*b^2*cos(c
/2 + (d*x)/2) - 12582912*a^15*cos(c/2 + (d*x)/2) + 679477248*root(729*a^2*b
^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)*a^11*b^5*sin(c/2 +
(d*x)/2) + 679477248*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^
4 + a^10, z, k)^2*a^9*b^8*cos(c/2 + (d*x)/2) - 42467328*root(729*a^2*b^14*z
^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^2*a^11*b^6*cos(c/2 + (d*
x)/2) - 402653184*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 +
 a^10, z, k)^2*a^13*b^4*cos(c/2 + (d*x)/2) + 4586471424*root(729*a^2*b^14*z
^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^3*a^8*b^10*cos(c/2 + (d*
x)/2) - 503316480*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 +
 a^10, z, k)^3*a^12*b^6*cos(c/2 + (d*x)/2) + 1911029760*root(729*a^2*b^14*z
^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^4*a^7*b^12*cos(c/2 + (d*
x)/2) + 1774190592*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4
+ a^10, z, k)^4*a^9*b^10*cos(c/2 + (d*x)/2) - 301989888*root(729*a^2*b^14*z
^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^4*a^11*b^8*cos(c/2 + (d*
x)/2) - 18345885696*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4
 + a^10, z, k)^5*a^4*b^16*cos(c/2 + (d*x)/2) + 17199267840*root(729*a^2*b^1
4*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^5*a^6*b^14*cos(c/2 +
(d*x)/2) + 32614907904*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*
z^4 + a^10, z, k)^5*a^8*b^12*cos(c/2 + (d*x)/2) + 9172942848*root(729*a^2*b
^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^6*a^5*b^16*cos(c/2
+ (d*x)/2) + 4416602112*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10
*z^4 + a^10, z, k)^6*a^7*b^14*cos(c/2 + (d*x)/2) - 130459631616*root(729*a^
2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^7*a^4*b^18*cos(c
/2 + (d*x)/2) + 122305904640*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4
*b^10*z^4 + a^10, z, k)^7*a^6*b^16*cos(c/2 + (d*x)/2) + 1613758464*root(729
*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^2*a^10*b^7*si
n(c/2 + (d*x)/2) + 1073741824*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^
4*b^10*z^4 + a^10, z, k)^2*a^12*b^5*sin(c/2 + (d*x)/2) - 4076863488*root(72
9*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^3*a^7*b^11*s
in(c/2 + (d*x)/2) + 2420637696*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a
^4*b^10*z^4 + a^10, z, k)^3*a^9*b^9*sin(c/2 + (d*x)/2) + 4831838208*root(72
9*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^3*a^11*b^7*s
in(c/2 + (d*x)/2) + 2293235712*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a
^4*b^10*z^4 + a^10, z, k)^4*a^8*b^11*sin(c/2 + (d*x)/2) + 11475615744*root(
```

```
729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^4*a^10*b^9
*sin(c/2 + (d*x)/2) - 2293235712*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243
*a^4*b^10*z^4 + a^10, z, k)^5*a^5*b^15*sin(c/2 + (d*x)/2) - 27844411392*roo
t(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^5*a^7*b^
13*sin(c/2 + (d*x)/2) + 25367150592*root(729*a^2*b^14*z^6 - 729*b^16*z^6 +
243*a^4*b^10*z^4 + a^10, z, k)^5*a^9*b^11*sin(c/2 + (d*x)/2) + 16307453952*
root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^6*a^8
*b^13*sin(c/2 + (d*x)/2) - 40768634880*root(729*a^2*b^14*z^6 - 729*b^16*z^6
 + 243*a^4*b^10*z^4 + a^10, z, k)^7*a^5*b^17*sin(c/2 + (d*x)/2) + 326149079
04*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^7*
a^7*b^15*sin(c/2 + (d*x)/2) + 33554432*root(729*a^2*b^14*z^6 - 729*b^16*z^6
 + 243*a^4*b^10*z^4 + a^10, z, k)*a^15*b*sin(c/2 + (d*x)/2) - 70778880*root
(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)*a^12*b^4*
cos(c/2 + (d*x)/2))/(b^9*cos(c/2 + (d*x)/2)))*root(729*a^2*b^14*z^6 - 729*b
^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k), k, 1, 6)/d + (log((cos(c/2 + (d*x
)/2)*1i + sin(c/2 + (d*x)/2))/cos(c/2 + (d*x)/2))*3i)/(8*b*d) - (log((cos(c
/2 + (d*x)/2)*1i - sin(c/2 + (d*x)/2))/cos(c/2 + (d*x)/2))*3i)/(8*b*d) - si
n(2*c + 2*d*x)/(4*b*d) + sin(4*c + 4*d*x)/(32*b*d) + (a*cos(c + d*x))/(b^2*
d)
```

3.183 $\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=273

$$\frac{x}{2b}-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{5/3}d}+\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,b^{5/3}d}+\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{\cdots}}{\sqrt{\sqrt[3]{-1}\,a\cdots}}\right.}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2\cdots}}}$$

```
[Out] 1/2*x/b-1/2*cos(d*x+c)*sin(d*x+c)/b/d-2/3*a*arctan((b^(1/3)+a^(1/3)*tan(1/2
*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(5/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3
*a*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2
/3)+b^(2/3))^(1/2))/b^(5/3)/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*a*arct
anh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^
(2/3))^(1/2))/b^(5/3)/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3299, 2715, 8, 2739, 632, 210, 212}

$$-\frac{2a\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{5/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}+\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] x/(2*b) - (2*a*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b
^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - (
-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])
/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3
) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]
])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(5/3)*d) - (Cos[c + d*x]*Sin[c +
 d*x])/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\sin^2(c+dx)}{b} - \frac{a\sin^2(c+dx)}{b\left(a+b\sin^3(c+dx)\right)}\right) dx$$

$$= \frac{\int \sin^2(c+dx)\,dx}{b} - \frac{a\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx}{b}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int 1\,dx}{2b} - \frac{a\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{}{3b^{2/3}\left(-\sqrt[3]{-\right.}}\right.}{}$$

$$= \frac{x}{2b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{a\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{5/3}} - \frac{a\int \frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\,\text{si}}}{3b^{5/3}}$$

$$= \frac{x}{2b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(2a)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}\,x+\sqrt[3]{a}\,x^2}\,dx, x, \tan\left(\frac{1}{2}(c+\right.\right.}{3b^{5/3}d}$$

$$= \frac{x}{2b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{(4a)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\,\text{t}\right.}{3b^{5/3}d}$$

$$= \frac{x}{2b} - \frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{5/3}d} + \frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right.}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,b^{5/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.20, size = 255, normalized size = 0.93

$$\frac{6(c+dx) - 2ia\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)-4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+2i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right] - 3\sin(2(c+dx))}{12bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (6*(c + d*x) - (2*I)*a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#
1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 -
2*Cos[c + d*x]*#1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
 + (2*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Co
s[c + d*x] - #1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 -
(4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] - 3*Sin[2*(c + d*x)])/(12*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.70, size = 141, normalized size = 0.52

method	result
derivativedivides	$\frac{\frac{8\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{4a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2}{\_R^5a+2}\right.}{3b}}{d}$
default	$\frac{\frac{8\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{4a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2}{\_R^5a+2}\right.}{3b}}{d}$
risch	$\frac{x}{2b}-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^{10}d^6-729b^{12}d^6\right)\_Z^6-248832a^2b^8d^4\_Z^4-28311552a^4b^4d^2\_Z^2-1073741824a^6\right)}\_R\ln\left(e^{i(dx}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(8/b*((1/8*tan(1/2*d*x+1/2*c)^3-1/8*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+
1/2*c)^2)^2+1/8*arctan(tan(1/2*d*x+1/2*c)))-4/3/b*a*sum(_R^2/(_R^5*a+2*_R^3
*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^
3*b+3*_Z^2*a+a)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b*d*integrate(2*(16*a^2*cos(3*d*x + 3*c)^2 + 16*a^2*sin(3*d*x + 3*c
)^2 + 3*a*b*cos(d*x + c)*sin(2*d*x + 2*c) - 3*a*b*cos(2*d*x + 2*c)*sin(d*x
+ c) + a*b*sin(d*x + c) - (a*b*sin(5*d*x + 5*c) - 2*a*b*sin(3*d*x + 3*c) +
a*b*sin(d*x + c))*cos(6*d*x + 6*c) - (8*a^2*cos(3*d*x + 3*c) + 3*a*b*sin(4*
d*x + 4*c) - 3*a*b*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 3*(2*a*b*sin(3*d*x
+ 3*c) - a*b*sin(d*x + c))*cos(4*d*x + 4*c) - 2*(4*a^2*cos(d*x + c) + 3*a*b
*sin(2*d*x + 2*c))*cos(3*d*x + 3*c) + (a*b*cos(5*d*x + 5*c) - 2*a*b*cos(3*d
*x + 3*c) + a*b*cos(d*x + c))*sin(6*d*x + 6*c) + (3*a*b*cos(4*d*x + 4*c) -
3*a*b*cos(2*d*x + 2*c) - 8*a^2*sin(3*d*x + 3*c) + a*b)*sin(5*d*x + 5*c) + 3
*(2*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*sin(4*d*x + 4*c) + 2*(3*a*b*co
s(2*d*x + 2*c) - 4*a^2*sin(d*x + c) - a*b)*sin(3*d*x + 3*c))/(b^3*cos(6*d*x
 + 6*c)^2 + 9*b^3*cos(4*d*x + 4*c)^2 + 64*a^2*b*cos(3*d*x + 3*c)^2 + 9*b^3*
cos(2*d*x + 2*c)^2 + b^3*sin(6*d*x + 6*c)^2 + 9*b^3*sin(4*d*x + 4*c)^2 + 64
*a^2*b*sin(3*d*x + 3*c)^2 - 48*a*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*
b^3*sin(2*d*x + 2*c)^2 - 6*b^3*cos(2*d*x + 2*c) + b^3 - 2*(3*b^3*cos(4*d*x
```

```
+ 4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*sin(3*d*x + 3*c) + b^3)*cos(6*d*x
 + 6*c) - 6*(3*b^3*cos(2*d*x + 2*c) + 8*a*b^2*sin(3*d*x + 3*c) - b^3)*cos(4
*d*x + 4*c) - 2*(8*a*b^2*cos(3*d*x + 3*c) + 3*b^3*sin(4*d*x + 4*c) - 3*b^3*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*a*b^2*cos(3*d*x + 3*c) - 3*b^3*si
n(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*(3*a*b^2*cos(2*d*x + 2*c) - a*b^2)*si
n(3*d*x + 3*c)), x) - 2*d*x + sin(2*d*x + 2*c))/(b*d)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.47, size = 29175, normalized size = 106.87

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2/3)*sqrt(1/6)*b*d*sqrt(((a^2*b^2 - b^4)*((a^4/(a^2*b^6*d^4 - b^
8*d^4) + a^4/(a^2*b^2*d^2 - b^4*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*a^6/(a^2*
b^10*d^6 - b^12*d^6) - 1/486*a^6/((a^2*b^6*d^4 - b^8*d^4)*(a^2*b^2*d^2 - b^
4*d^2)) - 1/729*a^6/(a^2*b^2*d^2 - b^4*d^2)^3 + 1/1458*a^8/((a^2 - b^2)^2*b
^10*d^6))^(1/3) + 81*(-1/1458*a^6/(a^2*b^10*d^6 - b^12*d^6) - 1/486*a^6/((a
^2*b^6*d^4 - b^8*d^4)*(a^2*b^2*d^2 - b^4*d^2)) - 1/729*a^6/(a^2*b^2*d^2 - b
^4*d^2)^3 + 1/1458*a^8/((a^2 - b^2)^2*b^10*d^6))^(1/3)*(I*sqrt(3) + 1) + 18
*a^2/(a^2*b^2*d^2 - b^4*d^2))*d^2 + 3*sqrt(1/3)*(a^2*b^2 - b^4)*d^2*sqrt(-(
(a^4*b^6 - 2*a^2*b^8 + b^10)*((a^4/(a^2*b^6*d^4 - b^8*d^4) + a^4/(a^2*b^2*d
^2 - b^4*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*a^6/(a^2*b^10*d^6 - b^12*d^6) -
1/486*a^6/((a^2*b^6*d^4 - b^8*d^4)*(a^2*b^2*d^2 - b^4*d^2)) - 1/729*a^6/(a^
2*b^2*d^2 - b^4*d^2)^3 + 1/1458*a^8/((a^2 - b^2)^2*b^10*d^6))^(1/3) + 81*(-
1/1458*a^6/(a^2*b^10*d^6 - b^12*d^6) - 1/486*a^6/((a^2*b^6*d^4 - b^8*d^4)*(
a^2*b^2*d^2 - b^4*d^2)) - ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^5/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 14.47, size = 1962, normalized size = 7.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(b*d + 2*b*d*tan(c/2 + (d*x)/2)^2 + b*d*tan(c/2 + (d*x
)/2)^4) - tan(c/2 + (d*x)/2)/(b*d + 2*b*d*tan(c/2 + (d*x)/2)^2 + b*d*tan(c/
2 + (d*x)/2)^4) + symsum(log((134217728*a^9*b^2 - 16777216*a^11 - 402653184
*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 +
a^6, z, k)*a^8*b^4 + 50331648*a^10*b*tan(c/2 + (d*x)/2) - 2415919104*root(7
29*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z,
 k)^2*a^7*b^6 + 914358272*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^
8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^2*a^9*b^4 + 7247757312*root(729*a^2*b^1
0*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^3*a^6*
b^8 - 478150656*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27
*a^4*b^4*z^2 + a^6, z, k)^3*a^8*b^6 + 10871635968*root(729*a^2*b^10*z^6 - 7
29*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^5*b^10 - 21
214789632*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b
^4*z^2 + a^6, z, k)^4*a^7*b^8 - 301989888*root(729*a^2*b^10*z^6 - 729*b^12*
z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^9*b^6 - 32614907904
*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 +
a^6, z, k)^5*a^4*b^12 + 59567505408*root(729*a^2*b^10*z^6 - 729*b^12*z^6 +
243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^5*a^6*b^10 + 4529848320*root(
729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z
, k)^5*a^8*b^8 + 55717134336*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2
*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^6*a^5*b^12 - 42127589376*root(729*a^
2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^6
*a^7*b^10 - 130459631616*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8
*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^7*a^4*b^14 + 122305904640*root(729*a^2*b
^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^7*a^
6*b^12 - 452984832*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 -
 27*a^4*b^4*z^2 + a^6, z, k)*a^9*b^3*tan(c/2 + (d*x)/2) + 1509949440*root(7
29*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z,
 k)^2*a^8*b^5*tan(c/2 + (d*x)/2) + 201326592*root(729*a^2*b^10*z^6 - 729*b^
12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^2*a^10*b^3*tan(c/2 +
 (d*x)/2) - 2717908992*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z
^4 - 27*a^4*b^4*z^2 + a^6, z, k)^3*a^7*b^7*tan(c/2 + (d*x)/2) - 2717908992*
root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a
^6, z, k)^3*a^9*b^5*tan(c/2 + (d*x)/2) + 4076863488*root(729*a^2*b^10*z^6 -
```

```
 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^6*b^9*tan
(c/2 + (d*x)/2) + 6039797760*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2
*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^8*b^7*tan(c/2 + (d*x)/2) - 40768
63488*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z
^2 + a^6, z, k)^5*a^5*b^11*tan(c/2 + (d*x)/2) - 679477248*root(729*a^2*b^10
*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^5*a^7*b
^9*tan(c/2 + (d*x)/2) + 16307453952*root(729*a^2*b^10*z^6 - 729*b^12*z^6 +
243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^6*a^6*b^11*tan(c/2 + (d*x)/2)
 - 40768634880*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*
a^4*b^4*z^2 + a^6, z, k)^7*a^5*b^13*tan(c/2 + (d*x)/2) + 32614907904*root(7
29*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z,
 k)^7*a^7*b^11*tan(c/2 + (d*x)/2) + 33554432*root(729*a^2*b^10*z^6 - 729*b^
12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)*a^11*b*tan(c/2 + (d*
x)/2))/b^5)*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4
*b^4*z^2 + a^6, z, k), k, 1, 6)/d - (log(tan(c/2 + (d*x)/2) - 1i)*1i)/(2*b*
d) + (log(tan(c/2 + (d*x)/2) + 1i)*1i)/(2*b*d)
```

3.184 $\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=259

$$\frac{x}{b}-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,bd}-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}\,bd}+\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{-1}\ldots}{\ldots}\right)}{3\sqrt{a^{2/3}\ldots}}$$

[Out] x/b-2/3*a^(1/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*a^(1/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*a^(1/3)*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)

Rubi [A]

time = 0.33, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3299, 3292, 2739, 632, 210}

$$-\frac{2\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}+\frac{2\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{x}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]

[Out] x/b - (2*a^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b*d) - (2*a^(1/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b*d) + (2*a^(1/3)*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{b}-\frac{a}{b\left(a+b\sin^3(c+dx)\right)}\right)\,dx$$

$$= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sin^3(c+dx)}\,dx}{b}$$

$$= \frac{x}{b}-\frac{a\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,\sin(c+dx)\right)}-\frac{}{3a^{2/3}\left(\right.}\right.}{b}$$

$$= \frac{x}{b}+\frac{\sqrt[3]{a}\,\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\,\sin(c+dx)}\,dx}{3b}+\frac{\sqrt[3]{a}\,\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,\sin(c+dx)}\,dx}{3b}+\frac{\sqrt[3]{a}\,\int}{}$$

$$= \frac{x}{b}+\frac{\left(2\sqrt[3]{a}\,\right)\text{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}\,x-\sqrt[3]{a}\,x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3bd}+\frac{\left(2\sqrt[3]{a}\,\right)\text{Su}}{}$$

$$= \frac{x}{b}-\frac{\left(4\sqrt[3]{a}\,\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}\,-2\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3bd}-$$

$$= \frac{x}{b}+\frac{2\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\,-\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,bd}-\frac{2\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{b}\,+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c\right.}{\sqrt{a^{2/3}-b^{2/3}}}\right.}{3\sqrt{a^{2/3}-b^{2/3}}\,bd}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.14, size = 140, normalized size = 0.54

$$\frac{3c+3dx+2ia\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*c + 3*d*x + (2*I)*a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#
1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/
(3*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.58, size = 104, normalized size = 0.40

method	result
derivativedivides	$-\frac{\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$
default	$-\frac{\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$
risch	$\frac{x}{b}+\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^6d^6-729b^8d^6\right)\_Z^6-15552a^2b^4d^4\_Z^4+110592a^2b^2d^2\_Z^2-262144a^2\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(\frac{243ia}{163}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/b*a*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/
2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+2/b*arctan
(tan(1/2*d*x+1/2*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] (8*a*b*integrate(-(8*a*cos(3*d*x + 3*c)^2 - b*cos(3*d*x + 3*c)*sin(6*d*x +
6*c) + 3*b*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) + b*cos(6*d*x + 6*c)*sin(3*d*x
 + 3*c) - 3*b*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) + 8*a*sin(3*d*x + 3*c)^2 -
3*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + (3*b*cos(2*d*x + 2*c) - b)*sin(3*d*
x + 3*c))/(b^3*cos(6*d*x + 6*c)^2 + 9*b^3*cos(4*d*x + 4*c)^2 + 64*a^2*b*cos
(3*d*x + 3*c)^2 + 9*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(6*d*x + 6*c)^2 + 9*b^3
*sin(4*d*x + 4*c)^2 + 64*a^2*b*sin(3*d*x + 3*c)^2 - 48*a*b^2*cos(3*d*x + 3*
c)*sin(2*d*x + 2*c) + 9*b^3*sin(2*d*x + 2*c)^2 - 6*b^3*cos(2*d*x + 2*c) + b
^3 - 2*(3*b^3*cos(4*d*x + 4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*sin(3*d*x
 + 3*c) + b^3)*cos(6*d*x + 6*c) - 6*(3*b^3*cos(2*d*x + 2*c) + 8*a*b^2*sin(3
*d*x + 3*c) - b^3)*cos(4*d*x + 4*c) - 2*(8*a*b^2*cos(3*d*x + 3*c) + 3*b^3*s
in(4*d*x + 4*c) - 3*b^3*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*a*b^2*cos
(3*d*x + 3*c) - 3*b^3*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*(3*a*b^2*cos(
2*d*x + 2*c) - a*b^2)*sin(3*d*x + 3*c)), x) + x)/b
```

Fricas [C] Result contains complex when optimal does not.
time = 1.44, size = 29221, normalized size = 112.82

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2/3)*sqrt(1/6)*b*sqrt(((a^2*b^2 - b^4)*((a^4/(a^2*b^2*d^2 - b^4*
d^2)^2 - a^2/(a^2*b^4*d^4 - b^6*d^4))*(-I*sqrt(3) + 1)/(-1/729*a^6/(a^2*b^2
*d^2 - b^4*d^2)^3 + 1/486*a^4/((a^2*b^4*d^4 - b^6*d^4)*(a^2*b^2*d^2 - b^4*d
^2)) - 1/1458*a^2/(a^2*b^6*d^6 - b^8*d^6) + 1/1458*a^2/((a^2 - b^2)^2*b^4*d
^6))^(1/3) + 81*(-1/729*a^6/(a^2*b^2*d^2 - b^4*d^2)^3 + 1/486*a^4/((a^2*b^4
*d^4 - b^6*d^4)*(a^2*b^2*d^2 - b^4*d^2)) - 1/1458*a^2/(a^2*b^6*d^6 - b^8*d^
6) + 1/1458*a^2/((a^2 - b^2)^2*b^4*d^6))^(1/3)*(I*sqrt(3) + 1) + 18*a^2/(a^
2*b^2*d^2 - b^4*d^2))*d^2 + 3*sqrt(1/3)*(a^2*b^2 - b^4)*d^2*sqrt(-((a^4*b^4
 - 2*a^2*b^6 + b^8)*((a^4/(a^2*b^2*d^2 - b^4*d^2)^2 - a^2/(a^2*b^4*d^4 - b^
6*d^4))*(-I*sqrt(3) + 1)/(-1/729*a^6/(a^2*b^2*d^2 - b^4*d^2)^3 + 1/486*a^4/
((a^2*b^4*d^4 - b^6*d^4)*(a^2*b^2*d^2 - b^4*d^2)) - 1/1458*a^2/(a^2*b^6*d^6
 - b^8*d^6) + 1/1458*a^2/((a^2 - b^2)^2*b^4*d^6))^(1/3) + 81*(-1/729*a^6/(a
^2*b^2*d^2 - b^4*d^2)^3 + 1/486*a^4/((a^2*b^4*d^4 - b^6*d^4)*(a^2*b^2*d^2 -
 b^4*d^2)) - 1/1458*a^2/( ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 14.90, size = 1672, normalized size = 6.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(134217728*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 +
 27*a^2*b^2*z^2 + a^2, z, k)*a^7*tan(c/2 + (d*x)/2) - 268435456*root(729*a^
2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^2*a
^7*b - 1073741824*a^6*tan(c/2 + (d*x)/2) + 4831838208*root(729*a^2*b^6*z^6
- 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^2*a^5*b^3 + 3
3722204160*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^
2*z^2 + a^2, z, k)^3*a^6*b^3 + 15703474176*root(729*a^2*b^6*z^6 - 729*b^8*z
^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^4*a^5*b^5 - 4831838208*r
oot(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2,
 z, k)^4*a^7*b^3 - 130459631616*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^
2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^5*a^4*b^7 + 154014842880*root(729*a
^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^5*
a^6*b^5 + 35332816896*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4
+ 27*a^2*b^2*z^2 + a^2, z, k)^6*a^5*b^7 - 21743271936*root(729*a^2*b^6*z^6
- 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^6*a^7*b^5 - 1
30459631616*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b
^2*z^2 + a^2, z, k)^7*a^4*b^9 + 122305904640*root(729*a^2*b^6*z^6 - 729*b^8
*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^7*a^6*b^7 + 2013265920
*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^
2, z, k)*a^6*b - 3221225472*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^
4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)*a^5*b^2*tan(c/2 + (d*x)/2) - 1858915532
8*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a
^2, z, k)^2*a^6*b^2*tan(c/2 + (d*x)/2) - 17716740096*root(729*a^2*b^6*z^6 -
 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^3*a^5*b^4*tan(
c/2 + (d*x)/2) + 2818572288*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^
4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^3*a^7*b^2*tan(c/2 + (d*x)/2) + 86973087
744*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 +
 a^2, z, k)^4*a^4*b^6*tan(c/2 + (d*x)/2) - 88181047296*root(729*a^2*b^6*z^6
 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^4*a^6*b^4*ta
n(c/2 + (d*x)/2) - 30802968576*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2
*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^5*a^5*b^6*tan(c/2 + (d*x)/2) + 18119
393280*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^
2 + a^2, z, k)^5*a^7*b^4*tan(c/2 + (d*x)/2) + 86973087744*root(729*a^2*b^6*
z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^6*a^4*b^8
*tan(c/2 + (d*x)/2) - 70665633792*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*
a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^6*a^6*b^6*tan(c/2 + (d*x)/2) - 40
768634880*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2
*z^2 + a^2, z, k)^7*a^5*b^8*tan(c/2 + (d*x)/2) + 32614907904*root(729*a^2*b
^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^7*a^7*
b^6*tan(c/2 + (d*x)/2))*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^
4 + 27*a^2*b^2*z^2 + a^2, z, k), k, 1, 6)/d - (log(tan(c/2 + (d*x)/2) - 1i)
```

```
*1i)/(b*d) + (log(tan(c/2 + (d*x)/2) + 1i)*1i)/(b*d)
```

3.185 $\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=267

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\,\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,\sqrt[3]{b}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\,\sqrt{a^{2/3}-b^{2/3}}\,\sqrt[3]{b}\,d}+\frac{2\sqrt[3]{-1}\,\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}\ \cdots}{\sqrt{a^{2/3}\ \cdots}}\right.}{3\sqrt[3]{a}\,\sqrt{a^{2/3}+\sqrt[3]{-1}\ \cdots}}$$

```
[Out] -2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a
^(1/3)/b^(1/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*(-1)^(1/3)*arctan(((-1)^(2/3)*
b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(
1/3)/b^(1/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*(-1)^(2/3)*arctan(((-
1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(
1/2))/a^(1/3)/b^(1/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3299, 2739, 632, 210}

$$\frac{2(-1)^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*(-1)^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a
^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)
]*b^(1/3)*d) - (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
- b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2*(-1)^(1/3)*
ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^
(1/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(1/3)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{1}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,\sin(c+\right.}\right.$$

$$= -\frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,\sin(c+dx)}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,\sin(c+dx)}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{}{\sqrt[3]{a}}}{3}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}\,x+\sqrt[3]{a}\,x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d} + \frac{\left(2\sqrt[3]{-1}\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{}}\right.}{}$$

$$= \frac{4\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d} - \frac{\left(4\sqrt[3]{-1}\right)\text{Sub}}{}$$

$$= \frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\,\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,\sqrt[3]{b}\,d} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right.}{3\sqrt[3]{a}\,\sqrt{a^{2/3}-b^{2/3}}\,\sqrt[3]{b}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 172, normalized size = 0.64

$$-\frac{\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/3*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & ,
 (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*Log[1 - 2*Co
s[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ]/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.64, size = 78, normalized size = 0.29

method	result
derivativedivides	$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3d}$
default	$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3d}$
risch	$-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(-64+\left(729a^4b^2d^6-729a^2b^4d^6\right)\_Z^6-972a^2b^2d^4\_Z^4\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(-\frac{243id^5b^2a^5}{32a^2+32b^2}+\frac{243id^5b^4a^3}{32a^2+32b^2}\right)\_R^5\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_
R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.50, size = 18879, normalized size = 70.71

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2/3)*sqrt(1/6)*sqrt(((a^2 - b^2)*(81*(I*sqrt(3) + 1)*(-1/1458/(a^
4*b^2*d^6 - a^2*b^4*d^6) - 1/729/(a^2*d^2 - b^2*d^2)^3 + 1/1458*(a^2 + b^2)
/((a^2 - b^2)^2*a^2*b^2*d^6))^(1/3) + 18/(a^2*d^2 - b^2*d^2) + (-I*sqrt(3)
+ 1)/((a^2*d^2 - b^2*d^2)^2*(-1/1458/(a^4*b^2*d^6 - a^2*b^4*d^6) - 1/729/(a
^2*d^2 - b^2*d^2)^3 + 1/1458*(a^2 + b^2)/((a^2 - b^2)^2*a^2*b^2*d^6))^(1/3)
))*d^2 + 3*sqrt(1/3)*(a^2 - b^2)*d^2*sqrt(-((a^4 - 2*a^2*b^2 + b^4)*(81*(I*
sqrt(3) + 1)*(-1/1458/(a^4*b^2*d^6 - a^2*b^4*d^6) - 1/729/(a^2*d^2 - b^2*d^
2)^3 + 1/1458*(a^2 + b^2)/((a^2 - b^2)^2*a^2*b^2*d^6))^(1/3) + 18/(a^2*d^2
- b^2*d^2) + (-I*sqrt(3) + 1)/((a^2*d^2 - b^2*d^2)^2*(-1/1458/(a^4*b^2*d^6
- a^2*b^4*d^6) - 1/729/(a^2*d^2 - b^2*d^2)^3 + 1/1458*(a^2 + b^2)/((a^2 - b
^2)^2*a^2*b^2*d^6))^(1/3)))^2*d^4 - 36*(a^2 - b^2)*(81*(I*sqrt(3) + 1)*(-1/
1458/(a^4*b^2*d^6 - a^2*b^4*d^6) - 1/729/(a^2*d^2 - b^2*d^2)^3 + 1/1458*(a^
2 + b^2)/((a^2 - b^2)^2*a^2*b^2*d^6))^(1/3) + 18/(a^2*d^2 - b^2*d^2) + (-I*
sqrt(3) + 1)/((a^2*d^2 -  ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

Mupad [**B**]
time = 16.12, size = 652, normalized size = 2.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(294912*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4
+ 1, d, k)^2*a^3*b^3 - 8192*a^3*b + 1548288*root(729*a^4*b^2*d^6 - 729*a^2*
b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^3*a^4*b^3 + 1990656*root(729*a^4*b^2*d
^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^4*a^5*b^3 - 7962624*root(
729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^5*a^4*b^5 +
5971968*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)
^5*a^6*b^3 + 65536*a^2*b^2*tan(c/2 + (d*x)/2) + 196608*root(729*a^4*b^2*d^6
 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)*a^3*b^2*tan(c/2 + (d*x)/2)
+ 294912*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k
)^2*a^4*b^2*tan(c/2 + (d*x)/2) - 1769472*root(729*a^4*b^2*d^6 - 729*a^2*b^4
*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^3*a^3*b^4*tan(c/2 + (d*x)/2) + 221184*roo
t(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^3*a^5*b^2*
tan(c/2 + (d*x)/2) + 2654208*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a
^2*b^2*d^4 + 1, d, k)^4*a^4*b^4*tan(c/2 + (d*x)/2) - 1990656*root(729*a^4*b
^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^5*a^5*b^4*tan(c/2 + (
d*x)/2))*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k
), k, 1, 6)/d
```

3.186 $\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{a^{2/3}-b^{2/3}}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3a\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,d}-$$

```
[Out] -arctanh(cos(d*x+c))/a/d-2/3*b^(1/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/
2*c))/(a^(2/3)-b^(2/3))^(1/2))/a/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*b^(1/3)*arct
anh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(
2/3))^(1/2))/a/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*b^(1/3)*arctanh((b^
(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^
(1/2))/a/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3299, 3855, 2739, 632, 210, 212}

$$-\frac{2\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3ad\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*b^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2
/3)]])/(3*a*Sqrt[a^(2/3) - b^(2/3)]*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (2*b
^(1/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^
(2/3)*a^(2/3)) + b^(2/3)]])/(3*a*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*d) +
 (2*b^(1/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-
1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\csc(c+dx)}{a} - \frac{b\sin^2(c+dx)}{a\,(a+b\sin^3(c+dx))}\right)\,dx$$

$$= \frac{\int \csc(c+dx)\,dx}{a} - \frac{b\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{b\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\right.}\right.}{a}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\sqrt[3]{b}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\,\mathrm{si}}}{3a}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\left(2\sqrt[3]{b}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}\,x+\sqrt[3]{a}\,x^2}\,dx, x, \tan\left(\frac{1}{2}(c+\right.\right.}{3ad}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\left(4\sqrt[3]{b}\right)\mathrm{Subst}\left(\int \frac{1}{-4(a^{2/3}-b^{2/3})-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\,\mathrm{ta}\right.}{3ad}$$

$$= -\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{2\sqrt[3]{b}\tanh^{-1}\left(\right.}{3a\sqrt{-}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.16, size = 264, normalized size = 1.00

$$-\frac{6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + ib\,\mathrm{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right) - i\log\left(1-2\cos(c+dx)\#1+\#1^2\right) - 4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2 + 2i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2 + 2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4 - i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right]}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/6*(6*Log[Cos[(c + d*x)/2]] - 6*Log[Sin[(c + d*x)/2]] + I*b*RootSum[(-I)*
b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c +
 d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 4*ArcTan
[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1 - 2*Cos[c + d*x]*#1 +
 #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*Log[1 - 2
*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) &
])/(a*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.75, size = 96, normalized size = 0.36

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}}{d}$
risch	$2i\left(\sum_{\_R=\mathrm{RootOf}\left((46656a^8d^6-46656b^2a^6d^6)\_Z^6-3888b^2a^4d^4\_Z^4-108a^2b^2d^2\_Z^2-b^2\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(-7\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tan(1/2*d*x+1/2*c))-4/3/a*b*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+
_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a
+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*d*integrate(2*(16*a*b*cos(3*d*x + 3*c)^2 + 16*a*b*sin(3*d*x + 3*c
)^2 + 3*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 3*b^2*cos(2*d*x + 2*c)*sin(d*x
+ c) + b^2*sin(d*x + c) - (b^2*sin(5*d*x + 5*c) - 2*b^2*sin(3*d*x + 3*c) +
b^2*sin(d*x + c))*cos(6*d*x + 6*c) - (8*a*b*cos(3*d*x + 3*c) + 3*b^2*sin(4*
d*x + 4*c) - 3*b^2*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 3*(2*b^2*sin(3*d*x
+ 3*c) - b^2*sin(d*x + c))*cos(4*d*x + 4*c) - 2*(4*a*b*cos(d*x + c) + 3*b^2
*sin(2*d*x + 2*c))*cos(3*d*x + 3*c) + (b^2*cos(5*d*x + 5*c) - 2*b^2*cos(3*d
*x + 3*c) + b^2*cos(d*x + c))*sin(6*d*x + 6*c) + (3*b^2*cos(4*d*x + 4*c) -
3*b^2*cos(2*d*x + 2*c) - 8*a*b*sin(3*d*x + 3*c) + b^2)*sin(5*d*x + 5*c) + 3
*(2*b^2*cos(3*d*x + 3*c) - b^2*cos(d*x + c))*sin(4*d*x + 4*c) + 2*(3*b^2*co
s(2*d*x + 2*c) - 4*a*b*sin(d*x + c) - b^2)*sin(3*d*x + 3*c))/(a*b^2*cos(6*d
*x + 6*c)^2 + 9*a*b^2*cos(4*d*x + 4*c)^2 + 64*a^3*cos(3*d*x + 3*c)^2 + 9*a*
b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(6*d*x + 6*c)^2 + 9*a*b^2*sin(4*d*x + 4*c
)^2 + 64*a^3*sin(3*d*x + 3*c)^2 - 48*a^2*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c
) + 9*a*b^2*sin(2*d*x + 2*c)^2 - 6*a*b^2*cos(2*d*x + 2*c) + a*b^2 - 2*(3*a*
```

```
b^2*cos(4*d*x + 4*c) - 3*a*b^2*cos(2*d*x + 2*c) - 8*a^2*b*sin(3*d*x + 3*c)
+ a*b^2)*cos(6*d*x + 6*c) - 6*(3*a*b^2*cos(2*d*x + 2*c) + 8*a^2*b*sin(3*d*x
 + 3*c) - a*b^2)*cos(4*d*x + 4*c) - 2*(8*a^2*b*cos(3*d*x + 3*c) + 3*a*b^2*s
in(4*d*x + 4*c) - 3*a*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*a^2*b*c
os(3*d*x + 3*c) - 3*a*b^2*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*(3*a^2*b*
cos(2*d*x + 2*c) - a^2*b)*sin(3*d*x + 3*c)), x) + log(cos(d*x)^2 + 2*cos(d*
x)*cos(c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) - log(cos
(d*x)^2 - 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 + 2*sin(d*x)*sin(c) + s
in(c)^2))/(a*d)
```

Fricas [C] Result contains complex when optimal does not.
time = 4.00, size = 29139, normalized size = 110.38

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2/3)*sqrt(1/6)*a*d*sqrt(((a^4 - a^2*b^2)*((b^4/(a^4*d^2 - a^2*b^
2*d^2)^2 + b^2/(a^6*d^4 - a^4*b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^6/(a^4*d
^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2
*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4
*d^6))^(1/3) + 81*(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d
^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 - a^6*b^2*
d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3)*(I*sqrt(3) + 1) + 18*b^2/(
a^4*d^2 - a^2*b^2*d^2))*d^2 + 3*sqrt(1/3)*(a^4 - a^2*b^2)*d^2*sqrt(-((a^8 -
 2*a^6*b^2 + a^4*b^4)*((b^4/(a^4*d^2 - a^2*b^2*d^2)^2 + b^2/(a^6*d^4 - a^4*
b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^
4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/1458*b^2/(a^8*d^6 -
 a^6*b^2*d^6) + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) + 81*(-1/729*b^6/
(a^4*d^2 - a^2*b^2*d^2)^3 - 1/486*b^4/((a^6*d^4 - a^4*b^2*d^4)*(a^4*d^2 - a
^2*b^2*d^2)) - 1/1458*b^2 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 15.66, size = 1439, normalized size = 5.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] symsum(log(98304*b^5 + 1048576*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4
*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)*b^6*tan(c/2 + (d*x)/2) - 98304*root(
729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z,
k)^2*a^2*b^5 + 5898240*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4
 + 27*a^2*b^2*z^2 - b^2, z, k)^3*a^3*b^5 - 7962624*root(729*a^6*b^2*z^6 - 7
29*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^4*a^4*b^5 - 6635
52*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 -
b^2, z, k)^4*a^6*b^3 - 5308416*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4
*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^5*a^5*b^5 + 10616832*root(729*a^6*b^
2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^5*a^7*b
^3 + 7962624*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*
b^2*z^2 - b^2, z, k)^6*a^6*b^5 - 9953280*root(729*a^6*b^2*z^6 - 729*a^8*z^6
 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^6*a^8*b^3 - 589824*root(72
9*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)
*a*b^5 - 24576*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^
2*b^2*z^2 - b^2, z, k)*a^2*b^4*tan(c/2 + (d*x)/2) - 3145728*root(729*a^6*b^
2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^2*a*b^6
*tan(c/2 + (d*x)/2) + 466944*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b
^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^2*a^3*b^4*tan(c/2 + (d*x)/2) - 1887436
8*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b
^2, z, k)^3*a^2*b^6*tan(c/2 + (d*x)/2) - 3981312*root(729*a^6*b^2*z^6 - 729
*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^3*a^4*b^4*tan(c/2
+ (d*x)/2) + 56623104*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4
+ 27*a^2*b^2*z^2 - b^2, z, k)^4*a^3*b^6*tan(c/2 + (d*x)/2) + 20791296*root(
729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z,
k)^4*a^5*b^4*tan(c/2 + (d*x)/2) + 84934656*root(729*a^6*b^2*z^6 - 729*a^8*z
^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^5*a^4*b^6*tan(c/2 + (d*x
)/2) - 78962688*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a
^2*b^2*z^2 - b^2, z, k)^5*a^6*b^4*tan(c/2 + (d*x)/2) - 254803968*root(729*a
^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^6*
a^5*b^6*tan(c/2 + (d*x)/2) + 252813312*root(729*a^6*b^2*z^6 - 729*a^8*z^6 -
 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^6*a^7*b^4*tan(c/2 + (d*x)/2)
```

```
)*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b
^2, z, k), k, 1, 6)/d + log(tan(c/2 + (d*x)/2))/(a*d)
```

3.187 $\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=287

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}-b^{2/3}}\,d}-\frac{2b\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}\,d}+\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\cdots\right.}{\sqrt{a^{2/3}-(-\cdots}}\right.}{3a^{5/3}\sqrt{a^{2/3}-(-1)^{2}\cdots}}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d-2/3*b*arctan((b^
(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^(5/3)/d/(a^(2/
3)-b^(2/3))^(1/2)-2/3*b*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*
c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(5/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/
3))^(1/2)+2/3*b*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1
/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(5/3)/d/(a^(2/3)-(-1)^(2/3)*b^
(2/3))^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3299, 3853, 3855, 3292, 2739, 632, 210}

$$-\frac{2b\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}+\frac{2b\text{ArcTan}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*b*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])
/(3*a^(5/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*b*ArcTan[((-1)^(2/3)*b^(1/3) +
a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(5/3)*S
qrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) + (2*b*ArcTan[((-1)^(1/3)*(b^(1/3) + (
-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(
3*a^(5/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - ArcTanh[Cos[c + d*x]]/(2*
a*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\csc^3(c+dx)}{a} - \frac{b}{a\,(a+b\sin^3(c+dx))}\right)dx$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a} - \frac{b\int \frac{1}{a+b\sin^3(c+dx)}\,dx}{a}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int \csc(c+dx)\,dx}{2a} - \frac{b\int\left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,\sin(c+dx)\right)}\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{b\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\,\sin(c+dx)}\,dx}{3a^{5/3}} + \cdots$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{(2b)\text{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}\,x-\cdots}\right.}{3a^{5/3}\cdots}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{(4b)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\cdots\right.}{}$$

$$= \frac{2b\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} - \frac{2b\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{2b\cdots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.29, size = 181, normalized size = 0.63

$$\frac{16ib\text{RootSum}\left[-b+3b\#1^2-8ia\#1^3-3b\#1^4+b\#1^6\&,\frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]-3\left(\csc^2\left(\frac{1}{2}(c+dx)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)\right)}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((16*I)*b*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (2*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ] - 3*(Csc[(c + d*x)/2]^2
+ 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] - Sec[(c + d*x)/2]^2))/
(24*a*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.96, size = 136, normalized size = 0.47

method	result
derivativedivides	$\frac{\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{}{8a\tan}}{d}$
default	$\frac{\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{}{8a\tan}}{d}$
risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^2}-8i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(191102976a^{12}d^6-191102976a^{10}b^2d^6\right)\_Z^6-995328a^8b^2d^4\_Z^4+1728a^4b^4d\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tan(1/2*d*x+1/2*c)^2/a-1/3/a*b*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*
a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3
*b+3*_Z^2*a+a))-1/8/a/tan(1/2*d*x+1/2*c)^2+1/2/a*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(cos(3*d*x + 3*c) + cos(d*x + c))*cos(4*d*x + 4*c) - 4*(2*cos(2*d*x
+ 2*c) - 1)*cos(3*d*x + 3*c) - 8*cos(2*d*x + 2*c)*cos(d*x + c) + 32*(a*b*d*
cos(4*d*x + 4*c)^2 + 4*a*b*d*cos(2*d*x + 2*c)^2 + a*b*d*sin(4*d*x + 4*c)^2
- 4*a*b*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*b*d*sin(2*d*x + 2*c)^2 -
4*a*b*d*cos(2*d*x + 2*c) + a*b*d - 2*(2*a*b*d*cos(2*d*x + 2*c) - a*b*d)*cos
(4*d*x + 4*c))*integrate(-(8*a*cos(3*d*x + 3*c)^2 - b*cos(3*d*x + 3*c)*sin(
6*d*x + 6*c) + 3*b*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) + b*cos(6*d*x + 6*c)*s
in(3*d*x + 3*c) - 3*b*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) + 8*a*sin(3*d*x + 3
*c)^2 - 3*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + (3*b*cos(2*d*x + 2*c) - b)*
sin(3*d*x + 3*c))/(a*b^2*cos(6*d*x + 6*c)^2 + 9*a*b^2*cos(4*d*x + 4*c)^2 +
64*a^3*cos(3*d*x + 3*c)^2 + 9*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(6*d*x +
6*c)^2 + 9*a*b^2*sin(4*d*x + 4*c)^2 + 64*a^3*sin(3*d*x + 3*c)^2 - 48*a^2*b*
cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*a*b^2*sin(2*d*x + 2*c)^2 - 6*a*b^2*co
s(2*d*x + 2*c) + a*b^2 - 2*(3*a*b^2*cos(4*d*x + 4*c) - 3*a*b^2*cos(2*d*x +
2*c) - 8*a^2*b*sin(3*d*x + 3*c) + a*b^2)*cos(6*d*x + 6*c) - 6*(3*a*b^2*cos(
```

```
2*d*x + 2*c) + 8*a^2*b*sin(3*d*x + 3*c) - a*b^2)*cos(4*d*x + 4*c) - 2*(8*a^
2*b*cos(3*d*x + 3*c) + 3*a*b^2*sin(4*d*x + 4*c) - 3*a*b^2*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c) + 6*(8*a^2*b*cos(3*d*x + 3*c) - 3*a*b^2*sin(2*d*x + 2*c))
*sin(4*d*x + 4*c) + 16*(3*a^2*b*cos(2*d*x + 2*c) - a^2*b)*sin(3*d*x + 3*c))
, x) + (2*(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 -
4*cos(2*d*x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) - 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log(cos(d*x)^2 + 2*co
s(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) - (2*
(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(2*d*
x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 4*s
in(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log(cos(d*x)^2 - 2*cos(d*x)*cos
(c) + cos(c)^2 + sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^2) + 4*(sin(3*d*x
+ 3*c) + sin(d*x + c))*sin(4*d*x + 4*c) - 8*sin(3*d*x + 3*c)*sin(2*d*x + 2*
c) - 8*sin(2*d*x + 2*c)*sin(d*x + c) + 4*cos(d*x + c))/(a*d*cos(4*d*x + 4*c
)^2 + 4*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(4*d*x + 4*c)^2 - 4*a*d*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*a*d*sin(2*d*x + 2*c)^2 - 4*a*d*cos(2*d*x + 2*c)
+ a*d - 2*(2*a*d*cos(2*d*x + 2*c) - a*d)*cos(4*d*x + 4*c))
```

Fricas [C] Result contains complex when optimal does not.
time = 14.64, size = 29431, normalized size = 102.55

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(3*sqrt(2/3)*sqrt(1/6)*(a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^2*b
^2)*((b^4/(a^8*d^4 - a^6*b^2*d^4) - b^4/(a^4*d^2 - a^2*b^2*d^2)^2)*(-I*sqrt
(3) + 1)/(-1/1458*b^6/(a^12*d^6 - a^10*b^2*d^6) + 1/486*b^6/((a^8*d^4 - a^6
*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) - 1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 +
1/1458*b^8/((a^2 - b^2)^2*a^10*d^6))^(1/3) - 81*(-1/1458*b^6/(a^12*d^6 - a^
10*b^2*d^6) + 1/486*b^6/((a^8*d^4 - a^6*b^2*d^4)*(a^4*d^2 - a^2*b^2*d^2)) -
 1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 + 1/1458*b^8/((a^2 - b^2)^2*a^10*d^6))
^(1/3)*(I*sqrt(3) + 1) - 18*b^2/(a^4*d^2 - a^2*b^2*d^2))*d^2 - 3*sqrt(1/3)*
(a^4 - a^2*b^2)*d^2*sqrt(-((a^10 - 2*a^8*b^2 + a^6*b^4)*((b^4/(a^8*d^4 - a^
6*b^2*d^4) - b^4/(a^4*d^2 - a^2*b^2*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*b^6/(
a^12*d^6 - a^10*b^2*d^6) + 1/486*b^6/((a^8*d^4 - a^6*b^2*d^4)*(a^4*d^2 - a^
2*b^2*d^2)) - 1/729*b^6/(a^4*d^2 - a^2*b^2*d^2)^3 + 1/1458*b^8/((a^2 - b^2)
^2*a^10*d^6))^(1/3) - 81*(-1/1458*b^6/(a^12*d^6 - a^10*b^2*d^6) + 1/486*b^6
/((a^8*d^4 - a^6*b^2*d^4) ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^3/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 15.03, size = 1573, normalized size = 5.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] symsum(log(-(65536*a*b^9 - 262144*b^10*tan(c/2 + (d*x)/2) - 131072*root(729
*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k
)*a^2*b^9 - 61440*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 -
27*a^4*b^4*z^2 - b^6, z, k)*a^4*b^7 + 860160*root(729*a^10*b^2*z^6 - 729*a^
12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^2*a^5*b^7 - 3244032*
root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b
^6, z, k)^3*a^6*b^7 - 1105920*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^
8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^3*a^8*b^5 + 3538944*root(729*a^10*b
^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^4*a^7
*b^7 + 3870720*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*
a^4*b^4*z^2 - b^6, z, k)^4*a^9*b^5 + 663552*root(729*a^10*b^2*z^6 - 729*a^1
2*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^5*a^10*b^5 - 4976640*
root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b
^6, z, k)^5*a^12*b^3 - 7962624*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a
^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^6*a^11*b^5 + 9953280*root(729*a^10
*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^6*a
^13*b^3 + 24576*a^2*b^8*tan(c/2 + (d*x)/2) + 540672*root(729*a^10*b^2*z^6 -
 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)*a^3*b^8*tan(c
/2 + (d*x)/2) - 7077888*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*
z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^2*a^4*b^8*tan(c/2 + (d*x)/2) + 442368*roo
t(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6,
 z, k)^2*a^6*b^6*tan(c/2 + (d*x)/2) - 2359296*root(729*a^10*b^2*z^6 - 729*a
```

```
^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^3*a^5*b^8*tan(c/2 +
 (d*x)/2) + 7741440*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4
- 27*a^4*b^4*z^2 - b^6, z, k)^3*a^7*b^6*tan(c/2 + (d*x)/2) - 80953344*root(
729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z
, k)^4*a^8*b^6*tan(c/2 + (d*x)/2) + 1990656*root(729*a^10*b^2*z^6 - 729*a^1
2*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^4*a^10*b^4*tan(c/2 +
(d*x)/2) - 31850496*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4
- 27*a^4*b^4*z^2 - b^6, z, k)^5*a^9*b^6*tan(c/2 + (d*x)/2) + 26873856*root(
729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z
, k)^5*a^11*b^4*tan(c/2 + (d*x)/2) + 254803968*root(729*a^10*b^2*z^6 - 729*
a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^6*a^10*b^6*tan(c/2
 + (d*x)/2) - 252813312*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*
z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^6*a^12*b^4*tan(c/2 + (d*x)/2))/a^5)*root(
729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z
, k), k, 1, 6)/d - cot(c/2 + (d*x)/2)^2/(8*a*d) + tan(c/2 + (d*x)/2)^2/(8*a
*d) + log(tan(c/2 + (d*x)/2))/(2*a*d)
```

3.188 $\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=344

$$\frac{2(-1)^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}-\frac{2b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2\sqrt[3]{-1}\,b^{5/3}\tan^{-1}}{3a^{7/3}\sqrt{}}$$

```
[Out] -3/8*arctanh(cos(d*x+c))/a/d+b*cot(d*x+c)/a^2/d-3/8*cot(d*x+c)*csc(d*x+c)/a
/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d-2/3*b^(5/3)*arctan((b^(1/3)+a^(1/3)*tan(
1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^(7/3)/d/(a^(2/3)-b^(2/3))^(1/2)+
2/3*(-1)^(1/3)*b^(5/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c
))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(7/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3
))^(1/2)+2/3*(-1)^(2/3)*b^(5/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*
d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(7/3)/d/(a^(2/3)-(-1)^(2/
3)*b^(2/3))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3299, 3852, 8, 3853, 3855, 2739, 632, 210}

$$\frac{2(-1)^{2/3}b^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2b^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\,b^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}+\frac{b\cot(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*(-1)^(2/3)*b^(5/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - (-1)^(2/3)
*b^(2/3)]*d) - (2*b^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
a^(2/3) - b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*(-1)^(1/3)*
b^(5/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
 + (-1)^(1/3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) -
 (3*ArcTanh[Cos[c + d*x]])/(8*a*d) + (b*Cot[c + d*x])/(a^2*d) - (3*Cot[c +
d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{b\csc^2(c+dx)}{a^2} + \frac{\csc^5(c+dx)}{a} + \frac{b^2\sin(c+dx)}{a^2\,(a+b\sin^3(c+dx))}\right)\,dx$$

$$= \frac{\int \csc^5(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{3\int \csc^3(c+dx)\,dx}{4a} + \frac{b^2\int \left(-\frac{1}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}\right.}{}$$

$$= \frac{b\cot(c+dx)}{a^2d} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{3\int \csc(}{}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{b\cot(c+dx)}{a^2d} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot(c+}{}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{b\cot(c+dx)}{a^2d} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot(c+}{}$$

$$= \frac{2(-1)^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} - \frac{2b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}\right.}{\sqrt{a^{2/3}-b}}\right.}{3a^{7/3}\sqrt{a^{2/3}-b^{2/3}}\,}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.41, size = 290, normalized size = 0.84

$$\frac{-64b^2\,\text{RootSum}\left[-b+3b\#1^2-8ia\#1^3-3b\#1^4+b\#1^6\&,\frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]+3\left(32b\cot\left(\frac{1}{2}(c+dx)\right)-6a\csc^2\left(\frac{1}{2}(c+dx)\right)-a\csc^4\left(\frac{1}{2}(c+dx)\right)-24a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+24a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6a\sec^2\left(\frac{1}{2}(c+dx)\right)+a\sec^4\left(\frac{1}{2}(c+dx)\right)-32b\tan\left(\frac{1}{2}(c+dx)\right)\right)}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-64*b^2*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (-2*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*Log[1 - 2*Cos[c +
d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*(32*b*Co
t[(c + d*x)/2] - 6*a*Csc[(c + d*x)/2]^2 - a*Csc[(c + d*x)/2]^4 - 24*a*Log[C
os[(c + d*x)/2]] + 24*a*Log[Sin[(c + d*x)/2]] + 6*a*Sec[(c + d*x)/2]^2 + a*
Sec[(c + d*x)/2]^4 - 32*b*Tan[(c + d*x)/2]))/(192*a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.07, size = 196, normalized size = 0.57

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^2}+\frac{2b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)}{\_R^5a+2\_}\right.}{3a^2}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^2}+\frac{2b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)}{\_R^5a+2\_}\right.}{3a^2}}{d}$
risch	$\frac{3a\,e^{7i(dx+c)}-11a\,e^{5i(dx+c)}+8ib\,e^{6i(dx+c)}-11a\,e^{3i(dx+c)}-24ib\,e^{4i(dx+c)}+3a\,e^{i(dx+c)}+24ib\,e^{2i(dx+c)}-8ib}{4a^2d\left(e^{2i(dx+c)}-1\right)^4}+32i\left(\sum_{\_R=}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/a^2*(1/4*a*tan(1/2*d*x+1/2*c)^4+2*a*tan(1/2*d*x+1/2*c)^2-8*b*tan(
1/2*d*x+1/2*c))+2/3*b^2/a^2*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*l
n(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-1/
64/a/tan(1/2*d*x+1/2*c)^4-1/8/a/tan(1/2*d*x+1/2*c)^2+3/8/a*ln(tan(1/2*d*x+1
/2*c))+1/2/a^2*b/tan(1/2*d*x+1/2*c))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**C**] Result contains complex when optimal does not.
time = 104.91, size = 21564, normalized size = 62.69

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/48*(18*a*cos(d*x + c)^3 + 4*sqrt(2/3)*sqrt(1/6)*(a^2*d*cos(d*x + c)^4 - 2
*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(-(54*b^4 - (b^8*(-I*sqrt(3) + 1)/((-1/1
```

```
458*b^10/(a^16*d^6 - a^14*b^2*d^6) - 1/729*b^12/(a^6*d^2 - a^4*b^2*d^2)^3 +
 1/1458*(a^2 + b^2)*b^10/((a^2 - b^2)^2*a^14*d^6))^(1/3)*(a^6*d^2 - a^4*b^2
*d^2)^2) + 18*b^4/(a^6*d^2 - a^4*b^2*d^2) + 81*(-1/1458*b^10/(a^16*d^6 - a^
14*b^2*d^6) - 1/729*b^12/(a^6*d^2 - a^4*b^2*d^2)^3 + 1/1458*(a^2 + b^2)*b^1
0/((a^2 - b^2)^2*a^14*d^6))^(1/3)*(I*sqrt(3) + 1))*(a^6 - a^4*b^2)*d^2 + 3*
sqrt(1/3)*(a^6 - a^4*b^2)*d^2*sqrt((972*b^8 - (a^12 - 2*a^10*b^2 + a^8*b^4)
*(b^8*(-I*sqrt(3) + 1)/((-1/1458*b^10/(a^16*d^6 - a^14*b^2*d^6) - 1/729*b^1
2/(a^6*d^2 - a^4*b^2*d^2)^3 + 1/1458*(a^2 + b^2)*b^10/((a^2 - b^2)^2*a^14*d
^6))^(1/3)*(a^6*d^2 - a^4*b^2*d^2)^2) + 18*b^4/(a^6*d^2 - a^4*b^2*d^2) + 81
*(-1/1458*b^10/(a^16*d^6 - a^14*b^2*d^6) - 1/729*b^12/(a^6*d^2 - a^4*b^2*d^
2)^3 + 1/1458*(a^2 + b^2)*b^10/((a^2 - b^2)^2*a^14*d^6))^(1/3)*(I*sqrt(3) +
 1))^2*d^4 + 36*(a^6*b^4  ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^5/(b*sin(d*x + c)^3 + a), x)
```

Mupad [**B**]
time = 14.60, size = 1560, normalized size = 4.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] symsum(log((262144*b^14*tan(c/2 + (d*x)/2) - 3072*a^3*b^11 + 155648*root(72
9*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)*a^4*b^11 - 3
93216*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)
^2*a^5*b^11 + 774144*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^
4 - b^10, z, k)^2*a^7*b^9 - 2064384*root(729*a^14*b^2*z^6 - 729*a^16*z^6 -
```

```
243*a^10*b^4*z^4 - b^10, z, k)^3*a^8*b^9 + 2073600*root(729*a^14*b^2*z^6 -
729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^3*a^10*b^7 - 9510912*root(729
*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^4*a^11*b^7 +
2737152*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z,
k)^4*a^13*b^5 + 10616832*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^
4*z^4 - b^10, z, k)^5*a^12*b^7 - 10285056*root(729*a^14*b^2*z^6 - 729*a^16*
z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^5*a^14*b^5 + 3732480*root(729*a^14*b^2
*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^5*a^16*b^3 + 7962624*r
oot(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^6*a^15
*b^5 - 9953280*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^
10, z, k)^6*a^17*b^3 + 98304*a^2*b^12*tan(c/2 + (d*x)/2) - 262144*root(729*
a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)*a^3*b^12*tan(c
/2 + (d*x)/2) + 165888*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*
z^4 - b^10, z, k)*a^5*b^10*tan(c/2 + (d*x)/2) - 1327104*root(729*a^14*b^2*z
^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^2*a^6*b^10*tan(c/2 + (d*
x)/2) + 165888*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^
10, z, k)^2*a^8*b^8*tan(c/2 + (d*x)/2) + 2359296*root(729*a^14*b^2*z^6 - 72
9*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^3*a^7*b^10*tan(c/2 + (d*x)/2) -
 7077888*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z,
 k)^3*a^9*b^8*tan(c/2 + (d*x)/2) + 82944*root(729*a^14*b^2*z^6 - 729*a^16*z
^6 - 243*a^10*b^4*z^4 - b^10, z, k)^3*a^11*b^6*tan(c/2 + (d*x)/2) + 8139571
2*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^4*a
^10*b^8*tan(c/2 + (d*x)/2) - 1714176*root(729*a^14*b^2*z^6 - 729*a^16*z^6 -
 243*a^10*b^4*z^4 - b^10, z, k)^4*a^12*b^6*tan(c/2 + (d*x)/2) + 27869184*ro
ot(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^5*a^13*
b^6*tan(c/2 + (d*x)/2) - 23141376*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 24
3*a^10*b^4*z^4 - b^10, z, k)^5*a^15*b^4*tan(c/2 + (d*x)/2) - 254803968*root
(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^6*a^14*b^
6*tan(c/2 + (d*x)/2) + 252813312*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243
*a^10*b^4*z^4 - b^10, z, k)^6*a^16*b^4*tan(c/2 + (d*x)/2))/a^9)*root(729*a^
14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k), k, 1, 6)/d - co
t(c/2 + (d*x)/2)^2/(8*a*d) - cot(c/2 + (d*x)/2)^4/(64*a*d) + tan(c/2 + (d*x
)/2)^2/(8*a*d) + tan(c/2 + (d*x)/2)^4/(64*a*d) + (3*log(tan(c/2 + (d*x)/2))
)/(8*a*d) + (b*cot(c/2 + (d*x)/2))/(2*a^2*d) - (b*tan(c/2 + (d*x)/2))/(2*a^
2*d)
```

3.189 $\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=293

$$-\frac{ax}{b^2}+\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^2d}+\frac{2a^{4/3}\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}\,b^2d}-\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-}\cdots}{\cdots}\right)}{3\sqrt{a^2\cdots}}$$

[Out] -a*x/b^2-cos(d*x+c)/b/d+1/3*cos(d*x+c)^3/b/d+2/3*a^(4/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^2/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*a^(4/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b^2/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*a^(4/3)*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^2/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)

Rubi [A]

time = 0.30, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3299, 2713, 3292, 2739, 632, 210}

$$\frac{2a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{ax}{b^2}+\frac{\cos^3(c+dx)}{3bd}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^3),x]

[Out] -((a*x)/b^2) + (2*a^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^2*d) + (2*a^(4/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^2*d) - (2*a^(4/3)*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) - Cos[c + d*x]/(b*d) + Cos[c + d*x]^3/(3*b*d)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{a}{b^2}+\frac{\sin^3(c+dx)}{b}+\frac{a^2}{b^2\left(a+b\sin^3(c+dx)\right)}\right) dx$$

$$= -\frac{ax}{b^2}+\frac{a^2\int \frac{1}{a+b\sin^3(c+dx)}\,dx}{b^2}+\frac{\int \sin^3(c+dx)\,dx}{b}$$

$$= -\frac{ax}{b^2}+\frac{a^2\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)}-\frac{}{3}\right.}{b^2}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}-\frac{a^{4/3}\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^2}-\frac{a^{4/3}\int \frac{}{-\sqrt[3]{}}}{}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}-\frac{\left(2a^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}\,x-\sqrt[3]{a}\,x^2}\,dx,\right.}{3b^2d}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}+\frac{\left(4a^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\right.}{3b^2d}$$

$$= -\frac{ax}{b^2}-\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^2d}+\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan(}{\sqrt{a^{2/3}-b}}\right.}{3\sqrt{a^{2/3}-b^{2/3}}\,b^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.20, size = 164, normalized size = 0.56

$$-\frac{12ac+12adx+9b\cos(c+dx)-b\cos(3(c+dx))+8ia^2\operatorname{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/12*(12*a*c + 12*a*d*x + 9*b*Cos[c + d*x] - b*Cos[3*(c + d*x)] + (8*I)*a^
2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2
*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(b^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.71, size = 143, normalized size = 0.49

method	result
derivativedivides	$\frac{\frac{a^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b^2}-\frac{2\left(\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}}{d}$
default	$\frac{\frac{a^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b^2}-\frac{2\left(\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}}{d}$
risch	$-\frac{ax}{b^2}-\frac{3\,\mathrm{e}^{i(dx+c)}}{8bd}-\frac{3\,\mathrm{e}^{-i(dx+c)}}{8bd}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^{12}d^6-729b^{14}d^6\right)\_Z^6+995328a^4b^8d^4\_Z^4+452984832a^6b^4d^2\_\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^2/b^2*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan
(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-2/b^2*(
(2*b*tan(1/2*d*x+1/2*c)^2+2/3*b)/(1+tan(1/2*d*x+1/2*c)^2)^3+a*arctan(tan(1/
2*d*x+1/2*c))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/12*(96*a^2*b^2*d*integrate(-(8*a*cos(3*d*x + 3*c)^2 - b*cos(3*d*x + 3*c)
*sin(6*d*x + 6*c) + 3*b*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) + b*cos(6*d*x + 6
*c)*sin(3*d*x + 3*c) - 3*b*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) + 8*a*sin(3*d*
x + 3*c)^2 - 3*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + (3*b*cos(2*d*x + 2*c)
- b)*sin(3*d*x + 3*c))/(b^4*cos(6*d*x + 6*c)^2 + 9*b^4*cos(4*d*x + 4*c)^2 +
 64*a^2*b^2*cos(3*d*x + 3*c)^2 + 9*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(6*d*x +
 6*c)^2 + 9*b^4*sin(4*d*x + 4*c)^2 + 64*a^2*b^2*sin(3*d*x + 3*c)^2 - 48*a*b
^3*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*b^4*sin(2*d*x + 2*c)^2 - 6*b^4*cos
(2*d*x + 2*c) + b^4 - 2*(3*b^4*cos(4*d*x + 4*c) - 3*b^4*cos(2*d*x + 2*c) -
8*a*b^3*sin(3*d*x + 3*c) + b^4)*cos(6*d*x + 6*c) - 6*(3*b^4*cos(2*d*x + 2*c
) + 8*a*b^3*sin(3*d*x + 3*c) - b^4)*cos(4*d*x + 4*c) - 2*(8*a*b^3*cos(3*d*x
 + 3*c) + 3*b^4*sin(4*d*x + 4*c) - 3*b^4*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
 + 6*(8*a*b^3*cos(3*d*x + 3*c) - 3*b^4*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) +
 16*(3*a*b^3*cos(2*d*x + 2*c) - a*b^3)*sin(3*d*x + 3*c)), x) + 12*a*d*x - b
*cos(3*d*x + 3*c) + 9*b*cos(d*x + c))/(b^2*d)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.53, size = 29350, normalized size = 100.17

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(2/3)*sqrt(1/6)*b^2*d*sqrt(-(54*a^4 - (a^2*b^4 - b^6)*(18*a^4/(a
^2*b^4*d^2 - b^6*d^2) + (a^8/(a^2*b^4*d^2 - b^6*d^2)^2 - a^6/(a^2*b^8*d^4 -
 b^10*d^4))*(-I*sqrt(3) + 1)/(-1/729*a^12/(a^2*b^4*d^2 - b^6*d^2)^3 + 1/486
*a^10/((a^2*b^8*d^4 - b^10*d^4)*(a^2*b^4*d^2 - b^6*d^2)) - 1/1458*a^8/(a^2*
b^12*d^6 - b^14*d^6) + 1/1458*a^8/((a^2 - b^2)^2*b^10*d^6))^(1/3) + 81*(-1/
729*a^12/(a^2*b^4*d^2 - b^6*d^2)^3 + 1/486*a^10/((a^2*b^8*d^4 - b^10*d^4)*(
a^2*b^4*d^2 - b^6*d^2)) - 1/1458*a^8/(a^2*b^12*d^6 - b^14*d^6) + 1/1458*a^8
/((a^2 - b^2)^2*b^10*d^6))^(1/3)*(I*sqrt(3) + 1))*d^2 - 3*sqrt(1/3)*(a^2*b^
4 - b^6)*d^2*sqrt(-(324*a^8 - 1296*a^6*b^2 + (a^4*b^8 - 2*a^2*b^10 + b^12)*
(18*a^4/(a^2*b^4*d^2 - b^6*d^2) + (a^8/(a^2*b^4*d^2 - b^6*d^2)^2 - a^6/(a^2
*b^8*d^4 - b^10*d^4))*(-I*sqrt(3) + 1)/(-1/729*a^12/(a^2*b^4*d^2 - b^6*d^2)
^3 + 1/486*a^10/((a^2*b^8*d^4 - b^10*d^4)*(a^2*b^4*d^2 - b^6*d^2)) - 1/1458
*a^8/(a^2*b^12*d^6 - b^14*d^6) + 1/1458*a^8/((a^2 - b^2)^2*b^10*d^6))^(1/3)
 + 81*(-1/729*a^12/(a^2*b ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^6/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 14.58, size = 1800, normalized size = 6.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log((1073741824*a^13*tan(c/2 + (d*x)/2) + 2013265920*root(729*a^2*b^
12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)*a^12*
b^2 - 4831838208*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 2
7*a^6*b^4*z^2 + a^8, z, k)^2*a^10*b^5 + 268435456*root(729*a^2*b^12*z^6 - 7
29*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^2*a^12*b^3 + 33
722204160*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b
^4*z^2 + a^8, z, k)^3*a^10*b^6 - 15703474176*root(729*a^2*b^12*z^6 - 729*b^
14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^4*a^8*b^9 + 48318382
08*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2
+ a^8, z, k)^4*a^10*b^7 - 130459631616*root(729*a^2*b^12*z^6 - 729*b^14*z^6
 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^5*a^6*b^12 + 154014842880*
root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a
^8, z, k)^5*a^8*b^10 - 35332816896*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 2
43*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^6*a^6*b^13 + 21743271936*root(
729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z
, k)^6*a^8*b^11 - 130459631616*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a
^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^7*a^4*b^16 + 122305904640*root(729
*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k
)^7*a^6*b^14 - 3221225472*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^
8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)*a^11*b^3*tan(c/2 + (d*x)/2) + 185891553
28*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2
+ a^8, z, k)^2*a^11*b^4*tan(c/2 + (d*x)/2) - 17716740096*root(729*a^2*b^12*
z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^3*a^9*b^
7*tan(c/2 + (d*x)/2) + 2818572288*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 24
3*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^3*a^11*b^5*tan(c/2 + (d*x)/2) -
 86973087744*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^
6*b^4*z^2 + a^8, z, k)^4*a^7*b^10*tan(c/2 + (d*x)/2) + 88181047296*root(729
*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k
)^4*a^9*b^8*tan(c/2 + (d*x)/2) - 30802968576*root(729*a^2*b^12*z^6 - 729*b^
14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^5*a^7*b^11*tan(c/2 +
 (d*x)/2) + 18119393280*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*
z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^5*a^9*b^9*tan(c/2 + (d*x)/2) - 8697308774
4*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 +
 a^8, z, k)^6*a^5*b^14*tan(c/2 + (d*x)/2) + 70665633792*root(729*a^2*b^12*z
^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^6*a^7*b^1
2*tan(c/2 + (d*x)/2) - 40768634880*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 2
43*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^7*a^5*b^15*tan(c/2 + (d*x)/2)
+ 32614907904*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a
^6*b^4*z^2 + a^8, z, k)^7*a^7*b^13*tan(c/2 + (d*x)/2) + 134217728*root(729*
a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)
*a^13*b*tan(c/2 + (d*x)/2))/b^7)*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243
```

```
*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k), k, 1, 6)/d - (4*tan(c/2 + (d*x)
/2)^2)/(b*d + 3*b*d*tan(c/2 + (d*x)/2)^2 + 3*b*d*tan(c/2 + (d*x)/2)^4 + b*d
*tan(c/2 + (d*x)/2)^6) - 4/(3*(b*d + 3*b*d*tan(c/2 + (d*x)/2)^2 + 3*b*d*tan
(c/2 + (d*x)/2)^4 + b*d*tan(c/2 + (d*x)/2)^6)) + (a*log(tan(c/2 + (d*x)/2)
- 1i)*1i)/(b^2*d) - (a*log(tan(c/2 + (d*x)/2) + 1i)*1i)/(b^2*d)
```

3.190 $\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d}+\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{4/3}d}-\frac{2\sqrt[3]{-1}\,a^{2/3}\tan^{-1}}{3\sqrt{a^2}}$$

```
[Out] -cos(d*x+c)/b/d+2/3*a^(2/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^
(2/3)-b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*(-1)^(1/3)*a^(2
/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1
/3)*b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-1)^(
2/3)*a^(2/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3
)-(-1)^(2/3)*b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3299, 2718, 2739, 632, 210}

$$-\frac{2(-1)^{2/3}a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}\,a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*(-1)^(2/3)*a^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*b^(4/3)*d) + (2*a^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt
[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(4/3)*d) - (2*(-1)^(1/3)
*a^(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3
) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d)
- Cos[c + d*x]/(b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\right.}\right.}{b}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+}}{3b^{4/3}}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{4/3}d} - \frac{(2}{}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\left(4a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+d\right.\right.}{3b^{4/3}d}$$

$$= -\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d} + \frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan(}{\sqrt{a^{2/3}-}}\right.}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{4/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.18, size = 186, normalized size = 0.66

$$\frac{-3\cos(c+dx)+a\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-3*Cos[c + d*x] + a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^
4 + I*b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2
*Cos[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
- I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*
#1^4) & ])/(3*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.69, size = 104, normalized size = 0.37

method	result
derivativedivides	$\frac{-\frac{2}{b\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b}}{d}$
default	$\frac{-\frac{2}{b\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b}}{d}$
risch	$-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^8d^6-729b^{10}d^6\right)\_Z^6+62208a^2b^6d^4\_Z^4+16777216a^4\right)}\_R\ln\left(e^{i(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b/(1+tan(1/2*d*x+1/2*c)^2)-2/3/b*a*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4
*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+
3*_Z^2*a+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] (b*d*integrate(-4*(3*a*b*cos(4*d*x + 4*c)^2 + 3*a*b*cos(2*d*x + 2*c)^2 + 3*
a*b*sin(4*d*x + 4*c)^2 + 8*a^2*cos(2*d*x + 2*c)*sin(3*d*x + 3*c) - 8*a^2*co
s(3*d*x + 3*c)*sin(2*d*x + 2*c) + 3*a*b*sin(2*d*x + 2*c)^2 - a*b*cos(2*d*x
+ 2*c) - (a*b*cos(4*d*x + 4*c) - a*b*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - (
6*a*b*cos(2*d*x + 2*c) + 8*a^2*sin(3*d*x + 3*c) - a*b)*cos(4*d*x + 4*c) - (
a*b*sin(4*d*x + 4*c) - a*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 2*(4*a^2*co
s(3*d*x + 3*c) - 3*a*b*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))/(b^3*cos(6*d*x +
 6*c)^2 + 9*b^3*cos(4*d*x + 4*c)^2 + 64*a^2*b*cos(3*d*x + 3*c)^2 + 9*b^3*co
s(2*d*x + 2*c)^2 + b^3*sin(6*d*x + 6*c)^2 + 9*b^3*sin(4*d*x + 4*c)^2 + 64*a
^2*b*sin(3*d*x + 3*c)^2 - 48*a*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*b^
3*sin(2*d*x + 2*c)^2 - 6*b^3*cos(2*d*x + 2*c) + b^3 - 2*(3*b^3*cos(4*d*x +
4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*sin(3*d*x + 3*c) + b^3)*cos(6*d*x +
 6*c) - 6*(3*b^3*cos(2*d*x + 2*c) + 8*a*b^2*sin(3*d*x + 3*c) - b^3)*cos(4*d
*x + 4*c) - 2*(8*a*b^2*cos(3*d*x + 3*c) + 3*b^3*sin(4*d*x + 4*c) - 3*b^3*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*a*b^2*cos(3*d*x + 3*c) - 3*b^3*sin(
```

```
2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*(3*a*b^2*cos(2*d*x + 2*c) - a*b^2)*sin(
3*d*x + 3*c)), x) - cos(d*x + c))/(b*d)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.57, size = 21185, normalized size = 75.39

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(2/3)*sqrt(1/6)*b*d*sqrt(((a^2*b^2 - b^4)*(a^4*(-I*sqrt(3) + 1)/
((a^2*b^2*d^2 - b^4*d^2)^2*(-1/1458*a^4/(a^2*b^8*d^6 - b^10*d^6) - 1/729*a^
6/(a^2*b^2*d^2 - b^4*d^2)^3 + 1/1458*(a^2 + b^2)*a^4/((a^2 - b^2)^2*b^8*d^6
))^(1/3)) + 81*(-1/1458*a^4/(a^2*b^8*d^6 - b^10*d^6) - 1/729*a^6/(a^2*b^2*d
^2 - b^4*d^2)^3 + 1/1458*(a^2 + b^2)*a^4/((a^2 - b^2)^2*b^8*d^6))^(1/3)*(I*
sqrt(3) + 1) + 18*a^2/(a^2*b^2*d^2 - b^4*d^2))*d^2 + 3*sqrt(1/3)*(a^2*b^2 -
 b^4)*d^2*sqrt(-((a^4*b^4 - 2*a^2*b^6 + b^8)*(a^4*(-I*sqrt(3) + 1)/((a^2*b^
2*d^2 - b^4*d^2)^2*(-1/1458*a^4/(a^2*b^8*d^6 - b^10*d^6) - 1/729*a^6/(a^2*b
^2*d^2 - b^4*d^2)^3 + 1/1458*(a^2 + b^2)*a^4/((a^2 - b^2)^2*b^8*d^6))^(1/3)
) + 81*(-1/1458*a^4/(a^2*b^8*d^6 - b^10*d^6) - 1/729*a^6/(a^2*b^2*d^2 - b^4
*d^2)^3 + 1/1458*(a^2 + b^2)*a^4/((a^2 - b^2)^2*b^8*d^6))^(1/3)*(I*sqrt(3)
+ 1) + 18*a^2/(a^2*b^2*d^2 - b^4*d^2))^2*d^4 - 972*a^4 - 36*(a^4*b^2 - a^2*
b^4)*(a^4*(-I*sqrt(3) + 1)/((a^2*b^2*d^2 - b^4*d^2)^2*(-1/1458*a^4/(a^2*b^8
*d^6 - b^10*d^6) - 1/729* ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 15.07, size = 665, normalized size = 2.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(8192*a^8*b^5 - 294912*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*
a^2*b^6*d^4 + a^4, d, k)^2*a^6*b^9 + 1548288*root(729*a^2*b^8*d^6 - 729*b^1
0*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^3*a^6*b^10 - 1990656*root(729*a^2*b^8*
d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^4*a^6*b^11 - 7962624*root
(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^5*a^4*b^14 +
 5971968*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)
^5*a^6*b^12 - 65536*a^7*b^6*tan(c/2 + (d*x)/2) + 196608*root(729*a^2*b^8*d^
6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)*a^7*b^7*tan(c/2 + (d*x)/2)
- 294912*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)
^2*a^7*b^8*tan(c/2 + (d*x)/2) - 1769472*root(729*a^2*b^8*d^6 - 729*b^10*d^6
 + 243*a^2*b^6*d^4 + a^4, d, k)^3*a^5*b^11*tan(c/2 + (d*x)/2) + 221184*root
(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^3*a^7*b^9*ta
n(c/2 + (d*x)/2) - 2654208*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^
6*d^4 + a^4, d, k)^4*a^5*b^12*tan(c/2 + (d*x)/2) - 1990656*root(729*a^2*b^8
*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^5*a^5*b^13*tan(c/2 + (d*
x)/2))*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k),
k, 1, 6)/d - 2/(b*d + b*d*tan(c/2 + (d*x)/2)^2)
```

3.191 $\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=240

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{2/3}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,b^{2/3}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}\,b^{2/3}d}$$

```
[Out] 2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^
(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan
(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d/((-1)^(1/3)*
a^(2/3)+b^(2/3))^(1/2)-2/3*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+
1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d/(-(-1)^(2/3)*a^(2/3)
+b^(2/3))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3299, 2739, 632, 210, 212}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1
/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1
)^(2/3)*a^(2/3)) + b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) + (-1)^(2/3)*a
^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^
(1/3)*a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\right.$$

$$=\frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{2/3}}+\frac{\int \frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{2/3}}+\frac{\int \frac{1}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\sin(c}}{3b^{2/3}}$$

$$=\frac{2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{2/3}d}+\frac{2\text{Subst}\left(\int \frac{}{-\sqrt[3]{-1}\sqrt[3]{a}+}\right.}{}$$

$$=-\frac{4\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{2/3}d}-\frac{4\text{Subst}\left(\int \frac{}{-}\right.}{}$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{2/3}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,b^{2/3}d}-\frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.14, size = 231, normalized size = 0.96

$$\frac{i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)-4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+2i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right]}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((I/6)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
 , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2
*b*#1^3 + b*#1^5) & ])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.58, size = 76, normalized size = 0.32

method	result
derivativedivides	$\frac{4\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3d}$
default	$\frac{4\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3d}$
risch	$-\frac{\left(\sum_{\_R=\text{RootOf}\left(4096+\left(729a^2b^4d^6-729b^6d^6\right)\_Z^6+3888b^4d^4\_Z^4-6912b^2d^2\_Z^2\right)}\_R\ln\left(e^{i(dx+c)}+\left(-\frac{243}{1024}b^3d^5a^2+\frac{243}{1024}b^5\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3/d*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R
=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.45, size = 25253, normalized size = 105.22

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2/3)*sqrt(1/6)*sqrt(((a^2 - b^2)*((-I*sqrt(3) + 1)*(1/(a^2*b^2*d
^4 - b^4*d^4) + 1/(a^2*d^2 - b^2*d^2)^2)/(-1/1458/(a^2*b^4*d^6 - b^6*d^6) -
 1/486/((a^2*b^2*d^4 - b^4*d^4)*(a^2*d^2 - b^2*d^2)) - 1/729/(a^2*d^2 - b^2
*d^2)^3 + 1/1458*a^2/((a^2 - b^2)^2*b^4*d^6))^(1/3) + 81*(I*sqrt(3) + 1)*(-
1/1458/(a^2*b^4*d^6 - b^6*d^6) - 1/486/((a^2*b^2*d^4 - b^4*d^4)*(a^2*d^2 -
b^2*d^2)) - 1/729/(a^2*d^2 - b^2*d^2)^3 + 1/1458*a^2/((a^2 - b^2)^2*b^4*d^6
))^(1/3) + 18/(a^2*d^2 - b^2*d^2))*d^2 + 3*sqrt(1/3)*(a^2 - b^2)*d^2*sqrt(-
((a^4*b^2 - 2*a^2*b^4 + b^6)*((-I*sqrt(3) + 1)*(1/(a^2*b^2*d^4 - b^4*d^4) +
 1/(a^2*d^2 - b^2*d^2)^2)/(-1/1458/(a^2*b^4*d^6 - b^6*d^6) - 1/486/((a^2*b^
2*d^4 - b^4*d^4)*(a^2*d^2 - b^2*d^2)) - 1/729/(a^2*d^2 - b^2*d^2)^3 + 1/145
8*a^2/((a^2 - b^2)^2*b^4*d^6))^(1/3) + 81*(I*sqrt(3) + 1)*(-1/1458/(a^2*b^4
*d^6 - b^6*d^6) - 1/486/((a^2*b^2*d^4 - b^4*d^4)*(a^2*d^2 - b^2*d^2)) - 1/7
29/(a^2*d^2 - b^2*d^2)^3 + 1/1458*a^2/((a^2 - b^2)^2*b^4*d^6))^(1/3) + 18/(
a^2*d^2 - b^2*d^2))^2*d^4 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 15.65, size = 590, normalized size = 2.46

$$\frac{\sum_{k=1}^{6}\ln\left(-\frac{8192\,a^4\,\left(-729\,a^2\,b^3-81\,a^2\,b^2\,\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)+243\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^4+324\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^3\,\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)+162\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2\,{\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)}^2+36\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b\,{\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)}^3+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,{\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)}^4+972\,b^5+324\,b^4\,\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)-216\,b^3\,{\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)}^2-72\,b^2\,{\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)}^3+12\,b\,{\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)}^4+4\,{\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)}^5\right)}{{\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,\left(a^2-b^2\right),d,k\right)}^5}\right)\,\mathrm{root}\left(729\,a^2\,b^4\,d^6-729\,b^6\,d^6+243\,b^4\,d^4-27\,b^2\,d^2+1,d,k\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(-(8192*a^4*(12*b*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(
a^2 - b^2), d, k)^4 + 324*b^4*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4
*(a^2 - b^2), d, k) + 972*b^5 + 4*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729
*b^4*(a^2 - b^2), d, k)^5 - 729*a^2*b^3 - 72*b^2*root(d^6 - 27*b^2*d^4 + 24
3*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^3 - 216*b^3*root(d^6 - 27*b^2*d^4 +
243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^2 - 81*a^2*b^2*root(d^6 - 27*b^2*d
^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k) + 243*a*b^4*tan(c/2 + (d*x)/2
) + 3*a*tan(c/2 + (d*x)/2)*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a
^2 - b^2), d, k)^4 + 162*a*b^2*tan(c/2 + (d*x)/2)*root(d^6 - 27*b^2*d^4 + 2
43*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^2 + 36*a*b*tan(c/2 + (d*x)/2)*root(
d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^3 + 324*a*tan(c
/2 + (d*x)/2)*b^3*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2)
, d, k)))/root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^
5)*root(729*a^2*b^4*d^6 - 729*b^6*d^6 + 243*b^4*d^4 - 27*b^2*d^2 + 1, d, k)
, k, 1, 6)/d
```

3.192 $\int \frac{1}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}$$

```
[Out] 2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^
(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(
1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+(-1)
^(1/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan
(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)-(-1
)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3292, 2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/
3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a
^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/
3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/
3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)}\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,\mathrm{s}}}{3a^{2/3}}$$

$$= -\frac{2\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}\,x-\sqrt[3]{a}\,x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}-\frac{2\mathrm{Subst}\left(\int \frac{}{-\sqrt[3]{a}+2\sqrt[3]{-}}\right.}{}$$

$$= \frac{4\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}+\frac{4\mathrm{Subst}\left(\int \frac{}{-4}\right.}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2\,\mathrm{ta}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.09, size = 126, normalized size = 0.51

$$-\frac{2i\mathrm{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (((-2*I)/3)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#
1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.52, size = 83, normalized size = 0.34

method	result
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(729a^6d^6-729a^4b^2d^6)\_Z^6+243a^4d^4\_Z^4+27a^2d^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(-\frac{486d^5a^6}{b}+48\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.42, size = 25429, normalized size = 103.79

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2/3)*sqrt(1/6)*sqrt(-((a^2 - b^2)*((-I*sqrt(3) + 1)*(1/(a^4*d^4 -
 a^2*b^2*d^4) - 1/(a^2*d^2 - b^2*d^2)^2)/(-1/1458/(a^6*d^6 - a^4*b^2*d^6) +
 1/486/((a^4*d^4 - a^2*b^2*d^4)*(a^2*d^2 - b^2*d^2)) - 1/729/(a^2*d^2 - b^2
*d^2)^3 + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) - 81*(I*sqrt(3) + 1)*(-
1/1458/(a^6*d^6 - a^4*b^2*d^6) + 1/486/((a^4*d^4 - a^2*b^2*d^4)*(a^2*d^2 -
b^2*d^2)) - 1/729/(a^2*d^2 - b^2*d^2)^3 + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6
))^(1/3) - 18/(a^2*d^2 - b^2*d^2))*d^2 - 3*sqrt(1/3)*(a^2 - b^2)*d^2*sqrt(-
((a^6 - 2*a^4*b^2 + a^2*b^4)*((-I*sqrt(3) + 1)*(1/(a^4*d^4 - a^2*b^2*d^4) -
 1/(a^2*d^2 - b^2*d^2)^2)/(-1/1458/(a^6*d^6 - a^4*b^2*d^6) + 1/486/((a^4*d^
4 - a^2*b^2*d^4)*(a^2*d^2 - b^2*d^2)) - 1/729/(a^2*d^2 - b^2*d^2)^3 + 1/145
8*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) - 81*(I*sqrt(3) + 1)*(-1/1458/(a^6*d^6
 - a^4*b^2*d^6) + 1/486/((a^4*d^4 - a^2*b^2*d^4)*(a^2*d^2 - b^2*d^2)) - 1/7
29/(a^2*d^2 - b^2*d^2)^3 + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) - 18/(
a^2*d^2 - b^2*d^2))^2*d^4 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin^3{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 15.88, size = 609, normalized size = 2.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(-(8192*a*b^3*(972*a^3*b^2 - 729*a^5 - 9*a*root(d^6 + 27*a^2*d^4
+ 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 162*a^3*root(d^6 + 27*a^2*d^
```

```
4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2 - 4*tan(c/2 + (d*x)/2)*root(
d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5 + 243*a^4*b*t
an(c/2 + (d*x)/2) - 324*tan(c/2 + (d*x)/2)*a^4*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 24*b*tan(c/2 + (d*x)/2)*root(d^6 + 2
7*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 72*a^2*tan(c/2 + (
d*x)/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3
+ 36*a*b*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3
 + 243*b*a^3*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d,
k) + 648*tan(c/2 + (d*x)/2)*a^2*b^2*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 7
29*a^4*(a^2 - b^2), d, k) + 216*a^2*b*tan(c/2 + (d*x)/2)*root(d^6 + 27*a^2*
d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^6 + 27*a^2*d^4 +
243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*b^2*d^6 - 729*a^6*
d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)/d
```

3.193 $\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}+\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}-b^{2/3}}\,d}-\frac{2\sqrt[3]{-1}\,b^{2/3}\tan^{-1}}{3a^{4/3}}$$

```
[Out] -cot(d*x+c)/a/d+2/3*b^(2/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^
(2/3)-b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*(-1)^(1/3)*b^(2
/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1
/3)*b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-1)^(
2/3)*b^(2/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3
)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3299, 3852, 8, 2739, 632, 210}

$$-\frac{2(-1)^{2/3}b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}\,b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*(-1)^(2/3)*b^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - (-1)^(2/3
)*b^(2/3)]*d) + (2*b^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt
[a^(2/3) - b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*(-1)^(1/3)
*b^(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3
) + (-1)^(1/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d)
- Cot[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\csc^2(c+dx)}{a} - \frac{b\sin(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc^2(c+dx)\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a}$$

$$= -\frac{b\int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} + \frac{\cdots}{3\sqrt[3]{\cdots}}\right.}{a}$$

$$= -\frac{\cot(c+dx)}{ad} + \frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}\,b^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+\cdots}}{3a^{4/3}}$$

$$= -\frac{\cot(c+dx)}{ad} + \frac{\left(2b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}\,x+\sqrt[3]{a}\,x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{4/3}d} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{\left(4b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+d\cdots\right.\right.}{3a^{4/3}d}$$

$$= -\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\cdots\right.}{\sqrt{a^{2/3}-b\cdots}}\right.}{3a^{4/3}\sqrt{a^{2/3}-b^{2/3}}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.20, size = 196, normalized size = 0.70

$$\frac{-3\cot\left(\frac{1}{2}(c+dx)\right)+2b\text{RootSum}\left[-b+3b\#1^2-8ia\#1^3-3b\#1^4+b\#1^6\&, \frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]+3\tan\left(\frac{1}{2}(c+dx)\right)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-3*Cot[(c + d*x)/2] + 2*b*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4
+ b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4
) & ] + 3*Tan[(c + d*x)/2])/(6*a*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.76, size = 114, normalized size = 0.41

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right.}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.}{3a}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right.}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.}{3a}}{d}$
risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}-4\left(\sum_{\_R=\mathrm{RootOf}\left(\left(2985984a^{10}d^6-2985984a^8b^2d^6\right)\_Z^6+62208a^6b^2d^4\_Z^4+b^4\right)}\_R\ln\left(e^{i(d}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tan(1/2*d*x+1/2*c)-1/2/a/tan(1/2*d*x+1/2*c)-2/3/a*b*sum((_R^3+_R
)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*
a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] ((a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2*a*d*cos(2*d*x + 2*c)
+ a*d)*integrate(-4*(3*b^2*cos(4*d*x + 4*c)^2 + 3*b^2*cos(2*d*x + 2*c)^2 +
3*b^2*sin(4*d*x + 4*c)^2 + 8*a*b*cos(2*d*x + 2*c)*sin(3*d*x + 3*c) - 8*a*b*
cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 3*b^2*sin(2*d*x + 2*c)^2 - b^2*cos(2*d*
x + 2*c) - (b^2*cos(4*d*x + 4*c) - b^2*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) -
 (6*b^2*cos(2*d*x + 2*c) + 8*a*b*sin(3*d*x + 3*c) - b^2)*cos(4*d*x + 4*c) -
 (b^2*sin(4*d*x + 4*c) - b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 2*(4*a*b*
cos(3*d*x + 3*c) - 3*b^2*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))/(a*b^2*cos(6*d
*x + 6*c)^2 + 9*a*b^2*cos(4*d*x + 4*c)^2 + 64*a^3*cos(3*d*x + 3*c)^2 + 9*a*
b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(6*d*x + 6*c)^2 + 9*a*b^2*sin(4*d*x + 4*c
)^2 + 64*a^3*sin(3*d*x + 3*c)^2 - 48*a^2*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c
) + 9*a*b^2*sin(2*d*x + 2*c)^2 - 6*a*b^2*cos(2*d*x + 2*c) + a*b^2 - 2*(3*a*
b^2*cos(4*d*x + 4*c) - 3*a*b^2*cos(2*d*x + 2*c) - 8*a^2*b*sin(3*d*x + 3*c)
+ a*b^2)*cos(6*d*x + 6*c) - 6*(3*a*b^2*cos(2*d*x + 2*c) + 8*a^2*b*sin(3*d*x
 + 3*c) - a*b^2)*cos(4*d*x + 4*c) - 2*(8*a^2*b*cos(3*d*x + 3*c) + 3*a*b^2*s
```

```
in(4*d*x + 4*c) - 3*a*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*a^2*b*c
os(3*d*x + 3*c) - 3*a*b^2*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*(3*a^2*b*
cos(2*d*x + 2*c) - a^2*b)*sin(3*d*x + 3*c)), x) - 2*sin(2*d*x + 2*c))/(a*d*
cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2*a*d*cos(2*d*x + 2*c) + a*d)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.55, size = 21243, normalized size = 75.60

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(2/3)*sqrt(1/6)*a*d*sqrt(((a^4 - a^2*b^2)*(b^4*(-I*sqrt(3) + 1)/
((a^4*d^2 - a^2*b^2*d^2)^2*(-1/1458*b^4/(a^10*d^6 - a^8*b^2*d^6) - 1/729*b^
6/(a^4*d^2 - a^2*b^2*d^2)^3 + 1/1458*(a^2 + b^2)*b^4/((a^2 - b^2)^2*a^8*d^6
))^(1/3)) + 81*(-1/1458*b^4/(a^10*d^6 - a^8*b^2*d^6) - 1/729*b^6/(a^4*d^2 -
 a^2*b^2*d^2)^3 + 1/1458*(a^2 + b^2)*b^4/((a^2 - b^2)^2*a^8*d^6))^(1/3)*(I*
sqrt(3) + 1) + 18*b^2/(a^4*d^2 - a^2*b^2*d^2))*d^2 + 3*sqrt(1/3)*(a^4 - a^2
*b^2)*d^2*sqrt(-((a^8 - 2*a^6*b^2 + a^4*b^4)*(b^4*(-I*sqrt(3) + 1)/((a^4*d^
2 - a^2*b^2*d^2)^2*(-1/1458*b^4/(a^10*d^6 - a^8*b^2*d^6) - 1/729*b^6/(a^4*d
^2 - a^2*b^2*d^2)^3 + 1/1458*(a^2 + b^2)*b^4/((a^2 - b^2)^2*a^8*d^6))^(1/3)
) + 81*(-1/1458*b^4/(a^10*d^6 - a^8*b^2*d^6) - 1/729*b^6/(a^4*d^2 - a^2*b^2
*d^2)^3 + 1/1458*(a^2 + b^2)*b^4/((a^2 - b^2)^2*a^8*d^6))^(1/3)*(I*sqrt(3)
+ 1) + 18*b^2/(a^4*d^2 - a^2*b^2*d^2))^2*d^4 - 972*b^4 - 36*(a^4*b^2 - a^2*
b^4)*(b^4*(-I*sqrt(3) + 1)/((a^4*d^2 - a^2*b^2*d^2)^2*(-1/1458*b^4/(a^10*d^
6 - a^8*b^2*d^6) - 1/729* ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 14.42, size = 697, normalized size = 2.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^3)),x)
[Out] (symsum(log(8192*a^7*b^6 - 294912*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243
*a^6*b^2*d^4 - b^4, d, k)^2*a^9*b^6 + 1548288*root(729*a^8*b^2*d^6 - 729*a^
10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^3*a^11*b^5 - 1990656*root(729*a^8*b^2
*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^4*a^13*b^4 - 7962624*roo
t(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^5*a^13*b^5
+ 5971968*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k
)^5*a^15*b^3 - 65536*a^6*b^7*tan(c/2 + (d*x)/2) + 196608*root(729*a^8*b^2*d
^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)*a^8*b^6*tan(c/2 + (d*x)/2)
 - 294912*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k
)^2*a^10*b^5*tan(c/2 + (d*x)/2) - 1769472*root(729*a^8*b^2*d^6 - 729*a^10*d
^6 - 243*a^6*b^2*d^4 - b^4, d, k)^3*a^10*b^6*tan(c/2 + (d*x)/2) + 221184*ro
ot(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^3*a^12*b^4
*tan(c/2 + (d*x)/2) - 2654208*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6
*b^2*d^4 - b^4, d, k)^4*a^12*b^5*tan(c/2 + (d*x)/2) - 1990656*root(729*a^8*
b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^5*a^14*b^4*tan(c/2 +
(d*x)/2))*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k
), k, 1, 6) - 1/(2*a*tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)/(2*a))/d
```

3.194 $\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=296

$$\frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3a^2\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,d}-\cdots 2$$

```
[Out] b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+2/3*b^(4/3)
*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^2/d
/(a^(2/3)-b^(2/3))^(1/2)-2/3*b^(4/3)*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*ta
n(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2))/a^2/d/((-1)^(1/3)*a^(
2/3)+b^(2/3))^(1/2)-2/3*b^(4/3)*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2
*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/a^2/d/(-(-1)^(2/3)*a^(2/3
)+b^(2/3))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3299, 3855, 3852, 2739, 632, 210, 212}

$$\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2b^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3a^2d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*b^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/
3)]])/(3*a^2*Sqrt[a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d)
 - (2*b^(4/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2]])/Sqrt[
-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2
/3)]*d) - (2*b^(4/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a^2*Sqrt[(-1)^(1/3)*a^(2/3) + b^(
2/3)]*d) - Cot[c + d*x]/(a*d) - Cot[c + d*x]^3/(3*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{b\csc(c+dx)}{a^2} + \frac{\csc^4(c+dx)}{a} + \frac{b^2\sin^2(c+dx)}{a^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc^4(c+dx)\,dx}{a} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{b^2\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\,\mathrm{si}\right.}\right.}{a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{b^{4/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\left(2b^{4/3}\right)\mathrm{Subst}\left(\int \frac{}{\sqrt[3]{a}+2\sqrt[3]{}}\right.}{}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} - \frac{\left(4b^{4/3}\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2}\right.}\right.}{}$$

$$= \frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{a^{2/3}-b^{2/3}}\,d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{}}{\sqrt{}}\right.}{3a^2\sqrt{-(-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.51, size = 333, normalized size = 1.12

$$\frac{-8a\cot\left(\frac{1}{2}(c+dx)\right) + 24b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 24b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4ib^2\,\mathrm{RootSum}\left[-b+3b\#1^2-8ia\#1^3-3b\#1^4+b\#1^6\&, \frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right) - i\log\left(1-2\cos(c+dx)\#1+\#1^2\right) - 4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2 + 2i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2 + 2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4 - i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right] + 8a\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right) - \frac{1}{2}a\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) + 8a\tan\left(\frac{1}{2}(c+dx)\right)}{24a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-8*a*Cot[(c + d*x)/2] + 24*b*Log[Cos[(c + d*x)/2]] - 24*b*Log[Sin[(c + d*x
)/2]] + (4*I)*b^2*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6
& , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1
)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 -
2*b*#1^3 + b*#1^5) & ] + 8*a*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (a*Csc[(c
+ d*x)/2]^4*Sin[c + d*x])/2 + 8*a*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.90, size = 162, normalized size = 0.55

method	result
derivativedivides	$\frac{\frac{\left(\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{4b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4\right.}\right.}{}}{d}$
default	$\frac{\frac{\left(\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{4b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4\right.}\right.}{}}{d}$
risch	$\frac{4i\left(3\,\mathrm{e}^{2i(dx+c)}-1\right)}{3da\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{a^2d}+16\left(\sum_{\_R=\mathrm{RootOf}\left(\left(12230590464a^{14}d^6-12230590464a^{12}b^2d^6\right)\_Z^6+1592\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a*(1/3*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c))-1/24/a/tan(1/2*d
*x+1/2*c)^3-3/8/a/tan(1/2*d*x+1/2*c)-1/a^2*b*ln(tan(1/2*d*x+1/2*c))+4/3*b^2
/a^2*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=
RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains complex when optimal does not.
time = 4.09, size = 29423, normalized size = 99.40

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(2/3)*sqrt(1/6)*(a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(54*b^4 - (
a^6 - a^4*b^2)*(18*b^4/(a^6*d^2 - a^4*b^2*d^2) + (b^8/(a^6*d^2 - a^4*b^2*d^
```

```
2)^2 + b^6/(a^10*d^4 - a^8*b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^12/(a^6*d^2
 - a^4*b^2*d^2)^3 - 1/486*b^10/((a^10*d^4 - a^8*b^2*d^4)*(a^6*d^2 - a^4*b^2
*d^2)) - 1/1458*b^8/(a^14*d^6 - a^12*b^2*d^6) + 1/1458*b^8/((a^2 - b^2)^2*a
^10*d^6))^(1/3) + 81*(-1/729*b^12/(a^6*d^2 - a^4*b^2*d^2)^3 - 1/486*b^10/((
a^10*d^4 - a^8*b^2*d^4)*(a^6*d^2 - a^4*b^2*d^2)) - 1/1458*b^8/(a^14*d^6 - a
^12*b^2*d^6) + 1/1458*b^8/((a^2 - b^2)^2*a^10*d^6))^(1/3)*(I*sqrt(3) + 1))*
d^2 + 3*sqrt(1/3)*(a^6 - a^4*b^2)*d^2*sqrt((1296*a^2*b^6 - 324*b^8 - (a^12
- 2*a^10*b^2 + a^8*b^4)*(18*b^4/(a^6*d^2 - a^4*b^2*d^2) + (b^8/(a^6*d^2 - a
^4*b^2*d^2)^2 + b^6/(a^10*d^4 - a^8*b^2*d^4))*(-I*sqrt(3) + 1)/(-1/729*b^12
/(a^6*d^2 - a^4*b^2*d^2)^3 - 1/486*b^10/((a^10*d^4 - a^8*b^2*d^4)*(a^6*d^2
- a^4*b^2*d^2)) - 1/1458*b^8/(a^14*d^6 - a^12*b^2*d^6) + 1/1458*b^8/((a^2 -
 b^2)^2*a^10*d^6))^(1/3)  ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 16.38, size = 1503, normalized size = 5.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] symsum(log((98304*b^11 + 589824*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*
a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)*a^2*b^10 - 98304*root(729*a^12*b^
2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^2*a^4*
b^9 - 5898240*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a
```

```
^4*b^6*z^2 - b^8, z, k)^3*a^6*b^8 - 7962624*root(729*a^12*b^2*z^6 - 729*a^1
4*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^4*a^8*b^7 - 663552*ro
ot(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8
, z, k)^4*a^10*b^5 + 5308416*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8
*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^5*a^10*b^6 - 10616832*root(729*a^12*
b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^5*a^
12*b^4 + 7962624*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 2
7*a^4*b^6*z^2 - b^8, z, k)^6*a^12*b^5 - 9953280*root(729*a^12*b^2*z^6 - 729
*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^6*a^14*b^3 + 2457
6*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 -
 b^8, z, k)*a^3*b^9*tan(c/2 + (d*x)/2) - 3145728*root(729*a^12*b^2*z^6 - 72
9*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^2*a^3*b^10*tan(c
/2 + (d*x)/2) + 466944*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z
^4 + 27*a^4*b^6*z^2 - b^8, z, k)^2*a^5*b^8*tan(c/2 + (d*x)/2) + 18874368*ro
ot(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8
, z, k)^3*a^5*b^9*tan(c/2 + (d*x)/2) + 3981312*root(729*a^12*b^2*z^6 - 729*
a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^3*a^7*b^7*tan(c/2
+ (d*x)/2) + 56623104*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^
4 + 27*a^4*b^6*z^2 - b^8, z, k)^4*a^7*b^8*tan(c/2 + (d*x)/2) + 20791296*roo
t(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8,
 z, k)^4*a^9*b^6*tan(c/2 + (d*x)/2) - 84934656*root(729*a^12*b^2*z^6 - 729*
a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^5*a^9*b^7*tan(c/2
+ (d*x)/2) + 78962688*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^
4 + 27*a^4*b^6*z^2 - b^8, z, k)^5*a^11*b^5*tan(c/2 + (d*x)/2) - 254803968*r
oot(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^
8, z, k)^6*a^11*b^6*tan(c/2 + (d*x)/2) + 252813312*root(729*a^12*b^2*z^6 -
729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^6*a^13*b^4*tan
(c/2 + (d*x)/2) - 1048576*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^
4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)*a*b^11*tan(c/2 + (d*x)/2))/a^6)*root(72
9*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z,
k), k, 1, 6)/d - (a*((3*cot(c/2 + (d*x)/2))/8 - (3*tan(c/2 + (d*x)/2))/8 +
cot(c/2 + (d*x)/2)^3/24 - tan(c/2 + (d*x)/2)^3/24) + b*log(tan(c/2 + (d*x)/
2)))/(a^2*d)
```

3.195 $\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=177

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{9/4}d}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{9/4}d}+\frac{(a+b)\cos(c+dx)}{b^2d}-\frac{2\cos^3(c+dx)}{3bd}+\frac{\cos^5}{}$$

```
[Out] (a+b)*cos(d*x+c)/b^2/d-2/3*cos(d*x+c)^3/b/d+1/5*cos(d*x+c)^5/b/d-1/2*a^(3/2
)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(9/4)/d/(a^(1/2)-b^(
1/2))^(1/2)-1/2*a^(3/2)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))
/b^(9/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1184, 1107, 211, 214}

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{(a+b)\cos(c+dx)}{b^2d}+\frac{\cos^5(c+dx)}{5bd}-\frac{2\cos^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*(a^(3/2)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(Sqrt
[Sqrt[a] - Sqrt[b]]*b^(9/4)*d) - (a^(3/2)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sq
rt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(9/4)*d) + ((a + b)*Co
s[c + d*x])/(b^2*d) - (2*Cos[c + d*x]^3)/(3*b*d) + Cos[c + d*x]^5/(5*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
```

```
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{a+b}{b^2}+\frac{2x^2}{b}-\frac{x^4}{b}+\frac{a^2}{b^2(a-b+2bx^2-bx^4)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{(a+b)\cos(c+dx)}{b^2d} - \frac{2\cos^3(c+dx)}{3bd} + \frac{\cos^5(c+dx)}{5bd} - \frac{a^2\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\, d\right.}{b^2d}$$

$$= \frac{(a+b)\cos(c+dx)}{b^2d} - \frac{2\cos^3(c+dx)}{3bd} + \frac{\cos^5(c+dx)}{5bd} + \frac{a^{3/2}\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+}\right.}{2b^3}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{9/4}d} - \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{9/4}d} + \frac{(a+b)\cos}{b^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.34, size = 228, normalized size = 1.29

$$\frac{\cos(c+dx)(120a+89b-28b\cos(2(c+dx))+3b\cos(4(c+dx)))+60ia^2\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^3-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^3}{-b-8a\#1^2+3b\#1^2-3b\#1^4+b\#1^6}\&\right]}{120b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (Cos[c + d*x]*(120*a + 89*b - 28*b*Cos[2*(c + d*x)] + 3*b*Cos[4*(c + d*x)])
 + (60*I)*a^2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1
^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^
4 + b*#1^6) & ])/(120*b^2*d)
```

Maple [A]
time = 0.48, size = 137, normalized size = 0.77

method	result
derivativedivides	$\frac{\frac{\frac{\left(\cos^5(dx+c)\right)b}{5}-\frac{2\left(\cos^3(dx+c)\right)b}{3}+a\cos(dx+c)+\cos(dx+c)b}{b^2}+\frac{a^2\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{b\cos(dx-}{\sqrt{\left(\sqrt{ab}\right.}}\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}\right.}}\right.}{b}}{d}$
default	$\frac{\frac{\frac{\left(\cos^5(dx+c)\right)b}{5}-\frac{2\left(\cos^3(dx+c)\right)b}{3}+a\cos(dx+c)+\cos(dx+c)b}{b^2}+\frac{a^2\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{b\cos(dx-}{\sqrt{\left(\sqrt{ab}\right.}}\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}\right.}}\right.}{b}}{d}$
risch	$\frac{a\,e^{i(dx+c)}}{2b^2d}+\frac{5\,e^{i(dx+c)}}{16bd}+\frac{a\,e^{-i(dx+c)}}{2b^2d}+\frac{5\,e^{-i(dx+c)}}{16bd}-\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a\,b^9d^4-b^{10}d^4\right)\_Z^4-32768a^3b^5d^2\_Z^2-268435\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*(1/5*cos(d*x+c)^5*b-2/3*cos(d*x+c)^3*b+a*cos(d*x+c)+cos(d*x+c)*b
)+a^2/b*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((
(a*b)^(1/2)+b)*b)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b
*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/240*(240*b^2*d*integrate(8*(4*a^2*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2
*(8*a^3 - 3*a^2*b)*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a^3 - 3*a^2*b)*
cos(4*d*x + 4*c)*sin(3*d*x + 3*c) - (a^2*b*sin(5*d*x + 5*c) - a^2*b*sin(3*d
*x + 3*c))*cos(8*d*x + 8*c) + 4*(a^2*b*sin(5*d*x + 5*c) - a^2*b*sin(3*d*x +
 3*c))*cos(6*d*x + 6*c) - 2*(2*a^2*b*sin(2*d*x + 2*c) + (8*a^3 - 3*a^2*b)*s
in(4*d*x + 4*c))*cos(5*d*x + 5*c) + (a^2*b*cos(5*d*x + 5*c) - a^2*b*cos(3*d
*x + 3*c))*sin(8*d*x + 8*c) - 4*(a^2*b*cos(5*d*x + 5*c) - a^2*b*cos(3*d*x +
 3*c))*sin(6*d*x + 6*c) + (4*a^2*b*cos(2*d*x + 2*c) - a^2*b + 2*(8*a^3 - 3*
a^2*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) - (4*a^2*b*cos(2*d*x + 2*c) - a^2
*b)*sin(3*d*x + 3*c))/(b^4*cos(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 +
 16*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*
c)^2 + 16*b^4*sin(2*d*x + 2*c)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2
*b^2 - 48*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*
b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) - 2*(4*b^4*cos(6*d*x + 6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b
^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c)
- b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^3
 - 3*b^4 - 4*(8*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^
4*sin(6*d*x + 6*c) + 2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x +
 4*c))*sin(8*d*x + 8*c) + 16*(2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*si
n(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + 3*b*cos(5*d*x + 5*c) - 25*b*cos(3*d
*x + 3*c) + 30*(8*a + 5*b)*cos(d*x + c))/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(133) = 266$.
time = 0.48, size = 872, normalized size = 4.93

$$\frac{12\,b\cos\left(dx+c\right)^{5}-15\,b^{2}d\sqrt{-\frac{\left(ab^{4}-b^{5}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{2}+a^{3}}{\left(ab^{4}-b^{5}\right)d^{2}}}\log\left(a^{5}\cos\left(dx+c\right)+\left(a^{4}b^{2}d-\left(ab^{7}-b^{8}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{3}\right)\sqrt{-\frac{\left(ab^{4}-b^{5}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{2}+a^{3}}{\left(ab^{4}-b^{5}\right)d^{2}}}\right)+15\,b^{2}d\sqrt{\frac{\left(ab^{4}-b^{5}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{2}-a^{3}}{\left(ab^{4}-b^{5}\right)d^{2}}}\log\left(a^{5}\cos\left(dx+c\right)-\left(a^{4}b^{2}d+\left(ab^{7}-b^{8}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{3}\right)\sqrt{\frac{\left(ab^{4}-b^{5}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{2}-a^{3}}{\left(ab^{4}-b^{5}\right)d^{2}}}\right)+15\,b^{2}d\sqrt{-\frac{\left(ab^{4}-b^{5}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{2}+a^{3}}{\left(ab^{4}-b^{5}\right)d^{2}}}\log\left(-a^{5}\cos\left(dx+c\right)+\left(a^{4}b^{2}d-\left(ab^{7}-b^{8}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{3}\right)\sqrt{-\frac{\left(ab^{4}-b^{5}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{2}+a^{3}}{\left(ab^{4}-b^{5}\right)d^{2}}}\right)-15\,b^{2}d\sqrt{\frac{\left(ab^{4}-b^{5}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{2}-a^{3}}{\left(ab^{4}-b^{5}\right)d^{2}}}\log\left(-a^{5}\cos\left(dx+c\right)-\left(a^{4}b^{2}d+\left(ab^{7}-b^{8}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{3}\right)\sqrt{\frac{\left(ab^{4}-b^{5}\right)\sqrt{\frac{a^{7}}{\left(a^{2}b^{9}-2\,ab^{10}+b^{11}\right)d^{4}}}\,d^{2}-a^{3}}{\left(ab^{4}-b^{5}\right)d^{2}}}\right)-40\,b\cos\left(dx+c\right)^{3}+60\,\left(a+b\right)\cos\left(dx+c\right)}{60\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b*cos(d*x + c)^5 - 15*b^2*d*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^
9 - 2*a*b^10 + b^11)*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))*log(a^5*cos(d*x
+ c) + (a^4*b^2*d - (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4
))*d^3)*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^
2 + a^3)/((a*b^4 - b^5)*d^2))) + 15*b^2*d*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^
```

```
2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))*log(a^5*cos(
d*x + c) - (a^4*b^2*d + (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)
*d^4))*d^3)*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))
*d^2 - a^3)/((a*b^4 - b^5)*d^2))) + 15*b^2*d*sqrt(-((a*b^4 - b^5)*sqrt(a^7/
((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))*log(-a^5
*cos(d*x + c) + (a^4*b^2*d - (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 +
b^11)*d^4))*d^3)*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)
*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))) - 15*b^2*d*sqrt(((a*b^4 - b^5)*sqrt
(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))*log
(-a^5*cos(d*x + c) - (a^4*b^2*d + (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^
10 + b^11)*d^4))*d^3)*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b
^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))) - 40*b*cos(d*x + c)^3 + 60*(a +
 b)*cos(d*x + c))/(b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[-36,13
]Warning, need t
```

Mupad [**B**]
time = 14.44, size = 1067, normalized size = 6.03

$$\frac{\cos(c+d\,x)^5}{5\,b\,d}-\frac{2\cos(c+d\,x)^3}{3\,b\,d}+\frac{\cos(c+d\,x)\,\left(\frac{a-b}{b^2}+\frac{2}{b}\right)}{d}+\frac{\mathrm{atan}\left(-\frac{a^4\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^7\,b^9}}{16\,(a\,b^9-b^{10})}-\frac{a^3\,b^5}{16\,(a\,b^9-b^{10})}}\,8{}\mathrm{i}}{\frac{2\,a^6\,b^7}{a\,b^9-b^{10}}+\frac{2\,a^3\,b^2\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}}+\frac{a^4\,b^9\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^7\,b^9}}{16\,(a\,b^9-b^{10})}-\frac{a^3\,b^5}{16\,(a\,b^9-b^{10})}}\,8{}\mathrm{i}}{\frac{2\,a^6\,b^{16}}{a\,b^9-b^{10}}-\frac{2\,a^7\,b^{15}}{a\,b^9-b^{10}}+\frac{2\,a^3\,b^{11}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}-\frac{2\,a^4\,b^{10}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}}+\frac{a\,b^4\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^7\,b^9}}{16\,(a\,b^9-b^{10})}-\frac{a^3\,b^5}{16\,(a\,b^9-b^{10})}}\,\sqrt{a^7\,b^9}\,8{}\mathrm{i}}{\frac{2\,a^6\,b^{16}}{a\,b^9-b^{10}}-\frac{2\,a^7\,b^{15}}{a\,b^9-b^{10}}+\frac{2\,a^3\,b^{11}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}-\frac{2\,a^4\,b^{10}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}}\right)\,\sqrt{-\frac{\sqrt{a^7\,b^9}+a^3\,b^5}{16\,(a\,b^9-b^{10})}}\,2{}\mathrm{i}}{d}-\frac{\mathrm{atan}\left(\frac{a^4\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a^7\,b^9}}{16\,(a\,b^9-b^{10})}-\frac{a^3\,b^5}{16\,(a\,b^9-b^{10})}}\,8{}\mathrm{i}}{\frac{2\,a^6\,b^7}{a\,b^9-b^{10}}-\frac{2\,a^3\,b^2\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}}-\frac{a^4\,b^9\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a^7\,b^9}}{16\,(a\,b^9-b^{10})}-\frac{a^3\,b^5}{16\,(a\,b^9-b^{10})}}\,8{}\mathrm{i}}{\frac{2\,a^6\,b^{16}}{a\,b^9-b^{10}}-\frac{2\,a^7\,b^{15}}{a\,b^9-b^{10}}-\frac{2\,a^3\,b^{11}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}+\frac{2\,a^4\,b^{10}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}}+\frac{a\,b^4\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a^7\,b^9}}{16\,(a\,b^9-b^{10})}-\frac{a^3\,b^5}{16\,(a\,b^9-b^{10})}}\,\sqrt{a^7\,b^9}\,8{}\mathrm{i}}{\frac{2\,a^6\,b^{16}}{a\,b^9-b^{10}}-\frac{2\,a^7\,b^{15}}{a\,b^9-b^{10}}-\frac{2\,a^3\,b^{11}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}+\frac{2\,a^4\,b^{10}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}}\right)\,\sqrt{\frac{\sqrt{a^7\,b^9}-a^3\,b^5}{16\,(a\,b^9-b^{10})}}\,2{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((a^4*b^9*cos(c + d*x)*(- (a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3*b
^5)/(16*(a*b^9 - b^10)))^(1/2)*8i)/((2*a^6*b^16)/(a*b^9 - b^10) - (2*a^7*b^
15)/(a*b^9 - b^10) + (2*a^3*b^11*(a^7*b^9)^(1/2))/(a*b^9 - b^10) - (2*a^4*b
^10*(a^7*b^9)^(1/2))/(a*b^9 - b^10)) - (a^4*cos(c + d*x)*(- (a^7*b^9)^(1/2)
/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b^10)))^(1/2)*8i)/((2*a^6*b^7
)/(a*b^9 - b^10) + (2*a^3*b^2*(a^7*b^9)^(1/2))/(a*b^9 - b^10)) + (a*b^4*cos
(c + d*x)*(- (a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b
^10)))^(1/2)*(a^7*b^9)^(1/2)*8i)/((2*a^6*b^16)/(a*b^9 - b^10) - (2*a^7*b^15
)/(a*b^9 - b^10) + (2*a^3*b^11*(a^7*b^9)^(1/2))/(a*b^9 - b^10) - (2*a^4*b^1
0*(a^7*b^9)^(1/2))/(a*b^9 - b^10)))*(-((a^7*b^9)^(1/2) + a^3*b^5)/(16*(a*b^
9 - b^10)))^(1/2)*2i)/d - (atan((a^4*cos(c + d*x)*((a^7*b^9)^(1/2)/(16*(a*b
^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b^10)))^(1/2)*8i)/((2*a^6*b^7)/(a*b^9
- b^10) - (2*a^3*b^2*(a^7*b^9)^(1/2))/(a*b^9 - b^10)) - (a^4*b^9*cos(c + d*
x)*((a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b^10)))^(1
/2)*8i)/((2*a^6*b^16)/(a*b^9 - b^10) - (2*a^7*b^15)/(a*b^9 - b^10) - (2*a^3
*b^11*(a^7*b^9)^(1/2))/(a*b^9 - b^10) + (2*a^4*b^10*(a^7*b^9)^(1/2))/(a*b^9
 - b^10)) + (a*b^4*cos(c + d*x)*((a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3
*b^5)/(16*(a*b^9 - b^10)))^(1/2)*(a^7*b^9)^(1/2)*8i)/((2*a^6*b^16)/(a*b^9 -
 b^10) - (2*a^7*b^15)/(a*b^9 - b^10) - (2*a^3*b^11*(a^7*b^9)^(1/2))/(a*b^9
- b^10) + (2*a^4*b^10*(a^7*b^9)^(1/2))/(a*b^9 - b^10)))*(((a^7*b^9)^(1/2) -
 a^3*b^5)/(16*(a*b^9 - b^10)))^(1/2)*2i)/d - (2*cos(c + d*x)^3)/(3*b*d) + c
os(c + d*x)^5/(5*b*d) + (cos(c + d*x)*((a - b)/b^2 + 2/b))/d
```

3.196 $\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{7/4}d}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{7/4}d}+\frac{\cos(c+dx)}{bd}-\frac{\cos^3(c+dx)}{3bd}$$

```
[Out] cos(d*x+c)/b/d-1/3*cos(d*x+c)^3/b/d-1/2*a*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2
)-b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*a*arctanh(b^(1/4)*c
os(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1184, 1180, 211, 214}

$$-\frac{a\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cos^3(c+dx)}{3bd}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*(a*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(Sqrt[Sqrt[
a] - Sqrt[b]]*b^(7/4)*d) + (a*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)*d) + Cos[c + d*x]/(b*d) - Co
s[c + d*x]^3/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{x^2}{b}+\frac{a-ax^2}{b(a-b+2bx^2-bx^4)}\right)\,dx, x, \cos(c+dx)\right)}{d}\\
&= \frac{\cos(c+dx)}{bd}-\frac{\cos^3(c+dx)}{3bd}-\frac{\text{Subst}\left(\int \frac{a-ax^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{bd}\\
&= \frac{\cos(c+dx)}{bd}-\frac{\cos^3(c+dx)}{3bd}+\frac{a\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2bd}+\\
&= -\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{7/4}d}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{7/4}d}+\frac{\cos(c+dx)}{bd}-
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.22, size = 310, normalized size = 2.09

$$\frac{18\cos(c+dx)-2\cos(3(c+dx))-3ia\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-3i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+3i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{24bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (18*Cos[c + d*x] - 2*Cos[3*(c + d*x)] - (3*I)*a*RootSum[b - 4*b*#1^2 - 16*a
*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#
1^5 + b*#1^7) & ])/(24*b*d)
```

Maple [A]

time = 0.42, size = 110, normalized size = 0.74

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\cos^3(dx+c)\right)}{3}-\cos(dx+c)}{b}-a\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}+\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
default	$\frac{-\frac{\frac{\left(\cos^3(dx+c)\right)}{3}-\cos(dx+c)}{b}-a\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}+\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
risch	$\frac{3\,e^{i(dx+c)}}{8bd}+\frac{3\,e^{-i(dx+c)}}{8bd}+\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a\,b^7d^4-b^8d^4\right)\_Z^4-2048a^2b^4d^2\_Z^2-1048576a^4\right)}\_R\ln\left(e^{2i(dx+c)}+\left(\left(-\frac{i}{16}\right.\right.\right.\right.}{128}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*(1/3*cos(d*x+c)^3-cos(d*x+c))-a*(-1/2/b/(((a*b)^(1/2)+b)*b)^(1/2)
*arctanh(b*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2))+1/2/b/(((a*b)^(1/2)-b)*b)^
(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/12*(12*b*d*integrate(-2*(12*a*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*a*
b*cos(d*x + c)*sin(2*d*x + 2*c) + 4*a*b*cos(2*d*x + 2*c)*sin(d*x + c) - a*b
*sin(d*x + c) + (a*b*sin(7*d*x + 7*c) - 3*a*b*sin(5*d*x + 5*c) + 3*a*b*sin(
3*d*x + 3*c) - a*b*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*a*b*sin(6*d*x + 6*
c) + 2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*cos(7*d*x +
 7*c) + 4*(3*a*b*sin(5*d*x + 5*c) - 3*a*b*sin(3*d*x + 3*c) + a*b*sin(d*x +
c))*cos(6*d*x + 6*c) - 6*(2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*
x + 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a^2 - 3*a*b)*sin(3*d*x + 3*c) - (8*a^2
 - 3*a*b)*sin(d*x + c))*cos(4*d*x + 4*c) - (a*b*cos(7*d*x + 7*c) - 3*a*b*co
s(5*d*x + 5*c) + 3*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*sin(8*d*x + 8*c
) - (4*a*b*cos(6*d*x + 6*c) + 4*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a
*b)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*a*b*cos(5*d*x + 5*c) - 3*a*b*
cos(3*d*x + 3*c) + a*b*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4*a*b*cos(2*d*x
+ 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) + 2*(3*
(8*a^2 - 3*a*b)*cos(3*d*x + 3*c) - (8*a^2 - 3*a*b)*cos(d*x + c))*sin(4*d*x
+ 4*c) - 3*(4*a*b*cos(2*d*x + 2*c) - a*b)*sin(3*d*x + 3*c))/(b^3*cos(8*d*x
+ 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(
8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*
b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*
c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3
*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3
*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x +
 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4
*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x
+ 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*
c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2
*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + c
os(3*d*x + 3*c) - 9*cos(d*x + c))/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 849 vs. 2(110) = 220.
time = 0.48, size = 849, normalized size = 5.74

$$\frac{3\,bd\sqrt{-\frac{(ab^3-b^4)d^2\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}+a^2}{(ab^3-b^4)d^2}}\log\left(a^3\cos(dx+c)+\left(a^2b^2d-(ab^5-b^6)d^3\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}\right)\sqrt{-\frac{(ab^3-b^4)d^2\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}+a^2}{(ab^3-b^4)d^2}}\right)-3\,bd\sqrt{\frac{(ab^3-b^4)d^2\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}-a^2}{(ab^3-b^4)d^2}}\log\left(a^3\cos(dx+c)-\left(a^2b^2d+(ab^5-b^6)d^3\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}\right)\sqrt{\frac{(ab^3-b^4)d^2\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}-a^2}{(ab^3-b^4)d^2}}\right)-3\,bd\sqrt{-\frac{(ab^3-b^4)d^2\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}+a^2}{(ab^3-b^4)d^2}}\log\left(-a^3\cos(dx+c)+\left(a^2b^2d-(ab^5-b^6)d^3\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}\right)\sqrt{-\frac{(ab^3-b^4)d^2\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}+a^2}{(ab^3-b^4)d^2}}\right)+3\,bd\sqrt{\frac{(ab^3-b^4)d^2\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}-a^2}{(ab^3-b^4)d^2}}\log\left(-a^3\cos(dx+c)-\left(a^2b^2d+(ab^5-b^6)d^3\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}\right)\sqrt{\frac{(ab^3-b^4)d^2\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}-a^2}{(ab^3-b^4)d^2}}\right)-4\cos(dx+c)^3+12\cos(dx+c)}{12\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4)) + a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cos(d*x + c) + (a^2*b^2*d - (a*b^5
- b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*
d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2)))
- 3*b*d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) -
 a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cos(d*x + c) - (a^2*b^2*d + (a*b^5 - b^6
)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)*d^2*sq
rt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) - 3*b*
d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)
/((a*b^3 - b^4)*d^2))*log(-a^3*cos(d*x + c) + (a^2*b^2*d - (a*b^5 - b^6)*d^
3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(
a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + 3*b*d*s
qrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a
*b^3 - b^4)*d^2))*log(-a^3*cos(d*x + c) - (a^2*b^2*d + (a*b^5 - b^6)*d^3*sq
rt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/(
(a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) - 4*cos(d*x +
c)^3 + 12*cos(d*x + c))/(b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[76,51]
Warning, need to
```

Mupad [**B**]
time = 14.27, size = 1119, normalized size = 7.56

$$\frac{\cos\left(c+d\,x\right)}{b\,d}-\frac{{\cos\left(c+d\,x\right)}^3}{3\,b\,d}+\frac{\mathrm{atan}\left(-\frac{a^3\,\cos\left(c+d\,x\right)\,\sqrt{-\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,8{}\mathrm{i}}{\frac{2\,a^4}{b^2}+\frac{2\,a^4\,b^6}{a\,b^7-b^8}+\frac{2\,a^2\,b^2\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}}+\frac{a^3\,b^8\,\cos\left(c+d\,x\right)\,\sqrt{-\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,8{}\mathrm{i}}{2\,a^4\,b^6-2\,a^5\,b^5+\frac{2\,a^4\,b^{14}}{a\,b^7-b^8}-\frac{2\,a^5\,b^{13}}{a\,b^7-b^8}+\frac{2\,a^2\,b^{10}\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}-\frac{2\,a^3\,b^9\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}}+\frac{a\,b^4\,\cos\left(c+d\,x\right)\,\sqrt{-\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,\sqrt{a^5\,b^7}\,8{}\mathrm{i}}{2\,a^4\,b^6-2\,a^5\,b^5+\frac{2\,a^4\,b^{14}}{a\,b^7-b^8}-\frac{2\,a^5\,b^{13}}{a\,b^7-b^8}+\frac{2\,a^2\,b^{10}\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}-\frac{2\,a^3\,b^9\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}}\right)\,\sqrt{-\frac{\sqrt{a^5\,b^7}+a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,2{}\mathrm{i}}{d}-\frac{\mathrm{atan}\left(\frac{a^3\,\cos\left(c+d\,x\right)\,\sqrt{\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,8{}\mathrm{i}}{\frac{2\,a^4}{b^2}+\frac{2\,a^4\,b^6}{a\,b^7-b^8}-\frac{2\,a^2\,b^2\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}}-\frac{a^3\,b^8\,\cos\left(c+d\,x\right)\,\sqrt{\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,8{}\mathrm{i}}{2\,a^4\,b^6-2\,a^5\,b^5+\frac{2\,a^4\,b^{14}}{a\,b^7-b^8}-\frac{2\,a^5\,b^{13}}{a\,b^7-b^8}-\frac{2\,a^2\,b^{10}\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}+\frac{2\,a^3\,b^9\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}}+\frac{a\,b^4\,\cos\left(c+d\,x\right)\,\sqrt{\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,\sqrt{a^5\,b^7}\,8{}\mathrm{i}}{2\,a^4\,b^6-2\,a^5\,b^5+\frac{2\,a^4\,b^{14}}{a\,b^7-b^8}-\frac{2\,a^5\,b^{13}}{a\,b^7-b^8}-\frac{2\,a^2\,b^{10}\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}+\frac{2\,a^3\,b^9\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}}\right)\,\sqrt{\frac{\sqrt{a^5\,b^7}-a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,2{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - b*sin(c + d*x)^4),x)
```

```
[Out] cos(c + d*x)/(b*d) - cos(c + d*x)^3/(3*b*d) + (atan((a^3*b^8*cos(c + d*x)*(
- (a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*
8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)/(a*b^7 - b^8) - (2*a^5*b^13)/(a*b
^7 - b^8) + (2*a^2*b^10*(a^5*b^7)^(1/2))/(a*b^7 - b^8) - (2*a^3*b^9*(a^5*b^
7)^(1/2))/(a*b^7 - b^8)) - (a^3*cos(c + d*x)*(- (a^5*b^7)^(1/2)/(16*(a*b^7
- b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*8i)/((2*a^4)/b^2 + (2*a^4*b^6
)/(a*b^7 - b^8) + (2*a^2*b^2*(a^5*b^7)^(1/2))/(a*b^7 - b^8)) + (a*b^4*cos(c
 + d*x)*(- (a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)
))^(1/2)*(a^5*b^7)^(1/2)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)/(a*b^7 -
 b^8) - (2*a^5*b^13)/(a*b^7 - b^8) + (2*a^2*b^10*(a^5*b^7)^(1/2))/(a*b^7 -
b^8) - (2*a^3*b^9*(a^5*b^7)^(1/2))/(a*b^7 - b^8)))*(-((a^5*b^7)^(1/2) + a^2
*b^4)/(16*(a*b^7 - b^8)))^(1/2)*2i)/d - (atan((a^3*cos(c + d*x)*((a^5*b^7)^
(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*8i)/((2*a^4)
/b^2 + (2*a^4*b^6)/(a*b^7 - b^8) - (2*a^2*b^2*(a^5*b^7)^(1/2))/(a*b^7 - b^8
)) - (a^3*b^8*cos(c + d*x)*((a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/
(16*(a*b^7 - b^8)))^(1/2)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)/(a*b^7
- b^8) - (2*a^5*b^13)/(a*b^7 - b^8) - (2*a^2*b^10*(a^5*b^7)^(1/2))/(a*b^7 -
 b^8) + (2*a^3*b^9*(a^5*b^7)^(1/2))/(a*b^7 - b^8)) + (a*b^4*cos(c + d*x)*((
a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*(a^
5*b^7)^(1/2)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)/(a*b^7 - b^8) - (2*a
^5*b^13)/(a*b^7 - b^8) - (2*a^2*b^10*(a^5*b^7)^(1/2))/(a*b^7 - b^8) + (2*a^
3*b^9*(a^5*b^7)^(1/2))/(a*b^7 - b^8)))*(((a^5*b^7)^(1/2) - a^2*b^4)/(16*(a*
b^7 - b^8)))^(1/2)*2i)/d
```

3.197 $\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{5/4}d}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{5/4}d}+\frac{\cos(c+dx)}{bd}$$

```
[Out] cos(d*x+c)/b/d-1/2*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*a^(1/
2)/b^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2
)+b^(1/2))^(1/2))*a^(1/2)/b^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3294, 1184, 1107, 211, 214}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(Sqrt
[Sqrt[a] - Sqrt[b]]*b^(5/4)*d) - (Sqrt[a]*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sq
rt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) + Cos[c + d*x
]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
```

```
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{a}{b(a-b+2bx^2-bx^4)}\right)\,dx, x, \cos(c+dx)\right)}{d}\\
&= \frac{\cos(c+dx)}{bd}-\frac{a\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{bd}\\
&= \frac{\cos(c+dx)}{bd}+\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2\sqrt{b}\,d}-\frac{\sqrt{a}\,\text{Subst}\left(\right.}{}\\
&= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{5/4}d}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{5/4}d}+\frac{\cos(c+dx}{bd}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.17, size = 198, normalized size = 1.43

$$\frac{2\cos(c+dx)+ia\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^3-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^3}{-b-8a\#1^2+3b\#1^2-3b\#1^4+b\#1^6}\&\right]}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (2*Cos[c + d*x] + I*a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^
6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#
1^3 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 -
 3*b*#1^4 + b*#1^6) & ])/(2*b*d)
```

Maple [A]
time = 0.50, size = 99, normalized size = 0.72

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)}{b}+a\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
default	$\frac{\frac{\cos(dx+c)}{b}+a\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
risch	$\frac{e^{i(dx+c)}}{2bd}+\frac{e^{-i(dx+c)}}{2bd}-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a\,b^5d^4-b^6d^4\right)\_Z^4-128a\,d^2\_Z^2b^3-4096a^2\right)}\_R\ln\left(e^{2i(dx+c)}+\left(\left(-\frac{ib^4d^3}{256a}+\frac{ib^5}{256}\right.\right.\right.\right.}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b*cos(d*x+c)+a*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b
*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^
(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

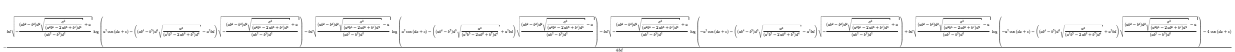
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] (b*d*integrate(8*(4*a*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2*(8*a^2 - 3*a*
b)*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c)*s
in(3*d*x + 3*c) - (a*b*sin(5*d*x + 5*c) - a*b*sin(3*d*x + 3*c))*cos(8*d*x +
 8*c) + 4*(a*b*sin(5*d*x + 5*c) - a*b*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) -
2*(2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*cos(5*d*x + 5
*c) + (a*b*cos(5*d*x + 5*c) - a*b*cos(3*d*x + 3*c))*sin(8*d*x + 8*c) - 4*(a
*b*cos(5*d*x + 5*c) - a*b*cos(3*d*x + 3*c))*sin(6*d*x + 6*c) + (4*a*b*cos(2
*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) -
(4*a*b*cos(2*d*x + 2*c) - a*b)*sin(3*d*x + 3*c))/(b^3*cos(8*d*x + 8*c)^2 +
16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c
)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d
*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(6
4*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x
+ 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(
4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*
d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos
(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b
^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c)
 + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + cos(d*x + c)
)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(98) = 196$.
time = 0.47, size = 815, normalized size = 5.91



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) + a)/((a*b^2 - b^3)*d^2))*log(a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2*b*d)*sqrt(-((a*b^2 - b^3)*d^2*sq
rt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) - b*d*sq
rt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^
2 - b^3)*d^2))*log(a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5
 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b
^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))) - b*d*sqrt(-((a*b^2 -
b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2)
)*log(-a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 +
 b^7)*d^4)) - a^2*b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^
```

```
6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) + b*d*sqrt(((a*b^2 - b^3)*d^2*sqr
t(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-a^2*c
os(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
+ a^2*b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) - a)/((a*b^2 - b^3)*d^2))) - 4*cos(d*x + c))/(b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[-36,13
]Warning, need t
```

Mupad [**B**]
time = 14.26, size = 1001, normalized size = 7.25

$$\frac{\cos(c+dx)}{b\,d}-\frac{2\,\mathrm{atanh}\left(\frac{8\,a^2\,b^7\,\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^3\,b^5}}{16\,(a\,b^5-b^6)}-\frac{a\,b^3}{16\,(a\,b^5-b^6)}}}{\frac{2\,a^3\,b^{11}}{a\,b^5-b^6}-\frac{2\,a^4\,b^{10}}{a\,b^5-b^6}+\frac{2\,a^2\,b^8\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}-\frac{2\,a^3\,b^7\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}}-\frac{8\,a^2\,b\,\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^3\,b^5}}{16\,(a\,b^5-b^6)}-\frac{a\,b^3}{16\,(a\,b^5-b^6)}}}{\frac{2\,a^3\,b^5}{a\,b^5-b^6}+\frac{2\,a^2\,b^2\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}}+\frac{8\,a\,b^4\,\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^3\,b^5}}{16\,(a\,b^5-b^6)}-\frac{a\,b^3}{16\,(a\,b^5-b^6)}}\,\sqrt{a^3\,b^5}}{\frac{2\,a^3\,b^{11}}{a\,b^5-b^6}-\frac{2\,a^4\,b^{10}}{a\,b^5-b^6}+\frac{2\,a^2\,b^8\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}-\frac{2\,a^3\,b^7\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}}\right)\,\sqrt{-\frac{\sqrt{a^3\,b^5}+a\,b^3}{16\,(a\,b^5-b^6)}}}{d}+\frac{2\,\mathrm{atanh}\left(\frac{8\,a^2\,b\,\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a^3\,b^5}}{16\,(a\,b^5-b^6)}-\frac{a\,b^3}{16\,(a\,b^5-b^6)}}}{\frac{2\,a^3\,b^5}{a\,b^5-b^6}-\frac{2\,a^2\,b^2\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}}-\frac{8\,a^2\,b^7\,\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a^3\,b^5}}{16\,(a\,b^5-b^6)}-\frac{a\,b^3}{16\,(a\,b^5-b^6)}}}{\frac{2\,a^3\,b^{11}}{a\,b^5-b^6}-\frac{2\,a^4\,b^{10}}{a\,b^5-b^6}-\frac{2\,a^2\,b^8\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}+\frac{2\,a^3\,b^7\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}}+\frac{8\,a\,b^4\,\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a^3\,b^5}}{16\,(a\,b^5-b^6)}-\frac{a\,b^3}{16\,(a\,b^5-b^6)}}\,\sqrt{a^3\,b^5}}{\frac{2\,a^3\,b^{11}}{a\,b^5-b^6}-\frac{2\,a^4\,b^{10}}{a\,b^5-b^6}-\frac{2\,a^2\,b^8\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}+\frac{2\,a^3\,b^7\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}}\right)\,\sqrt{\frac{\sqrt{a^3\,b^5}-a\,b^3}{16\,(a\,b^5-b^6)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - b*sin(c + d*x)^4),x)
```

```
[Out] cos(c + d*x)/(b*d) - (2*atanh((8*a^2*b^7*cos(c + d*x)*(- (a^3*b^5)^(1/2)/(1
6*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/((2*a^3*b^11)/(a*b^5
- b^6) - (2*a^4*b^10)/(a*b^5 - b^6) + (2*a^2*b^8*(a^3*b^5)^(1/2))/(a*b^5 -
b^6) - (2*a^3*b^7*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) - (8*a^2*b*cos(c + d*x)*(
- (a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/(
(2*a^3*b^5)/(a*b^5 - b^6) + (2*a^2*b^2*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) + (8
*a*b^4*cos(c + d*x)*(- (a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*
b^5 - b^6)))^(1/2)*(a^3*b^5)^(1/2))/((2*a^3*b^11)/(a*b^5 - b^6) - (2*a^4*b^
```

```
10)/(a*b^5 - b^6) + (2*a^2*b^8*(a^3*b^5)^(1/2))/(a*b^5 - b^6) - (2*a^3*b^7*
(a^3*b^5)^(1/2))/(a*b^5 - b^6)))*(-((a^3*b^5)^(1/2) + a*b^3)/(16*(a*b^5 - b
^6)))^(1/2))/d + (2*atanh((8*a^2*b*cos(c + d*x)*((a^3*b^5)^(1/2)/(16*(a*b^5
 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/((2*a^3*b^5)/(a*b^5 - b^6) -
(2*a^2*b^2*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) - (8*a^2*b^7*cos(c + d*x)*((a^3*
b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/((2*a^3*
b^11)/(a*b^5 - b^6) - (2*a^4*b^10)/(a*b^5 - b^6) - (2*a^2*b^8*(a^3*b^5)^(1/
2))/(a*b^5 - b^6) + (2*a^3*b^7*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) + (8*a*b^4*c
os(c + d*x)*((a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)
))^(1/2)*(a^3*b^5)^(1/2))/((2*a^3*b^11)/(a*b^5 - b^6) - (2*a^4*b^10)/(a*b^5
 - b^6) - (2*a^2*b^8*(a^3*b^5)^(1/2))/(a*b^5 - b^6) + (2*a^3*b^7*(a^3*b^5)^
(1/2))/(a*b^5 - b^6)))*(((a^3*b^5)^(1/2) - a*b^3)/(16*(a*b^5 - b^6)))^(1/2)
)/d
```

3.198 $\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/4}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/4}d}$$

```
[Out] -1/2*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/(a^(1/2)-
b^(1/2))^(1/2)+1/2*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^(3
/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3294, 1180, 211, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(Sqrt[Sqrt[a] -
 Sqrt[b]]*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b
]]]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{1-x^2}{a-b+2bx^2-bx^4}\,dx,x,\cos(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx,x,\cos(c+dx)\right)}{2d}+\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx,x,c\right.}{2d}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/4}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/4}d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 285, normalized size = 2.48

$$-\frac{i\mathrm{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-3i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+3i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((-1/8*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 &
 , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (3*I)*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1
)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d
*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(
-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/d
```

Maple [A]
time = 0.34, size = 83, normalized size = 0.72

method	result
derivativedivides	$\frac{b\left(\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
default	$\frac{b\left(\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
risch	$\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(-16+\left(a\,b^3d^4-b^4d^4\right)\_Z^4-8b^2d^2\_Z^2\right)}\_R\ln\left(e^{2i(dx+c)}+\left(\left(-\frac{1}{4}ia\,b^2d^3+\frac{1}{4}ib^3d^3\right)\_R^3+2ibd\_R\right)e^{i(dx+c)}\right.\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*b*(1/2/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/(((a*b)^(1/2)+b
)*b)^(1/2))-1/2/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/
2)-b)*b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sin(d*x + c)^3/(b*sin(d*x + c)^4 - a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(79) = 158.

time = 0.46, size = 703, normalized size = 6.11

$$\frac{1}{4}\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+1}{(ab-b^2)d^2}}\log\left(-\left((ab^2-b^3)d^3\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-bd\right)\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+1}{(ab-b^2)d^2}}+\cos(dx+c)\right)-\frac{1}{4}\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+1}{(ab-b^2)d^2}}\log\left(-\left((ab^2-b^3)d^3\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-bd\right)\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+1}{(ab-b^2)d^2}}-\cos(dx+c)\right)-\frac{1}{4}\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-1}{(ab-b^2)d^2}}\log\left(-\left((ab^2-b^3)d^3\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+bd\right)\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-1}{(ab-b^2)d^2}}+\cos(dx+c)\right)+\frac{1}{4}\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-1}{(ab-b^2)d^2}}\log\left(-\left((ab^2-b^3)d^3\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+bd\right)\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-1}{(ab-b^2)d^2}}-\cos(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a
*b - b^2)*d^2))*log(-((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d
^4)) - b*d)*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
+ 1)/((a*b - b^2)*d^2)) + cos(d*x + c)) - 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a
/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2))*log(-((a*b^2 - b^
3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - b*d)*sqrt(-((a*b - b^2)*d^
2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2)) - cos(d*x
 + c)) - 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
- 1)/((a*b - b^2)*d^2))*log(-((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4
+ b^5)*d^4)) + b*d)*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)
*d^4)) - 1)/((a*b - b^2)*d^2)) + cos(d*x + c)) + 1/4*sqrt(((a*b - b^2)*d^2*
sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2))*log(-((a*b^
2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(((a*b - b^
2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2)) - co
s(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(79) = 158.
time = 0.72, size = 166, normalized size = 1.44

$$\frac{\sqrt{-b^2-\sqrt{ab}\,b}\,\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2+\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(b+\sqrt{ab}\right)d|b|}+\frac{\sqrt{-b^2+\sqrt{ab}\,b}\,\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2-\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(b-\sqrt{ab}\right)d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-b^2 - sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d^2 + sqrt((a
- b)*b*d^4 + b^2*d^4))/(b*d^4))))/((b + sqrt(a*b))*d*abs(b)) + 1/2*sqrt(-b^
2 + sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d^2 - sqrt((a - b)*b*d^4 +
 b^2*d^4))/(b*d^4))))/((b - sqrt(a*b))*d*abs(b))
```

Mupad [B]
time = 0.51, size = 976, normalized size = 8.49

$$\frac{2\,\mathrm{atanh}\left(\frac{8\,a\,b^2\,\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a\,b^3}}{16\,(a\,b^3-b^4)}-\frac{b^2}{16\,(a\,b^3-b^4)}}}{2\,a\,b+\frac{2\,a\,b^5}{a\,b^3-b^4}-\frac{2\,a\,b^3\,\sqrt{a\,b^3}}{a\,b^3-b^4}}-\frac{8\,a\,b^6\,\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a\,b^3}}{16\,(a\,b^3-b^4)}-\frac{b^2}{16\,(a\,b^3-b^4)}}}{2\,a\,b^5-2\,a^2\,b^4-\frac{2\,a^2\,b^8}{a\,b^3-b^4}+\frac{2\,a\,b^9}{a\,b^3-b^4}+\frac{2\,a^2\,b^6\,\sqrt{a\,b^3}}{a\,b^3-b^4}-\frac{2\,a\,b^7\,\sqrt{a\,b^3}}{a\,b^3-b^4}}+\frac{8\,a\,b^4\,\cos(c+d\,x)\,\sqrt{a\,b^3}\,\sqrt{\frac{\sqrt{a\,b^3}}{16\,(a\,b^3-b^4)}-\frac{b^2}{16\,(a\,b^3-b^4)}}}{2\,a\,b^5-2\,a^2\,b^4-\frac{2\,a^2\,b^8}{a\,b^3-b^4}+\frac{2\,a\,b^9}{a\,b^3-b^4}+\frac{2\,a^2\,b^6\,\sqrt{a\,b^3}}{a\,b^3-b^4}-\frac{2\,a\,b^7\,\sqrt{a\,b^3}}{a\,b^3-b^4}}\right)\,\sqrt{-\frac{b^2-\sqrt{a\,b^3}}{16\,(a\,b^3-b^4)}}}{d}-\frac{2\,\mathrm{atanh}\left(\frac{8\,a\,b^6\,\cos(c+d\,x)\,\sqrt{-\frac{b^2}{16\,(a\,b^3-b^4)}-\frac{\sqrt{a\,b^3}}{16\,(a\,b^3-b^4)}}}{2\,a\,b^5-2\,a^2\,b^4-\frac{2\,a^2\,b^8}{a\,b^3-b^4}+\frac{2\,a\,b^9}{a\,b^3-b^4}-\frac{2\,a^2\,b^6\,\sqrt{a\,b^3}}{a\,b^3-b^4}+\frac{2\,a\,b^7\,\sqrt{a\,b^3}}{a\,b^3-b^4}}-\frac{8\,a\,b^2\,\cos(c+d\,x)\,\sqrt{-\frac{b^2}{16\,(a\,b^3-b^4)}-\frac{\sqrt{a\,b^3}}{16\,(a\,b^3-b^4)}}}{2\,a\,b+\frac{2\,a\,b^5}{a\,b^3-b^4}+\frac{2\,a\,b^3\,\sqrt{a\,b^3}}{a\,b^3-b^4}}+\frac{8\,a\,b^4\,\cos(c+d\,x)\,\sqrt{a\,b^3}\,\sqrt{-\frac{b^2}{16\,(a\,b^3-b^4)}-\frac{\sqrt{a\,b^3}}{16\,(a\,b^3-b^4)}}}{2\,a\,b^5-2\,a^2\,b^4-\frac{2\,a^2\,b^8}{a\,b^3-b^4}+\frac{2\,a\,b^9}{a\,b^3-b^4}-\frac{2\,a^2\,b^6\,\sqrt{a\,b^3}}{a\,b^3-b^4}+\frac{2\,a\,b^7\,\sqrt{a\,b^3}}{a\,b^3-b^4}}\right)\,\sqrt{-\frac{b^2+\sqrt{a\,b^3}}{16\,(a\,b^3-b^4)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (2*atanh((8*a*b^2*cos(c + d*x)*((a*b^3)^(1/2)/(16*(a*b^3 - b^4)) - b^2/(16*
(a*b^3 - b^4)))^(1/2))/(2*a*b + (2*a*b^5)/(a*b^3 - b^4) - (2*a*b^3*(a*b^3)^
(1/2))/(a*b^3 - b^4)) - (8*a*b^6*cos(c + d*x)*((a*b^3)^(1/2)/(16*(a*b^3 - b
^4)) - b^2/(16*(a*b^3 - b^4)))^(1/2))/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a
*b^3 - b^4) + (2*a*b^9)/(a*b^3 - b^4) + (2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 -
b^4) - (2*a*b^7*(a*b^3)^(1/2))/(a*b^3 - b^4)) + (8*a*b^4*cos(c + d*x)*(a*b^
3)^(1/2)*((a*b^3)^(1/2)/(16*(a*b^3 - b^4)) - b^2/(16*(a*b^3 - b^4)))^(1/2))
/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a*b^3 - b^4) + (2*a*b^9)/(a*b^3 - b^4)
 + (2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 - b^4) - (2*a*b^7*(a*b^3)^(1/2))/(a*b^3
 - b^4)))*(-(b^2 - (a*b^3)^(1/2))/(16*(a*b^3 - b^4)))^(1/2))/d - (2*atanh((
8*a*b^6*cos(c + d*x)*(- b^2/(16*(a*b^3 - b^4)) - (a*b^3)^(1/2)/(16*(a*b^3 -
 b^4)))^(1/2))/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a*b^3 - b^4) + (2*a*b^9)
/(a*b^3 - b^4) - (2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 - b^4) + (2*a*b^7*(a*b^3)
^(1/2))/(a*b^3 - b^4)) - (8*a*b^2*cos(c + d*x)*(- b^2/(16*(a*b^3 - b^4)) -
(a*b^3)^(1/2)/(16*(a*b^3 - b^4)))^(1/2))/(2*a*b + (2*a*b^5)/(a*b^3 - b^4) +
 (2*a*b^3*(a*b^3)^(1/2))/(a*b^3 - b^4)) + (8*a*b^4*cos(c + d*x)*(a*b^3)^(1/
2)*(- b^2/(16*(a*b^3 - b^4)) - (a*b^3)^(1/2)/(16*(a*b^3 - b^4)))^(1/2))/(2*
a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a*b^3 - b^4) + (2*a*b^9)/(a*b^3 - b^4) - (
2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 - b^4) + (2*a*b^7*(a*b^3)^(1/2))/(a*b^3 - b
^4)))*(-(b^2 + (a*b^3)^(1/2))/(16*(a*b^3 - b^4)))^(1/2))/d
```

3.199 $\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt[4]{b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt[4]{b}\,d}$$

```
[Out] -1/2*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(1/4)/d/a^(1/2)/(
a^(1/2)-b^(1/2))^(1/2)-1/2*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/
2))/b^(1/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3294, 1107, 211, 214}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(Sqrt[a]*Sqrt[S
qrt[a] - Sqrt[b]]*b^(1/4)*d) - ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
```

```
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx,x,\cos(c+dx)\right)}{d}\\ &= \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx,x,\cos(c+dx)\right)}{2\sqrt{a}\,d} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,d\right.}{2\sqrt{a}\,d}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt[4]{b}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt[4]{b}\,d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 183, normalized size = 1.46

$$\frac{i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^3-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^3}{-b-8a\#1^2+3b\#1^2-3b\#1^4+b\#1^6}\&\right]}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((I/2)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & ,
(-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#
1^6) & ])/d
```

Maple [A]
time = 0.39, size = 87, normalized size = 0.70

method	result
derivativedivides	$\frac{b\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
default	$\frac{b\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
risch	$-\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(-1+\left(16a^{3}b\,d^{4}-16a^{2}b^{2}d^{4}\right)\_Z^{4}-8a\,d^{2}\_Z^{2}b\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(\left(-16ia^{2}b\,d^{3}+16ia\,b^{2}d^{3}\right)\_R^{3}+(4iad\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*b*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/(((a
*b)^(1/2)+b)*b)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b*c
os(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sin(d*x + c)/(b*sin(d*x + c)^4 - a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(85) = 170$.
time = 0.45, size = 703, normalized size = 5.62

$$-\frac{1}{4}\sqrt{-\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+1}{(a^2-ab)d^2}}\log\left(-\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-ad\right)\sqrt{-\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+1}{(a^2-ab)d^2}}+\cos(dx+c)\right)+\frac{1}{4}\sqrt{-\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+1}{(a^2-ab)d^2}}\log\left(-\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-ad\right)\sqrt{-\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+1}{(a^2-ab)d^2}}-\cos(dx+c)\right)+\frac{1}{4}\sqrt{\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-1}{(a^2-ab)d^2}}\log\left(-\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+ad\right)\sqrt{\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-1}{(a^2-ab)d^2}}+\cos(dx+c)\right)-\frac{1}{4}\sqrt{\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-1}{(a^2-ab)d^2}}\log\left(-\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}+ad\right)\sqrt{\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2a^2b^2+ab^3)d^4}}-1}{(a^2-ab)d^2}}-\cos(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/
((a^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a
*b^3)*d^4)) - a*d)*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^
3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + cos(d*x + c)) + 1/4*sqrt(-((a^2 - a*b)*d
^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(-(
(a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - a*d)*sqrt(-
((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b
)*d^2)) - cos(d*x + c)) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*
b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1
/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a
^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + cos(d*x + c)) - 1
/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a
^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^
3)*d^4)) + a*d)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d
^4)) - 1)/((a^2 - a*b)*d^2)) - cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(85) = 170.
time = 0.98, size = 183, normalized size = 1.46

$$-\frac{\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2+\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(ab+\sqrt{ab}\,a\right)d|b|}+\frac{\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2-\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(ab-\sqrt{ab}\,a\right)d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d^2
 + sqrt((a - b)*b*d^4 + b^2*d^4))/(b*d^4))))/((a*b + sqrt(a*b)*a)*d*abs(b))
 + 1/2*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d
```

```
^2 - sqrt((a - b)*b*d^4 + b^2*d^4))/(b*d^4))))/((a*b - sqrt(a*b)*a)*d*abs(b
))
```

Mupad [B]
time = 15.11, size = 361, normalized size = 2.89

$$\frac{\ln\left(4\,a\,b^3\,\sqrt{\frac{1}{a\,b+\sqrt{a^3\,b}}}-4\,b^3\,\cos\left(c+d\,x\right)+\frac{4\,a\,b^4\,\cos(c+d\,x)}{a\,b+\sqrt{a^3\,b}}\right)\,\sqrt{-\frac{a\,b-\sqrt{a^3\,b}}{16\,\left(a^3\,b-a^2\,b^2\right)}}}{d}+\frac{\ln\left(4\,b^3\,\cos\left(c+d\,x\right)-4\,a\,b^3\,\sqrt{\frac{1}{a\,b-\sqrt{a^3\,b}}}-\frac{4\,a\,b^4\,\cos(c+d\,x)}{a\,b-\sqrt{a^3\,b}}\right)\,\sqrt{-\frac{a\,b+\sqrt{a^3\,b}}{16\,\left(a^3\,b-a^2\,b^2\right)}}}{d}-\frac{\ln\left(4\,b^3\,\cos\left(c+d\,x\right)+4\,a\,b^3\,\sqrt{\frac{1}{a\,b+\sqrt{a^3\,b}}}-\frac{4\,a\,b^4\,\cos(c+d\,x)}{a\,b+\sqrt{a^3\,b}}\right)\,\sqrt{\frac{1}{a\,b+\sqrt{a^3\,b}}}}{4\,d}-\frac{\ln\left(4\,b^3\,\cos\left(c+d\,x\right)+4\,a\,b^3\,\sqrt{\frac{1}{a\,b-\sqrt{a^3\,b}}}-\frac{4\,a\,b^4\,\cos(c+d\,x)}{a\,b-\sqrt{a^3\,b}}\right)\,\sqrt{\frac{1}{a\,b-\sqrt{a^3\,b}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (log(4*a*b^3*(1/(a*b + (a^3*b)^(1/2)))^(1/2) - 4*b^3*cos(c + d*x) + (4*a*b^
4*cos(c + d*x))/(a*b + (a^3*b)^(1/2)))*(-(a*b - (a^3*b)^(1/2))/(16*(a^3*b -
 a^2*b^2)))^(1/2))/d + (log(4*b^3*cos(c + d*x) - 4*a*b^3*(1/(a*b - (a^3*b)^
(1/2)))^(1/2) - (4*a*b^4*cos(c + d*x))/(a*b - (a^3*b)^(1/2)))*(-(a*b + (a^3
*b)^(1/2))/(16*(a^3*b - a^2*b^2)))^(1/2))/d - (log(4*b^3*cos(c + d*x) + 4*a
*b^3*(1/(a*b + (a^3*b)^(1/2)))^(1/2) - (4*a*b^4*cos(c + d*x))/(a*b + (a^3*b
)^(1/2)))*(1/(a*b + (a^3*b)^(1/2)))^(1/2))/(4*d) - (log(4*b^3*cos(c + d*x)
+ 4*a*b^3*(1/(a*b - (a^3*b)^(1/2)))^(1/2) - (4*a*b^4*cos(c + d*x))/(a*b - (
a^3*b)^(1/2)))*(1/(a*b - (a^3*b)^(1/2)))^(1/2))/(4*d)
```

3.200 $\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

```
[Out] -arctanh(cos(d*x+c))/a/d-1/2*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(
1/2))^(1/2))/a/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cos(d*
x+c)/(a^(1/2)+b^(1/2))^(1/2))/a/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3294, 1184, 213, 1180, 211, 214}

$$-\frac{\sqrt[4]{b}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*(b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(a*Sq
rt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (b^(1/4)*ArcTanh[(
b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt[b]
]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a(-1+x^2)}+\frac{b-bx^2}{a(a-b+2bx^2-bx^4)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{ad}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{b\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2ad} + \frac{b\text{Sub}}{}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos}{\sqrt{\sqrt{a}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.19, size = 318, normalized size = 2.34

$$-\frac{8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+ib\,\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-3i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+3i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/8*(8*Log[Cos[(c + d*x)/2]] - 8*Log[Sin[(c + d*x)/2]] + I*b*RootSum[b - 4
*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d
*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*
b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a*d)
```

Maple [A]

time = 0.52, size = 119, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{b^2\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}+\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{a}+\frac{\ln(\cos(dx+c)-1)}{2a}-\frac{\ln(1+\cos(dx+c))}{2a}}{d}$
default	$\frac{-\frac{b^2\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}+\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{a}+\frac{\ln(\cos(dx+c)-1)}{2a}-\frac{\ln(1+\cos(dx+c))}{2a}}{d}$
risch	$\frac{\ln\left(e^{i(dx+c)}-1\right)}{ad}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}+2i\left(\sum_{\_R=\text{RootOf}\left(\left(4096a^5d^4-4096a^4b\,d^4\right)\_Z^4-128a^2b\,d^2\_Z^2-b\right)}\_R\ln\left(e\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a*b^2*(-1/2/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/(((a*b
)^(1/2)+b)*b)^(1/2))+1/2/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*x+c)/((
(a*b)^(1/2)-b)*b)^(1/2)))+1/2/a*ln(cos(d*x+c)-1)-1/2/a*ln(1+cos(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*d*integrate(-2*(12*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*b^2*
cos(d*x + c)*sin(2*d*x + 2*c) + 4*b^2*cos(2*d*x + 2*c)*sin(d*x + c) - b^2*s
in(d*x + c) + (b^2*sin(7*d*x + 7*c) - 3*b^2*sin(5*d*x + 5*c) + 3*b^2*sin(3*
d*x + 3*c) - b^2*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x + 6*c)
 + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*cos(7*d*x + 7
*c) + 4*(3*b^2*sin(5*d*x + 5*c) - 3*b^2*sin(3*d*x + 3*c) + b^2*sin(d*x + c)
)*cos(6*d*x + 6*c) - 6*(2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x
+ 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a*b - 3*b^2)*sin(3*d*x + 3*c) - (8*a*b -
 3*b^2)*sin(d*x + c))*cos(4*d*x + 4*c) - (b^2*cos(7*d*x + 7*c) - 3*b^2*cos(
5*d*x + 5*c) + 3*b^2*cos(3*d*x + 3*c) - b^2*cos(d*x + c))*sin(8*d*x + 8*c)
- (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2
)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*b^2*cos(5*d*x + 5*c) - 3*b^2*co
s(3*d*x + 3*c) + b^2*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4*b^2*cos(2*d*x +
2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) + 2*(3*(8
*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*cos(d*x + c))*sin(4*d*x +
4*c) - 3*(4*b^2*cos(2*d*x + 2*c) - b^2)*sin(3*d*x + 3*c))/(a*b^2*cos(8*d*x
+ 8*c)^2 + 16*a*b^2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^
2*sin(8*d*x + 8*c)^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2
*c)^2 - 8*a*b^2*cos(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*
cos(4*d*x + 4*c)^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 1
6*(8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*
d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4
*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*
a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2
- 4*(8*a^2*b - 3*a*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin
(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x +
4*c))*sin(8*d*x + 8*c) + 16*(2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)
*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + log(cos(d*x)^2 + 2*cos(d*x)*cos(
c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) - log(cos(d*x)^2
 - 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^2
))/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(100) = 200$.
time = 0.51, size = 773, normalized size = 5.68

$$\frac{ad\sqrt{-\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}+b}{(a^3-a^2b)d^2}}\log\left(b\cos(dx+c)-\left((a^4-a^3b)d^3\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}-abd\right)\sqrt{-\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}+b}{(a^3-a^2b)d^2}}\right)-ad\sqrt{\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}-b}{(a^3-a^2b)d^2}}\log\left(b\cos(dx+c)-\left((a^4-a^3b)d^3\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}+abd\right)\sqrt{\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}-b}{(a^3-a^2b)d^2}}\right)-ad\sqrt{-\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}+b}{(a^3-a^2b)d^2}}\log\left(-b\cos(dx+c)-\left((a^4-a^3b)d^3\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}-abd\right)\sqrt{-\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}+b}{(a^3-a^2b)d^2}}\right)+ad\sqrt{\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}-b}{(a^3-a^2b)d^2}}\log\left(-b\cos(dx+c)-\left((a^4-a^3b)d^3\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}+abd\right)\sqrt{\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}-b}{(a^3-a^2b)d^2}}\right)-2\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +
 b)/((a^3 - a^2*b)*d^2))*log(b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sqrt(b/((a
^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^
5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - a*d*sqrt(((a^3 -
a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)
)*log(b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)
*d^4)) + a*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d
^4)) - b)/((a^3 - a^2*b)*d^2))) - a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5
 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(-b*cos(d*x + c) -
 ((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)*sqrt(-
((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2
*b)*d^2))) + a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*
d^4)) - b)/((a^3 - a^2*b)*d^2))*log(-b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sq
rt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt
(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))) - 2*log(1/2*
cos(d*x + c) + 1/2) + 2*log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(csc(c + d*x)/(a - b*sin(c + d*x)**4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[76,51]
Warning, need to
```

Mupad [B]

time = 15.36, size = 2031, normalized size = 14.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] - (atan((((((((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*(256*a^4*b^4
 - 192*a^3*b^5 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b + (a^5*b)
^(1/2))/(16*(a^4*b - a^5)))^(1/2)) - 144*a^2*b^5*cos(c + d*x))*((a^2*b + (a
^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 12*a*b^5)*((a^2*b + (a^5*b)^(1/2))
/(16*(a^4*b - a^5)))^(1/2) + 6*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(
16*(a^4*b - a^5)))^(1/2)*1i + (((((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)
))^(1/2)*(192*a^3*b^5 - 256*a^4*b^4 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b
^4)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)) - 144*a^2*b^5*cos(c
 + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 12*a*b^5)*((a
^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 6*b^5*cos(c + d*x))*((a^2
*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*1i)/((((((a^2*b + (a^5*b)^(1/
2))/(16*(a^4*b - a^5)))^(1/2)*(256*a^4*b^4 - 192*a^3*b^5 + cos(c + d*x)*(76
8*a^4*b^5 - 512*a^5*b^4)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)
) - 144*a^2*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^
(1/2) + 12*a*b^5)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 6*b^
5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - (((((a
^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*(192*a^3*b^5 - 256*a^4*b^4
+ cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b + (a^5*b)^(1/2))/(16*(a^
4*b - a^5)))^(1/2)) - 144*a^2*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(1
6*(a^4*b - a^5)))^(1/2) - 12*a*b^5)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a
^5)))^(1/2) + 6*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5
)))^(1/2)))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*2i)/d - atan
h((32*b^4*cos(c + d*x))/(32*b^4 - (18*b^5)/a) - (18*b^5*cos(c + d*x))/(32*a
*b^4 - 18*b^5))/(a*d) - (atan(((6*b^5*cos(c + d*x) + (((256*a^4*b^4 - 192*a
^3*b^5 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b - (a^5*b)^(1/2))/
(16*(a^4*b - a^5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/
2) - 144*a^2*b^5*cos(c + d*x))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))
^(1/2) + 12*a*b^5)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^
2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*1i + (6*b^5*cos(c + d*x) + (
((192*a^3*b^5 - 256*a^4*b^4 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^
2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(1
6*(a^4*b - a^5)))^(1/2) - 144*a^2*b^5*cos(c + d*x))*((a^2*b - (a^5*b)^(1/2)
```

```
)/(16*(a^4*b - a^5)))^(1/2) - 12*a*b^5)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b
 - a^5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*1i)/((6
*b^5*cos(c + d*x) + (((256*a^4*b^4 - 192*a^3*b^5 + cos(c + d*x)*(768*a^4*b^
5 - 512*a^5*b^4)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^2*
b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 144*a^2*b^5*cos(c + d*x))*((
a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 12*a*b^5)*((a^2*b - (a^5
*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b -
 a^5)))^(1/2) - (6*b^5*cos(c + d*x) + (((192*a^3*b^5 - 256*a^4*b^4 + cos(c
+ d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^
5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 144*a^2*b^
5*cos(c + d*x))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 12*a*b
^5)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^2*b - (a^5*b)^(
1/2))/(16*(a^4*b - a^5)))^(1/2)))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5
)))^(1/2)*2i)/d
```

3.201 $\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=184

$$-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{1}{4ad(1-\cos(c+d$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-1/4/a/d/(1-cos(d*x+c))+1/4/a/d/(1+cos(d*x+c))-
1/2*b^(3/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/2)/d/(a
^(1/2)-b^(1/2))^(1/2)-1/2*b^(3/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/
2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1184, 213, 1107, 211, 214}

$$-\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(\cos(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*(b^(3/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(a^(3
/2)*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(2*a*d) - (b^(3/4)*A
rcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqr
t[a] + Sqrt[b]]*d) - 1/(4*a*d*(1 - Cos[c + d*x])) + 1/(4*a*d*(1 + Cos[c + d
*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+2bx^2-bx^4)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}-\frac{1}{2a(-1+x^2)}+\frac{b}{a(a-b+2bx^2-bx^4)}\right)\,dx, x, \cos(c+dx)\right.}{d}$$

$$= -\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(1+\cos(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+d\right.}{2ad}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(1+\cos(c+dx))}+\frac{b^{3/2}\text{Sub}}{}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c}{\sqrt{\sqrt{a}+}}\right.}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.24, size = 242, normalized size = 1.32

$$\frac{-\csc^2\left(\frac{1}{2}(c+dx)\right)-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4ib\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^3-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^3}{-b-8a\#1^2+3b\#1^2-3b\#1^4+b\#1^6}\&\right]+\sec^2\left(\frac{1}{2}(c+dx)\right)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-Csc[(c + d*x)/2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]] +
(4*I)*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & ,
 (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*
#1^6) & ] + Sec[(c + d*x)/2]^2)/(8*a*d)
```

Maple [A]
time = 0.64, size = 152, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{1}{4a(\cos(dx+c)-1)}+\frac{\ln(\cos(dx+c)-1)}{4a}+\frac{1}{4a(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{4a}+\frac{b^2\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan \cdots}{2\sqrt{a \cdots}} \cdots\right)}{a}}{d}$
default	$\frac{\frac{1}{4a(\cos(dx+c)-1)}+\frac{\ln(\cos(dx+c)-1)}{4a}+\frac{1}{4a(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{4a}+\frac{b^2\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan \cdots}{2\sqrt{a \cdots}} \cdots\right)}{a}}{d}$
risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2ad}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}-8i\left(\sum_{\_R=\text{RootOf}\left(\left(1048576a^7d^4-1048576a^6b\,d^4\right)\_Z^4 \cdots\right.} \cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a/(cos(d*x+c)-1)+1/4/a*ln(cos(d*x+c)-1)+1/4/a/(1+cos(d*x+c))-1/4/a
*ln(1+cos(d*x+c))+1/a*b^2*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arcta
nh(b*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-b)
*b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(cos(3*d*x + 3*c) + cos(d*x + c))*cos(4*d*x + 4*c) - 4*(2*cos(2*d*x
+ 2*c) - 1)*cos(3*d*x + 3*c) - 8*cos(2*d*x + 2*c)*cos(d*x + c) + 4*(a*d*cos
(4*d*x + 4*c)^2 + 4*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(4*d*x + 4*c)^2 - 4*a*d
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*d*sin(2*d*x + 2*c)^2 - 4*a*d*cos(2
*d*x + 2*c) + a*d - 2*(2*a*d*cos(2*d*x + 2*c) - a*d)*cos(4*d*x + 4*c))*inte
grate(8*(4*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2*(8*a*b - 3*b^2)*cos(3*
d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(3*d*x
+ 3*c) - (b^2*sin(5*d*x + 5*c) - b^2*sin(3*d*x + 3*c))*cos(8*d*x + 8*c) + 4
*(b^2*sin(5*d*x + 5*c) - b^2*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) - 2*(2*b^2*
sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*cos(5*d*x + 5*c) + (b^
2*cos(5*d*x + 5*c) - b^2*cos(3*d*x + 3*c))*sin(8*d*x + 8*c) - 4*(b^2*cos(5*
d*x + 5*c) - b^2*cos(3*d*x + 3*c))*sin(6*d*x + 6*c) + (4*b^2*cos(2*d*x + 2*
c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) - (4*b^2*co
s(2*d*x + 2*c) - b^2)*sin(3*d*x + 3*c))/(a*b^2*cos(8*d*x + 8*c)^2 + 16*a*b^
2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(8*d*x + 8*c)
^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c)^2 - 8*a*b^2*co
s(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*cos(4*d*x + 4*c)^2
 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 3*a*b
^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*d*x + 6*c) + 4*a*b
^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(8
*d*x + 8*c) + 8*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*c
os(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2 - 4*(8*a^2*b - 3*a
*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin(6*d*x + 6*c) + 2*
a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(8*d*x +
8*c) + 16*(2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))
*sin(6*d*x + 6*c)), x) + (2*(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos
(4*d*x + 4*c)^2 - 4*cos(2*d*x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) - 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log
(cos(d*x)^2 + 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c)
 + sin(c)^2) - (2*(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4
*c)^2 - 4*cos(2*d*x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(
```

```
2*d*x + 2*c) - 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log(cos(d*x)^
2 - 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^
2) + 4*(sin(3*d*x + 3*c) + sin(d*x + c))*sin(4*d*x + 4*c) - 8*sin(3*d*x + 3
*c)*sin(2*d*x + 2*c) - 8*sin(2*d*x + 2*c)*sin(d*x + c) + 4*cos(d*x + c))/(a
*d*cos(4*d*x + 4*c)^2 + 4*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(4*d*x + 4*c)^2 -
 4*a*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*d*sin(2*d*x + 2*c)^2 - 4*a*d
*cos(2*d*x + 2*c) + a*d - 2*(2*a*d*cos(2*d*x + 2*c) - a*d)*cos(4*d*x + 4*c)
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(136) = 272$.

time $= 0.55$, size $= 924$, normalized size $= 5.02$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 -
2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(b^2*cos(d*x + c) -
 ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sq
rt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a
^4 - a^3*b)*d^2))) - (a*d*cos(d*x + c)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqr
t(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(b^2*
cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))
 + a^2*b*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4
)) - b^2)/((a^4 - a^3*b)*d^2))) - (a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 -
a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*
d^2))*log(-b^2*cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b +
 a^5*b^2)*d^4)) - a^2*b*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*
b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) + (a*d*cos(d*x + c)^2 - a*d
)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/
((a^4 - a^3*b)*d^2))*log(-b^2*cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((
a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/
((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) + (cos(d*x +
c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 - 1)*log(-1/2*cos(d
*x + c) + 1/2) - 2*cos(d*x + c))/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F]

time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \frac{\csc^3(c + dx)}{a - b\sin^4(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a - b*sin(c + d*x)**4), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[-35,-3
1]Warning, need
```

Mupad [**B**]
time = 15.19, size = 2779, normalized size = 15.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(cos(c + d*x)*1i)*1i)/(d*(2*a - 2*a*cos(c + d*x)^2)) - cos(c + d*x)/(d
*(2*a - 2*a*cos(c + d*x)^2)) - (atan(cos(c + d*x)*1i)*cos(c + d*x)^2*1i)/(d
*(2*a - 2*a*cos(c + d*x)^2)) + (a*atan((a^13*cos(c + d*x)*(((a^7*b^3)^(1/2)
 + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*(((a^7*b
^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*cos(c + d*x)*(
((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*7168i - a^4*b^5*cos(
c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*8i + a^5*b
^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*12i
 - a^7*b^2*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(
1/2)*4i + a^7*b^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*
a^7))^(3/2)*320i - a^8*b^3*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^
6*b - 16*a^7))^(3/2)*576i + a^9*b^2*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^
2)/(16*a^6*b - 16*a^7))^(3/2)*192i - a^10*b^3*cos(c + d*x)*(((a^7*b^3)^(1/2
) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*3072i + a^11*b^2*cos(c + d*x)*(((a^
7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*8192i)/(2*b^3*(a^7*b^3)^
(1/2) + a^3*b^5 + a^5*b^3 - a*b^2*(a^7*b^3)^(1/2) + a^2*b*(a^7*b^3)^(1/2)))
*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*4i)/(d*(2*a - 2*a*
cos(c + d*x)^2)) + (a*atan((a^13*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)
/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*(-((a^7*b^3)^(1/2)
- a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*cos(c + d*x)*(-((a^7*b^3
)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*7168i - a^4*b^5*cos(c + d*x)*
(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*8i + a^5*b^4*cos(c
 + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*12i - a^7*
```

b^2*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*4
i + a^7*b^4*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))
^(3/2)*320i - a^8*b^3*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b
- 16*a^7))^(3/2)*576i + a^9*b^2*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/
(16*a^6*b - 16*a^7))^(3/2)*192i - a^10*b^3*cos(c + d*x)*(-((a^7*b^3)^(1/2)
- a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*3072i + a^11*b^2*cos(c + d*x)*(-((a^7
*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*8192i)/(a^3*b^5 - 2*b^3*(
a^7*b^3)^(1/2) + a^5*b^3 + a*b^2*(a^7*b^3)^(1/2) - a^2*b*(a^7*b^3)^(1/2)))*
(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*4i)/(d*(2*a - 2*a*
cos(c + d*x)^2)) - (a*cos(c + d*x)^2*atan((a^13*cos(c + d*x)*(((a^7*b^3)^(1
/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*(((a^
7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*cos(c + d*x
)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*7168i - a^4*b^5*c
os(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*8i + a^
5*b^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*
12i - a^7*b^2*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7)
)^(1/2)*4i + a^7*b^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b -
16*a^7))^(3/2)*320i - a^8*b^3*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16
*a^6*b - 16*a^7))^(3/2)*576i + a^9*b^2*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3
*b^2)/(16*a^6*b - 16*a^7))^(3/2)*192i - a^10*b^3*cos(c + d*x)*(((a^7*b^3)^(
1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*3072i + a^11*b^2*cos(c + d*x)*((
(a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*8192i)/(2*b^3*(a^7*b^
3)^(1/2) + a^3*b^5 + a^5*b^3 - a*b^2*(a^7*b^3)^(1/2) + a^2*b*(a^7*b^3)^(1/2
)))*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*4i)/(d*(2*a - 2
*a*cos(c + d*x)^2)) - (a*cos(c + d*x)^2*atan((a^13*cos(c + d*x)*(-((a^7*b^3
)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*(
-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*cos(c
+ d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*7168i - a^4
*b^5*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*
8i + a^5*b^4*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7)
)^(1/2)*12i - a^7*b^2*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b
- 16*a^7))^(1/2)*4i + a^7*b^4*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(1
6*a^6*b - 16*a^7))^(3/2)*320i - a^8*b^3*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a
^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*576i + a^9*b^2*cos(c + d*x)*(-((a^7*b^3)
^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*192i - a^10*b^3*cos(c + d*x)*(
-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*3072i + a^11*b^2*co
s(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*8192i)/
(a^3*b^5 - 2*b^3*(a^7*b^3)^(1/2) + a^5*b^3 + a*b^2*(a^7*b^3)^(1/2) - a^2*b*
(a^7*b^3)^(1/2)))*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*
4i)/(d*(2*a - 2*a*cos(c + d*x)^2))

3.202 $\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=229

$$-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{\quad}{16ad(1-}$$

```
[Out] -1/8*(3*a+8*b)*arctanh(cos(d*x+c))/a^2/d-1/16/a/d/(1-cos(d*x+c))^2-3/16/a/d
/(1-cos(d*x+c))+1/16/a/d/(1+cos(d*x+c))^2+3/16/a/d/(1+cos(d*x+c))-1/2*b^(5/
4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^2/d/(a^(1/2)-b^(1/2
))^(1/2)+1/2*b^(5/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^
2/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1184, 213, 1180, 211, 214}

$$-\frac{b^{5/4}\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{3}{16ad(1-\cos(c+dx))}+\frac{3}{16ad(\cos(c+dx)+1)}-\frac{1}{16ad(1-\cos(c+dx))^2}+\frac{1}{16ad(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*(b^(5/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(a^2*
Sqrt[Sqrt[a] - Sqrt[b]]*d) - ((3*a + 8*b)*ArcTanh[Cos[c + d*x]])/(8*a^2*d)
+ (b^(5/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*
Sqrt[Sqrt[a] + Sqrt[b]]*d) - 1/(16*a*d*(1 - Cos[c + d*x])^2) - 3/(16*a*d*(1
 - Cos[c + d*x])) + 1/(16*a*d*(1 + Cos[c + d*x])^2) + 3/(16*a*d*(1 + Cos[c
+ d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a-b+2bx^2-bx^4)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{8a(-1+x)^3} + \frac{3}{16a(-1+x)^2} + \frac{1}{8a(1+x)^3} + \frac{3}{16a(1+x)^2} + \frac{-3a-8b}{8a^2(-1+x^2)} - \frac{b^2(-1\ldots}{a^2(a-b+2b\ldots}\right.\right.}{d}$$

$$= -\frac{1}{16ad(1-\cos(c+dx))^2} - \frac{3}{16ad(1-\cos(c+dx))} + \frac{1}{16ad(1+\cos(c+dx))^2} + \ldots$$

$$= -\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d} - \frac{1}{16ad(1-\cos(c+dx))^2} - \frac{3}{16ad(1-\cos(c+d\ldots}$$

$$= -\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d} + \frac{b^{5/4}\tanh^{-1}\left(\ldots\right.}{2a^2\sqrt{\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.81, size = 409, normalized size = 1.79

$$\frac{-6a\csc^2\left(\frac{1}{2}(c+dx)\right) - a\csc^4\left(\frac{1}{2}(c+dx)\right) - 24a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 64b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 24a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 64b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 8ib^2\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-3i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+3i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right] + 6a\sec^2\left(\frac{1}{2}(c+dx)\right) + a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-6*a*Csc[(c + d*x)/2]^2 - a*Csc[(c + d*x)/2]^4 - 24*a*Log[Cos[(c + d*x)/2]
] - 64*b*Log[Cos[(c + d*x)/2]] + 24*a*Log[Sin[(c + d*x)/2]] + 64*b*Log[Sin[
(c + d*x)/2]] - (8*I)*b^2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b
*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ] + 6*
a*Sec[(c + d*x)/2]^2 + a*Sec[(c + d*x)/2]^4)/(64*a^2*d)
```

Maple [A]
time = 0.65, size = 193, normalized size = 0.84

method	result

derivativedivides	$\frac{-\frac{1}{16a(\cos(dx+c)-1)^2}+\frac{3}{16a(\cos(dx+c)-1)}+\frac{(3a+8b)\ln(\cos(dx+c)-1)}{16a^2}-\frac{b^3\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}+\frac{\arctan\left(\ldots\right.}{2b\sqrt{\left(\ldots\right.}}\ldots\right.}{a^2}\ldots}{d}$
default	$\frac{-\frac{1}{16a(\cos(dx+c)-1)^2}+\frac{3}{16a(\cos(dx+c)-1)}+\frac{(3a+8b)\ln(\cos(dx+c)-1)}{16a^2}-\frac{b^3\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}+\frac{\arctan\left(\ldots\right.}{2b\sqrt{\left(\ldots\right.}}\ldots\right.}{a^2}\ldots}{d}$
risch	$\frac{3\,e^{7i(dx+c)}-11\,e^{5i(dx+c)}-11\,e^{3i(dx+c)}+3\,e^{i(dx+c)}}{4da\left(e^{2i(dx+c)}-1\right)^4}+\frac{3\ln\left(e^{i(dx+c)}-1\right)}{8ad}+\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{3\ln\left(e^{i(dx+c)}+1\right)}{8ad}-\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/16/a/(cos(d*x+c)-1)^2+3/16/a/(cos(d*x+c)-1)+1/16*(3*a+8*b)/a^2*ln(c
os(d*x+c)-1)-b^3/a^2*(-1/2/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)
/(((a*b)^(1/2)+b)*b)^(1/2))+1/2/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*
x+c)/(((a*b)^(1/2)-b)*b)^(1/2)))+1/16/a/(1+cos(d*x+c))^2+3/16/a/(1+cos(d*x+
c))+1/16/a^2*(-3*a-8*b)*ln(1+cos(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

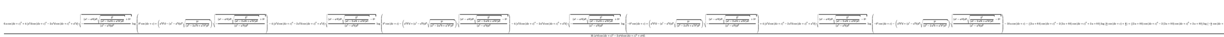
```
[In] integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/16*(48*a*cos(2*d*x + 2*c)*cos(d*x + c) - 176*a*sin(3*d*x + 3*c)*sin(2*d*
x + 2*c) + 48*a*sin(2*d*x + 2*c)*sin(d*x + c) - 4*(3*a*cos(7*d*x + 7*c) - 1
1*a*cos(5*d*x + 5*c) - 11*a*cos(3*d*x + 3*c) + 3*a*cos(d*x + c))*cos(8*d*x
+ 8*c) + 12*(4*a*cos(6*d*x + 6*c) - 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x +
```

```
2*c) - a)*cos(7*d*x + 7*c) - 16*(11*a*cos(5*d*x + 5*c) + 11*a*cos(3*d*x + 3
*c) - 3*a*cos(d*x + c))*cos(6*d*x + 6*c) + 44*(6*a*cos(4*d*x + 4*c) - 4*a*c
os(2*d*x + 2*c) + a)*cos(5*d*x + 5*c) + 24*(11*a*cos(3*d*x + 3*c) - 3*a*cos
(d*x + c))*cos(4*d*x + 4*c) - 44*(4*a*cos(2*d*x + 2*c) - a)*cos(3*d*x + 3*c
) - 12*a*cos(d*x + c) + 16*(a^2*d*cos(8*d*x + 8*c)^2 + 16*a^2*d*cos(6*d*x +
 6*c)^2 + 36*a^2*d*cos(4*d*x + 4*c)^2 + 16*a^2*d*cos(2*d*x + 2*c)^2 + a^2*d
*sin(8*d*x + 8*c)^2 + 16*a^2*d*sin(6*d*x + 6*c)^2 + 36*a^2*d*sin(4*d*x + 4*
c)^2 - 48*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*d*sin(2*d*x + 2*
c)^2 - 8*a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(4*a^2*d*cos(6*d*x + 6*c) - 6*a
^2*d*cos(4*d*x + 4*c) + 4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c)
 - 8*(6*a^2*d*cos(4*d*x + 4*c) - 4*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x
 + 6*c) - 12*(4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 4*(2*a^2
*d*sin(6*d*x + 6*c) - 3*a^2*d*sin(4*d*x + 4*c) + 2*a^2*d*sin(2*d*x + 2*c))*
sin(8*d*x + 8*c) - 16*(3*a^2*d*sin(4*d*x + 4*c) - 2*a^2*d*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c))*integrate(-2*(12*b^3*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) -
 4*b^3*cos(d*x + c)*sin(2*d*x + 2*c) + 4*b^3*cos(2*d*x + 2*c)*sin(d*x + c)
 - b^3*sin(d*x + c) + (b^3*sin(7*d*x + 7*c) - 3*b^3*sin(5*d*x + 5*c) + 3*b^3
*sin(3*d*x + 3*c) - b^3*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*b^3*sin(6*d*x
 + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*cos(
7*d*x + 7*c) + 4*(3*b^3*sin(5*d*x + 5*c) - 3*b^3*sin(3*d*x + 3*c) + b^3*sin
(d*x + c))*cos(6*d*x + 6*c) - 6*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)
*sin(4*d*x + 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a*b^2 - 3*b^3)*sin(3*d*x + 3*
c) - (8*a*b^2 - 3*b^3)*sin(d*x + c))*cos(4*d*x + 4*c) - (b^3*cos(7*d*x + 7*
c) - 3*b^3*cos(5*d*x + 5*c) + 3*b^3*cos(3*d*x + 3*c) - b^3*cos(d*x + c))*si
n(8*d*x + 8*c) - (4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2
*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*b^3*cos(5*d*x
+ 5*c) - 3*b^3*cos(3*d*x + 3*c) + b^3*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4
*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*sin(5*d
*x + 5*c) + 2*(3*(8*a*b^2 - 3*b^3)*cos(3*d*x + 3*c) - (8*a*b^2 - 3*b^3)*cos
(d*x + c))*sin(4*d*x + 4*c) - 3*(4*b^3*cos(2*d*x + 2*c) - b^3)*sin(3*d*x +
3*c))/(a^2*b^2*cos(8*d*x + 8*c)^2 + 16*a^2*b^2*cos(6*d*x + 6*c)^2 + 16*a^2*
b^2*cos(2*d*x + 2*c)^2 + a^2*b^2*sin(8*d*x + 8*c)^2 + 16*a^2*b^2*sin(6*d*x
+ 6*c)^2 + 16*a^2*b^2*sin(2*d*x + 2*c)^2 - 8*a^2*b^2*cos(2*d*x + 2*c) + a^2
*b^2 + 4*(64*a^4 - 48*a^3*b + 9*a^2*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^4 - 4
8*a^3*b + 9*a^2*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 3*a^2*b^2)*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a^2*b^2*cos(6*d*x + 6*c) + 4*a^2*b^2*cos(2
*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c))*cos(8*d*x
 + 8*c) + 8*(4*a^2*b^2*cos(2*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)
*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 3*a^2*b^2 - 4*(8*a^3*b -
 3*a^2*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a^2*b^2*sin(6*d*x + 6
*c) + 2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*
sin(8*d*x + 8*c) + 16*(2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*s
in(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + ((3*a + 8*b)*cos(8*d*x + 8*c)^2 +
16*(3*a + 8*b)*cos(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*cos(4*d*x + 4*c)^2 + 16*
(3*a + 8*b)*cos(2*d*x + 2*c)^2 + (3*a + 8*b)*sin(8*d*x + 8*c)^2 + 16*(3*a +
```

```
 8*b)*sin(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*sin(4*d*x + 4*c)^2 - 48*(3*a + 8*
b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(3*a + 8*b)*sin(2*d*x + 2*c)^2 -
2*(4*(3*a + 8*b)*cos(6*d*x + 6*c) - 6*(3*a + 8*b)*cos(4*d*x + 4*c) + 4*(3*a
 + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(8*d*x + 8*c) - 8*(6*(3*a + 8*b)*c
os(4*d*x + 4*c) - 4*(3*a + 8*b)*cos(2*d*x + 2*c) + 3*a + 8*b)*cos(6*d*x + 6
*c) - 12*(4*(3*a + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(4*d*x + 4*c) - 8*
(3*a + 8*b)*cos(2*d*x + 2*c) - 4*(2*(3*a + 8*b)*sin(6*d*x + 6*c) - 3*(3*a +
 8*b)*sin(4*d*x + 4*c) + 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) -
 16*(3*(3*a + 8*b)*sin(4*d*x + 4*c) - 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c) + 3*a + 8*b)*log(cos(d*x)^2 + 2*cos(d*x)*cos(c) + cos(c)^2 + si
n(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) - ((3*a + 8*b)*cos(8*d*x + 8*c)^2
+ 16*(3*a + 8*b)*cos(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*cos(4*d*x + 4*c)^2 + 1
6*(3*a + 8*b)*cos(2*d*x + 2*c)^2 + (3*a + 8*b)*sin(8*d*x + 8*c)^2 + 16*(3*a
 + 8*b)*sin(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*sin(4*d*x + 4*c)^2 - 48*(3*a +
8*b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(3*a + 8*b)*sin(2*d*x + 2*c)^2
- 2*(4*(3*a + 8*b)*cos(6*d*x + 6*c) - 6*(3*a + 8*b)*cos(4*d*x + 4*c) + 4*(3
*a + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(8*d*x + 8*c) - 8*(6*(3*a + 8*b)
*cos(4*d*x + 4*c) - 4*(3*a + 8*b)*cos(2*d*x + 2...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1089 vs. $2(179) = 358$.
time = 0.60, size = 1089, normalized size = 4.76



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^3 + 4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2
 + a^2*d)*sqrt(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)
) + b^3)/((a^5 - a^4*b)*d^2))*log(b^4*cos(d*x + c) + (a^2*b^3*d - (a^7 - a^
6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(-((a^5 - a^4*b)*d^
2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^3)/((a^5 - a^4*b)*d^2))) -
4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(((a^5 - a^4*
b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2)
)*log(b^4*cos(d*x + c) - (a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*
a^8*b + a^7*b^2)*d^4)))*sqrt(((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b +
a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2))) - 4*(a^2*d*cos(d*x + c)^4 - 2*a
^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^
8*b + a^7*b^2)*d^4)) + b^3)/((a^5 - a^4*b)*d^2))*log(-b^4*cos(d*x + c) + (a
^2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqr
t(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^3)/((a^
5 - a^4*b)*d^2))) + 4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*
d)*sqrt(((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)
/((a^5 - a^4*b)*d^2))*log(-b^4*cos(d*x + c) - (a^2*b^3*d + (a^7 - a^6*b)*d^
```

```
3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(((a^5 - a^4*b)*d^2*sqrt(b
^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2))) - 10*a*cos
(d*x + c) - ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a + 8*b)*cos(d*x + c)^2 + 3*
a + 8*b)*log(1/2*cos(d*x + c) + 1/2) + ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a
 + 8*b)*cos(d*x + c)^2 + 3*a + 8*b)*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d*co
s(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(csc(c + d*x)**5/(a - b*sin(c + d*x)**4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[76,51]
Warning, need to
```

Mupad [B]
time = 15.52, size = 2500, normalized size = 10.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(((((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a^
7*b^6 + 6144*a^9*b^4)/(64*a^5) - (cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^
4)*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(((a^9
*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(2304*a^4*
b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(1
6*(a^8*b - a^9)))^(1/2))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(
1/2) - (cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*(((a^9*b^5)
```

^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2)*1i - (((768*a^3*b^8 - 144*a^5*b
^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) +
(cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(((a^9*b^5)^(1/2) + a^4*b^3)/(
16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b
 - a^9)))^(1/2) - (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))
/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(((a^9*b
^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(48*a*b^8 +
96*b^9 + 9*a^2*b^7))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^
9)))^(1/2)*1i)/((((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 -
 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) - (cos(c + d*x)*(12288*a^8*b^5 - 81
92*a^9*b^4)*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4
))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(
2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))*(((a^9*b^5)^(1/2) + a^
4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b -
 a^9)))^(1/2) - (cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*((
(a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (((768*a^3*b^8 - 144
*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a
^5) + (cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(((a^9*b^5)^(1/2) + a^4*
b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*
(a^8*b - a^9)))^(1/2) - (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6
*b^5))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*((
(a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(48*a*
b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*
b - a^9)))^(1/2) + (9*a*b^8 + 24*b^9)/(32*a^5)))*(((a^9*b^5)^(1/2) + a^4*b^
3)/(16*(a^8*b - a^9)))^(1/2)*2i)/d + (atan(((((768*a^3*b^8 - 144*a^5*b^6)/(
64*a^5) + (((10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) - (cos(
c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(
a^8*b - a^9)))^(1/2))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b -
a^9)))^(1/2) + (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(1
6*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(-((a^9*b^
5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) - (cos(c + d*x)*(48*a*b^8 + 9
6*b^9 + 9*a^2*b^7))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^
9)))^(1/2)*1i - (((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 -
 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) + (cos(c + d*x)*(12288*a^8*b^5 - 81
92*a^9*b^4)*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^
4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) - (cos(c + d*x)
*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))*(-((a^9*b^5)^(1/2) -
 a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8
*b - a^9)))^(1/2) + (cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4)
)*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2)*1i)/((((768*a^3*b
^8 - 144*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^
4)/(64*a^5) - (cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(-((a^9*b^5)^(1/
2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4
b^3)/(16(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6
 + 144*a^6*b^5))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9))

```
)^(1/2))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) - (cos(c +
 d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b
^3)/(16*(a^8*b - a^9)))^(1/2) + (((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + ((
(10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) + (cos(c + d*x)*(12
288*a^8*b^5 - 8192*a^9*b^4)*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)
))^(1/2))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2)
 - (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))*(-((
a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(-((a^9*b^5)^(1/2) - a
^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^
2*b^7))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) +
 (9*a*b^8 + 24*b^9)/(32*a^5)))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a
^9)))^(1/2)*2i)/d - ((5*cos(c + d*x))/(8*a) - (3*cos(c + d*x)^3)/(8*a))/(d*
(cos(c + d*x)^4 - cos(c + d*x)^2 + sin(c + d*x)^2)) - (atan(((((cos(c + d*x
)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4) - (((12*a^3*b^8 - (9*a^5*b^6)/4
)/a^5 + ((3*a + 8*b)*((((160*a^8*b^5 - 192*a^7*...
```

3.203 $\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{5x}{8b}-\frac{(a+b)x}{b^2}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^2d}+\frac{5\cos(c+dx}{8}$$

```
[Out] 5/8*x/b-(a+b)*x/b^2+5/8*cos(d*x+c)*sin(d*x+c)/b/d-1/4*cos(d*x+c)^3*sin(d*x+
c)/b/d+1/2*a^(5/4)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^2/d
/(a^(1/2)-b^(1/2))^(1/2)+1/2*a^(5/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x
+c)/a^(1/4))/b^2/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3296, 1301, 205, 209, 1180, 211}

$$\frac{a^{5/4}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{5/4}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x(a+b)}{b^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}+\frac{5\sin(c+dx)\cos(c+dx)}{8bd}+\frac{5x}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (5*x)/(8*b) - ((a + b)*x)/b^2 + (a^(5/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Ta
n[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(5/4)*ArcTan[(
Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*
b^2*d) + (5*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]^3*Sin[c + d*
x])/(4*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)^3(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)^3}+\frac{2}{b(1+x^2)^2}+\frac{-a-b}{b^2(1+x^2)}+\frac{a^2(1+x^2)}{b^2(a+2ax^2+(a-b)x^4)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a^2\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{b^2d} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, \tan(c+d\right.}{bd}$$

$$= -\frac{(a+b)x}{b^2}+\frac{\cos(c+dx)\sin(c+dx)}{bd}-\frac{\cos^3(c+dx)\sin(c+dx)}{4bd}+\frac{\left(a^{3/2}\left(\sqrt{a}\right.\right.}{}$$

$$= \frac{x}{b}-\frac{(a+b)x}{b^2}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+}}{}\right.}{2\sqrt{\sqrt{a}+\sqrt{}}}$$

$$= \frac{5x}{8b}-\frac{(a+b)x}{b^2}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+}}{}\right.}{2\sqrt{\sqrt{a}+\sqrt{}}}$$

Mathematica [A]
time = 0.63, size = 172, normalized size = 0.93

$$-\frac{4(8a+3b)(c+dx)-\frac{16a^{3/2}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{16a^{3/2}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}-8b\sin(2(c+dx))+b\sin(4(c+dx))}{32b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/32*(4*(8*a + 3*b)*(c + d*x) - (16*a^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Ta
n[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (16*a^(
3/2)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]
)/Sqrt[-a + Sqrt[a]*Sqrt[b]] - 8*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(
b^2*d)
```

Maple [A]
time = 0.39, size = 209, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{a^2(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right.}{b^2}}{d}$
default	$\frac{\frac{a^2(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right.}{b^2}}{d}$
risch	$-\frac{ax}{b^2}-\frac{3x}{8b}-\frac{i\mathrm{e}^{2i(dx+c)}}{8bd}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8bd}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a\,b^8d^4-b^9d^4\right)\_Z^4+8192a^3b^4d^2\_Z^2+16777216a^5\right)}\_R\ln\left(\mathrm{e}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/b^2*(a-b)*(1/2*((a*b)^(1/2)-b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*
(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*(
(a*b)^(1/2)+b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)
*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2)))-1/b^2*((-5/8*b*tan(d*x+c)^3-3/8
*b*tan(d*x+c))/(tan(d*x+c)^2+1)^2+1/8*(8*a+3*b)*arctan(tan(d*x+c))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/32*(512*a^2*b^2*d*integrate((b*cos(8*d*x + 8*c)*cos(4*d*x + 4*c) - 4*b*c
os(6*d*x + 6*c)*cos(4*d*x + 4*c) - 2*(8*a - 3*b)*cos(4*d*x + 4*c)^2 + b*sin
(8*d*x + 8*c)*sin(4*d*x + 4*c) - 4*b*sin(6*d*x + 6*c)*sin(4*d*x + 4*c) - 2*
```

```
(8*a - 3*b)*sin(4*d*x + 4*c)^2 - 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (4
*b*cos(2*d*x + 2*c) - b)*cos(4*d*x + 4*c))/(b^4*cos(8*d*x + 8*c)^2 + 16*b^4
*cos(6*d*x + 6*c)^2 + 16*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(8*d*x + 8*c)^2 +
16*b^4*sin(6*d*x + 6*c)^2 + 16*b^4*sin(2*d*x + 2*c)^2 - 8*b^4*cos(2*d*x + 2
*c) + b^4 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^2 + 4*(64*a^
2*b^2 - 48*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^3 - 3*b^4)*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^4*cos(6*d*x + 6*c) + 4*b^4*cos(2*d*x +
2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*
b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(6*d*
x + 6*c) - 4*(8*a*b^3 - 3*b^4 - 4*(8*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*cos(4
*d*x + 4*c) - 4*(2*b^4*sin(6*d*x + 6*c) + 2*b^4*sin(2*d*x + 2*c) + (8*a*b^3
 - 3*b^4)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^4*sin(2*d*x + 2*c) +
 (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + 4*(8*a + 3*b)*
d*x + b*sin(4*d*x + 4*c) - 8*b*sin(2*d*x + 2*c))/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1311 vs. $2(142) = 284$.
time = 0.56, size = 1311, normalized size = 7.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*d*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4)) + a^3)/((a*b^4 - b^5)*d^2))*log(1/4*a^3*cos(d*x + c)^2 - 1/4*a^3 - 1/4*
(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4)*d^2)*sqrt(a^5/(
(a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + 1/2*(a^2*b^2*d*cos(d*x + c)*sin(d*x + c)
- (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4))*cos(d*x + c)*
sin(d*x + c))*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*
d^4)) + a^3)/((a*b^4 - b^5)*d^2))) - b^2*d*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^
5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^3)/((a*b^4 - b^5)*d^2))*log(1/4*a^3*
cos(d*x + c)^2 - 1/4*a^3 - 1/4*(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a
^2*b^3 - a*b^4)*d^2)*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - 1/2*(a^2*b
^2*d*cos(d*x + c)*sin(d*x + c) - (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a
*b^8 + b^9)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a*b^4 - b^5)*d^2*sqrt(
a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^3)/((a*b^4 - b^5)*d^2))) - b^2*d*s
qrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^3)/((a
*b^4 - b^5)*d^2))*log(-1/4*a^3*cos(d*x + c)^2 + 1/4*a^3 - 1/4*(2*(a^2*b^3 -
 a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4)*d^2)*sqrt(a^5/((a^2*b^7 - 2*
a*b^8 + b^9)*d^4)) + 1/2*(a^2*b^2*d*cos(d*x + c)*sin(d*x + c) + (a*b^5 - b^
6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4))*cos(d*x + c)*sin(d*x + c))
*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^3)/(
(a*b^4 - b^5)*d^2))) + b^2*d*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2
*a*b^8 + b^9)*d^4)) - a^3)/((a*b^4 - b^5)*d^2))*log(-1/4*a^3*cos(d*x + c)^2
```

```
 + 1/4*a^3 - 1/4*(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4
)*d^2)*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - 1/2*(a^2*b^2*d*cos(d*x +
 c)*sin(d*x + c) + (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4))*cos(d*x + c)*sin(d*x + c))*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 -
 2*a*b^8 + b^9)*d^4)) - a^3)/((a*b^4 - b^5)*d^2))) + (8*a + 3*b)*d*x + (2*b
*cos(d*x + c)^3 - 5*b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(142) = 284.
time = 1.11, size = 461, normalized size = 2.51

$$\frac{\frac{4\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^3-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^2b-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{ab^2+\sqrt{a^2b^4-(ab^2-b^3)ab^2}}{ab^2-b^3}}}\right)\right)|-a+b|}{3a^4b^2-12a^3b^3+14a^2b^4-4ab^5-b^6}+\frac{4\left(3\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,a^3-6\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,a^2b-\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{ab^2-\sqrt{a^2b^4-(ab^2-b^3)ab^2}}{ab^2-b^3}}}\right)\right)|-a+b|}{3a^4b^2-12a^3b^3+14a^2b^4-4ab^5-b^6}-\frac{(dx+c)(8a+3b)}{b^2}+\frac{5\tan(dx+c)^3+3\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2b}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(4*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3 - 6*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a^2*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^2)*(pi*floor
((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2 + sqrt(a^2*b^4 - (a*
b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^4*b^2 - 12*a^3*b^3 + 1
4*a^2*b^4 - 4*a*b^5 - b^6) + 4*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3 -
 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b - sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*a*b^2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*
b^2 - sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*
a^4*b^2 - 12*a^3*b^3 + 14*a^2*b^4 - 4*a*b^5 - b^6) - (d*x + c)*(8*a + 3*b)/
b^2 + (5*tan(d*x + c)^3 + 3*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*b))/d
```

Mupad [B]
time = 16.87, size = 2500, normalized size = 13.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(
64*b^5) - (tan(c + d*x)*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(
1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16
*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d
*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5
 + 2304*a^7*b^4))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)
))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a
^7*b^4 - 768*a^8*b^3)/(64*b^5))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 -
b^9)))^(1/2) + (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 +
 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*
b^8 - b^9)))^(1/2)*1i - (((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 +
 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*
(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12
288*a^5*b^8))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(
1/2) + (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b
^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/
(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648
*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(-((a^5*b^9)^(1/2) + a^3*b
^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8
*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(-((a^5*b^9)^(1/2) +
 a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*1i)/((((((2048*a^3*b^10 + 8192*a^4*b^9
- 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(-((a^5*b^9)^(1/2
) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 1
2288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*
(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4
*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(-((a^5*b^9)^
(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 1
12*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(-((a^5*b
^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(9*a^4*b^5 -
 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(-(
(a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (((((2048*a^3*b^10 +
 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*(-(
(a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 1228
8*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(-((a^5*b^9)^(1/2) +
 a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3
*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))
*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 6
24*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b
^5))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x
)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))
/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (63*a^
8*b - 216*a^9 + 27*a^6*b^3 + 126*a^7*b^2)/(32*b^5)))*(-((a^5*b^9)^(1/2) + a
^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*2i)/d + (atan((((((2048*a^3*b^10 + 8192*
a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(((a^5*b^
9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b
```

^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4
)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 8
80*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(((a^5*
b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^
7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(((a
^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(9*a^4*b
^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))
*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*1i - (((((2048*a^3*
b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) + (tan(c + d*
x)*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 -
 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(((a^5*b^9)^(1/
2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(432*a^2*b^9 + 1584
*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b
^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8
 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(6
4*b^5))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d
x)(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2
))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*1i)/(((
(((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) -
 (tan(c + d*x)*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(1228
8*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(((
a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(...

3.204 $\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{x}{2b}+\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/2}d}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/2}d}+\frac{\cos(c+dx)\sin(c+d}{2bd}$$

```
[Out] -1/2*x/b+1/2*cos(d*x+c)*sin(d*x+c)/b/d+1/2*a^(3/4)*arctan((a^(1/2)-b^(1/2))
^(1/2)*tan(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*a^(3/4)*ar
ctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1/2)
)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3296, 1301, 205, 209, 1144, 211}

$$\frac{a^{3/4}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/4}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*x/b + (a^(3/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])
/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) - (a^(3/4)*ArcTan[(Sqrt[Sqrt[a] + Sq
rt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) + (Cos
[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b(1+x^2)^2} - \frac{1}{b(1+x^2)} + \frac{ax^2}{b(a+2ax^2+(a-b)x^4)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd} + \frac{a\text{Sub}}{}$$

$$= -\frac{x}{b} + \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\left(a\left(\sqrt{a}+\sqrt{b}\right)\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\,\sqrt{b}+(a-b)x^2}\right.}{2b^{3/2}d}$$

$$= -\frac{x}{2b} + \frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/2}d} - \frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c}{\sqrt[4]{a}}\right.}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/2}d}$$

Mathematica [A]
time = 0.58, size = 157, normalized size = 1.01

$$\frac{-2\sqrt{b}\,(c+dx) - \frac{2a\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}} - \frac{2a\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}} + \sqrt{b}\,\sin(2(c+dx))}{4b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-2*Sqrt[b]*(c + d*x) - (2*a*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt
[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (2*a*ArcTanh[((Sqrt[a]
- Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqr
t[b]] + Sqrt[b]*Sin[2*(c + d*x)])/(4*b^(3/2)*d)
```

Maple [A]
time = 0.36, size = 186, normalized size = 1.20

method	result
risch	$-\frac{x}{2b} - \frac{i\mathrm{e}^{2i(dx+c)}}{8bd} + \frac{i\mathrm{e}^{-2i(dx+c)}}{8bd} - \frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a\,b^6d^4-b^7d^4\right)\_Z^4+512a^2b^3d^2\_Z^2+65536a^3\right)} \_R\ln\left(\mathrm{e}^{2i(dx+c)}+\right.\right.}{64}$

derivativedivides	$\frac{-\frac{-\frac{\tan(dx+c)}{2(\tan^2(dx+c)+1)}+\frac{\arctan(\tan(dx+c))}{2}}{b}+\frac{a(a-b)\left(\frac{(\sqrt{ab}+a)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{(\sqrt{ab}+a)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{(\sqrt{ab}+a)(a-b)}}+\frac{(\sqrt{ab}-a)\,\mathrm{a}}{2\sqrt{ab}\,(}\right.}{b}}{d}$
default	$\frac{-\frac{-\frac{\tan(dx+c)}{2(\tan^2(dx+c)+1)}+\frac{\arctan(\tan(dx+c))}{2}}{b}+\frac{a(a-b)\left(\frac{(\sqrt{ab}+a)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{(\sqrt{ab}+a)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{(\sqrt{ab}+a)(a-b)}}+\frac{(\sqrt{ab}-a)\,\mathrm{a}}{2\sqrt{ab}\,(}\right.}{b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*(-1/2*tan(d*x+c)/(tan(d*x+c)^2+1)+1/2*arctan(tan(d*x+c)))+1/b*a*(
a-b)*(1/2*((a*b)^(1/2)+a)/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*a
rctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*((a*b)^(1/2)-a)/(
a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/((
(a*b)^(1/2)-a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

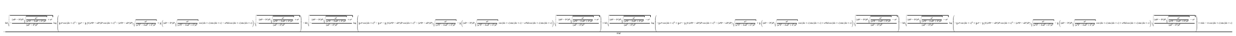
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b*d*integrate(-4*(4*a*b*cos(6*d*x + 6*c)^2 + 4*a*b*cos(2*d*x + 2*c)^
2 + 4*a*b*sin(6*d*x + 6*c)^2 + 4*a*b*sin(2*d*x + 2*c)^2 - 4*(8*a^2 - 3*a*b)
*cos(4*d*x + 4*c)^2 - a*b*cos(2*d*x + 2*c) - 4*(8*a^2 - 3*a*b)*sin(4*d*x +
4*c)^2 + 2*(8*a^2 - 7*a*b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (a*b*cos(6*d
*x + 6*c) - 2*a*b*cos(4*d*x + 4*c) + a*b*cos(2*d*x + 2*c))*cos(8*d*x + 8*c)
 + (8*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 7*a*b)*cos(4*d*x + 4*c))*cos(
6*d*x + 6*c) + 2*(a*b + (8*a^2 - 7*a*b)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c)
- (a*b*sin(6*d*x + 6*c) - 2*a*b*sin(4*d*x + 4*c) + a*b*sin(2*d*x + 2*c))*si
```

```
n(8*d*x + 8*c) + 2*(4*a*b*sin(2*d*x + 2*c) + (8*a^2 - 7*a*b)*sin(4*d*x + 4*
c))*sin(6*d*x + 6*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 +
 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*
c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2
*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)
*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 -
 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^
3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3
*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*si
n(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c
))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*
d*x + 4*c))*sin(6*d*x + 6*c)), x) - 2*d*x + sin(2*d*x + 2*c))/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(111) = 222$.
time = 0.61, size = 1275, normalized size = 8.23



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) + a^2)/((a*b^3 - b^4)*d^2))*log(1/4*a^2*cos(d*x + c)^2 - 1/4*a^2 - 1/4*(2
*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^3/((a
^2*b^5 - 2*a*b^6 + b^7)*d^4)) + 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 -
 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d*cos(d*x + c)*sin(
d*x + c))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) + a^2)/((a*b^3 - b^4)*d^2))) - b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))*log(1/4*a^2*cos(d*
x + c)^2 - 1/4*a^2 - 1/4*(2*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2
 - a*b^3)*d^2)*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - 1/2*((a*b^4 - b^
5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c)
- a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + b*d*sqrt(((a*b^
3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4
)*d^2))*log(-1/4*a^2*cos(d*x + c)^2 + 1/4*a^2 - 1/4*(2*(a^2*b^2 - a*b^3)*d^
2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^
7)*d^4)) + 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a*b^
3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4
)*d^2))) - b*d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*
d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(-1/4*a^2*cos(d*x + c)^2 + 1/4*a^2 - 1
/4*(2*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^
```

```
3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*
b^5 - 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)
*sin(d*x + c))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*
d^4)) - a^2)/((a*b^3 - b^4)*d^2))) + 4*d*x - 4*cos(d*x + c)*sin(d*x + c))/(
b*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. 2(111) = 222.
time = 1.26, size = 695, normalized size = 4.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/b + ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 -
 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a^3*b - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*a^2*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3)*abs(-a +
 b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b + sqrt(a
^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^2))))/((3*a^5*b^2 - 15*a^4*b^3 + 26*a^3
*b^4 - 18*a^2*b^5 + 3*a*b^6 + b^7)*abs(b)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a
*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (
3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 6*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
sqrt(a*b)*a*b^3)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*
x + c)/sqrt((a*b - sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^2))))/((3*a^5*
b^2 - 15*a^4*b^3 + 26*a^3*b^4 - 18*a^2*b^5 + 3*a*b^6 + b^7)*abs(b)) - tan(d
*x + c)/((tan(d*x + c)^2 + 1)*b))/d
```

Mupad [B]
time = 16.40, size = 1273, normalized size = 8.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^6/(a - b*sin(c + d*x)^4),x)

[Out] sin(2*c + 2*d*x)/(4*b*d) - (atan((a*b^7*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*4i - b^12*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(5/2)*3072i - b^10*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a*b^9*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a^2*b^6*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*24i + a^3*b^5*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*4i + a^4*b^4*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*8i + a^2*b^8*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*448i + a^3*b^7*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*320i + a^2*b^10*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(5/2)*3072i)/(a^2*b^5*cos(c + d*x) + a^3*b^4*cos(c + d*x) - a^4*b^3*cos(c + d*x) - a^2*cos(c + d*x)*(a^3*b^7)^(1/2) + 2*a*b*cos(c + d*x)*(a^3*b^7)^(1/2)))*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*2i)/d - (atan((a*b^7*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*4i - b^12*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(5/2)*3072i - b^10*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a*b^9*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a^2*b^6*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*24i + a^3*b^5*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*4i + a^4*b^4*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*8i + a^2*b^8*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*448i + a^3*b^7*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*320i + a^2*b^10*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(5/2)*3072i)/(a^2*b^5*cos(c + d*x) + a^3*b^4*cos(c + d*x) - a^4*b^3*cos(c + d*x) + a^2*cos(c + d*x)*(a^3*b^7)^(1/2) - 2*a*b*cos(c + d*x)*(a^3*b^7)^(1/2)))*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*2i)/d - atan(sin(c + d*x)/cos(c + d*x))/(2*b*d)

3.205 $\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=127

$$-\frac{x}{b}+\frac{\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,bd}+\frac{\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,bd}$$

```
[Out] -x/b+1/2*a^(1/4)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b/d/(a^
(1/2)-b^(1/2))^(1/2)+1/2*a^(1/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/
a^(1/4))/b/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1301, 209, 1180, 211}

$$\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(x/b) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(
2*Sqrt[Sqrt[a] - Sqrt[b]]*b*d) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)}+\frac{a(1+x^2)}{b(a+2ax^2+(a-b)x^4)}\right)\,dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd}+\frac{a\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right.}{bd}\\
&= -\frac{x}{b}+\frac{\left(a\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2bd}+\frac{\left(a\left(\right.\right.}{}\\
&= -\frac{x}{b}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,bd}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c\right.}{\sqrt[4]{a}}}{2\sqrt{\sqrt{a}+\sqrt{b}}\,bd}
\end{aligned}$$

Mathematica [A]

time = 0.28, size = 143, normalized size = 1.13

$$\frac{-2(c+dx)+\frac{\sqrt{a}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-2*(c + d*x) + (Sqrt[a]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
 Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (Sqrt[a]*ArcTanh[((Sqrt[a]
- Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqr
t[b]])/(2*b*d)
```

Maple [A]

time = 0.36, size = 163, normalized size = 1.28

method	result
risch	$-\frac{x}{b}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a\,b^4d^4-b^5d^4\right)\_Z^4+32a\,b^2d^2\_Z^2+256a\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(\frac{1}{32}ia\,b^2d^3-\frac{1}{32}ib^3d^3\right)\_R^3+\left(-\frac{1}{8}b\,d^2 \right.\right.\right.}{16}$
derivativedivides	$\frac{-\frac{\arctan(\tan(dx+c))}{b}+\frac{a(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)}{\sqrt{\left(\sqrt{ab}\right.}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+\right.}}\right.}{b}}{d}$
default	$\frac{-\frac{\arctan(\tan(dx+c))}{b}+\frac{a(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)}{\sqrt{\left(\sqrt{ab}\right.}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+\right.}}\right.}{b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*arctan(tan(d*x+c))+1/b*a*(a-b)*(1/2*((a*b)^(1/2)-b)/(a*b)^(1/2)/(
a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-
a)*(a-b))^(1/2))+1/2*((a*b)^(1/2)+b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-
b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -(16*a*b*integrate((b*cos(8*d*x + 8*c)*cos(4*d*x + 4*c) - 4*b*cos(6*d*x + 6
*c)*cos(4*d*x + 4*c) - 2*(8*a - 3*b)*cos(4*d*x + 4*c)^2 + b*sin(8*d*x + 8*c
)*sin(4*d*x + 4*c) - 4*b*sin(6*d*x + 6*c)*sin(4*d*x + 4*c) - 2*(8*a - 3*b)*
sin(4*d*x + 4*c)^2 - 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (4*b*cos(2*d*x
 + 2*c) - b)*cos(4*d*x + 4*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x +
 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6
*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 +
4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2
 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(
8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x +
2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*
a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*
(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*
d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^
3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + x)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. 2(91) = 182.
time = 0.53, size = 1125, normalized size = 8.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a
)/((a*b^2 - b^3)*d^2))*log(1/4*cos(d*x + c)^2 + 1/2*((a*b^2 - b^3)*d^3*sqrt
(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) - b*d*cos(d*x
 + c)*sin(d*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b
^5)*d^4)) + a)/((a*b^2 - b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2
```

```
 - (a*b - b^2)*d^2)*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1/4) - b*sqrt
(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a)/((a*b^2 -
 b^3)*d^2))*log(1/4*cos(d*x + c)^2 - 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^
3 - 2*a*b^4 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) - b*d*cos(d*x + c)*sin(d
*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) +
 a)/((a*b^2 - b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*b - b
^2)*d^2)*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1/4) + b*sqrt(((a*b^2 -
b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*
log(-1/4*cos(d*x + c)^2 + 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4
 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) + b*d*cos(d*x + c)*sin(d*x + c))*sq
rt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2
- b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*b - b^2)*d^2)*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1/4) - b*sqrt(((a*b^2 - b^3)*d^2*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-1/4*cos
(d*x + c)^2 - 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)
)*cos(d*x + c)*sin(d*x + c) + b*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a*b^2 -
 b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))
 - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*b - b^2)*d^2)*sqrt(a/((a^2*b^
3 - 2*a*b^4 + b^5)*d^4)) + 1/4) - 8*x)/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(91) = 182.
time = 0.91, size = 912, normalized size = 7.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/b + ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2
 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a^3*b - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 + 5*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*b^4)*abs(-a + b)*abs(b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^2*b^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a
```

```
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x +
c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b + sqrt(a^2*b^2 - (a*b - b^2)*a
*b))/(a*b - b^2))))/((3*a^5*b^2 - 15*a^4*b^3 + 26*a^3*b^4 - 18*a^2*b^5 + 3*
a*b^6 + b^7)*abs(b)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^
2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) + (3*sqrt(a^2 - a*b - sqr
t(a*b)*(a - b))*a^3*b - 9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 + 5*s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^3 + sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*b^4)*abs(-a + b)*abs(b) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a^2*b^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x +
 c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b - sqrt(a^2*b^2 - (a*b - b^2)*
a*b))/(a*b - b^2))))/((3*a^5*b^2 - 15*a^4*b^3 + 26*a^3*b^4 - 18*a^2*b^5 + 3
*a*b^6 + b^7)*abs(b)))/d
```

Mupad [B]
time = 16.27, size = 2991, normalized size = 23.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - b*sin(c + d*x)^4),x)
```

```
[Out] - atan((18*a^5*tan(c + d*x))/(18*a^5 - 50*a^4*b + 32*a^3*b^2) - (50*a^4*tan
(c + d*x))/(32*a^3*b - 50*a^4 + (18*a^5)/b) + (32*a^3*b*tan(c + d*x))/(32*a
^3*b - 50*a^4 + (18*a^5)/b))/(b*d) - (atan((((-(a*b^2 - (a*b^5)^(1/2))/(16*
(a*b^4 - b^5)))^(1/2)*(((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)
*(320*a^3*b^5 - 64*a^2*b^6 - 448*a^4*b^4 + 192*a^5*b^3 + tan(c + d*x)*(-(a*
b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 -
 768*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80
*a^4*b^3 + 144*a^5*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2
) + 12*a^5*b - 16*a^2*b^4 + 28*a^3*b^3 - 24*a^4*b^2) + tan(c + d*x)*(18*a^4
*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4
- b^5)))^(1/2)*1i + ((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*((
(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(64*a^2*b^6 - 320*a^3*b
^5 + 448*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 - (a*b^5)^(1/2))/(16
*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b
^4)) + tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2))
*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - 12*a^5*b + 16*a^2*b^
4 - 28*a^3*b^3 + 24*a^4*b^2) + tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 -
 20*a^3*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*1i)/(6*a^
3*b - 6*a^4 + ((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(((-(a*b
^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(320*a^3*b^5 - 64*a^2*b^6 - 4
48*a^4*b^4 + 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^
4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) +
```

tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) + 12*a^5*b - 16*a^2*b^4 + 28*a^3*b^3 - 24*a^4*b^2) + tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - ((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(64*a^2*b^6 - 320*a^3*b^5 + 448*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - 12*a^5*b + 16*a^2*b^4 - 28*a^3*b^3 + 24*a^4*b^2) + tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*2i)/d - (atan(((tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*((tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(320*a^3*b^5 - 64*a^2*b^6 - 448*a^4*b^4 + 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) + 12*a^5*b - 16*a^2*b^4 + 28*a^3*b^3 - 24*a^4*b^2))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*1i + (tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*((tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(64*a^2*b^6 - 320*a^3*b^5 + 448*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - 12*a^5*b + 16*a^2*b^4 - 28*a^3*b^3 + 24*a^4*b^2))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*1i)/((tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*((tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(320*a^3*b^5 - 64*a^2*b^6 - 448*a^4*b^4 + 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) + 12*a^5*b - 16*a^2*b^4 + 28*a^3*b^3 - 24*a^4*b^2))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - (tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*((tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(64*a^2*b^6 - 320*a^3*b^5 + 448*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - 12*a^5*b + 16*a^2*b^4 - 28*a^3*b^3 + 24*a^4*b^2))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) + 6*a^3*b - 6*a^4))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*2i)/d

3.206 $\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}\,d}$$

```
[Out] 1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(1/4)/d/b^(1/2)/(a
^(1/2)-b^(1/2))^(1/2)-1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4)
)/a^(1/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3296, 1144, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt
[a] - Sqrt[b]]*Sqrt[b]*d) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a
^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subs}}{}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}\,d} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}\,d}$$

Mathematica [A]
time = 0.22, size = 137, normalized size = 1.10

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}\,d} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(
Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]*d) - ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c +
 d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]*d)
```

Maple [A]
time = 0.38, size = 145, normalized size = 1.16

method	result

risch	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^2b^2d^4-a\,b^3d^4\right)\_Z^4+2a\,d^2\_Z^2b\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(2ia^2b\,d^3-2ia\,b^2d^3\right)\_R^3+\left(-2a^2d^2+2b\,d^2a\right)\right.\right.}{4}$
derivativedivides	$\frac{(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{2\sqrt{ab}(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{d}$
default	$\frac{(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{2\sqrt{ab}(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a-b)*(1/2*((a*b)^(1/2)+a)/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1
/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*((a*b)^(1/2)
-a)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+
c)/(((a*b)^(1/2)-a)*(a-b))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sin(d*x + c)^2/(b*sin(d*x + c)^4 - a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(85) = 170$.
time = 0.53, size = 1087, normalized size = 8.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/
((a*b - b^2)*d^2))*log(1/4*cos(d*x + c)^2 + 1/2*((a^2*b - a*b^2)*d^3*sqrt(1
/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) - a*d*cos(d*x
 + c)*sin(d*x + c))*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b
^3)*d^4)) + 1)/((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 -
 (a^2 - a*b)*d^2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1/4) + 1/8*sq
rt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a*b -
 b^2)*d^2))*log(1/4*cos(d*x + c)^2 - 1/2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*
b - 2*a^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) - a*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4
)) + 1)/((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 - (a^2 -
 a*b)*d^2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1/4) - 1/8*sqrt(((a*
b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^
2))*log(-1/4*cos(d*x + c)^2 + 1/2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a
^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c)*sin(d*x
+ c))*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/
((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 - (a^2 - a*b)*d^
2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1/4) + 1/8*sqrt(((a*b - b^2)
*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))*log(
-1/4*cos(d*x + c)^2 - 1/2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 +
 a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c)*sin(d*x + c))*sq
rt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b -
b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 - (a^2 - a*b)*d^2)*sqrt(
1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1/4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(85) = 170.
time = 1.00, size = 397, normalized size = 3.18

$$-\frac{\frac{\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a+\sqrt{-16(a-b)a+16a^2}}{a-b}}}\right)\right)|a-b|}{3a^5b-12a^4b^2+14a^3b^3-4a^2b^4-ab^5}+\frac{\left(3\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}a^2-6\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}ab-\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a-\sqrt{-16(a-b)a+16a^2}}{a-b}}}\right)\right)|a-b|}{3a^5b-12a^4b^2+14a^3b^3-4a^2b^4-ab^5}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a
*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))
*sqrt(a*b)*b^2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt(
(4*a + sqrt(-16*(a - b)*a + 16*a^2))/(a - b))))*abs(a - b)/(3*a^5*b - 12*a^
4*b^2 + 14*a^3*b^3 - 4*a^2*b^4 - a*b^5) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b -
 sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*(pi*floor((d*x + c)/pi
+ 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/(a
 - b))))*abs(a - b)/(3*a^5*b - 12*a^4*b^2 + 14*a^3*b^3 - 4*a^2*b^4 - a*b^5)
)/d
```

Mupad [B]
time = 16.19, size = 443, normalized size = 3.54

$$\frac{\ln\left(a\,b-a^2-\frac{a\tan(c+d\,x)\,(a-b)\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}\,\left(2\,a\,b^2+a\,\sqrt{a\,b^3}+b\,\sqrt{a\,b^3}\right)}{a\,b+\sqrt{a\,b^3}}\right)\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}}{4\,d}-\frac{\ln\left(a\,b-a^2-\frac{a\tan(c+d\,x)\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}\,(a-b)\,\left(a\,\sqrt{a\,b^3}-2\,a\,b^2+b\,\sqrt{a\,b^3}\right)}{a\,b-\sqrt{a\,b^3}}\right)\,\sqrt{\frac{a\,b+\sqrt{a\,b^3}}{16\,\left(a\,b^3-a^2\,b^2\right)}}}{d}+\frac{\ln\left(a\,b-a^2+\frac{a\tan(c+d\,x)\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}\,(a-b)\,\left(a\,\sqrt{a\,b^3}-2\,a\,b^2+b\,\sqrt{a\,b^3}\right)}{a\,b-\sqrt{a\,b^3}}\right)\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}}{4\,d}-\frac{\ln\left(a\,b-a^2+\frac{a\tan(c+d\,x)\,(a-b)\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}\,\left(2\,a\,b^2+a\,\sqrt{a\,b^3}+b\,\sqrt{a\,b^3}\right)}{a\,b+\sqrt{a\,b^3}}\right)\,\sqrt{\frac{a\,b-\sqrt{a\,b^3}}{16\,\left(a\,b^3-a^2\,b^2\right)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (log(a*b - a^2 - (a*tan(c + d*x)*(a - b)*(-1/(a*b + (a*b^3)^(1/2)))^(1/2)*(
2*a*b^2 + a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2)))/(a*b + (a*b^3)^(1/2)))*(-1/(a
*b + (a*b^3)^(1/2)))^(1/2))/(4*d) - (log(a*b - a^2 - (a*tan(c + d*x)*(-1/(a
*b - (a*b^3)^(1/2)))^(1/2)*(a - b)*(a*(a*b^3)^(1/2) - 2*a*b^2 + b*(a*b^3)^(
1/2)))/(a*b - (a*b^3)^(1/2)))*((a*b + (a*b^3)^(1/2))/(16*(a*b^3 - a^2*b^2))
)^(1/2))/d + (log(a*b - a^2 + (a*tan(c + d*x)*(-1/(a*b - (a*b^3)^(1/2)))^(1
/2)*(a - b)*(a*(a*b^3)^(1/2) - 2*a*b^2 + b*(a*b^3)^(1/2)))/(a*b - (a*b^3)^(
1/2)))*(-1/(a*b - (a*b^3)^(1/2)))^(1/2))/(4*d) - (log(a*b - a^2 + (a*tan(c
+ d*x)*(a - b)*(-1/(a*b + (a*b^3)^(1/2)))^(1/2)*(2*a*b^2 + a*(a*b^3)^(1/2)
+ b*(a*b^3)^(1/2)))/(a*b + (a*b^3)^(1/2)))*((a*b - (a*b^3)^(1/2))/(16*(a*b^
3 - a^2*b^2)))^(1/2))/d
```

3.207 $\int \frac{1}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

```
[Out] 1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-b
^(1/2))^(1/2)+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4
)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3288, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[c + d*x]^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sqrt
[a] - Sqrt[b]]*d) + ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/
(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\,\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Sub}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

Mathematica [A]
time = 0.17, size = 128, normalized size = 1.11

$$\frac{\frac{\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[c + d*x]^4)^(-1),x]
```

```
[Out] (ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/Sqrt[
a + Sqrt[a]*Sqrt[b]] - ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a +
 Sqrt[a]*Sqrt[b]]]/Sqrt[-a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*d)
```

Maple [A]
time = 0.35, size = 145, normalized size = 1.26

method	result
risch	$\sum_{\_R=\text{RootOf}\left(1+(256a^4d^4-256a^3b\,d^4)\_Z^4+32a^2d^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(\frac{128id^3a^4}{b}-128ia^3d^3\right)\_R^{3}\right.$

derivativedivides	$\frac{(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right.}{d}$
default	$\frac{(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a-b)*(1/2*((a*b)^(1/2)-b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1
/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*((a*b)^(1/
2)+b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+
c)/(((a*b)^(1/2)+a)*(a-b))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(d*x + c)^4 - a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(79) = 158$.
time = 0.54, size = 1079, normalized size = 9.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a
^2 - a*b)*d^2))*log(1/4*b*cos(d*x + c)^2 + 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((
a^5 - 2*a^4*b + a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a*b*d*cos(d*x +
c)*sin(d*x + c))*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d
^4)) + 1)/((a^2 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a
^3 - a^2*b)*d^2)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1/4*b) - 1/8*sqr
t(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a^2 - a*
b)*d^2))*log(1/4*b*cos(d*x + c)^2 - 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2
*a^4*b + a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a*b*d*cos(d*x + c)*sin(
d*x + c))*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +
1)/((a^2 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a^3 - a^
2*b)*d^2)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1/4*b) + 1/8*sqrt(((a^2
 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))
*log(-1/4*b*cos(d*x + c)^2 + 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b
+ a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) + a*b*d*cos(d*x + c)*sin(d*x + c
))*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2
 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a^3 - a^2*b)*d^2
)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1/4*b) - 1/8*sqrt(((a^2 - a*b)*
d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-1/
4*b*cos(d*x + c)^2 - 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^
2)*d^4))*cos(d*x + c)*sin(d*x + c) + a*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(
((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*
d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a^3 - a^2*b)*d^2)*sqrt(b
/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1/4*b)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(79) = 158.
time = 0.59, size = 361, normalized size = 3.14

$$\frac{\frac{\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^2-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,ab-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a+\sqrt{-16(a-b)a+16a^2}}{a-b}}}\right)\right)|a-b|}{3a^5-12a^4b+14a^3b^2-4a^2b^3-ab^4}+\frac{\left(3\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,a^2-6\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,ab-\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a-\sqrt{-16(a-b)a+16a^2}}{a-b}}}\right)\right)|a-b|}{3a^5-12a^4b+14a^3b^2-4a^2b^3-ab^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2 - 6*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*a*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^2)*(pi*floor((d*x
+ c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a + sqrt(-16*(a - b)*a + 16*
a^2))/(a - b))))*abs(a - b)/(3*a^5 - 12*a^4*b + 14*a^3*b^2 - 4*a^2*b^3 - a*
b^4) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2 - 6*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*a*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^2)*(pi*floor((d*
x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a - sqrt(-16*(a - b)*a + 1
6*a^2))/(a - b))))*abs(a - b)/(3*a^5 - 12*a^4*b + 14*a^3*b^2 - 4*a^2*b^3 -
a*b^4))/d
```

Mupad [B]
time = 14.99, size = 671, normalized size = 5.83

$$\frac{\mathrm{atan}\left(\frac{a^3\tan(c+dx)\sqrt{-\frac{1}{16a^2+16\sqrt{a^3b}}}\,4i+a^5\tan(c+dx)\left(-\frac{1}{16a^2+16\sqrt{a^3b}}\right)^{3/2}64i+a^3\tan(c+dx)\left(-\frac{1}{16a^2+16\sqrt{a^3b}}\right)^{3/2}\sqrt{a^3b}\,64i+a^2b\tan(c+dx)\sqrt{-\frac{1}{16a^2+16\sqrt{a^3b}}}\,4i-a^4b\tan(c+dx)\left(-\frac{1}{16a^2+16\sqrt{a^3b}}\right)^{3/2}64i+a\tan(c+dx)\sqrt{-\frac{1}{16a^2+16\sqrt{a^3b}}}\sqrt{a^3b}\,4i+b\tan(c+dx)\sqrt{-\frac{1}{16a^2+16\sqrt{a^3b}}}\sqrt{a^3b}\,4i-a^2b\tan(c+dx)\left(-\frac{1}{16a^2+16\sqrt{a^3b}}\right)^{3/2}\sqrt{a^3b}\,64i}{ab+\sqrt{a^3b}}\right)\sqrt{-\frac{1}{16a^2+16\sqrt{a^3b}}}\,2i}{d}+\frac{\mathrm{atan}\left(\frac{a^3\tan(c+dx)\sqrt{-\frac{1}{16a^2-16\sqrt{a^3b}}}\,4i+a^5\tan(c+dx)\left(-\frac{1}{16a^2-16\sqrt{a^3b}}\right)^{3/2}64i-a^3\tan(c+dx)\left(-\frac{1}{16a^2-16\sqrt{a^3b}}\right)^{3/2}\sqrt{a^3b}\,64i+a^2b\tan(c+dx)\sqrt{-\frac{1}{16a^2-16\sqrt{a^3b}}}\,4i-a^4b\tan(c+dx)\left(-\frac{1}{16a^2-16\sqrt{a^3b}}\right)^{3/2}64i-a\tan(c+dx)\sqrt{-\frac{1}{16a^2-16\sqrt{a^3b}}}\sqrt{a^3b}\,4i-b\tan(c+dx)\sqrt{-\frac{1}{16a^2-16\sqrt{a^3b}}}\sqrt{a^3b}\,4i+a^2b\tan(c+dx)\left(-\frac{1}{16a^2-16\sqrt{a^3b}}\right)^{3/2}\sqrt{a^3b}\,64i}{ab-\sqrt{a^3b}}\right)\sqrt{-\frac{1}{16a^2-16\sqrt{a^3b}}}\,2i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((a^3*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*4i + a^5*tan
(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(3/2)*64i + a^3*tan(c + d*x)*(-1
/(16*a^2 + 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i + a^2*b*tan(c + d*x)*
(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*4i - a^4*b*tan(c + d*x)*(-1/(16*a^2
+ 16*(a^3*b)^(1/2)))^(3/2)*64i + a*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1
/2)))^(1/2)*(a^3*b)^(1/2)*4i + b*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2
)))^(1/2)*(a^3*b)^(1/2)*4i - a^2*b*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1
/2)))^(3/2)*(a^3*b)^(1/2)*64i)/(a*b + (a^3*b)^(1/2)))*(-1/(16*a^2 + 16*(a^3
*b)^(1/2)))^(1/2)*2i)/d + (atan((a^3*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^
(1/2)))^(1/2)*4i + a^5*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*
64i - a^3*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)
*64i + a^2*b*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*4i - a^4*b
*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*64i - a*tan(c + d*x)*(
-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i - b*tan(c + d*x)*(-1
/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i + a^2*b*tan(c + d*x)*(
-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i)/(a*b - (a^3*b)^(1/
2)))*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*2i)/d
```

3.208 $\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d+1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1
/2)/a^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*ta
n(d*x+c)/a^(1/4))*b^(1/2)/a^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3296, 1301, 1144, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(5/4)
*Sqrt[Sqrt[a] - Sqrt[b]]*d) - (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[
c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Cot[c + d*x]/(a
*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{bx^2}{a(a+2ax^2+(a-b)x^4)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{ad}+\frac{b\text{Subst}\left(\int \frac{x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{\cot(c+dx)}{ad}+\frac{\left(\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)b\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2ad}$$
$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-$$

Mathematica [A]
time = 0.74, size = 143, normalized size = 1.03

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+2\cot(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*((Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*S
qrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*ArcTanh[((Sqrt[a] - Sqrt[b])
*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + 2*
Cot[c + d*x])/(a*d)
```

Maple [A]
time = 0.50, size = 164, normalized size = 1.18

method	result
risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}-4\left(\sum_{\_R=\mathrm{RootOf}\left(\left(65536a^{6}d^{4}-65536a^{5}b\,d^{4}\right)\_Z^{4}+512a^{3}b\,d^{2}\_Z^{2}+b^{2}\right)}\_R\ln\left(e^{2i(dx+c)}+\right.\right.$
derivativedivides	$\frac{\frac{b(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{a}}{d}$
default	$\frac{\frac{b(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{a}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*b*(a-b)*(1/2*((a*b)^(1/2)+a)/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a
-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*((a*b
)^(1/2)-a)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*t
an(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2)))-1/a/tan(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

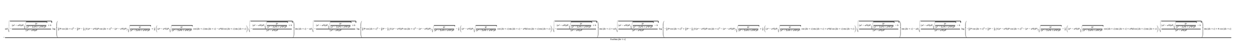
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] ((a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2*a*d*cos(2*d*x + 2*c)
+ a*d)*integrate(-4*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 +
4*b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2 - 4*(8*a*b - 3*b^2)*cos
(4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) - 4*(8*a*b - 3*b^2)*sin(4*d*x + 4*c)
^2 + 2*(8*a*b - 7*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^2*cos(6*d*x +
 6*c) - 2*b^2*cos(4*d*x + 4*c) + b^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (
8*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 7*b^2)*cos(4*d*x + 4*c))*cos(6*d*
x + 6*c) + 2*(b^2 + (8*a*b - 7*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b
^2*sin(6*d*x + 6*c) - 2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(8*
d*x + 8*c) + 2*(4*b^2*sin(2*d*x + 2*c) + (8*a*b - 7*b^2)*sin(4*d*x + 4*c))*
sin(6*d*x + 6*c))/(a*b^2*cos(8*d*x + 8*c)^2 + 16*a*b^2*cos(6*d*x + 6*c)^2 +
 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(8*d*x + 8*c)^2 + 16*a*b^2*sin(6*d*
x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c)^2 - 8*a*b^2*cos(2*d*x + 2*c) + a*b^2
 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^3 - 48*a^2*
b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) -
a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a*b
^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(6
*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2 - 4*(8*a^2*b - 3*a*b^2)*cos(2*d*x + 2*c)
)*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c)
 + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*a*b^2*sin
(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x)
 - 2*sin(2*d*x + 2*c))/(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2
*a*d*cos(2*d*x + 2*c) + a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(99) = 198$.
time = 0.56, size = 1229, normalized size = 8.84



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)
) + b)/((a^3 - a^2*b)*d^2))*log(1/4*b^2*cos(d*x + c)^2 - 1/4*b^2 - 1/4*(2*(
a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7 - 2*a^6
*b + a^5*b^2)*d^4)) + 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5
*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d*cos(d*x + c)*sin(d*x + c))*
sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a
^3 - a^2*b)*d^2)))*sin(d*x + c) - a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a
^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(1/4*b^2*cos(d*x
```

```
 + c)^2 - 1/4*b^2 - 1/4*(2*(a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)
*d^2)*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - 1/2*((a^5 - a^4*b)*d^3*sq
rt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d
*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6
*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)))*sin(d*x + c) + a*d*sqrt(((a^
3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b
)*d^2))*log(-1/4*b^2*cos(d*x + c)^2 + 1/4*b^2 - 1/4*(2*(a^4 - a^3*b)*d^2*co
s(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))
 + 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))*cos(d*x
 + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3 - a^2*b)
*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)))*s
in(d*x + c) - a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^
2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(-1/4*b^2*cos(d*x + c)^2 + 1/4*b^2 -
1/4*(2*(a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7
 - 2*a^6*b + a^5*b^2)*d^4)) - 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6
*b + a^5*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*
x + c))*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) -
 b)/((a^3 - a^2*b)*d^2)))*sin(d*x + c) + 8*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a - b*sin(c + d*x)**4), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(99) = 198.
time = 0.84, size = 672, normalized size = 4.83

$$-\frac{\frac{\left(\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2b-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab^2-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^3\right)a^2|a-b|-\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^5-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^4b-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^3b^2\right)|a-b|\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^2+\sqrt{a^4-(a^2-ab)a^2}}{a^2-ab}}}\right)\right)}{(3a^8-15a^7b+26a^6b^2-18a^5b^3+3a^4b^4+a^3b^5)|a|}-\frac{\left(\left(3\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}a^2b-6\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}ab^2-\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}b^3\right)a^2|a-b|-\left(3\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}a^5-6\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}a^4b-\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}a^3b^2\right)|a-b|\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^2-\sqrt{a^4-(a^2-ab)a^2}}{a^2-ab}}}\right)\right)}{(3a^8-15a^7b+26a^6b^2-18a^5b^3+3a^4b^4+a^3b^5)|a|}+\frac{2}{a\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*sqrt(a*b)*b^3)*a^2*abs(a - b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^5 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b - s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2)*abs(a - b))*(pi*floor
((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 + sqrt(a^4 - (a^2 - a*
b)*a^2))/(a^2 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^2 - 18*a^5*b^3 + 3*a^
```

```
4*b^4 + a^3*b^5)*abs(a)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b
)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) - (3*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^4*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2)*
abs(a - b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 -
 sqrt(a^4 - (a^2 - a*b)*a^2))/(a^2 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^
2 - 18*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*abs(a)) + 2/(a*tan(d*x + c)))/d
```

Mupad [B]
time = 14.51, size = 371, normalized size = 2.67

$$\frac{2\,\mathrm{atanh}\left(\frac{2\,\left(\tan(c+d\,x)\,\left(4\,a^4\,b^4-4\,a^6\,b^2\right)-\frac{\tan(c+d\,x)\,\left(\sqrt{a^5\,b^3}+a^3\,b\right)\,\left(64\,a^9\,b-128\,a^8\,b^2+64\,a^7\,b^3\right)}{16\,\left(a^5\,b-a^6\right)}\right)\,\sqrt{\frac{\sqrt{a^5\,b^3}+a^3\,b}{16\,\left(a^5\,b-a^6\right)}}}{2\,a^3\,b^4-2\,a^4\,b^3}\right)\,\sqrt{\frac{\sqrt{a^5\,b^3}+a^3\,b}{16\,\left(a^5\,b-a^6\right)}}}{d}+\frac{2\,\mathrm{atanh}\left(\frac{2\,\left(\tan(c+d\,x)\,\left(4\,a^4\,b^4-4\,a^6\,b^2\right)+\frac{\tan(c+d\,x)\,\left(\sqrt{a^5\,b^3}-a^3\,b\right)\,\left(64\,a^9\,b-128\,a^8\,b^2+64\,a^7\,b^3\right)}{16\,\left(a^5\,b-a^6\right)}\right)\,\sqrt{-\frac{\sqrt{a^5\,b^3}-a^3\,b}{16\,\left(a^5\,b-a^6\right)}}}{2\,a^3\,b^4-2\,a^4\,b^3}\right)\,\sqrt{-\frac{\sqrt{a^5\,b^3}-a^3\,b}{16\,\left(a^5\,b-a^6\right)}}}{d}-\frac{\cot(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (2*atanh((2*(tan(c + d*x)*(4*a^4*b^4 - 4*a^6*b^2) - (tan(c + d*x)*((a^5*b^3
)^(1/2) + a^3*b)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2))/(16*(a^5*b - a^6)))
*(((a^5*b^3)^(1/2) + a^3*b)/(16*(a^5*b - a^6)))^(1/2))/(2*a^3*b^4 - 2*a^4*b
^3))*(((a^5*b^3)^(1/2) + a^3*b)/(16*(a^5*b - a^6)))^(1/2))/d + (2*atanh((2*
(tan(c + d*x)*(4*a^4*b^4 - 4*a^6*b^2) + (tan(c + d*x)*((a^5*b^3)^(1/2) - a^
3*b)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2))/(16*(a^5*b - a^6)))*(-((a^5*b^3
)^(1/2) - a^3*b)/(16*(a^5*b - a^6)))^(1/2))/(2*a^3*b^4 - 2*a^4*b^3))*(-((a^
5*b^3)^(1/2) - a^3*b)/(16*(a^5*b - a^6)))^(1/2))/d - cot(c + d*x)/(a*d)
```

3.209 $\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}$$

```
[Out] -cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+1/2*b*arctan((a^(1/2)-b^(1/2))^(1/2)*t
an(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b*arctan((a^(1/2)+
b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3296, 1301, 1180, 211}

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqrt[
Sqrt[a] - Sqrt[b]]*d) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^
(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Cot[c + d*x]/(a*d) - Cot[c
+ d*x]^3/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^4(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{1}{ax^2}+\frac{b(1+x^2)}{a(a+2ax^2+(a-b)x^4)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{b\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad} + \frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx\right.}{2a^{3/2}d}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d} + \frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d} - \frac{\cot(c}{a}$$

Mathematica [A]
time = 1.08, size = 165, normalized size = 1.11

$$\frac{\frac{3b\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}} - \frac{3b\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}} - 4\sqrt{a}\cot(c+dx) - 2\sqrt{a}\cot(c+dx)\csc^2(c+dx)}{6a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((3*b*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])
/Sqrt[a + Sqrt[a]*Sqrt[b]] - (3*b*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x]
)/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] - 4*Sqrt[a]*Cot[c
 + d*x] - 2*Sqrt[a]*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^(3/2)*d)
```

Maple [A]
time = 0.54, size = 177, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{1}{3a\tan(dx+c)^3}-\frac{1}{a\tan(dx+c)}+\frac{b(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}+b\right)\arctan\left(\right.}{2\sqrt{ab}\,(a-b)\sqrt{(}}\right.}{a}}{d}$
default	$\frac{-\frac{1}{3a\tan(dx+c)^3}-\frac{1}{a\tan(dx+c)}+\frac{b(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}+b\right)\arctan\left(\right.}{2\sqrt{ab}\,(a-b)\sqrt{(}}\right.}{a}}{d}$
risch	$\frac{4i\left(3\,\mathrm{e}^{2i(dx+c)}-1\right)}{3da\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}+16\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16777216a^8d^4-16777216a^7b\,d^4\right)\_Z^4+8192a^4b^2d^2\_Z^2+b^4\right)}\_R\ln\left(\mathrm{e}^{2i}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/a/tan(d*x+c)^3-1/a/tan(d*x+c)+1/a*b*(a-b)*(1/2*((a*b)^(1/2)-b)/(a
*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((
a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*((a*b)^(1/2)+b)/(a*b)^(1/2)/(a-b)/(((a*b)^(
1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -4/3*(12*(a*b*d*cos(6*d*x + 6*c)^2 + 9*a*b*d*cos(4*d*x + 4*c)^2 + 9*a*b*d*c
os(2*d*x + 2*c)^2 + a*b*d*sin(6*d*x + 6*c)^2 + 9*a*b*d*sin(4*d*x + 4*c)^2 -
 18*a*b*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*b*d*sin(2*d*x + 2*c)^2 -
6*a*b*d*cos(2*d*x + 2*c) + a*b*d - 2*(3*a*b*d*cos(4*d*x + 4*c) - 3*a*b*d*co
s(2*d*x + 2*c) + a*b*d)*cos(6*d*x + 6*c) - 6*(3*a*b*d*cos(2*d*x + 2*c) - a*
b*d)*cos(4*d*x + 4*c) - 6*(a*b*d*sin(4*d*x + 4*c) - a*b*d*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c))*integrate((b*cos(8*d*x + 8*c)*cos(4*d*x + 4*c) - 4*b*cos
(6*d*x + 6*c)*cos(4*d*x + 4*c) - 2*(8*a - 3*b)*cos(4*d*x + 4*c)^2 + b*sin(8
*d*x + 8*c)*sin(4*d*x + 4*c) - 4*b*sin(6*d*x + 6*c)*sin(4*d*x + 4*c) - 2*(8
*a - 3*b)*sin(4*d*x + 4*c)^2 - 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (4*b
*cos(2*d*x + 2*c) - b)*cos(4*d*x + 4*c))/(a*b^2*cos(8*d*x + 8*c)^2 + 16*a*b
^2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(8*d*x + 8*c
)^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c)^2 - 8*a*b^2*c
os(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*cos(4*d*x + 4*c)^
2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 3*a*
b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*d*x + 6*c) + 4*a*
b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(
8*d*x + 8*c) + 8*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*
cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2 - 4*(8*a^2*b - 3*
a*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin(6*d*x + 6*c) + 2
*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(8*d*x +
 8*c) + 16*(2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c)
)*sin(6*d*x + 6*c)), x) - (3*cos(2*d*x + 2*c) - 1)*sin(6*d*x + 6*c) + 3*(3*
cos(2*d*x + 2*c) - 1)*sin(4*d*x + 4*c) + 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c
) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))/(a*d*cos(6*d*x + 6*c)^2 + 9*a*d*co
s(4*d*x + 4*c)^2 + 9*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(6*d*x + 6*c)^2 + 9*a*
d*sin(4*d*x + 4*c)^2 - 18*a*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*d*sin
(2*d*x + 2*c)^2 - 6*a*d*cos(2*d*x + 2*c) + a*d - 2*(3*a*d*cos(4*d*x + 4*c)
- 3*a*d*cos(2*d*x + 2*c) + a*d)*cos(6*d*x + 6*c) - 6*(3*a*d*cos(2*d*x + 2*c
) - a*d)*cos(4*d*x + 4*c) - 6*(a*d*sin(4*d*x + 4*c) - a*d*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(111) = 222$.
time = 0.58, size = 1365, normalized size = 9.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9
 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(1/4*b^4*cos(d*x
```

```
 + c)^2 - 1/4*b^4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b -
a^4*b^2)*d^2)*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 1/2*(a^2*b^3*d*co
s(d*x + c)*sin(d*x + c) - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*
b^2)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a
^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) - 3
*(a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8
*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(1/4*b^4*cos(d*x + c)^2
- 1/4*b^4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b - a^4*b^2)
*d^2)*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 1/2*(a^2*b^3*d*cos(d*x +
c)*sin(d*x + c) - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4
))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a
^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) - 3*(a*d*co
s(d*x + c)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*
b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(-1/4*b^4*cos(d*x + c)^2 + 1/4*b^
4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b - a^4*b^2)*d^2)*sq
rt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 1/2*(a^2*b^3*d*cos(d*x + c)*sin(d
*x + c) + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4))*cos(d
*x + c)*sin(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^
7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) + 3*(a*d*cos(d*x + c
)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)
) - b^2)/((a^4 - a^3*b)*d^2))*log(-1/4*b^4*cos(d*x + c)^2 + 1/4*b^4 - 1/4*(
2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b - a^4*b^2)*d^2)*sqrt(b^5/((
a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 1/2*(a^2*b^3*d*cos(d*x + c)*sin(d*x + c) +
 (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4))*cos(d*x + c)*s
in(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^
4)) - b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) + 16*cos(d*x + c)^3 - 24*cos(
d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

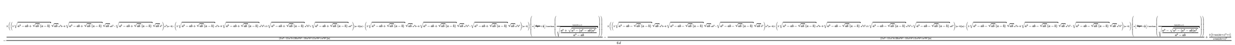
$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a - b*sin(c + d*x)**4), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(111) = 222.
time = 0.87, size = 938, normalized size = 6.30



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/6*(3*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a -
b))*a^4*b - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^2 + 5*sqrt(a^2 - a*
b + sqrt(a*b)*(a - b))*a^2*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4)
*abs(a - b)*abs(a) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b
 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 - sqrt(a^2 - a*b
 + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3)*abs(a - b))*(pi*floor((d*x + c)/pi
 + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 + sqrt(a^4 - (a^2 - a*b)*a^2))/(a^2
 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^2 - 18*a^5*b^3 + 3*a^4*b^4 + a^3*b
^5)*abs(a)) - 3*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6
*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b - sqr
t(a*b)*(a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) + (3*sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*a^4*b - 9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^2 + 5*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b)
)*a*b^4)*abs(a - b)*abs(a) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*
b)*a^4*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 - sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3)*abs(a - b))*(pi*floor((d*x
 + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 - sqrt(a^4 - (a^2 - a*b)*a^
2))/(a^2 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^2 - 18*a^5*b^3 + 3*a^4*b^4
 + a^3*b^5)*abs(a)) + 2*(3*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

Mupad [B]
time = 15.55, size = 1670, normalized size = 11.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan((((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b^4
- 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(
a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) - tan(c + d*x)*
(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(
1/2)*1i - ((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b
^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b^2)/(1
6*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) + tan(c + d*
x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8))
)^(1/2)*1i)/(((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^
5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b^2)
/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) - tan(c +
 d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^
8)))^(1/2) + ((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^
5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b^2)
```

```
/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) + tan(c +
 d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^
8)))^(1/2) - 2*a^2*b^5 + 2*a^3*b^4))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*
b - a^8)))^(1/2)*2i)/d + (atan((((-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b -
 a^8)))^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d*x)*(-((a^7*
b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 12
8*a^8*b^2)) - tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(-((a^7*b^5)^(1/2) - a^
4*b^2)/(16*(a^7*b - a^8)))^(1/2)*1i - ((-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a
^7*b - a^8)))^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c + d*x)*(-
((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^
3 - 128*a^8*b^2)) + tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(-((a^7*b^5)^(1/2
) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*1i)/(((-((a^7*b^5)^(1/2) - a^4*b^2)/
(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d
*x)*(-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*
a^7*b^3 - 128*a^8*b^2)) - tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(-((a^7*b^5
)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2) + ((-((a^7*b^5)^(1/2) - a^4*b^
2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c
+ d*x)*(-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b +
64*a^7*b^3 - 128*a^8*b^2)) + tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(-((a^7*
b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2) - 2*a^2*b^5 + 2*a^3*b^4))*(
-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*2i)/d - (1/(3*a) + t
an(c + d*x)^2/a)/(d*tan(c + d*x)^3)
```

3.210 $\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{2\cot^3(c}{3ad}$$

```
[Out] -(a+b)*cot(d*x+c)/a^2/d-2/3*cot(d*x+c)^3/a/d-1/5*cot(d*x+c)^5/a/d+1/2*b^(3/
2)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(9/4)/d/(a^(1/2)-b^
(1/2))^(1/2)-1/2*b^(3/2)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))
/a^(9/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3296, 1301, 1144, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(9/4)
*Sqrt[Sqrt[a] - Sqrt[b]]*d) - (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[
c + d*x])/a^(1/4)])/(2*a^(9/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - ((a + b)*Cot[c
+ d*x])/(a^2*d) - (2*Cot[c + d*x]^3)/(3*a*d) - Cot[c + d*x]^5/(5*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^6(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6}+\frac{2}{ax^4}+\frac{a+b}{a^2x^2}+\frac{b^2x^2}{a^2(a+2ax^2+(a-b)x^4)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d} - \frac{2\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{b^2\text{Subst}\left(\int \frac{x^2}{a+2ax^2+(a-b)\ldots}\right.}{a^2\ldots}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d} - \frac{2\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b^{3/2}\right)\text{S}\ldots}{\ldots}$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d} - \ldots$$

Mathematica [A]
time = 3.14, size = 174, normalized size = 0.98

$$-\frac{\frac{15b^{3/2}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{15b^{3/2}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+2\cot(c+dx)\left(8a+15b+4a\csc^2(c+dx)+3a\csc^4(c+dx)\right)}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

[Out] -1/30*((15*b^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (15*b^(3/2)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + 2*Cot[c + d*x]*(8*a + 15*b + 4*a*Csc[c + d*x]^2 + 3*a*Csc[c + d*x]^4))/(a^2*d)

Maple [A]
time = 0.53, size = 195, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{a+b}{a^2\tan(dx+c)}-\frac{2}{3a\tan(dx+c)^3}+\frac{b^2(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}\right.}{2\sqrt{}}\right.}{a^2}}{d}$
default	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{a+b}{a^2\tan(dx+c)}-\frac{2}{3a\tan(dx+c)^3}+\frac{b^2(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}\right.}{2\sqrt{}}\right.}{a^2}}{d}$
risch	$-\frac{2i\left(15b\,e^{8i(dx+c)}-60b\,e^{6i(dx+c)}+80a\,e^{4i(dx+c)}+90b\,e^{4i(dx+c)}-40a\,e^{2i(dx+c)}-60b\,e^{2i(dx+c)}+8a+15b\right)}{15d\,a^2\left(e^{2i(dx+c)}-1\right)^5}-64\Bigg(\_R=\mathrm{R}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/5/a/tan(d*x+c)^5-(a+b)/a^2/tan(d*x+c)-2/3/a/tan(d*x+c)^3+b^2/a^2*(a-b)*(1/2*((a*b)^(1/2)+a)/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*((a*b)^(1/2)-a)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")

[Out] 1/15*(300*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + 10*(3*b*sin(8*d*x + 8*c) - 12*b*sin(6*d*x + 6*c) + 2*(8*a + 9*b)*sin(4*d*x + 4*c) - 4*(2*a + 3*b)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + 50*(6*b*sin(6*d*x + 6*c) - 4*(4*a + 3*b)*sin(4*d*x + 4*c) + (8*a + 9*b)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 200*((8*a + 3*b)*sin(4*d*x + 4*c) - (4*a + 3*b)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 15*(a^2*d*cos(10*d*x + 10*c)^2 + 25*a^2*d*cos(8*d*x + 8*c)^2 + 100*a^2*d*cos(6*d*x + 6*c)^2 + 100*a^2*d*cos(4*d*x + 4*c)^2 + 25*a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(10*d*x + 10*c)^2 + 25*a^2*d*sin(8*d*x + 8*c)^2 + 100*a^2*d*sin(6*d*x + 6*c)^2 + 100*a^2*d*sin(4*d*x + 4*c)^2 - 100*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*d*sin(2*d*x + 2*c)^2 - 10*a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(5*a^2*d*cos(8*d*x + 8*c) - 10*a^2*d*cos(6*d*x + 6*c) + 10*a^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(10*d*x + 10*c) - 10*(10*a^2*d*cos(6*d*x + 6*c) - 10*a^2*d*cos(4*d*x + 4*c) + 5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c) - 20*(10*a^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x + 6*c) - 20*(5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 10*(a^2*d*sin(8*d*x + 8*c) - 2*a^2*d*sin(6*d*x + 6*c) + 2*a^2*d*sin(4*d*x + 4*c) - a^2*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) - 50*(2*a^2*d*sin(6*d*x + 6*c) - 2*a^2*d*sin(4*d*x + 4*c) + a^2*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 100*(2*a^2*d*sin(4*d*x + 4*c) - a^2*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-4*(4*b^3*cos(6*d*x + 6*c)^2 + 4*b^3*cos(2*d*x + 2*c)^2 + 4*b^3*sin(6*d*x + 6*c)^2 + 4*b^3*sin(2*d*x + 2*c)^2 - b^3*cos(2*d*x + 2*c) - 4*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c)^2 - 4*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)^2 + 2*(8*a*b^2 - 7*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^3*cos(6*d*x + 6*c) - 2*b^3*cos(4*d*x + 4*c) + b^3*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (8*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 7*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(b^3 + (8*a*b^2 - 7*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^3*sin(6*d*x + 6*c) - 2*b^3*sin(4*d*x + 4*c) + b^3*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*(4*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 7*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c))/(a^2*b^2*cos(8*d*x + 8*c)^2 + 16*a^2*b^2*cos(6*d*x + 6*c)^2 + 16*a^2*b^2*cos(2*d*x + 2*c)^2 + a^2*b^2*sin(8*d*x + 8*c)^2 + 16*a^2*b^2*sin(6*d*x + 6*c)^2 + 16*a^2*b^2*sin(2*d*x + 2*c)^2 - 8*a^2*b^2*cos(2*d*x + 2*c) + a^2*b^2 + 4*(64*a^4 - 48*a^3*b + 9*a^2*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^4 - 48*a^3*b + 9*a^2*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a^2*b^2*cos(6*d*x + 6*c) + 4*a^2*b^2*cos(2*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a^2*b^2*cos(2*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 3*a^2*b^2 - 4*(8*a^3*b - 3*a^2*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a^2*b^2*sin(6*d*x + 6*c) + 2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*

```
sin(6*d*x + 6*c)), x) - 2*(15*b*cos(8*d*x + 8*c) - 60*b*cos(6*d*x + 6*c) +
10*(8*a + 9*b)*cos(4*d*x + 4*c) - 20*(2*a + 3*b)*cos(2*d*x + 2*c) + 8*a + 1
5*b)*sin(10*d*x + 10*c) - 10*(30*b*cos(6*d*x + 6*c) - 20*(4*a + 3*b)*cos(4*
d*x + 4*c) + 5*(8*a + 9*b)*cos(2*d*x + 2*c) - 8*a - 12*b)*sin(8*d*x + 8*c)
- 20*(10*(8*a + 3*b)*cos(4*d*x + 4*c) - 10*(4*a + 3*b)*cos(2*d*x + 2*c) + 8
*a + 9*b)*sin(6*d*x + 6*c) - 60*(5*b*cos(2*d*x + 2*c) - 2*b)*sin(4*d*x + 4*
c) - 30*b*sin(2*d*x + 2*c))/(a^2*d*cos(10*d*x + 10*c)^2 + 25*a^2*d*cos(8*d*
x + 8*c)^2 + 100*a^2*d*cos(6*d*x + 6*c)^2 + 100*a^2*d*cos(4*d*x + 4*c)^2 +
25*a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(10*d*x + 10*c)^2 + 25*a^2*d*sin(8*d
*x + 8*c)^2 + 100*a^2*d*sin(6*d*x + 6*c)^2 + 100*a^2*d*sin(4*d*x + 4*c)^2 -
 100*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*d*sin(2*d*x + 2*c)^2
- 10*a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(5*a^2*d*cos(8*d*x + 8*c) - 10*a^2*
d*cos(6*d*x + 6*c) + 10*a^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) +
 a^2*d)*cos(10*d*x + 10*c) - 10*(10*a^2*d*cos(6*d*x + 6*c) - 10*a^2*d*cos(4
*d*x + 4*c) + 5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c) - 20*(10*a
^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x + 6*c)
- 20*(5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 10*(a^2*d*sin(8*
d*x + 8*c) - 2*a^2*d*sin(6*d*x + 6*c) + 2*a^2*d*sin(4*d*x + 4*c) - a^2*d*si
n(2*d*x + 2*c))*sin(10*d*x + 10*c) - 50*(2*a^2*d*sin(6*d*x + 6*c) - 2*a^2*d
*sin(4*d*x + 4*c) + a^2*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 100*(2*a^2*d
*sin(4*d*x + 4*c) - a^2*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(134) = 268$.
time = 0.58, size = 1477, normalized size = 8.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/120*(8*(8*a + 15*b)*cos(d*x + c)^5 - 80*(2*a + 3*b)*cos(d*x + c)^3 - 15*
(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(-((a^5 - a^4*b
)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 + b^3)/((a^5 - a^4*b)*d^2
))*log(1/4*b^5*cos(d*x + c)^2 - 1/4*b^5 - 1/4*(2*(a^6*b - a^5*b^2)*d^2*cos(
d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d
^4)) + 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x + c) - (a^8 - a^7*b)*sqrt(b^7/((
a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^5
 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 + b^3)/((a^5 - a^
4*b)*d^2)))*sin(d*x + c) + 15*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^
2 + a^2*d)*sqrt(-((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))
*d^2 + b^3)/((a^5 - a^4*b)*d^2))*log(1/4*b^5*cos(d*x + c)^2 - 1/4*b^5 - 1/4
*(2*(a^6*b - a^5*b^2)*d^2*cos(d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/
((a^11 - 2*a^10*b + a^9*b^2)*d^4)) - 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x +
c) - (a^8 - a^7*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x
```

```
+ c)*sin(d*x + c))*sqrt(-((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^
2)*d^4))*d^2 + b^3)/((a^5 - a^4*b)*d^2)))*sin(d*x + c) + 15*(a^2*d*cos(d*x
+ c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(((a^5 - a^4*b)*sqrt(b^7/((a^1
1 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5 - a^4*b)*d^2))*log(-1/4*b^5*
cos(d*x + c)^2 + 1/4*b^5 - 1/4*(2*(a^6*b - a^5*b^2)*d^2*cos(d*x + c)^2 - (a
^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4)) + 1/2*(a^3
*b^3*d*cos(d*x + c)*sin(d*x + c) + (a^8 - a^7*b)*sqrt(b^7/((a^11 - 2*a^10*b
 + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(((a^5 - a^4*b)*sqrt(b
^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5 - a^4*b)*d^2)))*sin(
d*x + c) - 15*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(
((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5
 - a^4*b)*d^2))*log(-1/4*b^5*cos(d*x + c)^2 + 1/4*b^5 - 1/4*(2*(a^6*b - a^5
*b^2)*d^2*cos(d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*
b + a^9*b^2)*d^4)) - 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x + c) + (a^8 - a^7*
b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c
))*sqrt(((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^
3)/((a^5 - a^4*b)*d^2)))*sin(d*x + c) + 120*(a + b)*cos(d*x + c))/((a^2*d*c
os(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(134) = 268.
time = 0.78, size = 471, normalized size = 2.65

$$-\frac{\frac{15\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2b-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab^2-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^3+\sqrt{a^6-(a^3-a^2b)a^3}}{a^3-a^2b}}}\right)\right)|a-b|}{3a^7-12a^6b+14a^5b^2-4a^4b^3-a^3b^4}+\frac{15\left(3\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}a^2b-6\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}ab^2-\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^3-\sqrt{a^6-(a^3-a^2b)a^3}}{a^3-a^2b}}}\right)\right)|a-b|}{3a^7-12a^6b+14a^5b^2-4a^4b^3-a^3b^4}+\frac{2\left(15a\tan(dx+c)^4+15b\tan(dx+c)^4+10a\tan(dx+c)^2+3a\right)}{a^2\tan(dx+c)^5}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)
/sqrt((a^3 + sqrt(a^6 - (a^3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*
a^7 - 12*a^6*b + 14*a^5*b^2 - 4*a^4*b^3 - a^3*b^4) + 15*(3*sqrt(a^2 - a*b -
 sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(pi*f
```

```
loor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 - sqrt(a^6 - (a^3
- a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*a^7 - 12*a^6*b + 14*a^5*b^2 -
 4*a^4*b^3 - a^3*b^4) + 2*(15*a*tan(d*x + c)^4 + 15*b*tan(d*x + c)^4 + 10*a
*tan(d*x + c)^2 + 3*a)/(a^2*tan(d*x + c)^5))/d
```

Mupad [B]
time = 15.26, size = 416, normalized size = 2.34

$$\frac{2\,\mathrm{atanh}\left(\frac{2\,\left(\tan(c+d\,x)\,\left(4\,a^7\,b^6-4\,a^9\,b^4\right)-\frac{\tan(c+d\,x)\,\left(\sqrt{a^9\,b^7}+a^5\,b^3\right)\,\left(64\,a^{14}\,b-128\,a^{13}\,b^2+64\,a^{12}\,b^3\right)}{16\,\left(a^9\,b-a^{10}\right)}\right)\,\sqrt{\frac{\sqrt{a^9\,b^7}+a^5\,b^3}{16\,\left(a^9\,b-a^{10}\right)}}}{2\,a^5\,b^7-2\,a^6\,b^6}\right)\,\sqrt{\frac{\sqrt{a^9\,b^7}+a^5\,b^3}{16\,\left(a^9\,b-a^{10}\right)}}}{d}+\frac{2\,\mathrm{atanh}\left(\frac{2\,\left(\tan(c+d\,x)\,\left(4\,a^7\,b^6-4\,a^9\,b^4\right)+\frac{\tan(c+d\,x)\,\left(\sqrt{a^9\,b^7}-a^5\,b^3\right)\,\left(64\,a^{14}\,b-128\,a^{13}\,b^2+64\,a^{12}\,b^3\right)}{16\,\left(a^9\,b-a^{10}\right)}\right)\,\sqrt{-\frac{\sqrt{a^9\,b^7}-a^5\,b^3}{16\,\left(a^9\,b-a^{10}\right)}}}{2\,a^5\,b^7-2\,a^6\,b^6}\right)\,\sqrt{-\frac{\sqrt{a^9\,b^7}-a^5\,b^3}{16\,\left(a^9\,b-a^{10}\right)}}}{d}-\frac{\frac{1}{5\,a}+\frac{2\,\tan(c+d\,x)^2}{3\,a}+\frac{\tan(c+d\,x)^4\,(a+b)}{a^2}}{d\,\tan\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (2*atanh((2*(tan(c + d*x)*(4*a^7*b^6 - 4*a^9*b^4) - (tan(c + d*x)*((a^9*b^7
)^(1/2) + a^5*b^3)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2))/(16*(a^9*b - a
^10)))*(((a^9*b^7)^(1/2) + a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/(2*a^5*b^7
- 2*a^6*b^6))*(((a^9*b^7)^(1/2) + a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/d +
(2*atanh((2*(tan(c + d*x)*(4*a^7*b^6 - 4*a^9*b^4) + (tan(c + d*x)*((a^9*b^7
)^(1/2) - a^5*b^3)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2))/(16*(a^9*b - a
^10)))*(-((a^9*b^7)^(1/2) - a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/(2*a^5*b^7
 - 2*a^6*b^6))*(-((a^9*b^7)^(1/2) - a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/d
- (1/(5*a) + (2*tan(c + d*x)^2)/(3*a) + (tan(c + d*x)^4*(a + b))/a^2)/(d*ta
n(c + d*x)^5)
```

3.211 $\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{(3a+b)\cot^3(c+dx)}{3a^2d}$$

```
[Out] -(a+b)*cot(d*x+c)/a^2/d-1/3*(3*a+b)*cot(d*x+c)^3/a^2/d-3/5*cot(d*x+c)^5/a/d
-1/7*cot(d*x+c)^7/a/d+1/2*b^2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(
1/4))/a^(11/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^2*arctan((a^(1/2)+b^(1/2))^(
1/2)*tan(d*x+c)/a^(1/4))/a^(11/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3296, 1301, 1180, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^2\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+b)\cot^3(c+dx)}{3a^2d}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^7(c+dx)}{7ad}-\frac{3\cot^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(11/4)*Sq
rt[Sqrt[a] - Sqrt[b]]*d) + (b^2*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x
])/a^(1/4)])/(2*a^(11/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - ((a + b)*Cot[c + d*x]
)/(a^2*d) - ((3*a + b)*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot[c + d*x]^5)/(5*a*
d) - Cot[c + d*x]^7/(7*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^5}{x^8(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^8}+\frac{3}{ax^6}+\frac{3a+b}{a^2x^4}+\frac{a+b}{a^2x^2}+\frac{b^2(1+x^2)}{a^2(a+2ax^2+(a-b)x^4)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d} - \frac{(3a+b)\cot^3(c+dx)}{3a^2d} - \frac{3\cot^5(c+dx)}{5ad} - \frac{\cot^7(c+dx)}{7ad} +$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d} - \frac{(3a+b)\cot^3(c+dx)}{3a^2d} - \frac{3\cot^5(c+dx)}{5ad} - \frac{\cot^7(c+dx)}{7ad} +$$

$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d} - \frac{(a-}{}$$

Mathematica [A]
time = 6.22, size = 277, normalized size = 1.41

$$\frac{b^2\tan^{-1}\left(\frac{\left(\sqrt{a}\sqrt{b}+b\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{2a^{5/2}\sqrt{a+\sqrt{a}\sqrt{b}}\,d} - \frac{b^2\tanh^{-1}\left(\frac{\left(\sqrt{a}\sqrt{b}-b\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{2a^{5/2}\sqrt{-a+\sqrt{a}\sqrt{b}}\,d} - \frac{2(24a\cos(c+dx)+35b\cos(c+dx))\csc(c+dx)}{105a^2d} + \frac{(-24a\cos(c+dx)-35b\cos(c+dx))\csc^3(c+dx)}{105a^2d} - \frac{6\cot(c+dx)\csc^4(c+dx)}{35ad} - \frac{\cot(c+dx)\csc^6(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

[Out] (b^2*ArcTan[((Sqrt[a]*Sqrt[b] + b)*Tan[c + d*x])/(Sqrt[a + Sqrt[a]*Sqrt[b]] *Sqrt[b])])/(2*a^(5/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*d) - (b^2*ArcTanh[((Sqrt[a]*Sqrt[b] - b)*Tan[c + d*x])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b])])/(2*a^(5 /2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]*d) - (2*(24*a*Cos[c + d*x] + 35*b*Cos[c + d* x])*Csc[c + d*x])/(105*a^2*d) + ((-24*a*Cos[c + d*x] - 35*b*Cos[c + d*x])*C sc[c + d*x]^3)/(105*a^2*d) - (6*Cot[c + d*x]*Csc[c + d*x]^4)/(35*a*d) - (Co t[c + d*x]*Csc[c + d*x]^6)/(7*a*d)

Maple [A]

time = 0.51, size = 213, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{1}{7a\tan(dx+c)^7}-\frac{a+b}{a^2\tan(dx+c)}-\frac{3a+b}{3a^2\tan(dx+c)^3}-\frac{3}{5a\tan(dx+c)^5}+\frac{b^2(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-}}\right.}{}}{d}$
default	$\frac{-\frac{1}{7a\tan(dx+c)^7}-\frac{a+b}{a^2\tan(dx+c)}-\frac{3a+b}{3a^2\tan(dx+c)^3}-\frac{3}{5a\tan(dx+c)^5}+\frac{b^2(a-b)\left(\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-}}\right.}{}}{d}$
risch	$\frac{4i\left(105b\,e^{10i(dx+c)}-455b\,e^{8i(dx+c)}+840a\,e^{6i(dx+c)}+770b\,e^{6i(dx+c)}-504a\,e^{4i(dx+c)}-630b\,e^{4i(dx+c)}+168a\,e^{2i(dx+c)}+24\right.}{105d\,a^2\left(e^{2i(dx+c)}-1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/7/a/tan(d*x+c)^7-(a+b)/a^2/tan(d*x+c)-1/3*(3*a+b)/a^2/tan(d*x+c)^3- 3/5/a/tan(d*x+c)^5+b^2/a^2*(a-b)*(1/2*((a*b)^(1/2)-b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*((a*b)^(1/2)+b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")

[Out] 4/105*(735*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 7*(15*b*sin(10*d*x + 10*c) - 65*b*sin(8*d*x + 8*c) + 10*(12*a + 11*b)*sin(6*d*x + 6*c) - 18*(4*a + 5*b)*sin(4*d*x + 4*c) + (24*a + 35*b)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) + 49*(15*b*sin(10*d*x + 10*c) - 65*b*sin(8*d*x + 8*c) + 10*(12*a + 11*b)*sin(6*d*x + 6*c) - 18*(4*a + 5*b)*sin(4*d*x + 4*c) + (24*a + 35*b)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 147*(40*b*sin(8*d*x + 8*c) - 5*(24*a + 17*b)*sin(6*d*x + 6*c) + 3*(24*a + 25*b)*sin(4*d*x + 4*c) - 6*(4*a + 5*b)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + 245*(15*(8*a + 3*b)*sin(6*d*x + 6*c) - 3*(24*a + 17*b)*sin(4*d*x + 4*c) + 2*(12*a + 11*b)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 245*(24*b*sin(4*d*x + 4*c) - 13*b*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 420*(a^2*b^2*d*cos(14*d*x + 14*c)^2 + 49*a^2*b^2*d*cos(12*d*x + 12*c)^2 + 441*a^2*b^2*d*cos(10*d*x + 10*c)^2 + 1225*a^2*b^2*d*cos(8*d*x + 8*c)^2 + 1225*a^2*b^2*d*cos(6*d*x + 6*c)^2 + 441*a^2*b^2*d*cos(4*d*x + 4*c)^2 + 49*a^2*b^2*d*cos(2*d*x + 2*c)^2 + a^2*b^2*d*sin(14*d*x + 14*c)^2 + 49*a^2*b^2*d*sin(12*d*x + 12*c)^2 + 441*a^2*b^2*d*sin(10*d*x + 10*c)^2 + 1225*a^2*b^2*d*sin(8*d*x + 8*c)^2 + 1225*a^2*b^2*d*sin(6*d*x + 6*c)^2 + 441*a^2*b^2*d*sin(4*d*x + 4*c)^2 - 294*a^2*b^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*a^2*b^2*d*sin(2*d*x + 2*c)^2 - 14*a^2*b^2*d*cos(2*d*x + 2*c) + a^2*b^2*d - 2*(7*a^2*b^2*d*cos(12*d*x + 12*c) - 21*a^2*b^2*d*cos(10*d*x + 10*c) + 35*a^2*b^2*d*cos(8*d*x + 8*c) - 35*a^2*b^2*d*cos(6*d*x + 6*c) + 21*a^2*b^2*d*cos(4*d*x + 4*c) - 7*a^2*b^2*d*cos(2*d*x + 2*c) + a^2*b^2*d)*cos(14*d*x + 14*c) - 14*(21*a^2*b^2*d*cos(10*d*x + 10*c) - 35*a^2*b^2*d*cos(8*d*x + 8*c) + 35*a^2*b^2*d*cos(6*d*x + 6*c) - 21*a^2*b^2*d*cos(4*d*x + 4*c) + 7*a^2*b^2*d*cos(2*d*x + 2*c) - a^2*b^2*d)*cos(12*d*x + 12*c) - 42*(35*a^2*b^2*d*cos(8*d*x + 8*c) - 35*a^2*b^2*d*cos(6*d*x + 6*c) + 21*a^2*b^2*d*cos(4*d*x + 4*c) - 7*a^2*b^2*d*cos(2*d*x + 2*c) + a^2*b^2*d)*cos(10*d*x + 10*c) - 70*(35*a^2*b^2*d*cos(6*d*x + 6*c) - 21*a^2*b^2*d*cos(4*d*x + 4*c) + 7*a^2*b^2*d*cos(2*d*x + 2*c) - a^2*b^2*d)*cos(8*d*x + 8*c) - 70*(21*a^2*b^2*d*cos(4*d*x + 4*c) - 7*a^2*b^2*d*cos(2*d*x + 2*c) + a^2*b^2*d)*cos(6*d*x + 6*c) - 42*(7*a^2*b^2*d*cos(2*d*x + 2*c) - a^2*b^2*d)*cos(4*d*x + 4*c) - 14*(a^2*b^2*d*sin(12*d*x + 12*c) - 3*a^2*b^2*d*sin(10*d*x + 10*c) + 5*a^2*b^2*d*sin(8*d*x + 8*c) - 5*a^2*b^2*d*sin(6*d*x + 6*c) + 3*a^2*b^2*d*sin(4*d*x + 4*c) - a^2*b^2*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 98*(3*a^2*b^2*d*sin(10*d*x + 10*c) - 5*a^2*b^2*d*sin(8*d*x + 8*c) + 5*a^2*b^2*d*sin(6*d*x + 6*c) - 3*a^2*b^2*d*sin(4*d*x + 4*c) + a^2*b^2*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) - 294*(5*a^2*b^2*d*sin(8*d*x + 8*c) - 5*a^2*b^2*d*sin(6*d*x + 6*c) + 3*a^2*b^2*d*sin(4*d*x + 4*c) - a^2*b^2*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) - 490*(5*a^2*b^2*d*sin(6*d*x + 6*c) - 3*a^2*b^2*d*sin(4*d*x + 4*c) + a^2*b^2*d*sin(2*d*x +

```
2*c))*sin(8*d*x + 8*c) - 490*(3*a^2*b^2*d*sin(4*d*x + 4*c) - a^2*b^2*d*sin(
2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate((b*cos(8*d*x + 8*c)*cos(4*d*x + 4
*c) - 4*b*cos(6*d*x + 6*c)*cos(4*d*x + 4*c) - 2*(8*a - 3*b)*cos(4*d*x + 4*c
)^2 + b*sin(8*d*x + 8*c)*sin(4*d*x + 4*c) - 4*b*sin(6*d*x + 6*c)*sin(4*d*x
+ 4*c) - 2*(8*a - 3*b)*sin(4*d*x + 4*c)^2 - 4*b*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) - (4*b*cos(2*d*x + 2*c) - b)*cos(4*d*x + 4*c))/(a^2*b^2*cos(8*d*x +
8*c)^2 + 16*a^2*b^2*cos(6*d*x + 6*c)^2 + 16*a^2*b^2*cos(2*d*x + 2*c)^2 + a^
2*b^2*sin(8*d*x + 8*c)^2 + 16*a^2*b^2*sin(6*d*x + 6*c)^2 + 16*a^2*b^2*sin(2
*d*x + 2*c)^2 - 8*a^2*b^2*cos(2*d*x + 2*c) + a^2*b^2 + 4*(64*a^4 - 48*a^3*b
 + 9*a^2*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^4 - 48*a^3*b + 9*a^2*b^2)*sin(4*
d*x + 4*c)^2 + 16*(8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) -
 2*(4*a^2*b^2*cos(6*d*x + 6*c) + 4*a^2*b^2*cos(2*d*x + 2*c) - a^2*b^2 + 2*(
8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a^2*b^2*cos(
2*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c))*cos(6*d*
x + 6*c) - 4*(8*a^3*b - 3*a^2*b^2 - 4*(8*a^3*b - 3*a^2*b^2)*cos(2*d*x + 2*c
))*cos(4*d*x + 4*c) - 4*(2*a^2*b^2*sin(6*d*x + 6*c) + 2*a^2*b^2*sin(2*d*x +
 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*a^
2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*sin(6*d*x
+ 6*c)), x) + (105*b*cos(10*d*x + 10*c) - 455*b*cos(8*d*x + 8*c) + 70*(12*a
 + 11*b)*cos(6*d*x + 6*c) - 126*(4*a + 5*b)*cos(4*d*x + 4*c) + 7*(24*a + 35
*b)*cos(2*d*x + 2*c) - 24*a - 35*b)*sin(14*d*x + 14*c) - 7*(105*b*cos(10*d*
x + 10*c) - 455*b*cos(8*d*x + 8*c) + 70*(12*a + 11*b)*cos(6*d*x + 6*c) - 12
6*(4*a + 5*b)*cos(4*d*x + 4*c) + 7*(24*a + 35*b)*cos(2*d*x + 2*c) - 24*a -
35*b)*sin(12*d*x + 12*c) - 21*(280*b*cos(8*d*x + 8*c) - 35*(24*a + 17*b)*co
s(6*d*x + 6*c) + 21*(24*a + 25*b)*cos(4*d*x + 4*c) - 42*(4*a + 5*b)*cos(2*d
*x + 2*c) + 24*a + 30*b)*sin(10*d*x + 10*c) - 35*(105*(8*a + 3*b)*cos(6*d*x
 + 6*c) - 21*(24*a + 17*b)*cos(4*d*x + 4*c) + 14*(12*a + 11*b)*cos(2*d*x +
2*c) - 24*a - 22*b)*sin(8*d*x + 8*c) - 35*(168*b*cos(4*d*x + 4*c) - 91*b*co
s(2*d*x + 2*c) + 13*b)*sin(6*d*x + 6*c) - 105*(...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1585 vs. $2(155) = 310$.
time = 0.63, size = 1585, normalized size = 8.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/840*(16*(24*a + 35*b)*cos(d*x + c)^7 - 56*(24*a + 35*b)*cos(d*x + c)^5 +
 560*(3*a + 4*b)*cos(d*x + c)^3 + 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d
*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 + (a^6 - a^5*b)*sqrt
(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(1/4*
b^7*cos(d*x + c)^2 - 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^
2 - (a^7*b^2 - a^6*b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) +
```

```
1/2*(a^3*b^5*d*cos(d*x + c)*sin(d*x + c) - (a^10 - a^9*b)*sqrt(b^9/((a^13
- 2*a^12*b + a^11*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 + (a
^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b
)*d^2)))*sin(d*x + c) - 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4
+ 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 + (a^6 - a^5*b)*sqrt(b^9/((a^1
3 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(1/4*b^7*cos(d*
x + c)^2 - 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^2 - (a^7*b
^2 - a^6*b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) - 1/2*(a^3*
b^5*d*cos(d*x + c)*sin(d*x + c) - (a^10 - a^9*b)*sqrt(b^9/((a^13 - 2*a^12*b
 + a^11*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 + (a^6 - a^5*b
)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2)))*s
in(d*x + c) - 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*
cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12
*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(-1/4*b^7*cos(d*x + c)^2
+ 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^2 - (a^7*b^2 - a^6*
b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) + 1/2*(a^3*b^5*d*cos
(d*x + c)*sin(d*x + c) + (a^10 - a^9*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b
^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^
9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2)))*sin(d*x +
c) + 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x +
 c)^2 - a^2*d)*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11
*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(-1/4*b^7*cos(d*x + c)^2 + 1/4*b^7
 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^2 - (a^7*b^2 - a^6*b^3)*d^2)
*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) - 1/2*(a^3*b^5*d*cos(d*x + c)
*sin(d*x + c) + (a^10 - a^9*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))
*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^9/((a^13
- 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2)))*sin(d*x + c) - 840*
(a + b)*cos(d*x + c))/((a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a
^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(155) = 310.
time = 0.83, size = 467, normalized size = 2.37

$$\frac{\frac{105\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^2b^2-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,ab^3-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^3+\sqrt{a^6-(a^3-a^2b)a^3}}{a^3-a^2b}}}\right)\right)|a-b|}{3a^7-12a^6b+14a^5b^2-4a^4b^3-a^3b^4}+\frac{105\left(3\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,a^2b^2-6\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,ab^3-\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^3-\sqrt{a^6-(a^3-a^2b)a^3}}{a^3-a^2b}}}\right)\right)|a-b|}{3a^7-12a^6b+14a^5b^2-4a^4b^3-a^3b^4}-\frac{2\left(105a\tan(dx+c)^6+105b\tan(dx+c)^6+105a\tan(dx+c)^4+35b\tan(dx+c)^4+63a\tan(dx+c)^2+15a\right)}{a^2\tan(dx+c)^7}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")

[Out] 1/210*(105*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 + sqrt(a^6 - (a^3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*a^7 - 12*a^6*b + 14*a^5*b^2 - 4*a^4*b^3 - a^3*b^4) + 105*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 - sqrt(a^6 - (a^3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*a^7 - 12*a^6*b + 14*a^5*b^2 - 4*a^4*b^3 - a^3*b^4) - 2*(105*a*tan(d*x + c)^6 + 105*b*tan(d*x + c)^6 + 105*a*tan(d*x + c)^4 + 35*b*tan(d*x + c)^4 + 63*a*tan(d*x + c)^2 + 15*a)/(a^2*tan(d*x + c)^7))/d

Mupad [B]
time = 17.08, size = 1704, normalized size = 8.65

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(c + d*x)^8*(a - b*sin(c + d*x)^4)),x)

[Out] (atan((((((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 + tan(c + d*x)*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) - tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*1i - ((((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 - tan(c + d*x)*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) + tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*1i)/(((((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 + tan(c + d*x)*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) - tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2) + ((((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 - tan(c + d*x)*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) + tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2) - 2*a^4*b^8 + 2*a^5*b^7))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*2i)/d + (atan((((-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 + tan(c + d*x)*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) - tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(-((a^11*b^9)^(1/2) - a^6*b^

```
4)/(16*(a^11*b - a^12)))^(1/2)*1i - ((-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^
11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 - tan(c + d*x)
*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64
*a^12*b^3 - 128*a^13*b^2)) + tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(-((a^11
*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*1i)/(((-((a^11*b^9)^(1/2
) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^1
1*b^3 + tan(c + d*x)*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(
1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) - tan(c + d*x)*(4*a^6*b^7 -
4*a^8*b^5))*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2) + ((
-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*
a^10*b^4 + 16*a^11*b^3 - tan(c + d*x)*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a
^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) + tan(c + d
*x)*(4*a^6*b^7 - 4*a^8*b^5))*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a
^12)))^(1/2) - 2*a^4*b^8 + 2*a^5*b^7))*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(
a^11*b - a^12)))^(1/2)*2i)/d - (1/(7*a) + (3*tan(c + d*x)^2)/(5*a) + (tan(c
 + d*x)^6*(a + b))/a^2 + (tan(c + d*x)^4*(3*a + b))/(3*a^2))/(d*tan(c + d*x
)^7)
```

3.212 $\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d}+\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{9/4}d}-\frac{\cos(c+}{b^2d}$$

```
[Out] -cos(d*x+c)/b^2/d-1/4*a*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b^2/d/(a-b+2*
b*cos(d*x+c)^2-b*cos(d*x+c)^4)+1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/
2))^(1/2))*a^(1/2)*(5*a^(1/2)-6*b^(1/2))/b^(9/4)/d/(a^(1/2)-b^(1/2))^(3/2)+
1/8*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*a^(1/2)*(5*a^(1/2)+
6*b^(1/2))/b^(9/4)/d/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A]

time = 0.34, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1219, 1690, 1180, 211, 214}

$$\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (Sqrt[a]*(5*Sqrt[a] - 6*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a]
 - Sqrt[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(9/4)*d) + (Sqrt[a]*(5*Sqrt[a]
 + 6*Sqrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(
Sqrt[a] + Sqrt[b])^(3/2)*b^(9/4)*d) - Cos[c + d*x]/(b^2*d) - (a*Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(4*(a - b)*b^2*d*(a - b + 2*b*Cos[c + d*x]^2
- b*Cos[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(a-b+2bx^2-bx^4)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\,(a+b-b\cos^2(c+dx))}{4(a-b)b^2d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))} + \frac{\text{Subst}\left(\int \frac{2a\left(a+\frac{a^2}{b}-\right.}{}\right.}{}$$

$$= -\frac{a\cos(c+dx)\,(a+b-b\cos^2(c+dx))}{4(a-b)b^2d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))} + \frac{\text{Subst}\left(\int \left(-\frac{8a(a-b)}{b}\right.\right.}{}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{a\cos(c+dx)\,(a+b-b\cos^2(c+dx))}{4(a-b)b^2d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))} + \frac{\text{Sub}}{}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{a\cos(c+dx)\,(a+b-b\cos^2(c+dx))}{4(a-b)b^2d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))} - \frac{\left(\sqrt{\right.}{}$$

$$= \frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d} + \frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-}}{8\left(\sqrt{a}+\sqrt{b}\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.82, size = 486, normalized size = 2.06

$$-\frac{32\cos(c+dx) + \frac{32a\cos(c+dx)(2a+b-b\cos(2(c+dx)))}{(a-b)(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))} + \frac{ia\,\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)-40a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+54b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+20ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-27ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2+40a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-54b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-20ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+27ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6-ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{a-b}}{32b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/32*(32*Cos[c + d*x] + (32*a*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))
/((a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (I*a*R
ootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1
^2] - 40*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 54*b*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (20*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2]*#1^2 - (27*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 40*a*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 54*b*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)]*#1^4 - (20*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (27*I)*
b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c +
```

```
d*x] - #1)]*#1^6 - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) -
8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(b^2*d)
```

Maple [A]
time = 1.06, size = 209, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)}{b^2}+\frac{a\left(\frac{\frac{\left(\cos^3(dx+c)\right)b}{4a-4b}-\frac{(a+b)\cos(dx+c)}{4(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{b\left(\frac{\left(-\sqrt{ab}-6b+5a\right)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(-\sqrt{ab}\cdots\right.}{\cdots}\right)}{4a-4b}\right)}{b^2}}{d}$
default	$\frac{-\frac{\cos(dx+c)}{b^2}+\frac{a\left(\frac{\frac{\left(\cos^3(dx+c)\right)b}{4a-4b}-\frac{(a+b)\cos(dx+c)}{4(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{b\left(\frac{\left(-\sqrt{ab}-6b+5a\right)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(-\sqrt{ab}\cdots\right.}{\cdots}\right)}{4a-4b}\right)}{b^2}}{d}$
risch	$-\frac{e^{i(dx+c)}}{2b^2d}-\frac{e^{-i(dx+c)}}{2b^2d}-\frac{a\left(b\,e^{7i(dx+c)}-4a\,e^{5i(dx+c)}-b\,e^{5i(dx+c)}-4a\,e^{3i(dx+c)}-b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}\right)}{2b^2(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\frac{i\left(\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/b^2*cos(d*x+c)+1/b^2*a*((1/4*b/(a-b)*cos(d*x+c)^3-1/4*(a+b)/(a-b)*cos(d*x+c))/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)+1/4/(a-b)*b*(1/2*(-(a*b)^(1/2)-6*b+5*a)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))-1/2*(-(a*b)^(1/2)+6*b-5*a)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2)))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")

[Out] 1/2*((2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c)*cos(d*x + c) - 4*(2*a*b^2 - 3*b^3)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + (2*a*b^2 - 3*b^3)*sin(2*d*x + 2*c)*sin(d*x + c) - ((a*b^2 - b^3)*cos(9*d*x + 9*c) - 4*(a*b^2 - b^3)*cos(7*d*x + 7*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(5*d*x + 5*c) - 4*(a*b^2 - b^3)*cos(3*d*x + 3*c) + (a*b^2 - b^3)*cos(d*x + c))*cos(10*d*x + 10*c) - (a*b^2 - b^3 - (2*a*b^2 - 3*b^3)*cos(8*d*x + 8*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(6*d*x + 6*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(4*d*x + 4*c) - (2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(9*d*x + 9*c) - (4*(2*a*b^2 - 3*b^3)*cos(7*d*x + 7*c) + 2*(16*a^2*b - 30*a*b^2 + 9*b^3)*cos(5*d*x + 5*c) + 4*(2*a*b^2 - 3*b^3)*cos(3*d*x + 3*c) - (2*a*b^2 - 3*b^3)*cos(d*x + c))*cos(8*d*x + 8*c) + 4*(a*b^2 - b^3 - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(6*d*x + 6*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(4*d*x + 4*c) - (2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(7*d*x + 7*c) - (2*(160*a^3 - 196*a^2*b + 67*a*b^2 - 6*b^3)*cos(5*d*x + 5*c) + 4*(20*a^2*b - 17*a*b^2 + 2*b^3)*cos(3*d*x + 3*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(d*x + c))*cos(6*d*x + 6*c) + 2*(8*a^2*b - 11*a*b^2 + 3*b^3 - (160*a^3 - 196*a^2*b + 67*a*b^2 - 6*b^3)*cos(4*d*x + 4*c) - (16*a^2*b - 30*a*b^2 + 9*b^3)*cos(2*d*x + 2*c))*cos(5*d*x + 5*c) - (4*(20*a^2*b - 17*a*b^2 + 2*b^3)*cos(3*d*x + 3*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(d*x + c))*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3 - (2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(3*d*x + 3*c) - (a*b^2 - b^3)*cos(d*x + c) + 2*((a*b^4 - b^5)*d*cos(9*d*x + 9*c)^2 + 16*(a*b^4 - b^5)*d*cos(7*d*x + 7*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*cos(5*d*x + 5*c)^2 + 16*(a*b^4 - b^5)*d*cos(3*d*x + 3*c)^2 - 8*(a*b^4 - b^5)*d*cos(3*d*x + 3*c)*cos(d*x + c) + (a*b^4 - b^5)*d*cos(d*x + c)^2 + (a*b^4 - b^5)*d*sin(9*d*x + 9*c)^2 + 16*(a*b^4 - b^5)*d*sin(7*d*x + 7*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*sin(5*d*x + 5*c)^2 + 16*(a*b^4 - b^5)*d*sin(3*d*x + 3*c)^2 - 8*(a*b^4 - b^5)*

```
d*sin(3*d*x + 3*c)*sin(d*x + c) + (a*b^4 - b^5)*d*sin(d*x + c)^2 - 2*(4*(a*
b^4 - b^5)*d*cos(7*d*x + 7*c) + 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(5*d*
x + 5*c) + 4*(a*b^4 - b^5)*d*cos(3*d*x + 3*c) - (a*b^4 - b^5)*d*cos(d*x + c
))*cos(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(5*d*x + 5*c
) + 4*(a*b^4 - b^5)*d*cos(3*d*x + 3*c) - (a*b^4 - b^5)*d*cos(d*x + c))*cos(
7*d*x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(3*d*x + 3*c) - (8*
a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(d*x + c))*cos(5*d*x + 5*c) - 2*(4*(a*b^4
- b^5)*d*sin(7*d*x + 7*c) + 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(5*d*x +
5*c) + 4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c) - (a*b^4 - b^5)*d*sin(d*x + c))*s
in(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(5*d*x + 5*c) +
4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c) - (a*b^4 - b^5)*d*sin(d*x + c))*sin(7*d*
x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(3*d*x + 3*c) - (8*a^2*
b^3 - 11*a*b^4 + 3*b^5)*d*sin(d*x + c))*sin(5*d*x + 5*c))*integrate(-1/2*(4
*a*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 4*a*b^2*cos(2*d*x + 2*c)*sin(d*x + c
) + a*b^2*sin(d*x + c) + 4*(20*a^2*b - 27*a*b^2)*cos(3*d*x + 3*c)*sin(2*d*x
 + 2*c) - (a*b^2*sin(7*d*x + 7*c) - a*b^2*sin(d*x + c) + (20*a^2*b - 27*a*b
^2)*sin(5*d*x + 5*c) - (20*a^2*b - 27*a*b^2)*sin(3*d*x + 3*c))*cos(8*d*x +
8*c) - 2*(2*a*b^2*sin(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b -
3*a*b^2)*sin(4*d*x + 4*c))*cos(7*d*x + 7*c) - 4*(a*b^2*sin(d*x + c) - (20*a
^2*b - 27*a*b^2)*sin(5*d*x + 5*c) + (20*a^2*b - 27*a*b^2)*sin(3*d*x + 3*c))
*cos(6*d*x + 6*c) - 2*((160*a^3 - 276*a^2*b + 81*a*b^2)*sin(4*d*x + 4*c) +
2*(20*a^2*b - 27*a*b^2)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 2*((160*a^3 -
276*a^2*b + 81*a*b^2)*sin(3*d*x + 3*c) + (8*a^2*b - 3*a*b^2)*sin(d*x + c))*
cos(4*d*x + 4*c) + (a*b^2*cos(7*d*x + 7*c) - a*b^2*cos(d*x + c) + (20*a^2*b
 - 27*a*b^2)*cos(5*d*x + 5*c) - (20*a^2*b - 27*a*b^2)*cos(3*d*x + 3*c))*sin
(8*d*x + 8*c) + (4*a*b^2*cos(6*d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) - a*b^
2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) + 4*(a*b^2*cos
(d*x + c) - (20*a^2*b - 27*a*b^2)*cos(5*d*x + 5*c) + (20*a^2*b - 27*a*b^2)*
cos(3*d*x + 3*c))*sin(6*d*x + 6*c) - (20*a^2*b - 27*a*b^2 - 2*(160*a^3 - 27
6*a^2*b + 81*a*b^2)*cos(4*d*x + 4*c) - 4*(20*a^2*b - 27*a*b^2)*cos(2*d*x +
2*c))*sin(5*d*x + 5*c) + 2*((160*a^3 - 276*a^2*b + 81*a*b^2)*cos(3*d*x + 3*
c) + (8*a^2*b - 3*a*b^2)*cos(d*x + c))*sin(4*d*x + 4*c) + (20*a^2*b - 27*a*
b^2 - 4*(20*a^2*b - 27*a*b^2)*cos(2*d*x + 2*c))*sin(3*d*x + 3*c))/(a*b^4 -
b^5 + (a*b^4 - b^5)*cos(8*d*x + 8*c)^2 + 16*(a*b^4 - b^5)*cos(6*d*x + 6*c)^
2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*cos(4*d*x + 4*c)^2 + 16
*(a*b^4 - b^5)*cos(2*d*x + 2*c)^2 + (a*b^4 - b^5)*sin(8*d*x + 8*c)^2 + 16*(
a*b^4 - b^5)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 -
9*b^5)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 16*(a*b^4 - b^5)*sin(2*d*x + 2*c)^2 + 2*(a*b^4 - b^5
 - 4*(a*b^4 - b^5)*cos(6*d*x + 6*c) - 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*cos(
4*d*x + 4*c) - 4*(a*b^4 - b^5)*cos(2*d*x + 2*c)...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2649 vs. $2(188) = 376$.

time = 0.73, size = 2649, normalized size = 11.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(16*(a*b - b^2)*cos(d*x + c)^5 - 4*(7*a*b - 8*b^2)*cos(d*x + c)^3 + (
(a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^
2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt
((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*
b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^1
5)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 -
b^7)*d^2))*log((625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x
 + c) + (2*(2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sq
rt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^
6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b
^15)*d^4)) - (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqr
t(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b +
7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4
*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^
2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - ((a*b^3 - b
^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^
3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 -
 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5
*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) -
15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*
log((625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x + c) + (2*
(2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sqrt((625*a^7
 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a
^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4))
+ (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqrt(((a^3*b^4
 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2
 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a
^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b
^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - ((a*b^3 - b^4)*d*cos(d*
x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*
sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b
 + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*
a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47
*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-(625*a
^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x + c) + (2*(2*a^4*b^7
 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sqrt((625*a^7 - 3450*a^
6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 +
15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - (125*a^5
```

```
*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqrt(-((a^3*b^4 - 3*a^2*
b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a
^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 +
 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3
*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + ((a*b^3 - b^4)*d*cos(d*x + c)^4
- 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^
3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^
5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 -
 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 3
6*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-(625*a^5 - 2625*
a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x + c) + (2*(2*a^4*b^7 - 9*a^3*b
^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sqrt((625*a^7 - 3450*a^6*b + 7161
*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^1
1 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + (125*a^5*b^2 - 520
*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b
^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 23
04*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^1
3 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^
2*b^5 + 3*a*b^6 - b^7)*d^2))) - 4*(5*a^2 - 7*a*b + 4*b^2)*cos(d*x + c))/((a
*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2
- 2*a*b^3 + b^4)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[-54,3]
Warning, need to
```

Mupad [B]
time = 16.00, size = 2500, normalized size = 10.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a - b*sin(c + d*x)^4)^2,x)
[Out] - cos(c + d*x)/(b^2*d) - ((cos(c + d*x)*(a*b + a^2))/(4*(a - b)) - (a*b*cos
(c + d*x)^3)/(4*(a - b)))/(d*(a*b^2 - b^3 + 2*b^3*cos(c + d*x)^2 - b^3*cos(
c + d*x)^4)) - (atan(((((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b
^5 - 2*a*b^4 + a^2*b^3)) - (cos(c + d*x)*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*
(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/
2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*
a^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/
2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(
a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) + (co
s(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*(
(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 -
15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a
^3*b^9)))^(1/2)*1i - (((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b^
5 - 2*a*b^4 + a^2*b^3)) + (cos(c + d*x)*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(
a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2
))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*a
^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2
) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a
^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) - (cos
(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*((
25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 1
5*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^
3*b^9)))^(1/2)*1i)/((36*a^3*b - 25*a^4)/(32*(b^5 - 2*a*b^4 + a^2*b^3)) + ((
(1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b^5 - 2*a*b^4 + a^2*b^3))
 - (cos(c + d*x)*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b
^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^1
2 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*a^2*b^6 + 256*a^3*b^5))/
(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1
/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*
a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) + (cos(c + d*x)*(25*a^4 - 59*
a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2)
+ 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3
*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) + (((179
2*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b^5 - 2*a*b^4 + a^2*b^3)) + (
cos(c + d*x)*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 +
 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 -
3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*a^2*b^6 + 256*a^3*b^5))/(4*(
```

```
a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2)
- 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^
11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) - (cos(c + d*x)*(25*a^4 - 59*a^3*
b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2) + 48
*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9
)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)))*((25*a^2*(
a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^
5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))
^(1/2)*2i)/d - (atan(((((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b
^5 - 2*a*b^4 + a^2*b^3)) - (cos(c + d*x)*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2
*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1
/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512
*a^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*(-(25*a^2*(a^3*b^9)^(
1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b
*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) + (
cos(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))
*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6
 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10
+ a^3*b^9)))^(1/2)*1i - (((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*
(b^5 - 2*a*b^4 + a^2*b^3)) + (cos(c + d*x)*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b
^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^
(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 5
12*a^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*(-(25*a^2*(a^3*b^9)
^(1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a
*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) -
 (cos(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)
))*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b
^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^1
0 + a^3*b^9)))^(1/2)*1i)/((36*a^3*b - 25*a^4)/(32*(b^5 - 2*a*b^4 + a^2*b^3)
) + (((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b...
```

3.213 $\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{7/4}d}-\frac{a\cos(c}{4(a-b)bd\,(a-b}$$

```
[Out] -1/4*a*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*
x+c)^4)+1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(1/2)-4
*b^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4)*cos(d*x+c)/
(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+4*b^(1/2))/b^(7/4)/d/(a^(1/2)+b^(1/2))^
(3/2)
```

Rubi [A]
time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1219, 1180, 211, 214}

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*Ar
cTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(Sqrt[a] + Sqrt[b
])^(3/2)*b^(7/4)*d) - (a*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*(a - b)*b*d*
(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=-\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^2}\,dx,x,\cos(c+dx)\right)}{d}$$

$$=-\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}+\frac{\text{Subst}\Big(\int \frac{4a(a-2b)-2a(3}{a-b+2bx^2-}}{8a(}$$

$$=-\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}-\frac{\left(3a-\sqrt{a}\sqrt{b}-4b\right)}{}$$

$$=\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\text{c}}{\sqrt{\sqrt{a}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{7/4}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.40, size = 565, normalized size = 2.69

$$\frac{\frac{16a(-5\cos(c+dx)+\cos(3(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}-i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{6a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-8b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-3ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+4ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)-10a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+24b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+5ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-12ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2+10a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-24b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-5ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+12ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4-6a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6+8b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6+3ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6-4ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{32(a-b)bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((16*a*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*
x)] - b*Cos[4*(c + d*x)]) - I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
 4*b*#1^6 + b*#1^8 & , (6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 8*b*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (3*I)*a*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2] + (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 10*a*ArcTan[Sin[c +
 d*x]/(Cos[c + d*x] - #1)]*#1^2 + 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
#1)]*#1^2 + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*b*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^4 - 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (5*I)*a*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (12*I)*b*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^4 - 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + 8*b*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (3*I)*a*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^6 - (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1)
 - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(32*(a - b)*b*d)
```

Maple [A]
time = 0.94, size = 214, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\cos^3(dx+c)\right)}{4b(a-b)}-\frac{a\cos(dx+c)}{2b(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{-\frac{\left(3a\sqrt{ab}-4\sqrt{ab}\,b+ab\right)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}+b\right)b}}+\frac{\left(3a\sqrt{ab}-4\sqrt{\cdots}\right.}{2\sqrt{\cdots}}}{4a-4b}}{d}$
default	$\frac{\frac{\frac{a\left(\cos^3(dx+c)\right)}{4b(a-b)}-\frac{a\cos(dx+c)}{2b(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{-\frac{\left(3a\sqrt{ab}-4\sqrt{ab}\,b+ab\right)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}+b\right)b}}+\frac{\left(3a\sqrt{ab}-4\sqrt{\cdots}\right.}{2\sqrt{\cdots}}}{4a-4b}}{d}$

risch	$-\frac{a\left(e^{7i(dx+c)}-5\,e^{5i(dx+c)}-5\,e^{3i(dx+c)}+e^{i(dx+c)}\right)}{2b(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^3b^7d^4-3a^2b^8d^4\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((1/4*a/b/(a-b)*cos(d*x+c)^3-1/2*a/b/(a-b)*cos(d*x+c))/(a-b+2*b*cos(d*x
+c)^2-b*cos(d*x+c)^4)+1/4/(a-b)*(-1/2*(3*a*(a*b)^(1/2)-4*(a*b)^(1/2)*b+a*b)
/(a*b)^(1/2)/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/(((a*b)^(1/2)
+b)*b)^(1/2))+1/2*(3*a*(a*b)^(1/2)-4*(a*b)^(1/2)*b-a*b)/(a*b)^(1/2)/b/(((a*
b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*cos(2*d*x + 2*c)*cos(d*x + c) - 20*a*b*sin(3*d*x + 3*c)*sin(2*d*
x + 2*c) + 4*a*b*sin(2*d*x + 2*c)*sin(d*x + c) - a*b*cos(d*x + c) - (a*b*co
s(7*d*x + 7*c) - 5*a*b*cos(5*d*x + 5*c) - 5*a*b*cos(3*d*x + 3*c) + a*b*cos(
d*x + c))*cos(8*d*x + 8*c) + (4*a*b*cos(6*d*x + 6*c) + 4*a*b*cos(2*d*x + 2*
c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*cos(7*d*x + 7*c) - 4*(5*a*b*
cos(5*d*x + 5*c) + 5*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*cos(6*d*x + 6
*c) - 5*(4*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))
*cos(5*d*x + 5*c) - 2*(5*(8*a^2 - 3*a*b)*cos(3*d*x + 3*c) - (8*a^2 - 3*a*b)
*cos(d*x + c))*cos(4*d*x + 4*c) - 5*(4*a*b*cos(2*d*x + 2*c) - a*b)*cos(3*d*
x + 3*c) + 2*((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6
*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x +
 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x
 + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b
^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*
b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2
*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3
- b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x +
4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*
c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^
4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2
 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)
*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11
*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*si
```

n(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*
(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-1/2*(4*(5*a*
b - 12*b^2)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*(3*a*b - 4*b^2)*cos(d*x +
c)*sin(2*d*x + 2*c) + 4*(3*a*b - 4*b^2)*cos(2*d*x + 2*c)*sin(d*x + c) + ((
3*a*b - 4*b^2)*sin(7*d*x + 7*c) - (5*a*b - 12*b^2)*sin(5*d*x + 5*c) + (5*a*
b - 12*b^2)*sin(3*d*x + 3*c) - (3*a*b - 4*b^2)*sin(d*x + c))*cos(8*d*x + 8*
c) + 2*(2*(3*a*b - 4*b^2)*sin(6*d*x + 6*c) + (24*a^2 - 41*a*b + 12*b^2)*sin
(4*d*x + 4*c) + 2*(3*a*b - 4*b^2)*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 4*((
5*a*b - 12*b^2)*sin(5*d*x + 5*c) - (5*a*b - 12*b^2)*sin(3*d*x + 3*c) + (3*a
*b - 4*b^2)*sin(d*x + c))*cos(6*d*x + 6*c) - 2*((40*a^2 - 111*a*b + 36*b^2)
*sin(4*d*x + 4*c) + 2*(5*a*b - 12*b^2)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) -
2*((40*a^2 - 111*a*b + 36*b^2)*sin(3*d*x + 3*c) - (24*a^2 - 41*a*b + 12*b^
2)*sin(d*x + c))*cos(4*d*x + 4*c) - ((3*a*b - 4*b^2)*cos(7*d*x + 7*c) - (5*
a*b - 12*b^2)*cos(5*d*x + 5*c) + (5*a*b - 12*b^2)*cos(3*d*x + 3*c) - (3*a*b
- 4*b^2)*cos(d*x + c))*sin(8*d*x + 8*c) + (3*a*b - 4*b^2 - 4*(3*a*b - 4*b^
2)*cos(6*d*x + 6*c) - 2*(24*a^2 - 41*a*b + 12*b^2)*cos(4*d*x + 4*c) - 4*(3*
a*b - 4*b^2)*cos(2*d*x + 2*c))*sin(7*d*x + 7*c) - 4*((5*a*b - 12*b^2)*cos(5
*d*x + 5*c) - (5*a*b - 12*b^2)*cos(3*d*x + 3*c) + (3*a*b - 4*b^2)*cos(d*x +
c))*sin(6*d*x + 6*c) - (5*a*b - 12*b^2 - 2*(40*a^2 - 111*a*b + 36*b^2)*cos
(4*d*x + 4*c) - 4*(5*a*b - 12*b^2)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) + 2*(
(40*a^2 - 111*a*b + 36*b^2)*cos(3*d*x + 3*c) - (24*a^2 - 41*a*b + 12*b^2)*c
os(d*x + c))*sin(4*d*x + 4*c) + (5*a*b - 12*b^2 - 4*(5*a*b - 12*b^2)*cos(2*
d*x + 2*c))*sin(3*d*x + 3*c) - (3*a*b - 4*b^2)*sin(d*x + c))/(a*b^3 - b^4 +
(a*b^3 - b^4)*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*cos(6*d*x + 6*c)^2 + 4
*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*cos(4*d*x + 4*c)^2 + 16*(a*b^3
- b^4)*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*sin(8*d*x + 8*c)^2 + 16*(a*b^3 -
b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*si
n(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 16*(a*b^3 - b^4)*sin(2*d*x + 2*c)^2 + 2*(a*b^3 - b^4 - 4*(a*b
^3 - b^4)*cos(6*d*x + 6*c) - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x + 4
c) - 4(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^3 - b^4 -
2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*
d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4 - 4*(8*a^2*b
^2 - 11*a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^3 - b^4)
*cos(2*d*x + 2*c) - 4*(2*(a*b^3 - b^4)*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a
*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(8*d*
x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 -
b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (a*b*sin(7*d*x + 7*c) - 5*a
*b*sin(5*d*x + 5*c) - 5*a*b*sin(3*d*x + 3*c) + a*b*sin(d*x + c))*sin(8*d*x
+ 8*c) + 2*(2*a*b*sin(6*d*x + 6*c) + 2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*
b)*sin(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(5*a*b*sin(5*d*x + 5*c) + 5*a*b*s
in(3*d*x + 3*c) - a*b*sin(d*x + c))*sin(6*d*x + 6*c) - 10*(2*a*b*sin(2*d*x
+ 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*sin(5*d*x + 5*c) - 2*(5*(8*a^2 -
3*a*b)*sin(3*d*x + 3*c) - (8*a^2 - 3*a*b)*sin(...

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. $2(161) = 322$.
time = 0.69, size = 2507, normalized size = 11.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*cos(d*x + c)^3 - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^
3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^
4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b
^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b
^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a^3*b^3 - 3*a^2*b
^4 + 3*a*b^5 - b^6)*d^2))*log((81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*co
s(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*
sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b
^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*
d^4)) - 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d)*sqrt(-((a^3*b^3 -
 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1
392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(
a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^3*b^3 -
 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1
392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log((81*a^3 - 405*a^2*b + 680*a*b^2 - 38
4*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*
b^9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4
)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
 + b^13)*d^4)) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d)*sqrt(((a
^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3
*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^
3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((
a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cos(d*x + c)
^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((
a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^
3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a
^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/(
(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-(81*a^3 - 405*a^2*b + 680*
a*b^2 - 384*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*
a*b^8 + 5*b^9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 +
 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11
- 6*a*b^12 + b^13)*d^4)) - 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d
```

```
)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b
 + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*
b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b +
 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - ((a*b^2 - b^3)*d*c
os(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*
d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b
 + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*
b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b -
 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-(81*a^3 - 405*a^
2*b + 680*a*b^2 - 384*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2
*b^7 - 18*a*b^8 + 5*b^9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392
*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 -
 48*b^5)*d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 -
522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8
+ 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 +
 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - 8*a*cos(d
*x + c))/((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2
 - (a^2*b - 2*a*b^2 + b^3)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[54,-78
]Warning, need t
```

Mupad [**B**]
time = 15.87, size = 2500, normalized size = 11.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] ((a*cos(c + d*x)^3)/(4*b*(a - b)) - (a*cos(c + d*x))/(2*b*(a - b)))/(d*(a -
 b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((1024*a*b^6 - 1536*
a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) - (cos(c + d*x)*(-(9*
a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 -
29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(
256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(
a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*
b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (cos
(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*
(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a
*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*1i -
(((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2))
+ (cos(c + d*x)*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 +
16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8
 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*
b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^
6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^
3*b^7)))^(1/2) - (cos(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a
*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b
^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a
^3*b^7)))^(1/2)*1i)/((((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 -
 2*a*b^3 + a^2*b^2)) - (cos(c + d*x)*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7
)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b
^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3
*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/
2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 -
b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (cos(c + d*x)*(16*a*b^2 - 23*a^2*b +
9*a^3))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1
/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 -
 b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (((1024*a*b^6 - 1536*a^2*b^5 + 512*a
^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) + (cos(c + d*x)*(-(9*a^2*(a*b^7)^(1/
2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^
(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512
*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) +
24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2)
)/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) - (cos(c + d*x)*(16*a
*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) +
 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2
))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) - (64*a*b^2 - 84*a^2
*b + 27*a^3)/(32*(b^4 - 2*a*b^3 + a^2*b^2))))*(-(9*a^2*(a*b^7)^(1/2) + 24*b
^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(2
```

56*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*2i)/d - (atan(((((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) - (cos(c + d*x)*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (cos(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*1i - (((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) + (cos(c + d*x)*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) - (cos(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*1i)/((((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) - (cos(c + d*x)*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (cos(c...

3.214 $\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

Optimal. Leaf size=217

$$\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d}+\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{5/4}d}-\frac{\cos(c+dx)\,(}{4(a-b)bd\,(a-b+2}$$

```
[Out] -1/4*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(
d*x+c)^4)+1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(a^(1/2)-2
*b^(1/2))/b^(5/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/4)*cos
(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+2*b^(1/2))/b^(5/4)/d/a^(1/2)/(a^(
1/2)+b^(1/2))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1219, 1180, 211, 214}

$$\frac{\left(\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]
]])/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) + ((Sqrt[a] + 2*Sqrt[b]
)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*(Sqrt
[a] + Sqrt[b])^(3/2)*b^(5/4)*d) - (Cos[c + d*x]*(a + b - b*Cos[c + d*x]^2))
/(4*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a-b+2bx^2-bx^4)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a(a-3b)+2a}{a-b+2bx^2-b}\right.}{8a(a}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} - \frac{\left(\sqrt{a}-2\sqrt{b}\right)\text{Sub}}{8}$$

$$= \frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d} + \frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos}{\sqrt{\sqrt{a}}}\right.}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{5/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.44, size = 469, normalized size = 2.16

$$-\frac{\frac{32\cos(c+dx)(2a+b-b\cos(2(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}+i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)-8a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+22b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+4ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-11ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2+8a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-22b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-4ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+11ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6-ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{32(a-b)bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/32*((32*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(8*a - 3*b + 4*b*Co
s[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 8*a*ArcTan[Sin[c + d*x]/(Co
s[c + d*x] - #1)]*#1^2 + 22*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
 + (4*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (11*I)*b*Log[1 - 2*Cos[
c + d*x]*#1 + #1^2]*#1^2 + 8*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
4 - 22*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (4*I)*a*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^4 + (11*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#
1^4 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*b*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1
^7) & ])/((a - b)*b*d)
```

Maple [A]
time = 0.88, size = 188, normalized size = 0.87

method	result
derivativedivides	$\frac{-\frac{-\frac{\cos^3(dx+c)}{4(a-b)}+\frac{(a+b)\cos(dx+c)}{4b(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}-\frac{\frac{\left(\sqrt{ab}+2b-a\right)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(\sqrt{ab}-2b+a\right)\text{arctanh}\left(\cdots\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}\cdots\right.}}}{4(a-b)}}{d}$
default	$\frac{-\frac{-\frac{\cos^3(dx+c)}{4(a-b)}+\frac{(a+b)\cos(dx+c)}{4b(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}-\frac{\frac{\left(\sqrt{ab}+2b-a\right)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(\sqrt{ab}-2b+a\right)\text{arctanh}\left(\cdots\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}\cdots\right.}}}{4(a-b)}}{d}$

risch	$-\frac{b\,e^{7i(dx+c)}-4a\,e^{5i(dx+c)}-b\,e^{5i(dx+c)}-4a\,e^{3i(dx+c)}-b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}{2b(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5b^5d^4-3a^4b^6\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(-1/4/(a-b)*cos(d*x+c)^3+1/4*(a+b)/b/(a-b)*cos(d*x+c))/(a-b+2*b*cos(d
*x+c)^2-b*cos(d*x+c)^4)-1/4/(a-b)*(1/2*((a*b)^(1/2)+2*b-a)/(a*b)^(1/2)/(((a
*b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))-1/2*((
a*b)^(1/2)-2*b+a)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c
)/(((a*b)^(1/2)+b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*b^2*cos(2*d*x + 2*c)*cos(d*x + c) + 4*b^2*sin(2*d*x + 2*c)*sin(d*x +
 c) - b^2*cos(d*x + c) - 4*(4*a*b + b^2)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c)
- (b^2*cos(7*d*x + 7*c) + b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c)
 - (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*
c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*cos
(7*d*x + 7*c) + 4*(b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c) - (4*a
*b + b^2)*cos(3*d*x + 3*c))*cos(6*d*x + 6*c) + (4*a*b + b^2 - 2*(32*a^2 - 4
*a*b - 3*b^2)*cos(4*d*x + 4*c) - 4*(4*a*b + b^2)*cos(2*d*x + 2*c))*cos(5*d*
x + 5*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*c
os(d*x + c))*cos(4*d*x + 4*c) + (4*a*b + b^2 - 4*(4*a*b + b^2)*cos(2*d*x +
2*c))*cos(3*d*x + 3*c) + 2*((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3
- b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)
*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^
4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3
*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 -
 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d
*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d
- 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*
cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) +
 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4
*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (
```

```
8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*
d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d
*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate
(-1/2*(4*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 4*b^2*cos(2*d*x + 2*c)*sin(d*x
 + c) + 4*(4*a*b - 11*b^2)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + b^2*sin(d*x
+ c) - (b^2*sin(7*d*x + 7*c) - b^2*sin(d*x + c) + (4*a*b - 11*b^2)*sin(5*d*
x + 5*c) - (4*a*b - 11*b^2)*sin(3*d*x + 3*c))*cos(8*d*x + 8*c) - 2*(2*b^2*s
in(6*d*x + 6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c)
)*cos(7*d*x + 7*c) - 4*(b^2*sin(d*x + c) - (4*a*b - 11*b^2)*sin(5*d*x + 5*c
) + (4*a*b - 11*b^2)*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) - 2*((32*a^2 - 100*
a*b + 33*b^2)*sin(4*d*x + 4*c) + 2*(4*a*b - 11*b^2)*sin(2*d*x + 2*c))*cos(5
*d*x + 5*c) - 2*((32*a^2 - 100*a*b + 33*b^2)*sin(3*d*x + 3*c) + (8*a*b - 3*
b^2)*sin(d*x + c))*cos(4*d*x + 4*c) + (b^2*cos(7*d*x + 7*c) - b^2*cos(d*x +
 c) + (4*a*b - 11*b^2)*cos(5*d*x + 5*c) - (4*a*b - 11*b^2)*cos(3*d*x + 3*c)
)*sin(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2
 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) + 4*(b^2*cos(d*x +
c) - (4*a*b - 11*b^2)*cos(5*d*x + 5*c) + (4*a*b - 11*b^2)*cos(3*d*x + 3*c))
*sin(6*d*x + 6*c) - (4*a*b - 11*b^2 - 2*(32*a^2 - 100*a*b + 33*b^2)*cos(4*d
*x + 4*c) - 4*(4*a*b - 11*b^2)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) + 2*((32*
a^2 - 100*a*b + 33*b^2)*cos(3*d*x + 3*c) + (8*a*b - 3*b^2)*cos(d*x + c))*si
n(4*d*x + 4*c) + (4*a*b - 11*b^2 - 4*(4*a*b - 11*b^2)*cos(2*d*x + 2*c))*sin
(3*d*x + 3*c))/(a*b^3 - b^4 + (a*b^3 - b^4)*cos(8*d*x + 8*c)^2 + 16*(a*b^3
- b^4)*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*c
os(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*sin
(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a
^2*b^2 + 57*a*b^3 - 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 +
3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*sin(2*d*x + 2*c
)^2 + 2*(a*b^3 - b^4 - 4*(a*b^3 - b^4)*cos(6*d*x + 6*c) - 2*(8*a^2*b^2 - 11
*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(8*
d*x + 8*c) - 8*(a*b^3 - b^4 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x +
4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 -
11*a*b^3 + 3*b^4 - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(4
*d*x + 4*c) - 8*(a*b^3 - b^4)*cos(2*d*x + 2*c) - 4*(2*(a*b^3 - b^4)*sin(6*d
*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^
4)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x)
- (b^2*sin(7*d*x + 7*c) + b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*c)
 - (4*a*b + b^2)*sin(3*d*x + 3*c))*sin(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x +
6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*sin(7*d*x
 + 7*c) + 4*(b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*c) - (4*a*b + b
^2)*sin(3*d*x + 3*c))*sin(6*d*x + 6*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*sin(4*
d*x + 4*c) + 2*(4*a*b + b^2)*sin(2*d*x + 2*c))*sin(5*d*x + 5*c) - 2*((32*a^
2 - 4*a*b - 3*b^2)*sin(3*d*x + 3*c) - (8*a*b - 3*b^2)*sin(d*x + c))*sin(4*d
*x + 4*c))/((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 ...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. $2(169) = 338$.

time = 0.66, size = 2507, normalized size = 11.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*b*cos(d*x + c)^3 - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^
3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3
 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64
*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*
b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4
 - a*b^5)*d^2))*log((a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cos(d*x + c) - (2*(
a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a^4 - 10*a^3*b + 41*a^2*
b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 +
15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 -
 24*a*b^4)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4
 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^
5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + a^2 - a*b -
4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))) + ((a*b^2 - b^3)*d
*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3
)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^
3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 -
20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/(
(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*log((a^3 - 9*a^2*b + 28*a*b
^2 - 32*b^3)*cos(d*x + c) - (2*(a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^
3*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*
b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + (
a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 +
 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b
^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^
10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 -
 a*b^5)*d^2))) + ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*
x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7
*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^
11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d
^2))*log(-(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cos(d*x + c) - (2*(a^4*b^5 -
3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a
*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9
 - 6*a^2*b^10 + a*b^11)*d^4)) - (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)
*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*
b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20
```

```
*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a
^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))) - ((a*b^2 - b^3)*d*cos(d*x +
 c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(
-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a
^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8
 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 -
 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*log(-(a^3 - 9*a^2*b + 28*a*b^2 - 32*b
^3)*cos(d*x + c) - (2*(a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a
^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*
a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + (a^4*b - 8
*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7
*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^
11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d
^2))) - 4*(a + b)*cos(d*x + c))/((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2
- b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[-57,-8
4]Warning, need
```

Mupad [**B**]
time = 16.63, size = 2500, normalized size = 11.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - b*sin(c + d*x)^4)^2,x)

[Out] (cos(c + d*x)^3/(4*(a - b)) - (cos(c + d*x)*(a + b))/(4*b*(a - b)))/(d*(a -
 b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((768*a*b^4 - 1024*a
^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*(-(a^2*(a^3*
b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a
^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*(25
6*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*
b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a
^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) + (
cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^
3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*
(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*1
i - (((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)) + (
cos(c + d*x)*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2
*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b
^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*
a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2
*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b
^6 - a^5*b^5)))^(1/2) - (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2 -
2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a
^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4
*b^6 - a^5*b^5)))^(1/2)*1i)/((((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64
*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b
^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*
b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 2
56*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b
^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*
b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) + (cos(c + d*x)*(a^2*b - 3*a
*b^2 + 4*b^3))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3
*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^
2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) - (a - 4*b)/(32*(a^2 - 2*a
*b + b^2)) + (((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b +
b^2)) + (cos(c + d*x)*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*
b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7
+ 3*a^4*b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*
(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*
b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7
+ 3*a^4*b^6 - a^5*b^5)))^(1/2) - (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(
4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*
a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^
7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)
^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8
 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*2i)/d - (atan(((((768*a*b^4 - 1
024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a^2*(
```

```
a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*
b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)
*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((a^2*(a
^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*b
*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)
+ (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2 - 2*a*b + b^2)))*((a^2*(
a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*
b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)
*1i - (((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)) +
 (cos(c + d*x)*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^
2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*
b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2
*a*b + b^2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^2
*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b
^6 - a^5*b^5)))^(1/2) - (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2 -
2*a*b + b^2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^
2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*
b^6 - a^5*b^5)))^(1/2)*1i)/((((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*
(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5
)^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^
8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256
*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)
^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8
 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) + (...
```

3.215 $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d}-\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c\right.}$$

```
[Out] -1/4*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)
^4)-1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/a^(1/2
)/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^
(1/2))/b^(3/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1192, 1180, 211, 214}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/8*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(Sqrt[a]*(Sqrt[
a] - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a
] + Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cos[c + d*
x]*(2 - Cos[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c
 + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-4ab+2abx^2}{a-b+2bx^2-bx^4}\right.}{8a(a-}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-}\right.}{8\sqrt{a}\left(\sqrt{a}\right.}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d} - \frac{\cos(}{4(a-b)d\,(a-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.22, size = 345, normalized size = 1.85

$$\frac{\frac{16(-5\cos(c+dx)+\cos(3(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))} - i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+14\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-7i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-14\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+7i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{32(a-b)d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((16*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)
] - b*Cos[4*(c + d*x)]) - I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4
*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2] + 14*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*
#1^2 - (7*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 14*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^4 + (7*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4
+ 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])
/(32*(a - b)*d)
```

Maple [A]

time = 0.64, size = 204, normalized size = 1.10

method	result
derivativedivides	$\frac{b^2\left(\frac{\sqrt{ab}\left(\frac{\cos(dx+c)}{2\left(\sqrt{ab}+b\right)\left(-b\left(\cos^2(dx+c)\right)+\sqrt{ab}+b\right)}+\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\left(\sqrt{ab}+b\right)\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{4a\,b^2}-\frac{\sqrt{ab}\left(\frac{}{2\left(\sqrt{ab}\right.}\right.}{}\right.}{d}$

default	$\frac{b^2\left(\frac{\sqrt{ab}\left(\frac{\cos(dx+c)}{2\left(\sqrt{ab}+b\right)\left(-b\left(\cos^2(dx+c)\right)+\sqrt{ab}+b\right)}+\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\left(\sqrt{ab}+b\right)\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{4a\,b^2}-\frac{\sqrt{ab}\left(\frac{}{2\left(\sqrt{ab}-b\right.}\right.}{}\right.}{d}$
risch	$-\frac{e^{7i(dx+c)}-5\,e^{5i(dx+c)}-5\,e^{3i(dx+c)}+e^{i(dx+c)}}{2(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(-1+\left(16a^5b^3d^4-48\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*b^2*(1/4*(a*b)^(1/2)/a/b^2*(1/2*cos(d*x+c)/((a*b)^(1/2)+b)/(-b*cos(d*x+
c)^2+(a*b)^(1/2)+b)+1/2/((a*b)^(1/2)+b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b
*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2)))-1/4*(a*b)^(1/2)/a/b^2*(1/2*cos(d*x+
c)/((a*b)^(1/2)-b)/(b*cos(d*x+c)^2-b+(a*b)^(1/2))+1/2/((a*b)^(1/2)-b)/(((a*
b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*b*cos(2*d*x + 2*c)*cos(d*x + c) - 20*b*sin(3*d*x + 3*c)*sin(2*d*x +
2*c) + 4*b*sin(2*d*x + 2*c)*sin(d*x + c) - (b*cos(7*d*x + 7*c) - 5*b*cos(5*
d*x + 5*c) - 5*b*cos(3*d*x + 3*c) + b*cos(d*x + c))*cos(8*d*x + 8*c) + (4*b
*cos(6*d*x + 6*c) + 2*(8*a - 3*b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) -
 b)*cos(7*d*x + 7*c) - 4*(5*b*cos(5*d*x + 5*c) + 5*b*cos(3*d*x + 3*c) - b*c
os(d*x + c))*cos(6*d*x + 6*c) - 5*(2*(8*a - 3*b)*cos(4*d*x + 4*c) + 4*b*cos
```

```
(2*d*x + 2*c) - b)*cos(5*d*x + 5*c) - 2*(5*(8*a - 3*b)*cos(3*d*x + 3*c) - (
8*a - 3*b)*cos(d*x + c))*cos(4*d*x + 4*c) - 5*(4*b*cos(2*d*x + 2*c) - b)*co
s(3*d*x + 3*c) - b*cos(d*x + c) + 2*((a*b^2 - b^3)*d*cos(8*d*x + 8*c)^2 + 1
6*(a*b^2 - b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9
*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^2 - b^3)*d*cos(2*d*x + 2*c)^2 + (a*b^2
 - b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*d*sin(6*d*x + 6*c)^2 + 4*(6
4*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b -
11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^2 - b^3)*d*
sin(2*d*x + 2*c)^2 - 8*(a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a*b^2 - b^3)*d -
 2*(4*(a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*c
os(4*d*x + 4*c) + 4*(a*b^2 - b^3)*d*cos(2*d*x + 2*c) - (a*b^2 - b^3)*d)*cos
(8*d*x + 8*c) + 8*(2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cos(4*d*x + 4*c) + 4*(a
*b^2 - b^3)*d*cos(2*d*x + 2*c) - (a*b^2 - b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(
8*a^2*b - 11*a*b^2 + 3*b^3)*d*cos(2*d*x + 2*c) - (8*a^2*b - 11*a*b^2 + 3*b^
3)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^2 - b^3)*d*sin(6*d*x + 6*c) + (8*a^2*b -
 11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*d*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + 16*((8*a^2*b - 11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c) + 2
*(a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(1/2*(28*b*co
s(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*b*cos(d*x + c)*sin(2*d*x + 2*c) + 4*b*c
os(2*d*x + 2*c)*sin(d*x + c) + (b*sin(7*d*x + 7*c) - 7*b*sin(5*d*x + 5*c) +
 7*b*sin(3*d*x + 3*c) - b*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*b*sin(6*d*x
 + 6*c) + (8*a - 3*b)*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(7*d*x +
7*c) + 4*(7*b*sin(5*d*x + 5*c) - 7*b*sin(3*d*x + 3*c) + b*sin(d*x + c))*cos
(6*d*x + 6*c) - 14*((8*a - 3*b)*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*co
s(5*d*x + 5*c) - 2*(7*(8*a - 3*b)*sin(3*d*x + 3*c) - (8*a - 3*b)*sin(d*x +
c))*cos(4*d*x + 4*c) - (b*cos(7*d*x + 7*c) - 7*b*cos(5*d*x + 5*c) + 7*b*cos
(3*d*x + 3*c) - b*cos(d*x + c))*sin(8*d*x + 8*c) - (4*b*cos(6*d*x + 6*c) +
2*(8*a - 3*b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) - b)*sin(7*d*x + 7*c)
 - 4*(7*b*cos(5*d*x + 5*c) - 7*b*cos(3*d*x + 3*c) + b*cos(d*x + c))*sin(6*d
*x + 6*c) + 7*(2*(8*a - 3*b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) - b)*s
in(5*d*x + 5*c) + 2*(7*(8*a - 3*b)*cos(3*d*x + 3*c) - (8*a - 3*b)*cos(d*x +
 c))*sin(4*d*x + 4*c) - 7*(4*b*cos(2*d*x + 2*c) - b)*sin(3*d*x + 3*c) - b*s
in(d*x + c))/(a*b^2 - b^3 + (a*b^2 - b^3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 -
b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d
*x + 4*c)^2 + 16*(a*b^2 - b^3)*cos(2*d*x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x
 + 8*c)^2 + 16*(a*b^2 - b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 5
7*a*b^2 - 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b
^2 - b^3 - 4*(a*b^2 - b^3)*cos(6*d*x + 6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3
)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8
*(a*b^2 - b^3 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2
- b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3 -
 4*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b
^2 - b^3)*cos(2*d*x + 2*c) - 4*(2*(a*b^2 - b^3)*sin(6*d*x + 6*c) + (8*a^2*b
 - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*s
```

```
in(8*d*x + 8*c) + 16*((8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*
b^2 - b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (b*sin(7*d*x + 7*c) -
5*b*sin(5*d*x + 5*c) - 5*b*sin(3*d*x + 3*c) + b*sin(d*x + c))*sin(8*d*x + 8
*c) + 2*(2*b*sin(6*d*x + 6*c) + (8*a - 3*b)*sin(4*d*x + 4*c) + 2*b*sin(2*d*
x + 2*c))*sin(7*d*x + 7*c) - 4*(5*b*sin(5*d*x + 5*c) + 5*b*sin(3*d*x + 3*c)
 - b*sin(d*x + c))*sin(6*d*x + 6*c) - 10*((8*a - 3*b)*sin(4*d*x + 4*c) + 2*
b*sin(2*d*x + 2*c))*sin(5*d*x + 5*c) - 2*(5*(8*a - 3*b)*sin(3*d*x + 3*c) -
(8*a - 3*b)*sin(d*x + c))*sin(4*d*x + 4*c))/((a*b^2 - b^3)*d*cos(8*d*x + 8*
c)^2 + 16*(a*b^2 - b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a
*b^2 - 9*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^2 - b^3)*d*cos(2*d*x + 2*c)^2
+ (a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*d*sin(6*d*x + 6*c)^
2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*
a^2*b - 11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^2 -
 b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a*b^2 -
b^3)*d - 2*(4*(a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b - 11*a*b^2 + 3*
b^3)*d*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3)*d*cos...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2049 vs. $2(141) = 282$.
time = 0.56, size = 2049, normalized size = 11.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*cos(d*x + c)^3 - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*c
os(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^
3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^
5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b -
 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cos(d*x + c) - ((a^5*b^
2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3
 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^
4)) - 2*(a^2*b + 3*a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4
*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*d^2))) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2
)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^
5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4
*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cos(d*x + c) - ((a^
5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7
*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9
)*d^4)) + 2*(a^2*b + 3*a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b
^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*
```

```
a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*
b^2 + 3*a^2*b^3 - a*b^4)*d^2))) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b -
b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 +
3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1
5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/(
(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-(a + 3*b)*cos(d*x + c) -
 ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/
((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 +
a*b^9)*d^4)) - 2*(a^2*b + 3*a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5
 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b -
3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))) - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(
a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^
4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a -
 b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-(a + 3*b)*cos(d*x +
 c) - ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*
b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b
^8 + a*b^9)*d^4)) + 2*(a^2*b + 3*a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5
*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*
b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))) - 8*cos(d*x + c))/((a*b - b^2)*d*
cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(141) = 282.
time = 0.91, size = 605, normalized size = 3.25

$$-\frac{\frac{\cos(dx+c)^3}{d}-\frac{2\cos(dx+c)}{d}}{4\left(b\cos(dx+c)^4-2b\cos(dx+c)^2-a+b\right)(a-b)}+\frac{\left(\left(a^2b-2ab^2+b^3\right)\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,d^4-2\left(a^2b-ab^2\right)\sqrt{-b^2+\sqrt{ab}\,b}\,d^2|-ad^2+bd^2|+\left(ad^2-bd^2\right)^2\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,a\right)\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{abd^2-b^2d^2+\sqrt{\left(abd^2-b^2d^2\right)^2+\left(abd^4-b^2d^4\right)\left(a^2-2ab+b^2\right)}}{abd^4-b^2d^4}}}\right)}{8\left(a^4b-3a^3b^2+3a^2b^3-ab^4\right)d^3|-ad^2+bd^2||b|}-\frac{\left(\left(a^2b-2ab^2+b^3\right)\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,d^4+2\left(a^2b-ab^2\right)\sqrt{-b^2-\sqrt{ab}\,b}\,d^2|-ad^2+bd^2|+\left(ad^2-bd^2\right)^2\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,a\right)\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{abd^2-b^2d^2-\sqrt{\left(abd^2-b^2d^2\right)^2+\left(abd^4-b^2d^4\right)\left(a^2-2ab+b^2\right)}}{abd^4-b^2d^4}}}\right)}{8\left(a^4b-3a^3b^2+3a^2b^3-ab^4\right)d^3|-ad^2+bd^2||b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(cos(d*x + c)^3/d - 2*cos(d*x + c)/d)/((b*cos(d*x + c)^4 - 2*b*cos(d*x
 + c)^2 - a + b)*(a - b)) + 1/8*((a^2*b - 2*a*b^2 + b^3)*sqrt(a*b)*sqrt(-b^
```

```
2 + sqrt(a*b)*b)*d^4 - 2*(a^2*b - a*b^2)*sqrt(-b^2 + sqrt(a*b)*b)*d^2*abs(-
a*d^2 + b*d^2) + (a*d^2 - b*d^2)^2*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a)*ar
ctan(cos(d*x + c)/(d*sqrt(-(a*b*d^2 - b^2*d^2 + sqrt((a*b*d^2 - b^2*d^2)^2
+ (a*b*d^4 - b^2*d^4)*(a^2 - 2*a*b + b^2)))/(a*b*d^4 - b^2*d^4))))/((a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^3*abs(-a*d^2 + b*d^2)*abs(b)) - 1/8*((a^
2*b - 2*a*b^2 + b^3)*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*d^4 + 2*(a^2*b - a*
b^2)*sqrt(-b^2 - sqrt(a*b)*b)*d^2*abs(-a*d^2 + b*d^2) + (a*d^2 - b*d^2)^2*s
qrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a)*arctan(cos(d*x + c)/(d*sqrt(-(a*b*d^2
- b^2*d^2 - sqrt((a*b*d^2 - b^2*d^2)^2 + (a*b*d^4 - b^2*d^4)*(a^2 - 2*a*b +
 b^2)))/(a*b*d^4 - b^2*d^4))))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^3
*abs(-a*d^2 + b*d^2)*abs(b))
```

Mupad [B]
time = 15.60, size = 3060, normalized size = 16.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] (cos(c + d*x)^3/(4*(a - b)) - cos(c + d*x)/(2*(a - b)))/(d*(a - b + 2*b*cos
(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((512*a*b^4 - 512*a^2*b^3)/(64*(
a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/
2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^
(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((a
*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 -
 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) + (cos(c + d*x)*(a*b^2 + b^3))/(4
*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 +
3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*1i - ((
(512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)*((a*(a^3
*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^
3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4
))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b
^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) -
(cos(c + d*x)*(a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) +
 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4
*b^4 - a^5*b^3)))^(1/2)*1i)/(b/(32*(a^2 - 2*a*b + b^2)) + (((512*a*b^4 - 51
2*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a*(a^3*b^3)^(1/2) + 3
*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b
^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*
a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)
/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) + (cos(c + d*x)*(
a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^
(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)
))^(1/2) + (((512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) + (cos(c +
```

d*x)*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a
^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5
+ 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3
)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^
3)))^(1/2) - (cos(c + d*x)*(a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3
*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^
3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^
(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)
))^(1/2)*2i)/d - (atan(((((512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)
) - (cos(c + d*x)*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^
2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6
- 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2)
 + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a
^4*b^4 - a^5*b^3)))^(1/2) + (cos(c + d*x)*(a*b^2 + b^3))/(4*(a^2 - 2*a*b +
b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256
*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*1i - (((512*a*b^4 - 51
2*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)*(-(a*(a^3*b^3)^(1/2) +
3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*
b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2
*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^
2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) - (cos(c + d*x)
*(a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^
3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b
^3)))^(1/2)*1i)/(b/(32*(a^2 - 2*a*b + b^2)) + (((512*a*b^4 - 512*a^2*b^3)/(
64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3
)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^
3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2))
)*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2
*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) + (cos(c + d*x)*(a*b^2 + b^
3))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a
*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)
+ (((512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)*(-(a
*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 -
 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^
3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2)
 - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1
/2) - (cos(c + d*x)*(a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^
(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5
+ 3*a^4*b^4 - a^5*b^3)))^(1/2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2)
 - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1
/2)*2i)/d

3.216 $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d}-\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d}-\frac{\cos(c+}{4a(a-b)d\,(a-}$$

```
[Out] -1/4*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(
d*x+c)^4)-1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(1/2)
-2*b^(1/2))/a^(3/2)/b^(1/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4)*c
os(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+2*b^(1/2))/a^(3/2)/b^(1/4)/d/
(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A]

time = 0.20, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3294, 1106, 1180, 211, 214}

$$-\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4ad(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/8*((3*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] -
Sqrt[b]]])/(a^(3/2)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(1/4)*d) - ((3*Sqrt[a] + 2*
Sqrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(3/2
)*(Sqrt[a] + Sqrt[b])^(3/2)*b^(1/4)*d) - (Cos[c + d*x]*(a + b - b*Cos[c + d
*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(a-b+2bx^2-bx^4)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2\cdots}{a\cdots}\right.\cdots}{\cdots}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\left(\left(3\sqrt{a}-2\sqrt{b}\right)\cdots\right.}{\cdots}$$

$$= -\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d} - \frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{\cdots}}{\sqrt{\cdots}}\right.}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.28, size = 469, normalized size = 2.12

$$-\frac{\frac{32\cos(c+dx)(2a+b-b\cos(2(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}+i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+24a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-10b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-12ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2+5ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-24a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+10b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+12ia\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4-5ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4+2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^6-ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^6}{-b\#1-8a\#1^3+3b\#1^3-3b\#1^5+b\#1^7}\&\right]}{32a(a-b)d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/32*((32*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(8*a - 3*b + 4*b*Co
s[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 24*a*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^2 - 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
2 - (12*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (5*I)*b*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2]*#1^2 - 24*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#
1^4 + 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (12*I)*a*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^4 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*
#1^7) & ])/(a*(a - b)*d)
```

Maple [A]

time = 1.06, size = 241, normalized size = 1.09

method	result
derivativedivides	$-\frac{b^2\left(\frac{\frac{(\sqrt{ab}+a)\cos(dx+c)}{2b(a-b)\left(\cos^2(dx+c)+\frac{\sqrt{ab}}{b}-1\right)}+\frac{(\sqrt{ab}+3a-2b)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{(\sqrt{ab}-b)b}}\right)}{2(a-b)\sqrt{(\sqrt{ab}-b)b}}}{4\sqrt{ab}\,ab}+\frac{-\frac{(-\sqrt{ab}+a)\cos(d}{2b(a-b)\left(\cos^2(dx+c)-1-\right.}}{}\right.}{d}$

default	$-\frac{b^2\left(\frac{\frac{\left(\sqrt{ab}+a\right)\cos(dx+c)}{2b(a-b)\left(\cos^2(dx+c)+\frac{\sqrt{ab}}{b}-1\right)}+\frac{\left(\sqrt{ab}+3a-2b\right)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2(a-b)\sqrt{\left(\sqrt{ab}-b\right)b}}}{4\sqrt{ab}\,ab}+\frac{-\frac{\left(-\sqrt{ab}+a\right)\cos\cdots}{2b(a-b)\left(\cos^2(dx+c)-1\cdots\right.}\cdots}{\cdots}\right.}{d}$
risch	$-\frac{b\,e^{7i(dx+c)}-4a\,e^{5i(dx+c)}-b\,e^{5i(dx+c)}-4a\,e^{3i(dx+c)}-b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}{2a(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4096a^9b\,d^4-12\cdots\right.\right.}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*b^2*(1/4/(a*b)^(1/2)/a/b*(1/2*((a*b)^(1/2)+a)/b/(a-b)*cos(d*x+c)/(cos(
d*x+c)^2+(a*b)^(1/2)/b-1)+1/2*((a*b)^(1/2)+3*a-2*b)/(a-b)/(((a*b)^(1/2)-b)*
b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2)))+1/4/(a*b)^(1/2)/a/
b*(-1/2*(-(a*b)^(1/2)+a)/b/(a-b)*cos(d*x+c)/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)-
1/2*((a*b)^(1/2)-3*a+2*b)/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x
+c)/(((a*b)^(1/2)+b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*b^2*cos(2*d*x + 2*c)*cos(d*x + c) + 4*b^2*sin(2*d*x + 2*c)*sin(d*x +
 c) - b^2*cos(d*x + c) - 4*(4*a*b + b^2)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c)
- (b^2*cos(7*d*x + 7*c) + b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c)
 - (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*
c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*cos
(7*d*x + 7*c) + 4*(b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c) - (4*a
*b + b^2)*cos(3*d*x + 3*c))*cos(6*d*x + 6*c) + (4*a*b + b^2 - 2*(32*a^2 - 4
*a*b - 3*b^2)*cos(4*d*x + 4*c) - 4*(4*a*b + b^2)*cos(2*d*x + 2*c))*cos(5*d*
x + 5*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*c
```

os(d*x + c))*cos(4*d*x + 4*c) + (4*a*b + b^2 - 4*(4*a*b + b^2)*cos(2*d*x + 2*c))*cos(3*d*x + 3*c) + 2*((a^2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)*d - 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x + 2*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-1/2*(4*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 4*b^2*cos(2*d*x + 2*c)*sin(d*x + c) - 4*(12*a*b - 5*b^2)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + b^2*sin(d*x + c) - (b^2*sin(7*d*x + 7*c) - b^2*sin(d*x + c) - (12*a*b - 5*b^2)*sin(5*d*x + 5*c) + (12*a*b - 5*b^2)*sin(3*d*x + 3*c))*cos(8*d*x + 8*c) - 2*(2*b^2*sin(6*d*x + 6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*cos(7*d*x + 7*c) - 4*(b^2*sin(d*x + c) + (12*a*b - 5*b^2)*sin(5*d*x + 5*c) - (12*a*b - 5*b^2)*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) + 2*((96*a^2 - 76*a*b + 15*b^2)*sin(4*d*x + 4*c) + 2*(12*a*b - 5*b^2)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) + 2*((96*a^2 - 76*a*b + 15*b^2)*sin(3*d*x + 3*c) - (8*a*b - 3*b^2)*sin(d*x + c))*cos(4*d*x + 4*c) + (b^2*cos(7*d*x + 7*c) - b^2*cos(d*x + c) - (12*a*b - 5*b^2)*cos(5*d*x + 5*c) + (12*a*b - 5*b^2)*cos(3*d*x + 3*c))*sin(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) + 4*(b^2*cos(d*x + c) + (12*a*b - 5*b^2)*cos(5*d*x + 5*c) - (12*a*b - 5*b^2)*cos(3*d*x + 3*c))*sin(6*d*x + 6*c) + (12*a*b - 5*b^2 - 2*(96*a^2 - 76*a*b + 15*b^2)*cos(4*d*x + 4*c) - 4*(12*a*b - 5*b^2)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) - 2*((96*a^2 - 76*a*b + 15*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*cos(d*x + c))*sin(4*d*x + 4*c) - (12*a*b - 5*b^2 - 4*(12*a*b - 5*b^2)*cos(2*d*x + 2*c))*sin(3*d*x + 3*c))/(a^2*b^2 - a*b^3 + (a^2*b^2 - a*b^3)*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - a*b^3 - 4*(a^2*b^2 - a*b^3)*cos(6*d*x + 6*c) - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^2 - a*b^3

```
- 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - a*b^3)
*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3 - 4
*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a
^2*b^2 - a*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^2*b^2 - a*b^3)*sin(6*d*x + 6*c)
+ (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*s
in(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*si
n(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x
) - (b^2*sin(7*d*x + 7*c) + b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*
c) - (4*a*b + b^2)*sin(3*d*x + 3*c))*sin(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x
+ 6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*sin(7*d
*x + 7*c) + 4*(b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*c) - (4*a*b +
 b^2)*sin(3*d*x + 3*c))*sin(6*d*x + 6*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*sin(
4*d*x + 4*c) + 2*(4*a*b + b^2)*sin(2*d*x + 2*c)...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2269 vs. $2(173) = 346$.
time = 0.66, size = 2269, normalized size = 10.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*b*cos(d*x + c)^3 - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b -
a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b
+ 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*
b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 1
5*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((8
1*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3
 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8
*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) -
(27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*
b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4
*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) + ((a^2*b - a*b^2)*d*co
s(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)
*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b +
 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4
*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^
2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*
b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b
+ 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^
4*b^6 + a^3*b^7)*d^4)) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(((a^6 - 3*
a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b -
6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4
```

```
)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)))
 + ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 -
 (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^
2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6
*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/(
(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-(81*a^2 - 81*a*b + 20*b^2)
*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*
d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a
^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (27*a^4 - 24*a^3*b + 5*a
^2*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 -
90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5
 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b +
3*a^4*b^2 - a^3*b^3)*d^2))) - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b
- a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(((a^6 - 3*a^5*b
 + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8
*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) -
15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-
(81*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b
^3 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a
^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))
+ (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3
*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3
- 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b -
4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) - 4*(a + b)*cos(d*x +
c))/((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2
- (a^3 - 2*a^2*b + a*b^2)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(173) = 346.
time = 0.79, size = 693, normalized size = 3.14

$$-\frac{\frac{b\cos\left(dx+c\right)^{3}}{d}-\frac{a\cos\left(dx+c\right)}{d}-\frac{b\cos\left(dx+c\right)}{d}}{4\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}-a+b\right)\left(a^{2}-ab\right)}+\frac{\left(\left(3\,a^{4}b-8\,a^{3}b^{2}+7\,a^{2}b^{3}-2\,ab^{4}\right)\sqrt{-b^{2}+\sqrt{ab}\,b}\,d^{4}-\left(3\,a^{2}-4\,ab+b^{2}\right)\sqrt{ab}\,\sqrt{-b^{2}+\sqrt{ab}\,b}\,d^{2}\left|-a^{2}d^{2}+abd^{2}\right|+\left(a^{2}d^{2}-abd^{2}\right)^{2}\sqrt{-b^{2}+\sqrt{ab}\,b}\,b\right)\arctan\left(\frac{\cos\left(dx+c\right)}{d\sqrt{-\frac{a^{2}bd^{2}-ab^{2}d^{2}+\sqrt{\left(a^{2}bd^{2}-ab^{2}d^{2}\right)^{2}+\left(a^{2}bd^{4}-ab^{2}d^{4}\right)\left(a^{3}-2\,a^{2}b+ab^{2}\right)}}{a^{2}bd^{4}-ab^{2}d^{4}}}}\right)}{8\left(a^{4}-3\,a^{3}b+3\,a^{2}b^{2}-ab^{3}\right)\sqrt{ab}\,d^{3}\left|-a^{2}d^{2}+abd^{2}\right|\left|b\right|}-\frac{\left(\left(3\,a^{4}b-8\,a^{3}b^{2}+7\,a^{2}b^{3}-2\,ab^{4}\right)\sqrt{-b^{2}-\sqrt{ab}\,b}\,d^{4}+\left(3\,a^{2}-4\,ab+b^{2}\right)\sqrt{ab}\,\sqrt{-b^{2}-\sqrt{ab}\,b}\,d^{2}\left|-a^{2}d^{2}+abd^{2}\right|+\left(a^{2}d^{2}-abd^{2}\right)^{2}\sqrt{-b^{2}-\sqrt{ab}\,b}\,b\right)\arctan\left(\frac{\cos\left(dx+c\right)}{d\sqrt{-\frac{a^{2}bd^{2}-ab^{2}d^{2}-\sqrt{\left(a^{2}bd^{2}-ab^{2}d^{2}\right)^{2}+\left(a^{2}bd^{4}-ab^{2}d^{4}\right)\left(a^{3}-2\,a^{2}b+ab^{2}\right)}}{a^{2}bd^{4}-ab^{2}d^{4}}}}\right)}{8\left(a^{4}-3\,a^{3}b+3\,a^{2}b^{2}-ab^{3}\right)\sqrt{ab}\,d^{3}\left|-a^{2}d^{2}+abd^{2}\right|\left|b\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*cos(d*x + c)^3/d - a*cos(d*x + c)/d - b*cos(d*x + c)/d)/((b*cos(d*x
 + c)^4 - 2*b*cos(d*x + c)^2 - a + b)*(a^2 - a*b)) + 1/8*((3*a^4*b - 8*a^3*
b^2 + 7*a^2*b^3 - 2*a*b^4)*sqrt(-b^2 + sqrt(a*b)*b)*d^4 - (3*a^2 - 4*a*b +
b^2)*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*d^2*abs(-a^2*d^2 + a*b*d^2) + (a^2*
d^2 - a*b*d^2)^2*sqrt(-b^2 + sqrt(a*b)*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(
a^2*b*d^2 - a*b^2*d^2 + sqrt((a^2*b*d^2 - a*b^2*d^2)^2 + (a^2*b*d^4 - a*b^2
*d^4)*(a^3 - 2*a^2*b + a*b^2)))/(a^2*b*d^4 - a*b^2*d^4))))/((a^4 - 3*a^3*b
+ 3*a^2*b^2 - a*b^3)*sqrt(a*b)*d^3*abs(-a^2*d^2 + a*b*d^2)*abs(b)) - 1/8*((
3*a^4*b - 8*a^3*b^2 + 7*a^2*b^3 - 2*a*b^4)*sqrt(-b^2 - sqrt(a*b)*b)*d^4 + (
3*a^2 - 4*a*b + b^2)*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*d^2*abs(-a^2*d^2 +
a*b*d^2) + (a^2*d^2 - a*b*d^2)^2*sqrt(-b^2 - sqrt(a*b)*b)*b)*arctan(cos(d*x
 + c)/(d*sqrt(-(a^2*b*d^2 - a*b^2*d^2 - sqrt((a^2*b*d^2 - a*b^2*d^2)^2 + (a
^2*b*d^4 - a*b^2*d^4)*(a^3 - 2*a^2*b + a*b^2)))/(a^2*b*d^4 - a*b^2*d^4))))/
((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)*d^3*abs(-a^2*d^2 + a*b*d^2)*
abs(b))
```

Mupad [B]
time = 16.79, size = 2500, normalized size = 11.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] ((b*cos(c + d*x)^3)/(4*a*(a - b)) - (cos(c + d*x)*(a + b))/(4*a*(a - b)))/(
d*(a - b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) + (atan(((((256*a^3*b^5
- 1024*a^4*b^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) - (cos(c + d*x
)*(256*a^3*b^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(1/2)
 + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^
7*b^3 - 3*a^8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*
a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b -
 a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (cos(c + d*x)*(4*b^5 - 11*a*b^4
 + 9*a^2*b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/
2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*
a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i - (((256*a^3*b^5 - 1024*a^4*b^4 + 768*a^5*b
^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) + (cos(c + d*x)*(256*a^3*b^6 - 512*a^4*b
^5 + 256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a
^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)
)/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9
*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a
^8*b^2)))^(1/2) - (cos(c + d*x)*(4*b^5 - 11*a*b^4 + 9*a^2*b^3))/(4*(a^4 - 2
*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4
*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/
2)*1i)/((9*a*b^3 - 4*b^4)/(32*(a^5 - 2*a^4*b + a^3*b^2)) + (((256*a^3*b^5 -
 1024*a^4*b^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) - (cos(c + d*x)
```

*(256*a^3*b^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(1/2)
+ 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7
*b^3 - 3*a^8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a
*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b -
a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (cos(c + d*x)*(4*b^5 - 11*a*b^4
+ 9*a^2*b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2
) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a
^7*b^3 - 3*a^8*b^2)))^(1/2) + (((256*a^3*b^5 - 1024*a^4*b^4 + 768*a^5*b^3)/
(64*(a^5 - 2*a^4*b + a^3*b^2)) + (cos(c + d*x)*(256*a^3*b^6 - 512*a^4*b^5 +
256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b
^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2))/(4
*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^
(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b
^2)))^(1/2) - (cos(c + d*x)*(4*b^5 - 11*a*b^4 + 9*a^2*b^3))/(4*(a^4 - 2*a^3
*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3
*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)))
*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b
^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)*2i)/d + (atan(((
((256*a^3*b^5 - 1024*a^4*b^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2))
- (cos(c + d*x)*(256*a^3*b^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b + 9*a
*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b -
a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-
(15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)
/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (cos(c + d*x)*(4*
b^5 - 11*a*b^4 + 9*a^2*b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9
a(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b
- a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i - (((256*a^3*b^5 - 1024*a^4*b
^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) + (cos(c + d*x)*(256*a^3*b
^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^9*
b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^
8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9*a*(a^9*b)^(1
/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3
*a^7*b^3 - 3*a^8*b^2)))^(1/2) - (cos(c + d*x)*(4*b^5 - 11*a*b^4 + 9*a^2*b^3
))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^
9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*
a^8*b^2)))^(1/2)*1i)/((9*a*b^3 - 4*b^4)/(32*(a^5 - 2*a^4*b + a^3*b^2)) + ((
(256*a^3*b^5 - 1024*a^4*b^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) -
(cos(c + d*x)*(256*a^3*b^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b + 9*a*
(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a
^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(
15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/
(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (cos(c + d*x)*(4*b
^5 - 11*a*b^4 + 9*a^2*b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9*
a*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b -
a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (((256*a^3*b^5 - 1024*a^4*b^4 +

```
768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) + ...
```

3.217 $\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\mathrm{c}}{\sqrt{\sqrt{a}}}\right.}{8a^{3/2}\left(\sqrt{a}+\sqrt{\right.}}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d-1/4*b*cos(d*x+c)*(2-cos(d*x+c)^2)/a/(a-b)/d/(a-b
+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/8*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^
(1/2)-b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*b^(1/4)*arctanh
(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1/2))^(3
/2)-1/2*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^2/d/(a
^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/
2))^(1/2))/a^2/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3294, 1252, 213, 1192, 1180, 211, 214}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4ad(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/8*(b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(a^(3
/2)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/S
qrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c
+ d*x]]/(a^2*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sq
rt[b]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4
)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b]]*d)
 - (b*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cos[c
+ d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2(-1+x^2)} + \frac{b-bx^2}{a(a-b+2bx^2-bx^4)^2} + \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)}\right)\,dx, x, \cos(c+dx)\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{a^2 d} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{a^2 d}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\ldots}{\sqrt{\sqrt{a}\ldots}}\right)}{2a^2\sqrt{\sqrt{a}+\ldots}}$$

$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos\ldots}{a^2\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.59, size = 600, normalized size = 1.85

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((16*a*b*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/((a - b)*(8*a - 3*b + 4*b*Co
s[2*(c + d*x)] - b*Cos[4*(c + d*x)])) - 32*Log[Cos[(c + d*x)/2]] + 32*Log[S
in[(c + d*x)/2]] - (I*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#
1^6 + b*#1^8 & , (-10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 8*b*ArcT
an[Sin[c + d*x]/(Cos[c + d*x] - #1)] + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2] - (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 38*a*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^2 - 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^2 - (19*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (12*I)*b*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 38*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^4 + 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (19*I)*a*Lo
```

```
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (12*I)*b*Log[1 - 2*Cos[c + d*x]*#1 +
 #1^2]*#1^4 + 10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 8*b*ArcT
an[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - (5*I)*a*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2]*#1^6 + (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1)
- 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32*a^2*d)
```

Maple [A]
time = 1.14, size = 243, normalized size = 0.75

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\frac{a\left(\cos^3(dx+c)\right)}{4a-4b}-\frac{a\cos(dx+c)}{2(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{b\left(\frac{\left(-5a\sqrt{ab}+4\sqrt{ab}\,b-ab\right)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(-5a\sqrt{ab}\right.}{}\right.}{4a-4b}\right.}{a^2}}{d}$
default	$\frac{\frac{b\left(\frac{\frac{a\left(\cos^3(dx+c)\right)}{4a-4b}-\frac{a\cos(dx+c)}{2(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{b\left(\frac{\left(-5a\sqrt{ab}+4\sqrt{ab}\,b-ab\right)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(-5a\sqrt{ab}\right.}{}\right.}{4a-4b}\right.}{a^2}}{d}$

risch	$\frac{b\left(e^{7i(dx+c)}-5\,e^{5i(dx+c)}-5\,e^{3i(dx+c)}+e^{i(dx+c)}\right)}{2a(-a+b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{\ln\left(e^{i(dx+c)}+\right.}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/a^2*((1/4*a/(a-b)*cos(d*x+c)^3-1/2*a/(a-b)*cos(d*x+c))/(a-b+2*b*cos(
d*x+c)^2-b*cos(d*x+c)^4)+1/4/(a-b)*b*(1/2*(-5*a*(a*b)^(1/2)+4*(a*b)^(1/2)*b
-a*b)/(a*b)^(1/2)/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(
1/2)-b)*b)^(1/2))-1/2*(-5*a*(a*b)^(1/2)+4*(a*b)^(1/2)*b+a*b)/(a*b)^(1/2)/b/
(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2)))
-1/2/a^2*ln(1+cos(d*x+c))+1/2/a^2*ln(cos(d*x+c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b^2*cos(2*d*x + 2*c)*cos(d*x + c) - 20*a*b^2*sin(3*d*x + 3*c)*sin(
2*d*x + 2*c) + 4*a*b^2*sin(2*d*x + 2*c)*sin(d*x + c) - a*b^2*cos(d*x + c) -
 (a*b^2*cos(7*d*x + 7*c) - 5*a*b^2*cos(5*d*x + 5*c) - 5*a*b^2*cos(3*d*x + 3
*c) + a*b^2*cos(d*x + c))*cos(8*d*x + 8*c) + (4*a*b^2*cos(6*d*x + 6*c) + 4*
a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*co
s(7*d*x + 7*c) - 4*(5*a*b^2*cos(5*d*x + 5*c) + 5*a*b^2*cos(3*d*x + 3*c) - a
*b^2*cos(d*x + c))*cos(6*d*x + 6*c) - 5*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 +
 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(5*d*x + 5*c) - 2*(5*(8*a^2*b -
 3*a*b^2)*cos(3*d*x + 3*c) - (8*a^2*b - 3*a*b^2)*cos(d*x + c))*cos(4*d*x +
4*c) - 5*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2)*cos(3*d*x + 3*c) - 2*((a^3*b^2
- a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*cos(6*d*x + 6*c)
^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*cos(4*d*x + 4*c)^2 +
 16*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(8*
d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 11
2*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a
^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2
*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^
3*b^2 - a^2*b^3)*d - 2*(4*(a^3*b^2 - a^2*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^4
*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4*d*x + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*c
os(2*d*x + 2*c) - (a^3*b^2 - a^2*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^4*b -
 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4*d*x + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*cos(2
*d*x + 2*c) - (a^3*b^2 - a^2*b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^4*b - 11*
a^3*b^2 + 3*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3
```

```
)*d)*cos(4*d*x + 4*c) - 4*(2*(a^3*b^2 - a^2*b^3)*d*sin(6*d*x + 6*c) + (8*a^
4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*d*
sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)
*d*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c))*integrate(-1/2*(4*(19*a*b^2 - 12*b^3)*cos(3*d*x + 3*c)*sin(2*d*x + 2
*c) - 4*(5*a*b^2 - 4*b^3)*cos(d*x + c)*sin(2*d*x + 2*c) + 4*(5*a*b^2 - 4*b^
3)*cos(2*d*x + 2*c)*sin(d*x + c) + ((5*a*b^2 - 4*b^3)*sin(7*d*x + 7*c) - (1
9*a*b^2 - 12*b^3)*sin(5*d*x + 5*c) + (19*a*b^2 - 12*b^3)*sin(3*d*x + 3*c) -
 (5*a*b^2 - 4*b^3)*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*(5*a*b^2 - 4*b^3)*
sin(6*d*x + 6*c) + (40*a^2*b - 47*a*b^2 + 12*b^3)*sin(4*d*x + 4*c) + 2*(5*a
*b^2 - 4*b^3)*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 4*((19*a*b^2 - 12*b^3)*s
in(5*d*x + 5*c) - (19*a*b^2 - 12*b^3)*sin(3*d*x + 3*c) + (5*a*b^2 - 4*b^3)*
sin(d*x + c))*cos(6*d*x + 6*c) - 2*((152*a^2*b - 153*a*b^2 + 36*b^3)*sin(4*
d*x + 4*c) + 2*(19*a*b^2 - 12*b^3)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 2*(
(152*a^2*b - 153*a*b^2 + 36*b^3)*sin(3*d*x + 3*c) - (40*a^2*b - 47*a*b^2 +
12*b^3)*sin(d*x + c))*cos(4*d*x + 4*c) - ((5*a*b^2 - 4*b^3)*cos(7*d*x + 7*c
) - (19*a*b^2 - 12*b^3)*cos(5*d*x + 5*c) + (19*a*b^2 - 12*b^3)*cos(3*d*x +
3*c) - (5*a*b^2 - 4*b^3)*cos(d*x + c))*sin(8*d*x + 8*c) + (5*a*b^2 - 4*b^3
- 4*(5*a*b^2 - 4*b^3)*cos(6*d*x + 6*c) - 2*(40*a^2*b - 47*a*b^2 + 12*b^3)*c
os(4*d*x + 4*c) - 4*(5*a*b^2 - 4*b^3)*cos(2*d*x + 2*c))*sin(7*d*x + 7*c) -
4*((19*a*b^2 - 12*b^3)*cos(5*d*x + 5*c) - (19*a*b^2 - 12*b^3)*cos(3*d*x + 3
*c) + (5*a*b^2 - 4*b^3)*cos(d*x + c))*sin(6*d*x + 6*c) - (19*a*b^2 - 12*b^3
 - 2*(152*a^2*b - 153*a*b^2 + 36*b^3)*cos(4*d*x + 4*c) - 4*(19*a*b^2 - 12*b
^3)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) + 2*((152*a^2*b - 153*a*b^2 + 36*b^3
)*cos(3*d*x + 3*c) - (40*a^2*b - 47*a*b^2 + 12*b^3)*cos(d*x + c))*sin(4*d*x
 + 4*c) + (19*a*b^2 - 12*b^3 - 4*(19*a*b^2 - 12*b^3)*cos(2*d*x + 2*c))*sin(
3*d*x + 3*c) - (5*a*b^2 - 4*b^3)*sin(d*x + c))/(a^3*b^2 - a^2*b^3 + (a^3*b^
2 - a^2*b^3)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c)^2
 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*cos(4*d*x + 4*c)^2 + 16*
(a^3*b^2 - a^2*b^3)*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*sin(8*d*x + 8*
c)^2 + 16*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b +
57*a^3*b^2 - 9*a^2*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a^3*b^2 + 3*a
^2*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2*b^3)*sin(2*d*
x + 2*c)^2 + 2*(a^3*b^2 - a^2*b^3 - 4*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c)
- 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*
b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - a^2*b^3 - 2*(8*a^4*b
 - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*b^3)*cos(2*d
*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3 - 4*(8*a^
4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^3*b
^2 - a^2*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c)
+ (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^
3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2*b
^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*sin(2*d*x + 2*c))*sin(6*d*x +
6*c)), x) - (a*b^2 - b^3 + (a*b^2 - b^3)*cos(8*...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2711 vs. $2(244) = 488$.
time = 0.96, size = 2711, normalized size = 8.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*b*cos(d*x + c)^3 - 8*a*b*cos(d*x + c) + ((a^3*b - a^2*b^2)*d*cos
(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b
^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1
450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^1
1*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b -
47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log((625*a^
3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10 - 18*a^9*
b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 1450*a^3*b^2
 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 2*(75*a^5*b - 137*a^4*
b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b
^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)
/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a
^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2
)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*a^6*b + 3*
a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*
a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4
 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*
b + 3*a^5*b^2 - a^4*b^3)*d^2))*log((625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 -
128*b^4)*cos(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a
^6*b^4)*d^3*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*
b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5
 + a^7*b^6)*d^4)) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)
*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3
*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 -
 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^
2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))) - ((a^3*b - a^2*b
^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*
b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*
a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b
 + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35
*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*lo
g(-(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10
 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 14
50*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11
```

```
*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 2*(75*a^5*b
- 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b
^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4
 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a
^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3
*a^5*b^2 - a^4*b^3)*d^2))) + ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b
 - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*
a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*
b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 1
5*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^
7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-(625*a^3*b - 1125*a^2*b^2 + 6
64*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^
7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*
a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4
- 6*a^8*b^5 + a^7*b^6)*d^4)) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*
a^2*b^4)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b
 - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15
*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*
b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))) + 8*((
a*b - b^2)*cos(d*x + c)^4 - 2*(a*b - b^2)*cos(d*x + c)^2 - a^2 + 2*a*b - b^
2)*log(1/2*cos(d*x + c) + 1/2) - 8*((a*b - b^2)*cos(d*x + c)^4 - 2*(a*b - b
^2)*cos(d*x + c)^2 - a^2 + 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3
*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^
4 - 2*a^3*b + a^2*b^2)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
```

```
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[-89,-8
2]Warning, need
```

Mupad [B]
time = 17.55, size = 2500, normalized size = 7.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - b*sin(c + d*x)^4)^2),x)
```

```
[Out] ((b*cos(c + d*x)^3)/(4*a*(a - b)) - (b*cos(c + d*x))/(2*a*(a - b)))/(d*(a -
 b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((3072*a^3*b^7 - 109
44*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (((49152*a^7*b
^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256*(a^7 - 2*a^6*b
+ a^5*b^2)) - (cos(c + d*x)*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) +
35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b -
 a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 262144*a^9*b^6 + 2293
76*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a
^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29
*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) +
(cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))/(128*(a^6 -
2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*
b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 +
 a^8*b^3 - 3*a^9*b^2)))^(1/2))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2)
 + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*
b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) + (cos(c + d*x)*(768*b^7 - 2048*a*b
^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2)
 + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b
)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*1i - (((3072*
a^3*b^7 - 10944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (
((49152*a^7*b^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256*(a
^7 - 2*a^6*b + a^5*b^2)) + (cos(c + d*x)*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^
9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(25
6*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 262144*a
^9*b^6 + 229376*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^2))
)*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47
*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^
2)))^(1/2) - (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))
/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1
/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^
10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*
(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/
(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) - (cos(c + d*x)*(768*b
^7 - 2048*a*b^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*
```

```
(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 -
29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*
1i)/((((3072*a^3*b^7 - 10944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b +
a^5*b^2)) - (((49152*a^7*b^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10
*b^4)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (cos(c + d*x)*((25*a^2*(a^9*b)^(1/2
) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*
b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b
^7 - 262144*a^9*b^6 + 229376*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*
b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16
*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b
^3 - 3*a^9*b^2)))^(1/2) + (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29
312*a^6*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b
^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2
))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2))*((25*a^2*(a^9*b)^(
1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a
^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) + (cos(c
+ d*x)*(768*b^7 - 2048*a*b^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2
)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 -
47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*
b^2)))^(1/2) - (125*a*b^5 - 80*b^6)/(128*(a^7 - 2*a^6*b + a^5*b^2)) + (((30
72*a^3*b^7 - 10944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2))
- (((49152*a^7*b^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256
*(a^7 - 2*a^6*b + a^5*b^2)) + (cos(c + d*x)*((25*a^2*(a^9*b)^(1/2) + 8*b^2*
(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/
(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 26214
4*a^9*b^6 + 229376*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^
2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 -
 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9
*b^2)))^(1/2) - (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^
5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)
^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3
*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2))*((25*a^2*(a^9*b)^(1/2) + 8*b
^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a...
```

3.218 $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=320

$$\frac{x}{b^2}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\ldots}{\sqrt[4]{\ldots}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\ldots}$$

```
[Out] x/b^2+1/8*a^(1/4)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^(3/2
)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*a^(1/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(
d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1/2))^(3/2)-1/2*a^(1/4)*arctan((a^(1/
2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^2/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*a^(
1/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^2/d/(a^(1/2)+b^(1
/2))^(1/2)-1/4*tan(d*x+c)/(a-b)/b/d+1/4*tan(d*x+c)^5/b/d/(a+2*a*tan(d*x+c)^
2+(a-b)*tan(d*x+c)^4)
```

Rubi [A]
time = 0.32, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3296, 1327, 1289, 12, 1136, 1180, 211, 1301, 209}

$$\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\tan^5(c+dx)}{4bd\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\tan(c+dx)}{4bd(a-b)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] x/b^2 - (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2
*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*
Tan[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1/4)*
ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + S
qrt[b]]*b^2*d) - (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tan[c + d*x]/(4*(a - b)*b*
d) + Tan[c + d*x]^5/(4*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1327

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[-f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
```

```
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2)), Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1)
/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)(a+2ax^2+(a-b)x^4)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(a+ax^2)}{(a+2ax^2+(a-b)x^4)^2}\,dx, x, \tan(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+2ax^2+(a-b)x^4)}\right.}{bd}$$

$$= \frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2abx^4}{a+2ax^2+(a-b)x^4}\right.}{8ab^2d}$$

$$= \frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+\right.}{b^2d}$$

$$= \frac{x}{b^2} - \frac{\tan(c+dx)}{4(a-b)bd} + \frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\left(a\left(1\right.\right.}{}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} - \frac{\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c\right.}{\sqrt[4]{a}}}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^2d}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} + \frac{\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c\right.}{\sqrt[4]{a}}}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d}$$

Mathematica [A]

time = 3.36, size = 262, normalized size = 0.82

$$\frac{8(c+dx)-\frac{\sqrt{a}\left(4\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{a+\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{a}\left(4\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{-a+\sqrt{a}\,\sqrt{b}}}+\frac{2ab(-6\sin(2(c+dx))+\sin(4(c+dx)))}{(a-b)(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))}}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (8*(c + d*x) - (Sqrt[a]*(4*Sqrt[a] + 5*Sqrt[b])*ArcTan[((Sqrt[a] + Sqrt[b])
*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sq
rt[a]*Sqrt[b]]) + (Sqrt[a]*(4*Sqrt[a] - 5*Sqrt[b])*ArcTanh[((Sqrt[a] - Sqrt
[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-
a + Sqrt[a]*Sqrt[b]]) + (2*a*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/((
a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])))/(8*b^2*d)
```

Maple [A]

time = 0.79, size = 266, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{b^2}-\frac{a\left(\frac{\frac{b\left(\tan^3(dx+c)\right)}{2a-2b}+\frac{b\tan(dx+c)}{4a-4b}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(4a\sqrt{ab}-6\sqrt{ab}\,b+3ab-5b^2\right)\arctan\left(\cdots\right)}{8\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\cdots\right.}}\cdots\right)}{\cdots}}{d}$
default	$\frac{\frac{\arctan(\tan(dx+c))}{b^2}-\frac{a\left(\frac{\frac{b\left(\tan^3(dx+c)\right)}{2a-2b}+\frac{b\tan(dx+c)}{4a-4b}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(4a\sqrt{ab}-6\sqrt{ab}\,b+3ab-5b^2\right)\arctan\left(\cdots\right)}{8\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\cdots\right.}}\cdots\right)}{\cdots}}{d}$
risch	$\frac{x}{b^2}-\frac{ia\left(b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-5b\,e^{2i(dx+c)}+b\right)}{2b^2(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\left(\_R=\mathrm{RootOf}\left(\left(a^3b^8d^4-3\cdots\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*arctan(tan(d*x+c))-a/b^2*((1/2*b/(a-b)*tan(d*x+c)^3+1/4*b/(a-b)*
tan(d*x+c))/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)+1/8*(4*a*(a*
b)^(1/2)-6*(a*b)^(1/2)*b+3*a*b-5*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a
-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/8*(4*a*
(a*b)^(1/2)-6*(a*b)^(1/2)*b-3*a*b+5*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)
*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*b^2 - b^3)*d*x*cos(8*d*x + 8*c)^2 + 32*(a*b^2 - b^3)*d*x*cos(6*d*
x + 6*c)^2 + 8*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*d*x*cos(4*d*x + 4*c)
^2 + 32*(a*b^2 - b^3)*d*x*cos(2*d*x + 2*c)^2 + 2*(a*b^2 - b^3)*d*x*sin(8*d*
x + 8*c)^2 + 32*(a*b^2 - b^3)*d*x*sin(6*d*x + 6*c)^2 + 8*(64*a^3 - 112*a^2*
b + 57*a*b^2 - 9*b^3)*d*x*sin(4*d*x + 4*c)^2 + 32*(a*b^2 - b^3)*d*x*sin(2*d
*x + 2*c)^2 - 16*(a*b^2 - b^3)*d*x*cos(2*d*x + 2*c) - a*b^2*sin(2*d*x + 2*c
) + 2*(a*b^2 - b^3)*d*x - (16*(a*b^2 - b^3)*d*x*cos(6*d*x + 6*c) + 8*(8*a^2
*b - 11*a*b^2 + 3*b^3)*d*x*cos(4*d*x + 4*c) + 16*(a*b^2 - b^3)*d*x*cos(2*d*
x + 2*c) - a*b^2*sin(6*d*x + 6*c) + 5*a*b^2*sin(2*d*x + 2*c) - 4*(a*b^2 - b
^3)*d*x + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*cos(8*d*x + 8*c) + 2*(16*(8
*a^2*b - 11*a*b^2 + 3*b^3)*d*x*cos(4*d*x + 4*c) + 32*(a*b^2 - b^3)*d*x*cos(
2*d*x + 2*c) + 12*a*b^2*sin(2*d*x + 2*c) - 8*(a*b^2 - b^3)*d*x + 3*(8*a^2*b
 - 3*a*b^2)*sin(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(16*(8*a^2*b - 11*a*b^2
+ 3*b^3)*d*x*cos(2*d*x + 2*c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*x + 3*(8*a
^2*b - 3*a*b^2)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - 2*((a*b^4 - b^5)*d*cos
(8*d*x + 8*c)^2 + 16*(a*b^4 - b^5)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b^2 - 1
12*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^4 - b^5)*d*co
s(2*d*x + 2*c)^2 + (a*b^4 - b^5)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^4 - b^5)*d*
sin(6*d*x + 6*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*sin(
4*d*x + 4*c)^2 + 16*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 16*(a*b^4 - b^5)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^4 - b^5)*d*cos
(2*d*x + 2*c) + (a*b^4 - b^5)*d - 2*(4*(a*b^4 - b^5)*d*cos(6*d*x + 6*c) + 2
*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(4*d*x + 4*c) + 4*(a*b^4 - b^5)*d*cos(
2*d*x + 2*c) - (a*b^4 - b^5)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^3 - 11*a*b
^4 + 3*b^5)*d*cos(4*d*x + 4*c) + 4*(a*b^4 - b^5)*d*cos(2*d*x + 2*c) - (a*b^
4 - b^5)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(2*
d*x + 2*c) - (8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b
^4 - b^5)*d*sin(6*d*x + 6*c) + (8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(4*d*x +
 4*c) + 2*(a*b^4 - b^5)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b
```

```
^3 - 11*a*b^4 + 3*b^5)*d*sin(4*d*x + 4*c) + 2*(a*b^4 - b^5)*d*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c))*integrate((4*a*b^2*cos(6*d*x + 6*c)^2 + 4*a*b^2*cos(
2*d*x + 2*c)^2 + 4*a*b^2*sin(6*d*x + 6*c)^2 + 4*a*b^2*sin(2*d*x + 2*c)^2 -
a*b^2*cos(2*d*x + 2*c) + 4*(64*a^3 - 112*a^2*b + 33*a*b^2)*cos(4*d*x + 4*c)
^2 + 4*(64*a^3 - 112*a^2*b + 33*a*b^2)*sin(4*d*x + 4*c)^2 + 2*(40*a^2*b - 4
7*a*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (a*b^2*cos(6*d*x + 6*c) + a*b^
2*cos(2*d*x + 2*c) + 2*(8*a^2*b - 11*a*b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8
*c) + (8*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(40*a^2*b - 47*a*b^2)*cos(4*d*x
 + 4*c))*cos(6*d*x + 6*c) - 2*(8*a^2*b - 11*a*b^2 - (40*a^2*b - 47*a*b^2)*c
os(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b^2*sin(6*d*x + 6*c) + a*b^2*sin(2*d
*x + 2*c) + 2*(8*a^2*b - 11*a*b^2)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 2*(
4*a*b^2*sin(2*d*x + 2*c) + (40*a^2*b - 47*a*b^2)*sin(4*d*x + 4*c))*sin(6*d*
x + 6*c))/(a*b^4 - b^5 + (a*b^4 - b^5)*cos(8*d*x + 8*c)^2 + 16*(a*b^4 - b^5
)*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*cos(
4*d*x + 4*c)^2 + 16*(a*b^4 - b^5)*cos(2*d*x + 2*c)^2 + (a*b^4 - b^5)*sin(8*
d*x + 8*c)^2 + 16*(a*b^4 - b^5)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b^2 - 112*a^
2*b^3 + 57*a*b^4 - 9*b^5)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^3 - 11*a*b^4 + 3
*b^5)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^4 - b^5)*sin(2*d*x + 2*c)
^2 + 2*(a*b^4 - b^5 - 4*(a*b^4 - b^5)*cos(6*d*x + 6*c) - 2*(8*a^2*b^3 - 11*
a*b^4 + 3*b^5)*cos(4*d*x + 4*c) - 4*(a*b^4 - b^5)*cos(2*d*x + 2*c))*cos(8*d
*x + 8*c) - 8*(a*b^4 - b^5 - 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*cos(4*d*x + 4
*c) - 4*(a*b^4 - b^5)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^3 - 1
1*a*b^4 + 3*b^5 - 4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*cos(2*d*x + 2*c))*cos(4*
d*x + 4*c) - 8*(a*b^4 - b^5)*cos(2*d*x + 2*c) - 4*(2*(a*b^4 - b^5)*sin(6*d*
x + 6*c) + (8*a^2*b^3 - 11*a*b^4 + 3*b^5)*sin(4*d*x + 4*c) + 2*(a*b^4 - b^5
)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^3 - 11*a*b^4 + 3*b^5)*s
in(4*d*x + 4*c) + 2*(a*b^4 - b^5)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) -
 (a*b^2*cos(6*d*x + 6*c) - 5*a*b^2*cos(2*d*x + 2*c) + 16*(a*b^2 - b^3)*d*x*
sin(6*d*x + 6*c) + 8*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*x*sin(4*d*x + 4*c) + 16
*(a*b^2 - b^3)*d*x*sin(2*d*x + 2*c) + a*b^2 - (8*a^2*b - 3*a*b^2)*cos(4*d*x
 + 4*c))*sin(8*d*x + 8*c) - (24*a*b^2*cos(2*d*x + 2*c) - 32*(8*a^2*b - 11*a
*b^2 + 3*b^3)*d*x*sin(4*d*x + 4*c) - 64*(a*b^2 - b^3)*d*x*sin(2*d*x + 2*c)
- 5*a*b^2 + 6*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*sin(6*d*x + 6*c) + (32*
(8*a^2*b - 11*a*b^2 + 3*b^3)*d*x*sin(2*d*x + 2*c) + 8*a^2*b - 3*a*b^2 - 6*(
8*a^2*b - 3*a*b^2)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c))/((a*b^4 - b^5)*d*cos
(8*d*x + 8*c)^2 + 16*(a*b^4 - b^5)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b^2 - 1
12*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^4 - b^5)*d*co
s(2*d*x + 2*c)^2 + (a*b^4 - b^5)*d*sin(8*d*x + ...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3544 vs. $2(240) = 480$.
time = 1.04, size = 3544, normalized size = 11.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
[Out] 1/32*(32*(a*b - b^2)*d*x*cos(d*x + c)^4 - 64*(a*b - b^2)*d*x*cos(d*x + c)^2
 - 32*(a^2 - 2*a*b + b^2)*d*x + ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3
- b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*
a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450
*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^
4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(32*a^3 - 166*a^2*b + 1125/4*a*b^2
- 625/4*b^3 - 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)
^2 + 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqr
t((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7
- 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4
))*cos(d*x + c)*sin(d*x + c) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*
b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b
^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)
/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
+ b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b
^6 - b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6
+ 25*b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*
a*b^6 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3
 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^1
1 - 6*a*b^12 + b^13)*d^4))) - ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 -
b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*a^
2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a
^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a
^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4
- 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(32*a^3 - 166*a^2*b + 1125/4*a*b^2 -
625/4*b^3 - 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)^2
 - 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt(
(64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 -
6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))
*cos(d*x + c)*sin(d*x + c) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^
5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7
)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/(
(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 +
b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6
 - b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 +
25*b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*
b^6 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 +
 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11
- 6*a*b^12 + b^13)*d^4))) + ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^
4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b
^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*
b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*
```

```
b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3
*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-32*a^3 + 166*a^2*b - 1125/4*a*b^2 + 62
5/4*b^3 + 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)^2 +
 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((6
4*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*
a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))*c
os(d*x + c)*sin(d*x + c) + (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)
*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d
^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^
6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^1
3)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 -
b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*
b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6
 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 62
5*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6
*a*b^12 + b^13)*d^4))) - ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*
d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5
+ 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3
 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^1
1 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^
2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-32*a^3 + 166*a^2*b - 1125/4*a*b^2 + 625/4
*b^3 + 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)^2 - 1/
2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a
^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

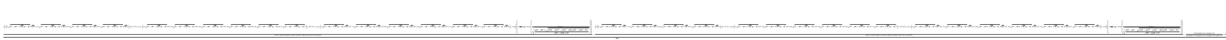
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1563 vs. $2(240) = 480$.
time = 0.92, size = 1563, normalized size = 4.88



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*((2*(6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 21*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 16*sqrt(a^2 - a*b - sqrt(a*b)*
```

```
(a - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
b^3)*(a*b^2 - b^3)^2*abs(-a + b) - (12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
a^5*b^2 - 63*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^3 + 116*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*a^3*b^4 - 86*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^
2*b^5 + 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^6 + 5*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*b^7)*abs(-a*b^2 + b^3)*abs(-a + b) - (9*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^4 - 51*sqrt(a^2 - a*b - sqrt(a*b)*(a - b
))*sqrt(a*b)*a^4*b^5 + 102*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^
3*b^6 - 82*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^7 + 17*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^8 + 5*sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*sqrt(a*b)*b^9)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arc
tan(tan(d*x + c)/sqrt((a^2*b^2 - a*b^3 + sqrt((a^2*b^2 - a*b^3)^2 - (a^2*b^
2 - a*b^3)*(a^2*b^2 - 2*a*b^3 + b^4)))/(a^2*b^2 - 2*a*b^3 + b^4))))/((3*a^7
*b^4 - 21*a^6*b^5 + 59*a^5*b^6 - 85*a^4*b^7 + 65*a^3*b^8 - 23*a^2*b^9 + a*b
^10 + b^11)*abs(-a*b^2 + b^3)) - (2*(6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a^3 - 21*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 16
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a*b^2 - b^3)^2*abs(-a + b) + (12*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*a^5*b^2 - 63*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*a^4*b^3 + 116*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^4 - 86*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*a^2*b^5 + 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
a*b^6 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^7)*abs(-a*b^2 + b^3)*abs(-a
 + b) - (9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^4 - 51*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^5 + 102*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*sqrt(a*b)*a^3*b^6 - 82*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a^2*b^7 + 17*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^8
+ 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^9)*abs(-a + b))*(pi*flo
or((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2*b^2 - a*b^3 - sqrt((
a^2*b^2 - a*b^3)^2 - (a^2*b^2 - a*b^3)*(a^2*b^2 - 2*a*b^3 + b^4)))/(a^2*b^2
 - 2*a*b^3 + b^4))))/((3*a^7*b^4 - 21*a^6*b^5 + 59*a^5*b^6 - 85*a^4*b^7 + 6
5*a^3*b^8 - 23*a^2*b^9 + a*b^10 + b^11)*abs(-a*b^2 + b^3)) - 2*(2*a*tan(d*x
 + c)^3 + a*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d
*x + c)^2 + a)*(a*b - b^2)) + 8*(d*x + c)/b^2)/d
```

Mupad [B]
time = 17.60, size = 2500, normalized size = 7.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] (atan(((((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 30
72*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208
896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) - (tan(c
```

+ d*x)*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*1i - (((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3072*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) + (tan(c + d*x)*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*1i)/((((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3072*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) - (tan(c + d*x)*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2))*((8*a^2*(a*b^9)^(1/2) + 25*b^2

```
*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))
/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(768*
a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*((8*
a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b
^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(
1/2) + (((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 30
72*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208
896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) + (tan(c
+ d*x)*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5
 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a
^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*
a^6*b^8))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2)
 - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^1
0 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(21376*a^2*b^8 - 84
864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 -
 b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^
5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 +
a^3*b^8)))^(1/2))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 +
 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2)...
```

3.219 $\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

Optimal. Leaf size=233

$$-\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d}+\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/2}d}-$$

```
[Out] -1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-3*b^(1/2
))/a^(1/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctan((a^(1/2)+b^(1/2))^(
1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+3*b^(1/2))/a^(1/4)/b^(3/2)/d/(a^(1/2)+b
^(1/2))^(3/2)-1/4*tan(d*x+c)/(a-b)/b/d+1/4*sec(d*x+c)^2*tan(d*x+c)^3/b/d/(a
+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A]
time = 0.22, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1134, 1293, 1180, 211}

$$-\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)}{4bd(a-b)}+\frac{\tan^3(c+dx)\sec^2(c+dx)}{4bd\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/8*((2*Sqrt[a] - 3*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])
/a^(1/4)])/(a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) + ((2*Sqrt[a] + 3*
Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(1/4)
*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tan[c + d*x]/(4*(a - b)*b*d) + (Sec
[c + d*x]^2*Tan[c + d*x]^3)/(4*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c
+ d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
```

```
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(a+2ax^2+(a-b)x^4)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{x^2(6a+2ax^2)}{a+2ax^2+(a-b)x^4}\,dx\right.}{8abd}$$

$$= -\frac{\tan(c+dx)}{4(a-b)bd} + \frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tan(c+dx)}{4(a-b)bd} + \frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))} - \frac{\left(a - \frac{2\sqrt{}}{}\right.}{}$$

$$= -\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} + \frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tan^{-1}}{8\sqrt[4]{a}\left(\sqrt{a}\right.}$$

Mathematica [A]

time = 1.77, size = 238, normalized size = 1.02

$$\frac{\frac{\left(2a+\sqrt{a}\sqrt{b}-3b\right)\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}} - \frac{\sqrt{b}\left(-2a+\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}} + \frac{4b(-2a-b+b\cos(2(c+dx)))\sin(2(c+dx))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}}{8(a-b)b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (((2*a + Sqrt[a]*Sqrt[b] - 3*b)*Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (Sqrt[b]*(-2
*a + Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt
[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (4*b*(-2*a - b + b*Co
s[2*(c + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos
[4*(c + d*x)]))/(8*(a - b)*b^2*d)
```

Maple [A]

time = 0.71, size = 262, normalized size = 1.12

method	result

derivativedivides	$\frac{\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4b(a-b)}-\frac{a\tan(dx+c)}{4b(a-b)}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(-a\sqrt{ab}+3\sqrt{ab}\,b-2a^2+4ab\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}}{d}$
default	$\frac{\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4b(a-b)}-\frac{a\tan(dx+c)}{4b(a-b)}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(-a\sqrt{ab}+3\sqrt{ab}\,b-2a^2+4ab\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}}{d}$
risch	$-\frac{i\left(2a\,e^{6i(dx+c)}-b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-3b\,e^{2i(dx+c)}+b\right)}{2b(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^4b^6d^4-3a^3b\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/4*(a+b)/b/(a-b)*tan(d*x+c)^3-1/4*a/b/(a-b)*tan(d*x+c))/(tan(d*x+c)
^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)+1/4/b*(1/2*(-a*(a*b)^(1/2)+3*(a*b)^
(1/2)*b-2*a^2+4*a*b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan
((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(-a*(a*b)^(1/2)+3*(a*b
)^(1/2)*b+2*a^2-4*a*b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arct
anh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((2*a*b
 - b^2)*sin(6*d*x + 6*c) - (8*a*b - 3*b^2)*sin(4*d*x + 4*c) - (2*a*b + 3*b^
2)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*((16*a^2 + 2*a*b - 3*b^2)*sin(4*d
*x + 4*c) + 4*(2*a*b + b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*((a*b^3
- b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64
*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 -
```

b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4 *(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin (4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2 *d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(2*a*b - 3*b^2)*cos(6*d*x + 6* c)^2 + 12*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(2*a*b - 3*b^2)*cos(2*d*x + 2*c)^2 + 4*(2*a*b - 3*b^2)*sin(6*d*x + 6*c)^2 + 12*(8*a*b - 3*b^2)*sin(4* d*x + 4*c)^2 + 2*(16*a^2 - 30*a*b + 21*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2* c) + 4*(2*a*b - 3*b^2)*sin(2*d*x + 2*c)^2 - (6*b^2*cos(4*d*x + 4*c) + (2*a* b - 3*b^2)*cos(6*d*x + 6*c) + (2*a*b - 3*b^2)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (2*a*b - 3*b^2 - 2*(16*a^2 - 30*a*b + 21*b^2)*cos(4*d*x + 4*c) - 8* (2*a*b - 3*b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*(3*b^2 - (16*a^2 - 3 0*a*b + 21*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (2*a*b - 3*b^2)*cos(2* d*x + 2*c) - (6*b^2*sin(4*d*x + 4*c) + (2*a*b - 3*b^2)*sin(6*d*x + 6*c) + (2*a*b - 3*b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((16*a^2 - 30*a*b + 2 1*b^2)*sin(4*d*x + 4*c) + 4*(2*a*b - 3*b^2)*sin(2*d*x + 2*c))*sin(6*d*x + 6 *c))/(a*b^3 - b^4 + (a*b^3 - b^4)*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*cos (6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*sin(8*d*x + 8 *c)^2 + 16*(a*b^3 - b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 5 7*a*b^3 - 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin (4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*sin(2*d*x + 2*c)^2 + 2*(a *b^3 - b^4 - 4*(a*b^3 - b^4)*cos(6*d*x + 6*c) - 2*(8*a^2*b^2 - 11*a*b^3 + 3 *b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^3 - b^4 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4 - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^3 - b^4)*cos(2*d*x + 2*c) - 4*(2*(a*b^3 - b^4)*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d *x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (b^2 + (2*a*b - b^2)*cos(6*d*x + 6*c) - (8*a*b - 3*b^2)*cos(4*d*x + 4*c) - (2*a*b + 3*b^2)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) + (2*a*b + 3*b^2 - 2*(16*a^2 + 2 *a*b - 3*b^2)*cos(4*d*x + 4*c) - 8*(2*a*b + b^2)*cos(2*d*x + 2*c))*sin(6*d* x + 6*c) + (8*a*b - 3*b^2 - 2*(16*a^2 + 2*a*b - 3*b^2)*cos(2*d*x + 2*c))*si n(4*d*x + 4*c) - (2*a*b - b^2)*sin(2*d*x + 2*c))/((a*b^3 - b^4)*d*cos(8*d*x

```
 + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b
^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x
+ 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*
x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*
c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2
*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^
2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*
c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4
)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d
)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c)
 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*
d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*
(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*
b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3135 vs. $2(181) = 362$.
time = 1.00, size = 3135, normalized size = 13.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 -
 (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^
2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a
^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/(
(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(1/4*(20*a^2 - 81*a*b + 81*b
^2)*cos(d*x + c)^2 - 5*a^2 + 81/4*a*b - 81/4*b^2 + 1/2*((a^5*b^3 - 6*a^4*b^
4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/
((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 +
a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(5*a^3*b - 19*a^2*b^2 + 18*a*b^3
)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)
*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 2
0*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2
)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)) + 1/4*(2*(4*a^5*b - 21*a^4*b
^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*cos(d*x + c)^2 - (4*a^5*b - 21*
a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2)*sqrt((25*a^2 - 90*a*b + 8
1*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2
*b^8 + a*b^9)*d^4))) - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*
cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3
*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1
```

```
5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15
*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(1/4*(20*a^2
 - 81*a*b + 81*b^2)*cos(d*x + c)^2 - 5*a^2 + 81/4*a*b - 81/4*b^2 - 1/2*((a^
5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*sqrt((25*a^2 - 9
0*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^
7 - 6*a^2*b^8 + a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(5*a^3*b - 19*a^
2*b^2 + 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4
+ 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4
+ 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 -
 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)) + 1/4*(2*(4*
a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*cos(d*x + c)^2
- (4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2)*sqrt((25*
a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15
*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) + ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*
(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3
- 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4
)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*
log(-1/4*(20*a^2 - 81*a*b + 81*b^2)*cos(d*x + c)^2 + 5*a^2 - 81/4*a*b + 81/
4*b^2 + 1/2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*
sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4
*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*
(5*a^3*b - 19*a^2*b^2 + 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3*b
^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b
^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*
d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2
)) + 1/4*(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*
cos(d*x + c)^2 - (4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)
*d^2)*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 -
20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) - ((a*b^2 - b^3)*d*cos(
d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*
sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*
b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b
^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^
5 - b^6)*d^2))*log(-1/4*(20*a^2 - 81*a*b + 81*b^2)*cos(d*x + c)^2 + 5*a^2 -
 81/4*a*b + 81/4*b^2 - 1/2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6
+ 3*a*b^7)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a
^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))*cos(d*x + c)*si
n(d*x + c) - 2*(5*a^3*b - 19*a^2*b^2 + 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c
))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b +
81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^
2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a
*b^5 - b^6)*d^2)) + 1/4*(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4
+ 9*a*b^5)*d^2*cos(d*x + c)^2 - (4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2
*b^4 + 9*a*b^5)*d^2)*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4
```

```
+ 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) + 8*(b*c
os(d*x + c)^3 - (a + b)*cos(d*x + c))*sin(d*x + c))/((a*b^2 - b^3)*d*cos(d*
x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1481 vs. 2(181) = 362.
time = 0.87, size = 1481, normalized size = 6.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*(((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2 -
a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 17*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^
3)*(a*b - b^2)^2*abs(-a + b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b
 - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^2 + 14*sqrt(a^2 - a*b - sqr
t(a*b)*(a - b))*a^3*b^3 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^4 - s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^5)*abs(-a*b + b^2)*abs(-a + b) - 2*(
3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b - 18*sqrt(a^2 - a*b -
 sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2 + 38*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^4*b^3 - 32*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^
3*b^4 + 7*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^5 + 2*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^6)*abs(-a + b))*(pi*floor((d*x +
 c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2*b - a*b^2 + sqrt((a^2*b - a*b
^2)^2 - (a^2*b - a*b^2)*(a^2*b - 2*a*b^2 + b^3)))/(a^2*b - 2*a*b^2 + b^3)))
)/((3*a^8*b^2 - 21*a^7*b^3 + 59*a^6*b^4 - 85*a^5*b^5 + 65*a^4*b^6 - 23*a^3*
b^7 + a^2*b^8 + a*b^9)*abs(-a*b + b^2)) - ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2
*b + 17*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a*b - b^2)^2*abs(-a + b) - (3*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b - 12*sqrt(a^2 - a*b + sqrt(a*b)*(a -
b))*a^4*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^3 - 4*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*a^2*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b
^5)*abs(-a*b + b^2)*abs(-a + b) - 2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
```

```
sqrt(a*b)*a^6*b - 18*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2
+ 38*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^3 - 32*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^4 + 7*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*a^2*b^5 + 2*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)
*a*b^6)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sq
rt((a^2*b - a*b^2 - sqrt((a^2*b - a*b^2)^2 - (a^2*b - a*b^2)*(a^2*b - 2*a*b
^2 + b^3)))/(a^2*b - 2*a*b^2 + b^3)))))/((3*a^8*b^2 - 21*a^7*b^3 + 59*a^6*b^
4 - 85*a^5*b^5 + 65*a^4*b^6 - 23*a^3*b^7 + a^2*b^8 + a*b^9)*abs(-a*b + b^2)
) - 2*(a*tan(d*x + c)^3 + b*tan(d*x + c)^3 + a*tan(d*x + c))/((a*tan(d*x +
c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c)^2 + a)*(a*b - b^2)))/d
```

Mupad [B]
time = 16.54, size = 2500, normalized size = 10.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] (atan(((((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) - (ta
n(c + d*x)*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4
+ 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^
2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a
*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256
*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) + (tan(c + d*x)*(9*a*b^3
 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b
^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2
*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*1i - (((256*a^2*b^5 - 512*a^3*b^4 + 256
*a^4*b^3)/(64*(a*b^3 - b^4)) + (tan(c + d*x)*((15*a*b^5 - 5*a*(a*b^9)^(1/2)
 + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*
a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a
^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1
/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6
)))^(1/2) - (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*
b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^
4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*1i)/(
(27*a*b^2 - 21*a^2*b + 4*a^3)/(32*(a*b^3 - b^4)) + (((256*a^2*b^5 - 512*a^3
*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) - (tan(c + d*x)*((15*a*b^5 - 5*a*(a*
b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^
2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b
^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*
(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7
 - a^4*b^6)))^(1/2) + (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^
2))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) -
 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(
```

1/2) + (((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) + (tan(c + d*x)*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) - (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*2i)/d + (atan(((((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) - (tan(c + d*x)*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) + (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*1i - (((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) + (tan(c + d*x)*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) - (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*1i)/((27*a*b^2 - 21*a^2*b + 4*a^3)/(32*(a*b^3 - b^4)) + (((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) - (tan(c + d*x)*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) + (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) + (((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) + (tan(c + d*x)*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^...

3.220 $\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

Optimal. Leaf size=195

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d}-\frac{\tan(c+dx)}{4a(a-b)d}+\frac{\tan^5}{4ad\left(a+2a\tan^2(c+\right.}$$

```
[Out] 1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-b
^(1/2))^(3/2)/b^(1/2)-1/8*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4)
)/a^(3/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(3/2)-1/4*tan(d*x+c)/a/(a-b)/d+1/4*ta
n(d*x+c)^5/a/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A]
time = 0.16, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3296, 1289, 12, 1136, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan^5(c+dx)}{4ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\tan(c+dx)}{4ad(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a]
- Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])
/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tan[c + d*x]/(4
*a*(a - b)*d) + Tan[c + d*x]^5/(4*a*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan
[c + d*x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4(1+x^2)}{(a+2ax^2+(a-b)x^4)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan^5(c+dx)}{4ad\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))} + \frac{\text{Subst}\left(\int -\frac{2bx^4}{a+2ax^2+(a-b)x^4}\right.}{8abd}$$

$$= \frac{\tan^5(c+dx)}{4ad\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{x^4}{a+2ax^2+(a-b)x^4}\,dx\right.}{4ad}$$

$$= -\frac{\tan(c+dx)}{4a(a-b)d} + \frac{\tan^5(c+dx)}{4ad\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tan(c+dx)}{4a(a-b)d} + \frac{\tan^5(c+dx)}{4ad\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))} + \frac{\left(2\sqrt{a}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d} - \frac{\tan(}{4a(}$$

Mathematica [A]
time = 2.98, size = 225, normalized size = 1.15

$$-\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{2(-6\sin(2(c+dx))+\sin(4(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}}{8(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/8*(((Sqrt[a] - Sqrt[b])*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a
 + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((Sqrt[
a] + Sqrt[b])*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*
Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (2*(-6*Sin[2*(c +
 d*x)] + Sin[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c
+ d*x)]))/((a - b)*d)
```

Maple [A]
time = 0.55, size = 218, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{-\frac{\tan^3(dx+c)}{2(a-b)}-\frac{\tan(dx+c)}{4(a-b)}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(-a-b+2\sqrt{ab}\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{8\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a+b+\right.}{8}}{d}$
default	$\frac{\frac{-\frac{\tan^3(dx+c)}{2(a-b)}-\frac{\tan(dx+c)}{4(a-b)}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(-a-b+2\sqrt{ab}\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{8\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a+b+\right.}{8}}{d}$
risch	$-\frac{i\left(b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-5b\,e^{2i(dx+c)}+b\right)}{2b(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(1+\left(a^6b^2d^4-3a^5b^3\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/2/(a-b)*tan(d*x+c)^3-1/4/(a-b)*tan(d*x+c))/(tan(d*x+c)^4*a-tan(d*x
+c)^4*b+2*a*tan(d*x+c)^2+a)+1/8*(-a-b+2*(a*b)^(1/2))/(a*b)^(1/2)/(a-b)/(((a
*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))
^(1/2))+1/8*(a+b+2*(a*b)^(1/2))/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(
1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(8*a - 3*b)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + (b*sin(6*d*x + 6*c)
- (8*a - 3*b)*sin(4*d*x + 4*c) - 5*b*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 6
*((8*a - 3*b)*sin(4*d*x + 4*c) + 4*b*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2
*((a*b^2 - b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*d*cos(6*d*x + 6*c)^
2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a*
b^2 - b^3)*d*cos(2*d*x + 2*c)^2 + (a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2 + 16*(
a*b^2 - b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^
```

```
3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c
)*sin(2*d*x + 2*c) + 16*(a*b^2 - b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^2 - b^3
)*d*cos(2*d*x + 2*c) + (a*b^2 - b^3)*d - 2*(4*(a*b^2 - b^3)*d*cos(6*d*x + 6
*c) + 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3)*d
*cos(2*d*x + 2*c) - (a*b^2 - b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b - 11*
a*b^2 + 3*b^3)*d*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3)*d*cos(2*d*x + 2*c) - (a
*b^2 - b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cos(2
*d*x + 2*c) - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^
2 - b^3)*d*sin(6*d*x + 6*c) + (8*a^2*b - 11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*
c) + 2*(a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b -
11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*d*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c))*integrate((4*b*cos(6*d*x + 6*c)^2 - 12*(8*a - 3*b)*cos(4*
d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + 4*b*sin(6*d*x + 6*c)^2 - 12*(8*a -
3*b)*sin(4*d*x + 4*c)^2 + 2*(8*a - 15*b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 4*b*sin(2*d*x + 2*c)^2 - (b*cos(6*d*x + 6*c) - 6*b*cos(4*d*x + 4*c) + b*c
os(2*d*x + 2*c))*cos(8*d*x + 8*c) + (2*(8*a - 15*b)*cos(4*d*x + 4*c) + 8*b*
cos(2*d*x + 2*c) - b)*cos(6*d*x + 6*c) + 2*((8*a - 15*b)*cos(2*d*x + 2*c) +
 3*b)*cos(4*d*x + 4*c) - b*cos(2*d*x + 2*c) - (b*sin(6*d*x + 6*c) - 6*b*sin
(4*d*x + 4*c) + b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a - 15*b)*sin(
4*d*x + 4*c) + 4*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a*b^2 - b^3 + (a*b^
2 - b^3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a
^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d*x + 4*c)^2 + 16*(a*b^2 - b^3)*co
s(2*d*x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*sin(
6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*sin(4*d*x + 4*c)
^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16
*(a*b^2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b^2 - b^3 - 4*(a*b^2 - b^3)*cos(6*
d*x + 6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b
^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^2 - b^3 - 2*(8*a^2*b - 11*a
*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(6*d*
x + 6*c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3 - 4*(8*a^2*b - 11*a*b^2 + 3*b^3)*c
os(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^2 - b^3)*cos(2*d*x + 2*c) - 4*(2
*(a*b^2 - b^3)*sin(6*d*x + 6*c) + (8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x +
4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b -
11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c)), x) - (b*cos(6*d*x + 6*c) - (8*a - 3*b)*cos(4*d*x + 4*c) - 5*
b*cos(2*d*x + 2*c) + b)*sin(8*d*x + 8*c) - (6*(8*a - 3*b)*cos(4*d*x + 4*c)
+ 24*b*cos(2*d*x + 2*c) - 5*b)*sin(6*d*x + 6*c) - (6*(8*a - 3*b)*cos(2*d*x
+ 2*c) - 8*a + 3*b)*sin(4*d*x + 4*c) - b*sin(2*d*x + 2*c))/((a*b^2 - b^3)*d
*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 1
12*a^2*b + 57*a*b^2 - 9*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^2 - b^3)*d*cos(
2*d*x + 2*c)^2 + (a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*d*si
n(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*d*sin(4*d*x +
4*c)^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*(a*b^2 - b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^2 - b^3)*d*cos(2*d*x + 2
*c) + (a*b^2 - b^3)*d - 2*(4*(a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b
```

```
- 11*a*b^2 + 3*b^3)*d*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3)*d*cos(2*d*x + 2*c)
 - (a*b^2 - b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*
cos(4*d*x + 4*c) + 4*(a*b^2 - b^3)*d*cos(2*d*x + 2*c) - (a*b^2 - b^3)*d)*co
s(6*d*x + 6*c) + 4*(4*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cos(2*d*x + 2*c) - (8*
a^2*b - 11*a*b^2 + 3*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^2 - b^3)*d*sin(6*
d*x + 6*c) + (8*a^2*b - 11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c) + 2*(a*b^2 - b
^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b - 11*a*b^2 + 3*b^3)
*d*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2796 vs. $2(151) = 302$.
time = 0.76, size = 2796, normalized size = 14.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^
2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt
((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a
^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*
b^3 - a*b^4)*d^2))*log(1/4*(3*a + b)*cos(d*x + c)^2 + 1/2*(2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*
b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))*cos
(d*x + c)*sin(d*x + c) - (3*a^3 + 4*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x +
 c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b
 + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*
b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^
2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^
5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b
- 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d
^4)) - 3/4*a - 1/4*b) - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos
(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
- a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(1/4*(3*a + b)*cos(d*x + c)^2 - 1/2*
(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)
/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a
^3*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^3 + 4*a^2*b + a*b^2)*d*cos(d
*x + c)*sin(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sq
rt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15
*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos
(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*
```

```
b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4
*b^6 + a^3*b^7)*d^4)) - 3/4*a - 1/4*b) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2*
(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^3*
b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2
 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a -
3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-1/4*(3*a + b)*cos(
d*x + c)^2 + 1/2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a
^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^
5 - 6*a^4*b^6 + a^3*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a^3 + 4*a^2*b
 + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b -
3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sq
rt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15
*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 3/4*a + 1/4*b) - ((a*b - b^2)*d*cos
(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(
((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a
^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b
^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-1/
4*(3*a + b)*cos(d*x + c)^2 - 1/2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^
4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6
*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) +
(3*a^3 + 4*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4*b - 3*a^3
*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^
2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a -
 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*
b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 3/4*a + 1/4*b) + 8*
(cos(d*x + c)^3 - 2*cos(d*x + c))*sin(d*x + c))/((a*b - b^2)*d*cos(d*x + c)
^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. $2(151) = 302$.

time = 0.89, size = 1262, normalized size = 6.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5 - 9*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^
2*b^3 - 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^2 -
a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*
b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a - b)^2 + (3*sqr
t(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a -
 b))*a^3*b^2 + 14*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 - 4*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^5)
*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2
 - a*b + sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - 2*a*
b + b^2))))/(3*a^8*b - 21*a^7*b^2 + 59*a^6*b^3 - 85*a^5*b^4 + 65*a^4*b^5 -
23*a^3*b^6 + a^2*b^7 + a*b^8) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt
(a*b)*a^5 - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sq
rt(a*b)*a*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*b^3)*(a - b)^2 + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 12*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*a^2*b^3 - 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*
b + sqrt(a*b)*(a - b))*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + ar
ctan(tan(d*x + c)/sqrt((a^2 - a*b - sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 -
 2*a*b + b^2)))/(a^2 - 2*a*b + b^2))))/(3*a^8*b - 21*a^7*b^2 + 59*a^6*b^3 -
 85*a^5*b^4 + 65*a^4*b^5 - 23*a^3*b^6 + a^2*b^7 + a*b^8) - 2*(2*tan(d*x + c
)^3 + tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c
)^2 + a)*(a - b)))/d
```

Mupad [B]
time = 15.99, size = 2980, normalized size = 15.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] - (atan(((((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) - (tan(c + d*
x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3
*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4 +
768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3
)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^
2)))^(1/2) - (tan(c + d*x)*(6*a*b + a^2 + b^2))/(4*(a - b)))*((3*a*(a^3*b^3
)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4
+ 3*a^5*b^3 - a^6*b^2)))^(1/2)*1i - (((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)
/(32*(a - b)) + (tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a
^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*
(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*((3*a*(
a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a
^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) + (tan(c + d*x)*(6*a*b + a^2 + b^2))/
(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)
/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*1i)/((((128*a*b^3
 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) - (tan(c + d*x)*((3*a*(a^3*b^3)^(1
/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*
a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4
*b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a
^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) - (tan(c +
 d*x)*(6*a*b + a^2 + b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)
^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2
)))^(1/2) + (((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) + (tan(c +
 d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(
a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4
 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*
b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6
*b^2)))^(1/2) + (tan(c + d*x)*(6*a*b + a^2 + b^2))/(4*(a - b)))*((3*a*(a^3*
b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b
^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) - 1/(16*(a - b))))*((3*a*(a^3*b^3)^(1/2)
+ b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*
b^3 - a^6*b^2)))^(1/2)*2i)/d - (atan(((((128*a*b^3 + 128*a^3*b - 256*a^2*b^
2)/(32*(a - b)) - (tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2)
- a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/
2)*(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*(-(3
*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 -
 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) - (tan(c + d*x)*(6*a*b + a^2 + b^
2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2
*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*1i - (((128*
a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) + (tan(c + d*x)*(-(3*a*(a^3*b
^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^
4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 7
68*a^4*b^2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*
b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) + (
tan(c + d*x)*(6*a*b + a^2 + b^2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(
```

```
a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 -
 a^6*b^2)))^(1/2)*1i)/((((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b))
 - (tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2
*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b -
 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1
/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*
a^5*b^3 - a^6*b^2)))^(1/2) - (tan(c + d*x)*(6*a*b + a^2 + b^2))/(4*(a - b))
)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3
*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) + (((128*a*b^3 + 128*a^3*b
- 256*a^2*b^2)/(32*(a - b)) + (tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3
*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^
6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a
 - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(25
6*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) + (tan(c + d*x)*(6*a*
b + a^2 + b^2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a
^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)
- 1/(16*(a - b))))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a
^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*2i)/d - (t
an(c + d*x)^3/(2*(a - b)) + tan(c + d*x)/(4*(a - b)))/(d*(a + 2*a*tan(c + d
*x)^2 + tan(c + d*x)^4*(a - b)))
```

3.221 $\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=219

$$\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d}-\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d}-\frac{}{4a(a}$$

```
[Out] 1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2))/
a^(5/4)/d/(a^(1/2)-b^(1/2))^(3/2)/b^(1/2)-1/8*arctan((a^(1/2)+b^(1/2))^(1/2
)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(5/4)/d/b^(1/2)/(a^(1/2)+b^(1/2
))^(3/2)-1/4*tan(d*x+c)*(a+(a+b)*tan(d*x+c)^2)/a/(a-b)/d/(a+2*a*tan(d*x+c)^
2+(a-b)*tan(d*x+c)^4)
```

Rubi [A]
time = 0.20, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3296, 1347, 1180, 211}

$$\frac{\left(2\sqrt{a}-\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)\left((a+b)\tan^2(c+dx)+a\right)}{4ad(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((2*Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4
)])/(8*a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ((2*Sqrt[a] + Sqrt[b]
)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(5/4)*(Sqrt[
a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - (Tan[c + d*x]*(a + (a + b)*Tan[c + d*x]^2)
)/(4*a*(a - b)*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1+x^2\right)^2}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b}{a-b}-\frac{2a}{}}{a+2ax^2+}\right.}{}$$

$$= -\frac{\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\left(2a+\sqrt{a}\sqrt{b}-\right.}{}$$

$$= \frac{\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d} - \frac{\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{8a^{5/4}\left(\sqrt{a}+\sqrt{}\right.}$$

Mathematica [A]

time = 1.43, size = 255, normalized size = 1.16

$$-\frac{\frac{\sqrt{a}\left(2a-\sqrt{a}\sqrt{b}-b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{a}\left(2a+\sqrt{a}\sqrt{b}-b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{4\sqrt{a}\left(2a+b-b\cos(2(c+dx))\right)\sin(2(c+dx))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}}{8a^{3/2}(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (-((Sqrt[a]*(2*a - Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b])) - (
Sqrt[a]*(2*a + Sqrt[a]*Sqrt[b] - b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*
x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (4*
Sqrt[a]*(2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*C
os[2*(c + d*x)] - b*Cos[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)
```

Maple [A]
time = 0.80, size = 250, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4(a-b)a}-\frac{\tan(dx+c)}{4(a-b)}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(3a\sqrt{ab}-\sqrt{ab}\,b+2a^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}}{d}$
default	$\frac{\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4(a-b)a}-\frac{\tan(dx+c)}{4(a-b)}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(3a\sqrt{ab}-\sqrt{ab}\,b+2a^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}}{d}$
risch	$-\frac{i\left(2a\,e^{6i(dx+c)}-b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-3b\,e^{2i(dx+c)}+b\right)}{2a(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\left(\_R=\mathrm{RootOf}\left(\left(256a^8b^2d^4\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/4*(a+b)/(a-b)/a*tan(d*x+c)^3-1/4/(a-b)*tan(d*x+c))/(tan(d*x+c)^4*a
-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)+1/4/a*(1/2*(3*a*(a*b)^(1/2)-(a*b)^(1/2)
*b+2*a^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(
d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(3*a*(a*b)^(1/2)-(a*b)^(1/2)*b-2*
a^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x
+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((2*a*b
 - b^2)*sin(6*d*x + 6*c) - (8*a*b - 3*b^2)*sin(4*d*x + 4*c) - (2*a*b + 3*b^
2)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*((16*a^2 + 2*a*b - 3*b^2)*sin(4*d
*x + 4*c) + 4*(2*a*b + b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*((a^2*b^
2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2
 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*
(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*
c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b +
57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a
*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*
x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)*d -
 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*
b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2
 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos
(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*
d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x + 2
*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*b^2
- a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x
+ 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*
a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(2*a*b - b^2)*cos(6*d*x + 6
*c)^2 - 4*(32*a^2 - 20*a*b + 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(2*a*b - b^2)*co
s(2*d*x + 2*c)^2 + 4*(2*a*b - b^2)*sin(6*d*x + 6*c)^2 - 4*(32*a^2 - 20*a*b
+ 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(16*a^2 - 30*a*b + 7*b^2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 4*(2*a*b - b^2)*sin(2*d*x + 2*c)^2 - ((2*a*b - b^2)*cos(
6*d*x + 6*c) - 2*(4*a*b - b^2)*cos(4*d*x + 4*c) + (2*a*b - b^2)*cos(2*d*x +
 2*c))*cos(8*d*x + 8*c) - (2*a*b - b^2 - 2*(16*a^2 - 30*a*b + 7*b^2)*cos(4*
d*x + 4*c) - 8*(2*a*b - b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(4*a*b
- b^2 + (16*a^2 - 30*a*b + 7*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (2*a
*b - b^2)*cos(2*d*x + 2*c) - ((2*a*b - b^2)*sin(6*d*x + 6*c) - 2*(4*a*b - b
```

```
^2)*sin(4*d*x + 4*c) + (2*a*b - b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2
*((16*a^2 - 30*a*b + 7*b^2)*sin(4*d*x + 4*c) + 4*(2*a*b - b^2)*sin(2*d*x +
2*c))*sin(6*d*x + 6*c))/(a^2*b^2 - a*b^3 + (a^2*b^2 - a*b^3)*cos(8*d*x + 8*
c)^2 + 16*(a^2*b^2 - a*b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57
*a^2*b^2 - 9*a*b^3)*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*cos(2*d*x + 2
*c)^2 + (a^2*b^2 - a*b^3)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*sin(6*d
*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*sin(4*d*x + 4*c
)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 16*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - a*b^3 - 4*(a^2*b^
2 - a*b^3)*cos(6*d*x + 6*c) - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x
+ 4*c) - 4*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^
2 - a*b^3 - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4*(a^2*b^
2 - a*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 11*a^2*b^2 + 3
*a*b^3 - 4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4
*c) - 8*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^2*b^2 - a*b^3)*sin(6*d
*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2
- a*b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3
*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c)), x) - (b^2 + (2*a*b - b^2)*cos(6*d*x + 6*c) - (8*a*b - 3*b^2)*cos(4
*d*x + 4*c) - (2*a*b + 3*b^2)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) + (2*a*b +
 3*b^2 - 2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c) - 8*(2*a*b + b^2)*cos(
2*d*x + 2*c))*sin(6*d*x + 6*c) + (8*a*b - 3*b^2 - 2*(16*a^2 + 2*a*b - 3*b^2
)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) - (2*a*b - b^2)*sin(2*d*x + 2*c))/((a^
2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*
c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 +
 16*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x
+ 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*
b + 57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 +
 3*a*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(
2*d*x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)
*d - 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 +
3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2
*b^2 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d
*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b
^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x
 + 2*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*
b^2 - a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3445 vs. $2(169) = 338$.
time = 0.99, size = 3445, normalized size = 15.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")

[Out] 1/32*(((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(8*a^3 - 7*a^2*b + 9/4*a*b^2 - 1/4*b^3 - 1/4*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cos(d*x + c)^2 + 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2)) + 1/4*(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*cos(d*x + c)^2 - (4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2)*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))) - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(8*a^3 - 7*a^2*b + 9/4*a*b^2 - 1/4*b^3 - 1/4*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cos(d*x + c)^2 - 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2)) + 1/4*(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*cos(d*x + c)^2 - (4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2)*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))) + ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(-8*a^3 + 7*a^2*b - 9/4*a*b^2 + 1/4*b^3 + 1/4*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cos(d*x + c)^2 + 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*

```
a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 +
 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(8*a
^5 - 5*a^4*b + a^3*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^5*b - 3*a^4*b
^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b
^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*
a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b
^3 - a^2*b^4)*d^2)) + 1/4*(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a
^3*b^4)*d^2*cos(d*x + c)^2 - (4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a
^3*b^4)*d^2)*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*
b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7
)*d^4))) - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x
+ c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 -
a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*
b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7
)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2
))*log(-8*a^3 + 7*a^2*b - 9/4*a*b^2 + 1/4*b^3 + 1/4*(32*a^3 - 28*a^2*b + 9*
a*b^2 - b^3)*cos(d*x + c)^2 - 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a
^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4
)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6
+ a^5*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(8*a^5 - 5*a^4*b + a^3*b^2)*
d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4
)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a
^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))
 - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1407 vs. $2(169) = 338$.
time = 0.89, size = 1407, normalized size = 6.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 21*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 3*sqrt(a^2 - a*b - sqrt(a*b
```

)*(a - b))*sqrt(a*b)*a*b^3 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
b^4)*(a^2 - a*b)^2*abs(-a + b) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6
*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b^2 + 14*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*a^4*b^3 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^4 -
 sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^5)*abs(-a^2 + a*b)*abs(-a + b) -
 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^8 - 12*sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b + 14*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^6*b^2 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5
*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^4)*abs(-a + b))*
(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 - a^2*b + sqr
t((a^3 - a^2*b)^2 - (a^3 - a^2*b)*(a^3 - 2*a^2*b + a*b^2)))/(a^3 - 2*a^2*b
+ a*b^2))))/((3*a^10*b - 21*a^9*b^2 + 59*a^8*b^3 - 85*a^7*b^4 + 65*a^6*b^5
- 23*a^5*b^6 + a^4*b^7 + a^3*b^8)*abs(-a^2 + a*b)) - ((9*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 21*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a^2*b^2 + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 +
 sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^2 - a*b)^2*abs(-a +
b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b - 12*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*a^5*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^3 -
4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*a^2*b^5)*abs(-a^2 + a*b)*abs(-a + b) - 2*(3*sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*sqrt(a*b)*a^8 - 12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*a^7*b + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^2 - 4*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^3 - sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a^4*b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2
) + arctan(tan(d*x + c)/sqrt((a^3 - a^2*b - sqrt((a^3 - a^2*b)^2 - (a^3 - a
^2*b)*(a^3 - 2*a^2*b + a*b^2)))/(a^3 - 2*a^2*b + a*b^2))))/((3*a^10*b - 21*
a^9*b^2 + 59*a^8*b^3 - 85*a^7*b^4 + 65*a^6*b^5 - 23*a^5*b^6 + a^4*b^7 + a^3
*b^8)*abs(-a^2 + a*b)) + 2*(a*tan(d*x + c)^3 + b*tan(d*x + c)^3 + a*tan(d*x
 + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c)^2 + a)*(a^2
 - a*b)))/d

Mupad [B]
time = 17.35, size = 2500, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] - (tan(c + d*x)/(4*(a - b)) + (tan(c + d*x)^3*(a + b))/(4*a*(a - b)))/(d*(a
 + 2*a*tan(c + d*x)^2 + tan(c + d*x)^4*(a - b))) - (atan(((((256*a^5*b + 25
6*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) - (tan(c + d*x)*(-(8*a^2*(a^5*b
^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*
b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a
^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*(-(8*a^2*
```

(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b
*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)
- (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*(-(8*a^
2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a
b(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2
)*1i - (((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) + (tan(
c + d*x)*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3
 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 -
 a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4
*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a
^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7
*b^3 - a^8*b^2)))^(1/2) + (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/
(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b +
 a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a
^7*b^3 - a^8*b^2)))^(1/2)*1i)/((((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(6
4*(a^2*b - a^3)) - (tan(c + d*x)*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(
1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 -
 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^
4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*
b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5
*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) - (tan(c + d*x)*(9*a^2*b -
6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^
5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a
^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) - (12*a^2 - 7*a*b + b^2)/
(32*(a^2*b - a^3)) + (((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b -
 a^3)) + (tan(c + d*x)*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a
^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4
 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 76
8*a^5*b^2))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2)
 - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a
^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) + (tan(c + d*x)*(9*a^2*b - 6*a*b^2 +
4*a^3 + b^3))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/
2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3
*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a
^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(
a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*2i)/d - (atan(((((256*a^
5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) - (tan(c + d*x)*((8*a^2
*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*
b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)
*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*((
8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 -
 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^
(1/2) - (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*(
(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2
- 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))

```
^(1/2)*1i - (((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) +
(tan(c + d*x)*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3
*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b
^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2)
)/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b
- a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*
a^7*b^3 - a^8*b^2)))^(1/2) + (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3
))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b
 - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3
*a^7*b^3 - a^8*b^2)))^(1/2)*1i)/((((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/
(64*(a^2*b - a^3)) - (tan(c + d*x)*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^
(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5
- 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a
^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*
b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5
*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)...
```

3.222 $\int \frac{1}{(a-b\sin^4(c+dx))^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}-\frac{}{4a}$$

```
[Out] 1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(4*a^(1/2)-3*b^(1/2)
)/a^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(
d*x+c)/a^(1/4))*(4*a^(1/2)+3*b^(1/2))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(3/2)-1/4
*b*tan(d*x+c)*(1+2*tan(d*x+c)^2)/a/(a-b)/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*
x+c)^4)
```

Rubi [A]
time = 0.17, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3288, 1219, 1180, 211}

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)\left(2\tan^2(c+dx)+1\right)}{4ad(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[c + d*x]^4)^(-2),x]
```

```
[Out] ((4*Sqrt[a] - 3*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(8*a^(7/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((4*Sqrt[a] + 3*Sqrt[b])*Ar
cTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(7/4)*(Sqrt[a] +
 Sqrt[b])^(3/2)*d) - (b*Tan[c + d*x]*(1 + 2*Tan[c + d*x]^2))/(4*a*(a - b)*d
*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$=-\frac{b\tan(c+dx)\left(1+2\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}-\frac{\text{Subst}\left(\int \frac{-\frac{2a(4a-3b}{a-b}}{a+2ax}\right.}{}$$

$$=-\frac{b\tan(c+dx)\left(1+2\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}+\frac{\left(4a-\sqrt{a}\sqrt{b}\right.-}{}$$

$$=\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\right.}{8a^{7/4}\left(\sqrt{a}\right.}$$

Mathematica [A]

time = 1.96, size = 230, normalized size = 1.10

$$\frac{\frac{\left(4a-\sqrt{a}\,\sqrt{b}-3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\,\sqrt{b}}}-\frac{\left(4a+\sqrt{a}\,\sqrt{b}-3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{a}\,b(-6\sin(2(c+dx))+\sin(4(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}}{8a^{3/2}(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[c + d*x]^4)^(-2),x]
```

```
[Out] (((4*a - Sqrt[a]*Sqrt[b] - 3*b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/S
qrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - ((4*a + Sqrt[a]*Sqrt
[b] - 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqr
t[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (2*Sqrt[a]*b*(-6*Sin[2*(c + d*x)] + Si
n[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]))/(
8*a^(3/2)*(a - b)*d)
```

Maple [A]

time = 0.82, size = 260, normalized size = 1.24

method	result
derivativedivides	$\frac{\frac{-\frac{b\left(\tan^3(dx+c)\right)}{2a(a-b)}-\frac{b\tan(dx+c)}{4a(a-b)}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(4a\sqrt{ab}-2\sqrt{ab}\,b-5ab+3b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}}{d}$
default	$\frac{\frac{-\frac{b\left(\tan^3(dx+c)\right)}{2a(a-b)}-\frac{b\tan(dx+c)}{4a(a-b)}}{\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(4a\sqrt{ab}-2\sqrt{ab}\,b-5ab+3b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}}{d}$
risch	$-\frac{i\left(b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-5b\,e^{2i(dx+c)}+b\right)}{2a(a-b)d\left(b\,e^{8i(dx+c)}-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\Bigg(\_R=\text{RootOf}((65536a^{10}d^4-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/2*b/a/(a-b)*tan(d*x+c)^3-1/4*b/a/(a-b)*tan(d*x+c))/(tan(d*x+c)^4*a
-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)+1/4/a*(1/2*(4*a*(a*b)^(1/2)-2*(a*b)^(1/
```

```
2)*b-5*a*b+3*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((
-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*(4*a*(a*b)^(1/2)-2*(a*b
)^(1/2)*b+5*a*b-3*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arct
an((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*sin(2*d*x + 2*c) - 6*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x +
 2*c) - (b^2*sin(6*d*x + 6*c) - 5*b^2*sin(2*d*x + 2*c) - (8*a*b - 3*b^2)*si
n(4*d*x + 4*c))*cos(8*d*x + 8*c) - 6*(4*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b
^2)*sin(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*((a^2*b^2 - a*b^3)*d*cos(8*d*x +
 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b
 + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(
2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b
^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*
sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^2*b^2
- a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)*d - 2*(4*(a^2*b^2 - a*b^3)*
d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c)
+ 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(8*d*x +
 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b
^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(6*d*x + 6*c) + 4*
(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x + 2*c) - (8*a^3*b - 11*a^2*
b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6
*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*
b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a
*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*
x + 6*c))*integrate((4*b^2*cos(6*d*x + 6*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 +
4*b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2 - 4*(64*a^2 - 64*a*b +
15*b^2)*cos(4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) - 4*(64*a^2 - 64*a*b + 15
*b^2)*sin(4*d*x + 4*c)^2 - 2*(24*a*b - 17*b^2)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) - (b^2*cos(6*d*x + 6*c) + b^2*cos(2*d*x + 2*c) - 2*(8*a*b - 5*b^2)*co
s(4*d*x + 4*c))*cos(8*d*x + 8*c) + (8*b^2*cos(2*d*x + 2*c) - b^2 - 2*(24*a*
b - 17*b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(8*a*b - 5*b^2 - (24*a*b
 - 17*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^2*sin(6*d*x + 6*c) + b^2
*sin(2*d*x + 2*c) - 2*(8*a*b - 5*b^2)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) +
2*(4*b^2*sin(2*d*x + 2*c) - (24*a*b - 17*b^2)*sin(4*d*x + 4*c))*sin(6*d*x +
 6*c))/(a^2*b^2 - a*b^3 + (a^2*b^2 - a*b^3)*cos(8*d*x + 8*c)^2 + 16*(a^2*b^
2 - a*b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^
```

```
3)*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c)^2 + (a^2*b^2
- a*b^3)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*sin(6*d*x + 6*c)^2 + 4*(
64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b
 - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 -
a*b^3)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - a*b^3 - 4*(a^2*b^2 - a*b^3)*cos(6*
d*x + 6*c) - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4*(a^2*b
^2 - a*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^2 - a*b^3 - 2*(8*
a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - a*b^3)*cos(2*
d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3 - 4*(8*a^3
*b - 11*a^2*b^2 + 3*a*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^2*b^2
 - a*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^2*b^2 - a*b^3)*sin(6*d*x + 6*c) + (8*a^
3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x
 + 4*c) + 2*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) + (b^
2*cos(6*d*x + 6*c) - 5*b^2*cos(2*d*x + 2*c) + b^2 - (8*a*b - 3*b^2)*cos(4*d
*x + 4*c))*sin(8*d*x + 8*c) + (24*b^2*cos(2*d*x + 2*c) - 5*b^2 + 6*(8*a*b -
 3*b^2)*cos(4*d*x + 4*c))*sin(6*d*x + 6*c) - (8*a*b - 3*b^2 - 6*(8*a*b - 3*
b^2)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c))/((a^2*b^2 - a*b^3)*d*cos(8*d*x + 8
*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b +
 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(2*
d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3
)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*si
n(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^2*b^2 -
a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)*d - 2*(4*(a^2*b^2 - a*b^3)*d*
cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) +
4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(8*d*x + 8
*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2
 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(6*d*x + 6*c) + 4*(4
*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x + 2*c) - (8*a^3*b - 11*a^2*b^
2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c
) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^
3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b
^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3477 vs. $2(164) = 328$.
time = 1.05, size = 3477, normalized size = 16.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)
^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3
)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/(
(a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7
*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^
3)*d^2))*log(96*a^3*b - 170*a^2*b^2 + 405/4*a*b^3 - 81/4*b^4 - 1/4*(384*a^3
*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 + 1/2*(2*(2*a^10 - 7*
a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2
 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c
) - (120*a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin
(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b
 - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15
*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2
- 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16
*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 -
 (16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a
^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b
+ 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) - ((
a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3
 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqr
t((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6
*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4
)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*
log(96*a^3*b - 170*a^2*b^2 + 405/4*a*b^3 - 81/4*b^4 - 1/4*(384*a^3*b - 680*
a^2*b^2 + 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 - 1/2*(2*(2*a^10 - 7*a^9*b + 9
*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a
^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3
+ 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - (120*
a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin(d*x + c)
)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a
^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2
 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b
+ 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16*a^8 - 57
*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 - (16*a^8
- 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a^4*b - 13
92*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11
*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) + ((a^2*b - a
*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*
b + a*b^2)*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4
*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b +
15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^
2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-96*a^
3*b + 170*a^2*b^2 - 405/4*a*b^3 + 81/4*b^4 + 1/4*(384*a^3*b - 680*a^2*b^2 +
 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 + 1/2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2
```

```
- 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 -
522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*
b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (120*a^5*b - 2
17*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((
a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1
273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10
*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/(
(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16*a^8 - 57*a^7*b + 7
5*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 - (16*a^8 - 57*a^7*b
 + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a^4*b - 1392*a^3*b^2
 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) - ((a^2*b - a*b^2)*d*co
s(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)
*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*
a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^
2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b
 - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-96*a^3*b + 170*
a^2*b^2 - 405/4*a*b^3 + 81/4*b^4 + 1/4*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3
 - 81*b^4)*cos(d*x + c)^2 - 1/2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^
3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4
+ 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^
8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

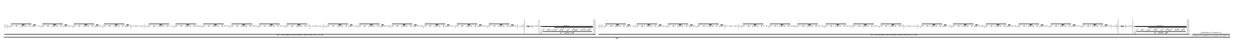
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(164) = 328$.
time = 0.56, size = 1506, normalized size = 7.17



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*((2*(6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 4*sqrt(a^2 - a*b - sqrt(a*b)*
(a - b))*sqrt(a*b)*a*b^2 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^
3)*(a^2 - a*b)^2*abs(-a + b) - (12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6
```

```
- 57*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b + 92*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*a^4*b^2 - 58*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^3 + 8*s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^4 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*a*b^5)*abs(-a^2 + a*b)*abs(-a + b) - (15*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^7 - 69*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
a^6*b + 106*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2 - 62*sqrt
(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^3 + 7*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a^3*b^4 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a^2*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*
x + c)/sqrt((a^3 - a^2*b + sqrt((a^3 - a^2*b)^2 - (a^3 - a^2*b)*(a^3 - 2*a^
2*b + a*b^2)))/(a^3 - 2*a^2*b + a*b^2))))/((3*a^10 - 21*a^9*b + 59*a^8*b^2
- 85*a^7*b^3 + 65*a^6*b^4 - 23*a^5*b^5 + a^4*b^6 + a^3*b^7)*abs(-a^2 + a*b)
) - (2*(6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 4*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*sqrt(a*b)*a*b^2 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)
*(a^2 - a*b)^2*abs(-a + b) + (12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6 -
57*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b + 92*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*a^4*b^2 - 58*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^3 + 8*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^4 + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*a*b^5)*abs(-a^2 + a*b)*abs(-a + b) - (15*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*a^7 - 69*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
6*b + 106*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2 - 62*sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^3 + 7*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a^3*b^4 + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt
(a*b)*a^2*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x
+ c)/sqrt((a^3 - a^2*b - sqrt((a^3 - a^2*b)^2 - (a^3 - a^2*b)*(a^3 - 2*a^2*
b + a*b^2)))/(a^3 - 2*a^2*b + a*b^2))))/((3*a^10 - 21*a^9*b + 59*a^8*b^2 -
85*a^7*b^3 + 65*a^6*b^4 - 23*a^5*b^5 + a^4*b^6 + a^3*b^7)*abs(-a^2 + a*b))
+ 2*(2*b*tan(d*x + c)^3 + b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x +
c)^4 + 2*a*tan(d*x + c)^2 + a)*(a^2 - a*b)))/d
```

Mupad [B]
time = 16.52, size = 2500, normalized size = 11.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] - (atan(((((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3
*b - a^4)) - (tan(c + d*x)*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 1
5*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 +
 a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*
a^6*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) -
 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10
```

+ a^7*b^3 - 3*a^8*b^2)))^(1/2) - (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i - (((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3*b - a^4)) + (tan(c + d*x)*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) + (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i)/((32*a^2*b - 34*a*b^2 + 9*b^3)/(16*(a^3*b - a^4)) + (((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3*b - a^4)) - (tan(c + d*x)*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) - (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) + (((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3*b - a^4)) + (tan(c + d*x)*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) + (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*2i)/d - (atan(((((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3*b - a^4)) - (tan(c + d*x)*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) - (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i - (((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^

```
2)/(32*(a^3*b - a^4)) + (tan(c + d*x)*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*
b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^
9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^
5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a
^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3
*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) + (tan(c + d*x)*(16*a^3*b - 26
*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/2) + 9
*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/
(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i)/((32*a^2*b - 34*a*b
^2 + 9*b^3)/(16*(a^3*b - a^4)) + (((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3
- 1408*a^5*b^2)/(32*(a^3*b - a^4)) - (tan(c + d*x)*(-(24*a^2*(a^7*b)^(1/2)
+ 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2
))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4
*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/
2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(
1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^...
```

3.223 $\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

Optimal. Leaf size=236

$$\frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}-\frac{\left(6\sqrt{a}+5\sqrt{b}\right)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}$$

```
[Out] -cot(d*x+c)/a^2/d+1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(6
*a^(1/2)-5*b^(1/2))*b^(1/2)/a^(9/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctan((a
^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1/2)*(6*a^(1/2)+5*b^(1/2))/a^(
9/4)/d/(a^(1/2)+b^(1/2))^(3/2)-1/4*b*tan(d*x+c)*(a+(a+b)*tan(d*x+c)^2)/a^2/
(a-b)/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A]

time = 0.36, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1348, 1678, 1180, 211}

$$\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)\left((a+b)\tan^2(c+dx)+a\right)}{4a^2d(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*
x])/a^(1/4)])/(8*a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - ((6*Sqrt[a] + 5*Sqr
t[b])*Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^
(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Cot[c + d*x]/(a^2*d) - (b*Tan[c + d*x]
*(a + (a + b)*Tan[c + d*x]^2))/(4*a^2*(a - b)*d*(a + 2*a*Tan[c + d*x]^2 + (
a - b)*Tan[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1348

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^2(a+2ax^2+(a-b)x^4)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-8ab-}{}\right.}{}$$

$$= -\frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(-\frac{8b}{x^2}\right.\right.}{}$$

$$= -\frac{\cot(c+dx)}{a^2d} - \frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} -$$

$$= -\frac{\cot(c+dx)}{a^2d} - \frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} +$$

$$= \frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\left(6\sqrt{a}+5\sqrt{b}\right)\sqrt{}}{8a^{9/}}$$

Mathematica [A]
time = 1.61, size = 274, normalized size = 1.16

$$\frac{-\frac{\left(6a\sqrt{b}+5\sqrt{a}\,b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{a+\sqrt{a}\sqrt{b}}} - \frac{\left(6a\sqrt{b}-5\sqrt{a}\,b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{-a+\sqrt{a}\sqrt{b}}} - 8\sqrt{a}\cot(c+dx) - \frac{4\sqrt{a}\,b(2a+b-b\cos(2(c+dx)))\sin(2(c+dx))}{(a-b)(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (-(((6*a*Sqrt[b] + 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/S
qrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt[b]]))
 - ((6*a*Sqrt[b] - 5*Sqrt[a]*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/
Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b]
]) - 8*Sqrt[a]*Cot[c + d*x] - (4*Sqrt[a]*b*(2*a + b - b*Cos[2*(c + d*x)])*S
in[2*(c + d*x)])/((a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c +
d*x)])))/(8*a^(5/2)*d)
```

Maple [A]
time = 0.88, size = 269, normalized size = 1.14 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2/tan(d*x+c)+b/a^2*((-1/4*(a+b)/(a-b)*tan(d*x+c)^3-1/4*a/(a-b)*ta
n(d*x+c))/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)+1/8*(7*a*(a*b)
^(1/2)-5*(a*b)^(1/2)*b-6*a^2+4*a*b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/8*(7*a*
(a*b)^(1/2)-5*(a*b)^(1/2)*b+6*a^2-4*a*b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)
*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(48*a^2*b - 5*a*b^2 - 25*b^3)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((
6*a*b^2 - 5*b^3)*sin(8*d*x + 8*c) - 2*(13*a*b^2 - 10*b^3)*sin(6*d*x + 6*c)
- 2*(32*a^2*b - 47*a*b^2 + 15*b^3)*sin(4*d*x + 4*c) - 2*(7*a*b^2 - 10*b^3)*
sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + (2*(48*a^2*b - 5*a*b^2 - 25*b^3)*sin
(6*d*x + 6*c) + 2*(112*a^2*b - 165*a*b^2 + 50*b^3)*sin(4*d*x + 4*c) + 5*(8*
a*b^2 - 15*b^3)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*(2*(256*a^3 - 432*a^
2*b + 210*a*b^2 - 25*b^3)*sin(4*d*x + 4*c) + (112*a^2*b - 165*a*b^2 + 50*b^
3)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*((a^3*b^2 - a^2*b^3)*d*cos(10*d*x
 + 10*c)^2 + 25*(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*
a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^
4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3*b^2 - a^2*b^
3)*d*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(10*d*x + 10*c)^2 + 25*(
a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b
^2 - 25*a^2*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2
 - 25*a^2*b^3)*d*sin(4*d*x + 4*c)^2 + 20*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)
*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x +
 2*c)^2 - 10*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d
 - 2*(5*(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c) + 2*(8*a^4*b - 13*a^3*b^2 +
5*a^2*b^3)*d*cos(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(
4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^
3)*d)*cos(10*d*x + 10*c) + 10*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(6
*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(4*d*x + 4*c) - 5*(
a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d)*cos(8*d*x +
8*c) - 4*(2*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(4*d*x + 4
*c) + 5*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b -
13*a^3*b^2 + 5*a^2*b^3)*d)*cos(6*d*x + 6*c) + 4*(5*(8*a^4*b - 13*a^3*b^2 +
5*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d)*cos(4
```

```
*d*x + 4*c) - 2*(5*(a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c) + 2*(8*a^4*b - 13
*a^3*b^2 + 5*a^2*b^3)*d*sin(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*
b^3)*d*sin(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(10*
d*x + 10*c) + 10*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(6*d*x + 6*c) -
 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x + 4*c) - 5*(a^3*b^2 - a^2
*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 4*(2*(64*a^5 - 144*a^4*b + 105
*a^3*b^2 - 25*a^2*b^3)*d*sin(4*d*x + 4*c) + 5*(8*a^4*b - 13*a^3*b^2 + 5*a^2
*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(6*a*b^2 - 5*b^3)
*cos(6*d*x + 6*c)^2 - 4*(64*a^2*b - 64*a*b^2 + 15*b^3)*cos(4*d*x + 4*c)^2 +
 4*(6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c)^2 + 4*(6*a*b^2 - 5*b^3)*sin(6*d*x + 6
*c)^2 - 4*(64*a^2*b - 64*a*b^2 + 15*b^3)*sin(4*d*x + 4*c)^2 + 2*(48*a^2*b -
 90*a*b^2 + 35*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(6*a*b^2 - 5*b^3)
*sin(2*d*x + 2*c)^2 - ((6*a*b^2 - 5*b^3)*cos(6*d*x + 6*c) - 2*(8*a*b^2 - 5*
b^3)*cos(4*d*x + 4*c) + (6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c
) - (6*a*b^2 - 5*b^3 - 2*(48*a^2*b - 90*a*b^2 + 35*b^3)*cos(4*d*x + 4*c) -
8*(6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(8*a*b^2 - 5*b^3
 + (48*a^2*b - 90*a*b^2 + 35*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (6*a
*b^2 - 5*b^3)*cos(2*d*x + 2*c) - ((6*a*b^2 - 5*b^3)*sin(6*d*x + 6*c) - 2*(8
*a*b^2 - 5*b^3)*sin(4*d*x + 4*c) + (6*a*b^2 - 5*b^3)*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + 2*((48*a^2*b - 90*a*b^2 + 35*b^3)*sin(4*d*x + 4*c) + 4*(6*a*
b^2 - 5*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^3*b^2 - a^2*b^3 + (a^3*
b^2 - a^2*b^3)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c)
^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*cos(4*d*x + 4*c)^2 + 1
6*(a^3*b^2 - a^2*b^3)*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*sin(8*d*x +
8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b
+ 57*a^3*b^2 - 9*a^2*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a^3*b^2 + 3
*a^2*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2*b^3)*sin(2*
d*x + 2*c)^2 + 2*(a^3*b^2 - a^2*b^3 - 4*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c
) - 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^
2*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - a^2*b^3 - 2*(8*a^4
*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*b^3)*cos(2
*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3 - 4*(8*
a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^3
*b^2 - a^2*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c
) + (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*
b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2
*b^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c)), x) - (4*a*b^2 - 5*b^3 + (6*a*b^2 - 5*b^3)*cos(8*d*x + 8*c) - 2*(13
*a*b^2 - 10*b^3)*cos(6*d*x + 6*c) - 2*(32*a^2*b - 47*a*b^2 + 15*b^3)*cos(4*
d*x + 4*c) - 2*(7*a*b^2 - 10*b^3)*cos(2*d*x + 2...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3648 vs. $2(186) = 372$.

time = 1.19, size = 3648, normalized size = 15.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(4*a*b - 5*b^2)*cos(d*x + c)^5 - 8*(7*a*b - 10*b^2)*cos(d*x + c)^3
 - ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)
^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b
^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625
*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*
b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^
5*b^2 - a^4*b^3)*d^2))*log(432*a^3*b^2 - 921*a^2*b^3 + 2625/4*a*b^4 - 625/4
*b^5 - 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c
)^2 + 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*s
qrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a
^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9
*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^
4*b^3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^7 - 3*a^6*b + 3*
a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3
450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^1
1*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 -
 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^
7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b
+ 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^
3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^
2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) +
 ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2
 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3
)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b
^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^
5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*
b^2 - a^4*b^3)*d^2))*log(432*a^3*b^2 - 921*a^2*b^3 + 2625/4*a*b^4 - 625/4*b
^5 - 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c)^
2 - 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqr
t((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^1
5 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b
^6)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*
b^3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^7 - 3*a^6*b + 3*a^
5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 345
0*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*
b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3
*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^7*
b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b +
183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^3*
b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2
- 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) - (
```

```
(a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 -
 (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d
^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)
/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 +
 a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2
 - a^4*b^3)*d^2))*log(-432*a^3*b^2 + 921*a^2*b^3 - 2625/4*a*b^4 + 625/4*b^5
 + 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c)^2
+ 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt(
(2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15
- 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6
)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^
3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b
^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a
*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4
 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^
6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^7*b^2
 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b + 183
*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^3*b^4
 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 2
0*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) + ((a^
3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a
^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*
sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((
a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^
9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 -
a^4*b^3)*d^2))*log(-432*a^3*b^2 + 921*a^2*b^3 -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

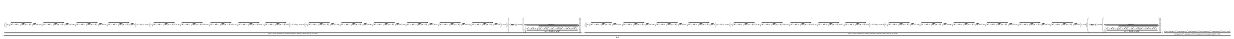
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(186) = 372$.
time = 0.87, size = 1545, normalized size = 6.55



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((21*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 57*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 23*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*b^4)*(a^3 - a^2*b)^2*abs(-a + b) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b
))*a^7*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6*b^2 + 14*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*a^5*b^3 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4
*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^5)*abs(-a^3 + a^2*b)*abs(-
a + b) - 2*(9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 - 42*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b + 66*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a^8*b^2 - 40*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqr
t(a*b)*a^7*b^3 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 +
2*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5)*abs(-a + b))*(pi*f
loor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4 - a^3*b + sqrt((a^
4 - a^3*b)^2 - (a^4 - a^3*b)*(a^4 - 2*a^3*b + a^2*b^2)))/(a^4 - 2*a^3*b + a
^2*b^2))))/((3*a^12 - 21*a^11*b + 59*a^10*b^2 - 85*a^9*b^3 + 65*a^8*b^4 - 2
3*a^7*b^5 + a^6*b^6 + a^5*b^7)*abs(-a^3 + a^2*b)) - ((21*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 57*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a^2*b^2 + 23*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3
+ 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^3 - a^2*b)^2*abs(
-a + b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b - 12*sqrt(a^2 - a*b
+ sqrt(a*b)*(a - b))*a^6*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b
^3 - 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^4 - sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*a^3*b^5)*abs(-a^3 + a^2*b)*abs(-a + b) - 2*(9*sqrt(a^2 - a*b
+ sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 - 42*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^9*b + 66*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^
2 - 40*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^3 + 5*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 + 2*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*a^5*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + ar
ctan(tan(d*x + c)/sqrt((a^4 - a^3*b - sqrt((a^4 - a^3*b)^2 - (a^4 - a^3*b)*
(a^4 - 2*a^3*b + a^2*b^2)))/(a^4 - 2*a^3*b + a^2*b^2))))/((3*a^12 - 21*a^11
*b + 59*a^10*b^2 - 85*a^9*b^3 + 65*a^8*b^4 - 23*a^7*b^5 + a^6*b^6 + a^5*b^7
)*abs(-a^3 + a^2*b)) + 2*(4*a^2*tan(d*x + c)^4 - 7*a*b*tan(d*x + c)^4 + 5*b
^2*tan(d*x + c)^4 + 8*a^2*tan(d*x + c)^2 - 7*a*b*tan(d*x + c)^2 + 4*a^2 - 4
*a*b)/((a*tan(d*x + c)^5 - b*tan(d*x + c)^5 + 2*a*tan(d*x + c)^3 + a*tan(d*
x + c))*(a^3 - a^2*b)))/d
```

Mupad [B]
time = 18.29, size = 2500, normalized size = 10.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - b*sin(c + d*x)^4)^2),x)
```

```
[Out] (atan((((-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*
a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*
```

b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 + tan(c + d*x)*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a^17*b^3 - 458752*a^18*b^2)) + tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 + 93952*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4 - 36608*a^13*b^3 + 9216*a^14*b^2))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*1i - ((-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 - tan(c + d*x)*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a^17*b^3 - 458752*a^18*b^2)) - tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 + 93952*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4 - 36608*a^13*b^3 + 9216*a^14*b^2))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*1i)/(((-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 + tan(c + d*x)*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a^17*b^3 - 458752*a^18*b^2)) + tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 + 93952*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4 - 36608*a^13*b^3 + 9216*a^14*b^2))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2) + ((-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 - tan(c + d*x)*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a^17*b^3 - 458752*a^18*b^2)) - tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 + 93952*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^

```
12*b^4 - 36608*a^13*b^3 + 9216*a^14*b^2))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^
2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(
1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2) - 4000*a^5*b^9
+ 27360*a^6*b^8 - 77504*a^7*b^7 + 116416*a^8*b^6 - 97824*a^9*b^5 + 43616*a^
10*b^4 - 8064*a^11*b^3))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2)
 - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^1
1*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*2i)/d - (1/a + (tan(c + d*x)^4*(
4*a^2 - 7*a*b + 5*b^2))/(4*a^2*(a - b)) + (tan(c + d*x)^2*(8*a - 7*b))/(4*a
*(a - b)))/(d*(a*tan(c + d*x) + 2*a*tan(c + d*x)^3 + tan(c + d*x)^5*(a - b)
)) + (atan(((((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) + 36*a^7*b +
 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 +
a^9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*
b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 + ta
n(c + d*x)*((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) + 36*a^7*b + 1
5*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^
9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^
7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a^17*b
^3 - 458752*a^18*b^2)) + tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 + 93952
*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4 - 36608*a^13*b
^3 + 9216*a^14*b^2))*((48*a^2*(a^9*b^3)^(1/2) +...
```

3.224 $\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^3} \, dx$

Optimal. Leaf size=315

$$-\frac{\left(5a-14\sqrt{a}\,\sqrt{b}+12b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d}-\frac{\left(5a+14\sqrt{a}\,\sqrt{b}+12b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\,\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{9/4}d}$$

```
[Out] -1/8*a*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b^2/d/(a-b+2*b*cos(d*x+c)^2-b*
cos(d*x+c)^4)^2+1/32*cos(d*x+c)*(9*a^2-11*a*b-10*b^2-2*(2*a-5*b)*b*cos(d*x+
c)^2)/(a-b)^2/b^2/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/64*arctan(b^(1/
4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5*a+12*b-14*a^(1/2)*b^(1/2))/b^(9/4
)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(5/2)-1/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2
)+b^(1/2))^(1/2))*(5*a+12*b+14*a^(1/2)*b^(1/2))/b^(9/4)/d/a^(1/2)/(a^(1/2)+
b^(1/2))^(5/2)
```

Rubi [A]
time = 0.39, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1219, 1692, 1180, 211, 214}

$$\frac{\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\left(-14\sqrt{a}\,\sqrt{b}+5a+12b\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(14\sqrt{a}\,\sqrt{b}+5a+12b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\,b^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] - Sqrt[b]]])/(Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) - ((5*a
 + 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) - (a*Cos[c + d
*x]*(a + b - b*Cos[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cos[c + d*x]^
2 - b*Cos[c + d*x]^4)^2) + (Cos[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(2*a
- 5*b)*b*Cos[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cos[c + d*x]^2 -
 b*Cos[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(a-b+2bx^2-bx^4)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2a\left(a^2+ab-\right.}{b}\right.}{}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(9a^2\right.}{32(a-b)^2b^2d\,(a}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(9a^2\right.}{32(a-b)^2b^2d\,(a}$$

$$= -\frac{\left(5a-14\sqrt{a}\sqrt{b}+12b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d} - \frac{\left(5a+14\sqrt{a}\sqrt{b}+1\right.}{64\sqrt{a}\,\left(\sqrt{\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.07, size = 785, normalized size = 2.49

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cos[c + d*x]*(-9*a^2 + 13*a*b + 5*b^2 + (2*a - 5*b)*b*Cos[2*(c + d*x)
]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) - (512*a*(a - b
)*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c +
 d*x)] + b*Cos[4*(c + d*x)])^2 + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#
1^4 - 4*b*#1^6 + b*#1^8 & , (-4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
] + 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + (2*I)*a*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2] - (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 20*a
^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 56*a*b*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 - 78*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^2 + (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (28*I)*a*b*L
og[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (39*I)*b^2*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2]*#1^2 + 20*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 56*
a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 78*b^2*ArcTan[Sin[c + d
```

```
*x]/(Cos[c + d*x] - #1)]*#1^4 - (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2
]*#1^4 + (28*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (39*I)*b^2*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*
x] - #1)]*#1^6 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - (2*
I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + (5*I)*b^2*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7)
& ])/(128*(a - b)^2*b^2*d)
```

Maple [A]
time = 1.75, size = 342, normalized size = 1.09

method	result
derivativedivides	$-\frac{-\frac{(2a-5b)\left(\cos^7(dx+c)\right)}{16\left(a^2-2ab+b^2\right)}+\frac{3\left(3a^2-ab-10b^2\right)\left(\cos^5(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{3\left(3a^2-2ab-5b^2\right)\left(\cos^3(dx+c)\right)}{16b\left(a^2-2ab+b^2\right)}-\frac{5\left(a^2-3ab-2b^2\right)\cos(dx+c)}{32b^2(a-b)}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}-\frac{\left(-4a \cdots\right.}{\cdots}$
default	$-\frac{-\frac{(2a-5b)\left(\cos^7(dx+c)\right)}{16\left(a^2-2ab+b^2\right)}+\frac{3\left(3a^2-ab-10b^2\right)\left(\cos^5(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{3\left(3a^2-2ab-5b^2\right)\left(\cos^3(dx+c)\right)}{16b\left(a^2-2ab+b^2\right)}-\frac{5\left(a^2-3ab-2b^2\right)\cos(dx+c)}{32b^2(a-b)}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}-\frac{\left(-4a \cdots\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(-1/16*(2*a-5*b)/(a^2-2*a*b+b^2)*cos(d*x+c)^7+3/32*(3*a^2-a*b-10*b^2)
/b/(a^2-2*a*b+b^2)*cos(d*x+c)^5-3/16*(3*a^2-2*a*b-5*b^2)/b/(a^2-2*a*b+b^2)*
cos(d*x+c)^3-5/32*(a^2-3*a*b-2*b^2)/b^2/(a-b)*cos(d*x+c))/(a-b+2*b*cos(d*x+
c)^2-b*cos(d*x+c)^4)^2-1/32/(a^2-2*a*b+b^2)/b*(1/2*(-4*a*(a*b)^(1/2)+10*(a*
b)^(1/2)*b+5*a^2-11*a*b+12*b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arcta
n(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))-1/2*(-4*a*(a*b)^(1/2)+10*(a*b)^(1
/2)*b-5*a^2+11*a*b-12*b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*
cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")

[Out] -1/8*(8*(2*a*b^4 - 5*b^5)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(18*a^2*b^3 - 20*a*b^4 - 25*b^5)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 8*(2*a*b^4 - 5*b^5)*sin(2*d*x + 2*c)*sin(d*x + c) - ((2*a*b^4 - 5*b^5)*cos(15*d*x + 15*c) - (18*a^2*b^3 - 20*a*b^4 - 25*b^5)*cos(13*d*x + 13*c) + 3*(18*a^2*b^3 - 8*a*b^4 - 15*b^5)*cos(11*d*x + 11*c) + (160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5)*cos(9*d*x + 9*c) + (160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5)*cos(7*d*x + 7*c) + 3*(18*a^2*b^3 - 8*a*b^4 - 15*b^5)*cos(5*d*x + 5*c) - (18*a^2*b^3 - 20*a*b^4 - 25*b^5)*cos(3*d*x + 3*c) + (2*a*b^4 - 5*b^5)*cos(d*x + c))*cos(16*d*x + 16*c) - (2*a*b^4 - 5*b^5 - 8*(2*a*b^4 - 5*b^5)*cos(14*d*x + 14*c) - 4*(16*a^2*b^3 - 54*a*b^4 + 35*b^5)*cos(12*d*x + 12*c) + 8*(32*a^2*b^3 - 94*a*b^4 + 35*b^5)*cos(10*d*x + 10*c) + 2*(256*a^3*b^2 - 832*a^2*b^3 + 550*a*b^4 - 175*b^5)*cos(8*d*x + 8*c) + 8*(32*a^2*b^3 - 94*a*b^4 + 35*b^5)*cos(6*d*x + 6*c) - 4*(16*a^2*b^3 - 54*a*b^4 + 35*b^5)*cos(4*d*x + 4*c) - 8*(2*a*b^4 - 5*b^5)*cos(2*d*x + 2*c))*cos(15*d*x + 15*c) - 8*((18*a^2*b^3 - 20*a*b^4 - 25*b^5)*cos(13*d*x + 13*c) - 3*(18*a^2*b^3 - 8*a*b^4 - 15*b^5)*cos(11*d*x + 11*c) - (160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5)*cos(9*d*x + 9*c) - (160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5)*cos(7*d*x + 7*c) - 3*(18*a^2*b^3 - 8*a*b^4 - 15*b^5)*cos(5*d*x + 5*c) + (18*a^2*b^3 - 20*a*b^4 - 25*b^5)*cos(3*d*x + 3*c) - (2*a*b^4 - 5*b^5)*cos(d*x + c))*cos(14*d*x + 14*c) + (18*a^2*b^3 - 20*a*b^4 - 25*b^5 - 4*(144*a^3*b^2 - 286*a^2*b^3 - 60*a*b^4 + 175*b^5)*cos(12*d*x + 12*c) + 8*(288*a^3*b^2 - 446*a^2*b^3 - 260*a*b^4 + 175*b^5)*cos(10*d*x + 10*c) + 2*(2304*a^4*b - 4288*a^3*b^2 - 650*a^2*b^3 + 1700*a*b^4 - 875*b^5)*cos(8*d*x + 8*c) + 8*(288*a^3*b^2 - 446*a^2*b^3 - 260*a*b^4 + 175*b^5)*cos(6*d*x + 6*c) - 4*(144*a^3*b^2 - 286*a^2*b^3 - 60*a*b^4 + 175*b^5)*cos(4*d*x + 4*c) - 8*(18*a^2*b^3 - 20*a*b^4 - 25*b^5)*cos(2*d*x + 2*c))*cos(13*d*x + 13*c) + 4*(3*(144*a^3*b^2 - 190*a^2*b^3 - 64*a*b^4 + 105*b^5)*cos(11*d*x + 11*c) + (1280*a^4*b - 4224*a^3*b^2 + 2732*a^2*b^3 + 186*a*b^4 - 175*b^5)*cos(9*d*x + 9*c) + (1280*a^4*b - 4224*a^3*b^2 + 2732*a^2*b^3 + 186*a*b^4 - 175*b^5)*cos(7*d*x + 7*c) + 3*(144*a^3*b^2 - 190*a^2*b^3 - 64*a*b^4 + 105*b^5)*cos(5*d*x + 5*c) - (144*a^3*b^2 - 286*a^2*b^3 - 60*a*b^4 + 175*b^5)*cos(3*d*x + 3*c) + (16*a^2*b^3 - 54*a*b^4 + 35*b^5)*cos(d*x + c))*cos(12*d*x + 12*c) - 3*(18*a^2*b^3 - 8*a*b^4 - 15*b^5 + 8*(288*a^3*b^2 - 254*a^2*b^3 - 184*a*b^4 + 105*b^5)*cos(10*d*x + 10*c) + 2*(2304*a^4*b - 2752*a^3*b^2 - 522*a^2*b^3 + 1160*a*b^4 - 525*b^5)*cos(8*d*x + 8*c) + 8*(288*a^3*b^2 - 254*a^2*b^3 - 184*a*b^4 + 105*b^5)*cos(6*d*x + 6*c) - 4*(144*a^3*b^2 - 190*a^2*b^3 - 64*a*b^4 + 105*b^5)*cos(4*d*x + 4*c) - 8*(18*a^2*b^3 - 8*a*b^4 - 15*b^5)*cos(2*d*x + 2*c))*cos(11*d*x + 11*c) - 8*((2560*a^4*b - 7328*a^3*b^2 + 2748*a^2*b^3 + 386*a*b^4 - 175*b^5)*cos(9*d*x + 9*c) + (2560*a^4*b - 7328*a^3*b^2 + 2748*a^2*b^3 + 386*a*b^4 - 175*b^5)*cos(7*d*x + 7*c) + 3*(288*a^3*b^2 - 254*a^2*b^3 - 184*a*b^4 + 105*b^5)*cos(5*d*x + 5*c) - (288*a^3*b^2 - 446*a^2*b^3 - 260*a*b^4 + 175*b^5)*cos(3*d*x + 3*

```
c) + (32*a^2*b^3 - 94*a*b^4 + 35*b^5)*cos(d*x + c))*cos(10*d*x + 10*c) - (1
60*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5 + 2*(20480*a^5 - 65024*a^4*b +
43104*a^3*b^2 - 10572*a^2*b^3 - 2330*a*b^4 + 875*b^5)*cos(8*d*x + 8*c) + 8*
(2560*a^4*b - 7328*a^3*b^2 + 2748*a^2*b^3 + 386*a*b^4 - 175*b^5)*cos(6*d*x
+ 6*c) - 4*(1280*a^4*b - 4224*a^3*b^2 + 2732*a^2*b^3 + 186*a*b^4 - 175*b^5)
*cos(4*d*x + 4*c) - 8*(160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5)*cos(2*
d*x + 2*c))*cos(9*d*x + 9*c) - 2*((20480*a^5 - 65024*a^4*b + 43104*a^3*b^2
- 10572*a^2*b^3 - 2330*a*b^4 + 875*b^5)*cos(7*d*x + 7*c) + 3*(2304*a^4*b -
2752*a^3*b^2 - 522*a^2*b^3 + 1160*a*b^4 - 525*b^5)*cos(5*d*x + 5*c) - (2304
*a^4*b - 4288*a^3*b^2 - 650*a^2*b^3 + 1700*a*b^4 - 875*b^5)*cos(3*d*x + 3*c
) + (256*a^3*b^2 - 832*a^2*b^3 + 550*a*b^4 - 175*b^5)*cos(d*x + c))*cos(8*d
*x + 8*c) - (160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5 + 8*(2560*a^4*b -
 7328*a^3*b^2 + 2748*a^2*b^3 + 386*a*b^4 - 175*b^5)*cos(6*d*x + 6*c) - 4*(1
280*a^4*b - 4224*a^3*b^2 + 2732*a^2*b^3 + 186*a*b^4 - 175*b^5)*cos(4*d*x +
4*c) - 8*(160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5)*cos(2*d*x + 2*c))*c
os(7*d*x + 7*c) - 8*(3*(288*a^3*b^2 - 254*a^2*b^3 - 184*a*b^4 + 105*b^5)*co
s(5*d*x + 5*c) - (288*a^3*b^2 - 446*a^2*b^3 - 260*a*b^4 + 175*b^5)*cos(3*d*
x + 3*c) + (32*a^2*b^3 - 94*a*b^4 + 35*b^5)*cos(d*x + c))*cos(6*d*x + 6*c)
- 3*(18*a^2*b^3 - 8*a*b^4 - 15*b^5 - 4*(144*a^3*b^2 - 190*a^2*b^3 - 64*a*b^
4 + 105*b^5)*cos(4*d*x + 4*c) - 8*(18*a^2*b^3 - 8*a*b^4 - 15*b^5)*cos(2*d*x
 + 2*c))*cos(5*d*x + 5*c) - 4*((144*a^3*b^2 - 286*a^2*b^3 - 60*a*b^4 + 175*
b^5)*cos(3*d*x + 3*c) - (16*a^2*b^3 - 54*a*b^4 + 35*b^5)*cos(d*x + c))*cos(
4*d*x + 4*c) + (18*a^2*b^3 - 20*a*b^4 - 25*b^5 - 8*(18*a^2*b^3 - 20*a*b^4 -
 25*b^5)*cos(2*d*x + 2*c))*cos(3*d*x + 3*c) - (2*a*b^4 - 5*b^5)*cos(d*x + c
) + 8*((a^2*b^6 - 2*a*b^7 + b^8)*d*cos(16*d*x + 16*c)^2 + 64*(a^2*b^6 - 2*a
*b^7 + b^8)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^4...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4640 vs. $2(264) = 528$.
time = 1.14, size = 4640, normalized size = 14.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(8*(2*a*b^2 - 5*b^3)*cos(d*x + c)^7 - 12*(3*a^2*b - a*b^2 - 10*b^3)*c
os(d*x + c)^5 + 24*(3*a^2*b - 2*a*b^2 - 5*b^3)*cos(d*x + c)^3 + ((a^2*b^4 -
 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x +
c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(a^3*
b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b^3 +
6*a^2*b^4 - 4*a*b^5 + b^6)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*
b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8
- a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 +
269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^
```

8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 -
252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*
b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 -
a*b^9)*d^2))*log((625*a^6 - 5250*a^5*b + 22509*a^4*b^2 - 57820*a^3*b^3 + 96
336*a^2*b^4 - 98304*a*b^5 + 55296*b^6)*cos(d*x + c) - ((a^8*b^7 - 6*a^7*b^8
 + 27*a^6*b^9 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 + 61*a^2*b^13 - 1
2*a*b^14)*d^3*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 +
 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b
^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 -
 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a
*b^19)*d^4)) + (125*a^7*b^2 - 1045*a^6*b^3 + 4305*a^5*b^4 - 10583*a^4*b^5 +
 16798*a^3*b^6 - 16320*a^2*b^7 + 8448*a*b^8)*d)*sqrt((15*a^4 - 94*a^3*b + 1
55*a^2*b^2 - 76*a*b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^
3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 -
 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640
*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12
 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17
- 10*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*
b^7 + 5*a^2*b^8 - a*b^9)*d^2))) - ((a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)
^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^6 - 2*(a^3*b^3 - 5*a^2*b^4
+ 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^
6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d)*
sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^4 - (a^6*b^4 - 5*a
^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6
700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^
5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 +
 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 -
120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5
*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))*log((625*a^6 - 52
50*a^5*b + 22509*a^4*b^2 - 57820*a^3*b^3 + 96336*a^2*b^4 - 98304*a*b^5 + 55
296*b^6)*cos(d*x + c) - ((a^8*b^7 - 6*a^7*b^8 + 27*a^6*b^9 - 80*a^5*b^10 +
135*a^4*b^11 - 126*a^3*b^12 + 61*a^2*b^13 - 12*a*b^14)*d^3*sqrt((625*a^8 -
6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b
^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10
+ 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 -
 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)) - (125*a^7*b^2 -
1045*a^6*b^3 + 4305*a^5*b^4 - 10583*a^4*b^5 + 16798*a^3*b^6 - 16320*a^2*b^7
 + 8448*a*b^8)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^
4 - (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2
*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b
^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^
9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14
 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))
/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))
) - ((a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^

```
6)*d*cos(d*x + c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x +
 c)^4 + 4*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2
 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d)*sqrt((15*a^4 - 94*a^3*b + 155*
a^2*b^2 - 76*a*b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b
^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 11
7532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*
b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 +
210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 1
0*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7
 + 5*a^2*b^8 - a*b^9)*d^2))*log(-(625*a^6 - 5250*a^5*b + 22509*a^4*b^2 - 57
820*a^3*b^3 + 96336*a^2*b^4 - 98304*a*b^5 + 55296*b^6)*cos(d*x + c) - ((a^8
*b^7 - 6*a^7*b^8 + 27*a^6*b^9 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 +
 61*a^2*b^13 - 12*a*b^14)*d^3*sqrt((625*a^8 - 6...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[-82,8]
Warning, need to
```

Mupad [B]
time = 19.29, size = 2500, normalized size = 7.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] ((cos(c + d*x)^7*(2*a - 5*b))/(16*(a^2 - 2*a*b + b^2)) + (3*cos(c + d*x)^5*
(a*b - 3*a^2 + 10*b^2))/(32*b*(a^2 - 2*a*b + b^2)) - (5*cos(c + d*x)*(3*a*b
```

- a^2 + 2*b^2))/(32*b^2*(a - b)) - (3*cos(c + d*x)^3*(2*a*b - 3*a^2 + 5*b^2))/(16*b*(a - b)^2))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b^2) - cos(c + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d*x)^8)) + (atan(((((180224*a*b^8 - 483328*a^2*b^7 + 466944*a^3*b^6 - 204800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) - (cos(c + d*x)*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b^7 - 65536*a^4*b^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2) + (cos(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161*a^2*b^2))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*1i - (((180224*a*b^8 - 483328*a^2*b^7 + 466944*a^3*b^6 - 204800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) + (cos(c + d*x)*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b^7 - 65536*a^4*b^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2) - (cos(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161*a^2*b^2))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*1i)/((((180224*a*b^8 - 483328*a^2*b^7 + 466944*a^3*b^6 - 204800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) - (cos(c + d*x)*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b^7 - 65536*a^4*b^6 + 16384* a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2) + (cos(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161*a^2*b^2))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2) + (((180224*a*b^8 - 483328*a^2*b^7 + 466944*a^3*b^6 - 204800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) + (cos(c + d*x)*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b^7 - 65536*a^4*b^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6...

3.225 $\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=290

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{7/4}d}-\frac{a\cos(c}{8(a-b)bd\,(a-b}$$

```
[Out] -1/8*a*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*
x+c)^4)^2+1/32*cos(d*x+c)*(5*a-17*b-3*(a-3*b)*cos(d*x+c)^2)/(a-b)^2/b/d/(a-
b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)+3/64*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-
b^(1/2))^(1/2))*(a^(1/2)-2*b^(1/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(5/
2)-3/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+2*b^(1
/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A]

time = 0.30, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1219, 1192, 1180, 211, 214}

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cos(c+dx)\left(-3(a-3b)\cos^2(c+dx)+5a-17b\right)}{32bd(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[a]
*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cos[c + d*x]*(2 - Cos[c + d*x]^2
))/(8*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) + (Cos
[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cos[c + d*x]^2))/(32*(a - b)^2*b*d*(a -
 b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a(a-4b)-2}{(a-b+2b}\right.}{1}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\,(5a}{32(a-b)^2bd\,(a-}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\,(5a}{32(a-b)^2bd\,(a-}$$

$$= \frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt{\sqrt{}}}\right.}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.77, size = 630, normalized size = 2.17

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cos[c + d*x]*(-7*a + 25*b + 3*(a - 3*b)*Cos[2*(c + d*x)]))/(8*a - 3*b
 + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + (512*a*(a - b)*(-5*Cos[c +
d*x] + Cos[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c +
 d*x)])^2 - (3*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 +
b*#1^8 & , (2*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 6*b*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)] - I*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (3*I
)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*
x] - #1)]*#1^2 + 34*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (3*I)
*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (17*I)*b*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^2 + 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 34*b
*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (3*I)*a*Log[1 - 2*Cos[c +
d*x]*#1 + #1^2]*#1^4 + (17*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 2*
a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + 6*b*ArcTan[Sin[c + d*x]/(
Cos[c + d*x] - #1)]*#1^6 + I*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 - (3*
```

```
I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3
 - 3*b*#1^5 + b*#1^7) & ])/(256*(a - b)^2*b*d)
```

Maple [A]
time = 1.50, size = 303, normalized size = 1.04 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((3/32*(a-3*b)/(a^2-2*a*b+b^2)*cos(d*x+c)^7-1/32*(11*a-35*b)/(a^2-2*a*b
+b^2)*cos(d*x+c)^5+1/32*(a^2+18*a*b-43*b^2)/b/(a^2-2*a*b+b^2)*cos(d*x+c)^3-
1/32*(3*a+17*b)/(a-b)/b*cos(d*x+c))/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)^2
+3/32/(a^2-2*a*b+b^2)*(1/2*(a*(a*b)^(1/2)-3*(a*b)^(1/2)*b-2*b^2)/(a*b)^(1/2
)/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2)
)-1/2*(a*(a*b)^(1/2)-3*(a*b)^(1/2)*b+2*b^2)/(a*b)^(1/2)/b/(((a*b)^(1/2)+b)*
b)^(1/2)*arctanh(b*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*(a*b^3 - 3*b^4)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(23*a*b^3 - 77*
b^4)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 24*(a*b^3 - 3*b^4)*sin(2*d*x + 2*c
)*sin(d*x + c) - (3*(a*b^3 - 3*b^4)*cos(15*d*x + 15*c) - (23*a*b^3 - 77*b^4
)*cos(13*d*x + 13*c) + (16*a^2*b^2 + 131*a*b^3 - 177*b^4)*cos(11*d*x + 11*c
) - (144*a^2*b^2 + 367*a*b^3 - 109*b^4)*cos(9*d*x + 9*c) - (144*a^2*b^2 + 3
67*a*b^3 - 109*b^4)*cos(7*d*x + 7*c) + (16*a^2*b^2 + 131*a*b^3 - 177*b^4)*c
os(5*d*x + 5*c) - (23*a*b^3 - 77*b^4)*cos(3*d*x + 3*c) + 3*(a*b^3 - 3*b^4)*
cos(d*x + c))*cos(16*d*x + 16*c) - 3*(a*b^3 - 3*b^4 - 8*(a*b^3 - 3*b^4)*cos
(14*d*x + 14*c) - 4*(8*a^2*b^2 - 31*a*b^3 + 21*b^4)*cos(12*d*x + 12*c) + 8*
(16*a^2*b^2 - 55*a*b^3 + 21*b^4)*cos(10*d*x + 10*c) + 2*(128*a^3*b - 480*a^
2*b^2 + 323*a*b^3 - 105*b^4)*cos(8*d*x + 8*c) + 8*(16*a^2*b^2 - 55*a*b^3 +
21*b^4)*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 - 31*a*b^3 + 21*b^4)*cos(4*d*x + 4*
c) - 8*(a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*cos(15*d*x + 15*c) - 8*((23*a*b^3
- 77*b^4)*cos(13*d*x + 13*c) - (16*a^2*b^2 + 131*a*b^3 - 177*b^4)*cos(11*d*
x + 11*c) + (144*a^2*b^2 + 367*a*b^3 - 109*b^4)*cos(9*d*x + 9*c) + (144*a^2
*b^2 + 367*a*b^3 - 109*b^4)*cos(7*d*x + 7*c) - (16*a^2*b^2 + 131*a*b^3 - 17
7*b^4)*cos(5*d*x + 5*c) + (23*a*b^3 - 77*b^4)*cos(3*d*x + 3*c) - 3*(a*b^3 -
 3*b^4)*cos(d*x + c))*cos(14*d*x + 14*c) + (23*a*b^3 - 77*b^4 - 4*(184*a^2*
b^2 - 777*a*b^3 + 539*b^4)*cos(12*d*x + 12*c) + 8*(368*a^2*b^2 - 1393*a*b^3
 + 539*b^4)*cos(10*d*x + 10*c) + 2*(2944*a^3*b - 12064*a^2*b^2 + 8197*a*b^3
 - 2695*b^4)*cos(8*d*x + 8*c) + 8*(368*a^2*b^2 - 1393*a*b^3 + 539*b^4)*cos(
```

6*d*x + 6*c) - 4*(184*a^2*b^2 - 777*a*b^3 + 539*b^4)*cos(4*d*x + 4*c) - 8*(
23*a*b^3 - 77*b^4)*cos(2*d*x + 2*c))*cos(13*d*x + 13*c) + 4*((128*a^3*b + 9
36*a^2*b^2 - 2333*a*b^3 + 1239*b^4)*cos(11*d*x + 11*c) - (1152*a^3*b + 1928
*a^2*b^2 - 3441*a*b^3 + 763*b^4)*cos(9*d*x + 9*c) - (1152*a^3*b + 1928*a^2*
b^2 - 3441*a*b^3 + 763*b^4)*cos(7*d*x + 7*c) + (128*a^3*b + 936*a^2*b^2 - 2
333*a*b^3 + 1239*b^4)*cos(5*d*x + 5*c) - (184*a^2*b^2 - 777*a*b^3 + 539*b^4
)*cos(3*d*x + 3*c) + 3*(8*a^2*b^2 - 31*a*b^3 + 21*b^4)*cos(d*x + c))*cos(12
*d*x + 12*c) - (16*a^2*b^2 + 131*a*b^3 - 177*b^4 + 8*(256*a^3*b + 1984*a^2*
b^2 - 3749*a*b^3 + 1239*b^4)*cos(10*d*x + 10*c) + 2*(2048*a^4 + 15232*a^3*b
 - 34672*a^2*b^2 + 21577*a*b^3 - 6195*b^4)*cos(8*d*x + 8*c) + 8*(256*a^3*b
 + 1984*a^2*b^2 - 3749*a*b^3 + 1239*b^4)*cos(6*d*x + 6*c) - 4*(128*a^3*b + 9
36*a^2*b^2 - 2333*a*b^3 + 1239*b^4)*cos(4*d*x + 4*c) - 8*(16*a^2*b^2 + 131*
a*b^3 - 177*b^4)*cos(2*d*x + 2*c))*cos(11*d*x + 11*c) + 8*((2304*a^3*b + 48
64*a^2*b^2 - 4313*a*b^3 + 763*b^4)*cos(9*d*x + 9*c) + (2304*a^3*b + 4864*a^
2*b^2 - 4313*a*b^3 + 763*b^4)*cos(7*d*x + 7*c) - (256*a^3*b + 1984*a^2*b^2
 - 3749*a*b^3 + 1239*b^4)*cos(5*d*x + 5*c) + (368*a^2*b^2 - 1393*a*b^3 + 539
*b^4)*cos(3*d*x + 3*c) - 3*(16*a^2*b^2 - 55*a*b^3 + 21*b^4)*cos(d*x + c))*c
os(10*d*x + 10*c) + (144*a^2*b^2 + 367*a*b^3 - 109*b^4 + 2*(18432*a^4 + 331
52*a^3*b - 44144*a^2*b^2 + 23309*a*b^3 - 3815*b^4)*cos(8*d*x + 8*c) + 8*(23
04*a^3*b + 4864*a^2*b^2 - 4313*a*b^3 + 763*b^4)*cos(6*d*x + 6*c) - 4*(1152*
a^3*b + 1928*a^2*b^2 - 3441*a*b^3 + 763*b^4)*cos(4*d*x + 4*c) - 8*(144*a^2*
b^2 + 367*a*b^3 - 109*b^4)*cos(2*d*x + 2*c))*cos(9*d*x + 9*c) + 2*((18432*a
^4 + 33152*a^3*b - 44144*a^2*b^2 + 23309*a*b^3 - 3815*b^4)*cos(7*d*x + 7*c)
 - (2048*a^4 + 15232*a^3*b - 34672*a^2*b^2 + 21577*a*b^3 - 6195*b^4)*cos(5*
d*x + 5*c) + (2944*a^3*b - 12064*a^2*b^2 + 8197*a*b^3 - 2695*b^4)*cos(3*d*x
 + 3*c) - 3*(128*a^3*b - 480*a^2*b^2 + 323*a*b^3 - 105*b^4)*cos(d*x + c))*c
os(8*d*x + 8*c) + (144*a^2*b^2 + 367*a*b^3 - 109*b^4 + 8*(2304*a^3*b + 4864
*a^2*b^2 - 4313*a*b^3 + 763*b^4)*cos(6*d*x + 6*c) - 4*(1152*a^3*b + 1928*a^
2*b^2 - 3441*a*b^3 + 763*b^4)*cos(4*d*x + 4*c) - 8*(144*a^2*b^2 + 367*a*b^3
 - 109*b^4)*cos(2*d*x + 2*c))*cos(7*d*x + 7*c) - 8*((256*a^3*b + 1984*a^2*b
^2 - 3749*a*b^3 + 1239*b^4)*cos(5*d*x + 5*c) - (368*a^2*b^2 - 1393*a*b^3 +
539*b^4)*cos(3*d*x + 3*c) + 3*(16*a^2*b^2 - 55*a*b^3 + 21*b^4)*cos(d*x + c)
)*cos(6*d*x + 6*c) - (16*a^2*b^2 + 131*a*b^3 - 177*b^4 - 4*(128*a^3*b + 936
*a^2*b^2 - 2333*a*b^3 + 1239*b^4)*cos(4*d*x + 4*c) - 8*(16*a^2*b^2 + 131*a*
b^3 - 177*b^4)*cos(2*d*x + 2*c))*cos(5*d*x + 5*c) - 4*((184*a^2*b^2 - 777*a
*b^3 + 539*b^4)*cos(3*d*x + 3*c) - 3*(8*a^2*b^2 - 31*a*b^3 + 21*b^4)*cos(d*
x + c))*cos(4*d*x + 4*c) + (23*a*b^3 - 77*b^4 - 8*(23*a*b^3 - 77*b^4)*cos(2
*d*x + 2*c))*cos(3*d*x + 3*c) - 3*(a*b^3 - 3*b^4)*cos(d*x + c) + 16*((a^2*b
^5 - 2*a*b^6 + b^7)*d*cos(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d
*cos(14*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*
b^6 + 49*b^7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*
a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 573
44*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1
225*b^7)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5
 - 322*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4

```
+ 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4185 vs. 2(234) = 468.
time = 0.88, size = 4185, normalized size = 14.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
[Out] 1/128*(12*(a*b - 3*b^2)*cos(d*x + c)^7 - 4*(11*a*b - 35*b^2)*cos(d*x + c)^5
 + 4*(a^2 + 18*a*b - 43*b^2)*cos(d*x + c)^3 + 3*((a^2*b^3 - 2*a*b^4 + b^5)*
d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^
2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3
+ 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^
4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b
^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^
4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^
10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^1
5 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 + 4*b^3)/((a^6*
b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(27
*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*b^3 - 64*b^4)*cos(d*x + c) + 27*((a^8*b
^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*a^2*
b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a
^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a
^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^
3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b^4 -
 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3
*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^
3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^
9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^1
4 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 +
 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8
)*d^2))) - 3*((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a
*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*
cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 +
 (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(((a^6*b^3 - 5*a^5*
b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b
 + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7
- 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 2
10*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - a^
3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^
3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(27*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*
b^3 - 64*b^4)*cos(d*x + c) + 27*((a^8*b^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5
```

```
*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*a^2*b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*
a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11
*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^1
2 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4))
 + (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b^4 - 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(((a^6
*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a
^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6
)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252
*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^1
7)*d^4)) - a^3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^
4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))) - 3*((a^2*b^3 - 2*a*b^4 + b^
5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3
*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b
^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a
*b^4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^
2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2
*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8
*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*
b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 + 4*b^3)/((a
^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log
(-27*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*b^3 - 64*b^4)*cos(d*x + c) + 27*((a
^8*b^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*
a^2*b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 1
67*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 1
20*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 4
5*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b
^4 - 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10
*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^
3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^
9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4
*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b
^2 + 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a
*b^8)*d^2))) + 3*((a^2*b^3 - 2*a*b^4 + b^5)*d*c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[61,-66
]Warning, need t
```

Mupad [B]
time = 19.62, size = 2500, normalized size = 8.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] ((3*cos(c + d*x)^7*(a - 3*b))/(32*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)^5*(1
1*a - 35*b))/(32*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)^3*(18*a*b + a^2 - 43*
b^2))/(32*b*(a - b)^2) - (cos(c + d*x)*(3*a + 17*b))/(32*b*(a - b)))/(d*(a^
2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b^2) - cos(c + d*x)^4*(2*a*b -
6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d*x)^8)) + (atan(((((3*(81920*a
*b^7 - 180224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6 - 4*a*
b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2)) - (cos(c + d*x)*((9*(a^3*(a^3*b^7)^
(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^
4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 -
5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*(16384
*a*b^8 - 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4*b^5 + 16384*a^5*b^4))/(2
56*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) +
 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a
*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b
^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) + (cos(c + d*
x)*(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^3*b - 4*a*b^3 +
b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*
b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b
*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9
 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*1i - (((3*(81920*a*b^7 - 180224*a^2*b^6 + 1
14688*a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b
^3 + a^4*b^2)) + (cos(c + d*x)*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^
(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/
2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10
- 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*(16384*a*b^8 - 65536*a^2*b^7 +
```

98304*a^3*b^6 - 65536*a^4*b^5 + 16384*a^5*b^4))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) - (cos(c + d*x)*(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*1i)/((((3*(81920*a*b^7 - 180224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2)) - (cos(c + d*x)*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*(16384*a*b^8 - 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4*b^5 + 16384*a^5*b^4))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) + (cos(c + d*x)*(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) + (((3*(81920*a*b^7 - 180224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2)) + (cos(c + d*x)*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*(16384*a*b^8 - 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4*b^5 + 16384*a^5*b^4))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) - (cos(c + d*x)*(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) + (3*(9*a^2 - 63*a*b + 108*b^2))/(16384*(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2))))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/...

3.226 $\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

Optimal. Leaf size=313

$$\frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d}+\frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{5/4}d}-\frac{}{8(a}$$

```
[Out] -1/8*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(
d*x+c)^4)^2+1/32*cos(d*x+c)*(a^2-11*a*b-2*b^2+2*b*(2*a+b)*cos(d*x+c)^2)/a/(
a-b)^2/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)+1/64*arctan(b^(1/4)*cos(d*
x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a+4*b-10*a^(1/2)*b^(1/2))/a^(3/2)/b^(5/4)/
d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))
^(1/2))*(3*a+4*b+10*a^(1/2)*b^(1/2))/a^(3/2)/b^(5/4)/d/(a^(1/2)+b^(1/2))^(5
/2)
```

Rubi [A]
time = 0.32, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3294, 1219, 1192, 1180, 211, 214}

$$\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\sqrt{b}+3a+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cos(c+dx)\left(a^2+2b(2a+b)\cos^2(c+dx)-11ab-2b^2\right)}{32abd(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a
] - Sqrt[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) + ((3*a + 1
0*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) - (Cos[c + d*x]*(a
+ b - b*Cos[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos
[c + d*x]^4)^2) + (Cos[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a + b)*Cos[c
 + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x
]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a-b+2bx^2-bx^4)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2a(a-7b)+10 \ldots}{(a-b+2bx^2-b \ldots}\right.}{16a(\ldots}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(a^2-11 \ldots\right.}{32a(a-b)^2bd\,(a-b \ldots}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(a^2-11 \ldots\right.}{32a(a-b)^2bd\,(a-b \ldots}$$

$$= \frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d} + \frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right)\text{ta}\ldots}{64a^{3/2}\left(\sqrt{a}+\ldots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.93, size = 786, normalized size = 2.51

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cos[c + d*x]*(a^2 - 9*a*b - b^2 + b*(2*a + b)*Cos[2*(c + d*x)]))/(a*(8
*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) - (512*(a - b)*Cos[c
 + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)]
+ b*Cos[4*(c + d*x)])^2 + (I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
4*b*#1^6 + b*#1^8 & , (4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b
^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (2*I)*a*b*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2] - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 12*a^2*ArcTan[Si
n[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 64*a*b*ArcTan[Sin[c + d*x]/(Cos[c +
d*x] - #1)]*#1^2 + 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (
6*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (32*I)*a*b*Log[1 - 2*Cos[
c + d*x]*#1 + #1^2]*#1^2 - (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2
 - 12*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 64*a*b*ArcTan[Sin
[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)]*#1^4 + (6*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (32*I)
```

```
*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (5*I)*b^2*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2]*#1^4 - 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 -
 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (2*I)*a*b*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^6 + I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6
)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/a)/(128*(a - b)^
2*b*d)
```

Maple [A]
time = 1.79, size = 347, normalized size = 1.11

method	result
derivativedivides	$-\frac{\frac{b(2a+b)\left(\cos^7(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(a^2-19ab-6b^2\right)\left(\cos^5(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{\left(5a^2-14ab-3b^2\right)\left(\cos^3(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(3a^2+15ab+2b^2\right)\cos(dx+c)}{32(a-b)ab}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}-\frac{\left(4a\cdots\right.}{\cdots}$
default	$-\frac{\frac{b(2a+b)\left(\cos^7(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(a^2-19ab-6b^2\right)\left(\cos^5(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{\left(5a^2-14ab-3b^2\right)\left(\cos^3(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(3a^2+15ab+2b^2\right)\cos(dx+c)}{32(a-b)ab}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}-\frac{\left(4a\cdots\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(1/16*b*(2*a+b)/a/(a^2-2*a*b+b^2)*cos(d*x+c)^7+1/32*(a^2-19*a*b-6*b^2
)/a/(a^2-2*a*b+b^2)*cos(d*x+c)^5-1/16*(5*a^2-14*a*b-3*b^2)/a/(a^2-2*a*b+b^2
)*cos(d*x+c)^3+1/32*(3*a^2+15*a*b+2*b^2)/(a-b)/a/b*cos(d*x+c))/(a-b+2*b*cos
(d*x+c)^2-b*cos(d*x+c)^4)^2-1/32/a/(a^2-2*a*b+b^2)*(-1/2*(4*a*(a*b)^(1/2)+2
*(a*b)^(1/2)*b+3*a^2-13*a*b+4*b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*ar
ctanh(b*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2))+1/2*(4*a*(a*b)^(1/2)+2*(a*b)^
(1/2)*b-3*a^2+13*a*b-4*b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b*
cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")

[Out] 1/8*(8*(2*a*b^4 + b^5)*cos(2*d*x + 2*c)*cos(d*x + c) + 8*(2*a^2*b^3 - 24*a*b^4 - 5*b^5)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 8*(2*a*b^4 + b^5)*sin(2*d*x + 2*c)*sin(d*x + c) - ((2*a*b^4 + b^5)*cos(15*d*x + 15*c) + (2*a^2*b^3 - 24*a*b^4 - 5*b^5)*cos(13*d*x + 13*c) - (70*a^2*b^3 - 76*a*b^4 - 9*b^5)*cos(11*d*x + 11*c) + (96*a^3*b^2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5)*cos(9*d*x + 9*c) + (96*a^3*b^2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5)*cos(7*d*x + 7*c) - (70*a^2*b^3 - 76*a*b^4 - 9*b^5)*cos(5*d*x + 5*c) + (2*a^2*b^3 - 24*a*b^4 - 5*b^5)*cos(3*d*x + 3*c) + (2*a*b^4 + b^5)*cos(d*x + c))*cos(16*d*x + 16*c) - (2*a*b^4 + b^5 - 8*(2*a*b^4 + b^5)*cos(14*d*x + 14*c) - 4*(16*a^2*b^3 - 6*a*b^4 - 7*b^5)*cos(12*d*x + 12*c) + 8*(32*a^2*b^3 + 2*a*b^4 - 7*b^5)*cos(10*d*x + 10*c) + 2*(256*a^3*b^2 - 64*a^2*b^3 - 26*a*b^4 + 35*b^5)*cos(8*d*x + 8*c) + 8*(32*a^2*b^3 + 2*a*b^4 - 7*b^5)*cos(6*d*x + 6*c) - 4*(16*a^2*b^3 - 6*a*b^4 - 7*b^5)*cos(4*d*x + 4*c) - 8*(2*a*b^4 + b^5)*cos(2*d*x + 2*c))*cos(15*d*x + 15*c) + 8*((2*a^2*b^3 - 24*a*b^4 - 5*b^5)*cos(13*d*x + 13*c) - (70*a^2*b^3 - 76*a*b^4 - 9*b^5)*cos(11*d*x + 11*c) + (96*a^3*b^2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5)*cos(9*d*x + 9*c) + (96*a^3*b^2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5)*cos(7*d*x + 7*c) - (70*a^2*b^3 - 76*a*b^4 - 9*b^5)*cos(5*d*x + 5*c) + (2*a^2*b^3 - 24*a*b^4 - 5*b^5)*cos(3*d*x + 3*c) + (2*a*b^4 + b^5)*cos(d*x + c))*cos(14*d*x + 14*c) - (2*a^2*b^3 - 24*a*b^4 - 5*b^5 - 4*(16*a^3*b^2 - 206*a^2*b^3 + 128*a*b^4 + 35*b^5)*cos(12*d*x + 12*c) + 8*(32*a^3*b^2 - 398*a^2*b^3 + 88*a*b^4 + 35*b^5)*cos(10*d*x + 10*c) + 2*(256*a^4*b - 3264*a^3*b^2 + 1734*a^2*b^3 - 360*a*b^4 - 175*b^5)*cos(8*d*x + 8*c) + 8*(32*a^3*b^2 - 398*a^2*b^3 + 88*a*b^4 + 35*b^5)*cos(6*d*x + 6*c) - 4*(16*a^3*b^2 - 206*a^2*b^3 + 128*a*b^4 + 35*b^5)*cos(4*d*x + 4*c) - 8*(2*a^2*b^3 - 24*a*b^4 - 5*b^5)*cos(2*d*x + 2*c))*cos(13*d*x + 13*c) - 4*((560*a^3*b^2 - 1098*a^2*b^3 + 460*a*b^4 + 63*b^5)*cos(11*d*x + 11*c) - (768*a^4*b + 640*a^3*b^2 - 1580*a^2*b^3 + 338*a*b^4 + 35*b^5)*cos(9*d*x + 9*c) - (768*a^4*b + 640*a^3*b^2 - 1580*a^2*b^3 + 338*a*b^4 + 35*b^5)*cos(7*d*x + 7*c) + (560*a^3*b^2 - 1098*a^2*b^3 + 460*a*b^4 + 63*b^5)*cos(5*d*x + 5*c) - (16*a^3*b^2 - 206*a^2*b^3 + 128*a*b^4 + 35*b^5)*cos(3*d*x + 3*c) - (16*a^2*b^3 - 6*a*b^4 - 7*b^5)*cos(d*x + c))*cos(12*d*x + 12*c) + (70*a^2*b^3 - 76*a*b^4 - 9*b^5 + 8*(1120*a^3*b^2 - 1706*a^2*b^3 + 388*a*b^4 + 63*b^5)*cos(10*d*x + 10*c) + 2*(8960*a^4*b - 16448*a^3*b^2 + 8594*a^2*b^3 - 1796*a*b^4 - 315*b^5)*cos(8*d*x + 8*c) + 8*(1120*a^3*b^2 - 1706*a^2*b^3 + 388*a*b^4 + 63*b^5)*cos(6*d*x + 6*c) - 4*(560*a^3*b^2 - 1098*a^2*b^3 + 460*a*b^4 + 63*b^5)*cos(4*d*x + 4*c) - 8*(70*a^2*b^3 - 76*a*b^4 - 9*b^5)*cos(2*d*x + 2*c))*cos(11*d*x + 11*c) - 8*((1536*a^4*b + 1952*a^3*b^2 - 2012*a^2*b^3 + 298*a*b^4 + 35*b^5)*cos(9*d*x + 9*c) + (1536*a^4*b + 1952*a^3*b^2 - 2012*a^2*b^3 + 298*a*b^4 + 35*b^5)*cos(7*d*x + 7*c) - (1120*a^3*b^2 - 1706*a^2*b^3 + 388*a*b^4 + 63*b^5)*cos(5*d*x + 5*c) + (32*a^3*b^2 - 398*a^2*b^3 + 88*a*b^4 + 35*b^5)*cos(3*d*x + 3*c) + (32*a^2*b^3 + 2*a*b^4 - 7*b^5)*cos(d*x + c))*cos(10*d*x + 10*c) - (96*a^

```
3*b^2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5 + 2*(12288*a^5 + 11776*a^4*b - 19296
*a^3*b^2 + 10284*a^2*b^3 - 1410*a*b^4 - 175*b^5)*cos(8*d*x + 8*c) + 8*(1536
*a^4*b + 1952*a^3*b^2 - 2012*a^2*b^3 + 298*a*b^4 + 35*b^5)*cos(6*d*x + 6*c)
 - 4*(768*a^4*b + 640*a^3*b^2 - 1580*a^2*b^3 + 338*a*b^4 + 35*b^5)*cos(4*d*
x + 4*c) - 8*(96*a^3*b^2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5)*cos(2*d*x + 2*c)
)*cos(9*d*x + 9*c) - 2*((12288*a^5 + 11776*a^4*b - 19296*a^3*b^2 + 10284*a^
2*b^3 - 1410*a*b^4 - 175*b^5)*cos(7*d*x + 7*c) - (8960*a^4*b - 16448*a^3*b^
2 + 8594*a^2*b^3 - 1796*a*b^4 - 315*b^5)*cos(5*d*x + 5*c) + (256*a^4*b - 32
64*a^3*b^2 + 1734*a^2*b^3 - 360*a*b^4 - 175*b^5)*cos(3*d*x + 3*c) + (256*a^
3*b^2 - 64*a^2*b^3 - 26*a*b^4 + 35*b^5)*cos(d*x + c))*cos(8*d*x + 8*c) - (9
6*a^3*b^2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5 + 8*(1536*a^4*b + 1952*a^3*b^2 -
 2012*a^2*b^3 + 298*a*b^4 + 35*b^5)*cos(6*d*x + 6*c) - 4*(768*a^4*b + 640*a
^3*b^2 - 1580*a^2*b^3 + 338*a*b^4 + 35*b^5)*cos(4*d*x + 4*c) - 8*(96*a^3*b^
2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5)*cos(2*d*x + 2*c))*cos(7*d*x + 7*c) + 8*
((1120*a^3*b^2 - 1706*a^2*b^3 + 388*a*b^4 + 63*b^5)*cos(5*d*x + 5*c) - (32*
a^3*b^2 - 398*a^2*b^3 + 88*a*b^4 + 35*b^5)*cos(3*d*x + 3*c) - (32*a^2*b^3 +
 2*a*b^4 - 7*b^5)*cos(d*x + c))*cos(6*d*x + 6*c) + (70*a^2*b^3 - 76*a*b^4 -
 9*b^5 - 4*(560*a^3*b^2 - 1098*a^2*b^3 + 460*a*b^4 + 63*b^5)*cos(4*d*x + 4*
c) - 8*(70*a^2*b^3 - 76*a*b^4 - 9*b^5)*cos(2*d*x + 2*c))*cos(5*d*x + 5*c) +
 4*((16*a^3*b^2 - 206*a^2*b^3 + 128*a*b^4 + 35*b^5)*cos(3*d*x + 3*c) + (16*
a^2*b^3 - 6*a*b^4 - 7*b^5)*cos(d*x + c))*cos(4*d*x + 4*c) - (2*a^2*b^3 - 24
*a*b^4 - 5*b^5 - 8*(2*a^2*b^3 - 24*a*b^4 - 5*b^5)*cos(2*d*x + 2*c))*cos(3*d
*x + 3*c) - (2*a*b^4 + b^5)*cos(d*x + c) - 8*((a^3*b^5 - 2*a^2*b^6 + a*b^7)
*d*cos(16*d*x + 16*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(14*d*x + 1
4*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^
7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^3 - 7...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4524 vs. $2(262) = 524$.
time = 1.06, size = 4524, normalized size = 14.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(8*(2*a*b^2 + b^3)*cos(d*x + c)^7 + 4*(a^2*b - 19*a*b^2 - 6*b^3)*cos
(d*x + c)^5 - 8*(5*a^2*b - 14*a*b^2 - 3*b^3)*cos(d*x + c)^3 + ((a^3*b^3 - 2
*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(
d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4
 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b -
4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229
*a^2*b^2 + 116*a*b^3 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*
b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 -
53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^
```

12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log((81*a^5 - 1458*a^4*b + 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)*cos(d*x + c) + ((a^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^6*b^8 + 114*a^5*b^9 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)) - (27*a^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*a^3*b^5 - 160*a^2*b^6)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))) - ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4 - (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log((81*a^5 - 1458*a^4*b + 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)*cos(d*x + c) + ((a^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^6*b^8 + 114*a^5*b^9 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)) + (27*a^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*a^3*b^5 - 160*a^2*b^6)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4 - (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))) - ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3

```
 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^
3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 1043
61*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7
 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12
+ 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a
^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log(-(81*a^5 - 1458*a^4*b
+ 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)*cos(d*x + c) + ((a
^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^6*b^8 + 114*a^5*b^9
 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2
- 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*
a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^
7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)) - (27*a
^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[-94,23
]Warning, need t
```

Mupad [**B**]
time = 20.15, size = 2500, normalized size = 7.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - ((cos(c + d*x)^3*(14*a*b - 5*a^2 + 3*b^2))/(16*a*(a - b)^2) - (cos(c + d*
x)^5*(19*a*b - a^2 + 6*b^2))/(32*a*(a^2 - 2*a*b + b^2)) + (b*cos(c + d*x)^7
```

*(2*a + b))/(16*a*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)*(15*a*b + 3*a^2 + 2*
b^2))/(32*a*b*(a - b)))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b
^2) - cos(c + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d
*x)^8)) - (atan(((((16384*a^3*b^6 - 172032*a^4*b^5 + 319488*a^5*b^4 - 18841
6*a^6*b^3 + 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*
a^5*b^2)) - (cos(c + d*x)*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2)
+ 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*
b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^
9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*(16384*a^3*b^8
 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^4))/(256*(a^
6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((80*b^3*(a^9*b^5)^(1/2) -
 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^
4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(163
84*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)
))^(1/2) + (cos(c + d*x)*(9*a^4*b - 100*a*b^4 + 16*b^5 + 209*a^2*b^3 - 62*a
^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((80*b^3*
(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^
5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9
b^5)^(1/2))/(16384(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^1
0*b^6 - a^11*b^5)))^(1/2)*1i - (((16384*a^3*b^6 - 172032*a^4*b^5 + 319488*a
^5*b^4 - 188416*a^6*b^3 + 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*b + a^3*b^4 -
4*a^4*b^3 + 6*a^5*b^2)) + (cos(c + d*x)*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a
^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^
7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b
^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*
(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*
b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((80*b^3*(a^
9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b
^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^
5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b
^6 - a^11*b^5)))^(1/2) - (cos(c + d*x)*(9*a^4*b - 100*a*b^4 + 16*b^5 + 209*
a^2*b^3 - 62*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^
2)))*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^
4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) +
 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a
^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*1i)/((((16384*a^3*b^6 - 172032*a^4*
b^5 + 319488*a^5*b^4 - 188416*a^6*b^3 + 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*
b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) - (cos(c + d*x)*((80*b^3*(a^9*b^5)^(1
/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a
^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))
/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11
b^5)))^(1/2)(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^
5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2))
)*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b
^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86

*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*
b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) + (cos(c + d*x)*(9*a^4*b - 100*a*b^4 +
16*b^5 + 209*a^2*b^3 - 62*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*
b^3 + 6*a^4*b^2)))*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^
3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^
9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*
a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) + (((16384*a^3*b^6 -
172032*a^4*b^5 + 319488*a^5*b^4 - 188416*a^6*b^3 + 24576*a^7*b^2)/(16384*(a
^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) + (cos(c + d*x)*((80*b^3*(
a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5
*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*
b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10
*b^6 - a^11*b^5)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 6
5536*a^6*b^5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 +
6*a^4*b^2)))*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7
- 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)
^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^
8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) - (cos(c + d*x)*(9*a^4*b -
100*a*b^4 + 16*b^5 + 209*a^2*b^3 - 62*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b
^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((80*b^3*(a^9*b^5...

3.227 $\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

Optimal. Leaf size=288

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/4}d}-\frac{\cos(c}{8(a-b)d\,(a-b}$$

```
[Out] -1/8*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)
^4)^2-1/32*cos(d*x+c)*(11*a+b-(5*a+b)*cos(d*x+c)^2)/a/(a-b)^2/d/(a-b+2*b*co
s(d*x+c)^2-b*cos(d*x+c)^4)-1/64*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))
^(1/2))*(5*a^(1/2)-2*b^(1/2))/a^(3/2)/b^(3/4)/d/(a^(1/2)-b^(1/2))^(5/2)+1/6
4*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(5*a^(1/2)+2*b^(1/2))
/a^(3/2)/b^(3/4)/d/(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A]
time = 0.34, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3294, 1192, 1180, 211, 214}

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cos(c+dx)\left(-\left((5a+b)\cos^2(c+dx)\right)+11a+b\right)}{32ad(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] -
 Sqrt[b]]])/(a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] + 2
*Sqrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*a^(3
/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cos[c + d*x]*(2 - Cos[c + d*x]^
2))/(8*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - (Cos[
c + d*x]*(11*a + b - (5*a + b)*Cos[c + d*x]^2))/(32*a*(a - b)^2*d*(a - b +
2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+2bx^2-bx^4)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\,(2-\cos^2(c+dx))}{8(a-b)d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))^2} + \frac{\text{Subst}\left(\int \frac{-12ab+10abx}{(a-b+2bx^2-bx}\right.}{16a(a}$$

$$= -\frac{\cos(c+dx)\,(2-\cos^2(c+dx))}{8(a-b)d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))^2} - \frac{\cos(c+dx)\,(11a}{32a(a-b)^2d\,(a-b+}$$

$$= -\frac{\cos(c+dx)\,(2-\cos^2(c+dx))}{8(a-b)d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))^2} - \frac{\cos(c+dx)\,(11a}{32a(a-b)^2d\,(a-b+}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d} + \frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt{\sqrt{}}}\right.}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.74, size = 631, normalized size = 2.19

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cos[c + d*x]*(-17*a - b + (5*a + b)*Cos[2*(c + d*x)]))/(a*(8*a - 3*b +
 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (512*(a - b)*(-5*Cos[c + d*x
] + Cos[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*
x)])^2 + (I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8
 & , (10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)] - (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - I*b*L
og[1 - 2*Cos[c + d*x]*#1 + #1^2] - 94*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)]*#1^2 + 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (47*I)*a*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^2 + 94*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 10*b*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (47*I)*a*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^4 + (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 10*a*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 2*b*ArcTan[Sin[c + d*x]/(Co
s[c + d*x] - #1)]*#1^6 + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + I
```

```
*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
 3*b*#1^5 + b*#1^7) & ])/a)/(256*(a - b)^2*d)
```

Maple [A]
time = 1.45, size = 403, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*b^3*(1/16/(a*b)^(1/2)/a/b^2*((1/4*(-5*a*(a*b)^(1/2)-(a*b)^(1/2)*b+8*a*b
-2*b^2)/b^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+1/4*(7*a-5*(a*b)^(1/2)-2*b)/b^2/(a
-b)*cos(d*x+c))/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2+1/4*(5*a*(a*b)^(1/2)+(a*b)
^(1/2)*b-8*a*b+2*b^2)/b/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b
*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2)))-1/16/(a*b)^(1/2)/a/b^2*((1/4*(5*a*(
a*b)^(1/2)+(a*b)^(1/2)*b+8*a*b-2*b^2)/b^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+1/4*
(7*a+5*(a*b)^(1/2)-2*b)/b^2/(a-b)*cos(d*x+c))/(cos(d*x+c)^2+(a*b)^(1/2)/b-1
)^2+1/4*(5*a*(a*b)^(1/2)+(a*b)^(1/2)*b+8*a*b-2*b^2)/b/(a^2-2*a*b+b^2)/(((a*
b)^(1/2)-b)*b)^(1/2)*arctan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(8*(5*a*b^3 + b^4)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(49*a*b^3 + 5*b^4
)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 8*(5*a*b^3 + b^4)*sin(2*d*x + 2*c)*si
n(d*x + c) - ((5*a*b^3 + b^4)*cos(15*d*x + 15*c) - (49*a*b^3 + 5*b^4)*cos(1
3*d*x + 13*c) - 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4)*cos(11*d*x + 11*c) + (784
*a^2*b^2 - 377*a*b^3 - 5*b^4)*cos(9*d*x + 9*c) + (784*a^2*b^2 - 377*a*b^3 -
 5*b^4)*cos(7*d*x + 7*c) - 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4)*cos(5*d*x + 5*
c) - (49*a*b^3 + 5*b^4)*cos(3*d*x + 3*c) + (5*a*b^3 + b^4)*cos(d*x + c))*co
s(16*d*x + 16*c) - (5*a*b^3 + b^4 - 8*(5*a*b^3 + b^4)*cos(14*d*x + 14*c) -
4*(40*a^2*b^2 - 27*a*b^3 - 7*b^4)*cos(12*d*x + 12*c) + 8*(80*a^2*b^2 - 19*a
*b^3 - 7*b^4)*cos(10*d*x + 10*c) + 2*(640*a^3*b - 352*a^2*b^2 + 79*a*b^3 +
35*b^4)*cos(8*d*x + 8*c) + 8*(80*a^2*b^2 - 19*a*b^3 - 7*b^4)*cos(6*d*x + 6*
c) - 4*(40*a^2*b^2 - 27*a*b^3 - 7*b^4)*cos(4*d*x + 4*c) - 8*(5*a*b^3 + b^4)
*cos(2*d*x + 2*c))*cos(15*d*x + 15*c) - 8*((49*a*b^3 + 5*b^4)*cos(13*d*x +
13*c) + 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4)*cos(11*d*x + 11*c) - (784*a^2*b^2
 - 377*a*b^3 - 5*b^4)*cos(9*d*x + 9*c) - (784*a^2*b^2 - 377*a*b^3 - 5*b^4)*
cos(7*d*x + 7*c) + 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4)*cos(5*d*x + 5*c) + (49
*a*b^3 + 5*b^4)*cos(3*d*x + 3*c) - (5*a*b^3 + b^4)*cos(d*x + c))*cos(14*d*x
 + 14*c) + (49*a*b^3 + 5*b^4 - 4*(392*a^2*b^2 - 303*a*b^3 - 35*b^4)*cos(12*
d*x + 12*c) + 8*(784*a^2*b^2 - 263*a*b^3 - 35*b^4)*cos(10*d*x + 10*c) + 2*(
```

6272*a^3*b - 4064*a^2*b^2 + 1235*a*b^3 + 175*b^4)*cos(8*d*x + 8*c) + 8*(784
*a^2*b^2 - 263*a*b^3 - 35*b^4)*cos(6*d*x + 6*c) - 4*(392*a^2*b^2 - 303*a*b^
3 - 35*b^4)*cos(4*d*x + 4*c) - 8*(49*a*b^3 + 5*b^4)*cos(2*d*x + 2*c))*cos(1
3*d*x + 13*c) - 4*(3*(384*a^3*b - 776*a^2*b^2 + 361*a*b^3 + 21*b^4)*cos(11*
d*x + 11*c) - (6272*a^3*b - 8504*a^2*b^2 + 2599*a*b^3 + 35*b^4)*cos(9*d*x +
 9*c) - (6272*a^3*b - 8504*a^2*b^2 + 2599*a*b^3 + 35*b^4)*cos(7*d*x + 7*c)
+ 3*(384*a^3*b - 776*a^2*b^2 + 361*a*b^3 + 21*b^4)*cos(5*d*x + 5*c) + (392*
a^2*b^2 - 303*a*b^3 - 35*b^4)*cos(3*d*x + 3*c) - (40*a^2*b^2 - 27*a*b^3 - 7
*b^4)*cos(d*x + c))*cos(12*d*x + 12*c) + 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4 +
 8*(768*a^3*b - 1216*a^2*b^2 + 337*a*b^3 + 21*b^4)*cos(10*d*x + 10*c) + 2*(
6144*a^4 - 11648*a^3*b + 6576*a^2*b^2 - 1637*a*b^3 - 105*b^4)*cos(8*d*x + 8
c) + 8(768*a^3*b - 1216*a^2*b^2 + 337*a*b^3 + 21*b^4)*cos(6*d*x + 6*c) -
4*(384*a^3*b - 776*a^2*b^2 + 361*a*b^3 + 21*b^4)*cos(4*d*x + 4*c) - 8*(48*a
^2*b^2 - 55*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*cos(11*d*x + 11*c) - 8*((12544
*a^3*b - 11520*a^2*b^2 + 2559*a*b^3 + 35*b^4)*cos(9*d*x + 9*c) + (12544*a^3
*b - 11520*a^2*b^2 + 2559*a*b^3 + 35*b^4)*cos(7*d*x + 7*c) - 3*(768*a^3*b -
 1216*a^2*b^2 + 337*a*b^3 + 21*b^4)*cos(5*d*x + 5*c) - (784*a^2*b^2 - 263*a
*b^3 - 35*b^4)*cos(3*d*x + 3*c) + (80*a^2*b^2 - 19*a*b^3 - 7*b^4)*cos(d*x +
 c))*cos(10*d*x + 10*c) - (784*a^2*b^2 - 377*a*b^3 - 5*b^4 + 2*(100352*a^4
- 123520*a^3*b + 62992*a^2*b^2 - 12715*a*b^3 - 175*b^4)*cos(8*d*x + 8*c) +
8*(12544*a^3*b - 11520*a^2*b^2 + 2559*a*b^3 + 35*b^4)*cos(6*d*x + 6*c) - 4*
(6272*a^3*b - 8504*a^2*b^2 + 2599*a*b^3 + 35*b^4)*cos(4*d*x + 4*c) - 8*(784
*a^2*b^2 - 377*a*b^3 - 5*b^4)*cos(2*d*x + 2*c))*cos(9*d*x + 9*c) - 2*((1003
52*a^4 - 123520*a^3*b + 62992*a^2*b^2 - 12715*a*b^3 - 175*b^4)*cos(7*d*x +
7*c) - 3*(6144*a^4 - 11648*a^3*b + 6576*a^2*b^2 - 1637*a*b^3 - 105*b^4)*cos
(5*d*x + 5*c) - (6272*a^3*b - 4064*a^2*b^2 + 1235*a*b^3 + 175*b^4)*cos(3*d*
x + 3*c) + (640*a^3*b - 352*a^2*b^2 + 79*a*b^3 + 35*b^4)*cos(d*x + c))*cos(
8*d*x + 8*c) - (784*a^2*b^2 - 377*a*b^3 - 5*b^4 + 8*(12544*a^3*b - 11520*a^
2*b^2 + 2559*a*b^3 + 35*b^4)*cos(6*d*x + 6*c) - 4*(6272*a^3*b - 8504*a^2*b^
2 + 2599*a*b^3 + 35*b^4)*cos(4*d*x + 4*c) - 8*(784*a^2*b^2 - 377*a*b^3 - 5*
b^4)*cos(2*d*x + 2*c))*cos(7*d*x + 7*c) + 8*(3*(768*a^3*b - 1216*a^2*b^2 +
337*a*b^3 + 21*b^4)*cos(5*d*x + 5*c) + (784*a^2*b^2 - 263*a*b^3 - 35*b^4)*c
os(3*d*x + 3*c) - (80*a^2*b^2 - 19*a*b^3 - 7*b^4)*cos(d*x + c))*cos(6*d*x +
 6*c) + 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4 - 4*(384*a^3*b - 776*a^2*b^2 + 361
*a*b^3 + 21*b^4)*cos(4*d*x + 4*c) - 8*(48*a^2*b^2 - 55*a*b^3 - 3*b^4)*cos(2
*d*x + 2*c))*cos(5*d*x + 5*c) - 4*((392*a^2*b^2 - 303*a*b^3 - 35*b^4)*cos(3
*d*x + 3*c) - (40*a^2*b^2 - 27*a*b^3 - 7*b^4)*cos(d*x + c))*cos(4*d*x + 4*c
) + (49*a*b^3 + 5*b^4 - 8*(49*a*b^3 + 5*b^4)*cos(2*d*x + 2*c))*cos(3*d*x +
3*c) - (5*a*b^3 + b^4)*cos(d*x + c) - 16*((a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*c
os(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(14*d*x + 14*c)
^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d
*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a
^2*b^5 + 49*a*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^7 - 57344*a^6*b + 83
712*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a^2*b^5 + 1225*a*b^6)*d*
cos(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a^2*

```
b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a
^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^3*b^4 - 2*a^2
*b^5 + a*b^6)*d*cos(2*d*x + 2*c)^2 + (a^3*b^4 -...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4050 vs. $2(233) = 466$.
time = 0.93, size = 4050, normalized size = 14.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(5*a*b + b^2)*cos(d*x + c)^7 - 12*(7*a*b + b^2)*cos(d*x + c)^5 -
12*(3*a^2 - 10*a*b - b^2)*cos(d*x + c)^3 + ((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d
*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4
*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a
^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*
a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a
*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 21
0*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4
*b^12 + a^3*b^13)*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*b -
5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log((625*a
^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2 - 16*a^
9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b^8 + 2*a
^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225
*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 -
 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3
*b^13)*d^4)) - (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4)*d)*sqr
t(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2
*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4)/((a^1
3*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^
8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)
) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3
 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) - ((a^3*b^2 - 2*a^2*b^3 + a*b^4
)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(
a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^
3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 -
4*a^2*b^3 + a*b^4)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 +
5*a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780
*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 +
210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a
^4*b^12 + a^3*b^13)*d^4)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a^8*b
- 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log((625
```

```
*a^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2 - 16*
a^9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b^8 + 2
*a^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 12
25*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7
 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a
^3*b^13)*d^4)) + (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4)*d)*s
qrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^
2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4)/((a^
13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b
^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4
)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^
3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) - ((a^3*b^2 - 2*a^2*b^3 + a*b^
4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*
(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 -
 4*a^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4
+ 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 107
80*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6
+ 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10
*a^4*b^12 + a^3*b^13)*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*
b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(-(
625*a^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2 -
16*a^9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b^8
+ 2*a^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 +
 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*
b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12
+ a^3*b^13)*d^4)) - (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4)*d
)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6
)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4)/
((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a
^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)
*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*a^
6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) + ((a^3*b^2 - 2*a^2*b^3 +
a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6
- 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(233) = 466$.
time = 1.29, size = 1076, normalized size = 3.74

$$-\frac{\frac{5ab\cos(dx+c)^7}{d}+\frac{b^2\cos(dx+c)^7}{d}-\frac{21ab\cos(dx+c)^5}{d}-\frac{3b^2\cos(dx+c)^5}{d}-\frac{9a^2\cos(dx+c)^3}{d}+\frac{30ab\cos(dx+c)^3}{d}+\frac{3b^2\cos(dx+c)^3}{d}+\frac{19a^2\cos(dx+c)}{d}-\frac{18ab\cos(dx+c)}{d}-\frac{b^2\cos(dx+c)}{d}}{32\left(b\cos(dx+c)^4-2b\cos(dx+c)^2-a+b\right)^2(a^3-2a^2b+ab^2)}-\frac{\left(2\left(4a^6b-17a^5b^2+28a^4b^3-22a^3b^4+8a^2b^5-ab^6\right)\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,d^4+\left(13a^4b-27a^3b^2+15a^2b^3-ab^4\right)\sqrt{-b^2+\sqrt{ab}\,b}\,d^2|a^3d^2-2a^2bd^2+ab^2d^2|+\left(a^3d^2-2a^2bd^2+ab^2d^2\right)^2\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,(5a+b)\right)\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{a^3bd^2-2a^2b^2d^2+ab^3d^2-\sqrt{(a^3bd^2-2a^2b^2d^2+ab^3d^2)^2+(a^3bd^4-2a^2b^2d^4+ab^3d^4)(a^4-3a^3b+3a^2b^2-ab^3)}}{a^3bd^4-2a^2b^2d^4+ab^3d^4}}}\right)}{64\left(a^7b-5a^6b^2+10a^5b^3-10a^4b^4+5a^3b^5-a^2b^6\right)d^3|a^3d^2-2a^2bd^2+ab^2d^2||b|}+\frac{\left(2\left(4a^6b-17a^5b^2+28a^4b^3-22a^3b^4+8a^2b^5-ab^6\right)\sqrt{-b^2-\sqrt{ab}\,b}\,d^4-\left(13a^3-27a^2b+15ab^2-b^3\right)\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,d^2|a^3d^2-2a^2bd^2+ab^2d^2|+\left(a^3d^2-2a^2bd^2+ab^2d^2\right)^2\sqrt{-b^2-\sqrt{ab}\,b}\,(5a+b)\right)\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{a^3bd^2-2a^2b^2d^2+ab^3d^2+\sqrt{(a^3bd^2-2a^2b^2d^2+ab^3d^2)^2+(a^3bd^4-2a^2b^2d^4+ab^3d^4)(a^4-3a^3b+3a^2b^2-ab^3)}}{a^3bd^4-2a^2b^2d^4+ab^3d^4}}}\right)}{64\left(a^6-5a^5b+10a^4b^2-10a^3b^3+5a^2b^4-ab^5\right)\sqrt{ab}\,d^3|a^3d^2-2a^2bd^2+ab^2d^2||b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/32*(5*a*b*cos(d*x + c)^7/d + b^2*cos(d*x + c)^7/d - 21*a*b*cos(d*x + c)^
5/d - 3*b^2*cos(d*x + c)^5/d - 9*a^2*cos(d*x + c)^3/d + 30*a*b*cos(d*x + c)
^3/d + 3*b^2*cos(d*x + c)^3/d + 19*a^2*cos(d*x + c)/d - 18*a*b*cos(d*x + c)
/d - b^2*cos(d*x + c)/d)/((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - a + b)^2
*(a^3 - 2*a^2*b + a*b^2)) - 1/64*(2*(4*a^6*b - 17*a^5*b^2 + 28*a^4*b^3 - 22
*a^3*b^4 + 8*a^2*b^5 - a*b^6)*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*d^4 + (13*
a^4*b - 27*a^3*b^2 + 15*a^2*b^3 - a*b^4)*sqrt(-b^2 + sqrt(a*b)*b)*d^2*abs(a
^3*d^2 - 2*a^2*b*d^2 + a*b^2*d^2) + (a^3*d^2 - 2*a^2*b*d^2 + a*b^2*d^2)^2*s
qrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*(5*a + b))*arctan(cos(d*x + c)/(d*sqrt(-(
a^3*b*d^2 - 2*a^2*b^2*d^2 + a*b^3*d^2 - sqrt((a^3*b*d^2 - 2*a^2*b^2*d^2 + a
*b^3*d^2)^2 + (a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4)*(a^4 - 3*a^3*b + 3*a^
2*b^2 - a*b^3)))/(a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4))))/((a^7*b - 5*a^6
*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^3*abs(a^3*d^2 - 2*a
^2*b*d^2 + a*b^2*d^2)*abs(b)) + 1/64*(2*(4*a^6*b - 17*a^5*b^2 + 28*a^4*b^3
- 22*a^3*b^4 + 8*a^2*b^5 - a*b^6)*sqrt(-b^2 - sqrt(a*b)*b)*d^4 - (13*a^3 -
27*a^2*b + 15*a*b^2 - b^3)*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*d^2*abs(a^3*d
^2 - 2*a^2*b*d^2 + a*b^2*d^2) + (a^3*d^2 - 2*a^2*b*d^2 + a*b^2*d^2)^2*sqrt(
-b^2 - sqrt(a*b)*b)*(5*a + b))*arctan(cos(d*x + c)/(d*sqrt(-(a^3*b*d^2 - 2*
a^2*b^2*d^2 + a*b^3*d^2 + sqrt((a^3*b*d^2 - 2*a^2*b^2*d^2 + a*b^3*d^2)^2 +
(a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4)*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)
))/(a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4))))/((a^6 - 5*a^5*b + 10*a^4*b^2
- 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*sqrt(a*b)*d^3*abs(a^3*d^2 - 2*a^2*b*d^2 +
 a*b^2*d^2)*abs(b))
```

Mupad [B]
time = 19.28, size = 2500, normalized size = 8.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - (atan(((((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b^4 - 212992*a^6*b^
3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) - (cos(c + d*x
)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^
4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5
```

*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*(16384*
a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^4))/(
256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^
(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*
a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6
 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) + (cos(c + d*x)*(4*b^5 - 31*
a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3
 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b
^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(1638
4*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))
^(1/2)*1i - (((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b^4 - 212992*a^6
b^3)/(32768(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) + (cos(c +
d*x)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4
*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8
- 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*(163
84*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^4)
)/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^
3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 1
05*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*
b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) - (cos(c + d*x)*(4*b^5 -
31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*
b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^
3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(1
6384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3
)))^(1/2)*1i)/((((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b^4 - 212992*
a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) - (cos(c
 + d*x)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*
a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b
^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*(
16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b
^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9
*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3
+ 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a
^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) + (cos(c + d*x)*(4*b^5
 - 31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a
^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4
*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))
/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*
b^3)))^(1/2) + (((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b^4 - 212992*
a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) + (cos(c
 + d*x)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*
a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b
^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*(
16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b
^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9

```
*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3
+ 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a
^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) - (cos(c + d*x)*(4*b^5
 - 31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a
^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4
*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))
/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*
b^3)))^(1/2) + (5*a*b^2 + 125*a^2*b - 4*b^3)/(16384*(a^7 - 4*a^6*b + a^3*b^
4 - 4*a^4*b^3 + 6*a^5*b^2))))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(
1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3
)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4
 - a^11*b^3)))^(1/2)*2i)/d - (atan(((((16384*a^3*b^6 - 245760*a^4*b^5 + 442
368*a^5*b^4 - 212992*a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 +
 6*a^5*b^2)) - (cos(c + d*x)*((25*a^2*(a^9*b^3)^(1/2) - 35*b^2*(a^9*b^3)^(1
/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154*a*b*(a^9*b^3)
^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4
- a^11*b^3)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*
a^6*b^5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b +...
```

3.228 $\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

Optimal. Leaf size=313

$$-\frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}\,d}-\frac{3\left(7a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt[4]{b}\,d}-$$

```
[Out] -1/8*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(
d*x+c)^4)^2-1/32*cos(d*x+c)*((7*a-3*b)*(a+2*b)-6*(2*a-b)*b*cos(d*x+c)^2)/a^
2/(a-b)^2/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-3/64*arctan(b^(1/4)*cos(d
*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(7*a+4*b-10*a^(1/2)*b^(1/2))/a^(5/2)/b^(1/4)
/d/(a^(1/2)-b^(1/2))^(5/2)-3/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2)
)^(1/2))*(7*a+4*b+10*a^(1/2)*b^(1/2))/a^(5/2)/b^(1/4)/d/(a^(1/2)+b^(1/2))^(
5/2)
```

Rubi [A]
time = 0.32, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3294, 1106, 1192, 1180, 211, 214}

$$-\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cos(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cos^2(c+dx)\right)}{32a^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8ad(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (-3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) - (3*(7*
a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) - (Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 -
b*Cos[c + d*x]^4)^2) - (Cos[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a - b)*b
*Cos[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c
 + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(a-b+2bx^2-bx^4)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\,(a+b-b\cos^2(c+dx))}{8a(a-b)d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))^2} + \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2\cdots}{(a-\cdots}\right.}{\cdots}$$

$$= -\frac{\cos(c+dx)\,(a+b-b\cos^2(c+dx))}{8a(a-b)d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))^2} - \frac{\cos(c+dx)\,((7a-\cdots}{32a^2(a-b)^2d\,(a-\cdots}$$

$$= -\frac{\cos(c+dx)\,(a+b-b\cos^2(c+dx))}{8a(a-b)d\,(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx))^2} - \frac{\cos(c+dx)\,((7a-\cdots}{32a^2(a-b)^2d\,(a-\cdots}$$

$$= -\frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}\,d} - \frac{3\left(7a+10\sqrt{a}\sqrt{b}+4\cdots\right.}{64a^{5/2}\left(\sqrt{a}\cdots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.85, size = 784, normalized size = 2.50

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cos[c + d*x]*(7*a^2 + 5*a*b - 3*b^2 + 3*b*(-2*a + b)*Cos[2*(c + d*x)]
))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) - (512*a*(a - b)
*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c +
d*x)] + b*Cos[4*(c + d*x)])^2 + (3*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*
b*#1^4 - 4*b*#1^6 + b*#1^8 & , (4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)] - 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (2*I)*a*b*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2] + I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 28*a^2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 24*a*b*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1^2 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^2 + (14*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*a*b*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1^2 + 28*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 24*a*b*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 10*b^2*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1^4 - (14*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1
```

```
^4 + (12*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (5*I)*b^2*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
#1)]*#1^6 + 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (2*I)*a*b
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
 #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(128*
a^2*(a - b)^2*d)
```

Maple [A]

time = 1.78, size = 430, normalized size = 1.37

method	result
derivativedivides	$b^{3}\left(\frac{\frac{\frac{3\left(-4a\sqrt{ab}+2\sqrt{ab}\,b+3a^{2}-ab\right)\left(\cos^{3}\left(dx+c\right)\right)}{4b^{2}\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(-11a\sqrt{ab}+6\sqrt{ab}\,b+5ab\right)\cos\left(dx+c\right)}{4b^{3}\left(a-b\right)}}{\left(\cos^{2}\left(dx+c\right)-1-\frac{\sqrt{ab}}{b}\right)^{2}}+\frac{3\left(4a\sqrt{ab}-2\sqrt{ab}\,b\right.}{4b\left(a^{2}\right.}}{16ba^{2}\sqrt{ab}}\right.$
default	$b^{3}\left(\frac{\frac{\frac{3\left(-4a\sqrt{ab}+2\sqrt{ab}\,b+3a^{2}-ab\right)\left(\cos^{3}\left(dx+c\right)\right)}{4b^{2}\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(-11a\sqrt{ab}+6\sqrt{ab}\,b+5ab\right)\cos\left(dx+c\right)}{4b^{3}\left(a-b\right)}}{\left(\cos^{2}\left(dx+c\right)-1-\frac{\sqrt{ab}}{b}\right)^{2}}+\frac{3\left(4a\sqrt{ab}-2\sqrt{ab}\,b\right.}{4b\left(a^{2}\right.}}{16ba^{2}\sqrt{ab}}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*b^3*(1/16/b/a^2/(a*b)^(1/2)*((3/4*(-4*a*(a*b)^(1/2)+2*(a*b)^(1/2)*b+3*a
^2-a*b)/b^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+1/4*(-11*a*(a*b)^(1/2)+6*(a*b)^(1/
```

```
2)*b+5*a*b)/b^3/(a-b)*cos(d*x+c))/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2+3/4*(4*a
*(a*b)^(1/2)-2*(a*b)^(1/2)*b-7*a^2+9*a*b-4*b^2)/b/(a^2-2*a*b+b^2)/(((a*b)^(
1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/(((a*b)^(1/2)+b)*b)^(1/2)))-1/16/b/a^
2/(a*b)^(1/2)*((3/4*(4*a*(a*b)^(1/2)-2*(a*b)^(1/2)*b+3*a^2-a*b)/b^2/(a^2-2*
a*b+b^2)*cos(d*x+c)^3+1/4*(11*a*(a*b)^(1/2)-6*(a*b)^(1/2)*b+5*a*b)/b^3/(a-b
)*cos(d*x+c))/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2+3/4*(4*a*(a*b)^(1/2)-2*(a*b)
^(1/2)*b+7*a^2-9*a*b+4*b^2)/b/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arc
tan(b*cos(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(24*(2*a*b^4 - b^5)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(14*a^2*b^3 + 28*
a*b^4 - 15*b^5)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 24*(2*a*b^4 - b^5)*sin(
2*d*x + 2*c)*sin(d*x + c) - (3*(2*a*b^4 - b^5)*cos(15*d*x + 15*c) - (14*a^2
*b^3 + 28*a*b^4 - 15*b^5)*cos(13*d*x + 13*c) - (86*a^2*b^3 - 128*a*b^4 + 27
*b^5)*cos(11*d*x + 11*c) + (352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5)*
cos(9*d*x + 9*c) + (352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5)*cos(7*d*
x + 7*c) - (86*a^2*b^3 - 128*a*b^4 + 27*b^5)*cos(5*d*x + 5*c) - (14*a^2*b^3
 + 28*a*b^4 - 15*b^5)*cos(3*d*x + 3*c) + 3*(2*a*b^4 - b^5)*cos(d*x + c))*co
s(16*d*x + 16*c) - 3*(2*a*b^4 - b^5 - 8*(2*a*b^4 - b^5)*cos(14*d*x + 14*c)
- 4*(16*a^2*b^3 - 22*a*b^4 + 7*b^5)*cos(12*d*x + 12*c) + 8*(32*a^2*b^3 - 30
*a*b^4 + 7*b^5)*cos(10*d*x + 10*c) + 2*(256*a^3*b^2 - 320*a^2*b^3 + 166*a*b
^4 - 35*b^5)*cos(8*d*x + 8*c) + 8*(32*a^2*b^3 - 30*a*b^4 + 7*b^5)*cos(6*d*x
 + 6*c) - 4*(16*a^2*b^3 - 22*a*b^4 + 7*b^5)*cos(4*d*x + 4*c) - 8*(2*a*b^4 -
 b^5)*cos(2*d*x + 2*c))*cos(15*d*x + 15*c) - 8*((14*a^2*b^3 + 28*a*b^4 - 15
*b^5)*cos(13*d*x + 13*c) + (86*a^2*b^3 - 128*a*b^4 + 27*b^5)*cos(11*d*x + 1
1*c) - (352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5)*cos(9*d*x + 9*c) - (
352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5)*cos(7*d*x + 7*c) + (86*a^2*b
^3 - 128*a*b^4 + 27*b^5)*cos(5*d*x + 5*c) + (14*a^2*b^3 + 28*a*b^4 - 15*b^5
)*cos(3*d*x + 3*c) - 3*(2*a*b^4 - b^5)*cos(d*x + c))*cos(14*d*x + 14*c) + (
14*a^2*b^3 + 28*a*b^4 - 15*b^5 - 4*(112*a^3*b^2 + 126*a^2*b^3 - 316*a*b^4 +
 105*b^5)*cos(12*d*x + 12*c) + 8*(224*a^3*b^2 + 350*a^2*b^3 - 436*a*b^4 + 1
05*b^5)*cos(10*d*x + 10*c) + 2*(1792*a^4*b + 2240*a^3*b^2 - 4118*a^2*b^3 +
2420*a*b^4 - 525*b^5)*cos(8*d*x + 8*c) + 8*(224*a^3*b^2 + 350*a^2*b^3 - 436
*a*b^4 + 105*b^5)*cos(6*d*x + 6*c) - 4*(112*a^3*b^2 + 126*a^2*b^3 - 316*a*b
^4 + 105*b^5)*cos(4*d*x + 4*c) - 8*(14*a^2*b^3 + 28*a*b^4 - 15*b^5)*cos(2*d
*x + 2*c))*cos(13*d*x + 13*c) - 4*((688*a^3*b^2 - 1626*a^2*b^3 + 1112*a*b^4
 - 189*b^5)*cos(11*d*x + 11*c) - (2816*a^4*b - 2944*a^3*b^2 - 428*a^2*b^3 +
 862*a*b^4 - 105*b^5)*cos(9*d*x + 9*c) - (2816*a^4*b - 2944*a^3*b^2 - 428*a
```

```
^2*b^3 + 862*a*b^4 - 105*b^5)*cos(7*d*x + 7*c) + (688*a^3*b^2 - 1626*a^2*b^
3 + 1112*a*b^4 - 189*b^5)*cos(5*d*x + 5*c) + (112*a^3*b^2 + 126*a^2*b^3 - 3
16*a*b^4 + 105*b^5)*cos(3*d*x + 3*c) - 3*(16*a^2*b^3 - 22*a*b^4 + 7*b^5)*co
s(d*x + c))*cos(12*d*x + 12*c) + (86*a^2*b^3 - 128*a*b^4 + 27*b^5 + 8*(1376
*a^3*b^2 - 2650*a^2*b^3 + 1328*a*b^4 - 189*b^5)*cos(10*d*x + 10*c) + 2*(110
08*a^4*b - 24640*a^3*b^2 + 18754*a^2*b^3 - 7072*a*b^4 + 945*b^5)*cos(8*d*x
+ 8*c) + 8*(1376*a^3*b^2 - 2650*a^2*b^3 + 1328*a*b^4 - 189*b^5)*cos(6*d*x +
 6*c) - 4*(688*a^3*b^2 - 1626*a^2*b^3 + 1112*a*b^4 - 189*b^5)*cos(4*d*x + 4
*c) - 8*(86*a^2*b^3 - 128*a*b^4 + 27*b^5)*cos(2*d*x + 2*c))*cos(11*d*x + 11
*c) - 8*((5632*a^4*b - 3424*a^3*b^2 - 1276*a^2*b^3 + 982*a*b^4 - 105*b^5)*c
os(9*d*x + 9*c) + (5632*a^4*b - 3424*a^3*b^2 - 1276*a^2*b^3 + 982*a*b^4 - 1
05*b^5)*cos(7*d*x + 7*c) - (1376*a^3*b^2 - 2650*a^2*b^3 + 1328*a*b^4 - 189*
b^5)*cos(5*d*x + 5*c) - (224*a^3*b^2 + 350*a^2*b^3 - 436*a*b^4 + 105*b^5)*c
os(3*d*x + 3*c) + 3*(32*a^2*b^3 - 30*a*b^4 + 7*b^5)*cos(d*x + c))*cos(10*d*
x + 10*c) - (352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5 + 2*(45056*a^5 -
 41472*a^4*b + 4512*a^3*b^2 + 9996*a^2*b^3 - 5150*a*b^4 + 525*b^5)*cos(8*d*
x + 8*c) + 8*(5632*a^4*b - 3424*a^3*b^2 - 1276*a^2*b^3 + 982*a*b^4 - 105*b^
5)*cos(6*d*x + 6*c) - 4*(2816*a^4*b - 2944*a^3*b^2 - 428*a^2*b^3 + 862*a*b^
4 - 105*b^5)*cos(4*d*x + 4*c) - 8*(352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 1
5*b^5)*cos(2*d*x + 2*c))*cos(9*d*x + 9*c) - 2*((45056*a^5 - 41472*a^4*b + 4
512*a^3*b^2 + 9996*a^2*b^3 - 5150*a*b^4 + 525*b^5)*cos(7*d*x + 7*c) - (1100
8*a^4*b - 24640*a^3*b^2 + 18754*a^2*b^3 - 7072*a*b^4 + 945*b^5)*cos(5*d*x +
 5*c) - (1792*a^4*b + 2240*a^3*b^2 - 4118*a^2*b^3 + 2420*a*b^4 - 525*b^5)*c
os(3*d*x + 3*c) + 3*(256*a^3*b^2 - 320*a^2*b^3 + 166*a*b^4 - 35*b^5)*cos(d*
x + c))*cos(8*d*x + 8*c) - (352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5 +
 8*(5632*a^4*b - 3424*a^3*b^2 - 1276*a^2*b^3 + 982*a*b^4 - 105*b^5)*cos(6*d
*x + 6*c) - 4*(2816*a^4*b - 2944*a^3*b^2 - 428*a^2*b^3 + 862*a*b^4 - 105*b^
5)*cos(4*d*x + 4*c) - 8*(352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5)*cos
(2*d*x + 2*c))*cos(7*d*x + 7*c) + 8*((1376*a^3*b^2 - 2650*a^2*b^3 + 1328*a*
b^4 - 189*b^5)*cos(5*d*x + 5*c) + (224*a^3*b^2 + 350*a^2*b^3 - 436*a*b^4 +
105*b^5)*cos(3*d*x + 3*c) - 3*(32*a^2*b^3 - 30*a*b^4 + 7*b^5)*cos(d*x + c))
*cos(6*d*x + 6*c) + (86*a^2*b^3 - 128*a*b^4 + 27*b^5 - 4*(688*a^3*b^2 - 162
6*a^2*b^3 + 1112*a*b^4 - 189*b^5)*cos(4*d*x + 4*c) - 8*(86*a^2*b^3 - 128*a*
b^4 + 27*b^5)*cos(2*d*x + 2*c))*cos(5*d*x + 5*c) - 4*((112*a^3*b^2 + 126*a^
2*b^3 - 316*a*b^4 + 105*b^5)*cos(3*d*x + 3*c) - 3*(16*a^2*b^3 - 22*a*b^4 +
7*b^5)*cos(d*x + c))*cos(4*d*x + 4*c) + (14*a^2*b^3 + 28*a*b^4 - 15*b^5 - 8
*(14*a^2*b^3 + 28*a*b^4 - 15*b^5)*cos(2*d*x + 2*c))*cos(3*d*x + 3*c) - 3*(2
*a*b^4 - b^5)*cos(d*x + c) + 8*((a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(16*d*
x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4160 vs. $2(263) = 526$.

time = 1.04, size = 4160, normalized size = 13.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")

[Out] -1/128*(24*(2*a*b^2 - b^3)*cos(d*x + c)^7 - 4*(7*a^2*b + 35*a*b^2 - 18*b^3)*cos(d*x + c)^5 - 8*(a^2*b - 22*a*b^2 + 9*b^3)*cos(d*x + c)^3 + 3*((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 - 1788*a*b^3 + 336*b^4)*cos(d*x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^10*b^3 - 240*a^9*b^4 + 205*a^8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*d^3*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)) - (343*a^7 - 623*a^6*b + 515*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))) - 3*((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 - 1788*a*b^3 + 336*b^4)*cos(d*x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^10*b^3 - 240*a^9*b^4 + 205*a^8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*d^3*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)) + (343*a^7 - 623*a^6*b + 515*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10

```
*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*
b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12
*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9
 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^
3 + 5*a^6*b^4 - a^5*b^5)*d^2))) - 3*((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(
d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b
- 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^
2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*
a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 +
 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*
d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^
15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b
^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4))
)/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*l
og(-27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 - 1788*a*b^3 + 336*b^4)*cos(d*
x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^10*b^3 - 240*a^9*b^4 + 205*a^
8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*d^3*sqrt((2401*a^4 - 5292*a^3
*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*
b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^
8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)) - (343*a^7 - 623*a^6*b + 515
*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^
2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a
^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b
^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^1
1*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10
 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(263) = 526.
time = 1.20, size = 766, normalized size = 2.45

$$-\frac{3\left(7a^3-5a^2b+2ab^2-(11a^2-11ab+4b^2)\sqrt{ab}\right)\sqrt{-b^2-\sqrt{ab}\,b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{a^4bd^2-2a^3b^2d^2+a^2b^3d^2+\sqrt{(a^4bd^2-2a^3b^2d^2+a^2b^3d^2)^2+(a^4bd^4-2a^3b^2d^4+a^2b^3d^4)(a^5-3a^4b+3a^3b^2-a^2b^3)}}{a^4bd^4-2a^3b^2d^4+a^2b^3d^4}}}\right)}{64(a^6-3a^5b+3a^4b^2-a^3b^3)d|b|}-\frac{3\left(7a^3-5a^2b+2ab^2+(11a^2-11ab+4b^2)\sqrt{ab}\right)\sqrt{-b^2+\sqrt{ab}\,b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{a^4bd^2-2a^3b^2d^2+a^2b^3d^2-\sqrt{(a^4bd^2-2a^3b^2d^2+a^2b^3d^2)^2+(a^4bd^4-2a^3b^2d^4+a^2b^3d^4)(a^5-3a^4b+3a^3b^2-a^2b^3)}}{a^4bd^4-2a^3b^2d^4+a^2b^3d^4}}}\right)}{64(a^6-3a^5b+3a^4b^2-a^3b^3)d|b|}-\frac{\frac{12ab^2\cos(dx+c)^7}{d}-\frac{6b^3\cos(dx+c)^7}{d}-\frac{7a^2b\cos(dx+c)^5}{d}-\frac{35ab^2\cos(dx+c)^5}{d}+\frac{18b^3\cos(dx+c)^5}{d}-\frac{2a^2b\cos(dx+c)^3}{d}+\frac{44ab^2\cos(dx+c)^3}{d}-\frac{18b^3\cos(dx+c)^3}{d}+\frac{11a^3\cos(dx+c)}{d}+\frac{4a^2b\cos(dx+c)}{d}-\frac{21ab^2\cos(dx+c)}{d}+\frac{6b^3\cos(dx+c)}{d}}{32\left(b\cos(dx+c)^4-2b\cos(dx+c)^2-a+b\right)^2(a^4-2a^3b+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -3/64*(7*a^3 - 5*a^2*b + 2*a*b^2 - (11*a^2 - 11*a*b + 4*b^2)*sqrt(a*b))*sqr
t(-b^2 - sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(a^4*b*d^2 - 2*a^3*b^2*d
^2 + a^2*b^3*d^2 + sqrt((a^4*b*d^2 - 2*a^3*b^2*d^2 + a^2*b^3*d^2)^2 + (a^4*
b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3))
)/(a^4*b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4))))/((a^6 - 3*a^5*b + 3*a^4*b^2
- a^3*b^3)*d*abs(b)) - 3/64*(7*a^3 - 5*a^2*b + 2*a*b^2 + (11*a^2 - 11*a*b +
 4*b^2)*sqrt(a*b))*sqrt(-b^2 + sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(a
^4*b*d^2 - 2*a^3*b^2*d^2 + a^2*b^3*d^2 - sqrt((a^4*b*d^2 - 2*a^3*b^2*d^2 +
a^2*b^3*d^2)^2 + (a^4*b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4)*(a^5 - 3*a^4*b +
 3*a^3*b^2 - a^2*b^3)))/(a^4*b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4))))/((a^6
- 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d*abs(b)) - 1/32*(12*a*b^2*cos(d*x + c)^7/
d - 6*b^3*cos(d*x + c)^7/d - 7*a^2*b*cos(d*x + c)^5/d - 35*a*b^2*cos(d*x +
c)^5/d + 18*b^3*cos(d*x + c)^5/d - 2*a^2*b*cos(d*x + c)^3/d + 44*a*b^2*cos(
d*x + c)^3/d - 18*b^3*cos(d*x + c)^3/d + 11*a^3*cos(d*x + c)/d + 4*a^2*b*co
s(d*x + c)/d - 21*a*b^2*cos(d*x + c)/d + 6*b^3*cos(d*x + c)/d)/((b*cos(d*x
+ c)^4 - 2*b*cos(d*x + c)^2 - a + b)^2*(a^4 - 2*a^3*b + a^2*b^2))
```

Mupad [B]
time = 18.91, size = 2500, normalized size = 7.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] (atan(((((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5 - 155648*a^8*b^
4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^
2)) - (cos(c + d*x)*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 10
5*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^1
5*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13
*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304*a^7*b^6 -
 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3
+ 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9
*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^
(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3
- 5*a^14*b^2)))^(1/2) + (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 1089*a^2*b^5 -
 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*
a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b -
 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2
)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*
a^14*b^2)))^(1/2)*1i - (((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5
 - 155648*a^8*b^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^
7*b^3 + 6*a^8*b^2)) + (cos(c + d*x)*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^
15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b
^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^
```

12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304*a^7*b^6 - 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2) - (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 1089*a^2*b^5 - 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*1i)/((3*(684*a*b^5 - 144*b^6 - 1233*a^2*b^4 + 882*a^3*b^3))/(8192*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)) + (((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5 - 155648*a^8*b^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)) - (cos(c + d*x)*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304*a^7*b^6 - 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2) + (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 1089*a^2*b^5 - 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2) + (((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5 - 155648*a^8*b^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)) + (cos(c + d*x)*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304*a^7*b^6 - 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2) - (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 1089*a^2*b^5 - 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4

```
 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*2i)/d - ((cos(c + d*x)*(15*a*b + 11*a^
2 - 6*b^2))/(32*a^2*(a - b)) - (cos(c + d*x)^3*...
```

3.229 $\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

Optimal. Leaf size=617

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+d}{\sqrt{\sqrt{a}-\sqrt{}}}\right.}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d}$$

```
[Out] -arctanh(cos(d*x+c))/a^3/d-1/8*b*cos(d*x+c)*(2-cos(d*x+c)^2)/a/(a-b)/d/(a-b
+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)^2-1/4*b*cos(d*x+c)*(2-cos(d*x+c)^2)/a^2/(
a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/32*b*cos(d*x+c)*(11*a+b-(5*a
+b)*cos(d*x+c)^2)/a^2/(a-b)^2/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/64*
b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5*a^(1/2)-2*b^(
1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(5/2)-1/8*b^(1/4)*arctan(b^(1/4)*cos(d*x+
c)/(a^(1/2)-b^(1/2))^(1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*b^(1/4)*a
rctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(5/2)/d/(a^(1/2)+b^(1/
2))^(3/2)+1/64*b^(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*
(5*a^(1/2)+2*b^(1/2))/a^(5/2)/d/(a^(1/2)+b^(1/2))^(5/2)-1/2*b^(1/4)*arctan(
b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^3/d/(a^(1/2)-b^(1/2))^(1/2)+1
/2*b^(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^3/d/(a^(1/
2)+b^(1/2))^(1/2)
```

Rubi [A]

time = 0.58, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3294, 1252, 213, 1192, 1180, 211, 214}

$$-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\left(5\sqrt{a}-2\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\sqrt[4]{b}\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{5/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4a^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{b\cos(c+dx)\left(-\left((5a+b)\cos^2(c+dx)\right)+11a+b\right)}{32a^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8ad(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[S
qrt[a] - Sqrt[b]]])/(a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (b^(1/4)*ArcTan
[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] - Sqr
t[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) +
(b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(5/2
)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sq
rt[Sqrt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]*d) + ((5*Sqrt[a] + 2
*Sqrt[b])*b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/
(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b*Cos[c + d*x]*(2 - Cos[c + d*x
```

```
]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - (
b*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*a^2*(a - b)*d*(a - b + 2*b*Cos[c +
d*x]^2 - b*Cos[c + d*x]^4)) - (b*Cos[c + d*x]*(11*a + b - (5*a + b)*Cos[c +
 d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^
4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^3(-1+x^2)} + \frac{b-bx^2}{a(a-b+2bx^2-bx^4)^3} + \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)^2} + \frac{b-bx^2}{a^3(a-b+2bx^2-bx^4 \cdots}\right.\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{a^3 d} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{a^3 d} \cdots$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3 d} - \frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx \cdots\right.}$$

$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{\sqrt[4]{b}\,\tanh^{-1}\left(\cdots\right)}{2a^3\sqrt{\sqrt{a}+\cdots}}$$

$$= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} d} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(c \cdots}{\cdots}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2} d} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\cos(c+dx \cdots}{\sqrt{\sqrt{a}-\sqrt{b \cdots}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.93, size = 920, normalized size = 1.49

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((32*a*b*Cos[c + d*x]*(-41*a + 23*b + (13*a - 7*b)*Cos[2*(c + d*x)]))/((a -
 b)^2*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (512*a^2*b
*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/((a - b)*(-8*a + 3*b - 4*b*Cos[2*(c
+ d*x)] + b*Cos[4*(c + d*x)])^2) - 256*Log[Cos[(c + d*x)/2]] + 256*Log[Sin[
(c + d*x)/2]] - (I*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6
 + b*#1^8 & , (-90*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 142*a*b*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 64*b^2*ArcTan[Sin[c + d*x]/(Cos[c
 + d*x] - #1)] + (45*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (71*I)*a*b*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (32*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2] + 398*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 506*a*b*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 192*b^2*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^2 - (199*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^
2 + (253*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (96*I)*b^2*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 398*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^4 + 506*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 192*
b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (199*I)*a^2*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^4 - (253*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^4 + (96*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 90*a^2*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 142*a*b*ArcTan[Sin[c + d*x]/(Cos[c
+ d*x] - #1)]*#1^6 + 64*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 -
 (45*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + (71*I)*a*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^6 - (32*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*
#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b)^2)/(
256*a^3*d)
```

Maple [A]
time = 2.01, size = 384, normalized size = 0.62

method	result

derivativedivides	$\frac{\ln(\cos(dx+c)-1)}{2a^3}+\frac{b\left(\frac{-\frac{ab(13a-7b)\left(\cos^7(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}+\frac{(53a-29b)ab\left(\cos^5(dx+c)\right)}{32a^2-64ab+32b^2}+\frac{a\left(17a^2-78ab+37b^2\right)\left(\cos^3(dx+c)\right)}{32a^2-64ab+32b^2}-\frac{5(7a-3b)a\cos}{32(a-}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}$
default	$\frac{\ln(\cos(dx+c)-1)}{2a^3}+\frac{b\left(\frac{-\frac{ab(13a-7b)\left(\cos^7(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}+\frac{(53a-29b)ab\left(\cos^5(dx+c)\right)}{32a^2-64ab+32b^2}+\frac{a\left(17a^2-78ab+37b^2\right)\left(\cos^3(dx+c)\right)}{32a^2-64ab+32b^2}-\frac{5(7a-3b)a\cos}{32(a-}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^3*ln(cos(d*x+c)-1)+b/a^3*((-1/32*a*b*(13*a-7*b)/(a^2-2*a*b+b^2)*
cos(d*x+c)^7+1/32*(53*a-29*b)*a*b/(a^2-2*a*b+b^2)*cos(d*x+c)^5+1/32*a*(17*a
^2-78*a*b+37*b^2)/(a^2-2*a*b+b^2)*cos(d*x+c)^3-5/32*(7*a-3*b)*a/(a-b)*cos(d
*x+c))/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)^2+1/32/(a^2-2*a*b+b^2)*b*(-1/2
```

```
*(-45*a^2*(a*b)^(1/2)+71*a*b*(a*b)^(1/2)-32*b^2*(a*b)^(1/2)+16*a^2*b-10*a*b
^2)/(a*b)^(1/2)/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/(((a*b)^(1
/2)+b)*b)^(1/2))+1/2*(-45*a^2*(a*b)^(1/2)+71*a*b*(a*b)^(1/2)-32*b^2*(a*b)^(
1/2)-16*a^2*b+10*a*b^2)/(a*b)^(1/2)/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(b*co
s(d*x+c)/(((a*b)^(1/2)-b)*b)^(1/2))))-1/2/a^3*ln(1+cos(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(8*(13*a^2*b^4 - 7*a*b^5)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(121*a^2*b
^4 - 67*a*b^5)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 8*(13*a^2*b^4 - 7*a*b^5)
*sin(2*d*x + 2*c)*sin(d*x + c) - ((13*a^2*b^4 - 7*a*b^5)*cos(15*d*x + 15*c)
 - (121*a^2*b^4 - 67*a*b^5)*cos(13*d*x + 13*c) - (272*a^3*b^3 - 461*a^2*b^4
 + 159*a*b^5)*cos(11*d*x + 11*c) + (1424*a^3*b^3 - 1121*a^2*b^4 + 99*a*b^5)
*cos(9*d*x + 9*c) + (1424*a^3*b^3 - 1121*a^2*b^4 + 99*a*b^5)*cos(7*d*x + 7*
c) - (272*a^3*b^3 - 461*a^2*b^4 + 159*a*b^5)*cos(5*d*x + 5*c) - (121*a^2*b^
4 - 67*a*b^5)*cos(3*d*x + 3*c) + (13*a^2*b^4 - 7*a*b^5)*cos(d*x + c))*cos(1
6*d*x + 16*c) - (13*a^2*b^4 - 7*a*b^5 - 8*(13*a^2*b^4 - 7*a*b^5)*cos(14*d*x
 + 14*c) - 4*(104*a^3*b^3 - 147*a^2*b^4 + 49*a*b^5)*cos(12*d*x + 12*c) + 8*
(208*a^3*b^3 - 203*a^2*b^4 + 49*a*b^5)*cos(10*d*x + 10*c) + 2*(1664*a^4*b^2
 - 2144*a^3*b^3 + 1127*a^2*b^4 - 245*a*b^5)*cos(8*d*x + 8*c) + 8*(208*a^3*b
^3 - 203*a^2*b^4 + 49*a*b^5)*cos(6*d*x + 6*c) - 4*(104*a^3*b^3 - 147*a^2*b^
4 + 49*a*b^5)*cos(4*d*x + 4*c) - 8*(13*a^2*b^4 - 7*a*b^5)*cos(2*d*x + 2*c))
*cos(15*d*x + 15*c) - 8*((121*a^2*b^4 - 67*a*b^5)*cos(13*d*x + 13*c) + (272
*a^3*b^3 - 461*a^2*b^4 + 159*a*b^5)*cos(11*d*x + 11*c) - (1424*a^3*b^3 - 11
21*a^2*b^4 + 99*a*b^5)*cos(9*d*x + 9*c) - (1424*a^3*b^3 - 1121*a^2*b^4 + 99
*a*b^5)*cos(7*d*x + 7*c) + (272*a^3*b^3 - 461*a^2*b^4 + 159*a*b^5)*cos(5*d*
x + 5*c) + (121*a^2*b^4 - 67*a*b^5)*cos(3*d*x + 3*c) - (13*a^2*b^4 - 7*a*b^
5)*cos(d*x + c))*cos(14*d*x + 14*c) + (121*a^2*b^4 - 67*a*b^5 - 4*(968*a^3*
b^3 - 1383*a^2*b^4 + 469*a*b^5)*cos(12*d*x + 12*c) + 8*(1936*a^3*b^3 - 1919
*a^2*b^4 + 469*a*b^5)*cos(10*d*x + 10*c) + 2*(15488*a^4*b^2 - 20192*a^3*b^3
 + 10667*a^2*b^4 - 2345*a*b^5)*cos(8*d*x + 8*c) + 8*(1936*a^3*b^3 - 1919*a^
2*b^4 + 469*a*b^5)*cos(6*d*x + 6*c) - 4*(968*a^3*b^3 - 1383*a^2*b^4 + 469*a
*b^5)*cos(4*d*x + 4*c) - 8*(121*a^2*b^4 - 67*a*b^5)*cos(2*d*x + 2*c))*cos(1
3*d*x + 13*c) - 4*((2176*a^4*b^2 - 5592*a^3*b^3 + 4499*a^2*b^4 - 1113*a*b^5
)*cos(11*d*x + 11*c) - (11392*a^4*b^2 - 18936*a^3*b^3 + 8639*a^2*b^4 - 693*
a*b^5)*cos(9*d*x + 9*c) - (11392*a^4*b^2 - 18936*a^3*b^3 + 8639*a^2*b^4 - 6
93*a*b^5)*cos(7*d*x + 7*c) + (2176*a^4*b^2 - 5592*a^3*b^3 + 4499*a^2*b^4 -
1113*a*b^5)*cos(5*d*x + 5*c) + (968*a^3*b^3 - 1383*a^2*b^4 + 469*a*b^5)*cos
(3*d*x + 3*c) - (104*a^3*b^3 - 147*a^2*b^4 + 49*a*b^5)*cos(d*x + c))*cos(12
```

```
*d*x + 12*c) + (272*a^3*b^3 - 461*a^2*b^4 + 159*a*b^5 + 8*(4352*a^4*b^2 - 9
280*a^3*b^3 + 5771*a^2*b^4 - 1113*a*b^5)*cos(10*d*x + 10*c) + 2*(34816*a^5*
b - 85120*a^4*b^2 + 74128*a^3*b^3 - 31399*a^2*b^4 + 5565*a*b^5)*cos(8*d*x +
 8*c) + 8*(4352*a^4*b^2 - 9280*a^3*b^3 + 5771*a^2*b^4 - 1113*a*b^5)*cos(6*d
*x + 6*c) - 4*(2176*a^4*b^2 - 5592*a^3*b^3 + 4499*a^2*b^4 - 1113*a*b^5)*cos
(4*d*x + 4*c) - 8*(272*a^3*b^3 - 461*a^2*b^4 + 159*a*b^5)*cos(2*d*x + 2*c))
*cos(11*d*x + 11*c) - 8*((22784*a^4*b^2 - 27904*a^3*b^3 + 9431*a^2*b^4 - 69
3*a*b^5)*cos(9*d*x + 9*c) + (22784*a^4*b^2 - 27904*a^3*b^3 + 9431*a^2*b^4 -
 693*a*b^5)*cos(7*d*x + 7*c) - (4352*a^4*b^2 - 9280*a^3*b^3 + 5771*a^2*b^4
- 1113*a*b^5)*cos(5*d*x + 5*c) - (1936*a^3*b^3 - 1919*a^2*b^4 + 469*a*b^5)*
cos(3*d*x + 3*c) + (208*a^3*b^3 - 203*a^2*b^4 + 49*a*b^5)*cos(d*x + c))*cos
(10*d*x + 10*c) - (1424*a^3*b^3 - 1121*a^2*b^4 + 99*a*b^5 + 2*(182272*a^5*b
 - 280192*a^4*b^2 + 170128*a^3*b^3 - 48739*a^2*b^4 + 3465*a*b^5)*cos(8*d*x
+ 8*c) + 8*(22784*a^4*b^2 - 27904*a^3*b^3 + 9431*a^2*b^4 - 693*a*b^5)*cos(6
*d*x + 6*c) - 4*(11392*a^4*b^2 - 18936*a^3*b^3 + 8639*a^2*b^4 - 693*a*b^5)*
cos(4*d*x + 4*c) - 8*(1424*a^3*b^3 - 1121*a^2*b^4 + 99*a*b^5)*cos(2*d*x + 2
*c))*cos(9*d*x + 9*c) - 2*((182272*a^5*b - 280192*a^4*b^2 + 170128*a^3*b^3
- 48739*a^2*b^4 + 3465*a*b^5)*cos(7*d*x + 7*c) - (34816*a^5*b - 85120*a^4*b
^2 + 74128*a^3*b^3 - 31399*a^2*b^4 + 5565*a*b^5)*cos(5*d*x + 5*c) - (15488*
a^4*b^2 - 20192*a^3*b^3 + 10667*a^2*b^4 - 2345*a*b^5)*cos(3*d*x + 3*c) + (1
664*a^4*b^2 - 2144*a^3*b^3 + 1127*a^2*b^4 - 245*a*b^5)*cos(d*x + c))*cos(8*
d*x + 8*c) - (1424*a^3*b^3 - 1121*a^2*b^4 + 99*a*b^5 + 8*(22784*a^4*b^2 - 2
7904*a^3*b^3 + 9431*a^2*b^4 - 693*a*b^5)*cos(6*d*x + 6*c) - 4*(11392*a^4*b^
2 - 18936*a^3*b^3 + 8639*a^2*b^4 - 693*a*b^5)*cos(4*d*x + 4*c) - 8*(1424*a^
3*b^3 - 1121*a^2*b^4 + 99*a*b^5)*cos(2*d*x + 2*c))*cos(7*d*x + 7*c) + 8*((4
352*a^4*b^2 - 9280*a^3*b^3 + 5771*a^2*b^4 - 1113*a*b^5)*cos(5*d*x + 5*c) +
(1936*a^3*b^3 - 1919*a^2*b^4 + 469*a*b^5)*cos(3*d*x + 3*c) - (208*a^3*b^3 -
 203*a^2*b^4 + 49*a*b^5)*cos(d*x + c))*cos(6*d*x + 6*c) + (272*a^3*b^3 - 46
1*a^2*b^4 + 159*a*b^5 - 4*(2176*a^4*b^2 - 5592*a^3*b^3 + 4499*a^2*b^4 - 111
3*a*b^5)*cos(4*d*x + 4*c) - 8*(272*a^3*b^3 - 461*a^2*b^4 + 159*a*b^5)*cos(2
*d*x + 2*c))*cos(5*d*x + 5*c) - 4*((968*a^3*b^3 - 1383*a^2*b^4 + 469*a*b^5)
*cos(3*d*x + 3*c) - (104*a^3*b^3 - 147*a^2*b^4 + 49*a*b^5)*cos(d*x + c))*co
s(4*d*x + 4*c) + (121*a^2*b^4 - 67*a*b^5 - 8*(121*a^2*b^4 - 67*a*b^5)*cos(2
*d*x + 2*c))*cos(3*d*x + 3*c) - (13*a^2*b^4 - 7*a*b^5)*cos(d*x + c) - 16*((
a^5*b^4 - 2*a^4*b^5 + a^3*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^5*b^4 - 2*a^4
*b^5 + a^3*b^6)*d*cos(14*d*x + 14*c)^2 + 16*(64...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5020 vs. $2(481) = 962$.
time = 2.24, size = 5020, normalized size = 8.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(13*a^2*b^2 - 7*a*b^3)*cos(d*x + c)^7 - 4*(53*a^2*b^2 - 29*a*b^3)
*cos(d*x + c)^5 - 4*(17*a^3*b - 78*a^2*b^2 + 37*a*b^3)*cos(d*x + c)^3 - ((a
^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a
^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*
cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)
^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(3465*a^4*b
 - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 + (a^11 - 5*a^10*b
+ 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b -
19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 -
 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 -
10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*
a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a
^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log((
4100625*a^6*b - 14762250*a^5*b^2 + 23227949*a^4*b^3 - 20354340*a^3*b^4 + 10
504896*a^2*b^5 - 3044864*a*b^6 + 393216*b^7)*cos(d*x + c) - ((45*a^16 - 280
*a^15*b + 747*a^14*b^2 - 1110*a^13*b^3 + 995*a^12*b^4 - 540*a^11*b^5 + 165*
a^10*b^6 - 22*a^9*b^7)*d^3*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 3997108
6*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 740147
2*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 -
120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 +
45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)) - (123525*a^9*b - 450359*a^8*b
^2 + 715183*a^7*b^3 - 630957*a^6*b^4 + 327152*a^5*b^5 - 95104*a^4*b^6 + 122
88*a^3*b^7)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^
4 + 1024*b^5 + (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6
*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679
452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 15360
00*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 21
0*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*
a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 +
5*a^7*b^4 - a^6*b^5)*d^2))) + ((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x +
c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^
5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*
a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^
3 + a^3*b^4)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b
^4 + 1024*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^
6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 4967
9452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536
000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 2
10*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10
*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 +
 5*a^7*b^4 - a^6*b^5)*d^2))*log((4100625*a^6*b - 14762250*a^5*b^2 + 2322794
9*a^4*b^3 - 20354340*a^3*b^4 + 10504896*a^2*b^5 - 3044864*a*b^6 + 393216*b^
7)*cos(d*x + c) - ((45*a^16 - 280*a^15*b + 747*a^14*b^2 - 1110*a^13*b^3 + 9
95*a^12*b^4 - 540*a^11*b^5 + 165*a^10*b^6 - 22*a^9*b^7)*d^3*sqrt((4100625*a
^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^
```

```
4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((
a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5
 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4
)) + (123525*a^9*b - 450359*a^8*b^2 + 715183*a^7*b^3 - 630957*a^6*b^4 + 327
152*a^5*b^5 - 95104*a^4*b^6 + 12288*a^3*b^7)*d)*sqrt(-(3465*a^4*b - 9306*a^
3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 - (a^11 - 5*a^10*b + 10*a^9*b
^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a
^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*
a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b
+ 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 -
 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^
10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))) + ((a^5*b^2 -
2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d
*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x +
 c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7
 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(3465*a^4*b - 9306*a
^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 + (a^11 - 5*a^10*b + 10*a^9*
b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*
a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992
*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b
 + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6
- 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa,sageVARb]=[53,-89
]Warning, need t
```

Mupad [B]
time = 20.83, size = 2500, normalized size = 4.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - b*sin(c + d*x)^4)^3),x)
```

```
[Out] - ((5*cos(c + d*x)*(7*a*b - 3*b^2))/(32*a^2*(a - b)) - (cos(c + d*x)^3*(17*
a^2*b - 78*a*b^2 + 37*b^3))/(32*a^2*(a - b)^2) - (cos(c + d*x)^5*(53*a*b^2
- 29*b^3))/(32*a^2*(a - b)^2) + (b*cos(c + d*x)^7*(13*a*b - 7*b^2))/(32*a^2
*(a - b)^2))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b^2) - cos(c
 + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d*x)^8)) - (
atan(((((((((192*a^11*b^9 - 990*a^12*b^8 + 2050*a^13*b^7 - 2154*a^14*b^6 + 1
158*a^15*b^5 - 256*a^16*b^4)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 +
6*a^12*b^2)) - (cos(c + d*x)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 34
89660928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a
^17*b^4))/(2097152*a^3*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)
))/(2*a^3) + (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 59274854
4*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(1048576*(a^12 - 4*a^1
1*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3) - (12*a^5*b^9 - (4311*a^6
*b^8)/64 + (307961*a^7*b^7)/2048 - (1290253*a^8*b^6)/8192 + (546059*a^9*b^5
)/8192)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))*1i)/(2*
a^3) - (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 230
76232*a^3*b^6 + 8247825*a^4*b^5)*1i)/(1048576*(a^12 - 4*a^11*b + a^8*b^4 -
4*a^9*b^3 + 6*a^10*b^2)))/a^3 - ((((((192*a^11*b^9 - 990*a^12*b^8 + 2050*a^
13*b^7 - 2154*a^14*b^6 + 1158*a^15*b^5 - 256*a^16*b^4)/(2*(a^14 - 4*a^13*b
+ a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) + (cos(c + d*x)*(402653184*a^12*b^9
- 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395
008*a^16*b^5 - 268435456*a^17*b^4))/(2097152*a^3*(a^12 - 4*a^11*b + a^8*b^4
 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3) - (cos(c + d*x)*(75497472*a^6*b^9 - 33
7215488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^
5))/(1048576*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3)
 - (12*a^5*b^9 - (4311*a^6*b^8)/64 + (307961*a^7*b^7)/2048 - (1290253*a^8*b
^6)/8192 + (546059*a^9*b^5)/8192)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b
^3 + 6*a^12*b^2)))*1i)/(2*a^3) + (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^
8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*b^5)*1i)/(1048576*(a^
12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/a^3)/((((((192*a^11*b^9
 - 990*a^12*b^8 + 2050*a^13*b^7 - 2154*a^14*b^6 + 1158*a^15*b^5 - 256*a^16*
b^4)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) - (cos(c +
d*x)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221
225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^4))/(2097152*a^3*(
a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3) + (cos(c + d*
x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^
9*b^6 + 163684352*a^10*b^5))/(1048576*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^
```

```
3 + 6*a^10*b^2)))/(2*a^3) - (12*a^5*b^9 - (4311*a^6*b^8)/64 + (307961*a^7*b
^7)/2048 - (1290253*a^8*b^6)/8192 + (546059*a^9*b^5)/8192)/(2*(a^14 - 4*a^1
3*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))/(2*a^3) - (cos(c + d*x)*(314572
8*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*
b^5))/(1048576*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/a^3 +
 (((((192*a^11*b^9 - 990*a^12*b^8 + 2050*a^13*b^7 - 2154*a^14*b^6 + 1158*a^
15*b^5 - 256*a^16*b^4)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12
*b^2)) + (cos(c + d*x)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 34896609
28*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^
4))/(2097152*a^3*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*
a^3) - (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 592748544*a^8*
b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(1048576*(a^12 - 4*a^11*b +
a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3) - (12*a^5*b^9 - (4311*a^6*b^8)/
64 + (307961*a^7*b^7)/2048 - (1290253*a^8*b^6)/8192 + (546059*a^9*b^5)/8192
)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))/(2*a^3) + (co
s(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*
b^6 + 8247825*a^4*b^5))/(1048576*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6
*a^10*b^2)))/a^3 + ((90009*a*b^7)/32768 - (81*b^8)/128 - (271845*a^2*b^6)/6
5536 + (1184625*a^3*b^5)/524288)/(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 +
 6*a^12*b^2)))*1i)/(a^3*d) - (atan((((((201326592*a^11*b^9 - 1038090240*a^
12*b^8 + 2149580800*a^13*b^7 - 2258632704*a^14*b^6 + 1214251008*a^15*b^5 -
268435456*a^16*b^4)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a
^12*b^2)) - (cos(c + d*x)*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/
2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b
^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(
a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^
3 - 10*a^15*b^2)))^(1/2)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 348966
0928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*
b^4))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(202
5*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) -...
```

3.230 $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=319

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d}-(a$$

```
[Out] -1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-5*b^(1/
2))/a^(3/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctan((a^(1/2)+b^(1/2))
^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+5*b^(1/2))/a^(3/4)/b^(3/2)/d/(a^(1/2)
+b^(1/2))^(5/2)-1/32*(a+5*b)*tan(d*x+c)/a/(a-b)^2/b/d+1/32*tan(d*x+c)^3/a/(
a-b)/b/d+1/8*tan(d*x+c)^9/a/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/3
2*sec(d*x+c)^2*tan(d*x+c)^5/a/b/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A]

time = 0.35, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3296, 1289, 12, 1134, 1293, 1180, 211}

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^3(c+dx)}{32abd(a-b)}+\frac{\tan^9(c+dx)}{8ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}-\frac{(a+5b)\tan(c+dx)}{32abd(a-b)^2}-\frac{\tan^5(c+dx)\sec^2(c+dx)}{32abd\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((2*Sqrt[a] - 5*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x]
)/a^(1/4)])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] + 5
*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(64*a^(3/
4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tan[c + d*x])/(32*a*(a
 - b)^2*b*d) + Tan[c + d*x]^3/(32*a*(a - b)*b*d) + Tan[c + d*x]^9/(8*a*d*(a
 + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Sec[c + d*x]^2*Tan[c
+ d*x]^5)/(32*a*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^8(1+x^2)}{(a+2ax^2+(a-b)x^4)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int -\frac{2bx^8}{(a+2ax^2+(a-b)x^4)}\right.}{16abd}$$

$$= \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^8}{(a+2ax^2+(a-b)x^4)^2}\right.}{8ad}$$

$$= \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\sec^2(c+dx)}{32abd\left(a+2a\tan^2(c+dx\right.}$$

$$= \frac{\tan^3(c+dx)}{32a(a-b)bd} + \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{}{32abd\,(a}$$

$$= -\frac{(a+5b)\tan(c+dx)}{32a(a-b)^2bd} + \frac{\tan^3(c+dx)}{32a(a-b)bd} + \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\,\text{ta}\right.}$$

$$= -\frac{(a+5b)\tan(c+dx)}{32a(a-b)^2bd} + \frac{\tan^3(c+dx)}{32a(a-b)bd} + \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\,\text{ta}\right.}$$

$$= -\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} + \frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tan^{-1}}{64a^{3/4}\left(\sqrt{a}\right.}$$

Mathematica [A]
time = 2.95, size = 331, normalized size = 1.04

$$\frac{\frac{\left(2a^{3/2}\sqrt{b}+ab-8\sqrt{a}\,b^{3/2}+5b^2\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}} + \frac{\left(2\sqrt{a}-5\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}} + \frac{8b(5a-14b+(-2a+5b)\cos(2(c+dx)))\sin(2(c+dx))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))} + \frac{64a(a-b)b(-6\sin(2(c+dx))+\sin(4(c+dx)))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}}{64(a-b)^2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (((2*a^(3/2)*Sqrt[b] + a*b - 8*Sqrt[a]*b^(3/2) + 5*b^2)*ArcTan[((Sqrt[a] +
Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a
]*Sqrt[b]]) + ((2*Sqrt[a] - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*Sqrt[b]*ArcTan
h[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*
```

```
Sqrt[-a + Sqrt[a]*Sqrt[b]]) + (8*b*(5*a - 14*b + (-2*a + 5*b)*Cos[2*(c + d*
x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x
)]) + (64*a*(a - b)*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*b
 - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)/(64*(a - b)^2*b^2*d)
```

Maple [A]
time = 1.16, size = 374, normalized size = 1.17

method	result
derivativedivides	$\frac{\frac{-\frac{(a+19b)\left(\tan^{7}(dx+c)\right)}{32(a-b)b}-\frac{3\left(a^{2}+10ab-3b^{2}\right)\left(\tan^{5}(dx+c)\right)}{32b\left(a^{2}-2ab+b^{2}\right)}-\frac{3(a+7b)a\left(\tan^{3}(dx+c)\right)}{32b\left(a^{2}-2ab+b^{2}\right)}-\frac{a(a+5b)\tan(dx+c)}{32b\left(a^{2}-2ab+b^{2}\right)}}{\left(\left(\tan^{4}(dx+c)\right)a-\left(\tan^{4}(dx+c)\right)b+2a\left(\tan^{2}(dx+c)\right)+a\right)^{2}}+\frac{(a-b)\left(\frac{\left(-a\sqrt{ab}+\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{-\frac{(a+19b)\left(\tan^{7}(dx+c)\right)}{32(a-b)b}-\frac{3\left(a^{2}+10ab-3b^{2}\right)\left(\tan^{5}(dx+c)\right)}{32b\left(a^{2}-2ab+b^{2}\right)}-\frac{3(a+7b)a\left(\tan^{3}(dx+c)\right)}{32b\left(a^{2}-2ab+b^{2}\right)}-\frac{a(a+5b)\tan(dx+c)}{32b\left(a^{2}-2ab+b^{2}\right)}}{\left(\left(\tan^{4}(dx+c)\right)a-\left(\tan^{4}(dx+c)\right)b+2a\left(\tan^{2}(dx+c)\right)+a\right)^{2}}+\frac{(a-b)\left(\frac{\left(-a\sqrt{ab}+\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/32*(a+19*b)/(a-b)/b*tan(d*x+c)^7-3/32*(a^2+10*a*b-3*b^2)/b/(a^2-2*
a*b+b^2)*tan(d*x+c)^5-3/32*(a+7*b)*a/b/(a^2-2*a*b+b^2)*tan(d*x+c)^3-1/32*a*
(a+5*b)/b/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*ta
n(d*x+c)^2+a)^2+1/32/b/(a^2-2*a*b+b^2)*(a-b)*(1/2*(-a*(a*b)^(1/2)+13*(a*b)^
(1/2)*b-2*a^2+9*a*b+5*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*
arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(-a*(a*b)^(1/2)+
13*(a*b)^(1/2)*b+2*a^2-9*a*b-5*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")

[Out] -1/8*(4*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((a*b^3 - 4*b^4)*sin(14*d*x + 14*c) - (32*a^2*b^2 - 58*a*b^3 - b^4)*sin(12*d*x + 12*c) + 3*(48*a^2*b^2 - 73*a*b^3 + 20*b^4)*sin(10*d*x + 10*c) + (256*a^3*b - 832*a^2*b^2 + 550*a*b^3 - 175*b^4)*sin(8*d*x + 8*c) + (112*a^2*b^2 - 533*a*b^3 + 220*b^4)*sin(6*d*x + 6*c) - (32*a^2*b^2 - 158*a*b^3 + 141*b^4)*sin(4*d*x + 4*c) - (17*a*b^3 - 44*b^4)*sin(2*d*x + 2*c))*cos(16*d*x + 16*c) + 2*(2*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*sin(12*d*x + 12*c) - 8*(80*a^2*b^2 - 145*a*b^3 + 44*b^4)*sin(10*d*x + 10*c) - 3*(384*a^3*b - 1312*a^2*b^2 + 873*a*b^3 - 280*b^4)*sin(8*d*x + 8*c) - 16*(32*a^2*b^2 - 151*a*b^3 + 62*b^4)*sin(6*d*x + 6*c) + 2*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*sin(4*d*x + 4*c) + 24*(3*a*b^3 - 8*b^4)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*sin(10*d*x + 10*c) - (6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^4)*sin(8*d*x + 8*c) - 2*(128*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*sin(6*d*x + 6*c) + 4*(400*a^2*b^2 - 918*a*b^3 + 497*b^4)*sin(4*d*x + 4*c) - 2*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) - 2*((2048*a^4 + 18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3 - 2800*b^4)*sin(8*d*x + 8*c) + 8*(256*a^3*b + 2400*a^2*b^2 - 2379*a*b^3 + 560*b^4)*sin(6*d*x + 6*c) - 2*(128*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*sin(4*d*x + 4*c) + 16*(32*a^2*b^2 - 151*a*b^3 + 62*b^4)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((2048*a^4 + 18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3 - 2800*b^4)*sin(6*d*x + 6*c) - (6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^4)*sin(4*d*x + 4*c) + 3*(384*a^3*b - 1312*a^2*b^2 + 873*a*b^3 - 280*b^4)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) - 4*((128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*sin(4*d*x + 4*c) + 4*(80*a^2*b^2 - 145*a*b^3 + 44*b^4)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 8*((a^2*b^5 - 2*a*b^6 + b^7)*d*cos(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^7)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c)^2 + (a^2*b^5 - 2*a*b^6 + b^7)*d*sin(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^7)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*

```
b^7)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*
d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*
d*x + 2*c)^2 - 16*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) + (a^2*b^5 -
 2*a*b^6 + b^7)*d - 2*(8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(14*d*x + 14*c) + 4
*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a
^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^4*b
^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d*x + 8*c) - 8
*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^3
*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(a^2*b^5 - 2*a
*b^6 + b^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*cos(16*d*x +
16*c) + 16*(4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12
*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) -
 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d
*x + 8*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*
c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(
a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*
cos(14*d*x + 14*c) - 8*(8*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*
b^6 + 49*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^5*b^2 - 3712*a^4*b^3 + 5304*
a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*cos(8*d*x + 8*c) + 8*(128*
a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c
) - 4*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4
*d*x + 4*c) - 8*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(2*d*x + 2
*c) + (8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d)*cos(12*d*x + 12*c) + 1
6*(2*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^
6 - 245*b^7)*d*cos(8*d*x + 8*c) + 8*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^
5 - 322*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c) - 4*...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5219 vs. $2(263) = 526$.
time = 1.47, size = 5219, normalized size = 16.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4
+ b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d
*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4
*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(((a^6*b^3 - 5*a^5*b^4 +
 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^
3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a
^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*
b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) - 4*a^3 + 35*a^2*b - 70*
a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^
```

```
7 - a*b^8)*d^2))*log(35*a^3 - 1491/4*a^2*b + 1875/2*a*b^2 + 625/4*b^3 - 1/4
*(140*a^3 - 1491*a^2*b + 3750*a*b^2 + 625*b^3)*cos(d*x + c)^2 + 1/2*((a^9*b
^3 - 18*a^8*b^4 + 75*a^7*b^5 - 140*a^6*b^6 + 135*a^5*b^7 - 66*a^4*b^8 + 13*
a^3*b^9)*d^3*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 62
5*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7
- 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^
3*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (70*a^5*b - 623*a^4*b^2 + 1161*a^
3*b^3 + 995*a^2*b^4 + 125*a*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^6*b^
3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225
*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*
a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7
*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) - 4*a^3 +
 35*a^2*b - 70*a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3
*b^6 + 5*a^2*b^7 - a*b^8)*d^2)) + 1/4*(2*(4*a^8*b - 45*a^7*b^2 + 165*a^6*b^
3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2*cos(d*x + c)^
2 - (4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a
^3*b^6 + 25*a^2*b^7)*d^2)*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 77
00*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 +
 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*
a^4*b^12 + a^3*b^13)*d^4))) - ((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 -
 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*
a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d
*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt((
(a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqr
t((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^
3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 +
210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) -
4*a^3 + 35*a^2*b - 70*a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 -
 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(35*a^3 - 1491/4*a^2*b + 1875/2*a
*b^2 + 625/4*b^3 - 1/4*(140*a^3 - 1491*a^2*b + 3750*a*b^2 + 625*b^3)*cos(d*
x + c)^2 - 1/2*((a^9*b^3 - 18*a^8*b^4 + 75*a^7*b^5 - 140*a^6*b^6 + 135*a^5*
b^7 - 66*a^4*b^8 + 13*a^3*b^9)*d^3*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2
*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a
^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b
^11 - 10*a^4*b^12 + a^3*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (70*a^5*b -
 623*a^4*b^2 + 1161*a^3*b^3 + 995*a^2*b^4 + 125*a*b^5)*d*cos(d*x + c)*sin(d
*x + c))*sqrt(((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 -
 a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625
*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 -
 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3
*b^13)*d^4)) - 4*a^3 + 35*a^2*b - 70*a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4
 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2)) + 1/4*(2*(4*a^8*b - 4
5*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*
b^7)*d^2*cos(d*x + c)^2 - (4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4
 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2)*sqrt((1225*a^4 - 10780*a^3*
```

```
b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^1
1*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^
10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4))) + ((a^2*b^3 - 2*a*b^4 + b
^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^
3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*
b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*
a*b^4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a
^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b
^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a
^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^
12 + a^3*b^13)*d^4)) + 4*a^3 - 35*a^2*b + 70*a*b^2 + 105*b^3)/((a^6*b^3 - 5
*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(-35*a^3 +
 1491/4*a^2*b - 1875/2*a*b^2 - 625/4*b^3 + 1/4*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1989 vs. 2(263) = 526.
time = 1.32, size = 1989, normalized size = 6.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/64*(((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 45*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 77*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*a*b^2 + 13*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)
*b^3)*(a^2*b - 2*a*b^2 + b^3)^2*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt(a*b)
*(a - b))*a^6*b - 49*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^3 + 112*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^4 - 87*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*a^2*b^5 + 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^6 + 5*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*b^7)*abs(a^2*b - 2*a*b^2 + b^3)*abs(-a + b) - (6*
sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b - 63*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^2 + 229*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*a^6*b^3 - 367*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
5*b^4 + 233*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^5 + 27*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^6 - 89*sqrt(a^2 - a*b + sqr
```

```
t(a*b)*(a - b))*sqrt(a*b)*a^2*b^7 + 19*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a*b^8 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^9)*abs(
-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3*b -
2*a^2*b^2 + a*b^3 + sqrt((a^3*b - 2*a^2*b^2 + a*b^3)^2 - (a^3*b - 2*a^2*b^2
 + a*b^3)*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)))/(a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4))))/((3*a^10*b^2 - 27*a^9*b^3 + 104*a^8*b^4 - 224*a^7*b^5 + 294*a^6
*b^6 - 238*a^5*b^7 + 112*a^4*b^8 - 24*a^3*b^9 - a^2*b^10 + a*b^11)*abs(a^2*
b - 2*a*b^2 + b^3)) + ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3
 - 45*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 77*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 13*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*sqrt(a*b)*b^3)*(a^2*b - 2*a*b^2 + b^3)^2*abs(-a + b) - (3*sqrt(a^2 -
a*b - sqrt(a*b)*(a - b))*a^6*b - 49*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4
*b^3 + 112*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^4 - 87*sqrt(a^2 - a*b
- sqrt(a*b)*(a - b))*a^2*b^5 + 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^6
 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^7)*abs(a^2*b - 2*a*b^2 + b^3)*ab
s(-a + b) - (6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b - 63*sqr
t(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^2 + 229*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^3 - 367*sqrt(a^2 - a*b - sqrt(a*b)*(a - b
))*sqrt(a*b)*a^5*b^4 + 233*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^
4*b^5 + 27*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^6 - 89*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^7 + 19*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a*b^8 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt
(a*b)*b^9)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)
/sqrt((a^3*b - 2*a^2*b^2 + a*b^3 - sqrt((a^3*b - 2*a^2*b^2 + a*b^3)^2 - (a^
3*b - 2*a^2*b^2 + a*b^3)*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)))/(a^3*b - 3*a
^2*b^2 + 3*a*b^3 - b^4))))/((3*a^10*b^2 - 27*a^9*b^3 + 104*a^8*b^4 - 224*a^
7*b^5 + 294*a^6*b^6 - 238*a^5*b^7 + 112*a^4*b^8 - 24*a^3*b^9 - a^2*b^10 + a
*b^11)*abs(a^2*b - 2*a*b^2 + b^3)) + 2*(a^2*tan(d*x + c)^7 + 18*a*b*tan(d*x
 + c)^7 - 19*b^2*tan(d*x + c)^7 + 3*a^2*tan(d*x + c)^5 + 30*a*b*tan(d*x + c
)^5 - 9*b^2*tan(d*x + c)^5 + 3*a^2*tan(d*x + c)^3 + 21*a*b*tan(d*x + c)^3 +
 a^2*tan(d*x + c) + 5*a*b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)
^4 + 2*a*tan(d*x + c)^2 + a)^2*(a^2*b - 2*a*b^2 + b^3)))/d
```

Mupad [B]
time = 19.65, size = 2500, normalized size = 7.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] (atan(((((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768*a^4*b^4 + 1
6384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) - (tan(c + d*x)
*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b
^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2)))/(16384*(a^3*b^11 - 5
```

*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*(16384*a
^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 81920*a^6*b^4 -
16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*
b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4
 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*
a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) + (tan(c + d*x)*(259*a*
b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 - b^5 - 3*a^2*
b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^
2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(163
84*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6))
)^(1/2)*1i - (((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768*a^4*b
^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) + (tan(c
+ d*x)*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70
*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^
11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*(1
6384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 81920*a^6*
b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2
*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a
^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10
 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) - (tan(c + d*x)*(
259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 - b^5 -
3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) +
105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2)
)/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8
*b^6)))^(1/2)*1i)/((4*a^2 - 77*a*b + 325*b^2)/(16384*(3*a*b^5 - b^6 - 3*a^2
*b^4 + a^3*b^3)) + (((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768
*a^4*b^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) - (
tan(c + d*x)*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^
6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(
a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1
/2)*(16384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 8192
0*a^6*b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((
25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5
- 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^
4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) + (tan(c +
d*x)*(259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 -
b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1
/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)
^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7
 - a^8*b^6)))^(1/2) + (((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32
768*a^4*b^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3))
+ (tan(c + d*x)*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2
*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(1638
4*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))
^(1/2)*(16384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 8

```
1920*a^6*b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))
*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b
^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5
*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) - (tan(c
 + d*x)*(259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4
 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)
^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b
^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*
b^7 - a^8*b^6)))^(1/2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2)
+ 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/
2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a
^8*b^6)))^(1/2)*2i)/d - ((tan(c + d*x)^7*(a + 19*b))/(32*(a*b - b^2)) + (3*
tan(c + d*x)^3*(7*a*b + a^2))/(32*(a^2*b - 2*a*b^2 + b^3)) + (a*tan(c + d*x
)*(a + 5*b))/(32*(a^2*b - 2*a*b^2 + b^3)) + (3*tan(c + d*x)^5*(10*a*b + a^2
 - 3*b^2))/(32*(a - b)*(a*b - b^2)))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2)
 + a^2 - tan(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) +
4*a^2*tan(c + d*x)^2)) + (atan(((((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^
3*b^5 + 32768*a^4*b^4 + 16384*a^5*b^3)/(32768*(...
```

3.231 $\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

Optimal. Leaf size=343

$$-\frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d}+\frac{\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}}{\sqrt[4]{a}}\right.}{64a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d}$$

```
[Out] -1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(4*a+3*b-10*a^(1/2
)*b^(1/2))/a^(5/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctan((a^(1/2)+b
^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(4*a+3*b+10*a^(1/2)*b^(1/2))/a^(5/4)/b^(3
/2)/d/(a^(1/2)+b^(1/2))^(5/2)-1/8*tan(d*x+c)*(a*(a+3*b)+(a^2+6*a*b+b^2)*tan
(d*x+c)^2)/(a-b)^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*tan(d*x
+c)*(2*a*(a^2-a*b-8*b^2)/(a-b)^3+(2*a^2+15*a*b+3*b^2)*tan(d*x+c)^2/(a-b)^2)
/a/b/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A]
time = 0.51, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1347, 1692, 1180, 211}

$$-\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tan(c+dx)\left(\frac{(2a^2+15ab+3b^2)\tan^2(c+dx)}{(a-b)^2}+\frac{2a(a^2-ab-8b^2)}{(a-b)^3}\right)}{32abd\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\tan(c+dx)\left((a^2+6ab+b^2)\tan^2(c+dx)+a(a+3b)\right)}{8d(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan
[c + d*x])/a^(1/4)])/(a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((4*a
+ 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a
^(1/4)])/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - (Tan[c + d*x]*(
a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/(8*(a - b)^3*d*(a + 2*a*
Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Tan[c + d*x]*((2*a*(a^2 - a*
b - 8*b^2))/(a - b)^3 + ((2*a^2 + 15*a*b + 3*b^2)*Tan[c + d*x]^2)/(a - b)^2
))/(32*a*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6(1+x^2)^2}{(a+2ax^2+(a-b)x^4)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\tan(c+dx)\left(a(a+3b)+(a^2+6ab+b^2)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^3b(c}{(a-}}{}\right.}{}$$

$$= -\frac{\tan(c+dx)\left(a(a+3b)+(a^2+6ab+b^2)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a}{}\right.}{32abd\,(a+2a\,\mathrm{t}}$$

$$= -\frac{\tan(c+dx)\left(a(a+3b)+(a^2+6ab+b^2)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a}{}\right.}{32abd\,(a+2a\,\mathrm{t}}$$

$$= -\frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} + \frac{\left(4a+10\sqrt{a}\right.}{}$$

Mathematica [A]
time = 2.58, size = 350, normalized size = 1.02

$$\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\sqrt{b}\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{a\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{b}\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{a\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{4b(4a^2-19ab-3b^2+3b(a+b)\cos(2(c+dx)))\sin(2(c+dx))}{a(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))}-\frac{128(a-b)b(2a+b-b\cos(2(c+dx)))\sin(2(c+dx))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}}{64(a-b)^2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (((Sqrt[a] - Sqrt[b])^2*Sqrt[b]*(4*a + 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((S
qrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a + Sqr
t[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])^2*Sqrt[b]*(4*a - 10*Sqrt[a]*Sqrt[b] +
 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]
])/(a*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + (4*b*(4*a^2 - 19*a*b - 3*b^2 + 3*b*(a +
 b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(a*(8*a - 3*b + 4*b*Cos[2*(c + d*x)
] - b*Cos[4*(c + d*x)])) - (128*(a - b)*b*(2*a + b - b*Cos[2*(c + d*x)])*Si
n[2*(c + d*x)])/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)
/(64*(a - b)^2*b^2*d)
```

Maple [A]
time = 1.73, size = 432, normalized size = 1.26

method	result
derivativedivides	$\frac{-\frac{\left(2a^2+15ab+3b^2\right)\left(\tan^7(dx+c)\right)}{32(a-b)ab}-\frac{\left(3a^2+14ab-5b^2\right)\left(\tan^5(dx+c)\right)}{16b\left(a^2-2ab+b^2\right)}-\frac{\left(6a^2+19ab-b^2\right)\left(\tan^3(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{a(a+2b)\tan(dx+c)}{16b\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\cdots\right.}{\cdots}$
default	$\frac{-\frac{\left(2a^2+15ab+3b^2\right)\left(\tan^7(dx+c)\right)}{32(a-b)ab}-\frac{\left(3a^2+14ab-5b^2\right)\left(\tan^5(dx+c)\right)}{16b\left(a^2-2ab+b^2\right)}-\frac{\left(6a^2+19ab-b^2\right)\left(\tan^3(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{a(a+2b)\tan(dx+c)}{16b\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\cdots\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/32*(2*a^2+15*a*b+3*b^2)/(a-b)/a/b*tan(d*x+c)^7-1/16*(3*a^2+14*a*b-
5*b^2)/b/(a^2-2*a*b+b^2)*tan(d*x+c)^5-1/32*(6*a^2+19*a*b-b^2)/b/(a^2-2*a*b+
b^2)*tan(d*x+c)^3-1/16*a*(a+2*b)/b/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+c)^
4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2+1/32/a/b/(a^2-2*a*b+b^2)*(a-b)*(1/
2*(-2*a^2*(a*b)^(1/2)+17*a*b*(a*b)^(1/2)-3*b^2*(a*b)^(1/2)-4*a^3+15*a^2*b+a
*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+
c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(-2*a^2*(a*b)^(1/2)+17*a*b*(a*b)^(1/2
)-3*b^2*(a*b)^(1/2)+4*a^3-15*a^2*b-a*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a
)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

[Out] -1/16*(4*(32*a^3*b^2 - 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c) + ((4*a^2*b^3 - 13*a*b^4 + 3*b^5)*sin(14*d*x + 14*c) - 3*(8*a
^2*b^3 - 33*a*b^4 + 7*b^5)*sin(12*d*x + 12*c) + (64*a^3*b^2 + 68*a^2*b^3 -
225*a*b^4 + 63*b^5)*sin(10*d*x + 10*c) - 3*(128*a^3*b^2 + 32*a^2*b^3 - 61*a
*b^4 + 35*b^5)*sin(8*d*x + 8*c) - (64*a^3*b^2 + 452*a^2*b^3 - 9*a*b^4 - 105
*b^5)*sin(6*d*x + 6*c) + 3*(40*a^2*b^3 - 29*a*b^4 - 21*b^5)*sin(4*d*x + 4*c
) - (4*a^2*b^3 - 37*a*b^4 - 21*b^5)*sin(2*d*x + 2*c))*cos(16*d*x + 16*c) +
2*(2*(32*a^3*b^2 - 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*sin(12*d*x + 12*c) - 8*(
64*a^3*b^2 - 84*a^2*b^3 - 43*a*b^4 + 21*b^5)*sin(10*d*x + 10*c) - (512*a^4*
b - 3584*a^3*b^2 + 1388*a^2*b^3 - 11*a*b^4 - 315*b^5)*sin(8*d*x + 8*c) + 16
*(172*a^2*b^3 - 37*a*b^4 - 21*b^5)*sin(6*d*x + 6*c) + 2*(32*a^3*b^2 - 372*a
^2*b^3 + 289*a*b^4 + 105*b^5)*sin(4*d*x + 4*c) + 8*(4*a^2*b^3 - 25*a*b^4 -
9*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(512*a^4*b - 672*a^3*b^2
 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*sin(10*d*x + 10*c) - 3*(3072*a^4*b -
6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4 - 245*b^5)*sin(8*d*x + 8*c) - 2*(51
2*a^4*b + 3936*a^3*b^2 - 6740*a^2*b^3 + 1281*a*b^4 + 441*b^5)*sin(6*d*x + 6
c) + 12(192*a^3*b^2 - 416*a^2*b^3 + 161*a*b^4 + 49*b^5)*sin(4*d*x + 4*c)
 - 2*(32*a^3*b^2 - 372*a^2*b^3 + 289*a*b^4 + 105*b^5)*sin(2*d*x + 2*c))*cos(
12*d*x + 12*c) - 2*((8192*a^5 + 27136*a^4*b - 37696*a^3*b^2 + 17644*a^2*b^3
 - 2079*a*b^4 - 735*b^5)*sin(8*d*x + 8*c) + 8*(1024*a^4*b + 3712*a^3*b^2 -
3692*a^2*b^3 + 483*a*b^4 + 147*b^5)*sin(6*d*x + 6*c) - 2*(512*a^4*b + 3936*
a^3*b^2 - 6740*a^2*b^3 + 1281*a*b^4 + 441*b^5)*sin(4*d*x + 4*c) - 16*(172*a
^2*b^3 - 37*a*b^4 - 21*b^5)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((8192
*a^5 + 27136*a^4*b - 37696*a^3*b^2 + 17644*a^2*b^3 - 2079*a*b^4 - 735*b^5)*
sin(6*d*x + 6*c) - 3*(3072*a^4*b - 6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4
 - 245*b^5)*sin(4*d*x + 4*c) + (512*a^4*b - 3584*a^3*b^2 + 1388*a^2*b^3 - 11
*a*b^4 - 315*b^5)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) - 4*((512*a^4*b - 672*
a^3*b^2 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*sin(4*d*x + 4*c) + 4*(64*a^3*b
^2 - 84*a^2*b^3 - 43*a*b^4 + 21*b^5)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 1
6*((a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(16*d*x + 16*c)^2 + 64*(a^3*b^5 - 2*a
^2*b^6 + a*b^7)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337
*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^3
 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*cos(10*d*x + 10*c)
^2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*a^5*b^3 - 67648*a^4*b^4 + 32841
*a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^5*b^
3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*cos(6*d*x + 6*c)^
2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*
cos(4*d*x + 4*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c)^2
 + (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(16*d*x + 16*c)^2 + 64*(a^3*b^5 - 2*a^
2*b^6 + a*b^7)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*
a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^5*b^3
 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*sin(10*d*x + 10*c)^
2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*a^5*b^3 - 67648*a^4*b^4 + 32841*
a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^5*b^3
 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*sin(6*d*x + 6*c)^2

```
 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*s
in(4*d*x + 4*c)^2 + 64*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(2*
d*x + 2*c)^2 - 16*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c) + (a^3*b
^5 - 2*a^2*b^6 + a*b^7)*d - 2*(8*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(14*d*x
 + 14*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(12*d*x +
 12*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(10*d*x +
10*c) - 2*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7
)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d
*cos(6*d*x + 6*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos
(4*d*x + 4*c) + 8*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c) - (a^3*b
^5 - 2*a^2*b^6 + a*b^7)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^4*b^4 - 23*a^3*b
^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^4 - 39*a^3*b^
5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^3 - 352*a^4*b
^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b
^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^4 -
 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(4*d*x + 4*c) + 8*(a^3*b^5 - 2*a^2
*b^6 + a*b^7)*d*cos(2*d*x + 2*c) - (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d)*cos(14*
d*x + 14*c) - 8*(8*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 +
 49*a*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^6*b^2 - 3712*a^5*b^3 + 5304*a^4
*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*cos(8*d*x + 8*c) + 8*(128
*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5961 vs. $2(291) = 582$.
time = 2.08, size = 5961, normalized size = 17.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2
*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b
^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*
x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt(-(16
*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 +
 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*
a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^
6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 2
52*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*
b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 -
a^2*b^8)*d^2))*log(320*a^5 - 2724*a^4*b + 6243*a^3*b^2 - 9389/4*a^2*b^3 + 7
29/2*a*b^4 - 81/4*b^5 - 1/4*(1280*a^5 - 10896*a^4*b + 24972*a^3*b^2 - 9389*
a^2*b^3 + 1458*a*b^4 - 81*b^5)*cos(d*x + c)^2 + 1/2*((2*a^11*b^3 - 27*a^10*
```

b^4 + 108*a^9*b^5 - 205*a^8*b^6 + 210*a^7*b^7 - 117*a^6*b^8 + 32*a^5*b^9 - 3*a^4*b^10)*d^3*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) + (320*a^7*b - 2404*a^6*b^2 + 4779*a^5*b^3 - 1025*a^4*b^4 + 49*a^3*b^5 + 9*a^2*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2)) - 1/4*(2*(16*a^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8)*d^2*cos(d*x + c)^2 - (16*a^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8)*d^2)*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))) - ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2))*log(320*a^5 - 2724*a^4*b + 6243*a^3*b^2 - 9389/4*a^2*b^3 + 729/2*a*b^4 - 81/4*b^5 - 1/4*(1280*a^5 - 10896*a^4*b + 24972*a^3*b^2 - 9389*a^2*b^3 + 1458*a*b^4 - 81*b^5)*cos(d*x + c)^2 - 1/2*((2*a^11*b^3 - 27*a^10*b^4 + 108*a^9*b^5 - 205*a^8*b^6 + 210*a^7*b^7 - 117*a^6*b^8 + 32*a^5*b^9 - 3*a^4*b^10)*d^3*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) + (320*a^7*b - 2404*a^6*b^2 + 4779*a^5*b^3 - 1025*a^4*b^4 + 49*a^3*b^5 + 9*a^2*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^

```
4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2)) - 1/4*(2*(16*a^10*b - 156*a^9*b^2 + 549*
a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8
)*d^2*cos(d*x + c)^2 - (16*a^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4
 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8)*d^2)*sqrt((6400*a^6
 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^
5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^1
1*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^
12 + a^5*b^13)*d^4))) + ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4
*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. 2(291) = 582.
time = 1.27, size = 2231, normalized size = 6.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*(((6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4 - 63*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b + 109*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*a^2*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*
a*b^3 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^3*b - 2*a^2
*b^2 + a*b^3)^2*abs(-a + b) + 2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^8*
b - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b^2 - 4*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a^6*b^3 + 34*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^4 - 3
3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^5 + 7*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*a^3*b^6 + 2*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^7)*abs(a^3
*b - 2*a^2*b^2 + a*b^3)*abs(-a + b) - (12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*a^11*b - 117*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^1
0*b^2 + 431*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b^3 - 773*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^4 + 703*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^5 - 279*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*a^6*b^6 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*
b^7 + 17*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^8 + sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^9)*abs(-a + b))*(pi*floor((d*x +
```

```
c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4*b - 2*a^3*b^2 + a^2*b^3 + sqrt
((a^4*b - 2*a^3*b^2 + a^2*b^3)^2 - (a^4*b - 2*a^3*b^2 + a^2*b^3)*(a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)))/(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4))))/
((3*a^12*b^2 - 27*a^11*b^3 + 104*a^10*b^4 - 224*a^9*b^5 + 294*a^8*b^6 - 238
*a^7*b^7 + 112*a^6*b^8 - 24*a^5*b^9 - a^4*b^10 + a^3*b^11)*abs(a^3*b - 2*a^
2*b^2 + a*b^3)) - ((6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4 - 6
3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b + 109*sqrt(a^2 - a*b
- sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b)
)*sqrt(a*b)*a*b^3 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a
^3*b - 2*a^2*b^2 + a*b^3)^2*abs(-a + b) - 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*a^8*b - 9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^7*b^2 - 4*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*a^6*b^3 + 34*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*a^5*b^4 - 33*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^5 + 7*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*a^3*b^6 + 2*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*
b^7)*abs(a^3*b - 2*a^2*b^2 + a*b^3)*abs(-a + b) - (12*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a^11*b - 117*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*s
qrt(a*b)*a^10*b^2 + 431*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b
^3 - 773*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^4 + 703*sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^5 - 279*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a^6*b^6 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a^5*b^7 + 17*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^8
+ sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^9)*abs(-a + b))*(pi*f
loor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4*b - 2*a^3*b^2 + a^
2*b^3 - sqrt((a^4*b - 2*a^3*b^2 + a^2*b^3)^2 - (a^4*b - 2*a^3*b^2 + a^2*b^3
)*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)))/(a^4*b - 3*a^3*b^2 + 3*a^2*b^3
- a*b^4))))/((3*a^12*b^2 - 27*a^11*b^3 + 104*a^10*b^4 - 224*a^9*b^5 + 294*a
^8*b^6 - 238*a^7*b^7 + 112*a^6*b^8 - 24*a^5*b^9 - a^4*b^10 + a^3*b^11)*abs(
a^3*b - 2*a^2*b^2 + a*b^3)) - 2*(2*a^3*tan(d*x + c)^7 + 13*a^2*b*tan(d*x +
c)^7 - 12*a*b^2*tan(d*x + c)^7 - 3*b^3*tan(d*x + c)^7 + 6*a^3*tan(d*x + c)^
5 + 28*a^2*b*tan(d*x + c)^5 - 10*a*b^2*tan(d*x + c)^5 + 6*a^3*tan(d*x + c)^
3 + 19*a^2*b*tan(d*x + c)^3 - a*b^2*tan(d*x + c)^3 + 2*a^3*tan(d*x + c) + 4
*a^2*b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x +
c)^2 + a)^2*(a^3*b - 2*a^2*b^2 + a*b^3)))/d
```

Mupad [B]
time = 20.34, size = 2500, normalized size = 7.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - ((tan(c + d*x)^3*(19*a*b + 6*a^2 - b^2))/(32*(a^2*b - 2*a*b^2 + b^3)) + (
a*tan(c + d*x)*(a + 2*b))/(16*(a^2*b - 2*a*b^2 + b^3)) + (tan(c + d*x)^7*(1
5*a*b + 2*a^2 + 3*b^2))/(32*a*(a*b - b^2)) + (tan(c + d*x)^5*(14*a*b + 3*a^
```

2 - 5*b^2))/(16*(a - b)*(a*b - b^2)))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2
) + a^2 - tan(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) +
4*a^2*tan(c + d*x)^2)) - (atan(((((65536*a^3*b^7 - 163840*a^4*b^6 + 98304*
a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a^2*b^6 - 3*a^3*b^5 + 3*a^
4*b^4 - a^5*b^3)) - (tan(c + d*x)*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9
)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3
- 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 -
5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*(16384
*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840*a^6*b^5 + 81920*a^7*b^4
- 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*
(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^
5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^
5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a
^9*b^7 - a^10*b^6)))^(1/2) + (tan(c + d*x)*(16*a^5 - 116*a^4*b - 101*a*b^4
+ 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 -
a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 +
30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(
1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^
9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*1i - (((65536*a^3*b^7 - 1638
40*a^4*b^6 + 98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a^2*b^6
- 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) + (tan(c + d*x)*((9*b^3*(a^5*b^9)^(1/2
) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^
6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/
(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*
b^6)))^(1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840*a^6*b
^5 + 81920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 -
a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 +
30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1
/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9
- 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) - (tan(c + d*x)*(16*a^5 - 116
*a^4*b - 101*a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))/(256*(a*b^5 - 3*a^
2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(
1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 8
6*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a
^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*1i)/((((6
5536*a^3*b^7 - 163840*a^4*b^6 + 98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b
^3)/(32768*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) - (tan(c + d*x)*((9
b^3(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 2
29*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*
b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8
+ 5*a^9*b^7 - a^10*b^6)))^(1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5
*b^6 - 163840*a^6*b^5 + 81920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2
*b^4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1
/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86
*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^

```
6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) + (tan(c +
 d*x)*(16*a^5 - 116*a^4*b - 101*a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))
/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) -
 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b
^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16
384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6
)))^(1/2) - (32*a^4 - 424*a^3*b - 381*a*b^3 + 27*b^4 + 1358*a^2*b^2)/(16384
*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) + (((65536*a^3*b^7 - 163840*a
^4*b^6 + 98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a^2*b^6 - 3
*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) + (tan(c + d*x)*((9*b^3*(a^5*b^9)^(1/2) -
80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^
4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(163
84*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)
))^(1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840*a^6*b^5 +
 81920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 - a^4*
b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a
^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2)
+ 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 1
0*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) - (ta...
```

3.232 $\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

Optimal. Leaf size=313

$$\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}\,d}-\frac{}{8(a}$$

```
[Out] 3/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2))
/a^(7/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)-3/64*arctan((a^(1/2)+b^(1/2))^(1
/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(7/4)/d/b^(1/2)/(a^(1/2)+b^(1
/2))^(5/2)-1/8*b*tan(d*x+c)*(3*a+b+4*(a+b)*tan(d*x+c)^2)/(a-b)^3/d/(a+2*a*t
an(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*tan(d*x+c)*((9*a^2-24*a*b-b^2)/(a-b)
^3+(17*a+3*b)*tan(d*x+c)^2/(a-b)^2)/a/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c
)^4)
```

Rubi [A]
time = 0.46, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3296, 1347, 1692, 1180, 211}

$$\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\frac{(17a+3b)\tan^2(c+dx)}{(a-b)^2}\right)}{32ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{b\tan(c+dx)\left(4(a+b)\tan^2(c+dx)+3a+b\right)}{8d(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] + Sq
rt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(64*a^(7/4)*
(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tan[c + d*x]*(3*a + b + 4*(a + b)
*Tan[c + d*x]^2))/(8*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c +
d*x]^4)^2) - (Tan[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 + ((17*a + 3*b
)*Tan[c + d*x]^2)/(a - b)^2))/(32*a*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan
[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4(1+x^2)^3}{(a+2ax^2+(a-b)x^4)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b^2(3a}{(a-b)}}{}\right.}{}$$

$$= -\frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(9\right.}{32ad\left(a+2a\tan^2\right.}$$

$$= -\frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(9\right.}{32ad\left(a+2a\tan^2\right.}$$

$$= \frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d} - \frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\right.}{64a^{7/4}\left(\sqrt{a}+\right.}$$

Mathematica [A]
time = 3.42, size = 316, normalized size = 1.01

$$\frac{-\frac{3\left(2a^{3/2}-3a\sqrt{b}+b^{3/2}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{a^{3/2}\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{3\left(2a^{3/2}+3a\sqrt{b}-b^{3/2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{a^{3/2}\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{8(-7a-2b+(2a+b)\cos(2(c+dx)))\sin(2(c+dx))}{a(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))} + \frac{64(a-b)(-6\sin(2(c+dx))+\sin(4(c+dx)))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}}{64(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-3*(2*a^(3/2) - 3*a*Sqrt[b] + b^(3/2))*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c
+ d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt
[b]) - (3*(2*a^(3/2) + 3*a*Sqrt[b] - b^(3/2))*ArcTanh[((Sqrt[a] - Sqrt[b])*
Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[-a + Sqrt[a]*Sqrt[
b]]*Sqrt[b]) + (8*(-7*a - 2*b + (2*a + b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)
])/(a*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (64*(a - b
)*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*
x)] + b*Cos[4*(c + d*x)])^2)/(64*(a - b)^2*d)
```

Maple [A]
time = 1.10, size = 372, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{(17a+3b)\left(\tan^7(dx+c)\right)}{32a(a-b)}-\frac{\left(43a^2-18ab-b^2\right)\left(\tan^5(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{(35a-11b)\left(\tan^3(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}-\frac{3(3a-b)\tan(dx+c)}{32\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{3(a-b)\left(\frac{\left(5a\sqrt{ab}\ \dots\right.}{\dots}\right.}{\dots}$
default	$\frac{-\frac{(17a+3b)\left(\tan^7(dx+c)\right)}{32a(a-b)}-\frac{\left(43a^2-18ab-b^2\right)\left(\tan^5(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{(35a-11b)\left(\tan^3(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}-\frac{3(3a-b)\tan(dx+c)}{32\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{3(a-b)\left(\frac{\left(5a\sqrt{ab}\ \dots\right.}{\dots}\right.}{\dots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/32*(17*a+3*b)/a/(a-b)*tan(d*x+c)^7-1/32*(43*a^2-18*a*b-b^2)/a/(a^2
-2*a*b+b^2)*tan(d*x+c)^5-1/32*(35*a-11*b)/(a^2-2*a*b+b^2)*tan(d*x+c)^3-3/32
*(3*a-b)/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan
(d*x+c)^2+a)^2+3/32/a/(a^2-2*a*b+b^2)*(a-b)*(1/2*(5*a*(a*b)^(1/2)-(a*b)^(1/
2)*b+2*a^2+3*a*b-b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arcta
n((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(5*a*(a*b)^(1/2)-(a*b
)^(1/2)*b-2*a^2-3*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*
arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*a*b^3*sin(2*d*x + 2*c) - 12*(8*a^2*b^2 + 13*a*b^3 - 2*b^4)*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c) - (3*a*b^3*sin(14*d*x + 14*c) - 3*(10*a*b^3 - b^4
```

)*sin(12*d*x + 12*c) - (80*a^2*b^2 - 111*a*b^3 + 16*b^4)*sin(10*d*x + 10*c) + (256*a^3*b - 64*a^2*b^2 - 26*a*b^3 + 35*b^4)*sin(8*d*x + 8*c) + (336*a^2 *b^2 - 95*a*b^3 - 40*b^4)*sin(6*d*x + 6*c) - (64*a^2*b^2 - 54*a*b^3 - 25*b^ 4)*sin(4*d*x + 4*c) - (19*a*b^3 + 8*b^4)*sin(2*d*x + 2*c))*cos(16*d*x + 16* c) - 2*(6*(8*a^2*b^2 + 13*a*b^3 - 2*b^4)*sin(12*d*x + 12*c) + 8*(16*a^2*b^2 - 45*a*b^3 + 8*b^4)*sin(10*d*x + 10*c) - (1408*a^3*b - 544*a^2*b^2 + a*b^3 + 140*b^4)*sin(8*d*x + 8*c) - 16*(96*a^2*b^2 - 29*a*b^3 - 10*b^4)*sin(6*d* x + 6*c) + 2*(152*a^2*b^2 - 129*a*b^3 - 50*b^4)*sin(4*d*x + 4*c) + 8*(11*a* b^3 + 4*b^4)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(640*a^3*b - 488*a ^2*b^2 + 389*a*b^3 - 70*b^4)*sin(10*d*x + 10*c) - (4096*a^4 - 8448*a^3*b + 3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*sin(8*d*x + 8*c) - 2*(2688*a^3*b - 4072 *a^2*b^2 + 861*a*b^3 + 238*b^4)*sin(6*d*x + 6*c) + 4*(256*a^3*b - 560*a^2*b ^2 + 206*a*b^3 + 77*b^4)*sin(4*d*x + 4*c) + 2*(152*a^2*b^2 - 129*a*b^3 - 50 *b^4)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) - 2*((26624*a^4 - 33152*a^3*b + 15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*sin(8*d*x + 8*c) + 8*(3328*a^3*b - 31 04*a^2*b^2 + 529*a*b^3 + 84*b^4)*sin(6*d*x + 6*c) - 2*(2688*a^3*b - 4072*a^ 2*b^2 + 861*a*b^3 + 238*b^4)*sin(4*d*x + 4*c) - 16*(96*a^2*b^2 - 29*a*b^3 - 10*b^4)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((26624*a^4 - 33152*a^3*b + 15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*sin(6*d*x + 6*c) - (4096*a^4 - 844 8*a^3*b + 3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*sin(4*d*x + 4*c) - (1408*a^3* b - 544*a^2*b^2 + a*b^3 + 140*b^4)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) - 4*((640*a^3*b - 488*a^2*b^2 + 389*a*b^3 - 70*b^4)*sin(4*d*x + 4*c) + 4*(16*a^2 *b^2 - 45*a*b^3 + 8*b^4)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 8*((a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^ 6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 21 0*a^2*b^5 + 49*a*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^ 3 + 753*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384 *a^7 - 57344*a^6*b + 83712*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a ^2*b^5 + 1225*a*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^5*b^ 2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c)^ 2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c)^2 + (a^3*b^4 - 2*a^ 2*b^5 + a*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d* sin(14*d*x + 14*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2 *b^5 + 49*a*b^6)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 7 53*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^7 - 57344*a^6*b + 83712*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a^2*b^ 5 + 1225*a*b^6)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753* a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^5*b^2 - 2 40*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c)^2 + 6 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c)*sin(2* d*x + 2*c) + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^ 3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c) + (a^3*b^4 - 2*a^2*b^5 + a*b^ 6)*d - 2*(8*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^4*b ^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^

```
3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^
2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*cos(8*d*x + 8*c)
- 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(6*d*x + 6*c) + 4
*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(4*d*x + 4*c) + 8*(a^
3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c) - (a^3*b^4 - 2*a^2*b^5 + a*b^
6)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a
*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*
b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 16
6*a^2*b^5 + 35*a*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*
a^2*b^5 - 7*a*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*
b^5 - 7*a*b^6)*d*cos(4*d*x + 4*c) + 8*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2
*d*x + 2*c) - (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(1
28*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(10*d
*x + 10*c) + 2*(1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1
442*a^2*b^5 - 245*a*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^5*b^2 - 424*a^4*b^3
+ 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^5*b^2
- 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) -
8*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) + (8*a
^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*(
2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5510 vs. $2(261) = 522$.
time = 1.47, size = 5510, normalized size = 17.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a
^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b
^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x
+ c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b -
 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a
^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((
a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12
*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d
^4)) + 4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3
- 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(432*a^4 + 27*a^3*b - 783/4*a^
2*b^2 + 135/2*a*b^3 - 27/4*b^4 - 27/4*(64*a^4 + 4*a^3*b - 29*a^2*b^2 + 10*a
*b^3 - b^4)*cos(d*x + c)^2 + 27/2*((5*a^12*b - 26*a^11*b^2 + 55*a^10*b^3 -
60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7)*d^3*sqrt((256*a^6 + 160*a^5
*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*
a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a
```

^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) - (32*a^7 + 58*a^6*b - 13*a^5*b^2 - 21*a^4*b^3 + 9*a^3*b^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) + 4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*cos(d*x + c)^2 - (4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2)*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))) - 3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) + 4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(432*a^4 + 27*a^3*b - 783/4*a^2*b^2 + 135/2*a*b^3 - 27/4*b^4 - 27/4*(64*a^4 + 4*a^3*b - 29*a^2*b^2 + 10*a*b^3 - b^4)*cos(d*x + c)^2 - 27/2*((5*a^12*b - 26*a^11*b^2 + 55*a^10*b^3 - 60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7)*d^3*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) - (32*a^7 + 58*a^6*b - 13*a^5*b^2 - 21*a^4*b^3 + 9*a^3*b^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) + 4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*cos(d*x + c)^2 - (4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2)*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))) + 3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2

```
 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4
)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b
^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 -
 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*
a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8
*b^10 + a^7*b^11)*d^4)) - 4*a^3 - 21*a^2*b + 10...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1986 vs. 2(261) = 522.
time = 1.41, size = 1986, normalized size = 6.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/64*(3*((15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 33*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b
)*b^4)*(a^3 - 2*a^2*b + a*b^2)^2*abs(-a + b) - (9*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*a^7*b - 48*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b^2 + 93*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^3 - 80*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*a^4*b^4 + 27*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^5 - sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*a*b^7)*abs(a^3 - 2*a^2*b + a*b^2)*abs(-a + b) - (
6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 - 27*sqrt(a^2 - a*b +
sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b + 25*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))
*sqrt(a*b)*a^8*b^2 + 53*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b
^3 - 131*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 + 103*sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 29*sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*sqrt(a*b)*a^4*b^6 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*a^3*b^7 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^8)*abs(-
a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4 - 2*a
^3*b + a^2*b^2 + sqrt((a^4 - 2*a^3*b + a^2*b^2)^2 - (a^4 - 2*a^3*b + a^2*b^
2)*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)))/(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3
))))/((3*a^12*b - 27*a^11*b^2 + 104*a^10*b^3 - 224*a^9*b^4 + 294*a^8*b^5 -
```

```
238*a^7*b^6 + 112*a^6*b^7 - 24*a^5*b^8 - a^4*b^9 + a^3*b^10)*abs(a^3 - 2*a^
2*b + a*b^2)) - 3*((15*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b
- 33*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*sqrt(a*b)*b^4)*(a^3 - 2*a^2*b + a*b^2)^2*abs(-a + b) + (9*sqrt(a^2 - a*b -
 sqrt(a*b)*(a - b))*a^7*b - 48*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6*b^2
+ 93*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b^3 - 80*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*a^4*b^4 + 27*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^5 - s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^7)*abs(a^3 - 2*a^2*b + a*b^2)*abs(-a
 + b) - (6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 - 27*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b + 25*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^8*b^2 + 53*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a^7*b^3 - 131*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 + 1
03*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 29*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^6 - sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^3*b^7 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b
^8)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((
a^4 - 2*a^3*b + a^2*b^2 - sqrt((a^4 - 2*a^3*b + a^2*b^2)^2 - (a^4 - 2*a^3*b
 + a^2*b^2)*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)))/(a^4 - 3*a^3*b + 3*a^2*b^
2 - a*b^3))))/((3*a^12*b - 27*a^11*b^2 + 104*a^10*b^3 - 224*a^9*b^4 + 294*a
^8*b^5 - 238*a^7*b^6 + 112*a^6*b^7 - 24*a^5*b^8 - a^4*b^9 + a^3*b^10)*abs(a
^3 - 2*a^2*b + a*b^2)) + 2*(17*a^2*tan(d*x + c)^7 - 14*a*b*tan(d*x + c)^7 -
 3*b^2*tan(d*x + c)^7 + 43*a^2*tan(d*x + c)^5 - 18*a*b*tan(d*x + c)^5 - b^2
*tan(d*x + c)^5 + 35*a^2*tan(d*x + c)^3 - 11*a*b*tan(d*x + c)^3 + 9*a^2*tan
(d*x + c) - 3*a*b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a
*tan(d*x + c)^2 + a)^2*(a^3 - 2*a^2*b + a*b^2)))/d
```

Mupad [B]
time = 19.50, size = 2500, normalized size = 7.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - (atan(((((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 + 196608*a^5*b^3
 - 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) - (tan(c
+ d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^
4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(
1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3
- a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 81920*a^5*b^5 - 163840*a
^6*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3
*a^3*b^2)))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a
^4*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b
^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11
```

*b^3 - a^12*b^2)))^(1/2) - (tan(c + d*x)*(333*a^3*b - 45*a*b^3 + 36*a^4 + 9
*b^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*((9*(16*a^
3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 - 10*a^5*b^3 +
21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^
7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/
2)*1i - (((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 + 196608*a^5*b^3
- 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) + (tan(c +
d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4
- 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1
/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 -
a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 81920*a^5*b^5 - 163840*a^
6*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*
a^3*b^2)))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^
4*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^
3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*
b^3 - a^12*b^2)))^(1/2) + (tan(c + d*x)*(333*a^3*b - 45*a*b^3 + 36*a^4 + 9*
b^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*((9*(16*a^3
*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 - 10*a^5*b^3 + 2
1*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7
*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2
)*1i)/((((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 + 196608*a^5*b^3 -
163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) - (tan(c +
d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4
- 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/
2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 -
a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 81920*a^5*b^5 - 163840*a^6
*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a
^3*b^2)))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4
*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3
)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b
^3 - a^12*b^2)))^(1/2) - (tan(c + d*x)*(333*a^3*b - 45*a*b^3 + 36*a^4 + 9*b
^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*((9*(16*a^3*
(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 - 10*a^5*b^3 + 21
*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*
b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)
- (3*(180*a^2 - 81*a*b + 9*b^2))/(16384*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b
^2)) + (((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 + 196608*a^5*b^3 -
163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) + (tan(c +
d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4
- 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/
2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 -
a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 81920*a^5*b^5 - 163840*a^6
*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a
^3*b^2)))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4
*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3

```
)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b
^3 - a^12*b^2)))^(1/2) + (tan(c + d*x)*(333*a^3*b - 45*a*b^3 + 36*a^4 + 9*b
^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*((9*(16*a^3*
(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 - 10*a^5*b^3 + 21
*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*
b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)
))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 -
10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)
))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^
12*b^2)))^(1/2)*2i)/d - (atan(((((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^
4*b^4 + 196608*a^5*b^3 - 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 -
 3*a^4*b^2)) - (tan(c + d*x)*(-(9*(16*a^3*(a^7*...
```

3.233 $\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

Optimal. Leaf size=347

$$\frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d}-\frac{\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}\,d}$$

```
[Out] 1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(12*a+5*b-14*a^(1/2
)*b^(1/2))/a^(9/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)-1/64*arctan((a^(1/2)+b
^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(12*a+5*b+14*a^(1/2)*b^(1/2))/a^(9/4)/d/b
^(1/2)/(a^(1/2)+b^(1/2))^(5/2)-1/8*b*tan(d*x+c)*(a*(a+3*b)+(a^2+6*a*b+b^2)*
tan(d*x+c)^2)/a/(a-b)^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*ta
n(d*x+c)*(2*a*(5*a^2-9*a*b-4*b^2)/(a-b)^3+5*(2*a^2+3*a*b-b^2)*tan(d*x+c)^2/
(a-b)^2)/a^2/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A]
time = 0.46, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3296, 1347, 1692, 1180, 211}

$$\frac{\left(-14\sqrt{a}\sqrt{b}+12a+5b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(14\sqrt{a}\sqrt{b}+12a+5b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tan(c+dx)\left(\frac{5(2a^2+3ab-b^2)\tan^2(c+dx)}{(a-b)^2}+\frac{2a(5a^2-9ab-4b^2)}{(a-b)^3}\right)}{32a^2d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{b\tan(c+dx)\left((a^2+6ab+b^2)\tan^2(c+dx)+a(a+3b)\right)}{8ad(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - ((12*a +
 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^
(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tan[c + d*x]*
(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/(8*a*(a - b)^3*d*(a + 2
*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Tan[c + d*x]*((2*a*(5*a^2
 - 9*a*b - 4*b^2))/(a - b)^3 + (5*(2*a^2 + 3*a*b - b^2)*Tan[c + d*x]^2)/(a
- b)^2))/(32*a^2*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1347

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(1+x^2)^4}{(a+2ax^2+(a-b)x^4)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(a(a+3b)+(a^2+6ab+b^2)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int -\frac{2a^2b\ldots}{(\ldots}\right.}{\ldots}$$

$$= -\frac{b\tan(c+dx)\left(a(a+3b)+(a^2+6ab+b^2)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\ldots\right.}{32a^2d\,(a+2a\ldots}$$

$$= -\frac{b\tan(c+dx)\left(a(a+3b)+(a^2+6ab+b^2)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\ldots\right.}{32a^2d\,(a+2a\ldots}$$

$$= \frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d} - \frac{\left(12a+14\sqrt{a}\ldots\right.}{\ldots}$$

Mathematica [A]
time = 5.37, size = 343, normalized size = 0.99

$$-\frac{\frac{(\sqrt{a}-\sqrt{b})^2(12a+14\sqrt{a}\sqrt{b}+5b)\tan^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{(\sqrt{a}+\sqrt{b})^2(12a-14\sqrt{a}\sqrt{b}+5b)\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{4(12a^2+11ab-5b^2+b(-11a+5b)\cos(2(c+dx)))\sin(2(c+dx))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}+\frac{128a(a-b)(2a+b-b\cos(2(c+dx)))\sin(2(c+dx))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}}{64a^2(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -1/64*(((Sqrt[a] - Sqrt[b])^2*(12*a + 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[((Sq
rt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a
]*Sqrt[b]]*Sqrt[b]) + ((Sqrt[a] + Sqrt[b])^2*(12*a - 14*Sqrt[a]*Sqrt[b] + 5
*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])
/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (4*(12*a^2 + 11*a*b - 5*b^2 + b*(-1
1*a + 5*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c +
d*x)] - b*Cos[4*(c + d*x)]) + (128*a*(a - b)*(2*a + b - b*Cos[2*(c + d*x)])
*Sin[2*(c + d*x)])/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])
^2)/(a^2*(a - b)^2*d)
```

Maple [A]
time = 1.65, size = 422, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{5\left(2a^2+3ab-b^2\right)\left(\tan^7(dx+c)\right)}{32a^2(a-b)}-\frac{3\left(5a^2+2ab-3b^2\right)\left(\tan^5(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}-\frac{3\left(10a^2+ab-3b^2\right)\left(\tan^3(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{(5a-2b)\tan(dx+c)}{16\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\right.}{}$
default	$\frac{-\frac{5\left(2a^2+3ab-b^2\right)\left(\tan^7(dx+c)\right)}{32a^2(a-b)}-\frac{3\left(5a^2+2ab-3b^2\right)\left(\tan^5(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}-\frac{3\left(10a^2+ab-3b^2\right)\left(\tan^3(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{(5a-2b)\tan(dx+c)}{16\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-5/32*(2*a^2+3*a*b-b^2)/a^2/(a-b)*tan(d*x+c)^7-3/16*(5*a^2+2*a*b-3*b^
2)/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5-3/32*(10*a^2+a*b-3*b^2)/a/(a^2-2*a*b+b^2)
*tan(d*x+c)^3-1/16*(5*a-2*b)/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+c)^4*a-ta
n(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2+1/32/a^2/(a^2-2*a*b+b^2)*(a-b)*(1/2*(22*
a^2*(a*b)^(1/2)-15*a*b*(a*b)^(1/2)+5*b^2*(a*b)^(1/2)-12*a^3+a^2*b-a*b^2)/(a
*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((
a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*(22*a^2*(a*b)^(1/2)-15*a*b*(a*b)^(1/2)+5*b^
2*(a*b)^(1/2)+12*a^3-a^2*b+a*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

[Out] 1/16*(4*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c) + ((12*a^2*b^3 - 11*a*b^4 + 5*b^5)*sin(14*d*x + 14*c) - (104*a
^2*b^3 - 85*a*b^4 + 35*b^5)*sin(12*d*x + 12*c) - (320*a^3*b^2 - 652*a^2*b^3
 + 407*a*b^4 - 105*b^5)*sin(10*d*x + 10*c) + (1408*a^3*b^2 - 1696*a^2*b^3 +
 865*a*b^4 - 175*b^5)*sin(8*d*x + 8*c) + (320*a^3*b^2 + 756*a^2*b^3 - 849*a
*b^4 + 175*b^5)*sin(6*d*x + 6*c) - (248*a^2*b^3 - 383*a*b^4 + 105*b^5)*sin(
4*d*x + 4*c) - (12*a^2*b^3 + 77*a*b^4 - 35*b^5)*sin(2*d*x + 2*c))*cos(16*d*
x + 16*c) + 2*(2*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*sin(12*d*x +
 12*c) + 8*(64*a^3*b^2 - 196*a^2*b^3 + 125*a*b^4 - 35*b^5)*sin(10*d*x + 10*
c) - 3*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4 - 175*b^5)*sin(
8*d*x + 8*c) - 16*(128*a^3*b^2 + 124*a^2*b^3 - 173*a*b^4 + 35*b^5)*sin(6*d*
x + 6*c) + 2*(96*a^3*b^2 + 324*a^2*b^3 - 649*a*b^4 + 175*b^5)*sin(4*d*x + 4
c) + 24(4*a^2*b^3 + 11*a*b^4 - 5*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c
) + 2*(2*(2560*a^4*b - 4128*a^3*b^2 + 3644*a^2*b^3 - 1379*a*b^4 + 245*b^5)*
sin(10*d*x + 10*c) - (9216*a^4*b - 25984*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b
^4 + 1225*b^5)*sin(8*d*x + 8*c) - 2*(2560*a^4*b + 480*a^3*b^2 - 7908*a^2*b^
3 + 5033*a*b^4 - 735*b^5)*sin(6*d*x + 6*c) + 4*(576*a^3*b^2 - 1696*a^2*b^3
+ 1323*a*b^4 - 245*b^5)*sin(4*d*x + 4*c) + 2*(96*a^3*b^2 + 324*a^2*b^3 - 64
9*a*b^4 + 175*b^5)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 2*((40960*a^5 - 2
4064*a^4*b - 22080*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*b^5)*sin(8*
d*x + 8*c) + 8*(5120*a^4*b - 1408*a^3*b^2 - 3900*a^2*b^3 + 2107*a*b^4 - 245
*b^5)*sin(6*d*x + 6*c) - 2*(2560*a^4*b + 480*a^3*b^2 - 7908*a^2*b^3 + 5033*
a*b^4 - 735*b^5)*sin(4*d*x + 4*c) - 16*(128*a^3*b^2 + 124*a^2*b^3 - 173*a*b
^4 + 35*b^5)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + 2*((40960*a^5 - 24064*a
^4*b - 22080*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*b^5)*sin(6*d*x +
6*c) - (9216*a^4*b - 25984*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b^4 + 1225*b^5)
*sin(4*d*x + 4*c) - 3*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4
- 175*b^5)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 4*((2560*a^4*b - 4128*a^3*b
^2 + 3644*a^2*b^3 - 1379*a*b^4 + 245*b^5)*sin(4*d*x + 4*c) + 4*(64*a^3*b^2
- 196*a^2*b^3 + 125*a*b^4 - 35*b^5)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 16
*((a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*
a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 +
337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^
6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x
+ 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32
841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a
^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x
+ 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^
2*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*
x + 2*c)^2 + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a
^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240
*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2 +
 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*
sin(10*d*x + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a
^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2

```
+ 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d
*sin(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*
b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^
3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a^
3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*
d*cos(2*d*x + 2*c) + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2*
a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^
3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a
^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 3
55*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 -
 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 2
3*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3
*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos
(16*d*x + 16*c) + 16*(4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d
*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*
d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3
*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3
*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b
^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*co
s(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8*
(8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*c
os(10*d*x + 10*c) + 2*(1024*a^7*b - 3712*a^6*b^...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6215 vs. $2(295) = 590$.
time = 2.43, size = 6215, normalized size = 17.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*
a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*
a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*c
os(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-
(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 - (a^9*b - 5*a^8*b^2
+ 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368
640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b
^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17
*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12
*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 +
10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2))*log(13824*a^6 - 24576*
a^5*b + 24084*a^4*b^2 - 14455*a^3*b^3 + 22509/4*a^2*b^4 - 2625/2*a*b^5 + 62
5/4*b^6 - 1/4*(55296*a^6 - 98304*a^5*b + 96336*a^4*b^2 - 57820*a^3*b^3 + 22
```

509*a^2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d*x + c)^2 + 1/2*((22*a^14*b - 125*
a^13*b^2 + 300*a^12*b^3 - 395*a^11*b^4 + 310*a^10*b^5 - 147*a^9*b^6 + 40*a^
8*b^7 - 5*a^7*b^8)*d^3*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 4
37952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^
7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15
*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b
^10 + a^9*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) + (4608*a^9 - 6144*a^8*b +
5052*a^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4 - 159*a^4*b^5 + 25*a^3*b^6)*d*cos
(d*x + c)*sin(d*x + c))*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3
- 15*b^4 - (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b
^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 +
269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((
a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14
*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)
*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6
)*d^2)) - 1/4*(2*(144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 17
30*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2*cos(d*x + c)^2 - (
144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a
^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2)*sqrt((147456*a^8 - 368640*a^7*b + 4
98432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^
2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^
16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^1
1*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*
d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(
a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*
a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^
2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*
b^3 - 15*b^4 - (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a
^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b
^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8
)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*
a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b
^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4
*b^6)*d^2))*log(13824*a^6 - 24576*a^5*b + 24084*a^4*b^2 - 14455*a^3*b^3 + 2
2509/4*a^2*b^4 - 2625/2*a*b^5 + 625/4*b^6 - 1/4*(55296*a^6 - 98304*a^5*b +
96336*a^4*b^2 - 57820*a^3*b^3 + 22509*a^2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d
x + c)^2 - 1/2((22*a^14*b - 125*a^13*b^2 + 300*a^12*b^3 - 395*a^11*b^4 +
310*a^10*b^5 - 147*a^9*b^6 + 40*a^8*b^7 - 5*a^7*b^8)*d^3*sqrt((147456*a^8 -
368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a
^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*
a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*
a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))*cos(d*x + c)*sin(d*
x + c) + (4608*a^9 - 6144*a^8*b + 5052*a^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4
- 159*a^4*b^5 + 25*a^3*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(144*a^4 +
76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 - (a^9*b - 5*a^8*b^2 + 10*a^7*b^

```
3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b +
 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*
a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*
a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a
^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3
- 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2)) - 1/4*(2*(144*a^12 - 796*a^11*b +
 1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 -
25*a^5*b^7)*d^2*cos(d*x + c)^2 - (144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2
325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. 2(295) = 590.
time = 1.41, size = 1184, normalized size = 3.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*((30*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 72*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*a^3*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^3
+ 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4 + 36*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a^4 - 105*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^3*b + 69*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 19*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 5*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*
x + c)/sqrt((a^5 - 2*a^4*b + a^3*b^2 + sqrt((a^5 - 2*a^4*b + a^3*b^2)^2 - (
a^5 - 2*a^4*b + a^3*b^2)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^5 - 3*a
^4*b + 3*a^3*b^2 - a^2*b^3))))*abs(-a + b)/(3*a^9*b - 18*a^8*b^2 + 41*a^7*b
^3 - 44*a^6*b^4 + 21*a^5*b^5 - 2*a^4*b^6 - a^3*b^7) + (30*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*a^4*b - 72*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^2 +
 14*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 + 4*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*a*b^4 - 36*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4 +
 105*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 69*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 19*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^
```

```
4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^5 - 2*a^4*b
+ a^3*b^2 - sqrt((a^5 - 2*a^4*b + a^3*b^2)^2 - (a^5 - 2*a^4*b + a^3*b^2)*(a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)
)))*abs(-a + b)/(3*a^9*b - 18*a^8*b^2 + 41*a^7*b^3 - 44*a^6*b^4 + 21*a^5*b^
5 - 2*a^4*b^6 - a^3*b^7) - 2*(10*a^3*tan(d*x + c)^7 + 5*a^2*b*tan(d*x + c)^
7 - 20*a*b^2*tan(d*x + c)^7 + 5*b^3*tan(d*x + c)^7 + 30*a^3*tan(d*x + c)^5
+ 12*a^2*b*tan(d*x + c)^5 - 18*a*b^2*tan(d*x + c)^5 + 30*a^3*tan(d*x + c)^3
 + 3*a^2*b*tan(d*x + c)^3 - 9*a*b^2*tan(d*x + c)^3 + 10*a^3*tan(d*x + c) -
4*a^2*b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x +
 c)^2 + a)^2*(a^4 - 2*a^3*b + a^2*b^2)))/d
```

Mupad [B]
time = 19.90, size = 2500, normalized size = 7.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - ((tan(c + d*x)*(5*a - 2*b))/(16*(a^2 - 2*a*b + b^2)) + (3*tan(c + d*x)^3*
(a*b + 10*a^2 - 3*b^2))/(32*a*(a^2 - 2*a*b + b^2)) + (5*tan(c + d*x)^7*(3*a
*b + 2*a^2 - b^2))/(32*a^2*(a - b)) + (3*tan(c + d*x)^5*(2*a*b + 5*a^2 - 3*
b^2))/(16*a*(a - b)^2))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2) + a^2 - tan(
c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) + 4*a^2*tan(c +
 d*x)^2)) - (atan(((((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 + 68812
8*a^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) -
 (tan(c + d*x)*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^
9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9
*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384
*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)
))^(1/2)*(16384*a^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^7*b^4 + 1
63840*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2))
)*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*
b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) -
 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5
*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) - (t
an(c + d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5 + 443*a^2*b^3 - 635*a
^3*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^
(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*
a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1
/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5
 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*1i - (((163840*a^9*b + 6553
6*a^5*b^5 - 360448*a^6*b^4 + 688128*a^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7
*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + (tan(c + d*x)*(-(384*a^4*(a^9*b^3)^(1/2)
 + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b
```

```
^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) -
 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10
*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*(16384*a^10*b - 16384*a^5*b^6 +
81920*a^6*b^5 - 163840*a^7*b^4 + 163840*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a
^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^
9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b
^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a
^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5
*a^13*b^3 - a^14*b^2)))^(1/2) + (tan(c + d*x)*(460*a^4*b - 149*a*b^4 + 144*
a^5 + 25*b^5 + 443*a^2*b^3 - 635*a^3*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 -
3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*
b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b
^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(
a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))
^(1/2)*1i)/((((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 + 688128*a^7*b
^3 - 557056*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) - (tan(c
 + d*x)*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 1
5*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(
1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b
^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2
)*(16384*a^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^7*b^4 + 163840*a
^8*b^3 - 81920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(38
4*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 9
4*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*
b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b
^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) - (tan(c +
d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5 + 443*a^2*b^3 - 635*a^3*b^2)
)/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) +
 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3
 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 4
80*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a
^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) - (3168*a^4 - 3832*a^3*b - 755*a*b
^3 + 125*b^4 + 2410*a^2*b^2)/(16384*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2))
+ (((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 + 688128*a^7*b^3 - 55705
6*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + (tan(c + d*x)*(-
(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5
- 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134
*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^1
0*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*(16384*a
^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^7*b^4 + 163840*a^8*b^3 - 8
1920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - ...
```

3.234 $\int \frac{1}{(a-b\sin^4(c+dx))^3}\,dx$

Optimal. Leaf size=319

$$\frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d}$$

```
[Out] 1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(32*a+21*b-50*a^(1/
2)*b^(1/2))/a^(11/4)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctan((a^(1/2)+b^(1/2)
)^(1/2)*tan(d*x+c)/a^(1/4))*(32*a+21*b+50*a^(1/2)*b^(1/2))/a^(11/4)/d/(a^(1
/2)+b^(1/2))^(5/2)-1/8*b^2*tan(d*x+c)*(3*a+b+4*(a+b)*tan(d*x+c)^2)/a/(a-b)^
3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*b*tan(d*x+c)*((17*a^2-40
*a*b+7*b^2)/(a-b)^3+(33*a-13*b)*tan(d*x+c)^2/(a-b)^2)/a^2/d/(a+2*a*tan(d*x+
c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A]

time = 0.42, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3288, 1219, 1692, 1180, 211}

$$\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{b\tan(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}+\frac{(33a-13b)\tan^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{b^2\tan(c+dx)\left(4(a+b)\tan^2(c+dx)+3a+b\right)}{8ad(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[c + d*x]^4)^(-3),x]
```

```
[Out] ((32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(64*a^(11/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + ((32*a + 50*Sq
rt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)
])/(64*a^(11/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b^2*Tan[c + d*x]*(3*a + b +
 4*(a + b)*Tan[c + d*x]^2))/(8*a*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a -
 b)*Tan[c + d*x]^4)^2) - (b*Tan[c + d*x]*((17*a^2 - 40*a*b + 7*b^2)/(a - b)
^3 + ((33*a - 13*b)*Tan[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a + 2*a*Tan[c +
d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a-b\sin^4(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^5}{(a+2ax^2+(a-b)x^4)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b^2\tan(c+dx)\,(3a+b+4(a+b)\tan^2(c+dx))}{8a(a-b)^3d\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))^2} - \frac{\text{Subst}\left(\int -\frac{2ab(\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{b^2\tan(c+dx)\,(3a+b+4(a+b)\tan^2(c+dx))}{8a(a-b)^3d\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))^2} - \frac{b\tan(c+dx)\cdots}{32a^2d\,(a+2a\cdots}$$

$$= -\frac{b^2\tan(c+dx)\,(3a+b+4(a+b)\tan^2(c+dx))}{8a(a-b)^3d\,(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx))^2} - \frac{b\tan(c+dx)\cdots}{32a^2d\,(a+2a\cdots}$$

$$= \frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} + \frac{\left(32a+50\sqrt{a}\cdots\right.}{\cdots}$$

Mathematica [A]
time = 2.10, size = 333, normalized size = 1.04

$$\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}} + \frac{8\sqrt{a}\,b(-19a+10b+(6a-3b)\cos(2(c+dx)))\sin(2(c+dx))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))} + \frac{64a^{3/2}(a-b)b(-6\sin(2(c+dx))+\sin(4(c+dx)))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}}{64a^{5/2}(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[c + d*x]^4)^(-3),x]
```

```
[Out] (((Sqrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[((Sqrt[a]
 + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt
[b]] - ((Sqrt[a] + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[((
Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqr
t[a]*Sqrt[b]] + (8*Sqrt[a]*b*(-19*a + 10*b + (6*a - 3*b)*Cos[2*(c + d*x)])*
Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) +
 (64*a^(3/2)*(a - b)*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*
b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)/(64*a^(5/2)*(a - b)^2*d)
```

Maple [A]
time = 1.66, size = 416, normalized size = 1.30

method	result
derivativedivides	$\frac{\frac{-\frac{(33a-13b)b\left(\tan^7(dx+c)\right)}{32a^2(a-b)}-\frac{b\left(83a^2-66ab+7b^2\right)\left(\tan^5(dx+c)\right)}{32a^2\left(a^2-2ab+b^2\right)}-\frac{(67a-43b)b\left(\tan^3(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{b(17a-11b)\tan(dx+c)}{32a\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\frac{\left(32a^2 \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{-\frac{(33a-13b)b\left(\tan^7(dx+c)\right)}{32a^2(a-b)}-\frac{b\left(83a^2-66ab+7b^2\right)\left(\tan^5(dx+c)\right)}{32a^2\left(a^2-2ab+b^2\right)}-\frac{(67a-43b)b\left(\tan^3(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{b(17a-11b)\tan(dx+c)}{32a\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\frac{\left(32a^2 \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/32*(33*a-13*b)/a^2*b/(a-b)*tan(d*x+c)^7-1/32/a^2*b*(83*a^2-66*a*b+
7*b^2)/(a^2-2*a*b+b^2)*tan(d*x+c)^5-1/32*(67*a-43*b)/a*b/(a^2-2*a*b+b^2)*ta
n(d*x+c)^3-1/32*b*(17*a-11*b)/a/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+c)^4*a
-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2+1/32/a^2/(a^2-2*a*b+b^2)*(a-b)*(1/2*(
32*a^2*(a*b)^(1/2)-33*a*b*(a*b)^(1/2)+13*b^2*(a*b)^(1/2)+46*a^2*b-55*a*b^2+
21*b^3)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*
x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(32*a^2*(a*b)^(1/2)-33*a*b*(a*b)^(1
/2)+13*b^2*(a*b)^(1/2)-46*a^2*b+55*a*b^2-21*b^3)/(a*b)^(1/2)/(a-b)/(((a*b)^
(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/
2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

[Out] 1/8*(4*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ ((7*a*b^4 - 4*b^5)*sin(14*d*x + 14*c) - (32*a^2*b^3 + 2*a*b^4 - 7*b^5)*si
n(12*d*x + 12*c) - (16*a^2*b^3 - 3*a*b^4 - 28*b^5)*sin(10*d*x + 10*c) + 3*(
256*a^3*b^2 - 320*a^2*b^3 + 166*a*b^4 - 35*b^5)*sin(8*d*x + 8*c) + (784*a^2
*b^3 - 723*a*b^4 + 140*b^5)*sin(6*d*x + 6*c) - (160*a^2*b^3 - 266*a*b^4 + 9
1*b^5)*sin(4*d*x + 4*c) - (55*a*b^4 - 28*b^5)*sin(2*d*x + 2*c))*cos(16*d*x
+ 16*c) + 2*(2*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*sin(12*d*x + 12*c) - 8*(48
*a^2*b^3 - 55*a*b^4 + 28*b^5)*sin(10*d*x + 10*c) - (3968*a^3*b^2 - 5024*a^2
*b^3 + 2621*a*b^4 - 560*b^5)*sin(8*d*x + 8*c) - 16*(224*a^2*b^3 - 209*a*b^4
+ 42*b^5)*sin(6*d*x + 6*c) + 2*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*sin(4*d
*x + 4*c) + 8*(31*a*b^4 - 16*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) + 2*
(2*(1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*sin(10*d*x + 10*c) -
(8192*a^4*b - 23296*a^3*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5)*sin(8*
d*x + 8*c) - 2*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4 - 1078*b^5)*sin(6
*d*x + 6*c) + 4*(512*a^3*b^2 - 1520*a^2*b^3 + 1330*a*b^4 - 343*b^5)*sin(4*d
*x + 4*c) + 2*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*sin(2*d*x + 2*c))*cos(12*
d*x + 12*c) + 2*((51200*a^4*b - 84864*a^3*b^2 + 56016*a^2*b^3 - 18081*a*b^4
+ 1960*b^5)*sin(8*d*x + 8*c) + 8*(6400*a^3*b^2 - 8608*a^2*b^3 + 3437*a*b^4
- 392*b^5)*sin(6*d*x + 6*c) - 2*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4
- 1078*b^5)*sin(4*d*x + 4*c) - 16*(224*a^2*b^3 - 209*a*b^4 + 42*b^5)*sin(2
*d*x + 2*c))*cos(10*d*x + 10*c) + 2*((51200*a^4*b - 84864*a^3*b^2 + 56016*a
^2*b^3 - 18081*a*b^4 + 1960*b^5)*sin(6*d*x + 6*c) - (8192*a^4*b - 23296*a^3
*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5)*sin(4*d*x + 4*c) - (3968*a^3*
b^2 - 5024*a^2*b^3 + 2621*a*b^4 - 560*b^5)*sin(2*d*x + 2*c))*cos(8*d*x + 8*
c) + 4*((1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*sin(4*d*x + 4*c)
- 4*(48*a^2*b^3 - 55*a*b^4 + 28*b^5)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) -
8*((a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2
*a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 +
337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a
^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x
+ 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 3
2841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*
a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x
+ 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a
^2*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d
*x + 2*c)^2 + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(
a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 24
0*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2
+ 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d
*sin(10*d*x + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*
a^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2
+ 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*
d*sin(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3
*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a

```
^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)
*d*cos(2*d*x + 2*c) + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2
*a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*
a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 +
355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3
- 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 -
23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^
3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*co
s(16*d*x + 16*c) + 16*(4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*
d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)
*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^
3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^
3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*
b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*c
os(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8
*(8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*
cos(10*d*x + 10*c) + 2*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4
*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^6*b^2 - 42
4*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*
(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4
*d*x + 4*c) - 8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6152 vs. $2(271) = 542$.
time = 2.60, size = 6152, normalized size = 19.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a
^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a
^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*co
s(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(
1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^10 - 5*a^9*
b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b
- 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5
 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21
- 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 21
0*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/(
(a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(
491520*a^6*b - 1742720*a^5*b^2 + 2747904*a^4*b^3 - 2435877*a^3*b^4 + 510698
9/4*a^2*b^5 - 750141/2*a*b^6 + 194481/4*b^7 - 1/4*(1966080*a^6*b - 6970880*
```

a^5*b^2 + 10991616*a^4*b^3 - 9743508*a^3*b^4 + 5106989*a^2*b^5 - 1500282*a*b^6 + 194481*b^7)*cos(d*x + c)^2 + 1/2*((32*a^16 - 193*a^15*b + 498*a^14*b^2 - 715*a^13*b^3 + 620*a^12*b^4 - 327*a^11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*d^3*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (88320*a^9*b - 319040*a^8*b^2 + 510294*a^7*b^3 - 457551*a^6*b^4 + 241865*a^5*b^5 - 71421*a^4*b^6 + 9261*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2)) - 1/4*(2*(1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2*cos(d*x + c)^2 - (1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2)*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(491520*a^6*b - 1742720*a^5*b^2 + 2747904*a^4*b^3 - 2435877*a^3*b^4 + 5106989/4*a^2*b^5 - 750141/2*a*b^6 + 194481/4*b^7 - 1/4*(1966080*a^6*b - 6970880*a^5*b^2 + 10991616*a^4*b^3 - 9743508*a^3*b^4 + 5106989*a^2*b^5 - 1500282*a*b^6 + 194481*b^7)*cos(d*x + c)^2 - 1/2*((32*a^16 - 193*a^15*b + 498*a^14*b^2 - 715*a^13*b^3 + 620*a^12*b^4 - 327*a^11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*d^3*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))*cos(d*x + c)*sin(

```
d*x + c) + (88320*a^9*b - 319040*a^8*b^2 + 510294*a^7*b^3 - 457551*a^6*b^4
+ 241865*a^5*b^5 - 71421*a^4*b^6 + 9261*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c
))*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^1
0 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686
400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 443358
81*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^
9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^1
6*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10
)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. 2(271) = 542.
time = 0.71, size = 1131, normalized size = 3.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*((96*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4 - 333*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*a^3*b + 313*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 -
79*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - 21*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*b^4 + 42*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 108
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 34*sqrt(a^2 - a*b +
sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 8*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^
5 - 2*a^4*b + a^3*b^2 + sqrt((a^5 - 2*a^4*b + a^3*b^2)^2 - (a^5 - 2*a^4*b +
 a^3*b^2)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^5 - 3*a^4*b + 3*a^3*b^
2 - a^2*b^3))))*abs(-a + b)/(3*a^9 - 18*a^8*b + 41*a^7*b^2 - 44*a^6*b^3 + 2
1*a^5*b^4 - 2*a^4*b^5 - a^3*b^6) + (96*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
a^4 - 333*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b + 313*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*a^2*b^2 - 79*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^3 -
 21*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4 - 42*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^3 + 108*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)
*a^2*b - 34*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - 8*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) +
```

```
 arctan(tan(d*x + c)/sqrt((a^5 - 2*a^4*b + a^3*b^2 - sqrt((a^5 - 2*a^4*b +
a^3*b^2)^2 - (a^5 - 2*a^4*b + a^3*b^2)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3
)))/(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3))))*abs(-a + b)/(3*a^9 - 18*a^8*b
+ 41*a^7*b^2 - 44*a^6*b^3 + 21*a^5*b^4 - 2*a^4*b^5 - a^3*b^6) - 2*(33*a^2*b
*tan(d*x + c)^7 - 46*a*b^2*tan(d*x + c)^7 + 13*b^3*tan(d*x + c)^7 + 83*a^2*
b*tan(d*x + c)^5 - 66*a*b^2*tan(d*x + c)^5 + 7*b^3*tan(d*x + c)^5 + 67*a^2*
b*tan(d*x + c)^3 - 43*a*b^2*tan(d*x + c)^3 + 17*a^2*b*tan(d*x + c) - 11*a*b
^2*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c)^2
 + a)^2*(a^4 - 2*a^3*b + a^2*b^2)))/d
```

Mupad [B]
time = 19.67, size = 2500, normalized size = 7.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - ((tan(c + d*x)^5*(83*a^2*b - 66*a*b^2 + 7*b^3))/(32*a^2*(a - b)^2) + (tan
(c + d*x)^7*(33*a*b - 13*b^2))/(32*a^2*(a - b)) + (tan(c + d*x)*(17*a*b - 1
1*b^2))/(32*a*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)^3*(67*a*b - 43*b^2))/(32
*a*(a^2 - 2*a*b + b^2)))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2) + a^2 - tan
(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) + 4*a^2*tan(c
+ d*x)^2)) - (atan(((((524288*a^10*b - 344064*a^5*b^6 + 1802240*a^6*b^5 - 3
866624*a^7*b^4 + 4227072*a^8*b^3 - 2342912*a^9*b^2)/(32768*(3*a^8*b - a^9 +
 a^6*b^3 - 3*a^7*b^2)) - (tan(c + d*x)*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*
(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*
a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*
b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^
13*b^3 - 10*a^14*b^2)))^(1/2)*(16384*a^11*b - 16384*a^6*b^6 + 81920*a^7*b^5
 - 163840*a^8*b^4 + 163840*a^9*b^3 - 81920*a^10*b^2))/(256*(3*a^6*b - a^7 +
 a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2)
- 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*
a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1
/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^1
4*b^2)))^(1/2) - (tan(c + d*x)*(1024*a^5*b - 2141*a*b^5 + 441*b^6 + 4099*a^
2*b^4 - 3139*a^3*b^3 + 4*a^4*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^
2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024
*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2
) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15
*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*1i -
 (((524288*a^10*b - 344064*a^5*b^6 + 1802240*a^6*b^5 - 3866624*a^7*b^4 + 42
27072*a^8*b^3 - 2342912*a^9*b^2)/(32768*(3*a^8*b - a^9 + a^6*b^3 - 3*a^7*b^
2)) + (tan(c + d*x)*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 19
16*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^
```

3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))
/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^
2)))^(1/2)*(16384*a^11*b - 16384*a^6*b^6 + 81920*a^7*b^5 - 163840*a^8*b^4 +
 163840*a^9*b^3 - 81920*a^10*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^
2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024
*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2
) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15
*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2) + (t
an(c + d*x)*(1024*a^5*b - 2141*a*b^5 + 441*b^6 + 4099*a^2*b^4 - 3139*a^3*b^
3 + 4*a^4*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^
11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4
 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^1
1*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^
5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*1i)/((((524288*a^10*b -
 344064*a^5*b^6 + 1802240*a^6*b^5 - 3866624*a^7*b^4 + 4227072*a^8*b^3 - 234
2912*a^9*b^2)/(32768*(3*a^8*b - a^9 + a^6*b^3 - 3*a^7*b^2)) - (tan(c + d*x)
*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^1
0 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) -
4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b -
 a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*(16384*a
^11*b - 16384*a^6*b^6 + 81920*a^7*b^5 - 163840*a^8*b^4 + 163840*a^9*b^3 - 8
1920*a^10*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^
11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4
 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^1
1*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^
5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2) - (tan(c + d*x)*(1024*a
^5*b - 2141*a*b^5 + 441*b^6 + 4099*a^2*b^4 - 3139*a^3*b^3 + 4*a^4*b^2))/(25
6*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b
^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 15
01*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a
^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10
*a^13*b^3 - 10*a^14*b^2)))^(1/2) + (((524288*a^10*b - 344064*a^5*b^6 + 1802
240*a^6*b^5 - 3866624*a^7*b^4 + 4227072*a^8*b^3 - 2342912*a^9*b^2)/(32768*(
3*a^8*b - a^9 + a^6*b^3 - 3*a^7*b^2)) + (tan(c + d*x)*((1920*a^4*(a^11*b)^(
1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*
a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1
/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a
^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*(16384*a^11*b - 16384*a^6*b^6
+ 81920*a^7*b^5 - 163840*a^8*b^4 + 163840*a^9*b^3 - 81920*a^10*b^2))/(256*(
3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*((1920*a...

3.235 $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=357

$$\frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}-\frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+15b\right)\tan^{-1}\left(\right.}{64a^{13/4}\left(\sqrt{a}+\sqrt{b}\right.}$$

```
[Out] -cot(d*x+c)/a^3/d+3/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b
^(1/2)*(20*a+15*b-34*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)-b^(1/2))^(5/2)-3/
64*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1/2)*(20*a+15*b+34
*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)+b^(1/2))^(5/2)-1/8*b^2*tan(d*x+c)*(a*
(a+3*b)+(a^2+6*a*b+b^2)*tan(d*x+c)^2)/a^2/(a-b)^3/d/(a+2*a*tan(d*x+c)^2+(a-
b)*tan(d*x+c)^4)^2-1/32*b*tan(d*x+c)*(2*a^2*(9*a-17*b)/(a-b)^3+(18*a^2+15*a
*b-13*b^2)*tan(d*x+c)^2/(a-b)^2)/a^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)
^4)
```

Rubi [A]

time = 0.87, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3296, 1348, 1683, 1678, 1180, 211}

$$\frac{3\sqrt{b}\left(-34\sqrt{a}\sqrt{b}+20a+15b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\sqrt{b}\left(34\sqrt{a}\sqrt{b}+20a+15b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cot(c+dx)}{a^3d}-\frac{b^2\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+dx)+a(a+3b)\right)}{8a^2d(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}-\frac{b\tan(c+dx)\left(\frac{\left(18a^2+15ab-13b^2\right)\tan^2(c+dx)}{(a-b)^2}+\frac{2a^2(9a-17b)}{(a-b)^3}\right)}{32a^3d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tan[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (3*S
qrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot[c +
d*x]/(a^3*d) - (b^2*Tan[c + d*x]*(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c +
 d*x]^2))/(8*a^2*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]
^4)^2) - (b*Tan[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 + ((18*a^2 + 15*a*
b - 13*b^2)*Tan[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a + 2*a*Tan[c + d*x]^2 +
 (a - b)*Tan[c + d*x]^4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1348

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 3296

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^
```

```
(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^6}{x^2\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-16a \cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx) \cdots}{32a^3d\,(a+2 \cdots}$$

$$= -\frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx) \cdots}{32a^3d\,(a+2 \cdots}$$

$$= -\frac{\cot(c+dx)}{a^3d} - \frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \cdots$$

$$= -\frac{\cot(c+dx)}{a^3d} - \frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \cdots$$

$$= \frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} - \frac{3\sqrt{b}\left(20 \cdots\right.}{\cdots}$$

Mathematica [A]

time = 3.51, size = 357, normalized size = 1.00

$$-\frac{\frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+15b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{a+\sqrt{a}\sqrt{b}}} + \frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)^2\sqrt{-a+\sqrt{a}\sqrt{b}}} + 64\cot(c+dx) + \frac{4b\left(28a^2+3ab-13b^2+b(-19a+13b)\cos(2(c+dx))\right)\sin(2(c+dx))}{(a-b)^2(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))} + \frac{128ab(2a+b-b\cos(2(c+dx)))\sin(2(c+dx))}{(a-b)(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}}{64a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[((Sqrt[a] + Sqr
t[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])^2*Sqrt
```

```
[a + Sqrt[a]*Sqrt[b]]) + (3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcT
anh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[
a] - Sqrt[b])^2*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 64*Cot[c + d*x] + (4*b*(28*a^
2 + 3*a*b - 13*b^2 + b*(-19*a + 13*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(
(a - b)^2*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (128*a
*b*(2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a - b)*(-8*a + 3*b -
4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2))/(a^3*d)
```

Maple [A]
time = 1.78, size = 437, normalized size = 1.22

method	result
derivativedivides	$-\frac{1}{a^3\tan(dx+c)}+\frac{b\left(\frac{-\frac{\left(18a^2+15ab-13b^2\right)\left(\tan^7(dx+c)\right)}{32(a-b)}-\frac{a\left(27a^2-2ab-13b^2\right)\left(\tan^5(dx+c)\right)}{16\left(a^2-2ab+b^2\right)}-\frac{\left(54a^2-13ab-17b^2\right)a\left(\tan^3(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}-}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}\right.}{}$

default	$-\frac{1}{a^3\tan(dx+c)}+\frac{b\left(\frac{-\frac{\left(18a^2+15ab-13b^2\right)\left(\tan^7(dx+c)\right)}{32(a-b)}-\frac{a\left(27a^2-2ab-13b^2\right)\left(\tan^5(dx+c)\right)}{16\left(a^2-2ab+b^2\right)}-\frac{\left(54a^2-13ab-17b^2\right)a\left(\tan^3(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)^2}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3/tan(d*x+c)+b/a^3*((-1/32*(18*a^2+15*a*b-13*b^2)/(a-b)*tan(d*x+c
)^7-1/16*a*(27*a^2-2*a*b-13*b^2)/(a^2-2*a*b+b^2)*tan(d*x+c)^5-1/32*(54*a^2-
13*a*b-17*b^2)*a/(a^2-2*a*b+b^2)*tan(d*x+c)^3-3/16*a^2*(3*a-2*b)/(a^2-2*a*b
+b^2)*tan(d*x+c))/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2+3/32
/(a^2-2*a*b+b^2)*(a-b)*(1/2*(26*a^2*(a*b)^(1/2)-37*a*b*(a*b)^(1/2)+15*b^2*(
a*b)^(1/2)+20*a^3-27*a^2*b+11*a*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-
b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(26*a^
2*(a*b)^(1/2)-37*a*b*(a*b)^(1/2)+15*b^2*(a*b)^(1/2)-20*a^3+27*a^2*b-11*a*b^
2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c
)/(((a*b)^(1/2)-a)*(a-b))^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(12*(160*a^3*b^3 - 57*a^2*b^4 - 195*a*b^5 + 135*b^6)*cos(4*d*x + 4*c)*
sin(2*d*x + 2*c) + (3*(20*a^2*b^4 - 33*a*b^5 + 15*b^6)*sin(16*d*x + 16*c) -
 12*(43*a^2*b^4 - 68*a*b^5 + 30*b^6)*sin(14*d*x + 14*c) - 4*(400*a^3*b^3 -
```

1137*a^2*b^4 + 1031*a*b^5 - 315*b^6)*sin(12*d*x + 12*c) + 12*(592*a^3*b^3 -
1237*a^2*b^4 + 886*a*b^5 - 210*b^6)*sin(10*d*x + 10*c) + 2*(4096*a^4*b^2 -
12192*a^3*b^3 + 13634*a^2*b^4 - 7113*a*b^5 + 1575*b^6)*sin(8*d*x + 8*c) +
4*(880*a^3*b^3 - 2855*a^2*b^4 + 2512*a*b^5 - 630*b^6)*sin(6*d*x + 6*c) - 4*
(256*a^3*b^3 - 823*a^2*b^4 + 903*a*b^5 - 315*b^6)*sin(4*d*x + 4*c) - 12*(19
*a^2*b^4 - 54*a*b^5 + 30*b^6)*sin(2*d*x + 2*c))*cos(18*d*x + 18*c) + 3*(4*(
160*a^3*b^3 - 57*a^2*b^4 - 195*a*b^5 + 135*b^6)*sin(14*d*x + 14*c) + 4*(400
*a^3*b^3 - 1671*a^2*b^4 + 1800*a*b^5 - 630*b^6)*sin(12*d*x + 12*c) - 2*(256
0*a^4*b^2 + 3232*a^3*b^3 - 13806*a^2*b^4 + 11469*a*b^5 - 2835*b^6)*sin(10*d
*x + 10*c) - 4*(4864*a^4*b^2 - 14576*a^3*b^3 + 16221*a^2*b^4 - 8430*a*b^5 +
1890*b^6)*sin(8*d*x + 8*c) - 4*(1840*a^3*b^3 - 6825*a^2*b^4 + 6243*a*b^5 -
1575*b^6)*sin(6*d*x + 6*c) + 4*(608*a^3*b^3 - 2025*a^2*b^4 + 2292*a*b^5 -
810*b^6)*sin(4*d*x + 4*c) + 9*(56*a^2*b^4 - 183*a*b^5 + 105*b^6)*sin(2*d*x
+ 2*c))*cos(16*d*x + 16*c) + 4*(4*(3200*a^4*b^2 - 7536*a^3*b^3 + 7612*a^2*b
^4 - 3915*a*b^5 + 945*b^6)*sin(12*d*x + 12*c) - 6*(3968*a^4*b^2 - 14864*a^3
*b^3 + 19013*a^2*b^4 - 10224*a*b^5 + 1890*b^6)*sin(10*d*x + 10*c) - 2*(3276
8*a^5*b - 117888*a^4*b^2 + 172048*a^3*b^3 - 127323*a^2*b^4 + 49365*a*b^5 -
8505*b^6)*sin(8*d*x + 8*c) - 8*(3520*a^4*b^2 - 12800*a^3*b^3 + 17461*a^2*b^
4 - 9882*a*b^5 + 1890*b^6)*sin(6*d*x + 6*c) + 4*(2048*a^4*b^2 - 7856*a^3*b^
3 + 11838*a^2*b^4 - 8091*a*b^5 + 2025*b^6)*sin(4*d*x + 4*c) + 3*(608*a^3*b^
3 - 2025*a^2*b^4 + 2292*a*b^5 - 810*b^6)*sin(2*d*x + 2*c))*cos(14*d*x + 14*
c) + 4*(2*(51200*a^5*b - 67456*a^4*b^2 - 32384*a^3*b^3 + 91591*a^2*b^4 - 46
683*a*b^5 + 6615*b^6)*sin(10*d*x + 10*c) + 2*(112640*a^5*b - 364160*a^4*b^2
+ 462304*a^3*b^3 - 293923*a^2*b^4 + 97020*a*b^5 - 13230*b^6)*sin(8*d*x + 8
c) + 4(19200*a^4*b^2 - 78800*a^3*b^3 + 95318*a^2*b^4 - 43701*a*b^5 + 6615
*b^6)*sin(6*d*x + 6*c) - 8*(3520*a^4*b^2 - 12800*a^3*b^3 + 17461*a^2*b^4 -
9882*a*b^5 + 1890*b^6)*sin(4*d*x + 4*c) - 3*(1840*a^3*b^3 - 6825*a^2*b^4 +
6243*a*b^5 - 1575*b^6)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 4*((524288*a^
6 - 1761280*a^5*b + 2435584*a^4*b^2 - 1768256*a^3*b^3 + 719196*a^2*b^4 - 16
3611*a*b^5 + 19845*b^6)*sin(8*d*x + 8*c) + 2*(112640*a^5*b - 364160*a^4*b^2
+ 462304*a^3*b^3 - 293923*a^2*b^4 + 97020*a*b^5 - 13230*b^6)*sin(6*d*x + 6
c) - 2(32768*a^5*b - 117888*a^4*b^2 + 172048*a^3*b^3 - 127323*a^2*b^4 + 4
9365*a*b^5 - 8505*b^6)*sin(4*d*x + 4*c) - 3*(4864*a^4*b^2 - 14576*a^3*b^3 +
16221*a^2*b^4 - 8430*a*b^5 + 1890*b^6)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c
) + 2*(4*(51200*a^5*b - 67456*a^4*b^2 - 32384*a^3*b^3 + 91591*a^2*b^4 - 466
83*a*b^5 + 6615*b^6)*sin(6*d*x + 6*c) - 12*(3968*a^4*b^2 - 14864*a^3*b^3 +
19013*a^2*b^4 - 10224*a*b^5 + 1890*b^6)*sin(4*d*x + 4*c) - 3*(2560*a^4*b^2
+ 3232*a^3*b^3 - 13806*a^2*b^4 + 11469*a*b^5 - 2835*b^6)*sin(2*d*x + 2*c))*
cos(8*d*x + 8*c) + 4*(4*(3200*a^4*b^2 - 7536*a^3*b^3 + 7612*a^2*b^4 - 3915*
a*b^5 + 945*b^6)*sin(4*d*x + 4*c) + 3*(400*a^3*b^3 - 1671*a^2*b^4 + 1800*a*
b^5 - 630*b^6)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 16*((a^5*b^4 - 2*a^4*b^
5 + a^3*b^6)*d*cos(18*d*x + 18*c)^2 + 81*(a^5*b^4 - 2*a^4*b^5 + a^3*b^6)*d*
cos(16*d*x + 16*c)^2 + 16*(64*a^7*b^2 - 272*a^6*b^3 + 433*a^5*b^4 - 306*a^4
*b^5 + 81*a^3*b^6)*d*cos(14*d*x + 14*c)^2 + 16*(1600*a^7*b^2 - 4880*a^6*b^3
+ 5401*a^5*b^4 - 2562*a^4*b^5 + 441*a^3*b^6)*d*cos(12*d*x + 12*c)^2 + 4*(1

```
6384*a^9 - 73728*a^8*b + 140032*a^7*b^2 - 144576*a^6*b^3 + 86017*a^5*b^4 -
28098*a^4*b^5 + 3969*a^3*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^9 - 73728
*a^8*b + 140032*a^7*b^2 - 144576*a^6*b^3 + 86017*a^5*b^4 - 28098*a^4*b^5 +
3969*a^3*b^6)*d*cos(8*d*x + 8*c)^2 + 16*(1600*a^7*b^2 - 4880*a^6*b^3 + 5401
*a^5*b^4 - 2562*a^4*b^5 + 441*a^3*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^7*b^
2 - 272*a^6*b^3 + 433*a^5*b^4 - 306*a^4*b^5 + 81*a^3*b^6)*d*cos(4*d*x + 4*c
)^2 + 81*(a^5*b^4 - 2*a^4*b^5 + a^3*b^6)*d*cos(2*d*x + 2*c)^2 + (a^5*b^4 -
2*a^4*b^5 + a^3*b^6)*d*sin(18*d*x + 18*c)^2 + 81*(a^5*b^4 - 2*a^4*b^5 + a^3
*b^6)*d*sin(16*d*x + 16*c)^2 + 16*(64*a^7*b^2 - 272*a^6*b^3 + 433*a^5*b^4 -
 306*a^4*b^5 + 81*a^3*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(1600*a^7*b^2 - 4880
*a^6*b^3 + 5401*a^5*b^4 - 2562*a^4*b^5 + 441*a^3*b^6)*d*sin(12*d*x + 12*c)^
2 + 4*(16384*a^9 - 73728*a^8*b + 140032*a^7*b^2 - 144576*a^6*b^3 + 86017*a^
5*b^4 - 28098*a^4*b^5 + 3969*a^3*b^6)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^9
 - 73728*a^8*b + 140032*a^7*b^2 - 144576*a^6*b^3 + 86017*a^5*b^4 - 28098*a^
4*b^5 + 3969*a^3*b^6)*d*sin(8*d*x + 8*c)^2 + 16*(1600*a^7*b^2 - 4880*a^6*b^
3 + 5401*a^5*b^4 - 2562*a^4*b^5 + 441*a^3*b^6)*d*sin(6*d*x + 6*c)^2 + 16*(6
4*a^7*b^2 - 272*a^6*b^3 + 433*a^5*b^4 - 306*a^4*b^5 + 81*a^3*b^6)*d*sin(4*d
*x + 4*c)^2 + 72*(8*a^6*b^3 - 25*a^5*b^4 + 26*a^4*b^5 - 9*a^3*b^6)*d*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 81*(a^5*b^4 - 2*a...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6323 vs. $2(305) = 610$.
time = 2.75, size = 6323, normalized size = 17.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(8*(32*a^2*b^2 - 83*a*b^3 + 45*b^4)*cos(d*x + c)^9 - 48*(19*a^2*b^2
- 54*a*b^3 + 30*b^4)*cos(d*x + c)^7 - 8*(64*a^3*b - 301*a^2*b^2 + 555*a*b^3
 - 270*b^4)*cos(d*x + c)^5 + 16*(55*a^3*b - 188*a^2*b^2 + 235*a*b^3 - 90*b^
4)*cos(d*x + c)^3 + 3*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4
*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 +
7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3
- a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*
b^4)*d)*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^
5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*s
qrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 +
 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 5062
5*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 25
2*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13
*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6
*b^5)*d^2))*log(1728000*a^6*b^2 - 7369920*a^5*b^3 + 13507020*a^4*b^4 - 1357
3305*a^3*b^5 + 31519503/4*a^2*b^6 - 5011875/2*a*b^7 + 1366875/4*b^8 - 27/4*
```

(256000*a^6*b^2 - 1091840*a^5*b^3 + 2001040*a^4*b^4 - 2010860*a^3*b^5 + 116
7389*a^2*b^6 - 371250*a*b^7 + 50625*b^8)*cos(d*x + c)^2 + 27/2*((26*a^17 -
167*a^16*b + 460*a^15*b^2 - 705*a^14*b^3 + 650*a^13*b^4 - 361*a^12*b^5 + 11
2*a^11*b^6 - 15*a^10*b^7)*d^3*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054
400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086
*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 1
20*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 4
5*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (12
800*a^10*b - 54080*a^9*b^2 + 98420*a^8*b^3 - 98415*a^7*b^4 + 56973*a^6*b^5
- 18109*a^5*b^6 + 2475*a^4*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(400*a^4
*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b +
10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 -
2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 538
9980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^
22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*
b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 -
5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2)) - 27/4*(2*
(400*a^14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 -
5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2*cos(d*x + c)^2 - (400*a^14 -
2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^
5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2)*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4
+ 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 +
2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*
b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*
b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))*sin(d*x + c) - 3*((a^5*
b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*
b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos
(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2
+ (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(400*a^4*b - 1
044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b + 10*a^
9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 235520
0*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a
^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b +
45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 -
120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^1
0*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log(1728000*a^6*
b^2 - 7369920*a^5*b^3 + 13507020*a^4*b^4 - 13573305*a^3*b^5 + 31519503/4*a^
2*b^6 - 5011875/2*a*b^7 + 1366875/4*b^8 - 27/4*(256000*a^6*b^2 - 1091840*a^
5*b^3 + 2001040*a^4*b^4 - 2010860*a^3*b^5 + 1167389*a^2*b^6 - 371250*a*b^7
+ 50625*b^8)*cos(d*x + c)^2 - 27/2*((26*a^17 - 167*a^16*b + 460*a^15*b^2 -
705*a^14*b^3 + 650*a^13*b^4 - 361*a^12*b^5 + 112*a^11*b^6 - 15*a^10*b^7)*d^
3*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^
6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 5
0625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 -
252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a

```
^13*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (12800*a^10*b - 54080*a^9*b^2 +
 98420*a^8*b^3 - 98415*a^7*b^4 + 56973*a^6*b^5 - 18109*a^5*b^6 + 2475*a^4*b
^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2
*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5
*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^
6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 538...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2203 vs. 2(305) = 610.
time = 1.37, size = 2203, normalized size = 6.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/64*(3*((78*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b - 267*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 241*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 53*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*sqrt(a*b)*a*b^4 - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^5)*(
a^5 - 2*a^4*b + a^3*b^2)^2*abs(-a + b) - 2*(9*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*a^10*b - 51*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^9*b^2 + 108*sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*a^8*b^3 - 106*sqrt(a^2 - a*b + sqrt(a*b)*(a -
 b))*a^7*b^4 + 45*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b^5 - 3*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*a^5*b^6 - 2*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
a^4*b^7)*abs(a^5 - 2*a^4*b + a^3*b^2)*abs(-a + b) - (60*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*sqrt(a*b)*a^15 - 441*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*s
qrt(a*b)*a^14*b + 1339*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^13*b
^2 - 2185*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^12*b^3 + 2059*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^11*b^4 - 1091*sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*sqrt(a*b)*a^10*b^5 + 265*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*sqrt(a*b)*a^9*b^6 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a
^8*b^7 - 11*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^8)*abs(-a +
 b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^6 - 2*a^5*
b + a^4*b^2 + sqrt((a^6 - 2*a^5*b + a^4*b^2)^2 - (a^6 - 2*a^5*b + a^4*b^2)*
(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)))/(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^
```

```
3))))/((3*a^16 - 27*a^15*b + 104*a^14*b^2 - 224*a^13*b^3 + 294*a^12*b^4 - 2
38*a^11*b^5 + 112*a^10*b^6 - 24*a^9*b^7 - a^8*b^8 + a^7*b^9)*abs(a^5 - 2*a^
4*b + a^3*b^2)) + 3*((78*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*
b - 267*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 241*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 53*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^4 - 15*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(
a*b)*b^5)*(a^5 - 2*a^4*b + a^3*b^2)^2*abs(-a + b) - 2*(9*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*a^10*b - 51*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^9*b^2 +
 108*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^8*b^3 - 106*sqrt(a^2 - a*b - sqr
t(a*b)*(a - b))*a^7*b^4 + 45*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6*b^5 -
3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b^6 - 2*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*a^4*b^7)*abs(a^5 - 2*a^4*b + a^3*b^2)*abs(-a + b) - (60*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^15 - 441*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^14*b + 1339*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(
a*b)*a^13*b^2 - 2185*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^12*b^3
 + 2059*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^11*b^4 - 1091*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^10*b^5 + 265*sqrt(a^2 - a*b - sq
rt(a*b)*(a - b))*sqrt(a*b)*a^9*b^6 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
sqrt(a*b)*a^8*b^7 - 11*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^
8)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a
^6 - 2*a^5*b + a^4*b^2 - sqrt((a^6 - 2*a^5*b + a^4*b^2)^2 - (a^6 - 2*a^5*b
+ a^4*b^2)*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)))/(a^6 - 3*a^5*b + 3*a^4*b
^2 - a^3*b^3))))/((3*a^16 - 27*a^15*b + 104*a^14*b^2 - 224*a^13*b^3 + 294*a
^12*b^4 - 238*a^11*b^5 + 112*a^10*b^6 - 24*a^9*b^7 - a^8*b^8 + a^7*b^9)*abs
(a^5 - 2*a^4*b + a^3*b^2)) + 2*(18*a^3*b*tan(d*x + c)^7 - 3*a^2*b^2*tan(d*x
 + c)^7 - 28*a*b^3*tan(d*x + c)^7 + 13*b^4*tan(d*x + c)^7 + 54*a^3*b*tan(d*
x + c)^5 - 4*a^2*b^2*tan(d*x + c)^5 - 26*a*b^3*tan(d*x + c)^5 + 54*a^3*b*ta
n(d*x + c)^3 - 13*a^2*b^2*tan(d*x + c)^3 - 17*a*b^3*tan(d*x + c)^3 + 18*a^3
*b*tan(d*x + c) - 12*a^2*b^2*tan(d*x + c))/((a^5 - 2*a^4*b + a^3*b^2)*(a*ta
n(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c)^2 + a)^2) + 64/(a^3*tan(
d*x + c)))/d
```

Mupad [B]
time = 20.80, size = 2500, normalized size = 7.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - b*sin(c + d*x)^4)^3),x)
```

```
[Out] (atan((((-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^1
1*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b
^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(
1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a
^16*b^2)))^(1/2)*(2315255808*a^15*b^12 - 201326592*a^14*b^13 - 12079595520*
```

a^16*b^11 + 37748736000*a^17*b^10 - 78517370880*a^18*b^9 + 114152177664*a^19*b^8 - 118380036096*a^20*b^7 + 87577067520*a^21*b^6 - 45298483200*a^22*b^5 + 15602810880*a^23*b^4 - 3221225472*a^24*b^3 + 301989888*a^25*b^2 + tan(c + d*x)*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2147483648*a^29*b + 2147483648*a^17*b^13 - 25769803776*a^18*b^12 + 141733920768*a^19*b^11 - 472446402560*a^20*b^10 + 1063004405760*a^21*b^9 - 1700807049216*a^22*b^8 + 1984274890752*a^23*b^7 - 1700807049216*a^24*b^6 + 1063004405760*a^25*b^5 - 472446402560*a^26*b^4 + 141733920768*a^27*b^3 - 25769803776*a^28*b^2)) + tan(c + d*x)*(3024617472*a^11*b^13 - 265420800*a^10*b^14 - 15574892544*a^12*b^12 + 47520940032*a^13*b^11 - 94402510848*a^14*b^10 + 125505110016*a^15*b^9 - 108421447680*a^16*b^8 + 51536461824*a^17*b^7 + 484835328*a^18*b^6 - 18454413312*a^19*b^5 + 12354453504*a^20*b^4 - 3779592192*a^21*b^3 + 471859200*a^22*b^2))*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*1i + ((-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(201326592*a^14*b^13 - 2315255808*a^15*b^12 + 12079595520*a^16*b^11 - 37748736000*a^17*b^10 + 78517370880*a^18*b^9 - 114152177664*a^19*b^8 + 118380036096*a^20*b^7 - 87577067520*a^21*b^6 + 45298483200*a^22*b^5 - 15602810880*a^23*b^4 + 3221225472*a^24*b^3 - 301989888*a^25*b^2 + tan(c + d*x)*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2147483648*a^29*b + 2147483648*a^17*b^13 - 25769803776*a^18*b^12 + 141733920768*a^19*b^11 - 472446402560*a^20*b^10 + 1063004405760*a^21*b^9 - 1700807049216*a^22*b^8 + 1984274890752*a^23*b^7 - 1700807049216*a^24*b^6 + 1063004405760*a^25*b^5 - 472446402560*a^26*b^4 + 141733920768*a^27*b^3 - 25769803776*a^28*b^2)) + tan(c + d*x)*(3024617472*a^11*b^13 - 265420800*a^10*b^14 - 15574892544*a^12*b^12 + 47520940032*a^13*b^11 - 94402510848*a^14*b^10 + 125505110016*a^15*b^9 - 108421447680*a^16*b^8 + 51536461824*a^17*b^7 + 484835328*a^18*b^6 - 18454413312*a^19*b^5 + 12354453504*a^20*b^4 - 3779592192*a^21*b^3 + 471859200*a^22*b^2))*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*1i)/(((-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4 *(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2315255808*a^15*b^12 - 201326592*a^14*b^13 - 12079595520*a^16*b^11 + 37748736000*a^17*b^10 - 78517370880*a^18*b^9 + 114152177664*a^19*b^8 - 118380036096*a^20*b^7 + 87577067520*a^21*b^6 - 45298483200*a^22*b^5 + 15602810880*a^23*b^4 - 3221225472*a^24*b^3 + 301989888*a^25*b^2 + tan(c + d*x)*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2147483648*a^29*b + 2147483648*a^17*b^13 - 25769803776*a^18*b^12 + 141733920768*a^19*b^11 - 472446402560*a^20*b^10 + 1063004405760*a^21*b^9 - 1700807049216*a^22*b^8 + 1984274890752*a^23*b^7 - 1700807049216*a^24*b^6 + 1063004405760*a^25*b^5 - 472446402560*a^26*b^4 + 141733920768*a^27*b^3 - 25769803776*a^28*b^2)) + tan(c + d*x)*(3024617472*a^11*b^13 - 265420800*a^10*b^14 - 1557...

3.236 $\int \frac{1}{1-\sin^4(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\sqrt{2}\,\tan(x)\right)}{2\sqrt{2}}+\frac{\tan(x)}{2}$$

```
[Out] 1/4*arctan(2^(1/2)*tan(x))*2^(1/2)+1/2*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.80, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3288, 396, 209}

$$\frac{\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{2\sqrt{2}}+\frac{x}{2\sqrt{2}}+\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^4)^(-1),x]
```

```
[Out] x/(2*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(2*Sqrt[2]
) + Tan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{1-\sin^4(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{1+2x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{x}{2\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{2\sqrt{2}} + \frac{\tan(x)}{2}$$

Mathematica [A]
time = 0.04, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\tan^{-1}\left(\sqrt{2}\tan(x)\right) + 2\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^4)^(-1),x]
[Out] (Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/4
```

Maple [A]
time = 0.15, size = 18, normalized size = 0.72

method	result	size
default	$\frac{\arctan\left(\sqrt{2}\,\tan(x)\right)\sqrt{2}}{4} + \frac{\tan(x)}{2}$	18
risch	$\frac{i}{e^{2ix}+1} + \frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{8} - \frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{8}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^4),x,method=_RETURNVERBOSE)
[Out] 1/4*arctan(2^(1/2)*tan(x))*2^(1/2)+1/2*tan(x)
```

Maxima [A]
time = 0.50, size = 17, normalized size = 0.68

$$\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) + \frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*tan(x)) + 1/2*tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(17) = 34$.
time = 0.40, size = 43, normalized size = 1.72

$$-\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)\cos(x) - 4\sin(x)}{8\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))*
cos(x) - 4*sin(x))/cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(20) = 40$.
time = 24.20, size = 724, normalized size = 28.96

$$\frac{54608393\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{x}{2}\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}} + \frac{77227930\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{x}{2}\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}} - \frac{77227930\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}} - \frac{54608393\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}} + \frac{9369319\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{x}{2}\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}} + \frac{13250218\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^2\left(\frac{x}{2}\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}} - \frac{13250218\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}} - \frac{9369319\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}} - \frac{90478148\tan\left(\frac{x}{2}\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}} - \frac{63977712\sqrt{2}\tan\left(\frac{x}{2}\right)}{63977712\sqrt{2}\tan^2\left(\frac{x}{2}\right)+90478148\tan^2\left(\frac{x}{2}\right)-90478148-63977712\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**4),x)
```

```
[Out] 54608393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) +
pi*floor((x/2 - pi/2)/pi))*tan(x/2)**2/(63977712*sqrt(2)*tan(x/2)**2 + 9047
8148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) + 77227930*sqrt(3 - 2*sqrt(
2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))*tan(x/
2)**2/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 639
77712*sqrt(2)) - 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqr
t(2))) + pi*floor((x/2 - pi/2)/pi))/(63977712*sqrt(2)*tan(x/2)**2 + 9047814
8*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) - 54608393*sqrt(2)*sqrt(3 - 2*
sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(
63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 63977712*s
qrt(2)) + 9369319*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2)
 + 3)) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**2/(63977712*sqrt(2)*tan(x/2)*
*2 + 90478148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) + 13250218*sqrt(2*
sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi
))*tan(x/2)**2/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478
148 - 63977712*sqrt(2)) - 13250218*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(
2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(63977712*sqrt(2)*tan(x/2)**2
+ 90478148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) - 9369319*sqrt(2)*sqr
t(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2
)/pi))/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 63
977712*sqrt(2)) - 90478148*tan(x/2)/(63977712*sqrt(2)*tan(x/2)**2 + 9047814
```

```
8*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) - 63977712*sqrt(2)*tan(x/2)/(6
3977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 63977712*sq
rt(2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(17) = 34.
time = 0.44, size = 51, normalized size = 2.04

$$\frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)+\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
+ sqrt(2) - 2*cos(2*x) + 2))) + 1/2*tan(x)
```

Mupad [B]
time = 14.34, size = 17, normalized size = 0.68

$$\frac{\tan(x)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)^4 - 1),x)
```

```
[Out] tan(x)/2 + (2^(1/2)*atan(2^(1/2)*tan(x)))/4
```

3.237 $\int \frac{1}{a+b\sin^4(x)}\,dx$

Optimal. Leaf size=487

$$-\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}-\sqrt{2}(a+b)^{3/4}\tan(x)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}+\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\cdots\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a\cdots}}$$

```
[Out] 1/8*ln((a+b)^(1/4)*a^(1/2)-a^(1/4)*2^(1/2)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)*
tan(x)+(a+b)^(3/4)*tan(x)^2)*(a^(1/2)-(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1
/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)-1/8*ln((a+b)^(1/4)*a^(1/2)+a^(1/4)*2^(1
/2)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)*tan(x)+(a+b)^(3/4)*tan(x)^2)*(a^(1/2)-(
a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)-1/4
*arctan((a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)-(a+b)^(3/4)*2^(1/2)*tan(x)
)/a^(1/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/(a
+b)^(1/4)*2^(1/2)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)+1/4*arctan((a^(1/4)*(a+b-
a^(1/2)*(a+b)^(1/2))^(1/2)+(a+b)^(3/4)*2^(1/2)*tan(x))/a^(1/4)/(a+b+a^(1/2)
*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a+b
+a^(1/2)*(a+b)^(1/2))^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3288, 1183, 648, 632, 210, 642}

$$-\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}-\sqrt{2}(a+b)^{3/4}\tan(x)}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\text{ArcTan}\left(\frac{\sqrt{2}(a+b)^{3/4}\tan(x)+\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left((a+b)^{3/4}\tan^2(x)-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\tan(x)+\sqrt{a}\sqrt[4]{a+b}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left((a+b)^{3/4}\tan^2(x)+\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\tan(x)+\sqrt{a}\sqrt[4]{a+b}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^4)^(-1),x]
```

```
[Out] -1/2*((Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a +
 b]] - Sqrt[2]*(a + b)^(3/4)*Tan[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a +
 b]])])/(Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) +
 ((Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
 + Sqrt[2]*(a + b)^(3/4)*Tan[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]
)])/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) + (
(Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a
+ b - Sqrt[a]*Sqrt[a + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/(4*Sqrt[2]*a^(
3/4)*(a + b)^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) - ((Sqrt[a] - Sqrt[a
+ b])*Log[Sqrt[a]*(a + b)^(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt
[a + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^(1/4)
*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a+b\sin^4(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a+b)x^4}\,dx, x, \tan(x)\right)$$

$$= \frac{\sqrt[4]{a+b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)x}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \tan(x)\right)}{2\sqrt{2}\,a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}} + $$

$$= \frac{\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \tan(x\right.}{4(a+b)}$$

$$= -\frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,\tan(x\right.}{4\sqrt{2}\,a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}$$

$$= -\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\sqrt{2}\tan(x)\right)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}} + \left(\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 148, normalized size = 0.30

$$\frac{\left(\sqrt{a}-i\sqrt{b}\right)\sqrt{a+i\sqrt{a}\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt{a+i\sqrt{a}\sqrt{b}}\,\tan(x)}{\sqrt{a}}\right)-\left(\sqrt{a}+i\sqrt{b}\right)\sqrt{-a+i\sqrt{a}\sqrt{b}}\,\tanh^{-1}\left(\frac{\sqrt{-a+i\sqrt{a}\sqrt{b}}\,\tan(x)}{\sqrt{a}}\right)}{2a(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^4)^(-1),x]
```

```
[Out] ((Sqrt[a] - I*Sqrt[b])*Sqrt[a + I*Sqrt[a]*Sqrt[b]]*ArcTan[(Sqrt[a + I*Sqrt[
a]*Sqrt[b]]*Tan[x])/Sqrt[a]] - (Sqrt[a] + I*Sqrt[b])*Sqrt[-a + I*Sqrt[a]*Sq
```

```
rt[b]]*ArcTanh[(Sqrt[-a + I*Sqrt[a]*Sqrt[b]]*Tan[x])/Sqrt[a]])/(2*a*(a + b)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(343) = 686$.

time = 0.33, size = 771, normalized size = 1.58

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(256a^4+256a^3b)\_Z^4+32a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{128ia^4}{b}-128ia^3\right)\_R^3+\left(\frac{32a^3}{b}+32a^2\right)\_R^2\right.$
default	$\frac{\left(-a^{\frac{5}{2}}\sqrt{2\sqrt{a^2+ab}-2a}-a^{\frac{3}{2}}\sqrt{2\sqrt{a^2+ab}-2a}\sqrt{a^2+ab}+\sqrt{a+b}\sqrt{2\sqrt{a^2+ab}-2a}\sqrt{a^2+a}\right.}{2\sqrt{a+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^(5/2)/b*(1/2*(-a^(5/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-a^(3/2)*(2*(a^2+
a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(
1/2)*(a^2+a*b)^(1/2)*a+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2)/(a+b)
^(1/2)*ln((a+b)^(1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+a^(1/2)
)+2*(2*a^2*b-1/2*(-a^(5/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-a^(3/2)*(2*(a^2+a*
b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/
2)*(a^2+a*b)^(1/2)*a+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2)*(2*(a*(
a+b))^(1/2)-2*a)^(1/2)/(a+b)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2
)+2*a)^(1/2)*arctan((2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4
*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)))+1/4/a^(5/2)/b*(-1/2*(-a
^(5/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-a^(3/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*
(a^2+a*b)^(1/2)+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a
+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)
*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-a^(1/2))+2*(-2*a^2*b+1/2*(-a
^(5/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-a^(3/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*
(a^2+a*b)^(1/2)+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a
+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2)*(2*(a*(a+b))^(1/2)-2*a)^(1/
2)/(a+b)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan(
(-2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2
)-2*(a*(a+b))^(1/2)+2*a)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^4 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(344) = 688$.
time = 0.55, size = 823, normalized size = 1.69

$$-\frac{1}{8}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\left(\frac{1}{4}b\cos(x)^2+\frac{1}{2}\left(ab\cos(x)\sin(x)+(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}-\frac{1}{4}\left(a^3+a^2b-2(a^3+a^2b)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-\frac{1}{4}b\right)+\frac{1}{8}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\left(\frac{1}{4}b\cos(x)^2-\frac{1}{2}\left(ab\cos(x)\sin(x)+(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}-\frac{1}{4}\left(a^3+a^2b-2(a^3+a^2b)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-\frac{1}{4}b\right)+\frac{1}{8}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}\log\left(-\frac{1}{4}b\cos(x)^2+\frac{1}{2}\left(ab\cos(x)\sin(x)-(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}-\frac{1}{4}\left(a^3+a^2b-2(a^3+a^2b)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+\frac{1}{4}b\right)-\frac{1}{8}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}\log\left(-\frac{1}{4}b\cos(x)^2-\frac{1}{2}\left(ab\cos(x)\sin(x)-(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}-\frac{1}{4}\left(a^3+a^2b-2(a^3+a^2b)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+\frac{1}{4}b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)
)*log(1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5
+ 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^
4*b + a^3*b^2)) + 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(
x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(-((a^2 + a*b)*
sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b))*log(1/4*b*cos(x)^2 - 1
/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*co
s(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^
2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b +
a^3*b^2)) - 1)/(a^2 + a*b))*log(-1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x) -
(a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2
+ a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - 1/4*(a^3 + a^
2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1/4*b)
 - 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b
))*log(-1/4*b*cos(x)^2 - 1/2*(a*b*cos(x)*sin(x) - (a^4 + a^3*b)*sqrt(-b/(a^
5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos
(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1/4*b)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.62, size = 318, normalized size = 0.65

$$\frac{\left(3\sqrt{a^2+ab+\sqrt{-ab}\,(a+b)}\,a^2+6\sqrt{a^2+ab+\sqrt{-ab}\,(a+b)}\,ab-\sqrt{a^2+ab+\sqrt{-ab}\,(a+b)}\,b^2\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a+\sqrt{-16(a+b)a+16a^2}}{a+b}}}\right)\right)|a+b|}{2\left(3a^5+12a^4b+14a^3b^2+4a^2b^3-ab^4\right)}+\frac{\left(3\sqrt{a^2+ab-\sqrt{-ab}\,(a+b)}\,a^2+6\sqrt{a^2+ab-\sqrt{-ab}\,(a+b)}\,ab-\sqrt{a^2+ab-\sqrt{-ab}\,(a+b)}\,b^2\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a-\sqrt{-16(a+b)a+16a^2}}{a+b}}}\right)\right)|a+b|}{2\left(3a^5+12a^4b+14a^3b^2+4a^2b^3-ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^4),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + a*b + sqrt(-a*b)*(a + b))*a^2 + 6*sqrt(a^2 + a*b + sqrt(-
a*b)*(a + b))*a*b - sqrt(a^2 + a*b + sqrt(-a*b)*(a + b))*b^2)*(pi*floor(x/p
i + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a + b)*a + 16*a^2))/(a + b
))))*abs(a + b)/(3*a^5 + 12*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 - a*b^4) + 1/2*(
3*sqrt(a^2 + a*b - sqrt(-a*b)*(a + b))*a^2 + 6*sqrt(a^2 + a*b - sqrt(-a*b)*
(a + b))*a*b - sqrt(a^2 + a*b - sqrt(-a*b)*(a + b))*b^2)*(pi*floor(x/pi + 1
/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a + b)*a + 16*a^2))/(a + b))))*
abs(a + b)/(3*a^5 + 12*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 - a*b^4)
```

Mupad [B]
time = 15.18, size = 407, normalized size = 0.84

$$\mathrm{atan}\left(\frac{a^3\,\tan(x)\,\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^5\,\tan(x)\,{\left(-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}\right)}^{3/2}\,64\mathrm{i}-a^2\,b\,\tan(x)\,\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^4\,b\,\tan(x)\,{\left(-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}\right)}^{3/2}\,64\mathrm{i}}{\sqrt{-a^3\,b}}\right)\,\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{a^3\,\tan(x)\,\sqrt{-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^5\,\tan(x)\,{\left(-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}\right)}^{3/2}\,64\mathrm{i}-a^2\,b\,\tan(x)\,\sqrt{-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^4\,b\,\tan(x)\,{\left(-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}\right)}^{3/2}\,64\mathrm{i}}{\sqrt{-a^3\,b}}\right)\,\sqrt{-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)^4),x)
```

```
[Out] atan((a^3*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a
^5*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i - a^2*b*t
an(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^4*b*tan(x)
*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i)/(-a^3*b)^(1/2))*(
-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*2i - atan((a^3*tan(x)*(-
(a^2 + (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^5*tan(x)*(-(a^2 +
(-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i - a^2*b*tan(x)*(-(a^2 + (-a^
3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^4*b*tan(x)*(-(a^2 + (-a^3*b)^
(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i)/(-a^3*b)^(1/2))*(-(a^2 + (-a^3*b)^(1
/2))/(16*a^3*b + 16*a^4))^(1/2)*2i
```

3.238 $\int \frac{1}{1+\sin^4(x)}\,dx$

Optimal. Leaf size=309

$$\frac{x}{2\sqrt{-1+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos^2(x)-\left(-2+\sqrt{2}\right)\cos(x)\sin(x)}{2+\sqrt{1+\sqrt{2}}+\left(-2+\sqrt{2}\right)\cos^2(x)-2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)}\right)}{4\sqrt{-1+\sqrt{2}}}-\frac{\tan^{-1}\left(\frac{\sqrt{-1+\sqrt{2}}}{2+\sqrt{1+\sqrt{2}}}\right.}{}$$

```
[Out] 1/2*x/(2^(1/2)-1)^(1/2)+1/4*arctan((-cos(x)*sin(x)*(-2+2^(1/2))+(2^(1/2)-1)
^(1/2)-2*cos(x)^2*(2^(1/2)-1)^(1/2))/(2+cos(x)^2*(-2+2^(1/2))-2*cos(x)*sin(
x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/2)))/(2^(1/2)-1)^(1/2)-1/4*arctan((cos(
x)*sin(x)*(-2+2^(1/2))+(2^(1/2)-1)^(1/2)-2*cos(x)^2*(2^(1/2)-1)^(1/2))/(2+c
os(x)^2*(-2+2^(1/2))+2*cos(x)*sin(x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/2)))/
(2^(1/2)-1)^(1/2)-1/8*ln(2^(1/2)-2*(2^(1/2)-1)^(1/2)*tan(x)+2*tan(x)^2)*(2^
(1/2)-1)^(1/2)+1/8*ln(1+(-2+2*2^(1/2))^(1/2)*tan(x)+2^(1/2)*tan(x)^2)*(2^(1
/2)-1)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3288, 1183, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{-2\sqrt{\sqrt{2}-1}\cos^2(x)-\left(\sqrt{2}-2\right)\sin(x)\cos(x)+\sqrt{\sqrt{2}-1}}{\left(\sqrt{2}-2\right)\cos^2(x)-2\sqrt{\sqrt{2}-1}\sin(x)\cos(x)+\sqrt{1+\sqrt{2}}+2}\right)}{4\sqrt{\sqrt{2}-1}}-\frac{\text{ArcTan}\left(\frac{-2\sqrt{\sqrt{2}-1}\cos^2(x)+\left(\sqrt{2}-2\right)\sin(x)\cos(x)+\sqrt{\sqrt{2}-1}}{\left(\sqrt{2}-2\right)\cos^2(x)+2\sqrt{\sqrt{2}-1}\sin(x)\cos(x)+\sqrt{1+\sqrt{2}}+2}\right)}{4\sqrt{\sqrt{2}-1}}+\frac{x}{2\sqrt{\sqrt{2}-1}}-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(2\tan^2(x)-2\sqrt{\sqrt{2}-1}\tan(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\sqrt{2}\tan^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\tan(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^4)^(-1),x]
```

```
[Out] x/(2*Sqrt[-1 + Sqrt[2]]) + ArcTan[(Sqrt[-1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]
]*Cos[x]^2 - (-2 + Sqrt[2])*Cos[x]*Sin[x])/(2 + Sqrt[1 + Sqrt[2]] + (-2 + S
qrt[2])*Cos[x]^2 - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x])]/(4*Sqrt[-1 + Sqrt[2
]]) - ArcTan[(Sqrt[-1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]^2 + (-2 + Sq
rt[2])*Cos[x]*Sin[x])/(2 + Sqrt[1 + Sqrt[2]] + (-2 + Sqrt[2])*Cos[x]^2 + 2*
Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x])]/(4*Sqrt[-1 + Sqrt[2]]) - (Sqrt[-1 + Sqrt
[2]]*Log[Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*Tan[x] + 2*Tan[x]^2])/8 + (Sqrt[-1
+ Sqrt[2]]*Log[1 + Sqrt[2*(-1 + Sqrt[2])]*Tan[x] + Sqrt[2]*Tan[x]^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{1+\sin^4(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{1+2x^2+2x^4}\,dx, x, \tan(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}-\left(1-\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \tan(x)\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}+\left(1-\frac{1}{\sqrt{2}}\right.}{\frac{1}{\sqrt{2}}+\sqrt{-1+\sqrt{2}}\,x+}\right.}{2\sqrt{2\left(-1+\sqrt{2}\right.}}$$

$$= -\left(\frac{1}{8}\sqrt{-1+\sqrt{2}}\,\text{Subst}\left(\int \frac{-\sqrt{-1+\sqrt{2}}+2x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \tan(x)\right)\right) + \frac{1}{8}\sqrt{-1+}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{2}}\,\log\left(\sqrt{2}-2\sqrt{-1+\sqrt{2}}\,\tan(x)+2\tan^2(x)\right) + \frac{1}{8}\sqrt{-1+\sqrt{2}}\,\log$$

$$= \frac{1}{2}\sqrt{1+\sqrt{2}}\,x + \frac{1}{4}\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{-1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\,\cos^2(x)+\left(2-\right.}{2+\sqrt{1+\sqrt{2}}-\left(2-\sqrt{2}\right)\cos^2(x)-2\sqrt{-1}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 45, normalized size = 0.15

$$\frac{\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{2\sqrt{1-i}} + \frac{\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^4)^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 - I]*Tan[x]]/(2*Sqrt[1 - I]) + ArcTan[Sqrt[1 + I]*Tan[x]]/(2*
Sqrt[1 + I])
```

Maple [A]
time = 0.44, size = 190, normalized size = 0.61

method	result
risch	$\frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}+i\sqrt{-2-2i}-\sqrt{-2-2i}-1-2i\right)}{8} - \frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}-i\sqrt{-2-2i}+\sqrt{-2-2i}-1\right.}{8}$

default	$\frac{\sqrt{2}\left(\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\sqrt{2}+2\left(\tan^2(x)\right)+\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\,\tan(x)\right)}{4}+\frac{\left(-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}}{4}+2\right)\arctan\left(\frac{4\tan(x)+\sqrt{\ }}{2}\right.}{\sqrt{1+\sqrt{2}}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*(1/4*(-2+2*2^(1/2))^(1/2)*ln(2^(1/2)+2*tan(x)^2+(-2+2*2^(1/2))^
(1/2)*2^(1/2)*tan(x))+(-1/4*(-2+2*2^(1/2))*2^(1/2)+2)/(1+2^(1/2))^(1/2)*arc
tan(1/2*(4*tan(x)+2^(1/2)*(-2+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)))+1/4*2^(
1/2)*(-1/4*(-2+2*2^(1/2))^(1/2)*ln(-(-2+2*2^(1/2))^(1/2)*2^(1/2)*tan(x)+2*t
an(x)^2+2^(1/2))+(-1/4*(-2+2*2^(1/2))*2^(1/2)+2)/(1+2^(1/2))^(1/2)*arctan(1
/2*(-2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x))/(1+2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sin(x)^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3830 vs. $2(229) = 458$.
time = 18.12, size = 3830, normalized size = 12.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^4),x, algorithm="fricas")
```

```
[Out] -1/32*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-(4*sqrt(2) - 5)*cos(x)
^4 + 2*(2*sqrt(2) - 3)*cos(x)^2 + (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2*2^(
1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 2) + 1/32*2^(1/4)*s
qrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-(4*sqrt(2) - 5)*cos(x)^4 + 2*(2*sqrt(
2) - 3)*cos(x)^2 - (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) -
 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 2) - 1/16*2^(1/4)*sqrt(2*sqrt(2) +
 4)*arctan(1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16
*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14 + 16*(sqrt(2)*(51*
sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt(2)*(41*sqrt(2) - 36) -
```

54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2) - 90) - 58*sqrt(2) + 1
32)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt(2) + 216)*cos(x)^6 - 4
(sqrt(2)(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4 + 4*(2*sqrt(2) - 15)*co
s(x)^2 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2) + 2))*cos(x)^
15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqrt(2) + 4))*cos(x)^13 +
4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2) - 14))*cos(x)^11 - 8*(
2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31))*cos(x)^9 + 2*(2^(3/4
)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(x)^7 - 2*(2^(3/4)*(9*sq
rt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (2*2^(3/4)*(sqrt(2) - 2)
 + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(x))*sqrt(2*sqrt(2) + 4)
sin(x) + (16(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 - 56*(sqr
t(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sqrt(2) -
62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 10*sqrt(2
) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*cos(x)^6
 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt(2)*(sqr
t(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt
(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) -
 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(2) - 94)
)*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 34))*cos(
x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*cos(x)^5 -
 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2^(1/4)*(
sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqrt(2) - 5
)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 + 4*(2^(1/4)*(3*sqrt(2) - 4)*cos(x)
^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 8) + 4)/(
112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(x)^10 - 152*cos(x)^
8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) + 1/16*2^(1/4)*sqrt(2*sq
rt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)
^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14 + 16*(sqrt
(2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt(2)*(41*sqrt(2)
 - 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2) - 90) - 58*sqr
t(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt(2) + 216)*cos(
x)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4 + 4*(2*sqrt(2)
 - 15)*cos(x)^2 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2) + 2))
cos(x)^15 - 8(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqrt(2) + 4))*cos(
x)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2) - 14))*cos(x)^
11 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31))*cos(x)^9 + 2
(2^(3/4)(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(x)^7 - 2*(2^(3/
4)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (2*2^(3/4)*(sqrt
(2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(x))*sqrt(2*sqrt
(2) + 4)*sin(x) - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 -
 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sq
rt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 1
0*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*
cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt
(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)

```
*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*s
qrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(
2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 3
4))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*co
s(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2
^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqr
t(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 + 4*(2^(1/4)*(3*sqrt(2) - 4
)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 8
) + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(x)^10 - 152
*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) - 1/16*2^(1/4)*s
qrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6
)*cos(x)^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14 +
16*(sqrt(2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**4),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 170, normalized size = 0.55

$$\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan\left(x\right)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}+\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan\left(x\right)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan\left(x\right)^{2}+\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan\left(x\right)+\sqrt{\frac{1}{2}}\right)-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan\left(x\right)^{2}-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan\left(x\right)+\sqrt{\frac{1}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^4),x, algorithm="giac")
```

```
[Out] 1/4*(pi*floor(x/pi + 1/2) + arctan(2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2)
 + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/4*(pi*floor(x/p
i + 1/2) + arctan(-2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2) + 2) - 2*tan(x)
)/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/8*sqrt(sqrt(2) - 1)*log(tan(x)^
2 + (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(1/2)) - 1/8*sqrt(sqrt(2) -
 1)*log(tan(x)^2 - (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(1/2))
```

Mupad [B]
time = 14.35, size = 236, normalized size = 0.76

$$\mathrm{atanh}\left(\frac{\tan\left(x\right)}{8\,\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}-\frac{\tan\left(x\right)}{8\,\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\sqrt{2}\,\tan\left(x\right)}{16\,\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\sqrt{2}\,\tan\left(x\right)}{16\,\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}\right)\,\left(2\,\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\,\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)+\mathrm{atanh}\left(\frac{\tan\left(x\right)}{8\,\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\tan\left(x\right)}{8\,\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\sqrt{2}\,\tan\left(x\right)}{16\,\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}-\frac{\sqrt{2}\,\tan\left(x\right)}{16\,\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}\right)\,\left(2\,\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+2\,\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)-\frac{\left(x-\mathrm{atan}\left(\tan\left(x\right)\right)\right)\,\left(\pi\,\left(2\,\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\,\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)\,1{}\mathrm{i}+\pi\,\left(2\,\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+2\,\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)\,1{}\mathrm{i}\right)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4 + 1),x)
```

```
[Out] atanh(tan(x)/(8*(- 2^(1/2)/64 - 1/64)^(1/2)) - tan(x)/(8*(2^(1/2)/64 - 1/64
)^(1/2)) + (2^(1/2)*tan(x))/(16*(- 2^(1/2)/64 - 1/64)^(1/2)) + (2^(1/2)*tan
(x))/(16*(2^(1/2)/64 - 1/64)^(1/2)))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^
(1/2)/64 - 1/64)^(1/2)) + atanh(tan(x)/(8*(- 2^(1/2)/64 - 1/64)^(1/2)) + ta
n(x)/(8*(2^(1/2)/64 - 1/64)^(1/2)) + (2^(1/2)*tan(x))/(16*(- 2^(1/2)/64 - 1
/64)^(1/2)) - (2^(1/2)*tan(x))/(16*(2^(1/2)/64 - 1/64)^(1/2)))*(2*(- 2^(1/2
)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2)) - ((x - atan(tan(x)))*(pi
*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/64 - 1/64)^(1/2))*1i + pi*(2*(
- 2^(1/2)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2))*1i))/pi
```

3.239 $\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=477

$$-\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d}+\frac{2\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos}}{3\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)}$$

```
[Out] -1/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d+2/3*cos(d*x+c
)*b^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d/(1+cos(d*x+c)^2*b^(
1/2)/(a+b)^(1/2))/(a+b)^(1/2)-2/3*b^(1/4)*(a+b)^(3/4)*(cos(2*arctan(b^(1/4)
*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1
/4)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1
/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(
d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2
)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+1/3*(a+b)^(3/4)*(cos(2*arct
an(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c
)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2
*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(
1/2)-(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c
)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x
+c)^4)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3294, 1105, 1211, 1117, 1209}

$$\frac{(a+b)^{3/4}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{3\sqrt[4]{b}\,d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{2\sqrt[4]{b}(a+b)^{3/4}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{3d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3d}+\frac{2\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/3*(Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/d +
 (2*Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4
])/(3*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - (2*b^(1/4
)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*
b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)],
 (1 + Sqrt[b]/Sqrt[a + b])/2])/(3*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4]) + ((a + b)^(3/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c
+ d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)
```

```
/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(3*b^
(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \sqrt{a+b-2bx^2+bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} - \frac{\text{Subst}\Big(}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} - \frac{\left(2\sqrt{b}\right.}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} + \frac{2\sqrt{b}\,\text{co}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 61.22, size = 47242, normalized size = 99.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 18.47, size = 439, normalized size = 0.92

method	result
default	$-\frac{\frac{4\cos(dx+c)\sqrt{a+b-2b\left(\cos^2(dx+c)\right)+b\left(\cos^4(dx+c)\right)}}{3}+\frac{4\left(\frac{2a}{3}+\frac{2b}{3}\right)\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^2(dx+c)\right)}{a+b}}}{\sqrt{\frac{i\sqrt{a}}{}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(4/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+4*(2/3*a
+2/3*b)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*
cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b
-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^
(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))+16/3*b*(a+b
)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*
x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*c
os(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos
(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+
b))^(1/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(
I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*sin(d*x + c), x)
```

Fricas [**F**]
time = 0.11, size = 35, normalized size = 0.07

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b}\,\sin\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sin(d*x + c),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*sin(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)\,\sqrt{b\,{\sin\left(c+d\,x\right)}^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.240 $\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=521

$$\frac{\sqrt{-a}\,\tan^{-1}\left(\frac{\sqrt{-a}\,\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2d}+\frac{\sqrt{b}\,\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b}}{\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\,\cos^2(c+dx)}{\sqrt{a+b}}\right)}$$

```
[Out] 1/2*arctan(cos(d*x+c)*(-a)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2
))*(-a)^(1/2)/d+cos(d*x+c)*b^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1
/2)/d/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-b^(1/4)*(a+b)^(3/4)*
(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*cos(d*x+c)/(a+b)^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^
(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^
(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)
/(a+b)^(1/2))^2)^(1/2)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/4*(a
+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*cos(d*
x+c)/(a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*(2+2
*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(1/2)-
(a+b)^(1/2))^2*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2
*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)
^4)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3294, 1222, 1211, 1117, 1209, 1230, 1720}

$$\frac{\sqrt{-a}\,\text{ArcTan}\left(\frac{\sqrt{-a}\,\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2d}-\frac{\sqrt[4]{b}\,(a+b)^{3/4}\left(\frac{\sqrt{b}\,\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\,\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)^2\left(\frac{\sqrt{b}\,\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\,\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{4\sqrt[4]{b}\,d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}+\frac{\sqrt{b}\,\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{d\sqrt{a+b}\left(\frac{\sqrt{b}\,\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Sqrt[-a]*ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 +
b*Cos[c + d*x]^4]])/(2*d) + (Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c +
d*x]^2 + b*Cos[c + d*x]^4])/(Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sq
rt[a + b])) - (b^(1/4)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a +
 b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sq
rt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c +
d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(d*Sqrt[a + b - 2*b*Cos
[c + d*x]^2 + b*Cos[c + d*x]^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])^2
```

```
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2
)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(
b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*b^(1
/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-2bx^2+bx^4}}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{-b+bx^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\left(\sqrt{b}\sqrt{a+b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a+b}}}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx\right.}{d}$$

$$= \frac{\sqrt{-a}\,\tan^{-1}\left(\frac{\sqrt{-a}\,\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2d} + \frac{\sqrt{b}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 41.54, size = 118912, normalized size = 228.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 1.32, size = 0, normalized size = 0.00

$$\int \csc \left(dx+c\right) \sqrt{a+b \left(\sin^{4}\left(dx+c\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*csc(d*x + c), x)
```

Fricas [F]
time = 0.16, size = 35, normalized size = 0.07

$$\mathrm{integral}\left(\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}\,\csc\left(dx+c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*csc(d*x + c),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^{4}\left(c+dx\right)}\,\csc\left(c+dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x)**4)*csc(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*csc(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\sin\left(c+d\,x\right)^4+a}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^4)^(1/2)/sin(c + d*x),x)
```

```
[Out] int((a + b*sin(c + d*x)^4)^(1/2)/sin(c + d*x), x)
```

3.241 $\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=484

$$-\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd}+\frac{2\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c}{3\sqrt{b}\,\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\,\cos^2(c+dx)}{\sqrt{a+b}}\right)}$$

```
[Out] -1/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/b/d+2/3*cos(d*x
+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d/b^(1/2)/(1+cos(d*x+c)^2*b
^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-2/3*(a+b)^(3/4)*(cos(2*arctan(b^(1/4)*cos(d
*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*
EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a
+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)
^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(3
/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+1/6*(a+b)^(1/4)*(cos(2*ar
ctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x
+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1
/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(a
-2*b+2*b^(1/2)*(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1
+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(5/4)/d/(a+b-2*b*cos(d*x+c)^2
+b*cos(d*x+c)^4)^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3294, 1220, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a+b}\left(2\sqrt{b}\sqrt{a+b}+a-2b\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{6b^{5/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{2(a+b)^{3/4}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{3b^{3/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3bd}+\frac{2\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3\sqrt{b}\,d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/3*(Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(b*
d) + (2*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(
3*Sqrt[b]*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - (2*(a
 + b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Co
s[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqr
t[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1
+ Sqrt[b]/Sqrt[a + b])/2])/(3*b^(3/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b
*Cos[c + d*x]^4]) + ((a + b)^(1/4)*(a - 2*b + 2*Sqrt[b]*Sqrt[a + b])*(1 + (
```

```
Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*C
os[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*Elli
pticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a +
 b])/2])/(6*b^(5/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} - \frac{\text{Subst}\left(\int \frac{-a}{\sqrt{a+b}}\right.}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} - \frac{\left(2\sqrt{a+b}\right)\text{Subst}}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} + \frac{2\cos(c+dx)\sqrt{a}}{3\sqrt{b}\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 61.22, size = 47246, normalized size = 97.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 20.28, size = 837, normalized size = 1.73

method	result

default	$-\frac{\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)(\cos^2(dx+c))}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)(\cos^2(dx+c))}{a+b}}\,\mathrm{EllipticF}\left(\cos(dx+c)\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}},\right.}{d\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\,\sqrt{a+b-2b\left(\cos^2\left(dx+c\right)\right)+b\left(\cos^4\left(dx+c\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos
(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*
b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/
2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-4/d*(a+b)/((I*
a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2
)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x
+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos(d*x+c
)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1
/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1
/2)*b^(1/2)-b)/(a+b))^(1/2)))-4/d*(1/12/b*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+
b*cos(d*x+c)^4)^(1/2)-1/12*(a+b)/b/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(
I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+
b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*Elliptic
F(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b
)/(a+b))^(1/2))-2/3*(a+b)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)
*b^(1/2)+b)/(a+b)*cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*
x+c)^2)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)
*b^(1/2))*(EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(
I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)
+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^5/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F]
time = 0.13, size = 55, normalized size = 0.11

$$\text{integral}\left(\frac{\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c)/sqrt(b*cos(d*
x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^5/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^5}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.242 $\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=431

$$\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{\sqrt{b}\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)}-\frac{(a+b)^{3/4}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c}{(a+b)\left(1\right.}}}{b^{3/4}d\sqrt{a+b}}$$

```
[Out] cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d/b^(1/2)/(1+cos(d*x
+c)^2*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-(a+b)^(3/4)*(cos(2*arctan(b^(1/4)*co
s(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)
))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)
/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x
+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b
^(3/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/2*(a+b)^(1/4)*(cos(2
*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(
d*x+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))
),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))
*(b^(1/2)-(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(
d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(3/4)/d/(a+b-2*b*cos(d*x+c)^2+b*co
s(d*x+c)^4)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3294, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2b^{3/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{(a+b)^{3/4}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{b^{3/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}+\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{\sqrt{b}\,d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(Sqrt[b]
*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - ((a + b)^(3/4)
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2
)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/S
qrt[a + b])/2])/(b^(3/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]
^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)
```

```
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)
*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*b^(3/4)*d*S
qrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\sqrt{a+b}\,\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a+b}}}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{\sqrt{b}\,d} - \frac{\left(1-\frac{\sqrt{a+}}{\sqrt{b}}\right.}{(a+b)^{3/4}\left(1+\frac{\sqrt{}}{}\right.}$$

$$= \frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{\sqrt{b}\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 51.30, size = 89374, normalized size = 207.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 20.52, size = 398, normalized size = 0.92

method	result
default	$-\frac{\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)\left(\cos^2(dx+c)\right)}{a+b}}\;\text{EllipticF}\left(\cos(dx+c)\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}},\right.}{d\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\;\sqrt{a+b-2b\left(\cos^2\left(dx+c\right)\right)+b\left(\cos^4\left(dx+c\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos
(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*
```

```
b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/
2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-2/d*(a+b)/((I*
a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2
)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x
+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos(d*x+c
)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1
/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1
/2)*b^(1/2)-b)/(a+b))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [**F**]
time = 0.10, size = 46, normalized size = 0.11

$$\text{integral}\left(-\frac{\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/sqrt(b*cos(d*x + c)^4 - 2*b*cos
(d*x + c)^2 + a + b), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^3}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.243 $\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt[4]{a+b}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(1-\right.\right.}{2\sqrt[4]{b}\,d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

```
[Out] -1/2*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)
^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos
(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*c
os(d*x+c)^4)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3294, 1117}

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2\sqrt[4]{b}\,d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*((a + b)^(1/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b
- 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x
]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/
4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]
^2 + b*Cos[c + d*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\sqrt[4]{a+b}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\,F}{2\sqrt[4]{b}\,d\sqrt{a+b-2b\cos^2(c+dx)+b\cos}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 45.06, size = 13300, normalized size = 77.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 17.55, size = 163, normalized size = 0.95

method	result
default	$-\frac{\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,\mathrm{EllipticF}\left(\cos(dx+c)\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}},\right.}{d\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\,\sqrt{a+b-2b\left(\cos^2(dx+c)\right)+b\left(\cos^4(dx+c)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos
(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*
```

```
b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/
2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F]
time = 0.11, size = 35, normalized size = 0.20

$$\mathrm{integral}\left(\frac{\sin\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] integral(sin(d*x + c)/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b),
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)}{\sqrt{b\,{\sin\left(c+d\,x\right)}^4+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(sin(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.244 $\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=469

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2\sqrt{-a}\,d}+\frac{\sqrt[4]{b}\,\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)}{2ad}$$

```
[Out] -1/2*arctan(cos(d*x+c)*(-a)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/
2))/d/(-a)^(1/2)+1/2*b^(1/4)*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(
a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*Ellipti
cF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/
2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(1/2)-(a+b)^(1/2))*((a+b
-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2)
)^2)^(1/2)/a/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/4*(a+b)^(1/4)*
(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*cos(d*x+c)/(a+b)^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)
^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*(2+2*b^(1/2)/(
a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(1/2)-(a+b)^(1/2
))^2*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(
a+b)^(1/2))^2)^(1/2)/a/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2
)
```

Rubi [A]
time = 0.29, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3294, 1230, 1117, 1720}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2\sqrt{-a}\,d}+\frac{\sqrt[4]{b}\,\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2ad\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)^2\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{4a\sqrt[4]{b}\,d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4]]/(Sqrt[-a]*d) + (b^(1/4)*(a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2
)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/S
qrt[a + b])/2])/(2*a*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
 - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])^2*(1 + (Sqrt[b]*Cos[c + d*x]^2)/S
qrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(
```

```
1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a
+ b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1
/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*a*b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c +
 d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\left((a+b)\left(-1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}\,x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c\right.}{ad}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2\sqrt{-a}\,d} + \frac{\sqrt[4]{b}\,\sqrt[4]{a+b}\left(\sqrt{b}-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 41.07, size = 63281, normalized size = 134.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (c + dx)}{\sqrt{a + b \sin^4 (c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(csc(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(csc(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin (c + d\,x)\,\sqrt{b\,{\sin (c + d\,x)}^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2)),x)
[Out] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2)), x)
```

3.245 $\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=776

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{4\sqrt{-a}\,d}-\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(}{2a\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)}$$

```
[Out] -1/4*arctan(cos(d*x+c)*(-a)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/
2))/d/(-a)^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+
c)^4)^(1/2)/a/d-1/2*cos(d*x+c)*b^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4
)^(1/2)/a/d/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)+1/2*b^(1/4)*(a
+b)^(3/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x
+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1
/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)
^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/a/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^
(1/2)-1/8*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticPi(sin(2*arctan(b^
(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1/2)/(a+b)^(1
/2),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2
))*(b^(1/2)-(a+b)^(1/2))^2*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+
cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/a/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^
2+b*cos(d*x+c)^4)^(1/2)-1/2*b^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^
(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticF(si
n(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(
1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(a+b-b^(1/2)*(a+b)^(1/2))*((a+b-
2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))
^2)^(1/2)/a/(a+b)^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)
```

Rubi [A]

time = 0.70, antiderivative size = 776, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3294, 1237, 1728, 1209, 1722, 1117, 1720}

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/4*ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4]]/(Sqrt[-a]*d) - (Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c +
```

```
d*x]^2 + b*Cos[c + d*x]^4])/(2*a*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^
2)/Sqrt[a + b])) - (Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]*Cot
[c + d*x]*Csc[c + d*x])/(2*a*d) + (b^(1/4)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[
c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^
4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcT
an[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*
a*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]) - (b^(1/4)*(a + b
- Sqrt[b]*Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a +
b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c +
d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^
(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*a*(a + b)^(1/4)*d*Sqrt[a + b - 2*b
*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b
])^2*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d
*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b
])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcT
an[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(8*
a*b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
```

```
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 3294

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}\,\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\text{Subs}}{}$$

$$= -\frac{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}\,\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\text{Subs}}{}$$

$$= -\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{2a\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)} - \sqrt{a+b-2}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{4\sqrt{-a}\,d} - \frac{\sqrt{b}\cos(c+dx)\sqrt{}}{2a\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 41.77, size = 119171, normalized size = 153.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
[Out] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F]
time = 0.12, size = 37, normalized size = 0.05

$$\text{integral}\left(\frac{\csc(dx+c)^3}{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] integral(csc(d*x + c)^3/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(csc(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.09Not invertible E
rror: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(c+d\,x\right)^3\,\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^4)^(1/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^4)^(1/2)), x)
```

3.246 $\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=499

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}}\right)\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(}{2\sqrt{b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] -1/2*arctan(b^(1/2)*tan(d*x+c)/(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)^(1/2
))*cos(d*x+c)^2*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)^(1/2)/d/b^(1/2)/(a+
b*sin(d*x+c)^4)^(1/2)-1/2*a^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*ta
n(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*E
llipticF(sin(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+
b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))*((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c
)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d
*x+c)^2)/b/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1/2)+1/4*cos(d*x+c)^2*(cos(2*a
rctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan((a+b)^(1/4)*ta
n(d*x+c)/a^(1/4)))*EllipticPi(sin(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))
,-1/4*(a^(1/2)-(a+b)^(1/2))^2/a^(1/2)/(a+b)^(1/2),1/2*(2-2*a^(1/2)/(a+b)^(1
/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))^2*((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4
)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+
c)^2)/a^(1/4)/b/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3298, 1333, 1117, 1720}

$$-\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}\right)}{2\sqrt{b}\,d\sqrt{a+b\sin^4(c+dx)}}-\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2bd\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)^2\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\,\Pi\left(-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)^2}{4\sqrt{a}\sqrt{a+b}};2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{4\sqrt[4]{a}\,bd\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*(ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*T
an[c + d*x]^4]]*Cos[c + d*x]^2*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c
+ d*x]^4])/(Sqrt[b]*d*Sqrt[a + b*Sin[c + d*x]^4]) - (a^(1/4)*(Sqrt[a] + Sqr
t[a + b])*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^
(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)
*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a +
 b]*Tan[c + d*x]^2)^2])/(2*b*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) +
((Sqrt[a] + Sqrt[a + b])^2*Cos[c + d*x]^2*EllipticPi[-1/4*(Sqrt[a] - Sqrt[a
 + b])^2/(Sqrt[a]*Sqrt[a + b]), 2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/
```

```
4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sq
rt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]
*Tan[c + d*x]^2)^2])/(4*a^(1/4)*b*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4
])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1333

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(-a)*((e + d*q)/(c*d^2 - a*e^2))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[a*d*((e + d*q)/(c*d^2 - a*e^
2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3298

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)
*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f*x]^2)^(2*p)/(f*Apart[a*(1 + Tan[e + f
*x]^2)^2 + b*Tan[e + f*x]^4]^p)), Subst[Int[x^m*(ExpandToSum[a*(1 + ff^2*x^
2)^2 + b*ff^4*x^4, x]^p/(1 + ff^2*x^2)^(m/2 + 2*p + 1)), x], x, Tan[e + f*x
]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[p - 1/
2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{}{(1+x^2)\,\cdots}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\left(a\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx}\right.}{bd\sqrt{a+b\sin^4(c+d}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}}\right)\cos^2(c+dx)\sqrt{a+}}{2\sqrt{b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.08, size = 287, normalized size = 0.58

$$-\frac{2i\cos^2(c+dx)\left(F\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)|\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)-\Pi\left(\frac{\sqrt{a}}{\sqrt{a}-i\sqrt{b}};i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)|\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)\right)\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{2+\left(2-\frac{2i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}}{\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\,d\sqrt{8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ((-2*I)*Cos[c + d*x]^2*(EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*T
an[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])] - EllipticPi[Sqr
t[a]/(Sqrt[a] - I*Sqrt[b]), I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c +
 d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[1 + (1 + (I*Sqrt
[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[2 + (2 - ((2*I)*Sqrt[b])/Sqrt[a])*Tan[c
+ d*x]^2])/(Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Cos[2*(c +
 d*x)] + b*Cos[4*(c + d*x)]])
```

Maple [A]
time = 46.24, size = 881, normalized size = 1.77

method	result

default	$-\frac{\sqrt{\left(4a+\left(\cos^{2}\left(2dx+2c\right)\right)b+b-2b\cos\left(2dx+2c\right)\right)\left(\sin^{2}\left(2dx+2c\right)\right)}\,\sqrt{-ab}\,\sqrt{\frac{\left(-b+\sqrt{-ab}\right)(}{\sqrt{-ab}\,(\cos}}}{2\left(-b+\sqrt{-ab}\right)\sqrt{\frac{(-1+}{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))*sin(2*d*x+2*c)^2)^(1/2)
*(-a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*
x+2*c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)
/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b
)/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*(EllipticF(((-b+(-a*b)^(1/2))*(-1+
cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),((b+(-a*b)^(1/2))/(-
b+(-a*b)^(1/2)))^(1/2))-2*EllipticPi(((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))
/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),(-a*b)^(1/2)/(-b+(-a*b)^(1/2)),((b+
(-a*b)^(1/2))/(-b+(-a*b)^(1/2)))^(1/2)))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d
*x+2*c))*(cos(2*d*x+2*c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d
*x+2*c)+2*(-a*b)^(1/2)-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)^2*b+b-2
*b*cos(2*d*x+2*c))^(1/2)/d-1/2*((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c
))*sin(2*d*x+2*c)^2)^(1/2)*(-a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*
c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*
d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2
*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*Elliptic
F(((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(
1/2),((b+(-a*b)^(1/2))/(-b+(-a*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1
+cos(2*d*x+2*c))*(cos(2*d*x+2*c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b
*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)
^2*b+b-2*b*cos(2*d*x+2*c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F]

time = 0.20, size = 40, normalized size = 0.08

$$\text{integral}\left(-\frac{\cos\left(dx+c\right)^{2}-1}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 +
 a + b), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^2}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.247 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{\cos^2(c+dx)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)\sqrt{\frac{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)^2}}}{2\sqrt[4]{a}\sqrt[4]{a+b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] 1/2*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/co
s(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan((a+b)^(1
/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+2*a*tan(d
*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^
(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(1/4)/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3289, 1117}

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}\,d\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (
1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a
+ 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c
 + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 3289

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f
*x]^2)^(2*p)/(f*(a + 2*a*Tan[e + f*x]^2 + (a + b)*Tan[e + f*x]^4)^p)), Subs
```

```
t[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x],
 x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\cos^2(c+dx)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+\right.}{2\sqrt[4]{a}\sqrt[4]{a+b}\,d\sqrt{a+b\,\text{si}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.18, size = 304, normalized size = 1.88

$$\frac{2\sqrt{2}\left(i\sqrt{a}+\sqrt{b}\right)\left(2\sqrt{a}-i\sqrt{b}+i\sqrt{b}\cos(2(c+dx))\right)\left(2i\sqrt{a}-\sqrt{b}+\sqrt{b}\cos(2(c+dx))\right)\sqrt{\left(1-\frac{2i\sqrt{a}}{\sqrt{b}}-\cos(2(c+dx))\right)\csc^2(c+dx)}\sqrt{\frac{\cot^2(c+dx)\left(i\sqrt{a}\sqrt{b}-a\csc^2(c+dx)\right)}{\left(\sqrt{a}-i\sqrt{b}\right)^2}}F\left(\sin^{-1}\left(\sqrt{\frac{-i\sqrt{b}+\sqrt{a}\csc^2(c+dx)}{\sqrt{a}-i\sqrt{b}}}\right)|\frac{1}{2}+\frac{i\sqrt{a}}{2\sqrt{b}}\right)\sin^2(c+dx)\tan(c+dx)}{\sqrt{a}\,d(8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (2*Sqrt[2]*(I*Sqrt[a] + Sqrt[b])*(2*Sqrt[a] - I*Sqrt[b] + I*Sqrt[b]*Cos[2*(
c + d*x)])*((2*I)*Sqrt[a] - Sqrt[b] + Sqrt[b]*Cos[2*(c + d*x)])*Sqrt[(1 - (
(2*I)*Sqrt[a])/Sqrt[b] - Cos[2*(c + d*x)])*Csc[c + d*x]^2]*Sqrt[(Cot[c + d*
x]^2*(I*Sqrt[a]*Sqrt[b] - a*Csc[c + d*x]^2))/(Sqrt[a] - I*Sqrt[b])^2]*Ellip
ticF[ArcSin[Sqrt[((-I)*Sqrt[b] + Sqrt[a]*Csc[c + d*x]^2)/(Sqrt[a] - I*Sqrt[
b])]], 1/2 + ((I/2)*Sqrt[a])/Sqrt[b]]*Sin[c + d*x]^2*Tan[c + d*x])/(Sqrt[a]
*d*(8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(181) = 362$.
time = 29.47, size = 396, normalized size = 2.44

method	result

default	$-\frac{\sqrt{(4a+(\cos^2(2dx+2c))b+b-2b\cos(2dx+2c))(\sin^2(2dx+2c))}\,\sqrt{-ab}\,\sqrt{\frac{\left(-b+\sqrt{-ab}\right)(\cdots}{\sqrt{-ab}\,(\cos\cdots}}}{\left(-b+\sqrt{-ab}\right)\sqrt{\frac{(-1+\cos(2dx+2c))(\cos(2dx+2c)+1)\left(-b\cos(2dx+2c)+\cdots\right.}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))*sin(2*d*x+2*c)^2)^(1/2)*(-a
*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*
c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a
*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-
a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*EllipticF(((-b+(-a*b)^(1/2))*(-1+cos(2
*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),((b+(-a*b)^(1/2))/(-b+(-a
*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d*x+2*c))*(cos(2*d*x+2*
c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)
-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))^(1/
2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F]
time = 0.10, size = 28, normalized size = 0.17

$$\text{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin^4\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(1/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.248 $\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=493

$$-\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}}+\frac{\sqrt{a+b}\,\cos(c+dx)\sin(c+dx)\,(a}{ad\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] -(a+b)^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)
^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arctan
((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+
2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^
(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/d/(a+b*sin(d*x+c)^4)^(1/2)
+1/2*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/c
os(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan((a+b)^(
1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*(a+b+a^(1/2)
*(a+b)^(1/2))*((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)
*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/(a+b)^(3
/4)/d/(a+b*sin(d*x+c)^4)^(1/2)-cos(d*x+c)^2*cot(d*x+c)*(a+2*a*tan(d*x+c)^2+
(a+b)*tan(d*x+c)^4)/a/d/(a+b*sin(d*x+c)^4)^(1/2)+cos(d*x+c)*sin(d*x+c)*(a+b
)^(1/2)*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/a/d/(a+b*sin(d*x+c)^4)^(1/2
)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)
```

Rubi [A]
time = 0.28, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3298, 1295, 1211, 1117, 1209}

$$\frac{\left(\sqrt{a}\sqrt{a+b}+a+b\right)\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2a^{3/4}d(a+b)^{3/4}\sqrt{a+b\sin^4(c+dx)}}-\frac{\sqrt[4]{a+b}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{a^{3/4}d\sqrt{a+b\sin^4(c+dx)}}+\frac{\sqrt{a+b}\sin(c+dx)\cos(c+dx)\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}{ad\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)}-\frac{\cos^2(c+dx)\cot(c+dx)\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}{ad\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((Cos[c + d*x]^2*Cot[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4))/(a*d*Sqrt[a + b*Sin[c + d*x]^4])) + (Sqrt[a + b]*Cos[c + d*x]*Sin[c
+ d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))/(a*d*Sqrt[a + b*S
in[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) - ((a + b)^(1/4)*Cos
[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 -
Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*
a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)^2])/(a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((a + b + Sqrt[a]*Sqrt[
a + b])*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1
```

```
/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*S
qrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b
]*Tan[c + d*x]^2)^2])/(2*a^(3/4)*(a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]
)
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 3298

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)
*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f*x]^2)^(2*p)/(f*Apart[a*(1 + Tan[e + f
*x]^2)^2 + b*Tan[e + f*x]^4]^p)), Subst[Int[x^m*(ExpandToSum[a*(1 + ff^2*x^
2)^2 + b*ff^4*x^4, x]^p/(1 + ff^2*x^2)^(m/2 + 2*p + 1)), x], x, Tan[e + f*x
]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[p - 1/
```

2]

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{}{x^2\sqrt{}}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} - $$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} - $$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + $$

Mathematica [**C**] Result contains complex when optimal does not.
time = 101.05, size = 432, normalized size = 0.88

$$-\frac{\cot(c+dx)\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\,(8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))\sec^2(c+dx)+4\sqrt{a}\left(i\sqrt{a}+\sqrt{b}\right)E\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle|\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)\sin(2(c+dx))\sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}-4\sqrt{a}\sqrt{b}F\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle|\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)\sin(2(c+dx))\sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\right)}{2a\sqrt{2-\frac{2i\sqrt{b}}{\sqrt{a}}}\,d\sqrt{8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*(Cot[c + d*x]*(Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*(8*a + 3*b - 4*b*Cos[2*(c
 + d*x)] + b*Cos[4*(c + d*x)])*Sec[c + d*x]^2 + 4*Sqrt[a]*(I*Sqrt[a] + Sqrt
[b])*EllipticE[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt
[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]*Sin[2*(c + d*x)]*Sqrt[1 + (1 - (I*S
qrt[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c +
 d*x]^2] - 4*Sqrt[a]*Sqrt[b]*EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[
a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]*Sin[2*(c +
d*x)]*Sqrt[1 + (1 - (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*S
qrt[b])/Sqrt[a])*Tan[c + d*x]^2]))/(a*Sqrt[2 - ((2*I)*Sqrt[b])/Sqrt[a]]*d*S
qrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]])
```

Maple [F]
time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F]
time = 0.09, size = 37, normalized size = 0.08

$$\text{integral}\left(\frac{\csc(dx+c)^2}{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(csc(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(c+d\,x\right)^2\,\sqrt{b\,\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^4)^(1/2)),x)
[Out] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^4)^(1/2)), x)
```

3.249 $\int \frac{1}{a+b\sin^5(x)}\,dx$

Optimal. Leaf size=384

$$\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}+\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}\sqrt[5]{b}+\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}\sqrt[5]{b}+\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\tan^{-1}}{}$$

```
[Out] 2/5*arctan((b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/(a
^(2/5)-b^(2/5))^(1/2)-2/5*arctan((-1)^(3/5)*(b^(1/5)+(-1)^(2/5)*a^(1/5)*tan
(1/2*x))/(a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(1/5)*b^
(2/5))^(1/2)-2/5*arctan((-1)^(1/5)*(b^(1/5)+(-1)^(4/5)*a^(1/5)*tan(1/2*x))/
(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1
/2)+2/5*arctan(((-1)^(4/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^(3/5)*
b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2)+2/5*arctan(((-1)
^(2/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2))/a^(4
/5)/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3292, 2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)+(-1)^{2/5}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)+(-1)^{4/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\text{ArcTan}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}\,b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}\,b^{2/5}}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5)
*Sqrt[a^(2/5) - b^(2/5)]) + (2*ArcTan[((-1)^(2/5)*b^(1/5) + a^(1/5)*Tan[x/2
])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/5
)*b^(2/5)]) + (2*ArcTan[((-1)^(4/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5
) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)]) -
(2*ArcTan[((-1)^(3/5)*(b^(1/5) + (-1)^(2/5)*a^(1/5)*Tan[x/2]))/Sqrt[a^(2/5)
 + (-1)^(1/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)]) - (
2*ArcTan[((-1)^(1/5)*(b^(1/5) + (-1)^(4/5)*a^(1/5)*Tan[x/2]))/Sqrt[a^(2/5)
- (-1)^(2/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\sin^5(x)}\,dx &= \int \left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\,\sin(x)\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}\,\sin(x)\right)}-\frac{}{5a^{4/5}\left(-\sqrt[5]{}\right.}\right.\\
&= -\frac{\int \frac{1}{-\sqrt[5]{a}-\sqrt[5]{b}\,\sin(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}\,\sin(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\,\sin(x)}\,dx}{5a^{4/5}}-\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{-\sqrt[5]{a}-2\sqrt[5]{b}\,x-\sqrt[5]{a}\,x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{2\text{Subst}\left(\int \frac{1}{-\sqrt[5]{a}+2\sqrt[5]{-1}\,\sqrt[5]{b}\,x-\sqrt[5]{a}}\right.}{5a^{4/5}}\\
&= \frac{4\text{Subst}\left(\int \frac{1}{-4\left(a^{2/5}-b^{2/5}\right)-x^2}\,dx,x,-2\sqrt[5]{b}-2\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{4\text{Subst}\left(\int \frac{1}{-4\left(a^{2/5}+\sqrt[5]{-1}\right.}\right.}{}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}\,\sqrt[5]{b}-\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\sqrt[5]{b}-\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}\,b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}\,b^{2/5}}}+\frac{2\tan^{-1}\left(\right.}{5a^{4/5}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.15, size = 149, normalized size = 0.39

$$\frac{8}{5}i\text{RootSum}\left[ib - 5ib\#1^2 + 10ib\#1^4 + 32a\#1^5 - 10ib\#1^6 + 5ib\#1^8 - ib\#1^{10}\&, \frac{2\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3 - i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b-4b\#1^2+16ia\#1^3+6b\#1^4-4b\#1^6+b\#1^8}\&\right]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[x]^5)^(-1),x]
```

```
[Out] ((8*I)/5)*RootSum[I*b - (5*I)*b*#1^2 + (10*I)*b*#1^4 + 32*a*#1^5 - (10*I)*b
*#1^6 + (5*I)*b*#1^8 - I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 -
 I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 + (16*I)*a*#1^3 + 6*b*#1
^4 - 4*b*#1^6 + b*#1^8) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.71, size = 109, normalized size = 0.28

method	result
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(a\_Z^{10}+5a\_Z^{8}+10a\_Z^{6}+32b\_Z^{5}+10a\_Z^{4}+5a\_Z^{2}+a\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a+4\_R^{7}a+6\_R^{5}a+16\_R^{4}b+4\_R^{3}a+\_Ra}\right)}{5}$
risch	$\sum_{\_R=\text{RootOf}\left(1+\left(9765625a^{10}-9765625a^{8}b^{2}\right)\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(e^{ix}+\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a+16*_R^4*b+4
*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a+32*_Z^
5*b+10*_Z^4*a+5*_Z^2*a+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^5),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^5 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**5),x)
```

```
[Out] Integral(1/(a + b*sin(x)**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(x)^5 + a), x)
```

Mupad [B]
time = 19.75, size = 1515, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)^5),x)
```

```
[Out] symsum(log(-10995116277760*a*b^7*(16*tan(x/2) + 56*root(9765625*a^8*b^2*d^1
0 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 1
25*a^2*d^2 - 1, d, k)*a + 5425*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1
0 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k
)^3*a^3 + 196875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 + 3171
875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 15625
0*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 19140625*root(976
5625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 -
6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 1560*root(9765625*a^8*b^2*d^1
0 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 1
25*a^2*d^2 - 1, d, k)^2*a^2*tan(x/2) + 57000*root(9765625*a^8*b^2*d^10 - 97
```

65625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^4*tan(x/2) + 925000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^6*tan(x/2) + 5625000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^8*tan(x/2) + 14000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^3*b + 175000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^5*b + 546875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^7*b + 128*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)*b*tan(x/2) + 1000000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^5*b^2 - 18750000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^7*b^2 + 320*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^2*a*b + 6400*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^2*b*tan(x/2) + 100000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^4*b*tan(x/2) + 500000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^6*b*tan(x/2) + 390625*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^8*b*tan(x/2) + 400000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^4*b^2*tan(x/2) - 5000000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^6*b^2*tan(x/2)))*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k), k, 1, 10)

3.250 $\int \frac{1}{a+b\sin^6(x)}\,dx$

Optimal. Leaf size=171

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] 1/3*arctan((a^(1/3)+b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)+b^(1/3)
)^(1/2)+1/3*arctan((a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/
6)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctan((a^(1/3)+(-1)^(2/3)*b^(1/3
))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3290, 3260, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^6)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) +
b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*
a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(
2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)
])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \frac{\int \frac{1}{1+\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{}}}{}\right)}{3a^{5/6}\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.17, size = 148, normalized size = 0.87

$$-\frac{8}{3}\text{RootSum}\left[b-6b\#1+15b\#1^2-64a\#1^3-20b\#1^3+15b\#1^4-6b\#1^5+b\#1^6\&, \frac{2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{-b+5b\#1-32a\#1^2-10b\#1^2+10b\#1^3-5b\#1^4+b\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^6)^(-1),x]
```

```
[Out] (-8*RootSum[b - 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*
b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*
Cos[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 - 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3
- 5*b*#1^4 + b*#1^5) & ])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.85, size = 68, normalized size = 0.40

method	result

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+3a\_Z^4+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5a+\_R^5b+2\_R^3a+\_Ra}\right)}{6}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^6+46656a^5b)\_Z^6+3888a^4\_Z^4+108a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{15552ia^6}{b}+15552ia^5\right)\_R^5+\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Root
Of((a+b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^6),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^6 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.94, size = 15501, normalized size = 90.65

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^6),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a^4 + a^3*b) - 1/(a^2 + a*b)^2)/(
-1/93312/(a^6 + a^5*b) + 1/31104/((a^4 + a^3*b)*(a^2 + a*b)) - 1/46656/(a^2
 + a*b)^3 + 1/93312*b/((a + b)^2*a^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/933
12/(a^6 + a^5*b) + 1/31104/((a^4 + a^3*b)*(a^2 + a*b)) - 1/46656/(a^2 + a*b
)^3 + 1/93312*b/((a + b)^2*a^5))^(1/3) - 72/(a^2 + a*b))*log(-1/5184*(a^5 +
 a^4*b - 2*(a^5 + a^4*b)*cos(x)^2)*((-I*sqrt(3) + 1)*(1/(a^4 + a^3*b) - 1/(
a^2 + a*b)^2)/(-1/93312/(a^6 + a^5*b) + 1/31104/((a^4 + a^3*b)*(a^2 + a*b))
 - 1/46656/(a^2 + a*b)^3 + 1/93312*b/((a + b)^2*a^5))^(1/3) - 1296*(I*sqrt(
3) + 1)*(-1/93312/(a^6 + a^5*b) + 1/31104/((a^4 + a^3*b)*(a^2 + a*b)) - 1/4
6656/(a^2 + a*b)^3 + 1/93312*b/((a + b)^2*a^5))^(1/3) - 72/(a^2 + a*b))^2 +
 (2*a + b)*cos(x)^2 + 1/15552*sqrt(1/2)*((a^6 + a^5*b)*((-I*sqrt(3) + 1)*(1
/(a^4 + a^3*b) - 1/(a^2 + a*b)^2)/(-1/93312/(a^6 + a^5*b) + 1/31104/((a^4 +
 a^3*b)*(a^2 + a*b)) - 1/46656/(a^2 + a*b)^3 + 1/93312*b/((a + b)^2*a^5))^(
1/3) - 1296*(I*sqrt(3) +  ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin^6(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**6),x)
```

```
[Out] Integral(1/(a + b*sin(x)**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^6),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(x)^6 + a), x)
```

Mupad [B]
time = 15.71, size = 513, normalized size = 3.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)^6),x)
```

```
[Out] symsum(log(-(3*b^3*(a + b)*(8*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a
^4*d^4 + 108*a^2*d^2 + 1, d, k)*a - cot(x) + 2*root(46656*a^5*b*d^6 + 46656
*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)*b + 504*root(46656*a^5*b*d
^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^3*a^3 + 7776*roo
t(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^5
*a^5 - 144*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^
2 + 1, d, k)^3*a^2*b + 7776*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4
*d^4 + 108*a^2*d^2 + 1, d, k)^5*a^4*b - 60*root(46656*a^5*b*d^6 + 46656*a^6
*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a^2*cot(x) - 864*root(46656*
a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^4*a^4*cot
(x) - 864*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2
 + 1, d, k)^4*a^3*b*cot(x) + 12*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888
*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a*b*cot(x)))/cot(x))*root(46656*a^5*b*d
^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k), k, 1, 6)
```

3.251 $\int \frac{1}{a+b\sin^8(x)}\,dx$

Optimal. Leaf size=245

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-$$

```
[Out] -1/4*arctan(((-a)^(1/4)-b^(1/4))^(1/2)*tan(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^
(1/4)-b^(1/4))^(1/2)-1/4*arctan(((-a)^(1/4)-I*b^(1/4))^(1/2)*tan(x)/(-a)^(1
/8))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)-1/4*arctan(((-a)^(1/4)+I*b^(1/
4))^(1/2)*tan(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)-1/4*ar
ctan(((-a)^(1/4)+b^(1/4))^(1/2)*tan(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+b
^(1/4))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3290, 3260, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\tan(x)}{(-a)^{5/8}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^8)^(-1),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tan[x])/(-a)^(1/8)]/((-a)^(7/8)*S
qrt[(-a)^(1/4) - I*b^(1/4)]) - ArcTan[(Sqrt[(-a)^(1/4) + I*b^(1/4)]*Tan[x])
/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4)]) - ArcTan[(Sqrt[(-a
)^(1/4) + b^(1/4)]*Tan[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + b^(1
/4)]) - ArcTan[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Tan[x])/(-a)^(5/8)]/(4*(-a)^(3
/8)*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+b\sin^8(x)}\,dx &= \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}+\frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}+\frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tan(x)\right)}{4a}+\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tan(x)\right)}{4a}+ \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{}}}{}\right)}{4(-a)^{7/8}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.22, size = 174, normalized size = 0.71

$$
8\text{RootSum}\left[b-8b\#1+28b\#1^2-56b\#1^3+256a\#1^4+70b\#1^4-56b\#1^5+28b\#1^6-8b\#1^7+b\#1^8\&,\frac{2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{-b+7b\#1-21b\#1^2+128a\#1^3+35b\#1^3-35b\#1^4+21b\#1^5-7b\#1^6+b\#1^7}\&\right]
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[x]^8)^(-1),x]
```

```
[Out] 8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 - 56*
b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #
1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2 +
 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.48, size = 85, normalized size = 0.35

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7a+\_R^7b+3\_R^5a+3\_R^3a+\_Ra}\right)}{8}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8+16777216a^7b)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{4194304ia^8}{b}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a+_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(ta
n(x)-_R),_R=RootOf((a+b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^8 + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 665483 vs. 2(165) = 330.
time = 6.29, size = 665483, normalized size = 2716.26

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^8),x, algorithm="fricas")
```

```
[Out] -1/384*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(((a^3*sqrt(-(2*a*b*sqrt(-b/a) + a*b
 - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + a^2*b*sqrt(-(2*a*b*sqrt(-
b/a) + a*b - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) - 3*a)*sqrt(-b/a)
 - b)^2*a/((a^3 + a^2*b)^2*b) - 3*(2*a^2*b*sqrt(-(2*a*b*sqrt(-b/a) + a*b -
b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + (2*a^3*sqrt(-(2*a*b*sqrt(-b/
a) + a*b - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) - 3*a)*sqrt(-b/a) -
 b)/((a^5 + a^4*b)*sqrt(-b/a)))/(-1/1572864*(2*a^2*b*sqrt(-(2*a*b*sqrt(-b/a
) + a*b - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + (2*a^3*sqrt(-(2*a*
b*sqrt(-b/a) + a*b - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) - 3*a)*sq
rt(-b/a) - b)*((a^3*sqrt(-(2*a*b*sqrt(-b/a) + a*b - b^2)/((a^6 + 2*a^5*b +
a^4*b^2)*sqrt(-b/a))) + a^2*b*sqrt(-(2*a*b*sqrt(-b/a) + a*b - b^2)/((a^6 +
```

```
2*a^5*b + a^4*b^2)*sqrt(-b/a))) - 3*a)*sqrt(-b/a) - b)*a/((a^5 + a^4*b)*(a^
3 + a^2*b)*b) + 1/524288*(2*a^2*b*sqrt(-(2*a*b*sqrt(-b/a) + a*b - b^2)/((a^
6 + 2*a^5*b + a^4*b^2)*sq ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin^8(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**8),x)
[Out] Integral(1/(a + b*sin(x)**8), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^8),x, algorithm="giac")
[Out] integrate(1/(b*sin(x)^8 + a), x)
```

Mupad [B]
time = 16.95, size = 816, normalized size = 3.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)^8),x)
[Out] symsum(log(-2*b^5*(a + b)*(800*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 +
 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^2*a^2 + 43008*roo
t(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 +
 256*a^2*d^2 + 1, d, k)^4*a^4 + 786432*root(16777216*a^7*b*d^8 + 16777216*a
^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^6*a^6 + 4
*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d
^4 + 256*a^2*d^2 + 1, d, k)*b*tan(x) - 6144*root(16777216*a^7*b*d^8 + 16777
216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^4*a^
3*b + 786432*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 +
 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^6*a^5*b - 9984*root(16777216*a^7*b*
d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1,
 d, k)^3*a^3*tan(x) - 557056*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1
048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^5*a^5*tan(x) - 1048
```

```
5760*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a
^4*d^4 + 256*a^2*d^2 + 1, d, k)^7*a^7*tan(x) + 32*root(16777216*a^7*b*d^8 +
 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k
)^2*a*b - 60*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 +
 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*a*tan(x) - 768*root(16777216*a^7*b*
d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1,
 d, k)^3*a^2*b*tan(x) + 98304*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 +
1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^5*a^4*b*tan(x) - 1
0485760*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 2457
6*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^7*a^6*b*tan(x) + 5))*root(16777216*a^7*b
*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1
, d, k), k, 1, 8)
```

3.252 $\int \frac{1}{a-b\sin^5(x)}\,dx$

Optimal. Leaf size=379

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}+\frac{2\tan^{-1}}{5a}$$

```
[Out] -2/5*arctan((b^(1/5)-a^(1/5)*tan(1/2*x))/(a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/(
a^(2/5)-b^(2/5))^(1/2)+2/5*arctan(((-1)^(3/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(
a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/
2)+2/5*arctan(((-1)^(1/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-(-1)^(2/5)*b
^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1/2)-2/5*arctan(((-1)^
(4/5)*b^(1/5)-a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2))/a^(4/
5)/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2)-2/5*arctan(((-1)^(2/5)*b^(1/5)-a^(1/5
)*tan(1/2*x))/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(4/
5)*b^(2/5))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3292, 2739, 632, 210}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\text{ArcTan}\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\text{ArcTan}\left(\frac{(-1)^{4/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{3/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTan[(b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5
)*Sqrt[a^(2/5) - b^(2/5)]) - (2*ArcTan[((-1)^(2/5)*b^(1/5) - a^(1/5)*Tan[x/
2])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/
5)*b^(2/5)]) - (2*ArcTan[((-1)^(4/5)*b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/
5) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)]) +
 (2*ArcTan[((-1)^(1/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - (-1)^(2/5
)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)]) + (2*ArcTan[((-
1)^(3/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)]])/(
5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a-b\sin^5(x)}\,dx = \int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\,\sin(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}\,\sin(x)\right)}+\frac{}{5a^{4/5}\left(\sqrt[5]{a}-(\cdots\right.}\right.$$

$$=\frac{\int \frac{1}{\sqrt[5]{a}-\sqrt[5]{b}\,\sin(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}+\sqrt[5]{-1}\,\sqrt[5]{b}\,\sin(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\,\sin(x)}\,dx}{5a^{4/5}}+\frac{\int \cdots}{\cdots}$$

$$=\frac{2\text{Subst}\left(\int \frac{1}{\sqrt[5]{a}-2\sqrt[5]{b}\,x+\sqrt[5]{a}\,x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt[5]{a}+2\sqrt[5]{-1}\,\sqrt[5]{b}\,x+\sqrt[5]{a}\,x^2}\,\cdots\right.}{5a^{4/5}}$$

$$=-\frac{4\text{Subst}\left(\int \frac{1}{-4\left(a^{2/5}-b^{2/5}\right)-x^2}\,dx,x,-2\sqrt[5]{b}+2\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{4\text{Subst}\left(\int \frac{1}{-4\left(a^{2/5}+\sqrt[5]{-\cdots}\right.}\right.}{\cdots}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}\cdots}{\sqrt{a^{2/5}\cdots}}\right)}{5a^{4/5}\sqrt{a^{2/5}\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.13, size = 149, normalized size = 0.39

$$-\frac{8}{5}i\text{RootSum}\left[-ib+5ib\#1^2-10ib\#1^4+32a\#1^5+10ib\#1^6-5ib\#1^8+ib\#1^{10}\&,\frac{2\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b-4b\#1^2-16ia\#1^3+6b\#1^4-4b\#1^6+b\#1^8}\&\right]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Sin[x]^5)^(-1),x]
```

```
[Out] ((-8*I)/5)*RootSum[(-I)*b + (5*I)*b*#1^2 - (10*I)*b*#1^4 + 32*a*#1^5 + (10*
I)*b*#1^6 - (5*I)*b*#1^8 + I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1
^3 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 - (16*I)*a*#1^3 + 6*
b*#1^4 - 4*b*#1^6 + b*#1^8) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.64, size = 109, normalized size = 0.29

method	result
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(a\_Z^{10}+5a\_Z^8+10a\_Z^6-32b\_Z^5+10a\_Z^4+5a\_Z^2+a\right)}\frac{\left(\_R^8+4\_R^6+6\_R^4+4\_R^2+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^9a+4\_R^7a+6\_R^5a-16\_R^4b+4\_R^3a+\_Ra}\right)}{5}$
risch	$\sum_{\_R=\text{RootOf}\left(1+\left(9765625a^{10}-9765625a^8b^2\right)\_Z^{10}+1953125a^8\_Z^8+156250a^6\_Z^6+6250a^4\_Z^4+125a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{ix}+\left(-\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a-16*_R^4*b+4
*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a-32*_Z^
5*b+10*_Z^4*a+5*_Z^2*a+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^5),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(x)^5 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b \sin^5(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)**5),x)
```

```
[Out] Integral(1/(a - b*sin(x)**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^5),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*sin(x)^5 - a), x)
```

Mupad [B]
time = 20.14, size = 1515, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(x)^5),x)
```

```
[Out] symsum(log(10995116277760*a*b^7*(16*tan(x/2) + 56*root(9765625*a^8*b^2*d^10
 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 12
5*a^2*d^2 - 1, d, k)*a + 5425*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10
 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)
^3*a^3 + 196875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 + 31718
75*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250
*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 19140625*root(9765
625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6
250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 1560*root(9765625*a^8*b^2*d^10
 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 12
5*a^2*d^2 - 1, d, k)^2*a^2*tan(x/2) + 57000*root(9765625*a^8*b^2*d^10 - 976
```

```
5625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*
d^2 - 1, d, k)^4*a^4*tan(x/2) + 925000*root(9765625*a^8*b^2*d^10 - 9765625*
a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 -
 1, d, k)^6*a^6*tan(x/2) + 5625000*root(9765625*a^8*b^2*d^10 - 9765625*a^10
*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
d, k)^8*a^8*tan(x/2) - 14000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10
- 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^
4*a^3*b - 175000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^5*b - 54
6875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1562
50*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^7*b - 128*root(97656
25*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 62
50*a^4*d^4 - 125*a^2*d^2 - 1, d, k)*b*tan(x/2) + 1000000*root(9765625*a^8*b
^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d
^4 - 125*a^2*d^2 - 1, d, k)^7*a^5*b^2 - 18750000*root(9765625*a^8*b^2*d^10
- 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125
*a^2*d^2 - 1, d, k)^9*a^7*b^2 - 320*root(9765625*a^8*b^2*d^10 - 9765625*a^1
0*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
 d, k)^2*a*b - 6400*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125
*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^2*b*t
an(x/2) - 100000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^4*b*tan(
x/2) - 500000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d
^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^6*b*tan(x/2
) - 390625*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8
- 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^8*b*tan(x/2) +
 400000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1
56250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^4*b^2*tan(x/2) -
5000000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1
56250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^6*b^2*tan(x/2)))*
root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^
6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k), k, 1, 10)
```

3.253 $\int \frac{1}{a-b\sin^6(x)}\,dx$

Optimal. Leaf size=175

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] 1/3*arctan((a^(1/3)-b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)-b^(1/3)
)^(1/2)+1/3*arctan((a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/
6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctan((a^(1/3)-(-1)^(2/3)*b^(1/3
))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3290, 3260, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[x]^6)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) -
b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*
a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(
2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)
])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{}}}{}\right)}{3a^{5/6}\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 148, normalized size = 0.85

$$\frac{8}{3}\text{RootSum}\left[b-6b\#1+15b\#1^2+64a\#1^3-20b\#1^3+15b\#1^4-6b\#1^5+b\#1^6\&, \frac{2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{-b+5b\#1+32a\#1^2-10b\#1^2+10b\#1^3-5b\#1^4+b\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[x]^6)^(-1),x]
```

```
[Out] (8*RootSum[b - 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*b
*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*C
os[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 + 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3 -
 5*b*#1^4 + b*#1^5) & ])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.92, size = 71, normalized size = 0.41

method	result

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^6+3a\_Z^4+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5a-\_R^5b+2\_R^3a+\_Ra}\right)}{6}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^6-46656a^5b)\_Z^6+3888a^4\_Z^4+108a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{15552ia^6}{b}+15552ia^5\right)\_R^5+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a-_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Root
Of((a-b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^6),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(x)^6 - a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.94, size = 16697, normalized size = 95.41

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^6),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a^4 - a^3*b) - 1/(a^2 - a*b)^2)/(-
1/93312/(a^6 - a^5*b) + 1/31104/((a^4 - a^3*b)*(a^2 - a*b)) - 1/46656/(a^2
- a*b)^3 + 1/93312*b/((a - b)^2*a^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/9331
2/(a^6 - a^5*b) + 1/31104/((a^4 - a^3*b)*(a^2 - a*b)) - 1/46656/(a^2 - a*b)
^3 + 1/93312*b/((a - b)^2*a^5))^(1/3) - 72/(a^2 - a*b))*log(1/5184*(a^5 - a
^4*b - 2*(a^5 - a^4*b)*cos(x)^2)*((-I*sqrt(3) + 1)*(1/(a^4 - a^3*b) - 1/(a^
2 - a*b)^2)/(-1/93312/(a^6 - a^5*b) + 1/31104/((a^4 - a^3*b)*(a^2 - a*b)) -
 1/46656/(a^2 - a*b)^3 + 1/93312*b/((a - b)^2*a^5))^(1/3) - 1296*(I*sqrt(3)
 + 1)*(-1/93312/(a^6 - a^5*b) + 1/31104/((a^4 - a^3*b)*(a^2 - a*b)) - 1/466
56/(a^2 - a*b)^3 + 1/93312*b/((a - b)^2*a^5))^(1/3) - 72/(a^2 - a*b))^2 - (
2*a - b)*cos(x)^2 + 1/15552*sqrt(1/2)*((a^6 - a^5*b)*((-I*sqrt(3) + 1)*(1/(
a^4 - a^3*b) - 1/(a^2 - a*b)^2)/(-1/93312/(a^6 - a^5*b) + 1/31104/((a^4 - a
^3*b)*(a^2 - a*b)) - 1/46656/(a^2 - a*b)^3 + 1/93312*b/((a - b)^2*a^5))^(1/
3) - 1296*(I*sqrt(3) + 1) ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\sin^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)**6),x)
```

```
[Out] Integral(1/(a - b*sin(x)**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^6),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*sin(x)^6 - a), x)
```

Mupad [B]
time = 16.07, size = 513, normalized size = 2.93

$$\sum_{k=1}^{6} \ln\left(-\frac{b^3\,(a-b)\,\left(\cot(x) - \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)\,a\,8 + \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)\,b\,2 - \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)^3\,a^3\,504 - \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)^5\,a^5\,7776 - \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)^3\,a^2\,b\,144 + \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)^5\,a^4\,b\,7776 + \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)^2\,a^2\cot(x)\,60 + \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)^4\,a^4\cot(x)\,864 - \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)^4\,a^3\,b\cot(x)\,864 + \mathrm{root}(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k)^2\,a\,b\cot(x)\,12\right)\,3}{\cot(x)}\right)\,\mathrm{root}\left(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 - 3888\,a^4\,d^4 - 108\,a^2\,d^2 - 1, d, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(x)^6),x)
```

```
[Out] symsum(log(-(3*b^3*(a - b)*(cot(x) - 8*root(46656*a^5*b*d^6 - 46656*a^6*d^6
 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*a + 2*root(46656*a^5*b*d^6 - 46656
*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*b - 504*root(46656*a^5*b*d
^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^3*a^3 - 7776*roo
t(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^5
*a^5 - 144*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^
2 - 1, d, k)^3*a^2*b + 7776*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4
*d^4 - 108*a^2*d^2 - 1, d, k)^5*a^4*b + 60*root(46656*a^5*b*d^6 - 46656*a^6
*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a^2*cot(x) + 864*root(46656*
a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^4*a^4*cot
(x) - 864*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2
 - 1, d, k)^4*a^3*b*cot(x) + 12*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888
*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a*b*cot(x)))/cot(x))*root(46656*a^5*b*d
^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k), k, 1, 6)
```

3.254 $\int \frac{1}{a-b\sin^8(x)}\,dx$

Optimal. Leaf size=213

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\cdots\right)}{4a\cdots}$$

```
[Out] 1/4*arctan((a^(1/4)-b^(1/4))^(1/2)*tan(x)/a^(1/8))/a^(7/8)/(a^(1/4)-b^(1/4)
)^(1/2)+1/4*arctan((a^(1/4)-I*b^(1/4))^(1/2)*tan(x)/a^(1/8))/a^(7/8)/(a^(1/
4)-I*b^(1/4))^(1/2)+1/4*arctan((a^(1/4)+I*b^(1/4))^(1/2)*tan(x)/a^(1/8))/a^
(7/8)/(a^(1/4)+I*b^(1/4))^(1/2)+1/4*arctan((a^(1/4)+b^(1/4))^(1/2)*tan(x)/a
^(1/8))/a^(7/8)/(a^(1/4)+b^(1/4))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3290, 3260, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[x]^8)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) -
b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*S
qrt[a^(1/4) - I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Tan[x])/a^(1/
8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + b^(1/4)]
*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a}+\frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a}+\frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx,x,\tan(x)\right)}{4a}+\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx,x,\tan(x)\right)}{4a}+\frac{\text{Su}}{}$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i}}{\sqrt[8]{}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.14, size = 174, normalized size = 0.82

$$-8\text{RootSum}\left[b-8b\#1+28b\#1^2-56b\#1^3-256a\#1^4+70b\#1^4-56b\#1^5+28b\#1^6-8b\#1^7+b\#1^8\&,\frac{2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{-b+7b\#1-21b\#1^2-128a\#1^3+35b\#1^3-35b\#1^4+21b\#1^5-7b\#1^6+b\#1^7}\&\right]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Sin[x]^8)^(-1),x]
```

```
[Out] -8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 - 56
*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] -
#1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2
- 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.49, size = 88, normalized size = 0.41

method	result

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7a-\_R^7b+3\_R^5a+3\_R^3a+\_Ra}\right)}{8}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8-16777216a^7b)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{4194304ia^8}{b}-\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a-_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(ta
n(x)-_R),_R=RootOf((a-b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^8),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(x)^8 - a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643307 vs. 2(133) = 266.
time = 6.38, size = 643307, normalized size = 3020.22

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^8),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(1/2)*sqrt(1/6)*sqrt((6*sqrt(1/2)*sqrt(1/6)*(a^2 - a*b)*sqrt(-((a^
5 - 2*a^4*b + a^3*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(((a^3*sqrt((2*a*b*s
qrt(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) - a^2*b*sqrt((
2*a*b*sqrt(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) - 3*a)*
sqrt(b/a) - b)^2*a/((a^3 - a^2*b)^2*b) + 3*(2*a^2*b*sqrt((2*a*b*sqrt(b/a) -
 a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) + (2*a^3*sqrt((2*a*b*sqr
t(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) - 3*a)*sqrt(b/a)
 - b)/((a^5 - a^4*b)*sqrt(b/a)))/(9*(2*a^2*b*sqrt((2*a*b*sqrt(b/a) - a*b -
b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) + (2*a^3*sqrt((2*a*b*sqrt(b/a)
- a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) - 3*a)*sqrt(b/a) - b)*(
(a^3*sqrt((2*a*b*sqrt(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a
))) - a^2*b*sqrt((2*a*b*sqrt(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*s
```

```
qrt(b/a))) - 3*a)*sqrt(b/a) - b)*a/((a^5 - a^4*b)*(a^3 - a^2*b)*b) + 27*(2*
a^2*b*sqrt((2*a*b*sqrt(b/ ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\sin^8(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)**8),x)
```

```
[Out] Integral(1/(a - b*sin(x)**8), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^8),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*sin(x)^8 - a), x)
```

Mupad [B]
time = 16.54, size = 818, normalized size = 3.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(x)^8),x)
```

```
[Out] symsum(log(-2*b^5*(a - b)*(4*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1
048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*b*tan(x) - 43008*ro
ot(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4
- 256*a^2*d^2 - 1, d, k)^4*a^4 - 786432*root(16777216*a^7*b*d^8 - 16777216*
a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^6*a^6 -
800*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^
4*d^4 - 256*a^2*d^2 - 1, d, k)^2*a^2 - 6144*root(16777216*a^7*b*d^8 - 16777
216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^4*a^
3*b + 786432*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 -
 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^6*a^5*b + 9984*root(16777216*a^7*b*
d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1,
 d, k)^3*a^3*tan(x) + 557056*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1
048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^5*a^5*tan(x) + 1048
5760*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a
```

```
^4*d^4 - 256*a^2*d^2 - 1, d, k)^7*a^7*tan(x) + 32*root(16777216*a^7*b*d^8 -
 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k
)^2*a*b + 60*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 -
 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*a*tan(x) - 768*root(16777216*a^7*b*
d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1,
 d, k)^3*a^2*b*tan(x) + 98304*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 -
1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^5*a^4*b*tan(x) - 1
0485760*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 2457
6*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^7*a^6*b*tan(x) - 5))*root(16777216*a^7*b
*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1
, d, k), k, 1, 8)
```

3.255 $\int \frac{1}{1+\sin^5(x)}\,dx$

Optimal. Leaf size=195

$$\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\left(1+(-1)^{2/5}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tan^{-1}\left(\sqrt[5]{-\ldots\right.}{5\sqrt{1\ldots}}$$

```
[Out] -1/5*cos(x)/(1+sin(x))-2/5*arctan((-1)^(3/5)*(1+(-1)^(2/5)*tan(1/2*x))/(1+(
-1)^(1/5))^(1/2))/(1+(-1)^(1/5))^(1/2)-2/5*arctan((-1)^(1/5)*(1+(-1)^(4/5)*
tan(1/2*x))/(1-(-1)^(2/5))^(1/2))/(1-(-1)^(2/5))^(1/2)+2/5*arctan(((-1)^(4/
5)+tan(1/2*x))/(1+(-1)^(3/5))^(1/2))/(1+(-1)^(3/5))^(1/2)+2/5*arctan(((-1)^
(2/5)+tan(1/2*x))/(1-(-1)^(4/5))^(1/2))/(1-(-1)^(4/5))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3292, 2727, 2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{2/5}}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\text{ArcTan}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\text{ArcTan}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}-\frac{\cos(x)}{5(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[((-1)^(2/5) + Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^(
4/5)]) + (2*ArcTan[((-1)^(4/5) + Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sqrt[1
 + (-1)^(3/5)]) - (2*ArcTan[((-1)^(3/5)*(1 + (-1)^(2/5)*Tan[x/2]))/Sqrt[1 +
 (-1)^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]) - (2*ArcTan[((-1)^(1/5)*(1 + (-1)^(
4/5)*Tan[x/2]))/Sqrt[1 - (-1)^(2/5)]])/(5*Sqrt[1 - (-1)^(2/5)]) - Cos[x]/(5
*(1 + Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sin^5(x)}\,dx &= \int \left(-\frac{1}{5(-1-\sin(x))}-\frac{1}{5\left(-1+\sqrt[5]{-1}\,\sin(x)\right)}-\frac{1}{5\left(-1-(-1)^{2/5}\sin(x)\right)}-\frac{}{5\left(-1+(\right.}\right.\\
&= -\left(\frac{1}{5}\int \frac{1}{-1-\sin(x)}\,dx\right)-\frac{1}{5}\int \frac{1}{-1+\sqrt[5]{-1}\,\sin(x)}\,dx-\frac{1}{5}\int \frac{1}{-1-(-1)^{2/5}\sin(x)}\,dx\\
&= -\frac{\cos(x)}{5(1+\sin(x))}-\frac{2}{5}\mathrm{Subst}\left(\int \frac{1}{-1+2\sqrt[5]{-1}\,x-x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)-\frac{2}{5}\mathrm{Subst}\left(\int \frac{}{-1}\right.\\
&= -\frac{\cos(x)}{5(1+\sin(x))}+\frac{4}{5}\mathrm{Subst}\left(\int \frac{1}{-4\left(1+\sqrt[5]{-1}\right)-x^2}\,dx,x,2(-1)^{3/5}-2\tan\left(\frac{x}{2}\right)\right)+\frac{4}{5}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}+\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.11, size = 411, normalized size = 2.11

$$-\frac{1}{10}i\,\mathrm{RootSum}\left[1+2i\#1-8\#1^2-14i\#1^3+30\#1^4+14i\#1^5-8\#1^6-2i\#1^7+\#1^8\&,\frac{-2\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)+i\log\left(1-2\cos(x)\#1+\#1^2\right)-8i\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-4\log\left(1-2\cos(x)\#1+\#1^2\right)\#1+30\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2-15i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^2+80i\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3+40\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3-30\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^4+15i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^4-8i\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^5-4\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^5+2\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^6-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^6}{i-8\#1-21i\#1^2+60\#1^3+35i\#1^4-24\#1^5-7i\#1^6+4\#1^7}\&\right]+\frac{2\sin\left(\frac{x}{2}\right)}{5\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^5)^(-1),x]
```

```
[Out] (-1/10*I)*RootSum[1 + (2*I)*#1 - 8*#1^2 - (14*I)*#1^3 + 30*#1^4 + (14*I)*#1
^5 - 8*#1^6 - (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*Lo
g[1 - 2*Cos[x]*#1 + #1^2] - (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - 4*Log[1
 - 2*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*L
og[1 - 2*Cos[x]*#1 + #1^2]*#1^2 + (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3
+ 40*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^
4 + (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 - (8*I)*ArcTan[Sin[x]/(Cos[x] -
 #1)]*#1^5 - 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] -
 #1)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(I - 8*#1 - (21*I)*#1^2 +
60*#1^3 + (35*I)*#1^4 - 24*#1^5 - (7*I)*#1^6 + 4*#1^7) & ] + (2*Sin[x/2])/(
5*(Cos[x/2] + Sin[x/2]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.67, size = 133, normalized size = 0.68

method	result
risch	$-\frac{2}{5(e^{ix}+i)}+\left(\sum_{\_R=\mathrm{RootOf}(1953125\_Z^8+156250\_Z^6+6250\_Z^4+125\_Z^2+1)} \_R\ln\left(e^{ix}+2343750\_R^7+234375i\right.\right.$
default	$-\frac{2}{5(\tan(\frac{x}{2})+1)}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}(\_Z^8-2\_Z^7+8\_Z^6-14\_Z^5+30\_Z^4-14\_Z^3+8\_Z^2-2\_Z+1)} \frac{(2\_R^6-3\_R^5+10\_R^4-10\_R^3+}{4\_R^7-7\_R^6+24\_R^5-35\_}\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(tan(1/2*x)+1)+2/5*sum((2*_R^6-3*_R^5+10*_R^4-10*_R^3+10*_R^2-3*_R+2)/
(4*_R^7-7*_R^6+24*_R^5-35*_R^4+60*_R^3-21*_R^2+8*_R-1)*ln(tan(1/2*x)-_R),_R
=RootOf(_Z^8-2*_Z^7+8*_Z^6-14*_Z^5+30*_Z^4-14*_Z^3+8*_Z^2-2*_Z+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^5),x, algorithm="maxima")
```

```
[Out] -1/5*(5*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(-2/5*((4*cos(6*x) -
40*cos(4*x) + 4*cos(2*x) - sin(7*x) + 15*sin(5*x) - 15*sin(3*x) + sin(x))*c
os(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) - 8*sin(6*x) + 55*sin(4*x
) - 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos(2*x) - 4
4*sin(5*x) + 44*sin(3*x) - 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2 + 2*(210*
```

```
cos(3*x) - 22*cos(x) - 505*sin(4*x) + 88*sin(2*x))*cos(5*x) - 210*cos(5*x)^
2 + 10*(44*cos(2*x) - 101*sin(3*x) + 11*sin(x) - 4)*cos(4*x) - 1200*cos(4*x
)^2 + 44*(cos(x) - 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 - 4*(4*sin(x) - 1)
*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 + (cos(7*x) - 15*cos(5*x) + 15*cos(3
*x) - cos(x) + 4*sin(6*x) - 40*sin(4*x) + 4*sin(2*x))*sin(8*x) + (16*cos(6*
x) - 110*cos(4*x) + 16*cos(2*x) + 44*sin(5*x) - 44*sin(3*x) + 4*sin(x) - 1)
*sin(7*x) - 2*sin(7*x)^2 + 8*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) + 55*sin
(4*x) - 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 + (1010*cos(4*x) - 176*cos(2*x
) + 420*sin(3*x) - 44*sin(x) + 15)*sin(5*x) - 210*sin(5*x)^2 + 10*(101*cos(
3*x) - 11*cos(x) + 44*sin(2*x))*sin(4*x) - 1200*sin(4*x)^2 + (176*cos(2*x)
+ 44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 + 16*cos(x)*sin(2*x) - 32*sin(2
*x)^2 - 2*sin(x)^2 + sin(x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*x) - 2*
sin(7*x) + 14*sin(5*x) - 14*sin(3*x) + 2*sin(x) - 1)*cos(8*x) - cos(8*x)^2
+ 8*(7*cos(5*x) - 7*cos(3*x) + cos(x) - 4*sin(6*x) + 15*sin(4*x) - 4*sin(2*
x))*cos(7*x) - 4*cos(7*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) - 14*sin(5*x) +
14*sin(3*x) - 2*sin(x) + 1)*cos(6*x) - 64*cos(6*x)^2 + 56*(7*cos(3*x) - cos
(x) - 15*sin(4*x) + 4*sin(2*x))*cos(5*x) - 196*cos(5*x)^2 + 60*(8*cos(2*x)
- 14*sin(3*x) + 2*sin(x) - 1)*cos(4*x) - 900*cos(4*x)^2 + 56*(cos(x) - 4*si
n(2*x))*cos(3*x) - 196*cos(3*x)^2 - 16*(2*sin(x) - 1)*cos(2*x) - 64*cos(2*x
)^2 - 4*cos(x)^2 + 4*(cos(7*x) - 7*cos(5*x) + 7*cos(3*x) - cos(x) + 4*sin(6
*x) - 15*sin(4*x) + 4*sin(2*x))*sin(8*x) - sin(8*x)^2 + 4*(8*cos(6*x) - 30*
cos(4*x) + 8*cos(2*x) + 14*sin(5*x) - 14*sin(3*x) + 2*sin(x) - 1)*sin(7*x)
- 4*sin(7*x)^2 + 32*(7*cos(5*x) - 7*cos(3*x) + cos(x) + 15*sin(4*x) - 4*sin
(2*x))*sin(6*x) - 64*sin(6*x)^2 + 28*(30*cos(4*x) - 8*cos(2*x) + 14*sin(3*x
) - 2*sin(x) + 1)*sin(5*x) - 196*sin(5*x)^2 + 120*(7*cos(3*x) - cos(x) + 4*
sin(2*x))*sin(4*x) - 900*sin(4*x)^2 + 28*(8*cos(2*x) + 2*sin(x) - 1)*sin(3*
x) - 196*sin(3*x)^2 + 32*cos(x)*sin(2*x) - 64*sin(2*x)^2 - 4*sin(x)^2 + 4*s
in(x) - 1), x) + 2*cos(x))/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sin(x)+1)(\sin^4(x)-\sin^3(x)+\sin^2(x)-\sin(x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**5),x)
```

```
[Out] Integral(1/((sin(x) + 1)*(sin(x)**4 - sin(x)**3 + sin(x)**2 - sin(x) + 1)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 15.25, size = 2500, normalized size = 12.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^5 + 1),x)
```

```
[Out] 2*atanh((989855744*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3019
89888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2
)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1
)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) +
 (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25
)) - (2030043136*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5
*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*
tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5^(1/2)
)/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^
(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 18454
9376/25)) + (1627389952*5^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/
2))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*
5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/
5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25
+ 184549376/25)) + (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^(1/2))/5
 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1
/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 -
(452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 -
 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5
^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (184549376*5^(1/2)*(- (2*5^(1/2
))/5 - 1)^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3019898
88*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(
```

- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(
1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (4
36207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25))
- (5083496448*5^(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2
))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (13086
22848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5
^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5
 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 +
 184549376/25)) - (553648128*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^
(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364
672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2
))/25 - (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1
/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)
*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (553648128*5^(1/2)*tan(
x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1
/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*
5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/
5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25
+ 184549376/25)))*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((
2030043136*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3
01989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(
x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5
- 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2
) - (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376
/25)) - (989855744*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((30
1989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x
/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 -
 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2)
 - (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/
25)) - (1627389952*5^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))
/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 - (1308622
848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(
1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 -
 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 1
84549376/25)) + (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5
- 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1/
2)*tan(x/2))/125 - (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (
452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 +
16777216*(- (2*5^(1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*(- (2*5^
(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (184549376*5^(1/2)*(- (2*5^(1/2)
)/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989
888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x/2)*
(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^...

3.256 $\int \frac{1}{1+\sin^6(x)}\,dx$

Optimal. Leaf size=103

$$\frac{x}{3\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

```
[Out] 1/6*x*2^(1/2)+1/6*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)+1/3*ar
ctan((1-(-1)^(1/3))^(1/2)*tan(x))/(1-(-1)^(1/3))^(1/2)+1/3*arctan((1+(-1)^(
2/3))^(1/2)*tan(x))/(1+(-1)^(2/3))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3290, 3260, 209}

$$\frac{\text{ArcTan}\left(\sqrt{1-\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\text{ArcTan}\left(\sqrt{1+(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}+\frac{\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{3\sqrt{2}}+\frac{x}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^6)^(-1),x]
```

```
[Out] x/(3*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(3*Sqrt[2]
) + ArcTan[Sqrt[1 - (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTan[S
qrt[1 + (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(2/3)])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\sin^6(x)}\,dx = \frac{1}{3}\int \frac{1}{1+\sin^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\,\sin^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1+(-1)^{2/3}\sin^2(x)}\,dx$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(x)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[3]{-1}\right)x^2}\,dx, x, \tan(x)\right) +$$

$$= \frac{x}{3\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{3\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1+(-1\right.}{3\sqrt{1+(-}$$

Mathematica [A]

time = 0.12, size = 79, normalized size = 0.77

$$\frac{1}{12}\left(-2\sqrt{3}\,\tan^{-1}\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)+2\sqrt{2}\,\tan^{-1}\left(\sqrt{2}\,\tan(x)\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{1+2\tan(x)}{\sqrt{3}}\right)-\log(2-\sin(2x))+\log(2+\sin(2x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - 2*Tan[x])/Sqrt[3]] + 2*Sqrt[2]*ArcTan[Sqrt[2]*Tan[x
]] + 2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] - Log[2 - Sin[2*x]] + Log[2 +
 Sin[2*x]])/12
```

Maple [A]

time = 0.22, size = 72, normalized size = 0.70

method	result
default	$\frac{\arctan\left(\sqrt{2}\,\tan(x)\right)\sqrt{2}}{6}-\frac{\ln(\tan^2(x)-\tan(x)+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(\tan^2(x)+\tan(x)+1)}{12}+\frac{\sqrt{3}}{}$
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{12}+\frac{\ln\left(e^{2ix}+2i+i\sqrt{3}\right)}{12}+\frac{i\ln\left(e^{2ix}+2i+i\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(e^{2i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(2^(1/2)*tan(x))*2^(1/2)-1/12*ln(tan(x)^2-tan(x)+1)+1/6*3^(1/2)*a
rctan(1/3*(2*tan(x)-1)*3^(1/2))+1/12*ln(tan(x)^2+tan(x)+1)+1/6*3^(1/2)*arct
an(1/3*(2*tan(x)+1)*3^(1/2))
```

Maxima [A]

time = 0.50, size = 71, normalized size = 0.69

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\tan(x)+1)\right)+\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\tan(x)-1)\right)+\frac{1}{6}\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan(x)\right)+\frac{1}{12}\log\left(\tan(x)^2+\tan(x)+1\right)-\frac{1}{12}\log\left(\tan(x)^2-\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^6),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) + 1)) + 1/6*sqrt(3)*arctan(1/3*sqr
t(3)*(2*tan(x) - 1)) + 1/6*sqrt(2)*arctan(sqrt(2)*tan(x)) + 1/12*log(tan(x)
^2 + tan(x) + 1) - 1/12*log(tan(x)^2 - tan(x) + 1)
```

Fricas [**A**]
time = 0.42, size = 138, normalized size = 1.34

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos\left(x\right)\sin\left(x\right)+\sqrt{3}}{3\left(2\cos\left(x\right)^{2}-1\right)}\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos\left(x\right)\sin\left(x\right)-\sqrt{3}}{3\left(2\cos\left(x\right)^{2}-1\right)}\right)-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(x\right)^{2}-2\sqrt{2}}{4\cos\left(x\right)\sin\left(x\right)}\right)+\frac{1}{24}\log\left(-\cos\left(x\right)^{4}+\cos\left(x\right)^{2}+2\cos\left(x\right)\sin\left(x\right)+1\right)-\frac{1}{24}\log\left(-\cos\left(x\right)^{4}+\cos\left(x\right)^{2}-2\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^6),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 - 1
)) + 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^
2 - 1)) - 1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*
sin(x))) + 1/24*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) - 1/24*log(
-cos(x)^4 + cos(x)^2 - 2*cos(x)*sin(x) + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**6),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(73) = 146.
time = 0.41, size = 185, normalized size = 1.80

$$\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2x\right)+\cos\left(2x\right)-2\sin\left(2x\right)+1}{\sqrt{3}\cos\left(2x\right)+\sqrt{3}-2\cos\left(2x\right)-\sin\left(2x\right)+2}\right)\right)+\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2x\right)-\cos\left(2x\right)-2\sin\left(2x\right)-1}{\sqrt{3}\cos\left(2x\right)+\sqrt{3}-2\cos\left(2x\right)+\sin\left(2x\right)+2}\right)\right)+\frac{1}{6}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-2\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-2\cos\left(2x\right)+2}\right)\right)+\frac{1}{12}\log\left(\tan\left(x\right)^{2}+\tan\left(x\right)+1\right)-\frac{1}{12}\log\left(\tan\left(x\right)^{2}-\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^6),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(sq
rt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2))) + 1/6*sqrt(3)*(x +
arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(2*x) +
sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) + 1/6*sqrt(2)*(x + arctan(-(sqrt(2)*
sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2))) + 1/
12*log(tan(x)^2 + tan(x) + 1) - 1/12*log(tan(x)^2 - tan(x) + 1)
```

Mupad [B]
time = 14.23, size = 98, normalized size = 0.95

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(x)\right)}{6}+\mathrm{atan}\left(\frac{\sqrt{3}\,\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)-\mathrm{atan}\left(-\frac{\sqrt{3}\,\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)+\frac{\left(x-\mathrm{atan}(\tan(x))\right)\left(\frac{\pi\sqrt{2}}{6}+\pi\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)+\pi\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)\right)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^6 + 1),x)
```

```
[Out] atan((tan(x)*1i)/2 + (3^(1/2)*tan(x))/2)*(3^(1/2)/6 - 1i/6) - atan((tan(x)*
1i)/2 - (3^(1/2)*tan(x))/2)*(3^(1/2)/6 + 1i/6) + (2^(1/2)*atan(2^(1/2)*tan(
x)))/6 + ((x - atan(tan(x)))*((2^(1/2)*pi)/6 + pi*(3^(1/2)/6 - 1i/6) + pi*(
3^(1/2)/6 + 1i/6)))/pi
```

3.257 $\int \frac{1}{1+\sin^8(x)}\,dx$

Optimal. Leaf size=218

$$\frac{1}{8}\left(\sqrt{1+\sqrt{4-2\sqrt{2}}}+\sqrt{2+2\sqrt[4]{2}+2\sqrt{1+\sqrt{2}}+2\sqrt{2+\sqrt{2}}}+\sqrt{1+\sqrt{4+2\sqrt{2}}}\right)\left(x-\tan^{-1}(\right.$$

```
[Out] 1/4*arctan((1-(-1)^(1/4))^(1/2)*tan(x))/(1-(-1)^(1/4))^(1/2)+1/4*arctan((1+
(-1)^(1/4))^(1/2)*tan(x))/(1+(-1)^(1/4))^(1/2)+1/4*arctan((1-(-1)^(3/4))^(1
/2)*tan(x))/(1-(-1)^(3/4))^(1/2)+1/4*arctan((1+(-1)^(3/4))^(1/2)*tan(x))/(1
+(-1)^(3/4))^(1/2)+1/8*(x-arctan(tan(x)))*((1+(4-2*2^(1/2))^(1/2))^(1/2)+(2
+2*2^(1/4)+2*(1+2^(1/2))^(1/2)+2*(2+2^(1/2))^(1/2))^(1/2)+(1+(4+2*2^(1/2))^
(1/2))^(1/2))
```

Rubi [A]
time = 0.14, antiderivative size = 129, normalized size of antiderivative = 0.59, number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3290, 3260, 209}

$$\frac{\text{ArcTan}\left(\sqrt{1-\sqrt[4]{-1}}\,\tan(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\text{ArcTan}\left(\sqrt{1+\sqrt[4]{-1}}\,\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\text{ArcTan}\left(\sqrt{1-(-1)^{3/4}}\,\tan(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\text{ArcTan}\left(\sqrt{1+(-1)^{3/4}}\,\tan(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^8)^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 - (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTan[Sqrt[
1 + (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTan[Sqrt[1 - (-1)^(3/
4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTan[Sqrt[1 + (-1)^(3/4)]*Tan[x]]/
(4*Sqrt[1 + (-1)^(3/4)])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\sin^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\,\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt[4]{-1}\,\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sin^2(x)}\,dx +$$

$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[4]{-1}\right)x^2}\,dx, x, \tan(x)\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[4]{-1}\right)x^2}\,dx, x\right.$$

$$= \frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\tan(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1-(-1)\right.}{4\sqrt{1-(-1}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 141, normalized size = 0.65

$$8\text{RootSum}\left[1-8\#1+28\#1^2-56\#1^3+326\#1^4-56\#1^5+28\#1^6-8\#1^7+\#1^8\&, \frac{2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{-1+7\#1-21\#1^2+163\#1^3-35\#1^4+21\#1^5-7\#1^6+\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^8)^(-1),x]
[Out] 8*RootSum[1 - 8*#1 + 28*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8*#
1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Cos[2
*x]*#1 + #1^2]*#1^3)/(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^5 -
7*#1^6 + #1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.55, size = 71, normalized size = 0.33

method	result
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(2\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+1\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{2\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$
risch	$\left(\sum_{\_R=\text{RootOf}\left(8192\_Z^4+(128-128i)\_Z^2+1-i\right)}\_R\ln\left(e^{2ix}+(-1024-1024i)\_R^3+(128-128i)\_R^2+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(2*_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=R
ootOf(2*_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(sin(x)^8 + 1), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^8(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**8),x)
```

```
[Out] Integral(1/(sin(x)**8 + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^8),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 14.92, size = 945, normalized size = 4.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^8 + 1),x)
```

```
[Out] atan((tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/
2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (2^(1/2)
*tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(-
 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (tan(x)*(- 2*
2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*7i)/(256*((3
*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) + (2
^(1/2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)
^(1/2)*5i)/(256*((3*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) -
3)^(1/2)/512)))*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i - atan((tan(
x)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(- 2*
2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (2^(1/2)*tan(x)*
(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(- 2*2^(
1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) + (tan(x)*(- 2*2^(1/2
) - 3)^(1/2)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*7i)/(256*((3*2^(
1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (2^(1/
2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(
1/2)*5i)/(256*((3*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)
^(1/2)/512)))*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i + atan((tan(
x)*(- (2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(2*2^(1
/2) - 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(1/2)/512)) + (2^(1/2)*tan(x)*(- (2*
2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^
(1/2))/2048 + (2*2^(1/2) - 3)^(1/2)/512)) + (tan(x)*(2*2^(1/2) - 3)^(1/2)*(
- (2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*7i)/(256*((3*2^(1/2)*(2*2^(1/2)
- 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(1/2)/512)) + (2^(1/2)*tan(x)*(2*2^(1/2)
 - 3)^(1/2)*(- (2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*5i)/(256*((3*2^(1/2
)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(1/2)/512)))*(- (2*2^(1/2)
- 3)^(1/2)/128 - 1/128)^(1/2)*2i - atan((tan(x)*((2*2^(1/2) - 3)^(1/2)/128
- 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(1/2
) - 3)^(1/2)/512)) + (2^(1/2)*tan(x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1
/2)*1i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(1/2
)/512)) - (tan(x)*(2*2^(1/2) - 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)
^(1/2)*7i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(
1/2)/512)) - (2^(1/2)*tan(x)*(2*2^(1/2) - 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/1
28 - 1/128)^(1/2)*5i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(
1/2) - 3)^(1/2)/512)))*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i
```

3.258 $\int \frac{1}{1-\sin^5(x)}\,dx$

Optimal. Leaf size=187

$$-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}+\cdots}{\sqrt{1+\cdots}}\right)}{5\sqrt{1+\sqrt[5]{-\cdots}}}$$

```
[Out] 1/5*cos(x)/(1-sin(x))+2/5*arctan(((-1)^(3/5)+tan(1/2*x))/(1+(-1)^(1/5))^(1/
2))/(1+(-1)^(1/5))^(1/2)+2/5*arctan(((-1)^(1/5)+tan(1/2*x))/(1-(-1)^(2/5))^
(1/2))/(1-(-1)^(2/5))^(1/2)-2/5*arctan(((-1)^(4/5)-tan(1/2*x))/(1+(-1)^(3/5
))^(1/2))/(1+(-1)^(3/5))^(1/2)-2/5*arctan(((-1)^(2/5)-tan(1/2*x))/(1-(-1)^(
4/5))^(1/2))/(1-(-1)^(4/5))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3292, 2727, 2739, 632, 210}

$$-\frac{2\text{ArcTan}\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}-\frac{2\text{ArcTan}\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{2\text{ArcTan}\left(\frac{\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\text{ArcTan}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{3/5}}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}+\frac{\cos(x)}{5(1-\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTan[((-1)^(2/5) - Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^
(4/5)]) - (2*ArcTan[((-1)^(4/5) - Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sqrt[
1 + (-1)^(3/5)]) + (2*ArcTan[((-1)^(1/5) + Tan[x/2])/Sqrt[1 - (-1)^(2/5)]])
/(5*Sqrt[1 - (-1)^(2/5)]) + (2*ArcTan[((-1)^(3/5) + Tan[x/2])/Sqrt[1 + (-1)
^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]) + Cos[x]/(5*(1 - Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-\sin^5(x)}\,dx &= \int \left(\frac{1}{5(1-\sin(x))}+\frac{1}{5\left(1+\sqrt[5]{-1}\,\sin(x)\right)}+\frac{1}{5\left(1-(-1)^{2/5}\sin(x)\right)}+\frac{1}{5\left(1+(-1)^{3/5}\sin\right.} \\
&= \frac{1}{5}\int \frac{1}{1-\sin(x)}\,dx+\frac{1}{5}\int \frac{1}{1+\sqrt[5]{-1}\,\sin(x)}\,dx+\frac{1}{5}\int \frac{1}{1-(-1)^{2/5}\sin(x)}\,dx+\frac{1}{5}\int \frac{}{1+} \\
&= \frac{\cos(x)}{5(1-\sin(x))}+\frac{2}{5}\text{Subst}\left(\int \frac{1}{1+2\sqrt[5]{-1}\,x+x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)+\frac{2}{5}\text{Subst}\left(\int \frac{}{1-2(-} \right. \\
&= \frac{\cos(x)}{5(1-\sin(x))}-\frac{4}{5}\text{Subst}\left(\int \frac{1}{-4\left(1+\sqrt[5]{-1}\,\right)-x^2}\,dx,x,2(-1)^{3/5}+2\tan\left(\frac{x}{2}\right)\right)-\frac{4}{5}\text{Su} \\
&= -\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}\,+\tan\left(\frac{x}{2}\right.}{\sqrt{1-(-1)^{2/}}}\right.}{5\sqrt{1-(-1)^{2/5}}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.10, size = 413, normalized size = 2.21

$$\frac{1}{10}i\text{RootSum}\left[1-2i\#1-8\#1^2+14i\#1^3+30\#1^4-14i\#1^5-8\#1^6+2i\#1^7+\#1^8\&,\frac{-2\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)+i\log\left(1-2\cos(x)\#1+\#1^2\right)+8i\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1+4\log\left(1-2\cos(x)\#1+\#1^2\right)\#1+30\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2-15i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^2-80i\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-40\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3-30\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^4+15i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^4+8i\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^5+4\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^5+2\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^6-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^6}{-i-8\#1+21i\#1^2+60\#1^3-35i\#1^4-24\#1^5+7i\#1^6+4\#1^7}\&\right]+\frac{2\sin\left(\frac{x}{2}\right)}{5\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^5)^(-1),x]
```

```
[Out] (I/10)*RootSum[1 - (2*I)*#1 - 8*#1^2 + (14*I)*#1^3 + 30*#1^4 - (14*I)*#1^5
- 8*#1^6 + (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*Log[1
 - 2*Cos[x]*#1 + #1^2] + (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 + 4*Log[1 -
2*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[
1 - 2*Cos[x]*#1 + #1^2]*#1^2 - (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - 4
0*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 +
 (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + (8*I)*ArcTan[Sin[x]/(Cos[x] - #1
)]*#1^5 + 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1
)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(-I - 8*#1 + (21*I)*#1^2 + 60
*#1^3 - (35*I)*#1^4 - 24*#1^5 + (7*I)*#1^6 + 4*#1^7) & ] + (2*Sin[x/2])/(5*
(Cos[x/2] - Sin[x/2]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.66, size = 133, normalized size = 0.71

method	result
risch	$\frac{2}{5(e^{ix}-i)} + \left(\sum_{\_R=\mathrm{RootOf}(1953125\_Z^8+156250\_Z^6+6250\_Z^4+125\_Z^2+1)} \_R\ln\left(e^{ix} - 2343750\_R^7 - 234375i\_\right.\right.$
default	$\frac{2\left(\sum_{\_R=\mathrm{RootOf}(\_Z^8+2\_Z^7+8\_Z^6+14\_Z^5+30\_Z^4+14\_Z^3+8\_Z^2+2\_Z+1)} \frac{\left(2\_R^6+3\_R^5+10\_R^4+10\_R^3+10\_R^2+3\_R+2\right)\ln}{4\_R^7+7\_R^6+24\_R^5+35\_R^4+60\_R^3+21\_R}\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*sum((2*_R^6+3*_R^5+10*_R^4+10*_R^3+10*_R^2+3*_R+2)/(4*_R^7+7*_R^6+24*_R
^5+35*_R^4+60*_R^3+21*_R^2+8*_R+1)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^8+2*_Z^7+
8*_Z^6+14*_Z^5+30*_Z^4+14*_Z^3+8*_Z^2+2*_Z+1))-2/5/(tan(1/2*x)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^5),x, algorithm="maxima")
```

```
[Out] 1/5*(5*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(2/5*((4*cos(6*x) - 40
*cos(4*x) + 4*cos(2*x) + sin(7*x) - 15*sin(5*x) + 15*sin(3*x) - sin(x))*cos
(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) + 8*sin(6*x) - 55*sin(4*x)
+ 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos(2*x) + 44*
sin(5*x) - 44*sin(3*x) + 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2 + 2*(210*co
```

```
s(3*x) - 22*cos(x) + 505*sin(4*x) - 88*sin(2*x))*cos(5*x) - 210*cos(5*x)^2
+ 10*(44*cos(2*x) + 101*sin(3*x) - 11*sin(x) - 4)*cos(4*x) - 1200*cos(4*x)^
2 + 44*(cos(x) + 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 + 4*(4*sin(x) + 1)*c
os(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 - (cos(7*x) - 15*cos(5*x) + 15*cos(3*x
) - cos(x) - 4*sin(6*x) + 40*sin(4*x) - 4*sin(2*x))*sin(8*x) - (16*cos(6*x)
 - 110*cos(4*x) + 16*cos(2*x) - 44*sin(5*x) + 44*sin(3*x) - 4*sin(x) - 1)*s
in(7*x) - 2*sin(7*x)^2 - 8*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) - 55*sin(4
*x) + 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 - (1010*cos(4*x) - 176*cos(2*x)
- 420*sin(3*x) + 44*sin(x) + 15)*sin(5*x) - 210*sin(5*x)^2 - 10*(101*cos(3*
x) - 11*cos(x) - 44*sin(2*x))*sin(4*x) - 1200*sin(4*x)^2 - (176*cos(2*x) -
44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 - 16*cos(x)*sin(2*x) - 32*sin(2*x
)^2 - 2*sin(x)^2 - sin(x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*x) + 2*si
n(7*x) - 14*sin(5*x) + 14*sin(3*x) - 2*sin(x) - 1)*cos(8*x) - cos(8*x)^2 +
8*(7*cos(5*x) - 7*cos(3*x) + cos(x) + 4*sin(6*x) - 15*sin(4*x) + 4*sin(2*x)
)*cos(7*x) - 4*cos(7*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) + 14*sin(5*x) - 14
*sin(3*x) + 2*sin(x) + 1)*cos(6*x) - 64*cos(6*x)^2 + 56*(7*cos(3*x) - cos(x
) + 15*sin(4*x) - 4*sin(2*x))*cos(5*x) - 196*cos(5*x)^2 + 60*(8*cos(2*x) +
14*sin(3*x) - 2*sin(x) - 1)*cos(4*x) - 900*cos(4*x)^2 + 56*(cos(x) + 4*sin(
2*x))*cos(3*x) - 196*cos(3*x)^2 + 16*(2*sin(x) + 1)*cos(2*x) - 64*cos(2*x)^
2 - 4*cos(x)^2 - 4*(cos(7*x) - 7*cos(5*x) + 7*cos(3*x) - cos(x) - 4*sin(6*x
) + 15*sin(4*x) - 4*sin(2*x))*sin(8*x) - sin(8*x)^2 - 4*(8*cos(6*x) - 30*co
s(4*x) + 8*cos(2*x) - 14*sin(5*x) + 14*sin(3*x) - 2*sin(x) - 1)*sin(7*x) -
4*sin(7*x)^2 - 32*(7*cos(5*x) - 7*cos(3*x) + cos(x) - 15*sin(4*x) + 4*sin(2
*x))*sin(6*x) - 64*sin(6*x)^2 - 28*(30*cos(4*x) - 8*cos(2*x) - 14*sin(3*x)
+ 2*sin(x) + 1)*sin(5*x) - 196*sin(5*x)^2 - 120*(7*cos(3*x) - cos(x) - 4*si
n(2*x))*sin(4*x) - 900*sin(4*x)^2 - 28*(8*cos(2*x) - 2*sin(x) - 1)*sin(3*x)
 - 196*sin(3*x)^2 - 32*cos(x)*sin(2*x) - 64*sin(2*x)^2 - 4*sin(x)^2 - 4*sin
(x) - 1), x) + 2*cos(x))/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sin^5(x) - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**5),x)
```

```
[Out] -Integral(1/(sin(x)**5 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 14.44, size = 2500, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)^5 - 1),x)
```

```
[Out] 2*atanh((989855744*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3019
89888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2
)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1
)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) +
 (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25
)) + (2030043136*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5
*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*
tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2)
)/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^
(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - 18454
9376/25)) + (1627389952*5^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/
2))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*
5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/
5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25
- 184549376/25)) + (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^(1/2))/5
 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1
/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 +
(452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 +
 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5
^(1/2))/5 - 1)^(1/2))/25 - 184549376/25)) + (184549376*5^(1/2)*(- (2*5^(1/2
))/5 - 1)^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3019898
88*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(
- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(
```

1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (4
36207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25))
+ (5083496448*5^(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2
))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (13086
22848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5
^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5
 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 -
 184549376/25)) + (553648128*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^
(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364
672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2
))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1
/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)
*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25)) - (553648128*5^(1/2)*tan(
x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1
/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*
5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/
5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25
- 184549376/25)))*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((
989855744*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((1308622848*
tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (2382364672*5^(1/2)*tan(x/2))/12
5 - (301989888*tan(x/2))/5 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2)
)/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (43620
7616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (2
030043136*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((13
08622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (2382364672*5^(1/2)*tan
(x/2))/125 - (301989888*tan(x/2))/5 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 -
 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2)
 + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/
25)) + (1627389952*5^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))
/(25*((1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (2382364672*5^
(1/2)*tan(x/2))/125 - (301989888*tan(x/2))/5 + (452984832*5^(1/2)*(- (2*5^(
1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 -
 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 1
84549376/25)) - (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5
- 1)^(1/2)/50 - 1/50)^(1/2))/(5*((1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)
^(1/2))/25 - (2382364672*5^(1/2)*tan(x/2))/125 - (301989888*tan(x/2))/5 + (
452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 +
16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^
(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) - (184549376*5^(1/2)*(- (2*5^(1/2)
)/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((130862
2848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (2382364672*5^(1/2)*tan(x/2
))/125 - (301989888*tan(x/2))/5 + (452984832*5^...

3.259 $\int \frac{1}{1-\sin^6(x)} \, dx$

Optimal. Leaf size=71

$$\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}$$

```
[Out] 1/3*arctan((1+(-1)^(1/3))^(1/2)*tan(x))/(1+(-1)^(1/3))^(1/2)+1/3*arctan((1-
(-1)^(2/3))^(1/2)*tan(x))/(1-(-1)^(2/3))^(1/2)+1/3*tan(x)
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3290, 3260, 209, 3254, 3852, 8}

$$\frac{\text{ArcTan}\left(\sqrt{1+\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\text{ArcTan}\left(\sqrt{1-(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^6)^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 + (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTan[Sqrt[
1 - (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tan[x]/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sin^6(x)}\,dx &= \frac{1}{3}\int \frac{1}{1-\sin^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\,\sin^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin^2(x)}\,dx\\
&= \frac{1}{3}\int \sec^2(x)\,dx+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[3]{-1}\,\right)x^2}\,dx,x,\tan(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{}{1+(1}\right.\\
&= \frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}-\frac{1}{3}\text{Subst}(\int 1\,dx,x,\\
&= \frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 117, normalized size = 1.65

$$\frac{\cos(x)(15-8\cos(2x)+\cos(4x))\left(i\sqrt[4]{-3}\left(3i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}\left(-3i+\sqrt{3}\right)\tan(x)\right)\cos(x)+\sqrt[4]{-3}\left(-3i+\sqrt{3}\right)\tan^{-1}\left(\frac{(-1)^{3/4}\left(3i+\sqrt{3}\right)\tan(x)}{2\sqrt[4]{3}}\right)\cos(x)-6\sin(x)\right)}{144\left(-1+\sin^6(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^6)^(-1),x]
```

```
[Out] (Cos[x]*(15 - 8*Cos[2*x] + Cos[4*x])*(I*(-3)^(1/4)*(3*I + Sqrt[3])*ArcTan[(
(-1/3)^(1/4)*(-3*I + Sqrt[3])*Tan[x])/2]*Cos[x] + (-3)^(1/4)*(-3*I + Sqrt[3
])*ArcTan[((-1)^(3/4)*(3*I + Sqrt[3])*Tan[x])/(2*3^(1/4))]*Cos[x] - 6*Sin[x
]))/(144*(-1 + Sin[x]^6))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(49) = 98$.
time = 0.31, size = 202, normalized size = 2.85

method	result
risch	$\frac{2i}{3(e^{2ix}+1)} + \left(\sum_{\_R=\text{RootOf}(3888\_Z^4+108\_Z^2+1)} \_R \ln\left(e^{2ix} - 1296i\_R^3 + 216\_R^2 + 1\right)\right)$
default	$\frac{\tan(x)}{3} + \frac{\sqrt{3}\left(\frac{\sqrt{2\sqrt{3}-3}\,\ln\left(\sqrt{3}+3(\tan^2(x))+\sqrt{2\sqrt{3}-3}\,\sqrt{3}\,\tan(x)\right)}{6} + \frac{2\left(-\frac{(2\sqrt{3}-3)\sqrt{3}}{6}+2\right)\arctan\left(\frac{6\tan\ldots}{\ldots}\right)}{\sqrt{6\sqrt{3}+\ldots}}\ldots\right)}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*tan(x)+1/6*3^(1/2)*(1/6*(2*3^(1/2)-3)^(1/2)*ln(3^(1/2)+3*tan(x)^2+(2*3^
(1/2)-3)^(1/2)*3^(1/2)*tan(x))+2*(-1/6*(2*3^(1/2)-3)*3^(1/2)+2)/(6*3^(1/2)+
9)^(1/2)*arctan((6*tan(x)+(2*3^(1/2)-3)^(1/2)*3^(1/2))/(6*3^(1/2)+9)^(1/2))
)+1/6*3^(1/2)*(-1/6*(2*3^(1/2)-3)^(1/2)*ln(-(2*3^(1/2)-3)^(1/2)*3^(1/2)*tan
(x)+3*tan(x)^2+3^(1/2))+2*(-1/6*(2*3^(1/2)-3)*3^(1/2)+2)/(6*3^(1/2)+9)^(1/2
)*arctan((-(2*3^(1/2)-3)^(1/2)*3^(1/2)+6*tan(x))/(6*3^(1/2)+9)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^6),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*integrate(-4/3*((cos(6*x
) - 10*cos(4*x) + cos(2*x))*cos(8*x) + (110*cos(4*x) - 16*cos(2*x) + 1)*cos
(6*x) - 8*cos(6*x)^2 + 10*(11*cos(2*x) - 1)*cos(4*x) - 300*cos(4*x)^2 - 8*c
os(2*x)^2 + (sin(6*x) - 10*sin(4*x) + sin(2*x))*sin(8*x) + 2*(55*sin(4*x) -
 8*sin(2*x))*sin(6*x) - 8*sin(6*x)^2 - 300*sin(4*x)^2 + 110*sin(4*x)*sin(2*
x) - 8*sin(2*x)^2 + cos(2*x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*x) - 1
)*cos(8*x) - cos(8*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) + 1)*cos(6*x) - 64*c
os(6*x)^2 + 60*(8*cos(2*x) - 1)*cos(4*x) - 900*cos(4*x)^2 - 64*cos(2*x)^2 +
 4*(4*sin(6*x) - 15*sin(4*x) + 4*sin(2*x))*sin(8*x) - sin(8*x)^2 + 32*(15*s
in(4*x) - 4*sin(2*x))*sin(6*x) - 64*sin(6*x)^2 - 900*sin(4*x)^2 + 480*sin(4
*x)*sin(2*x) - 64*sin(2*x)^2 + 16*cos(2*x) - 1), x) - 2*sin(2*x))/(cos(2*x)
^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**6),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(49) = 98.
time = 0.48, size = 197, normalized size = 2.77

$$\frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-\arctan\left(-\frac{3\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)+4\tan\left(x\right)\right)}{\sqrt{6}+\sqrt{2}}\right)\right)\sqrt{6\sqrt{3}+9}+\frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{3\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)-4\tan\left(x\right)\right)}{\sqrt{6}+\sqrt{2}}\right)\right)\sqrt{6\sqrt{3}+9}+\frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(\frac{1}{2}\left(\sqrt{6}\left(\frac{1}{3}\right)^{\frac{1}{4}}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\right)\tan\left(x\right)+\tan\left(x\right)^{2}+\sqrt{\frac{1}{3}}\right)-\frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(-\frac{1}{2}\left(\sqrt{6}\left(\frac{1}{3}\right)^{\frac{1}{4}}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\right)\tan\left(x\right)+\tan\left(x\right)^{2}+\sqrt{\frac{1}{3}}\right)+\frac{1}{3}\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^6),x, algorithm="giac")
```

```
[Out] 1/18*(pi*floor(x/pi + 1/2) - arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6) -
sqrt(2)) + 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/18*(pi*f
loor(x/pi + 1/2) + arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6) - sqrt(2)) -
 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/36*sqrt(6*sqrt(3)
- 9)*log(1/2*(sqrt(6)*(1/3)^(1/4) - sqrt(2)*(1/3)^(1/4))*tan(x) + tan(x)^2
+ sqrt(1/3)) - 1/36*sqrt(6*sqrt(3) - 9)*log(-1/2*(sqrt(6)*(1/3)^(1/4) - sqr
t(2)*(1/3)^(1/4))*tan(x) + tan(x)^2 + sqrt(1/3)) + 1/3*tan(x)
```

Mupad [**B**]
time = 14.21, size = 99, normalized size = 1.39

$$\frac{\tan(x)}{3}-\frac{\sqrt{6}\,\mathrm{atan}\left(3^{1/4}\,\sqrt{6}\,\tan(x)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+3^{3/4}\,\sqrt{6}\,\tan(x)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)\right)\,\left(3^{1/4}\,\left(1+1\mathrm{i}\right)+3^{3/4}\,\left(-1+1\mathrm{i}\right)\right)\,1\mathrm{i}}{36}+\frac{\sqrt{6}\,\mathrm{atan}\left(3^{1/4}\,\sqrt{6}\,\tan(x)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+3^{3/4}\,\sqrt{6}\,\tan(x)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)\right)\,\left(3^{1/4}\,\left(1-\mathrm{i}\right)+3^{3/4}\,\left(-1-\mathrm{i}\right)\right)\,1\mathrm{i}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)^6 - 1),x)
```

```
[Out] tan(x)/3 - (6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x)*(1/4 - 1i/4) + 3^(3/4)*6^(1
/2)*tan(x)*(1/12 + 1i/12))*(3^(1/4)*(1 + 1i) - 3^(3/4)*(1 - 1i))*1i)/36 + (
6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x)*(1/4 + 1i/4) + 3^(3/4)*6^(1/2)*tan(x)*(
1/12 - 1i/12))*(3^(1/4)*(1 - 1i) - 3^(3/4)*(1 + 1i))*1i)/36
```

3.260 $\int \frac{1}{1-\sin^8(x)}\,dx$

Optimal. Leaf size=89

$$\frac{x}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{4\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{4\sqrt{1+i}}+\frac{\tan(x)}{4}$$

```
[Out] 1/4*arctan((1-I)^(1/2)*tan(x))/(1-I)^(1/2)+1/4*arctan((1+I)^(1/2)*tan(x))/(
1+I)^(1/2)+1/8*x*2^(1/2)+1/8*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(
1/2)+1/4*tan(x)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3290, 3260, 209, 3254, 3852, 8}

$$\frac{\text{ArcTan}\left(\sqrt{1-i}\,\tan(x)\right)}{4\sqrt{1-i}}+\frac{\text{ArcTan}\left(\sqrt{1+i}\,\tan(x)\right)}{4\sqrt{1+i}}+\frac{\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}+\frac{x}{4\sqrt{2}}+\frac{\tan(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^8)^(-1),x]
```

```
[Out] x/(4*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(4*Sqrt[2]
) + ArcTan[Sqrt[1 - I]*Tan[x]]/(4*Sqrt[1 - I]) + ArcTan[Sqrt[1 + I]*Tan[x]]
/(4*Sqrt[1 + I]) + Tan[x]/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-\sin^8(x)}\,dx &= \frac{1}{4}\int \frac{1}{1-\sin^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1-i\sin^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1+i\sin^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1+\sin^2(x)}\,d\\
&= \frac{1}{4}\int \sec^2(x)\,dx+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+(1-i)x^2}\,dx,x,\tan(x)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+(1+i)x^2}\right.\\
&= \frac{x}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{4\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{4\sqrt{1+i}}\\
&= \frac{x}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{4\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{4\sqrt{1+i}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 64, normalized size = 0.72

$$\frac{1}{8}\left(\frac{2\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{\sqrt{1-i}}+\frac{2\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{\sqrt{1+i}}+\sqrt{2}\,\tan^{-1}\left(\sqrt{2}\,\tan(x)\right)+2\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^8)^(-1),x]
```

```
[Out] ((2*ArcTan[Sqrt[1 - I]*Tan[x]])/Sqrt[1 - I] + (2*ArcTan[Sqrt[1 + I]*Tan[x]]
)/Sqrt[1 + I] + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(65) = 130.
time = 0.39, size = 206, normalized size = 2.31

method	result
risch	$\frac{i}{2\,\mathrm{e}^{2ix}+2}+\frac{\sqrt{-2+2i}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{-2+2i}+\sqrt{-2+2i}-1+2i\right)}{16}-\frac{\sqrt{-2+2i}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{-2+2i}-\sqrt{-2}\right.}{16}$
default	$\frac{\tan(x)}{4}+\frac{\arctan\left(\sqrt{2}\,\tan(x)\right)\sqrt{2}}{8}+\frac{\sqrt{2}\left(\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\sqrt{2}+2\left(\tan^{2}(x)\right)+\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\,\tan(x)\right)}{4}+\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*tan(x)+1/8*arctan(2^(1/2)*tan(x))*2^(1/2)+1/8*2^(1/2)*(1/4*(-2+2*2^(1/2
))^(1/2)*ln(2^(1/2)+2*tan(x)^2+(-2+2*2^(1/2))^(1/2)*2^(1/2)*tan(x))+(-1/4*(
-2+2*2^(1/2))*2^(1/2)+2)/(1+2^(1/2))^(1/2)*arctan(1/2*(4*tan(x)+2^(1/2)*(-2
+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)))+1/8*2^(1/2)*(-1/4*(-2+2*2^(1/2))^(1/
2)*ln(-(-2+2*2^(1/2))^(1/2)*2^(1/2)*tan(x)+2*tan(x)^2+2^(1/2))+(-1/4*(-2+2*
2^(1/2))*2^(1/2)+2)/(1+2^(1/2))^(1/2)*arctan(1/2*(-2^(1/2)*(-2+2*2^(1/2))^(
1/2)+4*tan(x))/(1+2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^8),x, algorithm="maxima")
```

```
[Out] 1/16*((sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt(
2))*arctan2(2*sqrt(2)*sin(x)/(2*(sqrt(2) + 1)*cos(x) + cos(x)^2 + sin(x)^2
+ 2*sqrt(2) + 3), (cos(x)^2 + sin(x)^2 + 2*cos(x) - 1)/(2*(sqrt(2) + 1)*cos
(x) + cos(x)^2 + sin(x)^2 + 2*sqrt(2) + 3)) - (sqrt(2)*cos(2*x)^2 + sqrt(2)
*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt(2))*arctan2(2*sqrt(2)*sin(x)/(2*(sq
rt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 - 2*sqrt(2) + 3), (cos(x)^2 + sin(x
)^2 - 2*cos(x) - 1)/(2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 - 2*sqrt(
2) + 3)) + 128*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*integrate(((4*cos
(2*x) - 1)*cos(4*x) - cos(8*x)*cos(4*x) + 4*cos(6*x)*cos(4*x) - 22*cos(4*x)
^2 - sin(8*x)*sin(4*x) + 4*sin(6*x)*sin(4*x) - 22*sin(4*x)^2 + 4*sin(4*x)*s
in(2*x))/(2*(4*cos(6*x) - 22*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)
```

```
^2 + 8*(22*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 44*(4*cos(
2*x) - 1)*cos(4*x) - 484*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 11*si
n(4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(11*sin(4*x) - 2*sin(2*x))*
sin(6*x) - 16*sin(6*x)^2 - 484*sin(4*x)^2 + 176*sin(4*x)*sin(2*x) - 16*sin(
2*x)^2 + 8*cos(2*x) - 1), x) + 8*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos
(2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3884 vs. $2(57) = 114$.
time = 18.06, size = 3884, normalized size = 43.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^8),x, algorithm="fricas")
```

```
[Out] -1/64*(2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*cos(x)*log(-(4*sqrt(2) - 5
)*cos(x)^4 + 2*(2*sqrt(2) - 3)*cos(x)^2 + (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3
 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 2) - 2^(1/4
)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*cos(x)*log(-(4*sqrt(2) - 5)*cos(x)^4 +
2*(2*sqrt(2) - 3)*cos(x)^2 - (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2*2^(1/4)*
(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 2) + 2*2^(1/4)*sqrt(2*sq
rt(2) + 4)*arctan(1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^
16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14 + 16*(sqrt(
2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt(2)*(41*sqrt(2)
- 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2) - 90) - 58*sqrt
(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt(2) + 216)*cos(x
)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4 + 4*(2*sqrt(2) -
 15)*cos(x)^2 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2) + 2))*
cos(x)^15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqrt(2) + 4))*cos(x
)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2) - 14))*cos(x)^1
1 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31))*cos(x)^9 + 2*
(2^(3/4)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(x)^7 - 2*(2^(3/4
)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (2*2^(3/4)*(sqrt(
2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(x))*sqrt(2*sqrt(
2) + 4)*sin(x) + (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 -
56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sqr
t(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 10
*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*c
os(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt(
2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*
(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sq
rt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(2
) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 34
))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*cos
```

```
(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2^
(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqrt
(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 + 4*(2^(1/4)*(3*sqrt(2) - 4)
*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 8)
 + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(x)^10 - 152*
cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1))*cos(x) - 2*2^(1/4
)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2)
- 6)*cos(x)^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14
 + 16*(sqrt(2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt(2)*
(41*sqrt(2) - 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2) - 9
0) - 58*sqrt(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt(2)
+ 216)*cos(x)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4 + 4*
(2*sqrt(2) - 15)*cos(x)^2 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sq
rt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqrt(2)
 + 4))*cos(x)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2) - 1
4))*cos(x)^11 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31))*c
os(x)^9 + 2*(2^(3/4)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(x)^7
 - 2*(2^(3/4)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (2*2^
(3/4)*(sqrt(2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(x))*
sqrt(2*sqrt(2) + 4)*sin(x) - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*
cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqr
t(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2
) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqr
t(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^
4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2) - 11)
- 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2
^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4
)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*
sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(
2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*c
os(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqr
t(-4*(4*sqrt(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 + 4*(2^(1/4)*(3*
sqrt(2) - 4)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)
*sin(x) + 8) + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(
x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1))*cos(x
) + 2*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2)
- 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 4
4)*cos(x)^14 + 16*(sqrt(2)*(51*sqrt(2) - 4) - 5...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**8),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(57) = 114.
time = 0.56, size = 220, normalized size = 2.47

$$\frac{1}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}+\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2+\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)-\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)+\frac{1}{4}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^8),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
+ sqrt(2) - 2*cos(2*x) + 2))) + 1/8*(pi*floor(x/pi + 1/2) + arctan(2*(1/2)^
(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2) + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(
sqrt(2) + 1) + 1/8*(pi*floor(x/pi + 1/2) + arctan(-2*(1/2)^(3/4)*((1/2)^(1/
4)*sqrt(-sqrt(2) + 2) - 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1
/16*sqrt(sqrt(2) - 1)*log(tan(x)^2 + (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x)
+ sqrt(1/2)) - 1/16*sqrt(sqrt(2) - 1)*log(tan(x)^2 - (1/2)^(1/4)*sqrt(-sqrt
(2) + 2)*tan(x) + sqrt(1/2)) + 1/4*tan(x)
```

Mupad [B]
time = 14.04, size = 141, normalized size = 1.58

$$\frac{\tan(x)}{4}+\mathrm{atan}\left(\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i}-\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right)+\mathrm{atan}\left(\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i}+\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right)+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)^8 - 1),x)
```

```
[Out] tan(x)/4 + atan(2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(1/2)*8i - 2^(1/2)*t
an(x)*(2^(1/2)/256 - 1/256)^(1/2)*8i)*((- 2^(1/2)/256 - 1/256)^(1/2)*2i + (
2^(1/2)/256 - 1/256)^(1/2)*2i) + atan(2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256
)^(1/2)*8i + 2^(1/2)*tan(x)*(2^(1/2)/256 - 1/256)^(1/2)*8i)*((- 2^(1/2)/256
 - 1/256)^(1/2)*2i - (2^(1/2)/256 - 1/256)^(1/2)*2i) + (2^(1/2)*atan(2^(1/2
)*tan(x)))/8
```

3.261 $\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=38

$$\frac{\sin(x)}{a}-\frac{\sin^3(x)}{a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^7(x)}{7a}$$

```
[Out] sin(x)/a-sin(x)^3/a+3/5*sin(x)^5/a-1/7*sin(x)^7/a
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2713}

$$-\frac{\sin^7(x)}{7a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^3(x)}{a}+\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^9/(a - a*Sin[x]^2),x]
```

```
[Out] Sin[x]/a - Sin[x]^3/a + (3*Sin[x]^5)/(5*a) - Sin[x]^7/(7*a)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^7(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(x)\right)}{a}\\ &= \frac{\sin(x)}{a}-\frac{\sin^3(x)}{a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^7(x)}{7a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.92

$$\frac{\frac{35\sin(x)}{64}+\frac{7}{64}\sin(3x)+\frac{7}{320}\sin(5x)+\frac{1}{448}\sin(7x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^9/(a - a*Sin[x]^2),x]
[Out] ((35*Sin[x])/64 + (7*Sin[3*x])/64 + (7*Sin[5*x])/320 + Sin[7*x]/448)/a
```

Maple [A]
time = 0.10, size = 26, normalized size = 0.68

method	result	size
default	$\frac{-\frac{\left(\sin^7(x)\right)}{7}+\frac{3\left(\sin^5(x)\right)}{5}-\left(\sin^3(x)\right)+\sin(x)}{a}$	26
risch	$\frac{35\sin(x)}{64a}+\frac{\sin(7x)}{448a}+\frac{7\sin(5x)}{320a}+\frac{7\sin(3x)}{64a}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(-1/7*sin(x)^7+3/5*sin(x)^5-sin(x)^3+sin(x))
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.74

$$-\frac{5\sin(x)^7-21\sin(x)^5+35\sin(x)^3-35\sin(x)}{35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] -1/35*(5*sin(x)^7 - 21*sin(x)^5 + 35*sin(x)^3 - 35*sin(x))/a
```

Fricas [A]
time = 0.41, size = 27, normalized size = 0.71

$$\frac{\left(5\cos(x)^6+6\cos(x)^4+8\cos(x)^2+16\right)\sin(x)}{35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="fricas")
[Out] 1/35*(5*cos(x)^6 + 6*cos(x)^4 + 8*cos(x)^2 + 16)*sin(x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(29) = 58$.

time = 15.01, size = 580, normalized size = 15.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**9/(a-a*sin(x)**2),x)
```

```
[Out] 70*tan(x/2)**13/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**1
0 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan
(x/2)**2 + 35*a) + 140*tan(x/2)**11/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12
 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan
(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 602*tan(x/2)**9/(35*a*tan(x/2)**14 +
 245*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(
x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 424*tan(x/2)**7/(
35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/
2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a)
+ 602*tan(x/2)**5/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)*
*10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*t
an(x/2)**2 + 35*a) + 140*tan(x/2)**3/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**1
2 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*ta
n(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 70*tan(x/2)/(35*a*tan(x/2)**14 + 24
5*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2
)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a)
```

Giac [A]

time = 0.44, size = 28, normalized size = 0.74

$$-\frac{5\sin(x)^{7} - 21\sin(x)^{5} + 35\sin(x)^{3} - 35\sin(x)}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/35*(5*sin(x)^7 - 21*sin(x)^5 + 35*sin(x)^3 - 35*sin(x))/a
```

Mupad [B]

time = 0.10, size = 34, normalized size = 0.89

$$\frac{\sin(x)}{a} - \frac{\sin(x)^{3}}{a} + \frac{3\sin(x)^{5}}{5\,a} - \frac{\sin(x)^{7}}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a - a*sin(x)^2),x)
```

```
[Out] sin(x)/a - sin(x)^3/a + (3*sin(x)^5)/(5*a) - sin(x)^7/(7*a)
```

3.262 $\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\sin(x)}{a}-\frac{2\sin^3(x)}{3a}+\frac{\sin^5(x)}{5a}$$

```
[Out] sin(x)/a-2/3*sin(x)^3/a+1/5*sin(x)^5/a
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3254, 2713}

$$\frac{\sin^5(x)}{5a}-\frac{2\sin^3(x)}{3a}+\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/(a - a*Sin[x]^2),x]
```

```
[Out] Sin[x]/a - (2*Sin[x]^3)/(3*a) + Sin[x]^5/(5*a)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^5(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(x)\right)}{a}\\ &= \frac{\sin(x)}{a}-\frac{2\sin^3(x)}{3a}+\frac{\sin^5(x)}{5a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 0.93

$$\frac{\frac{5\sin(x)}{8}+\frac{5}{48}\sin(3x)+\frac{1}{80}\sin(5x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/(a - a*Sin[x]^2),x]
[Out] ((5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80)/a
```

Maple [A]
time = 0.15, size = 20, normalized size = 0.69

method	result	size
default	$\frac{\frac{\left(\sin^5(x)\right)}{5}-\frac{2\left(\sin^3(x)\right)}{3}+\sin(x)}{a}$	20
risch	$\frac{5\sin(x)}{8a}+\frac{\sin(5x)}{80a}+\frac{5\sin(3x)}{48a}$	27
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{14\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{42\left(\tan^5\left(\frac{x}{2}\right)\right)}{5a}-\frac{86\left(\tan^7\left(\frac{x}{2}\right)\right)}{15a}+\frac{86\left(\tan^9\left(\frac{x}{2}\right)\right)}{15a}+\frac{42\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{5a}+\frac{14\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{3a}+\frac{2\left(\tan^{15}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^7\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/5*sin(x)^5-2/3*sin(x)^3+sin(x))
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.76

$$\frac{3\sin(x)^5-10\sin(x)^3+15\sin(x)}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a
```

Fricas [A]
time = 0.40, size = 21, normalized size = 0.72

$$\frac{\left(3\cos(x)^4+4\cos(x)^2+8\right)\sin(x)}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(22) = 44$.
time = 6.49, size = 311, normalized size = 10.72

$$\frac{30\tan^{9}\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^{8}\left(\frac{x}{2}\right)+150a\tan^{6}\left(\frac{x}{2}\right)+150a\tan^{4}\left(\frac{x}{2}\right)+75a\tan^{2}\left(\frac{x}{2}\right)+15a}+\frac{40\tan^{7}\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^{8}\left(\frac{x}{2}\right)+150a\tan^{6}\left(\frac{x}{2}\right)+150a\tan^{4}\left(\frac{x}{2}\right)+75a\tan^{2}\left(\frac{x}{2}\right)+15a}+\frac{116\tan^{5}\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^{8}\left(\frac{x}{2}\right)+150a\tan^{6}\left(\frac{x}{2}\right)+150a\tan^{4}\left(\frac{x}{2}\right)+75a\tan^{2}\left(\frac{x}{2}\right)+15a}+\frac{40\tan^{3}\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^{8}\left(\frac{x}{2}\right)+150a\tan^{6}\left(\frac{x}{2}\right)+150a\tan^{4}\left(\frac{x}{2}\right)+75a\tan^{2}\left(\frac{x}{2}\right)+15a}+\frac{30\tan\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^{8}\left(\frac{x}{2}\right)+150a\tan^{6}\left(\frac{x}{2}\right)+150a\tan^{4}\left(\frac{x}{2}\right)+75a\tan^{2}\left(\frac{x}{2}\right)+15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a-a*sin(x)**2),x)
```

```
[Out] 30*tan(x/2)**9/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 +
150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 40*tan(x/2)**7/(15*a*tan(x/2
)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan
(x/2)**2 + 15*a) + 116*tan(x/2)**5/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 +
150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 40*tan(x
/2)**3/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*ta
n(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 30*tan(x/2)/(15*a*tan(x/2)**10 + 75*
a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 +
15*a)
```

Giac [A]
time = 0.44, size = 22, normalized size = 0.76

$$\frac{3\,\sin\left(x\right)^{5}-10\,\sin\left(x\right)^{3}+15\,\sin\left(x\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a
```

Mupad [B]
time = 0.08, size = 19, normalized size = 0.66

$$\frac{\frac{\sin(x)^5}{5}-\frac{2\sin(x)^3}{3}+\sin\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a - a*sin(x)^2),x)
```

```
[Out] (sin(x) - (2*sin(x)^3)/3 + sin(x)^5/5)/a
```

3.263 $\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}$$

```
[Out] sin(x)/a-1/3*sin(x)^3/a
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3254, 2713}

$$\frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a - a*Sin[x]^2),x]
```

```
[Out] Sin[x]/a - Sin[x]^3/(3*a)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^3(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(x)\right)}{a}\\ &= \frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.06

$$\frac{\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a - a*Sin[x]^2),x]
[Out] ((3*Sin[x])/4 + Sin[3*x]/12)/a
```

Maple [A]
time = 0.15, size = 14, normalized size = 0.78

method	result	size
default	$\frac{-\frac{\left(\sin^{3}(x)\right)}{3}+\sin(x)}{a}$	14
risch	$\frac{3\sin(x)}{4a}+\frac{\sin(3x)}{12a}$	18
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{10\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3a}-\frac{4\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{3a}+\frac{4\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{3a}+\frac{10\left(\tan^{9}\left(\frac{x}{2}\right)\right)}{3a}+\frac{2\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{5}\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(-1/3*sin(x)^3+sin(x))
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.78

$$-\frac{\sin\left(x\right)^{3}-3\,\sin\left(x\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a
```

Fricas [A]
time = 0.41, size = 13, normalized size = 0.72

$$\frac{\left(\cos\left(x\right)^{2}+2\right)\sin\left(x\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(12) = 24$.
time = 2.47, size = 124, normalized size = 6.89

$$\frac{6\tan^5\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right)+9a\tan^4\left(\frac{x}{2}\right)+9a\tan^2\left(\frac{x}{2}\right)+3a}+\frac{4\tan^3\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right)+9a\tan^4\left(\frac{x}{2}\right)+9a\tan^2\left(\frac{x}{2}\right)+3a}+\frac{6\tan\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right)+9a\tan^4\left(\frac{x}{2}\right)+9a\tan^2\left(\frac{x}{2}\right)+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a-a*sin(x)**2),x)
```

```
[Out] 6*tan(x/2)**5/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a) +
 4*tan(x/2)**3/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
+ 6*tan(x/2)/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
```

Giac [A]
time = 0.46, size = 14, normalized size = 0.78

$$-\frac{\sin(x)^3 - 3\sin(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.89

$$\frac{3\sin(x) - \sin(x)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a - a*sin(x)^2),x)
```

```
[Out] (3*sin(x) - sin(x)^3)/(3*a)
```

3.264 $\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=6

$$\frac{\sin(x)}{a}$$

[Out] sin(x)/a

Rubi [A]
time = 0.03, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2717}

$$\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3/(a - a*Sin[x]^2),x]

[Out] Sin[x]/a

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos(x)\,dx}{a}\\ &= \frac{\sin(x)}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a - a*Sin[x]^2),x]
```

```
[Out] Sin[x]/a
```

Maple [A]
time = 0.11, size = 7, normalized size = 1.17

method	result	size
derivativedivides	$\frac{\sin(x)}{a}$	7
default	$\frac{\sin(x)}{a}$	7
risch	$\frac{\sin(x)}{a}$	7
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^7\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)/a
```

Maxima [A]
time = 0.27, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] sin(x)/a
```

Fricas [A]
time = 0.41, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] sin(x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.
time = 0.82, size = 15, normalized size = 2.50

$$\frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a-a*sin(x)**2),x)
```

```
[Out] 2*tan(x/2)/(a*tan(x/2)**2 + a)
```

Giac [A]
time = 0.45, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] sin(x)/a
```

Mupad [B]
time = 13.76, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a - a*sin(x)^2),x)
```

```
[Out] sin(x)/a
```

3.265 $\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=7

$$\frac{\tanh^{-1}(\sin(x))}{a}$$

```
[Out] arctanh(sin(x))/a
```

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3254, 3855}

$$\frac{\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a - a*Sin[x]^2),x]
```

```
[Out] ArcTanh[Sin[x]]/a
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec(x)\,dx}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(7) = 14.
time = 0.00, size = 37, normalized size = 5.29

$$\frac{-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a - a*Sin[x]^2),x]
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a
```

Maple [A]
time = 0.09, size = 8, normalized size = 1.14

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(\sin(x))}{a}$	8
default	$\frac{\operatorname{arctanh}(\sin(x))}{a}$	8
norman	$\frac{\ln(\tan(\frac{x}{2})+1)}{a}-\frac{\ln(\tan(\frac{x}{2})-1)}{a}$	25
risch	$\frac{\ln(e^{ix}+i)}{a}-\frac{\ln(e^{ix}-i)}{a}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] arctanh(sin(x))/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.
time = 0.28, size = 21, normalized size = 3.00

$$\frac{\log(\sin(x)+1)}{2a}-\frac{\log(\sin(x)-1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] 1/2*log(sin(x) + 1)/a - 1/2*log(sin(x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.
time = 0.40, size = 20, normalized size = 2.86

$$\frac{\log(\sin(x)+1)-\log(-\sin(x)+1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="fricas")
[Out] 1/2*(log(sin(x) + 1) - log(-sin(x) + 1))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(5) = 10.
time = 0.08, size = 19, normalized size = 2.71

$$-\frac{\log(\sin(x)-1)}{2a}+\frac{\log(\sin(x)+1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)**2),x)
```

```
[Out] -log(sin(x) - 1)/(2*a) + log(sin(x) + 1)/(2*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(7) = 14$.
time = 0.43, size = 23, normalized size = 3.29

$$\frac{\log\left(\sin\left(x\right) + 1\right)}{2\,a} - \frac{\log\left(-\sin\left(x\right) + 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x) + 1)/a - 1/2*log(-sin(x) + 1)/a
```

Mupad [B]
time = 13.60, size = 7, normalized size = 1.00

$$\frac{\mathrm{atanh}(\sin\left(x\right))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a - a*sin(x)^2),x)
```

```
[Out] atanh(sin(x))/a
```

3.266 $\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=35

$$\frac{3\tanh^{-1}(\sin(x))}{8a}+\frac{3\sec(x)\tan(x)}{8a}+\frac{\sec^3(x)\tan(x)}{4a}$$

```
[Out] 3/8*arctanh(sin(x))/a+3/8*sec(x)*tan(x)/a+1/4*sec(x)^3*tan(x)/a
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3254, 3853, 3855}

$$\frac{3\tanh^{-1}(\sin(x))}{8a}+\frac{\tan(x)\sec^3(x)}{4a}+\frac{3\tan(x)\sec(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a - a*Sin[x]^2),x]
```

```
[Out] (3*ArcTanh[Sin[x]])/(8*a) + (3*Sec[x]*Tan[x])/(8*a) + (Sec[x]^3*Tan[x])/(4*
a)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx = \frac{\int \sec^5(x)\,dx}{a}$$
$$= \frac{\sec^3(x)\tan(x)}{4a} + \frac{3\int \sec^3(x)\,dx}{4a}$$
$$= \frac{3\sec(x)\tan(x)}{8a} + \frac{\sec^3(x)\tan(x)}{4a} + \frac{3\int \sec(x)\,dx}{8a}$$
$$= \frac{3\tanh^{-1}(\sin(x))}{8a} + \frac{3\sec(x)\tan(x)}{8a} + \frac{\sec^3(x)\tan(x)}{4a}$$

Mathematica [A]
time = 0.10, size = 61, normalized size = 1.74

$$\frac{-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\sec^4(x)(11\sin(x)+3\sin(3x))}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a - a*Sin[x]^2),x]
[Out] (-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4*(11*S
in[x] + 3*Sin[3*x]))/2)/(16*a)
```

Maple [A]
time = 0.17, size = 52, normalized size = 1.49

method	result	size
default	$\frac{-\frac{1}{16(1+\sin(x))^2}-\frac{3}{16(1+\sin(x))}+\frac{3\ln(1+\sin(x))}{16}+\frac{1}{16(\sin(x)-1)^2}-\frac{3}{16(\sin(x)-1)}-\frac{3\ln(\sin(x)-1)}{16}}{a}$	52
risch	$-\frac{i(3\,e^{7ix}+11\,e^{5ix}-11\,e^{3ix}-3\,e^{ix})}{4(e^{2ix}+1)^4 a}-\frac{3\ln(e^{ix}-i)}{8a}+\frac{3\ln(e^{ix}+i)}{8a}$	74
norman	$\frac{\frac{5\tan\left(\frac{x}{2}\right)}{4a}+\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)}{4a}+\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4a}+\frac{5\left(\tan^7\left(\frac{x}{2}\right)\right)}{4a}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^4}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{8a}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{8a}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(-1/16/(1+sin(x))^2-3/16/(1+sin(x))+3/16*ln(1+sin(x))+1/16/(sin(x)-1)^2
-3/16/(sin(x)-1)-3/16*ln(sin(x)-1))
```

Maxima [A]
time = 0.27, size = 51, normalized size = 1.46

$$-\frac{3\,\sin\left(x\right)^3-5\,\sin\left(x\right)}{8\,\left(a\sin\left(x\right)^4-2\,a\sin\left(x\right)^2+a\right)}+\frac{3\,\log\left(\sin\left(x\right)+1\right)}{16\,a}-\frac{3\,\log\left(\sin\left(x\right)-1\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(a*sin(x)^4 - 2*a*sin(x)^2 + a) + 3/16*log(sin
(x) + 1)/a - 3/16*log(sin(x) - 1)/a
```

Fricas [A]
time = 0.40, size = 46, normalized size = 1.31

$$\frac{3 \cos(x)^4 \log(\sin(x)+1) - 3 \cos(x)^4 \log(-\sin(x)+1) + 2\left(3 \cos(x)^2+2\right)\sin(x)}{16\, a \cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(x
)^2 + 2)*sin(x))/(a*cos(x)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^3(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)**3/(sin(x)**2 - 1), x)/a
```

Giac [A]
time = 0.44, size = 47, normalized size = 1.34

$$\frac{3 \log(\sin(x)+1)}{16\, a} - \frac{3 \log(-\sin(x)+1)}{16\, a} - \frac{3 \sin(x)^3 - 5 \sin(x)}{8\left(\sin(x)^2-1\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 3/16*log(sin(x) + 1)/a - 3/16*log(-sin(x) + 1)/a - 1/8*(3*sin(x)^3 - 5*sin(
x))/((sin(x)^2 - 1)^2*a)
```

Mupad [B]
time = 13.88, size = 31, normalized size = 0.89

$$\frac{3\,\mathrm{atanh}(\sin(x))}{8\,a} + \frac{3 \sin(x)}{8\,a \cos(x)^2} + \frac{\sin(x)}{4\,a \cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^3*(a - a*sin(x)^2)),x)
```

```
[Out] (3*atanh(sin(x)))/(8*a) + (3*sin(x))/(8*a*cos(x)^2) + sin(x)/(4*a*cos(x)^4)
```

3.267 $\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=33

$$\frac{3x}{8a}+\frac{3\cos(x)\sin(x)}{8a}+\frac{\cos^3(x)\sin(x)}{4a}$$

```
[Out] 3/8*x/a+3/8*cos(x)*sin(x)/a+1/4*cos(x)^3*sin(x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3254, 2715, 8}

$$\frac{3x}{8a}+\frac{\sin(x)\cos^3(x)}{4a}+\frac{3\sin(x)\cos(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a - a*Sin[x]^2),x]
```

```
[Out] (3*x)/(8*a) + (3*Cos[x]*Sin[x])/(8*a) + (Cos[x]^3*Sin[x])/(4*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(x)}{a - a\sin^2(x)}\,dx = \frac{\int \cos^4(x)\,dx}{a}$$
$$= \frac{\cos^3(x)\sin(x)}{4a} + \frac{3\int \cos^2(x)\,dx}{4a}$$
$$= \frac{3\cos(x)\sin(x)}{8a} + \frac{\cos^3(x)\sin(x)}{4a} + \frac{3\int 1\,dx}{8a}$$
$$= \frac{3x}{8a} + \frac{3\cos(x)\sin(x)}{8a} + \frac{\cos^3(x)\sin(x)}{4a}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.79

$$\frac{\frac{3x}{8} + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a - a*Sin[x]^2),x]
[Out] ((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)/a
```

Maple [A]
time = 0.16, size = 35, normalized size = 1.06

method	result
risch	$\frac{3x}{8a} + \frac{\sin(4x)}{32a} + \frac{\sin(2x)}{4a}$
default	$\frac{\frac{\tan(x)}{4\left(\tan^2(x)+1\right)^2} + \frac{3\tan(x)}{8\left(\tan^2(x)+1\right)} + \frac{3\arctan(\tan(x))}{8}}{a}$
norman	$\frac{\frac{\tan^7\left(\frac{x}{2}\right)}{a} - \frac{5\tan\left(\frac{x}{2}\right)}{4a} - \frac{\tan^3\left(\frac{x}{2}\right)}{2a} + \frac{5\left(\tan^5\left(\frac{x}{2}\right)\right)}{4a} + \frac{5\left(\tan^9\left(\frac{x}{2}\right)\right)}{4a} - \frac{\tan^{11}\left(\frac{x}{2}\right)}{2a} - \frac{5\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{4a} - \frac{3x}{8a} - \frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8a} - \frac{27x\left(\tan^4\left(\frac{x}{2}\right)\right)}{8a} - \frac{15x\left(\tan\right.}{8}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/4*tan(x)/(tan(x)^2+1)^2+3/8*tan(x)/(tan(x)^2+1)+3/8*arctan(tan(x)))
```

Maxima [A]
time = 0.51, size = 37, normalized size = 1.12

$$\frac{3\,\tan\left(x\right)^3 + 5\,\tan\left(x\right)}{8\,\left(a\tan\left(x\right)^4 + 2\,a\tan\left(x\right)^2 + a\right)} + \frac{3\,x}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*tan(x)^3 + 5*tan(x))/(a*tan(x)^4 + 2*a*tan(x)^2 + a) + 3/8*x/a
```

Fricas [A]
time = 0.41, size = 23, normalized size = 0.70

$$\frac{\left(2\,\cos\left(x\right)^{3}+3\,\cos\left(x\right)\right)\sin\left(x\right)+3\,x}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*cos(x)^3 + 3*cos(x))*sin(x) + 3*x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(29) = 58.
time = 4.07, size = 473, normalized size = 14.33

$$\frac{3x\tan^{8}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{12x\tan^{6}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{18x\tan^{4}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{12x\tan^{2}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{3x}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}-\frac{10\tan^{7}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{6\tan^{5}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}-\frac{6\tan^{3}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{10\tan\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a-a*sin(x)**2),x)
```

```
[Out] 3*x*tan(x/2)**8/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 12*x*tan(x/2)**6/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**
6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 18*x*tan(x/2)**4/(8*a*tan(
x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 1
2*x*tan(x/2)**2/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 3*x/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(
x/2)**4 + 32*a*tan(x/2)**2 + 8*a) - 10*tan(x/2)**7/(8*a*tan(x/2)**8 + 32*a*
tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 6*tan(x/2)**5/(8
*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8
*a) - 6*tan(x/2)**3/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4
+ 32*a*tan(x/2)**2 + 8*a) + 10*tan(x/2)/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6
 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a)
```

Giac [A]
time = 0.47, size = 36, normalized size = 1.09

$$\frac{3\,\arctan\left(\tan\left(x\right)\right)}{8\,a}+\frac{\frac{3\,\tan\left(x\right)^{3}}{a}+\frac{5\,\tan\left(x\right)}{a}}{8\,{\left(\tan\left(x\right)^{2}+1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 3/8*arctan(tan(x))/a + 1/8*(3*tan(x)^3/a + 5*tan(x)/a)/(tan(x)^2 + 1)^2
```

Mupad [B]
time = 13.61, size = 25, normalized size = 0.76

$$\frac{\sin(2x)}{4a}+\frac{\sin(4x)}{32a}+\frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a - a*sin(x)^2),x)
```

```
[Out] sin(2*x)/(4*a) + sin(4*x)/(32*a) + (3*x)/(8*a)
```

3.268 $\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a}+\frac{\cos(x)\sin(x)}{2a}$$

[Out] 1/2*x/a+1/2*cos(x)*sin(x)/a

Rubi [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3254, 2715, 8}

$$\frac{x}{2a}+\frac{\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^4/(a - a*Sin[x]^2),x]

[Out] x/(2*a) + (Cos[x]*Sin[x])/(2*a)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^2(x)\,dx}{a}\\ &= \frac{\cos(x)\sin(x)}{2a}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{\cos(x)\sin(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 0.90

$$\frac{\frac{x}{2}+\frac{1}{4}\sin(2x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a - a*Sin[x]^2),x]
[Out] (x/2 + Sin[2*x]/4)/a
```

Maple [A]
time = 0.14, size = 23, normalized size = 1.15

method	result	size
risch	$\frac{x}{2a}+\frac{\sin(2x)}{4a}$	17
default	$\frac{\frac{\tan(x)}{2\left(\tan^2(x)\right)+2}+\frac{\arctan(\tan(x))}{2}}{a}$	23
norman	$\frac{\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}-\frac{\tan\left(\frac{x}{2}\right)}{a}+\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}-\frac{\tan^9\left(\frac{x}{2}\right)}{a}-\frac{x}{2a}-\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2a}-\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{2a}+\frac{x\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{2a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/2*tan(x)/(tan(x)^2+1)+1/2*arctan(tan(x)))
```

Maxima [A]
time = 0.49, size = 21, normalized size = 1.05

$$\frac{x}{2a}+\frac{\tan(x)}{2\left(a\tan(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] 1/2*x/a + 1/2*tan(x)/(a*tan(x)^2 + a)
```

Fricas [A]
time = 0.40, size = 12, normalized size = 0.60

$$\frac{\cos(x)\sin(x)+x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="fricas")
```

[Out] 1/2*(cos(x)*sin(x) + x)/a

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(14) = 28$.
time = 1.46, size = 153, normalized size = 7.65

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}+\frac{2x\tan^2\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}+\frac{x}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}-\frac{2\tan^3\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}+\frac{2\tan\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**4/(a-a*sin(x)**2),x)

[Out] x*tan(x/2)**4/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*x*tan(x/2)**2/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + x/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) - 2*tan(x/2)**3/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*tan(x/2)/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a)

Giac [A]
time = 0.45, size = 24, normalized size = 1.20

$$\frac{\arctan\left(\tan\left(x\right)\right)}{2\,a}+\frac{\tan\left(x\right)}{2\,{\left(\tan\left(x\right)^{2}+1\right)}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="giac")

[Out] 1/2*arctan(tan(x))/a + 1/2*tan(x)/((tan(x)^2 + 1)*a)

Mupad [B]
time = 13.81, size = 13, normalized size = 0.65

$$\frac{2\,x+\sin\left(2\,x\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^4/(a - a*sin(x)^2),x)

[Out] (2*x + sin(2*x))/(4*a)

3.269 $\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a}$$

```
[Out] x/a
```

Rubi [A]
time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 8}

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a - a*Sin[x]^2),x]
```

```
[Out] x/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx &= \frac{\int 1\,dx}{a}\\ &= \frac{x}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a - a*Sin[x]^2),x]
```

```
[Out] x/a
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.14, size = 8, normalized size = 1.60

method	result	size
risch	$\frac{x}{a}$	6
default	$\frac{\arctan(\tan(x))}{a}$	8
norman	$\frac{\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}-\frac{x}{a}-\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*arctan(tan(x))
```

Maxima [A]
time = 0.50, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] x/a
```

Fricas [A]
time = 0.39, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] x/a
```

Sympy [A]
time = 0.45, size = 2, normalized size = 0.40

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a-a*sin(x)**2),x)
```

```
[Out] x/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(5) = 10$.
time = 0.45, size = 14, normalized size = 2.80

$$\frac{\arctan\left(\frac{|a|\tan(x)}{a}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] arctan(abs(a)*tan(x)/a)/abs(a)
```

Mupad [B]
time = 13.81, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a - a*sin(x)^2),x)
```

```
[Out] x/a
```

3.270 $\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tanh^{-1}(\sin(x))}{2a}+\frac{\sec(x)\tan(x)}{2a}$$

[Out] 1/2*arctanh(sin(x))/a+1/2*sec(x)*tan(x)/a

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {3254, 3853, 3855}

$$\frac{\tanh^{-1}(\sin(x))}{2a}+\frac{\tan(x)\sec(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]/(a - a*Sin[x]^2),x]

[Out] ArcTanh[Sin[x]]/(2*a) + (Sec[x]*Tan[x])/(2*a)

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(x)}{a - a\sin^2(x)}\,dx = \frac{\int \sec^3(x)\,dx}{a}$$
$$= \frac{\sec(x)\tan(x)}{2a} + \frac{\int \sec(x)\,dx}{2a}$$
$$= \frac{\tanh^{-1}(\sin(x))}{2a} + \frac{\sec(x)\tan(x)}{2a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.
time = 0.03, size = 45, normalized size = 2.05

$$\frac{-\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right) + \sec(x)\tan(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a - a*Sin[x]^2),x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] + Sec[x]*Tan[x])/(2*a
)
```

Maple [A]
time = 0.16, size = 36, normalized size = 1.64

method	result	size
default	$\frac{-\frac{1}{4(1+\sin(x))}+\frac{\ln(1+\sin(x))}{4}-\frac{1}{4(\sin(x)-1)}-\frac{\ln(\sin(x)-1)}{4}}{a}$	36
norman	$\frac{\frac{\tan\left(\frac{x}{2}\right)}{a}+\frac{\tan^3\left(\frac{x}{2}\right)}{a}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^2} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2a} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{2a}$	56
risch	$-\frac{i\left(e^{3ix}-e^{ix}\right)}{\left(e^{2ix}+1\right)^2 a} - \frac{\ln\left(e^{ix}-i\right)}{2a} + \frac{\ln\left(e^{ix}+i\right)}{2a}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/4/(1+sin(x))+1/4*ln(1+sin(x))-1/4/(sin(x)-1)-1/4*ln(sin(x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.
time = 0.28, size = 37, normalized size = 1.68

$$\frac{\log\left(\sin\left(x\right)+1\right)}{4\,a} - \frac{\log\left(\sin\left(x\right)-1\right)}{4\,a} - \frac{\sin\left(x\right)}{2\,\left(a\sin\left(x\right)^2 - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*log(sin(x) + 1)/a - 1/4*log(sin(x) - 1)/a - 1/2*sin(x)/(a*sin(x)^2 - a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.
time = 0.41, size = 37, normalized size = 1.68

$$\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)+2\sin(x)}{4\,a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/(a*co
s(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sec(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)/(sin(x)**2 - 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.
time = 0.46, size = 38, normalized size = 1.73

$$\frac{\log(\sin(x)+1)}{4\,a}-\frac{\log(-\sin(x)+1)}{4\,a}-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1)/a - 1/4*log(-sin(x) + 1)/a - 1/2*sin(x)/((sin(x)^2 - 1)
*a)
```

Mupad [B]
time = 13.87, size = 25, normalized size = 1.14

$$\frac{\mathrm{atanh}(\sin(x))}{2\,a}+\frac{\sin(x)}{2\,\left(a-a\sin(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a - a*sin(x)^2)),x)
```

```
[Out] atanh(sin(x))/(2*a) + sin(x)/(2*(a - a*sin(x)^2))
```

3.271 $\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\tan(x)}{a}+\frac{\tan^3(x)}{3a}$$

```
[Out] tan(x)/a+1/3*tan(x)^3/a
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3254, 3852}

$$\frac{\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a - a*Sin[x]^2),x]
```

```
[Out] Tan[x]/a + Tan[x]^3/(3*a)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^4(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(x)\right)}{a}\\ &= \frac{\tan(x)}{a}+\frac{\tan^3(x)}{3a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.17

$$\frac{\frac{2\tan(x)}{3}+\frac{1}{3}\sec^2(x)\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a - a*Sin[x]^2),x]
[Out] ((2*Tan[x])/3 + (Sec[x]^2*Tan[x])/3)/a
```

Maple [A]
time = 0.16, size = 14, normalized size = 0.78

method	result	size
default	$\frac{\frac{\left(\tan^3(x)\right)}{3}+\tan(x)}{a}$	14
risch	$\frac{4i(3\,e^{2ix}+1)}{3(e^{2ix}+1)^3a}$	25
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/3*tan(x)^3+tan(x))
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.78

$$\frac{\tan(x)^3+3\tan(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] 1/3*(tan(x)^3 + 3*tan(x))/a
```

Fricas [A]
time = 0.37, size = 19, normalized size = 1.06

$$\frac{\left(2\cos(x)^2+1\right)\sin(x)}{3\,a\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)/(a*cos(x)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^2(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)**2/(sin(x)**2 - 1), x)/a
```

Giac [**A**]
time = 0.46, size = 14, normalized size = 0.78

$$\frac{\tan(x)^3 + 3\tan(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/3*(tan(x)^3 + 3*tan(x))/a
```

Mupad [**B**]
time = 13.87, size = 13, normalized size = 0.72

$$\frac{\tan(x)\,\left(\tan(x)^2+3\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a - a*sin(x)^2)),x)
```

```
[Out] (tan(x)*(tan(x)^2 + 3))/(3*a)
```

3.272 $\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\tan(x)}{a}+\frac{2\tan^3(x)}{3a}+\frac{\tan^5(x)}{5a}$$

```
[Out] tan(x)/a+2/3*tan(x)^3/a+1/5*tan(x)^5/a
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3254, 3852}

$$\frac{\tan^5(x)}{5a}+\frac{2\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a - a*Sin[x]^2),x]
```

```
[Out] Tan[x]/a + (2*Tan[x]^3)/(3*a) + Tan[x]^5/(5*a)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^6(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(x)\right)}{a}\\ &= \frac{\tan(x)}{a}+\frac{2\tan^3(x)}{3a}+\frac{\tan^5(x)}{5a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.07

$$\frac{\frac{8\tan(x)}{15}+\frac{4}{15}\sec^2(x)\tan(x)+\frac{1}{5}\sec^4(x)\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a - a*Sin[x]^2),x]
[Out] ((8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5)/a
```

Maple [A]
time = 0.15, size = 20, normalized size = 0.69

method	result	size
default	$\frac{\frac{(\tan^5(x))}{5}+\frac{2(\tan^3(x))}{3}+\tan(x)}{a}$	20
risch	$\frac{16i(10\,\mathrm{e}^{4ix}+5\,\mathrm{e}^{2ix}+1)}{15(\mathrm{e}^{2ix}+1)^5 a}$	32
norman	$\frac{-\frac{2\tan(\frac{x}{2})}{a}+\frac{8(\tan^3(\frac{x}{2}))}{3a}-\frac{116(\tan^5(\frac{x}{2}))}{15a}+\frac{8(\tan^7(\frac{x}{2}))}{3a}-\frac{2(\tan^9(\frac{x}{2}))}{a}}{(\tan^2(\frac{x}{2})-1)^5}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.76

$$\frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a
```

Fricas [A]
time = 0.38, size = 25, normalized size = 0.86

$$\frac{\left(8\,\cos(x)^4+4\,\cos(x)^2+3\right)\sin(x)}{15\,a\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a*cos(x)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^4(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)**4/(sin(x)**2 - 1), x)/a
```

Giac [A]
time = 0.46, size = 22, normalized size = 0.76

$$\frac{3\,\tan(x)^5 + 10\,\tan(x)^3 + 15\,\tan(x)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a
```

Mupad [B]
time = 13.92, size = 21, normalized size = 0.72

$$\frac{\tan(x)\,\left(3\tan(x)^4 + 10\tan(x)^2 + 15\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a - a*sin(x)^2)),x)
```

```
[Out] (tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a)
```

3.273 $\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=29

$$\frac{\sin(x)}{a^2}-\frac{2\sin^3(x)}{3a^2}+\frac{\sin^5(x)}{5a^2}$$

```
[Out] sin(x)/a^2-2/3*sin(x)^3/a^2+1/5*sin(x)^5/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3254, 2713}

$$\frac{\sin^5(x)}{5a^2}-\frac{2\sin^3(x)}{3a^2}+\frac{\sin(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^9/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Sin[x]/a^2 - (2*Sin[x]^3)/(3*a^2) + Sin[x]^5/(5*a^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx &= \frac{\int \cos^5(x)\,dx}{a^2}\\ &= -\frac{\text{Subst}\left(\int (1-2x^2+x^4)\,dx,x,-\sin(x)\right)}{a^2}\\ &= \frac{\sin(x)}{a^2}-\frac{2\sin^3(x)}{3a^2}+\frac{\sin^5(x)}{5a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 0.93

$$\frac{\frac{5\sin(x)}{8}+\frac{5}{48}\sin(3x)+\frac{1}{80}\sin(5x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^9/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80)/a^2
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.69

method	result	size
default	$\frac{\frac{\left(\sin^5(x)\right)}{5}-\frac{2\left(\sin^3(x)\right)}{3}+\sin(x)}{a^2}$	20
risch	$\frac{5\sin(x)}{8a^2}+\frac{\sin(5x)}{80a^2}+\frac{5\sin(3x)}{48a^2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/5*sin(x)^5-2/3*sin(x)^3+sin(x))
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.76

$$\frac{3\sin(x)^5-10\sin(x)^3+15\sin(x)}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a^2
```

Fricas [A]
time = 0.43, size = 21, normalized size = 0.72

$$\frac{\left(3\cos(x)^4+4\cos(x)^2+8\right)\sin(x)}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)/a^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(27) = 54$.
time = 32.98, size = 362, normalized size = 12.48

$$\frac{30\tan^{9}\left(\frac{x}{2}\right)}{15a^{2}\tan^{10}\left(\frac{x}{2}\right)+75a^{2}\tan^{8}\left(\frac{x}{2}\right)+150a^{2}\tan^{6}\left(\frac{x}{2}\right)+150a^{2}\tan^{4}\left(\frac{x}{2}\right)+75a^{2}\tan^{2}\left(\frac{x}{2}\right)+15a^{2}}+\frac{40\tan^{7}\left(\frac{x}{2}\right)}{15a^{2}\tan^{10}\left(\frac{x}{2}\right)+75a^{2}\tan^{8}\left(\frac{x}{2}\right)+150a^{2}\tan^{6}\left(\frac{x}{2}\right)+150a^{2}\tan^{4}\left(\frac{x}{2}\right)+75a^{2}\tan^{2}\left(\frac{x}{2}\right)+15a^{2}}+\frac{116\tan^{5}\left(\frac{x}{2}\right)}{15a^{2}\tan^{10}\left(\frac{x}{2}\right)+75a^{2}\tan^{8}\left(\frac{x}{2}\right)+150a^{2}\tan^{6}\left(\frac{x}{2}\right)+150a^{2}\tan^{4}\left(\frac{x}{2}\right)+75a^{2}\tan^{2}\left(\frac{x}{2}\right)+15a^{2}}+\frac{40\tan^{3}\left(\frac{x}{2}\right)}{15a^{2}\tan^{10}\left(\frac{x}{2}\right)+75a^{2}\tan^{8}\left(\frac{x}{2}\right)+150a^{2}\tan^{6}\left(\frac{x}{2}\right)+150a^{2}\tan^{4}\left(\frac{x}{2}\right)+75a^{2}\tan^{2}\left(\frac{x}{2}\right)+15a^{2}}+\frac{30\tan\left(\frac{x}{2}\right)}{15a^{2}\tan^{10}\left(\frac{x}{2}\right)+75a^{2}\tan^{8}\left(\frac{x}{2}\right)+150a^{2}\tan^{6}\left(\frac{x}{2}\right)+150a^{2}\tan^{4}\left(\frac{x}{2}\right)+75a^{2}\tan^{2}\left(\frac{x}{2}\right)+15a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**9/(a-a*sin(x)**2)**2,x)
```

```
[Out] 30*tan(x/2)**9/(15*a**2*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x
/2)**6 + 150*a**2*tan(x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 40*tan(x/2
)**7/(15*a**2*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 1
50*a**2*tan(x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 116*tan(x/2)**5/(15*
a**2*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 150*a**2*t
an(x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 40*tan(x/2)**3/(15*a**2*tan(x
/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 150*a**2*tan(x/2)**4
 + 75*a**2*tan(x/2)**2 + 15*a**2) + 30*tan(x/2)/(15*a**2*tan(x/2)**10 + 75*
a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 150*a**2*tan(x/2)**4 + 75*a**2*ta
n(x/2)**2 + 15*a**2)
```

Giac [A]
time = 0.41, size = 22, normalized size = 0.76

$$\frac{3\sin\left(x\right)^{5}-10\sin\left(x\right)^{3}+15\sin\left(x\right)}{15\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a^2
```

Mupad [B]
time = 14.00, size = 19, normalized size = 0.66

$$\frac{\frac{\sin(x)^{5}}{5}-\frac{2\sin(x)^{3}}{3}+\sin\left(x\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a - a*sin(x)^2)^2,x)
```

```
[Out] (sin(x) - (2*sin(x)^3)/3 + sin(x)^5/5)/a^2
```

3.274 $\int \frac{\cos^7(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=18

$$\frac{\sin(x)}{a^2}-\frac{\sin^3(x)}{3a^2}$$

```
[Out] sin(x)/a^2-1/3*sin(x)^3/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3254, 2713}

$$\frac{\sin(x)}{a^2}-\frac{\sin^3(x)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Sin[x]/a^2 - Sin[x]^3/(3*a^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(x)}{(a-a\sin^2(x))^2}\,dx&=\frac{\int \cos^3(x)\,dx}{a^2}\\&=-\frac{\operatorname{Subst}\left(\int (1-x^2)\,dx,x,-\sin(x)\right)}{a^2}\\&=\frac{\sin(x)}{a^2}-\frac{\sin^3(x)}{3a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.06

$$\frac{\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((3*Sin[x])/4 + Sin[3*x]/12)/a^2
```

Maple [A]
time = 0.09, size = 14, normalized size = 0.78

method	result	size
default	$\frac{-\frac{\left(\sin^3(x)\right)}{3}+\sin(x)}{a^2}$	14
risch	$\frac{3\sin(x)}{4a^2}+\frac{\sin(3x)}{12a^2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/3*sin(x)^3+sin(x))
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.78

$$-\frac{\sin\left(x\right)^{3}-3\,\sin\left(x\right)}{3\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a^2
```

Fricas [A]
time = 0.38, size = 13, normalized size = 0.72

$$\frac{\left(\cos\left(x\right)^{2}+2\right)\sin\left(x\right)}{3\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)/a^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(15) = 30$.
time = 15.19, size = 144, normalized size = 8.00

$$\frac{6\tan^5\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)+9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+3a^2}+\frac{4\tan^3\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)+9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+3a^2}+\frac{6\tan\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)+9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a-a*sin(x)**2)**2,x)
```

```
[Out] 6*tan(x/2)**5/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)**4 + 9*a**2*tan(x/2)**2
 + 3*a**2) + 4*tan(x/2)**3/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)**4 + 9*a**
2*tan(x/2)**2 + 3*a**2) + 6*tan(x/2)/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)*
*4 + 9*a**2*tan(x/2)**2 + 3*a**2)
```

Giac [A]
time = 0.44, size = 14, normalized size = 0.78

$$-\frac{\sin(x)^3-3\sin(x)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a^2
```

Mupad [B]
time = 0.04, size = 16, normalized size = 0.89

$$\frac{3\sin(x)-\sin(x)^3}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a - a*sin(x)^2)^2,x)
```

```
[Out] (3*sin(x) - sin(x)^3)/(3*a^2)
```

3.275 $\int \frac{\cos^5(x)}{(a - a\sin^2(x))^2} \, dx$

Optimal. Leaf size=6

$$\frac{\sin(x)}{a^2}$$

[Out] sin(x)/a^2

Rubi [A]
time = 0.03, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2717}

$$\frac{\sin(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^5/(a - a*Sin[x]^2)^2,x]

[Out] Sin[x]/a^2

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^5(x)}{(a - a\sin^2(x))^2} \, dx &= \frac{\int \cos(x) \, dx}{a^2} \\ &= \frac{\sin(x)}{a^2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a - a*Sin[x]^2)^2,x]
[Out] Sin[x]/a^2
```

Maple [A]
time = 0.16, size = 7, normalized size = 1.17

method	result	size
derivativedivides	$\frac{\sin(x)}{a^2}$	7
default	$\frac{\sin(x)}{a^2}$	7
risch	$\frac{\sin(x)}{a^2}$	7
norman	$\frac{-\frac{2\tan(\frac{x}{2})}{a}-\frac{2(\tan^3(\frac{x}{2}))}{a}+\frac{6(\tan^5(\frac{x}{2}))}{a}+\frac{6(\tan^7(\frac{x}{2}))}{a}-\frac{6(\tan^9(\frac{x}{2}))}{a}-\frac{6(\tan^{11}(\frac{x}{2}))}{a}+\frac{2(\tan^{13}(\frac{x}{2}))}{a}+\frac{2(\tan^{15}(\frac{x}{2}))}{a}}{(1+\tan^2(\frac{x}{2}))^5 a(\tan^2(\frac{x}{2})-1)^3}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
[Out] sin(x)/a^2
```

Maxima [A]
time = 0.27, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] sin(x)/a^2
```

Fricas [A]
time = 0.38, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="fricas")
[Out] sin(x)/a^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.
time = 6.60, size = 19, normalized size = 3.17

$$\frac{2\tan\left(\frac{x}{2}\right)}{a^2\tan^2\left(\frac{x}{2}\right)+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a-a*sin(x)**2)**2,x)
```

```
[Out] 2*tan(x/2)/(a**2*tan(x/2)**2 + a**2)
```

Giac [A]
time = 0.43, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] sin(x)/a^2
```

Mupad [B]
time = 0.02, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a - a*sin(x)^2)^2,x)
```

```
[Out] sin(x)/a^2
```

3.276 $\int \frac{\cos^3(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=7

$$\frac{\tanh^{-1}(\sin(x))}{a^2}$$

[Out] arctanh(sin(x))/a^2

Rubi [A]
time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 3855}

$$\frac{\tanh^{-1}(\sin(x))}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3/(a - a*Sin[x]^2)^2,x]

[Out] ArcTanh[Sin[x]]/a^2

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{(a-a\sin^2(x))^2}\,dx &= \frac{\int \sec(x)\,dx}{a^2}\\ &= \frac{\tanh^{-1}(\sin(x))}{a^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(7) = 14.
time = 0.00, size = 37, normalized size = 5.29

$$\frac{-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a^2
```

Maple [A]
time = 0.16, size = 8, normalized size = 1.14

method	result	size
default	$\frac{\operatorname{arctanh}(\sin(x))}{a^2}$	8
norman	$\frac{\ln(\tan(\frac{x}{2})+1)}{a^2}-\frac{\ln(\tan(\frac{x}{2})-1)}{a^2}$	25
risch	$\frac{\ln(e^{ix}+i)}{a^2}-\frac{\ln(e^{ix}-i)}{a^2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(sin(x))/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.
time = 0.27, size = 21, normalized size = 3.00

$$\frac{\log\left(\sin\left(x\right)+1\right)}{2\,a^2}-\frac{\log\left(\sin\left(x\right)-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1)/a^2 - 1/2*log(sin(x) - 1)/a^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(7) = 14$.
time = 0.38, size = 20, normalized size = 2.86

$$\frac{\log\left(\sin\left(x\right)+1\right)-\log\left(-\sin\left(x\right)+1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(x) + 1) - log(-sin(x) + 1))/a^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.
time = 2.32, size = 22, normalized size = 3.14

$$-\frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a-a*sin(x)**2)**2,x)
```

```
[Out] -log(tan(x/2) - 1)/a**2 + log(tan(x/2) + 1)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(7) = 14.
time = 0.43, size = 23, normalized size = 3.29

$$\frac{\log\left(\sin\left(x\right)+1\right)}{2\,a^2}-\frac{\log\left(-\sin\left(x\right)+1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x) + 1)/a^2 - 1/2*log(-sin(x) + 1)/a^2
```

Mupad [B]
time = 0.06, size = 7, normalized size = 1.00

$$\frac{\mathrm{atanh}(\sin\left(x\right))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a - a*sin(x)^2)^2,x)
```

```
[Out] atanh(sin(x))/a^2
```

3.277 $\int \frac{\cos(x)}{(a-a\sin^2(x))^2} \, dx$

Optimal. Leaf size=22

$$\frac{\tanh^{-1}(\sin(x))}{2a^2}+\frac{\sec(x)\tan(x)}{2a^2}$$

```
[Out] 1/2*arctanh(sin(x))/a^2+1/2*sec(x)*tan(x)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3254, 3853, 3855}

$$\frac{\tanh^{-1}(\sin(x))}{2a^2}+\frac{\tan(x)\sec(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[x]]/(2*a^2) + (Sec[x]*Tan[x])/(2*a^2)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\int \sec^3(x)\,dx}{a^2}$$
$$= \frac{\sec(x)\tan(x)}{2a^2} + \frac{\int \sec(x)\,dx}{2a^2}$$
$$= \frac{\tanh^{-1}(\sin(x))}{2a^2} + \frac{\sec(x)\tan(x)}{2a^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.
time = 0.01, size = 45, normalized size = 2.05

$$\frac{-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sec(x)\tan(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] + Sec[x]*Tan[x])/(2*a
^2)
```

Maple [A]
time = 0.13, size = 36, normalized size = 1.64

method	result	size
derivativedivides	$\frac{-\frac{1}{4(1+\sin(x))}+\frac{\ln(1+\sin(x))}{4}-\frac{1}{4(\sin(x)-1)}-\frac{\ln(\sin(x)-1)}{4}}{a^2}$	36
default	$\frac{-\frac{1}{4(1+\sin(x))}+\frac{\ln(1+\sin(x))}{4}-\frac{1}{4(\sin(x)-1)}-\frac{\ln(\sin(x)-1)}{4}}{a^2}$	36
risch	$-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2a^2}-\frac{\ln(e^{ix}-i)}{2a^2}+\frac{\ln(e^{ix}+i)}{2a^2}$	58
norman	$\frac{\frac{\tan^5(\frac{x}{2})}{a}+\frac{\tan^7(\frac{x}{2})}{a}-\frac{\tan(\frac{x}{2})}{a}-\frac{\tan^3(\frac{x}{2})}{a}}{(1+\tan^2(\frac{x}{2}))a(\tan^2(\frac{x}{2})-1)^3}-\frac{\ln(\tan(\frac{x}{2})-1)}{2a^2}+\frac{\ln(\tan(\frac{x}{2})+1)}{2a^2}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4/(1+sin(x))+1/4*ln(1+sin(x))-1/4/(sin(x)-1)-1/4*ln(sin(x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(18) = 36.
time = 0.29, size = 41, normalized size = 1.86

$$-\frac{\sin(x)}{2\left(a^2\sin(x)^2-a^2\right)}+\frac{\log(\sin(x)+1)}{4\,a^2}-\frac{\log(\sin(x)-1)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)/(a^2*sin(x)^2 - a^2) + 1/4*log(sin(x) + 1)/a^2 - 1/4*log(sin(x)
 - 1)/a^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.
time = 0.39, size = 37, normalized size = 1.68

$$\frac{\cos\left(x\right)^{2}\log\left(\sin\left(x\right)+1\right)-\cos\left(x\right)^{2}\log\left(-\sin\left(x\right)+1\right)+2\,\sin\left(x\right)}{4\,a^{2}\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/(a^2*
cos(x)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(20) = 40$.
time = 0.24, size = 117, normalized size = 5.32

$$-\frac{\log\left(\sin\left(x\right)-1\right)\sin^{2}\left(x\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}+\frac{\log\left(\sin\left(x\right)-1\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}+\frac{\log\left(\sin\left(x\right)+1\right)\sin^{2}\left(x\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}-\frac{\log\left(\sin\left(x\right)+1\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}-\frac{2\sin\left(x\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)**2)**2,x)
```

```
[Out] -log(sin(x) - 1)*sin(x)**2/(4*a**2*sin(x)**2 - 4*a**2) + log(sin(x) - 1)/(4
*a**2*sin(x)**2 - 4*a**2) + log(sin(x) + 1)*sin(x)**2/(4*a**2*sin(x)**2 - 4
*a**2) - log(sin(x) + 1)/(4*a**2*sin(x)**2 - 4*a**2) - 2*sin(x)/(4*a**2*sin
(x)**2 - 4*a**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.
time = 0.45, size = 38, normalized size = 1.73

$$\frac{\log\left(\sin\left(x\right)+1\right)}{4\,a^{2}}-\frac{\log\left(-\sin\left(x\right)+1\right)}{4\,a^{2}}-\frac{\sin\left(x\right)}{2\,\left(\sin\left(x\right)^{2}-1\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1)/a^2 - 1/4*log(-sin(x) + 1)/a^2 - 1/2*sin(x)/((sin(x)^2
- 1)*a^2)
```

Mupad [B]
time = 0.08, size = 30, normalized size = 1.36

$$\frac{\operatorname{atanh}(\sin(x))}{2a^2} - \frac{\sin(x)}{2\left(a^2\sin(x)^2 - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a - a*sin(x)^2)^2,x)
[Out] atanh(sin(x))/(2*a^2) - sin(x)/(2*(a^2*sin(x)^2 - a^2))
```

3.278 $\int \frac{\sec(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=35

$$\frac{3\tanh^{-1}(\sin(x))}{8a^2}+\frac{3\sec(x)\tan(x)}{8a^2}+\frac{\sec^3(x)\tan(x)}{4a^2}$$

```
[Out] 3/8*arctanh(sin(x))/a^2+3/8*sec(x)*tan(x)/a^2+1/4*sec(x)^3*tan(x)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3254, 3853, 3855}

$$\frac{3\tanh^{-1}(\sin(x))}{8a^2}+\frac{\tan(x)\sec^3(x)}{4a^2}+\frac{3\tan(x)\sec(x)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (3*ArcTanh[Sin[x]])/(8*a^2) + (3*Sec[x]*Tan[x])/(8*a^2) + (Sec[x]^3*Tan[x])
/(4*a^2)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(x)}{(a - a\sin^2(x))^2}\,dx = \frac{\int \sec^5(x)\,dx}{a^2}$$
$$= \frac{\sec^3(x)\tan(x)}{4a^2} + \frac{3\int \sec^3(x)\,dx}{4a^2}$$
$$= \frac{3\sec(x)\tan(x)}{8a^2} + \frac{\sec^3(x)\tan(x)}{4a^2} + \frac{3\int \sec(x)\,dx}{8a^2}$$
$$= \frac{3\tanh^{-1}(\sin(x))}{8a^2} + \frac{3\sec(x)\tan(x)}{8a^2} + \frac{\sec^3(x)\tan(x)}{4a^2}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 1.74

$$\frac{-6\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + 6\log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right) + \frac{1}{2}\sec^4(x)(11\sin(x) + 3\sin(3x))}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4*(11*S
in[x] + 3*Sin[3*x]))/2)/(16*a^2)
```

Maple [A]
time = 0.16, size = 52, normalized size = 1.49

method	result	size
default	$\frac{-\frac{1}{16(1+\sin(x))^2}-\frac{3}{16(1+\sin(x))}+\frac{3\ln(1+\sin(x))}{16}+\frac{1}{16(\sin(x)-1)^2}-\frac{3}{16(\sin(x)-1)}-\frac{3\ln(\sin(x)-1)}{16}}{a^2}$	52
risch	$-\frac{i(3\,e^{7ix}+11\,e^{5ix}-11\,e^{3ix}-3\,e^{ix})}{4(e^{2ix}+1)^4a^2}-\frac{3\ln(e^{ix}-i)}{8a^2}+\frac{3\ln(e^{ix}+i)}{8a^2}$	74
norman	$\frac{\frac{5\tan(\frac{x}{2})}{4a}+\frac{3(\tan^3(\frac{x}{2}))}{4a}+\frac{3(\tan^5(\frac{x}{2}))}{4a}+\frac{5(\tan^7(\frac{x}{2}))}{4a}}{(\tan^2(\frac{x}{2})-1)^4a}-\frac{3\ln(\tan(\frac{x}{2})-1)}{8a^2}+\frac{3\ln(\tan(\frac{x}{2})+1)}{8a^2}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/16/(1+sin(x))^2-3/16/(1+sin(x))+3/16*ln(1+sin(x))+1/16/(sin(x)-1)
^2-3/16/(sin(x)-1)-3/16*ln(sin(x)-1))
```

Maxima [A]
time = 0.29, size = 57, normalized size = 1.63

$$-\frac{3\,\sin(x)^3 - 5\,\sin(x)}{8\left(a^2\sin(x)^4 - 2\,a^2\sin(x)^2 + a^2\right)} + \frac{3\,\log(\sin(x) + 1)}{16\,a^2} - \frac{3\,\log(\sin(x) - 1)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(a^2*sin(x)^4 - 2*a^2*sin(x)^2 + a^2) + 3/16*l
og(sin(x) + 1)/a^2 - 3/16*log(sin(x) - 1)/a^2
```

Fricas [A]
time = 0.39, size = 46, normalized size = 1.31

$$\frac{3 \cos\left(x\right)^{4} \log\left(\sin\left(x\right) + 1\right) - 3 \cos\left(x\right)^{4} \log\left(-\sin\left(x\right) + 1\right) + 2 \left(3 \cos\left(x\right)^{2} + 2\right) \sin\left(x\right)}{16 a^{2} \cos\left(x\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(x
)^2 + 2)*sin(x))/(a^2*cos(x)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(x\right)}{\sin^{4}\left(x\right) - 2\sin^{2}\left(x\right) + 1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

Giac [A]
time = 0.45, size = 47, normalized size = 1.34

$$\frac{3 \log\left(\sin\left(x\right) + 1\right)}{16 a^{2}} - \frac{3 \log\left(-\sin\left(x\right) + 1\right)}{16 a^{2}} - \frac{3 \sin\left(x\right)^{3} - 5 \sin\left(x\right)}{8 \left(\sin\left(x\right)^{2} - 1\right)^{2} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 3/16*log(sin(x) + 1)/a^2 - 3/16*log(-sin(x) + 1)/a^2 - 1/8*(3*sin(x)^3 - 5*
sin(x))/((sin(x)^2 - 1)^2*a^2)
```

Mupad [B]
time = 13.98, size = 31, normalized size = 0.89

$$\frac{3\,\mathrm{atanh}(\sin\left(x\right))}{8\,a^2} + \frac{3\,\sin\left(x\right)}{8\,a^2\cos\left(x\right)^2} + \frac{\sin\left(x\right)}{4\,a^2\cos\left(x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a - a*sin(x)^2)^2),x)
```

```
[Out] (3*atanh(sin(x)))/(8*a^2) + (3*sin(x))/(8*a^2*cos(x)^2) + sin(x)/(4*a^2*cos
(x)^4)
```

3.279 $\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=33

$$\frac{3x}{8a^2}+\frac{3\cos(x)\sin(x)}{8a^2}+\frac{\cos^3(x)\sin(x)}{4a^2}$$

[Out] 3/8*x/a^2+3/8*cos(x)*sin(x)/a^2+1/4*cos(x)^3*sin(x)/a^2

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {3254, 2715, 8}

$$\frac{3x}{8a^2}+\frac{\sin(x)\cos^3(x)}{4a^2}+\frac{3\sin(x)\cos(x)}{8a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^8/(a - a*Sin[x]^2)^2,x]

[Out] (3*x)/(8*a^2) + (3*Cos[x]*Sin[x])/(8*a^2) + (Cos[x]^3*Sin[x])/(4*a^2)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\int \cos^4(x)\,dx}{a^2}$$
$$= \frac{\cos^3(x)\sin(x)}{4a^2} + \frac{3\int \cos^2(x)\,dx}{4a^2}$$
$$= \frac{3\cos(x)\sin(x)}{8a^2} + \frac{\cos^3(x)\sin(x)}{4a^2} + \frac{3\int 1\,dx}{8a^2}$$
$$= \frac{3x}{8a^2} + \frac{3\cos(x)\sin(x)}{8a^2} + \frac{\cos^3(x)\sin(x)}{4a^2}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.79

$$\frac{\frac{3x}{8} + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^8/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)/a^2
```

Maple [A]
time = 0.10, size = 35, normalized size = 1.06

method	result	size
risch	$\frac{3x}{8a^2} + \frac{\sin(4x)}{32a^2} + \frac{\sin(2x)}{4a^2}$	26
default	$\frac{\frac{\tan(x)}{4(\tan^2(x)+1)^2} + \frac{3\tan(x)}{8(\tan^2(x)+1)} + \frac{3\arctan(\tan(x))}{8}}{a^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^8/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/4*tan(x)/(tan(x)^2+1)^2+3/8*tan(x)/(tan(x)^2+1)+3/8*arctan(tan(x))
)
```

Maxima [A]
time = 0.54, size = 43, normalized size = 1.30

$$\frac{3\tan(x)^3 + 5\tan(x)}{8\left(a^2\tan(x)^4 + 2a^2\tan(x)^2 + a^2\right)} + \frac{3x}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(3*tan(x)^3 + 5*tan(x))/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2) + 3/8*x/a
^2
```

Fricas [A]
time = 0.39, size = 23, normalized size = 0.70

$$\frac{\left(2\,\cos\left(x\right)^{3}+3\,\cos\left(x\right)\right)\sin\left(x\right)+3\,x}{8\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((2*cos(x)^3 + 3*cos(x))*sin(x) + 3*x)/a^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(34) = 68$.
time = 23.25, size = 549, normalized size = 16.64

$$\frac{3x\tan^{8}\left(\frac{x}{2}\right)}{8a^{2}\tan^{8}\left(\frac{x}{2}\right)+32a^{2}\tan^{6}\left(\frac{x}{2}\right)+48a^{2}\tan^{4}\left(\frac{x}{2}\right)+32a^{2}\tan^{2}\left(\frac{x}{2}\right)+8a^{2}}+\frac{12x\tan^{6}\left(\frac{x}{2}\right)}{8a^{2}\tan^{8}\left(\frac{x}{2}\right)+32a^{2}\tan^{6}\left(\frac{x}{2}\right)+48a^{2}\tan^{4}\left(\frac{x}{2}\right)+32a^{2}\tan^{2}\left(\frac{x}{2}\right)+8a^{2}}+\frac{18x\tan^{4}\left(\frac{x}{2}\right)}{8a^{2}\tan^{8}\left(\frac{x}{2}\right)+32a^{2}\tan^{6}\left(\frac{x}{2}\right)+48a^{2}\tan^{4}\left(\frac{x}{2}\right)+32a^{2}\tan^{2}\left(\frac{x}{2}\right)+8a^{2}}+\frac{12x\tan^{2}\left(\frac{x}{2}\right)}{8a^{2}\tan^{8}\left(\frac{x}{2}\right)+32a^{2}\tan^{6}\left(\frac{x}{2}\right)+48a^{2}\tan^{4}\left(\frac{x}{2}\right)+32a^{2}\tan^{2}\left(\frac{x}{2}\right)+8a^{2}}+\frac{3x}{8a^{2}\tan^{8}\left(\frac{x}{2}\right)+32a^{2}\tan^{6}\left(\frac{x}{2}\right)+48a^{2}\tan^{4}\left(\frac{x}{2}\right)+32a^{2}\tan^{2}\left(\frac{x}{2}\right)+8a^{2}}-\frac{10\tan^{7}\left(\frac{x}{2}\right)}{8a^{2}\tan^{8}\left(\frac{x}{2}\right)+32a^{2}\tan^{6}\left(\frac{x}{2}\right)+48a^{2}\tan^{4}\left(\frac{x}{2}\right)+32a^{2}\tan^{2}\left(\frac{x}{2}\right)+8a^{2}}+\frac{6\tan^{5}\left(\frac{x}{2}\right)}{8a^{2}\tan^{8}\left(\frac{x}{2}\right)+32a^{2}\tan^{6}\left(\frac{x}{2}\right)+48a^{2}\tan^{4}\left(\frac{x}{2}\right)+32a^{2}\tan^{2}\left(\frac{x}{2}\right)+8a^{2}}-\frac{6\tan^{3}\left(\frac{x}{2}\right)}{8a^{2}\tan^{8}\left(\frac{x}{2}\right)+32a^{2}\tan^{6}\left(\frac{x}{2}\right)+48a^{2}\tan^{4}\left(\frac{x}{2}\right)+32a^{2}\tan^{2}\left(\frac{x}{2}\right)+8a^{2}}+\frac{10\tan\left(\frac{x}{2}\right)}{8a^{2}\tan^{8}\left(\frac{x}{2}\right)+32a^{2}\tan^{6}\left(\frac{x}{2}\right)+48a^{2}\tan^{4}\left(\frac{x}{2}\right)+32a^{2}\tan^{2}\left(\frac{x}{2}\right)+8a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**8/(a-a*sin(x)**2)**2,x)
```

```
[Out] 3*x*tan(x/2)**8/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2
)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 12*x*tan(x/2)**6/(8*a**2*tan(x/2)**8
 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2
) + 18*x*tan(x/2)**4/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*ta
n(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 12*x*tan(x/2)**2/(8*a**2*tan(x/
2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8
*a**2) + 3*x/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**
4 + 32*a**2*tan(x/2)**2 + 8*a**2) - 10*tan(x/2)**7/(8*a**2*tan(x/2)**8 + 32
*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 6
*tan(x/2)**5/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**
4 + 32*a**2*tan(x/2)**2 + 8*a**2) - 6*tan(x/2)**3/(8*a**2*tan(x/2)**8 + 32*
a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 10
*tan(x/2)/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 +
 32*a**2*tan(x/2)**2 + 8*a**2)
```

Giac [A]
time = 0.45, size = 31, normalized size = 0.94

$$\frac{3\,x}{8\,a^{2}}+\frac{3\,\tan\left(x\right)^{3}+5\,\tan\left(x\right)}{8\,{\left(\tan\left(x\right)^{2}+1\right)}^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="giac")
[Out] 3/8*x/a^2 + 1/8*(3*tan(x)^3 + 5*tan(x))/((tan(x)^2 + 1)^2*a^2)
```

Mupad [B]
time = 13.82, size = 29, normalized size = 0.88

$$\frac{3\,x}{8\,a^2}+\frac{3\,\cos\left(x\right)\,\sin\left(x\right)^3}{8\,a^2}+\frac{5\cos\left(x\right)^3\,\sin\left(x\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^8/(a - a*sin(x)^2)^2,x)
[Out] (3*x)/(8*a^2) + (3*cos(x)*sin(x)^3)/(8*a^2) + (5*cos(x)^3*sin(x))/(8*a^2)
```

3.280 $\int \frac{\cos^6(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a^2}+\frac{\cos(x)\sin(x)}{2a^2}$$

```
[Out] 1/2*x/a^2+1/2*cos(x)*sin(x)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3254, 2715, 8}

$$\frac{x}{2a^2}+\frac{\sin(x)\cos(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a - a*Sin[x]^2)^2,x]
```

```
[Out] x/(2*a^2) + (Cos[x]*Sin[x])/(2*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(x)}{(a-a\sin^2(x))^2}\,dx &= \frac{\int \cos^2(x)\,dx}{a^2}\\ &= \frac{\cos(x)\sin(x)}{2a^2}+\frac{\int 1\,dx}{2a^2}\\ &= \frac{x}{2a^2}+\frac{\cos(x)\sin(x)}{2a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 0.90

$$\frac{\frac{x}{2}+\frac{1}{4}\sin(2x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (x/2 + Sin[2*x]/4)/a^2
```

Maple [A]
time = 0.09, size = 23, normalized size = 1.15

method	result	size
risch	$\frac{x}{2a^2}+\frac{\sin(2x)}{4a^2}$	17
default	$\frac{\frac{\tan(x)}{2\left(\tan^2(x)\right)+2}+\frac{\arctan(\tan(x))}{2}}{a^2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*tan(x)/(tan(x)^2+1)+1/2*arctan(tan(x)))
```

Maxima [A]
time = 0.52, size = 25, normalized size = 1.25

$$\frac{\tan(x)}{2\left(a^2\tan(x)^2+a^2\right)}+\frac{x}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)/(a^2*tan(x)^2 + a^2) + 1/2*x/a^2
```

Fricas [A]
time = 0.38, size = 12, normalized size = 0.60

$$\frac{\cos(x)\sin(x)+x}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)*sin(x) + x)/a^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(17) = 34$.
time = 10.24, size = 178, normalized size = 8.90

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right)+4a^2\tan^2\left(\frac{x}{2}\right)+2a^2}+\frac{2x\tan^2\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right)+4a^2\tan^2\left(\frac{x}{2}\right)+2a^2}+\frac{x}{2a^2\tan^4\left(\frac{x}{2}\right)+4a^2\tan^2\left(\frac{x}{2}\right)+2a^2}-\frac{2\tan^3\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right)+4a^2\tan^2\left(\frac{x}{2}\right)+2a^2}+\frac{2\tan\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right)+4a^2\tan^2\left(\frac{x}{2}\right)+2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a-a*sin(x)**2)**2,x)
```

```
[Out] x*tan(x/2)**4/(2*a**2*tan(x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) + 2*x*tan(
x/2)**2/(2*a**2*tan(x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) + x/(2*a**2*tan(
x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) - 2*tan(x/2)**3/(2*a**2*tan(x/2)**4
+ 4*a**2*tan(x/2)**2 + 2*a**2) + 2*tan(x/2)/(2*a**2*tan(x/2)**4 + 4*a**2*ta
n(x/2)**2 + 2*a**2)
```

Giac [A]
time = 0.45, size = 22, normalized size = 1.10

$$\frac{x}{2\,a^2}+\frac{\tan\left(x\right)}{2\,\left(\tan\left(x\right)^2+1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/a^2 + 1/2*tan(x)/((tan(x)^2 + 1)*a^2)
```

Mupad [B]
time = 13.93, size = 13, normalized size = 0.65

$$\frac{2\,x+\sin\left(2\,x\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a - a*sin(x)^2)^2,x)
```

```
[Out] (2*x + sin(2*x))/(4*a^2)
```

3.281 $\int \frac{\cos^4(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a^2}$$

```
[Out] x/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 8}

$$\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] x/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(x)}{(a-a\sin^2(x))^2}\,dx &= \frac{\int 1\,dx}{a^2}\\ &= \frac{x}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] x/a^2
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.18, size = 8, normalized size = 1.60

method	result	size
risch	$\frac{x}{a^2}$	6
default	$\frac{\arctan(\tan(x))}{a^2}$	8
norman	$\frac{\frac{x\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{a}+\frac{x\left(\tan^{14}\left(\frac{x}{2}\right)\right)}{a}-\frac{x}{a}-\frac{x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{a}-\frac{3x\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{a}-\frac{3x\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4 a\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*arctan(tan(x))
```

Maxima [A]
time = 0.49, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x/a^2
```

Fricas [A]
time = 0.37, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x/a^2
```

Sympy [A]
time = 4.12, size = 3, normalized size = 0.60

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a-a*sin(x)**2)**2,x)
```

```
[Out] x/a**2
```

Giac [A]
time = 0.44, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] x/a^2
```

Mupad [B]
time = 13.94, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a - a*sin(x)^2)^2,x)
```

```
[Out] x/a^2
```

3.282 $\int \frac{\cos^2(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=6

$$\frac{\tan(x)}{a^2}$$

```
[Out] tan(x)/a^2
```

Rubi [A]

time = 0.03, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3254, 3852, 8}

$$\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Tan[x]/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(x)}{(a-a\sin^2(x))^2}\,dx &= \frac{\int \sec^2(x)\,dx}{a^2}\\
&= -\frac{\text{Subst}(\int 1\,dx, x, -\tan(x))}{a^2}\\
&= \frac{\tan(x)}{a^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Tan[x]/a^2
```

Maple [A]
time = 0.16, size = 7, normalized size = 1.17

method	result	size
default	$\frac{\tan(x)}{a^2}$	7
risch	$\frac{2i}{a^2(e^{2ix}+1)}$	16
norman	$\frac{-\frac{2\tan(\frac{x}{2})}{a}+\frac{4(\tan^5(\frac{x}{2}))}{a}-\frac{2(\tan^9(\frac{x}{2}))}{a}}{a(1+\tan^2(\frac{x}{2}))^2(\tan^2(\frac{x}{2})-1)^3}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] tan(x)/a^2
```

Maxima [A]
time = 0.30, size = 6, normalized size = 1.00

$$\frac{\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] tan(x)/a^2
```

Fricas [A]
time = 0.37, size = 10, normalized size = 1.67

$$\frac{\sin(x)}{a^2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] sin(x)/(a^2*cos(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(5) = 10$.
time = 1.66, size = 20, normalized size = 3.33

$$-\frac{2\tan\left(\frac{x}{2}\right)}{a^2\tan^2\left(\frac{x}{2}\right)-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a-a*sin(x)**2)**2,x)
```

```
[Out] -2*tan(x/2)/(a**2*tan(x/2)**2 - a**2)
```

Giac [A]
time = 0.46, size = 6, normalized size = 1.00

$$\frac{\tan\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] tan(x)/a^2
```

Mupad [B]
time = 13.78, size = 6, normalized size = 1.00

$$\frac{\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a - a*sin(x)^2)^2,x)
```

```
[Out] tan(x)/a^2
```

3.283 $\int \frac{\sec^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=29

$$\frac{\tan(x)}{a^2}+\frac{2\tan^3(x)}{3a^2}+\frac{\tan^5(x)}{5a^2}$$

```
[Out] tan(x)/a^2+2/3*tan(x)^3/a^2+1/5*tan(x)^5/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 3852}

$$\frac{\tan^5(x)}{5a^2}+\frac{2\tan^3(x)}{3a^2}+\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Tan[x]/a^2 + (2*Tan[x]^3)/(3*a^2) + Tan[x]^5/(5*a^2)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec^6(x)\,dx}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(x)\right)}{a^2}\\
&= \frac{\tan(x)}{a^2}+\frac{2\tan^3(x)}{3a^2}+\frac{\tan^5(x)}{5a^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.07

$$\frac{\frac{8\tan(x)}{15}+\frac{4}{15}\sec^2(x)\tan(x)+\frac{1}{5}\sec^4(x)\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a - a*Sin[x]^2)^2,x]
[Out] ((8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5)/a^2
```

Maple [A]
time = 0.17, size = 20, normalized size = 0.69

method	result	size
default	$\frac{\frac{\left(\tan^5(x)\right)}{5}+\frac{2\left(\tan^3(x)\right)}{3}+\tan(x)}{a^2}$	20
risch	$\frac{16i\left(10\,e^{4ix}+5\,e^{2ix}+1\right)}{15\left(e^{2ix}+1\right)^5a^2}$	32
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{116\left(\tan^5\left(\frac{x}{2}\right)\right)}{15a}+\frac{8\left(\tan^7\left(\frac{x}{2}\right)\right)}{3a}-\frac{2\left(\tan^9\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^5a}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/a^2*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.76

$$\frac{3\,\tan\left(x\right)^5+10\,\tan\left(x\right)^3+15\,\tan\left(x\right)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^2
```

Fricas [A]
time = 0.43, size = 25, normalized size = 0.86

$$\frac{\left(8\,\cos\left(x\right)^4+4\,\cos\left(x\right)^2+3\right)\sin\left(x\right)}{15\,a^2\cos\left(x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^2*cos(x)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(x)}{\sin^4(x)-2\sin^2(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a-a*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)**2/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

Giac [A]
time = 0.44, size = 22, normalized size = 0.76

$$\frac{3\tan(x)^5+10\tan(x)^3+15\tan(x)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^2
```

Mupad [B]
time = 13.80, size = 21, normalized size = 0.72

$$\frac{\tan(x)\,\left(3\tan(x)^4+10\tan(x)^2+15\right)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a - a*sin(x)^2)^2),x)
```

```
[Out] (tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a^2)
```

3.284 $\int \frac{\sec^4(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=37

$$\frac{\tan(x)}{a^2}+\frac{\tan^3(x)}{a^2}+\frac{3\tan^5(x)}{5a^2}+\frac{\tan^7(x)}{7a^2}$$

```
[Out] tan(x)/a^2+tan(x)^3/a^2+3/5*tan(x)^5/a^2+1/7*tan(x)^7/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 3852}

$$\frac{\tan^7(x)}{7a^2}+\frac{3\tan^5(x)}{5a^2}+\frac{\tan^3(x)}{a^2}+\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Tan[x]/a^2 + Tan[x]^3/a^2 + (3*Tan[x]^5)/(5*a^2) + Tan[x]^7/(7*a^2)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(x)}{(a-a\sin^2(x))^2}\,dx &= \frac{\int \sec^8(x)\,dx}{a^2}\\ &= -\frac{\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx,x,-\tan(x)\right)}{a^2}\\ &= \frac{\tan(x)}{a^2}+\frac{\tan^3(x)}{a^2}+\frac{3\tan^5(x)}{5a^2}+\frac{\tan^7(x)}{7a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 41, normalized size = 1.11

$$\frac{\frac{16\tan(x)}{35}+\frac{8}{35}\sec^2(x)\tan(x)+\frac{6}{35}\sec^4(x)\tan(x)+\frac{1}{7}\sec^6(x)\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((16*Tan[x])/35 + (8*Sec[x]^2*Tan[x])/35 + (6*Sec[x]^4*Tan[x])/35 + (Sec[x]
^6*Tan[x])/7)/a^2
```

Maple [A]
time = 0.22, size = 24, normalized size = 0.65

method	result	size
default	$\frac{\frac{\left(\tan^7(x)\right)}{7}+\frac{3\left(\tan^5(x)\right)}{5}+\tan^3(x)+\tan(x)}{a^2}$	24
risch	$\frac{32i\left(35\,\mathrm{e}^{6ix}+21\,\mathrm{e}^{4ix}+7\,\mathrm{e}^{2ix}+1\right)}{35\left(\mathrm{e}^{2ix}+1\right)^7a^2}$	39
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{86\left(\tan^5\left(\frac{x}{2}\right)\right)}{5a}+\frac{424\left(\tan^7\left(\frac{x}{2}\right)\right)}{35a}-\frac{86\left(\tan^9\left(\frac{x}{2}\right)\right)}{5a}+\frac{4\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^7a}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/7*tan(x)^7+3/5*tan(x)^5+tan(x)^3+tan(x))
```

Maxima [A]
time = 0.32, size = 28, normalized size = 0.76

$$\frac{5\,\tan\left(x\right)^7+21\,\tan\left(x\right)^5+35\,\tan\left(x\right)^3+35\,\tan\left(x\right)}{35\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^2
```

Fricas [A]
time = 0.40, size = 31, normalized size = 0.84

$$\frac{\left(16\,\cos\left(x\right)^6+8\,\cos\left(x\right)^4+6\,\cos\left(x\right)^2+5\right)\sin\left(x\right)}{35\,a^2\cos\left(x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/35*(16*cos(x)^6 + 8*cos(x)^4 + 6*cos(x)^2 + 5)*sin(x)/(a^2*cos(x)^7)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(x)}{\sin^4(x)-2\sin^2(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a-a*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)**4/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

Giac [**A**]
time = 0.43, size = 28, normalized size = 0.76

$$\frac{5\,\tan\left(x\right)^{7}+21\,\tan\left(x\right)^{5}+35\,\tan\left(x\right)^{3}+35\,\tan\left(x\right)}{35\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^2
```

Mupad [**B**]
time = 13.84, size = 33, normalized size = 0.89

$$\frac{\tan(x)}{a^2}+\frac{\tan(x)^3}{a^2}+\frac{3\tan(x)^5}{5\,a^2}+\frac{\tan(x)^7}{7\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a - a*sin(x)^2)^2),x)
```

```
[Out] tan(x)/a^2 + tan(x)^3/a^2 + (3*tan(x)^5)/(5*a^2) + tan(x)^7/(7*a^2)
```

3.285 $\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=109

$$\frac{5}{128}(8a+b)x+\frac{5(8a+b)\cos(e+fx)\sin(e+fx)}{128f}+\frac{5(8a+b)\cos^3(e+fx)\sin(e+fx)}{192f}+\frac{(8a+b)\cos^5(e+fx)\sin(e+fx)}{48f}$$

```
[Out] 5/128*(8*a+b)*x+5/128*(8*a+b)*cos(f*x+e)*sin(f*x+e)/f+5/192*(8*a+b)*cos(f*x
+e)^3*sin(f*x+e)/f+1/48*(8*a+b)*cos(f*x+e)^5*sin(f*x+e)/f-1/8*b*cos(f*x+e)^
7*sin(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3270, 393, 205, 209}

$$\frac{(8a+b)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5(8a+b)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5(8a+b)\sin(e+fx)\cos(e+fx)}{128f}+\frac{5}{128}x(8a+b)-\frac{b\sin(e+fx)\cos^7(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (5*(8*a + b)*x)/128 + (5*(8*a + b)*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*
(8*a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + ((8*a + b)*Cos[e + f*x]^5*
Sin[e + f*x])/(48*f) - (b*Cos[e + f*x]^7*Sin[e + f*x])/(8*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{\mathrm{Subst}\left(\int\frac{a+(a+b)x^2}{(1+x^2)^5}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{b\cos^7(e+fx)\sin(e+fx)}{8f}+\frac{(8a+b)\mathrm{Subst}\left(\int\frac{1}{(1+x^2)^4}\,dx,x,\mathrm{tan}\right.}{8f}$$

$$=\frac{(8a+b)\cos^5(e+fx)\sin(e+fx)}{48f}-\frac{b\cos^7(e+fx)\sin(e+fx)}{8f}+$$

$$=\frac{5(8a+b)\cos^3(e+fx)\sin(e+fx)}{192f}+\frac{(8a+b)\cos^5(e+fx)\sin(e}{48f}$$

$$=\frac{5(8a+b)\cos(e+fx)\sin(e+fx)}{128f}+\frac{5(8a+b)\cos^3(e+fx)\sin(e}{192f}$$

$$=\frac{5}{128}(8a+b)x+\frac{5(8a+b)\cos(e+fx)\sin(e+fx)}{128f}+\frac{5(8a+b)\,\mathrm{co}}{}$$

Mathematica [A]
time = 0.22, size = 87, normalized size = 0.80

$$\frac{960ae+960afx+120bfx+48(15a+b)\sin(2(e+fx))+24(6a-b)\sin(4(e+fx))+16a\sin(6(e+fx))-16b\sin(6(e+fx))-3b\sin(8(e+fx))}{3072f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (960*a*e + 960*a*f*x + 120*b*f*x + 48*(15*a + b)*Sin[2*(e + f*x)] + 24*(6*a
 - b)*Sin[4*(e + f*x)] + 16*a*Sin[6*(e + f*x)] - 16*b*Sin[6*(e + f*x)] - 3*
b*Sin[8*(e + f*x)])/(3072*f)
```

Maple [A]
time = 0.48, size = 112, normalized size = 1.03

method	result

derivativedivides	$\frac{b\left(-\frac{\sin(fx+e)\left(\cos^7(fx+e)\right)}{8}+\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3(fx+e)\right)}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{48}+\frac{5fx}{128}+\frac{5e}{128}\right)+a\left(\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3\right.}{}\right.}{}\right.}{f}$
default	$\frac{b\left(-\frac{\sin(fx+e)\left(\cos^7(fx+e)\right)}{8}+\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3(fx+e)\right)}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{48}+\frac{5fx}{128}+\frac{5e}{128}\right)+a\left(\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3\right.}{}\right.}{}\right.}{f}$
risch	$\frac{5ax}{16}+\frac{5bx}{128}-\frac{b\sin(8fx+8e)}{1024f}+\frac{\sin(6fx+6e)a}{192f}-\frac{\sin(6fx+6e)b}{192f}+\frac{3\sin(4fx+4e)a}{64f}-\frac{\sin(4fx+4e)b}{128f}+\frac{15\sin(2fx}{64f}$
norman	$\frac{\left(\frac{5a}{16}+\frac{5b}{128}\right)x+\left(\frac{5a}{2}+\frac{5b}{16}\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{5a}{2}+\frac{5b}{16}\right)x\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{5a}{16}+\frac{5b}{128}\right)x\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{35a}{2}+\frac{35b}{16}\right)x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(-1/8*sin(f*x+e)*cos(f*x+e)^7+1/48*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15
/8*cos(f*x+e))*sin(f*x+e)+5/128*f*x+5/128*e)+a*(1/6*(cos(f*x+e)^5+5/4*cos(f
*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e))
```

Maxima [A]
time = 0.53, size = 131, normalized size = 1.20

$$\frac{15\,(fx+e)(8\,a+b)+\frac{15\,(8\,a+b)\tan(fx+e)^7+55\,(8\,a+b)\tan(fx+e)^5+73\,(8\,a+b)\tan(fx+e)^3+3\,(88\,a-5\,b)\tan(fx+e)}{\tan(fx+e)^8+4\,\tan(fx+e)^6+6\,\tan(fx+e)^4+4\,\tan(fx+e)^2+1}}{384\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*(f*x + e)*(8*a + b) + (15*(8*a + b)*tan(f*x + e)^7 + 55*(8*a + b)
*tan(f*x + e)^5 + 73*(8*a + b)*tan(f*x + e)^3 + 3*(88*a - 5*b)*tan(f*x + e)
)/(tan(f*x + e)^8 + 4*tan(f*x + e)^6 + 6*tan(f*x + e)^4 + 4*tan(f*x + e)^2
+ 1))/f
```

Fricas [A]
time = 0.40, size = 78, normalized size = 0.72

$$\frac{15\,(8\,a+b)fx-\left(48\,b\cos\left(fx+e\right)^7-8\,(8\,a+b)\cos\left(fx+e\right)^5-10\,(8\,a+b)\cos\left(fx+e\right)^3-15\,(8\,a+b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{384\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/384*(15*(8*a + b)*f*x - (48*b*cos(f*x + e)^7 - 8*(8*a + b)*cos(f*x + e)^5
 - 10*(8*a + b)*cos(f*x + e)^3 - 15*(8*a + b)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(107) = 214$.

time = 0.92, size = 354, normalized size = 3.25

$$\begin{cases} \frac{5ax\sin^6(e+fx)}{16} + \frac{15ax\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{15ax\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{5ax\cos^6(e+fx)}{16} + \frac{5a\sin^5(e+fx)\cos(e+fx)}{16f} + \frac{5a\sin^3(e+fx)\cos^3(e+fx)}{6f} + \frac{11a\sin(e+fx)\cos^5(e+fx)}{16f} + \frac{5bx\sin^8(e+fx)}{128} + \frac{5bx\sin^6(e+fx)\cos^2(e+fx)}{32} + \frac{15bx\sin^4(e+fx)\cos^4(e+fx)}{64} + \frac{5bx\sin^2(e+fx)\cos^6(e+fx)}{32} + \frac{5bx\cos^8(e+fx)}{128} + \frac{5b\sin^7(e+fx)\cos(e+fx)}{128f} + \frac{55b\sin^5(e+fx)\cos^3(e+fx)}{384f} + \frac{73b\sin^3(e+fx)\cos^5(e+fx)}{384f} - \frac{5b\sin(e+fx)\cos^7(e+fx)}{128f} & \text{for } f \neq 0 \\ x(a+b\sin^2(e))\cos^6(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Piecewise((5*a*x*sin(e + f*x)**6/16 + 15*a*x*sin(e + f*x)**4*cos(e + f*x)**
2/16 + 15*a*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 5*a*x*cos(e + f*x)**6/16
 + 5*a*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 5*a*sin(e + f*x)**3*cos(e + f*
x)**3/(6*f) + 11*a*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 5*b*x*sin(e + f*x)
**8/128 + 5*b*x*sin(e + f*x)**6*cos(e + f*x)**2/32 + 15*b*x*sin(e + f*x)**4
*cos(e + f*x)**4/64 + 5*b*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 5*b*x*cos(
e + f*x)**8/128 + 5*b*sin(e + f*x)**7*cos(e + f*x)/(128*f) + 55*b*sin(e + f
*x)**5*cos(e + f*x)**3/(384*f) + 73*b*sin(e + f*x)**3*cos(e + f*x)**5/(384*
f) - 5*b*sin(e + f*x)*cos(e + f*x)**7/(128*f), Ne(f, 0)), (x*(a + b*sin(e)*
*2)*cos(e)**6, True))
```

Giac [A]

time = 0.52, size = 87, normalized size = 0.80

$$\frac{5}{128}(8a+b)x - \frac{b\sin(8fx+8e)}{1024f} + \frac{(a-b)\sin(6fx+6e)}{192f} + \frac{(6a-b)\sin(4fx+4e)}{128f} + \frac{(15a+b)\sin(2fx+2e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 5/128*(8*a + b)*x - 1/1024*b*sin(8*f*x + 8*e)/f + 1/192*(a - b)*sin(6*f*x +
 6*e)/f + 1/128*(6*a - b)*sin(4*f*x + 4*e)/f + 1/64*(15*a + b)*sin(2*f*x +
2*e)/f
```

Mupad [B]

time = 15.22, size = 119, normalized size = 1.09

$$x\left(\frac{5a}{16} + \frac{5b}{128}\right) + \frac{\left(\frac{5a}{16} + \frac{5b}{128}\right)\tan(e+fx)^7 + \left(\frac{55a}{48} + \frac{55b}{384}\right)\tan(e+fx)^5 + \left(\frac{73a}{48} + \frac{73b}{384}\right)\tan(e+fx)^3 + \left(\frac{11a}{16} - \frac{5b}{128}\right)\tan(e+fx)}{f\left(\tan(e+fx)^8 + 4\tan(e+fx)^6 + 6\tan(e+fx)^4 + 4\tan(e+fx)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b*sin(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 + (5*b)/128) + (tan(e + f*x)^7*((5*a)/16 + (5*b)/128) + tan(e +
 f*x)^5*((55*a)/48 + (55*b)/384) + tan(e + f*x)^3*((73*a)/48 + (73*b)/384)
+ tan(e + f*x)*((11*a)/16 - (5*b)/128))/(f*(4*tan(e + f*x)^2 + 6*tan(e + f*
x)^4 + 4*tan(e + f*x)^6 + tan(e + f*x)^8 + 1))
```

3.286 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=83

$$\frac{1}{16}(6a+b)x+\frac{(6a+b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(6a+b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{b\cos^5(e+fx)\sin(e+fx)}{6f}$$

```
[Out] 1/16*(6*a+b)*x+1/16*(6*a+b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(6*a+b)*cos(f*x+e)
^3*sin(f*x+e)/f-1/6*b*cos(f*x+e)^5*sin(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3270, 393, 205, 209}

$$\frac{(6a+b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(6a+b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(6a+b)-\frac{b\sin(e+fx)\cos^5(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] ((6*a + b)*x)/16 + ((6*a + b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((6*a + b
)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (b*Cos[e + f*x]^5*Sin[e + f*x])/(6*
f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^4}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=-\frac{b\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{(6a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx,x,\tan\right.}{6f}$$
$$=\frac{(6a+b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{b\cos^5(e+fx)\sin(e+fx)}{6f}+$$
$$=\frac{(6a+b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(6a+b)\cos^3(e+fx)\sin(e+}{24f}$$
$$=\frac{1}{16}(6a+b)x+\frac{(6a+b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(6a+b)\cos^3(}{}$$

Mathematica [A]
time = 0.12, size = 64, normalized size = 0.77

$$\frac{72ae+72afx+12bfx+3(16a+b)\sin(2(e+fx))+(6a-3b)\sin(4(e+fx))-b\sin(6(e+fx))}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (72*a*e + 72*a*f*x + 12*b*f*x + 3*(16*a + b)*Sin[2*(e + f*x)] + (6*a - 3*b)
*Sin[4*(e + f*x)] - b*Sin[6*(e + f*x)])/(192*f)
```

Maple [A]
time = 0.38, size = 92, normalized size = 1.11

method	result
risch	$\frac{3ax}{8}+\frac{bx}{16}-\frac{\sin(6fx+6e)b}{192f}+\frac{\sin(4fx+4e)a}{32f}-\frac{\sin(4fx+4e)b}{64f}+\frac{\sin(2fx+2e)a}{4f}+\frac{\sin(2fx+2e)b}{64f}$
derivativedivides	$\frac{b\left(-\frac{\sin(fx+e)\left(\cos^5(fx+e)\right)}{6}+\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{24}+\frac{fx}{16}+\frac{e}{16}\right)+a\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\right.}{f}$

default	$\frac{b\left(-\frac{\sin(fx+e)\left(\cos^5(fx+e)\right)}{6}+\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{24}+\frac{fx}{16}+\frac{e}{16}\right)+a\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3}{}\right.}{f}$
norman	$\left(\frac{3a}{8}+\frac{b}{16}\right)x+\left(\frac{3a}{8}+\frac{b}{16}\right)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{9a}{4}+\frac{3b}{8}\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{9a}{4}+\frac{3b}{8}\right)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{15a}{2}+\frac{5b}{4}\right)x\left(\text{ta}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(-1/6*sin(f*x+e)*cos(f*x+e)^5+1/24*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin
(f*x+e)+1/16*f*x+1/16*e)+a*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/
8*f*x+3/8*e))
```

Maxima [A]
time = 0.52, size = 104, normalized size = 1.25

$$\frac{3\,(fx+e)(6\,a+b)+\frac{3\,(6\,a+b)\tan(fx+e)^5+8\,(6\,a+b)\tan(fx+e)^3+3\,(10\,a-b)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(f*x + e)*(6*a + b) + (3*(6*a + b)*tan(f*x + e)^5 + 8*(6*a + b)*tan
(f*x + e)^3 + 3*(10*a - b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4
 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 0.39, size = 63, normalized size = 0.76

$$\frac{3\,(6\,a+b)fx-\left(8\,b\cos\left(fx+e\right)^5-2\,(6\,a+b)\cos\left(fx+e\right)^3-3\,(6\,a+b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(6*a + b)*f*x - (8*b*cos(f*x + e)^5 - 2*(6*a + b)*cos(f*x + e)^3 -
3*(6*a + b)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.
time = 0.42, size = 250, normalized size = 3.01

$$\begin{cases} \frac{3ax\sin^4(e+fx)}{8}+\frac{3ax\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{3ax\cos^4(e+fx)}{8}+\frac{3a\sin^3(e+fx)\cos(e+fx)}{8f}+\frac{5a\sin(e+fx)\cos^3(e+fx)}{8f}+\frac{bx\sin^6(e+fx)}{16}+\frac{3bx\sin^4(e+fx)\cos^2(e+fx)}{16}+\frac{3bx\sin^2(e+fx)\cos^4(e+fx)}{16}+\frac{bx\cos^6(e+fx)}{16}+\frac{b\sin^5(e+fx)\cos(e+fx)}{16f}+\frac{b\sin^3(e+fx)\cos^3(e+fx)}{6f}-\frac{b\sin(e+fx)\cos^5(e+fx)}{16f} & \text{for } f\neq 0 \\ x(a+b\sin^2(e))\cos^4(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Piecewise((3*a*x*sin(e + f*x)**4/8 + 3*a*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*a*x*cos(e + f*x)**4/8 + 3*a*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*a*
sin(e + f*x)*cos(e + f*x)**3/(8*f) + b*x*sin(e + f*x)**6/16 + 3*b*x*sin(e +
 f*x)**4*cos(e + f*x)**2/16 + 3*b*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + b*
x*cos(e + f*x)**6/16 + b*sin(e + f*x)**5*cos(e + f*x)/(16*f) + b*sin(e + f*
x)**3*cos(e + f*x)**3/(6*f) - b*sin(e + f*x)*cos(e + f*x)**5/(16*f), Ne(f,
0)), (x*(a + b*sin(e)**2)*cos(e)**4, True))
```

Giac [**A**]
time = 0.46, size = 67, normalized size = 0.81

$$\frac{1}{16}(6a+b)x-\frac{b\sin(6fx+6e)}{192f}+\frac{(2a-b)\sin(4fx+4e)}{64f}+\frac{(16a+b)\sin(2fx+2e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a + b)*x - 1/192*b*sin(6*f*x + 6*e)/f + 1/64*(2*a - b)*sin(4*f*x +
4*e)/f + 1/64*(16*a + b)*sin(2*f*x + 2*e)/f
```

Mupad [**B**]
time = 14.20, size = 91, normalized size = 1.10

$$x\left(\frac{3a}{8}+\frac{b}{16}\right)+\frac{\left(\frac{3a}{8}+\frac{b}{16}\right)\tan(e+fx)^5+\left(a+\frac{b}{6}\right)\tan(e+fx)^3+\left(\frac{5a}{8}-\frac{b}{16}\right)\tan(e+fx)}{f\left(\tan(e+fx)^6+3\tan(e+fx)^4+3\tan(e+fx)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2),x)
```

```
[Out] x*((3*a)/8 + b/16) + (tan(e + f*x)^5*((3*a)/8 + b/16) + tan(e + f*x)*((5*a)
/8 - b/16) + tan(e + f*x)^3*(a + b/6))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x
)^4 + tan(e + f*x)^6 + 1))
```

3.287 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=57

$$\frac{1}{8}(4a+b)x+\frac{(4a+b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{b\cos^3(e+fx)\sin(e+fx)}{4f}$$

```
[Out] 1/8*(4*a+b)*x+1/8*(4*a+b)*cos(f*x+e)*sin(f*x+e)/f-1/4*b*cos(f*x+e)^3*sin(f*
x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3270, 393, 205, 209}

$$\frac{(4a+b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(4a+b)-\frac{b\sin(e+fx)\cos^3(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] ((4*a + b)*x)/8 + ((4*a + b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (b*Cos[e +
f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{(4a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, \tan\right.}{4f}$$

$$= \frac{(4a+b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\cos^3(e+fx)\sin(e+fx)}{4f} +$$

$$= \frac{1}{8}(4a+b)x + \frac{(4a+b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\cos^3(e+fx)}{4f}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.81

$$\frac{4(4ae+4afx+bfx)+8a\sin(2(e+fx))-b\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (4*(4*a*e + 4*a*f*x + b*f*x) + 8*a*Sin[2*(e + f*x)] - b*Sin[4*(e + f*x)])/(
32*f)
```

Maple [A]
time = 0.25, size = 70, normalized size = 1.23

method	result
risch	$\frac{ax}{2}+\frac{bx}{8}-\frac{\sin(4fx+4e)b}{32f}+\frac{\sin(2fx+2e)a}{4f}$
derivativedivides	$\frac{b\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}+\frac{\cos(fx+e)\sin(fx+e)}{8}+\frac{fx}{8}+\frac{e}{8}\right)+a\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
default	$\frac{b\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}+\frac{\cos(fx+e)\sin(fx+e)}{8}+\frac{fx}{8}+\frac{e}{8}\right)+a\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$

norman	$\frac{\left(\frac{a}{2}+\frac{b}{8}\right)x+\left(2a+\frac{b}{2}\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(2a+\frac{b}{2}\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(3a+\frac{3b}{4}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{a}{2}+\frac{b}{8}\right)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(-1/4*cos(f*x+e)^3*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8*
e)+a*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.52, size = 74, normalized size = 1.30

$$\frac{(fx+e)(4\,a+b)+\frac{(4\,a+b)\tan(fx+e)^3+(4\,a-b)\tan(fx+e)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((f*x + e)*(4*a + b) + ((4*a + b)*tan(f*x + e)^3 + (4*a - b)*tan(f*x +
e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 0.39, size = 47, normalized size = 0.82

$$\frac{(4\,a+b)fx-\left(2\,b\cos\left(fx+e\right)^3-(4\,a+b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((4*a + b)*f*x - (2*b*cos(f*x + e)^3 - (4*a + b)*cos(f*x + e))*sin(f*x
+ e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(49) = 98$.
time = 0.18, size = 150, normalized size = 2.63

$$\begin{cases} \frac{ax\sin^2(e+fx)}{2}+\frac{ax\cos^2(e+fx)}{2}+\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx\sin^4(e+fx)}{8}+\frac{bx\sin^2(e+fx)\cos^2(e+fx)}{4}+\frac{bx\cos^4(e+fx)}{8}+\frac{b\sin^3(e+fx)\cos(e+fx)}{8f}-\frac{b\sin(e+fx)\cos^3(e+fx)}{8f} & \text{for } f\neq 0 \\ x(a+b\sin^2(e))\cos^2(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Piecewise((a*x*sin(e + f*x)**2/2 + a*x*cos(e + f*x)**2/2 + a*sin(e + f*x)*c
os(e + f*x)/(2*f) + b*x*sin(e + f*x)**4/8 + b*x*sin(e + f*x)**2*cos(e + f*x
```

```
)**2/4 + b*x*cos(e + f*x)**4/8 + b*sin(e + f*x)**3*cos(e + f*x)/(8*f) - b*s
in(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e)**2)*cos(e)**
2, True))
```

Giac [A]
time = 0.48, size = 41, normalized size = 0.72

$$\frac{1}{8}(4a+b)x - \frac{b\sin(4fx+4e)}{32f} + \frac{a\sin(2fx+2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*a + b)*x - 1/32*b*sin(4*f*x + 4*e)/f + 1/4*a*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 13.66, size = 67, normalized size = 1.18

$$x\left(\frac{a}{2}+\frac{b}{8}\right)+\frac{\left(\frac{a}{2}+\frac{b}{8}\right)\,\tan(e+f\,x)^3+\left(\frac{a}{2}-\frac{b}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2),x)
```

```
[Out] x*(a/2 + b/8) + (tan(e + f*x)^3*(a/2 + b/8) + tan(e + f*x)*(a/2 - b/8))/(f*
(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

3.288 $\int (a+b\sin^2(e+fx))\,dx$

Optimal. Leaf size=30

$$ax+\frac{bx}{2}-\frac{b\cos(e+fx)\sin(e+fx)}{2f}$$

```
[Out] a*x+1/2*b*x-1/2*b*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2715, 8}

$$ax-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[e + f*x]^2,x]
[Out] a*x + (b*x)/2 - (b*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin^2(e+fx))\,dx &= ax+b\int \sin^2(e+fx)\,dx\\ &= ax-\frac{b\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}b\int 1\,dx\\ &= ax+\frac{bx}{2}-\frac{b\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.10

$$ax+\frac{b(e+fx)}{2f}-\frac{b\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[e + f*x]^2,x]
[Out] a*x + (b*(e + f*x))/(2*f) - (b*Sin[2*(e + f*x)])/(4*f)
```

Maple [A]
time = 0.11, size = 32, normalized size = 1.07

method	result	size
risch	$ax+\frac{bx}{2}-\frac{\sin(2fx+2e)b}{4f}$	24
default	$ax+\frac{b\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$	32
derivativedivides	$\frac{(fx+e)a+b\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$	37
norman	$\frac{\left(a+\frac{b}{2}\right)x+\frac{b\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\left(a+\frac{b}{2}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2a+b)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] a*x+b/f*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)
```

Maxima [A]
time = 0.29, size = 31, normalized size = 1.03

$$ax+\frac{(2\,fx+2\,e-\sin{(2\,fx+2\,e)})b}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)^2,x, algorithm="maxima")
[Out] a*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b/f
```

Fricas [A]
time = 0.39, size = 29, normalized size = 0.97

$$\frac{(2\,a+b)fx-b\cos{(fx+e)}\sin{(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)^2,x, algorithm="fricas")
[Out] 1/2*((2*a + b)*f*x - b*cos(f*x + e)*sin(f*x + e))/f
```

Sympy [A]
time = 0.07, size = 51, normalized size = 1.70

$$ax+b\left(\begin{cases}\frac{x\sin^2(e+fx)}{2}+\frac{x\cos^2(e+fx)}{2}-\frac{\sin(e+fx)\cos(e+fx)}{2f} & \text{for } f\neq 0\\ x\sin^2(e) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)**2,x)
```

```
[Out] a*x + b*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 - sin(e + f*x)
*cos(e + f*x)/(2*f), Ne(f, 0)), (x*sin(e)**2, True))
```

Giac [A]
time = 0.45, size = 26, normalized size = 0.87

$$\frac{1}{4}\,b\left(2\,x-\frac{\sin\left(2\,fx+2\,e\right)}{f}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*(2*x - sin(2*f*x + 2*e)/f) + a*x
```

Mupad [B]
time = 13.65, size = 27, normalized size = 0.90

$$-\frac{\frac{b\,\sin(2\,e+2\,f\,x)}{4}-f\,x\,\left(a+\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(e + f*x)^2,x)
```

```
[Out] -((b*sin(2*e + 2*f*x))/4 - f*x*(a + b/2))/f
```

3.289 $\int \sec^2(e + fx)\left(a + b\sin^2(e + fx)\right)\,dx$

Optimal. Leaf size=18

$$-bx + \frac{(a+b)\tan(e+fx)}{f}$$

[Out] -b*x+(a+b)*tan(f*x+e)/f

Rubi [A]

time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3270, 396, 209}

$$\frac{(a+b)\tan(e+fx)}{f} - bx$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]

[Out] -(b*x) + ((a + b)*Tan[e + f*x])/f

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(a+b)\tan(e+fx)}{f} - \frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -bx + \frac{(a+b)\tan(e+fx)}{f}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 2.00

$$-\frac{b\tan^{-1}(\tan(e+fx))}{f} + \frac{a\tan(e+fx)}{f} + \frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
[Out] -((b*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.27, size = 30, normalized size = 1.67

method	result
derivativedivides	$\frac{\tan(fx+e)a+b(\tan(fx+e)-fx-e)}{f}$
default	$\frac{\tan(fx+e)a+b(\tan(fx+e)-fx-e)}{f}$
risch	$-bx+\frac{2ia}{f\left(e^{2i(fx+e)}+1\right)}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$
norman	$\frac{bx+bx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-bx\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-bx\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{4(a+b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2(a+b)\left(\tan\right.}{}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(tan(f*x+e)*a+b*(tan(f*x+e)-f*x-e))
```

Maxima [A]
time = 0.50, size = 33, normalized size = 1.83

$$-\frac{(fx+e-\tan(fx+e))b-a\tan(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -((f*x + e - tan(f*x + e))*b - a*tan(f*x + e))/f
```

Fricas [A]
time = 0.39, size = 35, normalized size = 1.94

$$-\frac{bfx\cos(fx+e) - (a+b)\sin(fx+e)}{f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -(b*f*x*cos(f*x + e) - (a + b)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^2(e+fx)\right)\sec^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)*sec(e + f*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(19) = 38.
time = 0.45, size = 49, normalized size = 2.72

$$-\frac{\left(fx - \pi\left\lfloor\frac{fx+e}{\pi} + \frac{1}{2}\right\rfloor + e - \tan(fx+e)\right)b - a\tan(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((f*x - pi*floor((f*x + e)/pi + 1/2) + e - tan(f*x + e))*b - a*tan(f*x + e
))/f
```

Mupad [B]
time = 13.64, size = 26, normalized size = 1.44

$$\frac{a\tan(e+f\,x) + b\tan(e+f\,x) - b\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/cos(e + f*x)^2,x)
```

```
[Out] (a*tan(e + f*x) + b*tan(e + f*x) - b*f*x)/f
```

3.290 $\int \sec^4(e+fx) \left(a+b\sin^2(e+fx)\right) \, dx$

Optimal. Leaf size=30

$$\frac{a\tan(e+fx)}{f}+\frac{(a+b)\tan^3(e+fx)}{3f}$$

```
[Out] a*tan(f*x+e)/f+1/3*(a+b)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3270}

$$\frac{(a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (a*Tan[e + f*x])/f + ((a + b)*Tan[e + f*x]^3)/(3*f)
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a\tan(e+fx)}{f}+\frac{(a+b)\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 1.37

$$\frac{b\tan^3(e+fx)}{3f}+\frac{a\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]

[Out] (b*Tan[e + f*x]^3)/(3*f) + (a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f

Maple [A]
time = 0.28, size = 46, normalized size = 1.53

method	result	size
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+\frac{b\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}}{f}$	46
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+\frac{b\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}}{f}$	46
risch	$-\frac{2i\left(3\,e^{4i(fx+e)}b-6a\,e^{2i(fx+e)}-2a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	49
norman	$\frac{-\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2a\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{8(a+b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{8(a+b)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{4(4b+a)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$	124

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)

[Out] 1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+1/3*b*sin(f*x+e)^3/cos(f*x+e)^3)

Maxima [A]
time = 0.28, size = 29, normalized size = 0.97

$$\frac{(a+b)\tan(fx+e)^3+3\,a\tan(fx+e)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="maxima")

[Out] 1/3*((a + b)*tan(f*x + e)^3 + 3*a*tan(f*x + e))/f

Fricas [A]
time = 0.39, size = 38, normalized size = 1.27

$$\frac{\left((2\,a-b)\cos(fx+e)^2+a+b\right)\sin(fx+e)}{3\,f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="fricas")

[Out] 1/3*((2*a - b)*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(f*cos(f*x + e)^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^2\left(e+fx\right)\right)\sec^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)*sec(e + f*x)**4, x)
```

Giac [A]
time = 0.45, size = 38, normalized size = 1.27

$$\frac{a\tan\left(fx+e\right)^3+b\tan\left(fx+e\right)^3+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(a*tan(f*x + e)^3 + b*tan(f*x + e)^3 + 3*a*tan(f*x + e))/f
```

Mupad [B]
time = 14.16, size = 31, normalized size = 1.03

$$\frac{\tan(e+f\,x)^3\,\left(\frac{a}{3}+\frac{b}{3}\right)}{f}+\frac{a\tan(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/cos(e + f*x)^4,x)
```

```
[Out] (tan(e + f*x)^3*(a/3 + b/3))/f + (a*tan(e + f*x))/f
```

3.291 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=50

$$\frac{a\tan(e+fx)}{f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)\tan^5(e+fx)}{5f}$$

```
[Out] a*tan(f*x+e)/f+1/3*(2*a+b)*tan(f*x+e)^3/f+1/5*(a+b)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3270, 380}

$$\frac{(a+b)\tan^5(e+fx)}{5f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (a*Tan[e + f*x])/f + ((2*a + b)*Tan[e + f*x]^3)/(3*f) + ((a + b)*Tan[e + f*
x]^5)/(5*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+(a+b)x^2\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a+(2a+b)x^2+(a+b)x^4\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a\tan(e+fx)}{f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)\tan^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 64, normalized size = 1.28

$$\frac{\tan(e+fx)\left(15a-2b-b\sec^2(e+fx)+3b\sec^4(e+fx)+10a\tan^2(e+fx)+3a\tan^4(e+fx)\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (Tan[e + f*x]*(15*a - 2*b - b*Sec[e + f*x]^2 + 3*b*Sec[e + f*x]^4 + 10*a*Ta
n[e + f*x]^2 + 3*a*Tan[e + f*x]^4))/(15*f)
```

Maple [A]
time = 0.31, size = 76, normalized size = 1.52

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)+b\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)}{f}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)+b\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)}{f}$
risch	$-\frac{4i\left(15b\,e^{6i(fx+e)}-40a\,e^{4i(fx+e)}-5\,e^{4i(fx+e)}b-20a\,e^{2i(fx+e)}+5\,e^{2i(fx+e)}b-4a+b\right)}{15f\left(e^{2i(fx+e)}+1\right)^5}$
norman	$\frac{-\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2a\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{4(a+2b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{4(a+2b)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{2(11a+16b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5f}-2(11}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+b*(1/5*sin(f*
x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3))
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.92

$$\frac{3\,(a+b)\tan\left(fx+e\right)^5+5\,(2\,a+b)\tan\left(fx+e\right)^3+15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a + b)*tan(f*x + e)^5 + 5*(2*a + b)*tan(f*x + e)^3 + 15*a*tan(f*x
+ e))/f
```

Fricas [A]
time = 0.39, size = 59, normalized size = 1.18

$$\frac{\left(2\,{\left(4\,a-b\right)}\cos\left(fx+e\right)^{4}+{\left(4\,a-b\right)}\cos\left(fx+e\right)^{2}+3\,a+3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(4*a - b)*cos(f*x + e)^4 + (4*a - b)*cos(f*x + e)^2 + 3*a + 3*b)*si
n(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 64, normalized size = 1.28

$$\frac{3\,a\tan\left(fx+e\right)^{5}+3\,b\tan\left(fx+e\right)^{5}+10\,a\tan\left(fx+e\right)^{3}+5\,b\tan\left(fx+e\right)^{3}+15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*a*tan(f*x + e)^5 + 3*b*tan(f*x + e)^5 + 10*a*tan(f*x + e)^3 + 5*b*t
an(f*x + e)^3 + 15*a*tan(f*x + e))/f
```

Mupad [B]
time = 13.90, size = 45, normalized size = 0.90

$$\frac{\left(\frac{a}{5}+\frac{b}{5}\right)\,{\mathrm{tan}\left(e+f\,x\right)}^5+\left(\frac{2\,a}{3}+\frac{b}{3}\right)\,{\mathrm{tan}\left(e+f\,x\right)}^3+a\,\mathrm{tan}\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/cos(e + f*x)^6,x)
```

```
[Out] (tan(e + f*x)^3*((2*a)/3 + b/3) + tan(e + f*x)^5*(a/5 + b/5) + a*tan(e + f*
x))/f
```

3.292 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=72

$$\frac{a\tan(e+fx)}{f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}+\frac{(a+b)\tan^7(e+fx)}{7f}$$

```
[Out] a*tan(f*x+e)/f+1/3*(3*a+b)*tan(f*x+e)^3/f+1/5*(3*a+2*b)*tan(f*x+e)^5/f+1/7*
(a+b)*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3270, 380}

$$\frac{(a+b)\tan^7(e+fx)}{7f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (a*Tan[e + f*x])/f + ((3*a + b)*Tan[e + f*x]^3)/(3*f) + ((3*a + 2*b)*Tan[e
+ f*x]^5)/(5*f) + ((a + b)*Tan[e + f*x]^7)/(7*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx&=\frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+(a+b)x^2\right)\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int \left(a+(3a+b)x^2+(3a+2b)x^4+(a+b)x^6\right)\,dx,x,\tan(e+\right.}{f}\\&=\frac{a\tan(e+fx)}{f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 86, normalized size = 1.19

$$\frac{\tan(e+fx)\left(105a-8b-4b\sec^2(e+fx)-3b\sec^4(e+fx)+15b\sec^6(e+fx)+105a\tan^2(e+fx)+63a\tan^4(e+fx)+15a\tan^6(e+fx)\right)}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2),x]
[Out] (Tan[e + f*x]*(105*a - 8*b - 4*b*Sec[e + f*x]^2 - 3*b*Sec[e + f*x]^4 + 15*b
*Sec[e + f*x]^6 + 105*a*Tan[e + f*x]^2 + 63*a*Tan[e + f*x]^4 + 15*a*Tan[e +
 f*x]^6))/(105*f)
```

Maple [A]
time = 0.29, size = 104, normalized size = 1.44

method	result
derivativedivides	$\frac{-a\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+b\left(\frac{\sin^3(fx+e)}{7\cos(fx+e)^7}+\frac{4\left(\sin^3(fx+e)\right)}{35\cos(fx+e)^5}+\frac{8\left(\sin^3(fx+e)\right)}{105\cos(fx+e)^3}\right.}{f}$
default	$\frac{-a\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+b\left(\frac{\sin^3(fx+e)}{7\cos(fx+e)^7}+\frac{4\left(\sin^3(fx+e)\right)}{35\cos(fx+e)^5}+\frac{8\left(\sin^3(fx+e)\right)}{105\cos(fx+e)^3}\right.}{f}$
risch	$-\frac{16i\left(70b\,e^{8i(fx+e)}-210a\,e^{6i(fx+e)}-35b\,e^{6i(fx+e)}-126a\,e^{4i(fx+e)}+21\,e^{4i(fx+e)}b-42a\,e^{2i(fx+e)}+7\,e^{2i(fx+e)}b-6a+b\right.}{105f\left(e^{2i(fx+e)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(-a*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f
*x+e)+b*(1/7*sin(f*x+e)^3/cos(f*x+e)^7+4/35*sin(f*x+e)^3/cos(f*x+e)^5+8/105
*sin(f*x+e)^3/cos(f*x+e)^3))
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.89

$$\frac{15\,(a+b)\tan\left(fx+e\right)^7+21\,(3\,a+2\,b)\tan\left(fx+e\right)^5+35\,(3\,a+b)\tan\left(fx+e\right)^3+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
[Out] 1/105*(15*(a + b)*tan(f*x + e)^7 + 21*(3*a + 2*b)*tan(f*x + e)^5 + 35*(3*a
+ b)*tan(f*x + e)^3 + 105*a*tan(f*x + e))/f
```

Fricas [A]
time = 0.37, size = 77, normalized size = 1.07

$$\frac{\left(8\,(6\,a-b)\cos\left(fx+e\right)^6+4\,(6\,a-b)\cos\left(fx+e\right)^4+3\,(6\,a-b)\cos\left(fx+e\right)^2+15\,a+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*a - b)*cos(f*x + e)^6 + 4*(6*a - b)*cos(f*x + e)^4 + 3*(6*a - b
)*cos(f*x + e)^2 + 15*a + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**8*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.51, size = 88, normalized size = 1.22

$$\frac{15\,a\tan\left(fx+e\right)^{7}+15\,b\tan\left(fx+e\right)^{7}+63\,a\tan\left(fx+e\right)^{5}+42\,b\tan\left(fx+e\right)^{5}+105\,a\tan\left(fx+e\right)^{3}+35\,b\tan\left(fx+e\right)^{3}+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*a*tan(f*x + e)^7 + 15*b*tan(f*x + e)^7 + 63*a*tan(f*x + e)^5 + 42
*b*tan(f*x + e)^5 + 105*a*tan(f*x + e)^3 + 35*b*tan(f*x + e)^3 + 105*a*tan(
f*x + e))/f
```

Mupad [B]
time = 13.84, size = 59, normalized size = 0.82

$$\frac{\left(\frac{a}{7}+\frac{b}{7}\right)\,{\mathrm{tan}\left(e+f\,x\right)}^7+\left(\frac{3\,a}{5}+\frac{2\,b}{5}\right)\,{\mathrm{tan}\left(e+f\,x\right)}^5+\left(a+\frac{b}{3}\right)\,{\mathrm{tan}\left(e+f\,x\right)}^3+a\,\mathrm{tan}\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/cos(e + f*x)^8,x)
```

```
[Out] (tan(e + f*x)^5*((3*a)/5 + (2*b)/5) + tan(e + f*x)^7*(a/7 + b/7) + a*tan(e
+ f*x) + tan(e + f*x)^3*(a + b/3))/f
```

3.293 $\int \cos^4(e+fx) \left(a+b\sin^2(e+fx)\right)^2 \, dx$

Optimal. Leaf size=156

$$\frac{1}{128}\left(48a^2+16ab+3b^2\right)x+\frac{\left(48a^2+16ab+3b^2\right)\cos(e+fx)\sin(e+fx)}{128f}+\frac{\left(48a^2+16ab+3b^2\right)\cos^3(e+fx)}{192f}$$

```
[Out] 1/128*(48*a^2+16*a*b+3*b^2)*x+1/128*(48*a^2+16*a*b+3*b^2)*cos(f*x+e)*sin(f*
x+e)/f+1/192*(48*a^2+16*a*b+3*b^2)*cos(f*x+e)^3*sin(f*x+e)/f-1/48*b*(10*a+3
*b)*cos(f*x+e)^5*sin(f*x+e)/f-1/8*b*cos(f*x+e)^7*sin(f*x+e)*(a+(a+b)*tan(f*
x+e)^2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3270, 424, 393, 205, 209}

$$\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos(e+fx)}{128f}+\frac{1}{128}x\left(48a^2+16ab+3b^2\right)-\frac{b(10a+3b)\sin(e+fx)\cos^5(e+fx)}{48f}-\frac{b\sin(e+fx)\cos^7(e+fx)\left((a+b)\tan^2(e+fx)+a\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((48*a^2 + 16*a*b + 3*b^2)*x)/128 + ((48*a^2 + 16*a*b + 3*b^2)*Cos[e + f*x]
*Sin[e + f*x])/(128*f) + ((48*a^2 + 16*a*b + 3*b^2)*Cos[e + f*x]^3*Sin[e +
f*x])/(192*f) - (b*(10*a + 3*b)*Cos[e + f*x]^5*Sin[e + f*x])/(48*f) - (b*Co
s[e + f*x]^7*Sin[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(8*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^5}\,dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{b\cos^7(e+fx)\sin(e+fx)\left(a+(a+b)\tan^2(e+fx)\right)}{8f}+\frac{\text{Subst}\big(\\
&= -\frac{b(10a+3b)\cos^5(e+fx)\sin(e+fx)}{48f}-\frac{b\cos^7(e+fx)\sin(e+f\\
&= \frac{\left(48a^2+16ab+3b^2\right)\cos^3(e+fx)\sin(e+fx)}{192f}-\frac{b(10a+3b)\cos^5(\\
&= \frac{\left(48a^2+16ab+3b^2\right)\cos(e+fx)\sin(e+fx)}{128f}+\frac{(48a^2+16ab+3b\\
&= \frac{1}{128}\left(48a^2+16ab+3b^2\right)x+\frac{\left(48a^2+16ab+3b^2\right)\cos(e+fx)\sin(\\
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 96, normalized size = 0.62

$$\frac{24\left(48a^2+16ab+3b^2\right)(e+fx)+96a(8a+b)\sin(2(e+fx))+24\left(4a^2-4ab-b^2\right)\sin(4(e+fx))-32ab\sin(6(e+fx))+3b^2\sin(8(e+fx))}{3072f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (24*(48*a^2 + 16*a*b + 3*b^2)*(e + f*x) + 96*a*(8*a + b)*Sin[2*(e + f*x)] +
 24*(4*a^2 - 4*a*b - b^2)*Sin[4*(e + f*x)] - 32*a*b*Sin[6*(e + f*x)] + 3*b^
2*Sin[8*(e + f*x)])/(3072*f)
```

Maple [A]
time = 0.50, size = 167, normalized size = 1.07

method	result
risch	$\frac{3a^2x}{8}+\frac{abx}{8}+\frac{3b^2x}{128}+\frac{b^2\sin(8fx+8e)}{1024f}-\frac{ab\sin(6fx+6e)}{96f}+\frac{\sin(4fx+4e)a^2}{32f}-\frac{\sin(4fx+4e)ab}{32f}-\frac{\sin(4fx+4e)}{128f}$
derivativedivides	$b^2\left(-\frac{\left(\sin^3(fx+e)\right)\left(\cos^5(fx+e)\right)}{8}-\frac{\sin(fx+e)\left(\cos^5(fx+e)\right)}{16}+\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{64}+\frac{3fx}{128}+\frac{3e}{128}\right)+2ab\left(-\frac{\sin}{}\right.$
default	$b^2\left(-\frac{\left(\sin^3(fx+e)\right)\left(\cos^5(fx+e)\right)}{8}-\frac{\sin(fx+e)\left(\cos^5(fx+e)\right)}{16}+\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{64}+\frac{3fx}{128}+\frac{3e}{128}\right)+2ab\left(-\frac{\sin}{}\right.$
norman	$(\frac{3}{8}a^2+\frac{1}{8}ab+\frac{3}{128}b^2)x+(3a^2+ab+\frac{3}{16}b^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a^2+ab+\frac{3}{16}b^2)x\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(21a^2+7ab+\frac{21}{16}b^2)x\left(\mathrm{t}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(-1/8*sin(f*x+e)^3*cos(f*x+e)^5-1/16*sin(f*x+e)*cos(f*x+e)^5+1/64*
(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/128*f*x+3/128*e)+2*a*b*(-1/6*sin
(f*x+e)*cos(f*x+e)^5+1/24*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+1/16*f*x
+1/16*e)+a^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A]
time = 0.49, size = 178, normalized size = 1.14

$$\frac{3\,(48\,a^2+16\,ab+3\,b^2)(fx+e)+\frac{3\,(48\,a^2+16\,ab+3\,b^2)\tan(fx+e)^7+11\,(48\,a^2+16\,ab+3\,b^2)\tan(fx+e)^5+(624\,a^2+80\,ab-33\,b^2)\tan(fx+e)^3+3\,(80\,a^2-16\,ab-3\,b^2)\tan(fx+e)}{\tan(fx+e)^8+4\tan(fx+e)^6+6\tan(fx+e)^4+4\tan(fx+e)^2+1}}{384\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/384*(3*(48*a^2 + 16*a*b + 3*b^2)*(f*x + e) + (3*(48*a^2 + 16*a*b + 3*b^2)
*tan(f*x + e)^7 + 11*(48*a^2 + 16*a*b + 3*b^2)*tan(f*x + e)^5 + (624*a^2 +
80*a*b - 33*b^2)*tan(f*x + e)^3 + 3*(80*a^2 - 16*a*b - 3*b^2)*tan(f*x + e))
/(tan(f*x + e)^8 + 4*tan(f*x + e)^6 + 6*tan(f*x + e)^4 + 4*tan(f*x + e)^2 +
 1))/f
```

Fricas [A]
time = 0.40, size = 114, normalized size = 0.73

$$\frac{3\,(48\,a^2+16\,ab+3\,b^2)fx+(48\,b^2\cos{(fx+e)}^7-8\,(16\,ab+9\,b^2)\cos{(fx+e)}^5+2\,(48\,a^2+16\,ab+3\,b^2)\cos{(fx+e)}^3+3\,(48\,a^2+16\,ab+3\,b^2)\cos{(fx+e)})\sin{(fx+e)}}{384\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*(48*a^2 + 16*a*b + 3*b^2)*f*x + (48*b^2*cos(f*x + e)^7 - 8*(16*a*b
 + 9*b^2)*cos(f*x + e)^5 + 2*(48*a^2 + 16*a*b + 3*b^2)*cos(f*x + e)^3 + 3*(
48*a^2 + 16*a*b + 3*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(146) = 292.
time = 0.95, size = 481, normalized size = 3.08

$$\begin{cases} \frac{3a^2x\sin^4{(e+fx)}}{8}+\frac{3a^2x\sin^2{(e+fx)}\cos^2{(e+fx)}}{4}+\frac{3a^2x\cos^4{(e+fx)}}{8}+\frac{3a^2\sin^3{(e+fx)}\cos{(e+fx)}}{8f}+\frac{5a^2\sin{(e+fx)}\cos^3{(e+fx)}}{8f}+\frac{abx\sin^6{(e+fx)}}{8}+\frac{3abx\sin^4{(e+fx)}\cos^2{(e+fx)}}{8}+\frac{3abx\sin^2{(e+fx)}\cos^4{(e+fx)}}{8}+\frac{abx\cos^6{(e+fx)}}{8}+\frac{ab\sin^5{(e+fx)}\cos{(e+fx)}}{8f}+\frac{ab\sin^3{(e+fx)}\cos^3{(e+fx)}}{3f}-\frac{ab\sin{(e+fx)}\cos^5{(e+fx)}}{8f}+\frac{3b^2x\sin^8{(e+fx)}}{128}+\frac{3b^2x\sin^6{(e+fx)}\cos^2{(e+fx)}}{32}+\frac{9b^2x\sin^4{(e+fx)}\cos^4{(e+fx)}}{64}+\frac{3b^2x\sin^2{(e+fx)}\cos^6{(e+fx)}}{32}+\frac{3b^2x\cos^8{(e+fx)}}{128}+\frac{3b^2\sin^7{(e+fx)}\cos{(e+fx)}}{128f}+\frac{11b^2\sin^5{(e+fx)}\cos^3{(e+fx)}}{128f}-\frac{11b^2\sin^3{(e+fx)}\cos^5{(e+fx)}}{128f}-\frac{3b^2\sin{(e+fx)}\cos^7{(e+fx)}}{128f} & \text{for}\ f\neq 0 \\ x(a+b\sin^2{(e)})^2\cos^4{(e)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(e + f*x)**4/8 + 3*a**2*x*sin(e + f*x)**2*cos(e + f*
x)**2/4 + 3*a**2*x*cos(e + f*x)**4/8 + 3*a**2*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + 5*a**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a*b*x*sin(e + f*x)**6/8
 + 3*a*b*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 3*a*b*x*sin(e + f*x)**2*cos(
e + f*x)**4/8 + a*b*x*cos(e + f*x)**6/8 + a*b*sin(e + f*x)**5*cos(e + f*x)/
(8*f) + a*b*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - a*b*sin(e + f*x)*cos(e
+ f*x)**5/(8*f) + 3*b**2*x*sin(e + f*x)**8/128 + 3*b**2*x*sin(e + f*x)**6*c
os(e + f*x)**2/32 + 9*b**2*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 3*b**2*x*
sin(e + f*x)**2*cos(e + f*x)**6/32 + 3*b**2*x*cos(e + f*x)**8/128 + 3*b**2*
sin(e + f*x)**7*cos(e + f*x)/(128*f) + 11*b**2*sin(e + f*x)**5*cos(e + f*x)
**3/(128*f) - 11*b**2*sin(e + f*x)**3*cos(e + f*x)**5/(128*f) - 3*b**2*sin(
e + f*x)*cos(e + f*x)**7/(128*f), Ne(f, 0)), (x*(a + b*sin(e)**2)**2*cos(e)
**4, True))
```

Giac [A]
time = 0.51, size = 108, normalized size = 0.69

$$\frac{1}{128}\,(48\,a^2+16\,ab+3\,b^2)x+\frac{b^2\sin{(8\,fx+8\,e)}}{1024\,f}-\frac{ab\sin{(6\,fx+6\,e)}}{96\,f}+\frac{(4\,a^2-4\,ab-b^2)\sin{(4\,fx+4\,e)}}{128\,f}+\frac{(8\,a^2+ab)\sin{(2\,fx+2\,e)}}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/128*(48*a^2 + 16*a*b + 3*b^2)*x + 1/1024*b^2*sin(8*f*x + 8*e)/f - 1/96*a*
b*sin(6*f*x + 6*e)/f + 1/128*(4*a^2 - 4*a*b - b^2)*sin(4*f*x + 4*e)/f + 1/3
2*(8*a^2 + a*b)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 15.49, size = 160, normalized size = 1.03

$$x\left(\frac{3a^2}{8}+\frac{ab}{8}+\frac{3b^2}{128}\right)+\frac{\left(\frac{3a^2}{8}+\frac{ab}{8}+\frac{3b^2}{128}\right)\tan(e+fx)^7+\left(\frac{11a^2}{8}+\frac{11ab}{24}+\frac{11b^2}{128}\right)\tan(e+fx)^5+\left(\frac{13a^2}{8}+\frac{5ab}{24}-\frac{11b^2}{128}\right)\tan(e+fx)^3+\left(\frac{5a^2}{8}-\frac{ab}{8}-\frac{3b^2}{128}\right)\tan(e+fx)}{f\left(\tan(e+fx)^8+4\tan(e+fx)^6+6\tan(e+fx)^4+4\tan(e+fx)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^2,x)
```

```
[Out] x*((a*b)/8 + (3*a^2)/8 + (3*b^2)/128) + (tan(e + f*x)^7*((a*b)/8 + (3*a^2)/
8 + (3*b^2)/128) - tan(e + f*x)*((a*b)/8 - (5*a^2)/8 + (3*b^2)/128) + tan(e
 + f*x)^3*((5*a*b)/24 + (13*a^2)/8 - (11*b^2)/128) + tan(e + f*x)^5*((11*a*
b)/24 + (11*a^2)/8 + (11*b^2)/128))/(f*(4*tan(e + f*x)^2 + 6*tan(e + f*x)^4
 + 4*tan(e + f*x)^6 + tan(e + f*x)^8 + 1))
```

3.294 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=116

$$\frac{1}{16}\left(8a^2+4ab+b^2\right)x+\frac{\left(8a^2+4ab+b^2\right)\cos(e+fx)\sin(e+fx)}{16f}-\frac{b(8a+3b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{b\cos}{}$$

```
[Out] 1/16*(8*a^2+4*a*b+b^2)*x+1/16*(8*a^2+4*a*b+b^2)*cos(f*x+e)*sin(f*x+e)/f-1/2
4*b*(8*a+3*b)*cos(f*x+e)^3*sin(f*x+e)/f-1/6*b*cos(f*x+e)^5*sin(f*x+e)*(a+(a
+b)*tan(f*x+e)^2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3270, 424, 393, 205, 209}

$$\frac{\left(8a^2+4ab+b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(8a^2+4ab+b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{b\sin(e+fx)\cos^5(e+fx)\left((a+b)\tan^2(e+fx)+a\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 4*a*b + b^2)*x)/16 + ((8*a^2 + 4*a*b + b^2)*Cos[e + f*x]*Sin[e +
f*x])/(16*f) - (b*(8*a + 3*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (b*Cos[
e + f*x]^5*Sin[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(6*f)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^5(e+fx)\sin(e+fx)\left(a+(a+b)\tan^2(e+fx)\right)}{6f} + \frac{\text{Subst}}{}$$

$$= -\frac{b(8a+3b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{b\cos^5(e+fx)\sin(e+}{}$$

$$= \frac{\left(8a^2+4ab+b^2\right)\cos(e+fx)\sin(e+fx)}{16f} - \frac{b(8a+3b)\cos^3(e+}{24f}$$

$$= \frac{1}{16}\left(8a^2+4ab+b^2\right)x + \frac{\left(8a^2+4ab+b^2\right)\cos(e+fx)\sin(e+fx}{16f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 79, normalized size = 0.68

$$\frac{12((2-2i)a+b)((2+2i)a+b)(e+fx)+3(4a-b)(4a+b)\sin(2(e+fx))-3b(4a+b)\sin(4(e+fx))+b^2\sin(6(e+fx))}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (12*((2 - 2*I)*a + b)*((2 + 2*I)*a + b)*(e + f*x) + 3*(4*a - b)*(4*a + b)*S
in[2*(e + f*x)] - 3*b*(4*a + b)*Sin[4*(e + f*x)] + b^2*Sin[6*(e + f*x)])/(1
92*f)
```

Maple [A]
time = 0.35, size = 134, normalized size = 1.16

method	result
risch	$\frac{a^2x}{2}+\frac{abx}{4}+\frac{b^2x}{16}+\frac{b^2\sin(6fx+6e)}{192f}-\frac{\sin(4fx+4e)ab}{16f}-\frac{\sin(4fx+4e)b^2}{64f}+\frac{\sin(2fx+2e)a^2}{4f}-\frac{\sin(2fx+2e)b^2}{64f}$
derivativedivides	$\frac{b^2\left(-\frac{\left(\sin^3(fx+e)\right)\left(\cos^3(fx+e)\right)}{6}-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{8}+\frac{\cos(fx+e)\sin(fx+e)}{16}+\frac{fx}{16}+\frac{e}{16}\right)+2ab\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}\right.}{f}$
default	$\frac{b^2\left(-\frac{\left(\sin^3(fx+e)\right)\left(\cos^3(fx+e)\right)}{6}-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{8}+\frac{\cos(fx+e)\sin(fx+e)}{16}+\frac{fx}{16}+\frac{e}{16}\right)+2ab\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}\right.}{f}$
norman	$\frac{(\frac{1}{2}a^2+\frac{1}{4}ab+\frac{1}{16}b^2)x+(3a^2+\frac{3}{2}ab+\frac{3}{8}b^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(3a^2+\frac{3}{2}ab+\frac{3}{8}b^2)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(10a^2+5ab+\frac{5}{4}b^2)x\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(-1/6*sin(f*x+e)^3*cos(f*x+e)^3-1/8*cos(f*x+e)^3*sin(f*x+e)+1/16*c
os(f*x+e)*sin(f*x+e)+1/16*f*x+1/16*e)+2*a*b*(-1/4*cos(f*x+e)^3*sin(f*x+e)+1
/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/8*e)+a^2*(1/2*cos(f*x+e)*sin(f*x+e)+1/2*
f*x+1/2*e))
```

Maxima [A]
time = 0.49, size = 134, normalized size = 1.16

$$\frac{3\,(8\,a^2+4\,ab+b^2)(fx+e)+\frac{3\,(8\,a^2+4\,ab+b^2)\tan(fx+e)^5+8\,(6\,a^2-b^2)\tan(fx+e)^3+3\,(8\,a^2-4\,ab-b^2)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(8*a^2 + 4*a*b + b^2)*(f*x + e) + (3*(8*a^2 + 4*a*b + b^2)*tan(f*x
+ e)^5 + 8*(6*a^2 - b^2)*tan(f*x + e)^3 + 3*(8*a^2 - 4*a*b - b^2)*tan(f*x +
 e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 0.40, size = 85, normalized size = 0.73

$$\frac{3\,(8\,a^2+4\,ab+b^2)fx+\left(8\,b^2\cos{(fx+e)}^5-2\,(12\,ab+7\,b^2)\cos{(fx+e)}^3+3\,(8\,a^2+4\,ab+b^2)\cos{(fx+e)}\right)\sin{(fx+e)}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(8*a^2 + 4*a*b + b^2)*f*x + (8*b^2*cos(f*x + e)^5 - 2*(12*a*b + 7*b
^2)*cos(f*x + e)^3 + 3*(8*a^2 + 4*a*b + b^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(107) = 214$.
time = 0.44, size = 314, normalized size = 2.71

$$\begin{cases} \frac{a^2x\sin^2(e+fx)}{2} + \frac{a^2x\cos^2(e+fx)}{2} + \frac{a^2\sin(e+fx)\cos(e+fx)}{2f} + \frac{abx\sin^4(e+fx)}{4} + \frac{abx\sin^2(e+fx)\cos^2(e+fx)}{2} + \frac{abx\cos^4(e+fx)}{4} + \frac{ab\sin^3(e+fx)\cos(e+fx)}{4f} - \frac{ab\sin(e+fx)\cos^3(e+fx)}{4f} + \frac{b^2x\sin^6(e+fx)}{16} + \frac{3b^2x\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{3b^2x\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{b^2x\cos^6(e+fx)}{16} + \frac{b^2\sin^5(e+fx)\cos(e+fx)}{16f} - \frac{b^2\sin^3(e+fx)\cos^3(e+fx)}{6f} - \frac{b^2\sin(e+fx)\cos^5(e+fx)}{16f} & \text{for } f \neq 0 \\ x(a+b\sin^2(e))^2\cos^2(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 + a**2*sin(e
 + f*x)*cos(e + f*x)/(2*f) + a*b*x*sin(e + f*x)**4/4 + a*b*x*sin(e + f*x)**
2*cos(e + f*x)**2/2 + a*b*x*cos(e + f*x)**4/4 + a*b*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) - a*b*sin(e + f*x)*cos(e + f*x)**3/(4*f) + b**2*x*sin(e + f*x)*
*6/16 + 3*b**2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*b**2*x*sin(e + f*x)
**2*cos(e + f*x)**4/16 + b**2*x*cos(e + f*x)**6/16 + b**2*sin(e + f*x)**5*c
os(e + f*x)/(16*f) - b**2*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - b**2*sin(
e + f*x)*cos(e + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e)**2)**2*cos(e)*
*2, True))
```

Giac [A]
time = 0.46, size = 84, normalized size = 0.72

$$\frac{1}{16}\left(8a^2 + 4ab + b^2\right)x + \frac{b^2\sin\left(6fx + 6e\right)}{192f} - \frac{\left(4ab + b^2\right)\sin\left(4fx + 4e\right)}{64f} + \frac{\left(16a^2 - b^2\right)\sin\left(2fx + 2e\right)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^2 + 4*a*b + b^2)*x + 1/192*b^2*sin(6*f*x + 6*e)/f - 1/64*(4*a*b +
 b^2)*sin(4*f*x + 4*e)/f + 1/64*(16*a^2 - b^2)*sin(2*f*x + 2*e)/f
```

Mupad [B]
time = 14.73, size = 120, normalized size = 1.03

$$x\left(\frac{a^2}{2} + \frac{a\,b}{4} + \frac{b^2}{16}\right) + \frac{\left(\frac{a^2}{2} + \frac{a\,b}{4} + \frac{b^2}{16}\right)\,\tan(e+f\,x)^5 + \left(a^2 - \frac{b^2}{6}\right)\,\tan(e+f\,x)^3 + \left(\frac{a^2}{2} - \frac{a\,b}{4} - \frac{b^2}{16}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6 + 3\tan\left(e+f\,x\right)^4 + 3\tan\left(e+f\,x\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^2,x)
```

```
[Out] x*((a*b)/4 + a^2/2 + b^2/16) + (tan(e + f*x)^3*(a^2 - b^2/6) - tan(e + f*x)
*((a*b)/4 - a^2/2 + b^2/16) + tan(e + f*x)^5*((a*b)/4 + a^2/2 + b^2/16))/(f
*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1))
```

3.295 $\int (a+b\sin^2(e+fx))^2 \, dx$

Optimal. Leaf size=72

$$\frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{b(8a+3b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{b^2\cos(e+fx)\sin^3(e+fx)}{4f}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*x-1/8*b*(8*a+3*b)*cos(f*x+e)*sin(f*x+e)/f-1/4*b^2*c
os(f*x+e)*sin(f*x+e)^3/f
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3258}

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos(e+fx)}{8f}-\frac{b^2\sin^3(e+fx)\cos(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rule 3258

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^2 \, dx=\frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{b(8a+3b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{b^2\cos(e+fx)\,\text{si}}{4f}$$

Mathematica [A]
time = 0.09, size = 58, normalized size = 0.81

$$\frac{4\left(8a^2+8ab+3b^2\right)(e+fx)-8b(2a+b)\sin(2(e+fx))+b^2\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (4*(8*a^2 + 8*a*b + 3*b^2)*(e + f*x) - 8*b*(2*a + b)*Sin[2*(e + f*x)] + b^2
*Sin[4*(e + f*x)])/(32*f)
```

Maple [A]
time = 0.23, size = 78, normalized size = 1.08

method	result
risch	$a^2x+abx+\frac{3b^2x}{8}+\frac{\sin(4fx+4e)b^2}{32f}-\frac{\sin(2fx+2e)ab}{2f}-\frac{\sin(2fx+2e)b^2}{4f}$
derivativedivides	$\frac{b^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a^2(fx+e)}{f}$
default	$\frac{b^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a^2(fx+e)}{f}$
norman	$(a^2+ab+\frac{3}{8}b^2)x+(a^2+ab+\frac{3}{8}b^2)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4a^2+4ab+\frac{3}{2}b^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(4a^2+4ab+\frac{3}{2}b^2)x\left(\tan^6\left(\frac{fx}{2}+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a*
b*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+a^2*(f*x+e))
```

Maxima [A]
time = 0.29, size = 73, normalized size = 1.01

$$a^2x+\frac{(2\,fx+2\,e-\sin(2\,fx+2\,e))ab}{2\,f}+\frac{(12\,fx+12\,e+\sin(4\,fx+4\,e)-8\,\sin(2\,fx+2\,e))b^2}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b/f + 1/32*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^2/f
```

Fricas [A]
time = 0.40, size = 63, normalized size = 0.88

$$\frac{(8\,a^2+8\,ab+3\,b^2)fx+\left(2\,b^2\cos(fx+e)^3-(8\,ab+5\,b^2)\cos(fx+e)\right)\sin(fx+e)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

[Out] 1/8*((8*a^2 + 8*a*b + 3*b^2)*f*x + (2*b^2*cos(f*x + e)^3 - (8*a*b + 5*b^2)*cos(f*x + e))*sin(f*x + e))/f

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(61) = 122$.
time = 0.18, size = 168, normalized size = 2.33

$$\begin{cases} a^2x + abx\sin^2(e+fx) + abx\cos^2(e+fx) - \frac{ab\sin(e+fx)\cos(e+fx)}{f} + \frac{3b^2x\sin^4(e+fx)}{8} + \frac{3b^2x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3b^2x\cos^4(e+fx)}{8} - \frac{5b^2\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{3b^2\sin(e+fx)\cos^3(e+fx)}{8f} & \text{for } f \neq 0 \\ x(a+b\sin^2(e))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e)**2)**2,x)

[Out] Piecewise((a**2*x + a*b*x*sin(e + f*x)**2 + a*b*x*cos(e + f*x)**2 - a*b*sin(e + f*x)*cos(e + f*x)/f + 3*b**2*x*sin(e + f*x)**4/8 + 3*b**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*b**2*x*cos(e + f*x)**4/8 - 5*b**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e)**2)**2, True))

Giac [A]
time = 0.41, size = 60, normalized size = 0.83

$$\frac{1}{8}\left(8a^2 + 8ab + 3b^2\right)x + \frac{b^2\sin(4fx + 4e)}{32f} - \frac{(2ab + b^2)\sin(2fx + 2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="giac")

[Out] 1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/32*b^2*sin(4*f*x + 4*e)/f - 1/4*(2*a*b + b^2)*sin(2*f*x + 2*e)/f

Mupad [B]
time = 14.49, size = 77, normalized size = 1.07

$$x\left(a^2 + a\,b + \frac{3\,b^2}{8}\right) - \frac{\left(\frac{5\,b^2}{8} + a\,b\right)\,\tan(e + f\,x)^3 + \left(\frac{3\,b^2}{8} + a\,b\right)\,\tan(e + f\,x)}{f\,\left(\tan(e + f\,x)^4 + 2\tan(e + f\,x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(e + f*x)^2)^2,x)

[Out] x*(a*b + a^2 + (3*b^2)/8) - (tan(e + f*x)*(a*b + (3*b^2)/8) + tan(e + f*x)^3*(a*b + (5*b^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))

3.296 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}b(4a+3b)x+\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{(a+b)^2\tan(e+fx)}{f}$$

```
[Out] -1/2*b*(4*a+3*b)*x+1/2*b^2*cos(f*x+e)*sin(f*x+e)/f+(a+b)^2*tan(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3270, 398, 393, 209}

$$\frac{(a+b)^2\tan(e+fx)}{f}-\frac{1}{2}bx(4a+3b)+\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(b*(4*a + 3*b)*x) + (b^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + ((a + b)^2
*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
```

```
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left((a+b)^2-\frac{b(2a+b)+2b(a+b)x^2}{\left(1+x^2\right)^2}\right)\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{(a+b)^2\tan(e+fx)}{f}-\frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{b^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{(a+b)^2\tan(e+fx)}{f}-\frac{(b(4a+3b))}{}$$

$$= -\frac{1}{2}b(4a+3b)x+\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{(a+b)^2\tan(e+f}{f}$$

Mathematica [A]
time = 0.23, size = 48, normalized size = 0.94

$$\frac{-2b(4a+3b)(e+fx)+b^2\sin(2(e+fx))+4(a+b)^2\tan(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
[Out] (-2*b*(4*a + 3*b)*(e + f*x) + b^2*Sin[2*(e + f*x)] + 4*(a + b)^2*Tan[e + f*
x])/(4*f)
```

Maple [A]
time = 0.38, size = 87, normalized size = 1.71

method	result
derivativedivides	$\frac{a^2\tan(fx+e)+2ab(\tan(fx+e)-fx-e)+b^2\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$
default	$\frac{a^2\tan(fx+e)+2ab(\tan(fx+e)-fx-e)+b^2\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$
risch	$-2abx-\frac{3b^2x}{2}-\frac{ib^2e^{2i(fx+e)}}{8f}+\frac{ib^2e^{-2i(fx+e)}}{8f}+\frac{2ia^2}{f\left(e^{2i(fx+e)}+1\right)}+\frac{4iab}{f\left(e^{2i(fx+e)}+1\right)}+\frac{2ib^2}{f\left(e^{2i(fx+e)}+1\right)}$

norman	$\frac{(2ab+\frac{3}{2}b^2)x+(-6ab-\frac{9}{2}b^2)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-2ab-\frac{3}{2}b^2)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6ab+\frac{9}{2}b^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4ab}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(a^2*tan(f*x+e)+2*a*b*(tan(f*x+e)-f*x-e)+b^2*(sin(f*x+e)^5/cos(f*x+e)+(
sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e))
```

Maxima [A]
time = 0.52, size = 81, normalized size = 1.59

$$-\frac{4\left(fx+e-\tan\left(fx+e\right)\right)ab+\left(3\,fx-\frac{\tan(fx+e)}{\tan(fx+e)^2+1}+3\,e-2\,\tan\left(fx+e\right)\right)b^2-2\,a^2\tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
[Out] -1/2*(4*(f*x + e - tan(f*x + e))*a*b + (3*f*x - tan(f*x + e)/(tan(f*x + e)^
2 + 1) + 3*e - 2*tan(f*x + e))*b^2 - 2*a^2*tan(f*x + e))/f
```

Fricas [A]
time = 0.42, size = 68, normalized size = 1.33

$$-\frac{(4\,ab+3\,b^2)fx\cos\left(fx+e\right)-\left(b^2\cos\left(fx+e\right)^2+2\,a^2+4\,ab+2\,b^2\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
[Out] -1/2*((4*a*b + 3*b^2)*f*x*cos(f*x + e) - (b^2*cos(f*x + e)^2 + 2*a^2 + 4*a*
b + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^2\left(e+fx\right)\right)^2\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**2,x)
[Out] Integral((a + b*sin(e + f*x)**2)**2*sec(e + f*x)**2, x)
```

Giac [A]
time = 0.49, size = 99, normalized size = 1.94

$$\frac{2\,a^2\tan\left(fx+e\right)+4\,ab\tan\left(fx+e\right)+2\,b^2\tan\left(fx+e\right)-\left(4\,ab+3\,b^2\right)\left(fx-\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+e\right)+\frac{b^2\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*tan(f*x + e) + 4*a*b*tan(f*x + e) + 2*b^2*tan(f*x + e) - (4*a*b
+ 3*b^2)*(f*x - pi*floor((f*x + e)/pi + 1/2) + e) + b^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

Mupad [B]
time = 14.09, size = 74, normalized size = 1.45

$$\frac{\tan(e+f\,x)\,\left(a+b\right)^2}{f}+\frac{b^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{b\,\mathrm{atan}\left(\frac{b\,\tan(e+f\,x)\,\left(4\,a+3\,b\right)}{2\,\left(\frac{3\,b^2}{2}+2\,a\,b\right)}\right)\,\left(4\,a+3\,b\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^2,x)
```

```
[Out] (tan(e + f*x)*(a + b)^2)/f + (b^2*sin(2*e + 2*f*x))/(4*f) - (b*atan((b*tan(
e + f*x)*(4*a + 3*b))/(2*(2*a*b + (3*b^2)/2)))*(4*a + 3*b))/(2*f)
```

3.297 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=45

$$b^2x+\frac{(a^2-b^2)\tan(e+fx)}{f}+\frac{(a+b)^2\tan^3(e+fx)}{3f}$$

```
[Out] b^2*x+(a^2-b^2)*tan(f*x+e)/f+1/3*(a+b)^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3270, 398, 209}

$$\frac{(a^2-b^2)\tan(e+fx)}{f}+\frac{(a+b)^2\tan^3(e+fx)}{3f}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] b^2*x + ((a^2 - b^2)*Tan[e + f*x])/f + ((a + b)^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-b^2+(a+b)^2x^2+\frac{b^2}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(a^2-b^2)\tan(e+fx)}{f} + \frac{(a+b)^2\tan^3(e+fx)}{3f} + \frac{b^2\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{}$$
$$= b^2x + \frac{(a^2-b^2)\tan(e+fx)}{f} + \frac{(a+b)^2\tan^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.27, size = 57, normalized size = 1.27

$$\frac{3b^2(e+fx)+(a+b)(2a-b+(a-2b)\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (3*b^2*(e + f*x) + (a + b)*(2*a - b + (a - 2*b)*Cos[2*(e + f*x)])*Sec[e + f
*x]^2*Tan[e + f*x])/(3*f)
```

Maple [A]
time = 0.40, size = 76, normalized size = 1.69

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+\frac{2ab\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}+b^2\left(\frac{\left(\tan^3(fx+e)\right)}{3}-\tan(fx+e)+fx+e\right)}{f}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+\frac{2ab\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}+b^2\left(\frac{\left(\tan^3(fx+e)\right)}{3}-\tan(fx+e)+fx+e\right)}{f}$
risch	$b^2x+\frac{4i\left(-3ab\,e^{4i(fx+e)}-3b^2e^{4i(fx+e)}+3a^2e^{2i(fx+e)}-3b^2e^{2i(fx+e)}+a^2-ab-2b^2\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
norman	$b^2x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+b^2x\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b^2x-b^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3b^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3b^2x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2/3*a*b*sin(f*x+e)^3/cos(f*x+e
)^3+b^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e))
```

Maxima [**A**]
time = 0.48, size = 56, normalized size = 1.24

$$\frac{(a^2+2\,ab+b^2)\tan\left(fx+e\right)^3+3\,(fx+e)b^2+3\,(a^2-b^2)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((a^2 + 2*a*b + b^2)*tan(f*x + e)^3 + 3*(f*x + e)*b^2 + 3*(a^2 - b^2)*t
an(f*x + e))/f
```

Fricas [**A**]
time = 0.42, size = 70, normalized size = 1.56

$$\frac{3\,b^2fx\cos\left(fx+e\right)^3+\left(2\,(a^2-ab-2\,b^2)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*f*x*cos(f*x + e)^3 + (2*(a^2 - a*b - 2*b^2)*cos(f*x + e)^2 + a^2
 + 2*a*b + b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 80, normalized size = 1.78

$$\frac{a^2\tan\left(fx+e\right)^3+2\,ab\tan\left(fx+e\right)^3+b^2\tan\left(fx+e\right)^3+3\,(fx+e)b^2+3\,a^2\tan\left(fx+e\right)-3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*tan(f*x + e)^3 + 2*a*b*tan(f*x + e)^3 + b^2*tan(f*x + e)^3 + 3*(f*
x + e)*b^2 + 3*a^2*tan(f*x + e) - 3*b^2*tan(f*x + e))/f
```

Mupad [B]
time = 13.76, size = 46, normalized size = 1.02

$$\frac{\frac{\tan(e+f\,x)^3\,(a+b)^2}{3}-\tan(e+f\,x)\,\left((a+b)^2-2\,a\,(a+b)\right)+b^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^4,x)
```

```
[Out] ((tan(e + f*x)^3*(a + b)^2)/3 - tan(e + f*x)*((a + b)^2 - 2*a*(a + b)) + b^
2*f*x)/f
```

3.298 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{a^2\tan(e+fx)}{f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}$$

```
[Out] a^2*tan(f*x+e)/f+2/3*a*(a+b)*tan(f*x+e)^3/f+1/5*(a+b)^2*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3270, 200}

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/f + (2*a*(a + b)*Tan[e + f*x]^3)/(3*f) + ((a + b)^2*Tan[
e + f*x]^5)/(5*f)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(a+(a+b)x^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int\left(a^2+2a(a+b)x^2+(a+b)^2x^4\right)\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{a^2\tan(e+fx)}{f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 67, normalized size = 1.26

$$\frac{(8a^2-4ab+3b^2+(4a^2-2ab-6b^2)\sec^2(e+fx)+3(a+b)^2\sec^4(e+fx))\tan(e+fx)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 4*a*b + 3*b^2 + (4*a^2 - 2*a*b - 6*b^2)*Sec[e + f*x]^2 + 3*(a + b
)^2*Sec[e + f*x]^4)*Tan[e + f*x])/(15*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(49) = 98$.
time = 0.34, size = 101, normalized size = 1.91

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)+\frac{b^2\left(\sin^5(fx+e)\right)}{5\cos(fx+e)^5}}{f}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)+\frac{b^2\left(\sin^5(fx+e)\right)}{5\cos(fx+e)^5}}{f}$
risch	$\frac{2i(15b^2 e^{8i(fx+e)}-60ab\, e^{6i(fx+e)}+80a^2 e^{4i(fx+e)}+20ab\, e^{4i(fx+e)}+30b^2 e^{4i(fx+e)}+40a^2 e^{2i(fx+e)}-20ab\, e^{2i(fx+e)}+8a^2-}{15f\left(e^{2i(fx+e)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/5*
sin(f*x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3)+1/5*b^2*sin(f*x+e
)^5/cos(f*x+e)^5)
```

Maxima [A]
time = 0.27, size = 58, normalized size = 1.09

$$\frac{3\,(a^2+2\,ab+b^2)\tan\left(fx+e\right)^5+10\,(a^2+ab)\tan\left(fx+e\right)^3+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a^2 + 2*a*b + b^2)*tan(f*x + e)^5 + 10*(a^2 + a*b)*tan(f*x + e)^3
+ 15*a^2*tan(f*x + e))/f
```

Fricas [A]
time = 0.40, size = 83, normalized size = 1.57

$$\frac{\left((8\,a^2-4\,ab+3\,b^2)\cos\left(fx+e\right)^4+2\,(2\,a^2-ab-3\,b^2)\cos\left(fx+e\right)^2+3\,a^2+6\,ab+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((8*a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^4 + 2*(2*a^2 - a*b - 3*b^2)*cos(
f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.49, size = 86, normalized size = 1.62

$$\frac{3\,a^2\tan\left(fx+e\right)^5+6\,ab\tan\left(fx+e\right)^5+3\,b^2\tan\left(fx+e\right)^5+10\,a^2\tan\left(fx+e\right)^3+10\,ab\tan\left(fx+e\right)^3+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^2*tan(f*x + e)^5 + 6*a*b*tan(f*x + e)^5 + 3*b^2*tan(f*x + e)^5 +
10*a^2*tan(f*x + e)^3 + 10*a*b*tan(f*x + e)^3 + 15*a^2*tan(f*x + e))/f
```

Mupad [B]
time = 15.83, size = 44, normalized size = 0.83

$$\frac{a^2\tan(e+f\,x)+\frac{\tan(e+f\,x)^5\,{(a+b)}^2}{5}+\frac{2\,a\tan(e+f\,x)^3\,(a+b)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^6,x)
```

```
[Out] (a^2*tan(e + f*x) + (tan(e + f*x)^5*(a + b)^2)/5 + (2*a*tan(e + f*x)^3*(a +
 b))/3)/f
```

3.299 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{a^2\tan(e+fx)}{f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)(3a+b)\tan^5(e+fx)}{5f}+\frac{(a+b)^2\tan^7(e+fx)}{7f}$$

```
[Out] a^2*tan(f*x+e)/f+1/3*a*(3*a+2*b)*tan(f*x+e)^3/f+1/5*(a+b)*(3*a+b)*tan(f*x+e
)^5/f+1/7*(a+b)^2*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3270, 380}

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^7(e+fx)}{7f}+\frac{(a+b)(3a+b)\tan^5(e+fx)}{5f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/f + (a*(3*a + 2*b)*Tan[e + f*x]^3)/(3*f) + ((a + b)*(3*a
 + b)*Tan[e + f*x]^5)/(5*f) + ((a + b)^2*Tan[e + f*x]^7)/(7*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+(a+b)x^2\right)^2\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^2+a(3a+2b)x^2+(a+b)(3a+b)x^4+(a+b)^2x^6\right)\,dx\right.}{f}\\ &= \frac{a^2\tan(e+fx)}{f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)(3a+b)\tan^5}{5f}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 92, normalized size = 1.15

$$\frac{(48a^2-16ab+6b^2+(24a^2-8ab+3b^2)\sec^2(e+fx)+6(3a^2-ab-4b^2)\sec^4(e+fx)+15(a+b)^2\sec^6(e+fx))\tan(e+fx)}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((48*a^2 - 16*a*b + 6*b^2 + (24*a^2 - 8*a*b + 3*b^2)*Sec[e + f*x]^2 + 6*(3*
a^2 - a*b - 4*b^2)*Sec[e + f*x]^4 + 15*(a + b)^2*Sec[e + f*x]^6)*Tan[e + f*
x])/(105*f)
```

Maple [A]
time = 0.37, size = 149, normalized size = 1.86

method	result
derivativedivides	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{7\cos(fx+e)^7}+\frac{4\left(\sin^3(fx+e)\right)}{35\cos(fx+e)^5}+\frac{8\left(\sin^3(fx+\right.}{105\cos(fx+}\right.}{f}$
default	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{7\cos(fx+e)^7}+\frac{4\left(\sin^3(fx+e)\right)}{35\cos(fx+e)^5}+\frac{8\left(\sin^3(fx+\right.}{105\cos(fx+}\right.}{f}$
risch	$\frac{4i\left(105b^2e^{10i(fx+e)}-560ab\,e^{8i(fx+e)}-105b^2e^{8i(fx+e)}+840a^2e^{6i(fx+e)}+280ab\,e^{6i(fx+e)}+210b^2e^{6i(fx+e)}+504a^2e^{4i(fx\right.}{105f\left(e^{2i(fx+e)}+1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan
(f*x+e)+2*a*b*(1/7*sin(f*x+e)^3/cos(f*x+e)^7+4/35*sin(f*x+e)^3/cos(f*x+e)^5
+8/105*sin(f*x+e)^3/cos(f*x+e)^3)+b^2*(1/7*sin(f*x+e)^5/cos(f*x+e)^7+2/35*s
in(f*x+e)^5/cos(f*x+e)^5))
```

Maxima [A]
time = 0.28, size = 85, normalized size = 1.06

$$\frac{15\,(a^2+2\,ab+b^2)\tan\left(fx+e\right)^7+21\,(3\,a^2+4\,ab+b^2)\tan\left(fx+e\right)^5+35\,(3\,a^2+2\,ab)\tan\left(fx+e\right)^3+105\,a^2\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*(a^2 + 2*a*b + b^2)*tan(f*x + e)^7 + 21*(3*a^2 + 4*a*b + b^2)*tan
(f*x + e)^5 + 35*(3*a^2 + 2*a*b)*tan(f*x + e)^3 + 105*a^2*tan(f*x + e))/f
```

Fricas [A]
time = 0.43, size = 108, normalized size = 1.35

$$\frac{\left(2\left(24a^2-8ab+3b^2\right)\cos\left(fx+e\right)^6+\left(24a^2-8ab+3b^2\right)\cos\left(fx+e\right)^4+6\left(3a^2-ab-4b^2\right)\cos\left(fx+e\right)^2+15a^2+30ab+15b^2\right)\sin\left(fx+e\right)}{105f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(24*a^2 - 8*a*b + 3*b^2)*cos(f*x + e)^6 + (24*a^2 - 8*a*b + 3*b^2)
*cos(f*x + e)^4 + 6*(3*a^2 - a*b - 4*b^2)*cos(f*x + e)^2 + 15*a^2 + 30*a*b
+ 15*b^2)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**8*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.48, size = 127, normalized size = 1.59

$$\frac{15a^2\tan\left(fx+e\right)^7+30ab\tan\left(fx+e\right)^7+15b^2\tan\left(fx+e\right)^7+63a^2\tan\left(fx+e\right)^5+84ab\tan\left(fx+e\right)^5+21b^2\tan\left(fx+e\right)^5+105a^2\tan\left(fx+e\right)^3+70ab\tan\left(fx+e\right)^3+105a^2\tan\left(fx+e\right)}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*a^2*tan(f*x + e)^7 + 30*a*b*tan(f*x + e)^7 + 15*b^2*tan(f*x + e)^
7 + 63*a^2*tan(f*x + e)^5 + 84*a*b*tan(f*x + e)^5 + 21*b^2*tan(f*x + e)^5 +
 105*a^2*tan(f*x + e)^3 + 70*a*b*tan(f*x + e)^3 + 105*a^2*tan(f*x + e))/f
```

Mupad [B]
time = 15.14, size = 72, normalized size = 0.90

$$\frac{a^2\tan(e+f\,x)+\frac{\tan(e+f\,x)^7\,(a+b)^2}{7}+\tan(e+f\,x)^5\,\left(\frac{3\,a^2}{5}+\frac{4\,a\,b}{5}+\frac{b^2}{5}\right)+\frac{a\tan(e+f\,x)^3\,(3\,a+2\,b)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^8,x)
```

```
[Out] (a^2*tan(e + f*x) + (tan(e + f*x)^7*(a + b)^2)/7 + tan(e + f*x)^5*((4*a*b)/
5 + (3*a^2)/5 + b^2/5) + (a*tan(e + f*x)^3*(3*a + 2*b))/3)/f
```

3.300 $\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=106

$$\frac{a^2\tan(e+fx)}{f}+\frac{2a(2a+b)\tan^3(e+fx)}{3f}+\frac{(6a^2+6ab+b^2)\tan^5(e+fx)}{5f}+\frac{2(a+b)(2a+b)\tan^7(e+fx)}{7f}+$$

```
[Out] a^2*tan(f*x+e)/f+2/3*a*(2*a+b)*tan(f*x+e)^3/f+1/5*(6*a^2+6*a*b+b^2)*tan(f*x
+e)^5/f+2/7*(a+b)*(2*a+b)*tan(f*x+e)^7/f+1/9*(a+b)^2*tan(f*x+e)^9/f
```

Rubi [A]

time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3270, 380}

$$\frac{(6a^2+6ab+b^2)\tan^5(e+fx)}{5f}+\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^9(e+fx)}{9f}+\frac{2(a+b)(2a+b)\tan^7(e+fx)}{7f}+\frac{2a(2a+b)\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^10*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/f + (2*a*(2*a + b)*Tan[e + f*x]^3)/(3*f) + ((6*a^2 + 6*a
*b + b^2)*Tan[e + f*x]^5)/(5*f) + (2*(a + b)*(2*a + b)*Tan[e + f*x]^7)/(7*f
) + ((a + b)^2*Tan[e + f*x]^9)/(9*f)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int (1+x^2)^2\left(a+(a+b)x^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^2+2a(2a+b)x^2+(6a^2+6ab+b^2)\,x^4+2(a+b)(2\right.\right.}{f}\\ &= \frac{a^2\tan(e+fx)}{f}+\frac{2a(2a+b)\tan^3(e+fx)}{3f}+\frac{(6a^2+6ab+b^2)\,\text{t}}{5f}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 107, normalized size = 1.01

$$\frac{\sec^9(e+fx)\left(252(8a^2+8ab+3b^2)\sin(e+fx)+336(4a^2-ab-b^2)\sin(3(e+fx))+(16a^2-4ab+b^2)(36\sin(5(e+fx))+9\sin(7(e+fx))+\sin(9(e+fx)))\right)}{10080f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^10*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (Sec[e + f*x]^9*(252*(8*a^2 + 8*a*b + 3*b^2)*Sin[e + f*x] + 336*(4*a^2 - a*
b - b^2)*Sin[3*(e + f*x)] + (16*a^2 - 4*a*b + b^2)*(36*Sin[5*(e + f*x)] + 9
*Sin[7*(e + f*x)] + Sin[9*(e + f*x)])))/(10080*f)
```

Maple [A]
time = 0.38, size = 195, normalized size = 1.84

method	result
derivativedivides	$\frac{-a^2\left(-\frac{128}{315}-\frac{\left(\sec^8(fx+e)\right)}{9}-\frac{8\left(\sec^6(fx+e)\right)}{63}-\frac{16\left(\sec^4(fx+e)\right)}{105}-\frac{64\left(\sec^2(fx+e)\right)}{315}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{9\cos(fx+e)^9}+\frac{2\left(\sin^3(fx\right.}{21\cos(fx\right.}\right.}{f}$
default	$\frac{-a^2\left(-\frac{128}{315}-\frac{\left(\sec^8(fx+e)\right)}{9}-\frac{8\left(\sec^6(fx+e)\right)}{63}-\frac{16\left(\sec^4(fx+e)\right)}{105}-\frac{64\left(\sec^2(fx+e)\right)}{315}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{9\cos(fx+e)^9}+\frac{2\left(\sin^3(fx\right.}{21\cos(fx\right.}\right.}{f}$
risch	$16i\left(210b^2e^{12i(fx+e)}-1260ab\,e^{10i(fx+e)}-315b^2e^{10i(fx+e)}+2016a^2e^{8i(fx+e)}+756ab\,e^{8i(fx+e)}+441b^2e^{8i(fx+e)}+1344a^2e\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^2*(-128/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4-
64/315*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/9*sin(f*x+e)^3/cos(f*x+e)^9+2/21*s
in(f*x+e)^3/cos(f*x+e)^7+8/105*sin(f*x+e)^3/cos(f*x+e)^5+16/315*sin(f*x+e)^
3/cos(f*x+e)^3)+b^2*(1/9*sin(f*x+e)^5/cos(f*x+e)^9+4/63*sin(f*x+e)^5/cos(f*
x+e)^7+8/315*sin(f*x+e)^5/cos(f*x+e)^5))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 1.02

$$\frac{35\,(a^2+2\,ab+b^2)\tan(fx+e)^9+90\,(2\,a^2+3\,ab+b^2)\tan(fx+e)^7+63\,(6\,a^2+6\,ab+b^2)\tan(fx+e)^5+210\,(2\,a^2+ab)\tan(fx+e)^3+315\,a^2\tan(fx+e)}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(a^2 + 2*a*b + b^2)*tan(f*x + e)^9 + 90*(2*a^2 + 3*a*b + b^2)*tan
(f*x + e)^7 + 63*(6*a^2 + 6*a*b + b^2)*tan(f*x + e)^5 + 210*(2*a^2 + a*b)*t
an(f*x + e)^3 + 315*a^2*tan(f*x + e))/f
```

Fricas [A]
time = 0.41, size = 128, normalized size = 1.21

$$\frac{\left(8 \left(16 a^{2}-4 a b+b^{2}\right) \cos \left(f x+e\right)^{8}+4 \left(16 a^{2}-4 a b+b^{2}\right) \cos \left(f x+e\right)^{6}+3 \left(16 a^{2}-4 a b+b^{2}\right) \cos \left(f x+e\right)^{4}+10 \left(4 a^{2}-a b-5 b^{2}\right) \cos \left(f x+e\right)^{2}+35 a^{2}+70 a b+35 b^{2}\right) \sin \left(f x+e\right)}{315 f \cos \left(f x+e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(16*a^2 - 4*a*b + b^2)*cos(f*x + e)^8 + 4*(16*a^2 - 4*a*b + b^2)*c
os(f*x + e)^6 + 3*(16*a^2 - 4*a*b + b^2)*cos(f*x + e)^4 + 10*(4*a^2 - a*b -
 5*b^2)*cos(f*x + e)^2 + 35*a^2 + 70*a*b + 35*b^2)*sin(f*x + e)/(f*cos(f*x
+ e)^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**10*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 168, normalized size = 1.58

$$\frac{35 a^{2} \tan \left(f x+e\right)^{9}+70 a b \tan \left(f x+e\right)^{9}+35 b^{2} \tan \left(f x+e\right)^{9}+180 a^{2} \tan \left(f x+e\right)^{7}+270 a b \tan \left(f x+e\right)^{7}+90 b^{2} \tan \left(f x+e\right)^{7}+378 a^{2} \tan \left(f x+e\right)^{5}+378 a b \tan \left(f x+e\right)^{5}+63 b^{2} \tan \left(f x+e\right)^{5}+420 a^{2} \tan \left(f x+e\right)^{3}+210 a b \tan \left(f x+e\right)^{3}+315 a^{2} \tan \left(f x+e\right)}{315 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*a^2*tan(f*x + e)^9 + 70*a*b*tan(f*x + e)^9 + 35*b^2*tan(f*x + e)^
9 + 180*a^2*tan(f*x + e)^7 + 270*a*b*tan(f*x + e)^7 + 90*b^2*tan(f*x + e)^7
 + 378*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)^5
+ 420*a^2*tan(f*x + e)^3 + 210*a*b*tan(f*x + e)^3 + 315*a^2*tan(f*x + e))/f
```

Mupad [B]
time = 14.22, size = 94, normalized size = 0.89

$$\frac{a^2 \tan(e+f\,x)+\frac{\tan(e+f\,x)^9\,(a+b)^2}{9}+\tan(e+f\,x)^5\,\left(\frac{6\,a^2}{5}+\frac{6\,a\,b}{5}+\frac{b^2}{5}\right)+\tan(e+f\,x)^7\,\left(\frac{4\,a^2}{7}+\frac{6\,a\,b}{7}+\frac{2\,b^2}{7}\right)+\frac{2\,a\,\tan(e+f\,x)^3\,(2\,a+b)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^10,x)
```

```
[Out] (a^2*tan(e + f*x) + (tan(e + f*x)^9*(a + b)^2)/9 + tan(e + f*x)^5*((6*a*b)/
5 + (6*a^2)/5 + b^2/5) + tan(e + f*x)^7*((6*a*b)/7 + (4*a^2)/7 + (2*b^2)/7)
 + (2*a*tan(e + f*x)^3*(2*a + b))/3)/f
```

3.301 $\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=78

$$\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{(a^2+3ab+3b^2)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}$$

```
[Out] -(a^2+3*a*b+3*b^2)*sin(x)/b^3+1/3*(a+3*b)*sin(x)^3/b^2-1/5*sin(x)^5/b+(a+b)
^3*arctan(sin(x)*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3269, 398, 211}

$$-\frac{(a^2+3ab+3b^2)\sin(x)}{b^3}+\frac{(a+b)^3\text{ArcTan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)^3*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)) - ((a^2 + 3*
a*b + 3*b^2)*Sin[x])/b^3 + ((a + 3*b)*Sin[x]^3)/(3*b^2) - Sin[x]^5/(5*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{(1-x^2)^3}{a+bx^2}\,dx, x, \sin(x)\right)$$

$$= \text{Subst}\left(\int \left(-\frac{a^2+3ab+3b^2}{b^3}+\frac{(a+3b)x^2}{b^2}-\frac{x^4}{b}+\frac{a^3+3a^2b+3ab^2+b^3}{b^3\,(a+bx^2)}\right)\,dx, x, \text{sin}$$

$$= -\frac{(a^2+3ab+3b^2)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}+\frac{(a+b)^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x\right.}{b^3}$$

$$= \frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{(a^2+3ab+3b^2)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}$$

Mathematica [A]
time = 0.20, size = 109, normalized size = 1.40

$$\frac{-120(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\,\csc(x)}{\sqrt{b}}\right)+120(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)-2\sqrt{a}\,\sqrt{b}\,(120a^2+340ab+309b^2+4b(5a+12b)\cos(2x)+3b^2\cos(4x))\sin(x)}{240\sqrt{a}\,b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/(a + b*Sin[x]^2),x]
```

```
[Out] (-120*(a + b)^3*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]] + 120*(a + b)^3*ArcTan[(Sq
rt[b]*Sin[x])/Sqrt[a]] - 2*Sqrt[a]*Sqrt[b]*(120*a^2 + 340*a*b + 309*b^2 + 4
*b*(5*a + 12*b)*Cos[2*x] + 3*b^2*Cos[4*x])*Sin[x])/(240*Sqrt[a]*b^(7/2))
```

Maple [A]
time = 0.33, size = 91, normalized size = 1.17

method	result
default	$-\frac{\frac{(\sin^5(x))b^2}{5}-\frac{ab(\sin^3(x))}{3}-b^2(\sin^3(x))+a^2\sin(x)+3ab\sin(x)+3b^2\sin(x)}{b^3}+\frac{(a^3+3a^2b+3a\,b^2+b^3)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{ie^{ix}a^2}{2b^3}+\frac{11ie^{ix}a}{8b^2}+\frac{19ie^{ix}}{16b}-\frac{ie^{-ix}a^2}{2b^3}-\frac{11ie^{-ix}a}{8b^2}-\frac{19ie^{-ix}}{16b}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)a^3}{2\sqrt{-ab}\,b^3}-\frac{3\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(1/5*sin(x)^5*b^2-1/3*a*b*sin(x)^3-b^2*sin(x)^3+a^2*sin(x)+3*a*b*sin
(x)+3*b^2*sin(x))+(a^3+3*a^2*b+3*a*b^2+b^3)/b^3/(a*b)^(1/2)*arctan(b*sin(x)
/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 86, normalized size = 1.10

$$\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}-\frac{3\,b^2\sin(x)^5-5\,(ab+3\,b^2)\sin(x)^3+15\,(a^2+3\,ab+3\,b^2)\sin(x)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
 - 1/15*(3*b^2*sin(x)^5 - 5*(a*b + 3*b^2)*sin(x)^3 + 15*(a^2 + 3*a*b + 3*b^2
)*sin(x))/b^3
```

Fricas [A]
time = 0.43, size = 233, normalized size = 2.99

$$\left[-\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\,(3\,ab^3\cos(x)^4+15\,a^3b+40\,a^2b^2+33\,ab^3+(5\,a^2b^2+9\,ab^3)\cos(x)^2)\sin(x)}{30\,ab^4},\ \frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)-(3\,ab^3\cos(x)^4+15\,a^3b+40\,a^2b^2+33\,ab^3+(5\,a^2b^2+9\,ab^3)\cos(x)^2)\sin(x)}{15\,ab^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*
sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(3*a*b^3*cos(x)^4 + 15
*a^3*b + 40*a^2*b^2 + 33*a*b^3 + (5*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/(a
*b^4), 1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*arctan(sqrt(a*b)*
sin(x)/a) - (3*a*b^3*cos(x)^4 + 15*a^3*b + 40*a^2*b^2 + 33*a*b^3 + (5*a^2*b
^2 + 9*a*b^3)*cos(x)^2)*sin(x))/(a*b^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 98, normalized size = 1.26

$$\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}-\frac{3\,b^4\sin(x)^5-5\,ab^3\sin(x)^3-15\,b^4\sin(x)^3+15\,a^2b^2\sin(x)+45\,ab^3\sin(x)+45\,b^4\sin(x)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
- 1/15*(3*b^4*sin(x)^5 - 5*a*b^3*sin(x)^3 - 15*b^4*sin(x)^3 + 15*a^2*b^2*si
n(x) + 45*a*b^3*sin(x) + 45*b^4*sin(x))/b^5
```

Mupad [B]
time = 0.12, size = 99, normalized size = 1.27

$$\sin(x)^3\left(\frac{a}{3\,b^2}+\frac{1}{b}\right)-\sin(x)\left(\frac{3}{b}+\frac{a\left(\frac{a}{b^2}+\frac{3}{b}\right)}{b}\right)-\frac{\sin(x)^5}{5\,b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,(a+b)^3}{\sqrt{a}\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}\right)\,(a+b)^3}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a + b*sin(x)^2),x)
```

```
[Out] sin(x)^3*(a/(3*b^2) + 1/b) - sin(x)*(3/b + (a*(a/b^2 + 3/b))/b) - sin(x)^5/
(5*b) + (atan((b^(1/2)*sin(x)*(a + b)^3)/(a^(1/2)*(3*a*b^2 + 3*a^2*b + a^3
+ b^3)))*(a + b)^3)/(a^(1/2)*b^(7/2))
```

3.302 $\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=87

$$-\frac{(8a^2+20ab+15b^2)\,x}{8b^3}+\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}-\frac{(4a+7b)\cos(x)\sin(x)}{8b^2}-\frac{\cos^3(x)\sin(x)}{4b}$$

```
[Out] -1/8*(8*a^2+20*a*b+15*b^2)*x/b^3-1/8*(4*a+7*b)*cos(x)*sin(x)/b^2-1/4*cos(x)
^3*sin(x)/b+(a+b)^(5/2)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/b^3/a^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3270, 425, 541, 536, 209, 211}

$$-\frac{x(8a^2+20ab+15b^2)}{8b^3}+\frac{(a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}-\frac{(4a+7b)\sin(x)\cos(x)}{8b^2}-\frac{\sin(x)\cos^3(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] -1/8*((8*a^2 + 20*a*b + 15*b^2)*x)/b^3 + ((a + b)^(5/2)*ArcTan[(Sqrt[a + b]
*Tan[x])/Sqrt[a]])/(Sqrt[a]*b^3) - ((4*a + 7*b)*Cos[x]*Sin[x])/(8*b^2) - (C
os[x]^3*Sin[x])/(4*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)^3\,(a+(a+b)x^2)}\,dx, x, \tan(x)\right) \\
&= -\frac{\cos^3(x)\sin(x)}{4b} + \frac{\text{Subst}\left(\int \frac{a+4b-3(a+b)x^2}{(1+x^2)^2(a+(a+b)x^2)}\,dx, x, \tan(x)\right)}{4b} \\
&= -\frac{(4a+7b)\cos(x)\sin(x)}{8b^2} - \frac{\cos^3(x)\sin(x)}{4b} + \frac{\text{Subst}\left(\int \frac{4a^2+9ab+8b^2-(a+b)(4a+7b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \text{t}\right.}{8b^2} \\
&= -\frac{(4a+7b)\cos(x)\sin(x)}{8b^2} - \frac{\cos^3(x)\sin(x)}{4b} + \frac{(a+b)^3\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{b^3} \\
&= -\frac{(8a^2+20ab+15b^2)\,x}{8b^3} + \frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3} - \frac{(4a+7b)\cos(x)\sin(x}{8b^2}
\end{aligned}$$

Mathematica [A]

time = 0.14, size = 79, normalized size = 0.91

$$\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}-\frac{4(8a^2+20ab+15b^2)\,x+8b(a+2b)\sin(2x)+b^2\sin(4x)}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b^3) - (4*(8*
a^2 + 20*a*b + 15*b^2)*x + 8*b*(a + 2*b)*Sin[2*x] + b^2*Sin[4*x])/(32*b^3)
```

Maple [A]
time = 0.29, size = 96, normalized size = 1.10

method	result
default	$\frac{(a+b)^3\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{b^3\sqrt{a(a+b)}}-\frac{\frac{\left(\frac{1}{2}ab+\frac{7}{8}b^2\right)\left(\tan^3(x)\right)+\left(\frac{1}{2}ab+\frac{9}{8}b^2\right)\tan(x)}{\left(\tan^2(x)+1\right)^2}+\frac{\left(8a^2+20ab+15b^2\right)\arctan(\tan(x))}{8}}{b^3}$
risch	$-\frac{x\,a^2}{b^3}-\frac{5ax}{2b^2}-\frac{15x}{8b}+\frac{i\mathrm{e}^{2ix}a}{8b^2}+\frac{i\mathrm{e}^{2ix}}{4b}-\frac{i\mathrm{e}^{-2ix}a}{8b^2}-\frac{i\mathrm{e}^{-2ix}}{4b}+\frac{a\sqrt{-a(a+b)}\,\ln\left(\mathrm{e}^{2ix}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] (a+b)^3/b^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))-1/b^3*(((1
/2*a*b+7/8*b^2)*tan(x)^3+(1/2*a*b+9/8*b^2)*tan(x))/(tan(x)^2+1)^2+1/8*(8*a^
2+20*a*b+15*b^2)*arctan(tan(x)))
```

Maxima [A]
time = 0.50, size = 114, normalized size = 1.31

$$-\frac{(4\,a+7\,b)\tan(x)^3+(4\,a+9\,b)\tan(x)}{8\left(b^2\tan(x)^4+2\,b^2\tan(x)^2+b^2\right)}-\frac{(8\,a^2+20\,ab+15\,b^2)x}{8\,b^3}+\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a + 7*b)*tan(x)^3 + (4*a + 9*b)*tan(x))/(b^2*tan(x)^4 + 2*b^2*tan(
x)^2 + b^2) - 1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^3)
```

Fricas [A]
time = 0.43, size = 312, normalized size = 3.59

$$\left[\frac{2\,(a^2+2\,ab+b^2)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+5\,ab+b^2)\cos(x)^2-4\,\left((2\,a^2+ab)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2+2\,ab+b^2}{b^2\cos(x)^4-2\,(ab+b^2)\cos(x)^2+a^2+2\,ab+b^2}\right)-(8\,a^2+20\,ab+15\,b^2)x-\left(2\,b^2\cos(x)^3+(4\,ab+7\,b^2)\cos(x)\right)\sin(x)}{8\,b^3}, -\frac{4\,(a^2+2\,ab+b^2)\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2\,(a+b)\cos(x)\sin(x)}\right)+(8\,a^2+20\,ab+15\,b^2)x+\left(2\,b^2\cos(x)^3+(4\,ab+7\,b^2)\cos(x)\right)\sin(x)}{8\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos
(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2
 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4
- 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - (8*a^2 + 20*a*b + 15*b^2)*
x - (2*b^2*cos(x)^3 + (4*a*b + 7*b^2)*cos(x))*sin(x))/b^3, -1/8*(4*(a^2 + 2
*a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)*sqrt((a
 + b)/a)/((a + b)*cos(x)*sin(x))) + (8*a^2 + 20*a*b + 15*b^2)*x + (2*b^2*co
s(x)^3 + (4*a*b + 7*b^2)*cos(x))*sin(x))/b^3]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 131, normalized size = 1.51

$$-\frac{(8\,a^2+20\,ab+15\,b^2)x}{8\,b^3}+\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}\,b^3}-\frac{4\,a\tan(x)^3+7\,b\tan(x)^3+4\,a\tan(x)+9\,b\tan(x)}{8\,\left(\tan(x)^2+1\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*
floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^2 +
a*b)))/(sqrt(a^2 + a*b)*b^3) - 1/8*(4*a*tan(x)^3 + 7*b*tan(x)^3 + 4*a*tan(x
) + 9*b*tan(x))/((tan(x)^2 + 1)^2*b^2)
```

Mupad [B]
time = 15.45, size = 1804, normalized size = 20.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^6/(a + b*sin(x)^2),x)

[Out] (atan(((((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(32*b^4) - ((((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/b^6 - (tan(x)*(a*b*20i + a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3))*(a*b*20i + a^2*8i + b^2*15i)*1i)/(16*b^3) + (((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(32*b^4) + ((((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/b^6 + (tan(x)*(a*b*20i + a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3))*(a*b*20i + a^2*8i + b^2*15i)*1i)/(16*b^3))/(((725*a*b^7)/32 + (37*a^7*b)/4 + a^8 + (105*b^8)/32 + (1093*a^2*b^6)/16 + (1881*a^3*b^5)/16 + (4045*a^4*b^4)/32 + (2785*a^5*b^3)/32 + (75*a^6*b^2)/2)/b^6 - (((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(32*b^4) - ((((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/b^6 - (tan(x)*(a*b*20i + a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3) + (((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(32*b^4) + ((((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/b^6 + (tan(x)*(a*b*20i + a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3)))*(a*b*20i + a^2*8i + b^2*15i)*1i)/(8*b^3) - ((tan(x)^3*(4*a + 7*b))/(8*b^2) + (tan(x)*(4*a + 9*b))/(8*b^2))/(2*tan(x)^2 + tan(x)^4 + 1) + (atan((((-a*(a + b)^5)^(1/2)*((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(64*b^4) - ((-a*(a + b)^5)^(1/2)*(((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*(-a*(a + b)^5)^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*a*b^7)))/(2*a*b^3))*1i)/(a*b^3) + ((-a*(a + b)^5)^(1/2)*((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(64*b^4) + ((-a*(a + b)^5)^(1/2)*(((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) + (tan(x)*(-a*(a + b)^5)^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*a*b^7)))/(2*a*b^3))*1i)/(a*b^3))/(((725*a*b^7)/32 + (37*a^7*b)/4 + a^8 + (105*b^8)/32 + (1093*a^2*b^6)/16 + (1881*a^3*b^5)/16 + (4045*a^4*b^4)/32 + (2785*a^5*b^3)/32 + (75*a^6*b^2)/2)/b^6 - ((-a*(a + b)^5)^(1/2)*((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(64*b^4) - ((-a*(a + b)^5)^(1/2)*(((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*(-a*(a + b)^5)^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*a*b^

```
7)))/(2*a*b^3)))/(a*b^3) + ((-a*(a + b)^5)^(1/2)*((tan(x)*(1723*a*b^6 + 960
*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3
136*a^5*b^2))/(64*b^4) + ((-a*(a + b)^5)^(1/2)*(((25*a*b^9)/2 + 4*b^10 + 15
*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) + (tan(x)*(-a*(a + b)^5)^(1/
2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*a*b^7)))/(2*a*
b^3)))/(a*b^3)))*(-a*(a + b)^5)^(1/2)*1i)/(a*b^3)
```

3.303 $\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=54

$$\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}-\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}$$

```
[Out] -(a+2*b)*sin(x)/b^2+1/3*sin(x)^3/b+(a+b)^2*arctan(sin(x)*b^(1/2)/a^(1/2))/b
^(5/2)/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 398, 211}

$$\frac{(a+b)^2\text{ArcTan}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}-\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - ((a + 2*b)
*Sin[x])/b^2 + Sin[x]^3/(3*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{(1-x^2)^2}{a+bx^2}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{a+2b}{b^2}+\frac{x^2}{b}+\frac{a^2+2ab+b^2}{b^2(a+bx^2)}\right)dx, x, \sin(x)\right)\\
&= -\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}+\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{b^2}\\
&= \frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}-\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 84, normalized size = 1.56

$$\frac{-6(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)+6(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)-2\sqrt{a}\sqrt{b}\,(6a+11b+b\cos(2x))\sin(x)}{12\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] (-6*(a + b)^2*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]] + 6*(a + b)^2*ArcTan[(Sqrt[b
]*Sin[x])/Sqrt[a]] - 2*Sqrt[a]*Sqrt[b]*(6*a + 11*b + b*Cos[2*x])*Sin[x])/(1
2*Sqrt[a]*b^(5/2))
```

Maple [A]
time = 0.27, size = 54, normalized size = 1.00

method	result
default	$-\frac{-\frac{b\left(\sin^3(x)\right)}{3}+\sin(x)a+2\sin(x)b}{b^2}+\frac{(a^2+2ab+b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{ie^{ix}a}{2b^2}+\frac{7ie^{ix}}{8b}-\frac{ie^{-ix}a}{2b^2}-\frac{7ie^{-ix}}{8b}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)a^2}{2\sqrt{-ab}\,b^2}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)a}{\sqrt{-ab}\,b}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/3*b*sin(x)^3+sin(x)*a+2*sin(x)*b)+(a^2+2*a*b+b^2)/b^2/(a*b)^(1/2
)*arctan(b*sin(x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 52, normalized size = 0.96

$$\frac{\left(a^{2}+2\,ab+b^{2}\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{b\sin\left(x\right)^{3}-3\left(a+2\,b\right)\sin\left(x\right)}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] (a^2 + 2*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*sin
(x)^3 - 3*(a + 2*b)*sin(x))/b^2
```

Fricas [A]
time = 0.42, size = 159, normalized size = 2.94

$$\left[-\frac{3\left(a^{2}+2\,ab+b^{2}\right)\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^{2}+2\,\sqrt{-ab}\,\sin(x)+a-b}{b\cos(x)^{2}-a-b}\right)+2\left(ab^{2}\cos\left(x\right)^{2}+3\,a^{2}b+5\,ab^{2}\right)\sin\left(x\right)}{6\,ab^{3}},\frac{3\left(a^{2}+2\,ab+b^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,\sin(x)}{a}\right)-\left(ab^{2}\cos\left(x\right)^{2}+3\,a^{2}b+5\,ab^{2}\right)\sin\left(x\right)}{3\,ab^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2 + 2*a*b + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin
(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(a*b^2*cos(x)^2 + 3*a^2*b + 5*a*b^2)
*sin(x))/(a*b^3), 1/3*(3*(a^2 + 2*a*b + b^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin
(x)/a) - (a*b^2*cos(x)^2 + 3*a^2*b + 5*a*b^2)*sin(x))/(a*b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 58, normalized size = 1.07

$$\frac{\left(a^{2}+2\,ab+b^{2}\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{b^{2}\sin\left(x\right)^{3}-3\,ab\sin\left(x\right)-6\,b^{2}\sin\left(x\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (a^2 + 2*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*s
in(x)^3 - 3*a*b*sin(x) - 6*b^2*sin(x))/b^3
```

Mupad [B]
time = 14.30, size = 65, normalized size = 1.20

$$\frac{\sin(x)^3}{3\,b}-\sin(x)\,\left(\frac{a}{b^2}+\frac{2}{b}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,(a+b)^2}{\sqrt{a}\,(a^2+2\,a\,b+b^2)}\right)\,(a+b)^2}{\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a + b*sin(x)^2),x)
```

```
[Out] sin(x)^3/(3*b) - sin(x)*(a/b^2 + 2/b) + (atan((b^(1/2)*sin(x)*(a + b)^2)/(a
^(1/2)*(2*a*b + a^2 + b^2)))*(a + b)^2)/(a^(1/2)*b^(5/2))
```

3.304 $\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{(2a+3b)x}{2b^2}+\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}-\frac{\cos(x)\sin(x)}{2b}$$

```
[Out] -1/2*(2*a+3*b)*x/b^2-1/2*cos(x)*sin(x)/b+(a+b)^(3/2)*arctan((a+b)^(1/2)*tan
(x)/a^(1/2))/b^2/a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3270, 425, 536, 209, 211}

$$\frac{(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}-\frac{x(2a+3b)}{2b^2}-\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] -1/2*((2*a + 3*b)*x)/b^2 + ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[
a]])/(Sqrt[a]*b^2) - (Cos[x]*Sin[x])/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)^2\,(a+(a+b)x^2)}\,dx, x, \tan(x)\right)\\
&= -\frac{\cos(x)\sin(x)}{2b} + \frac{\text{Subst}\left(\int \frac{a+2b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \tan(x)\right)}{2b}\\
&= -\frac{\cos(x)\sin(x)}{2b} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{b^2} - \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{2b^2}\\
&= -\frac{(2a+3b)x}{2b^2} + \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2} - \frac{\cos(x)\sin(x)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 0.93

$$\frac{\frac{4(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}} - 2(2ax+3bx+b\cos(x)\sin(x))}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] ((4*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/Sqrt[a] - 2*(2*a*x
+ 3*b*x + b*Cos[x]*Sin[x]))/(4*b^2)
```

Maple [A]
time = 0.26, size = 63, normalized size = 1.07

method	result
default	$-\frac{\frac{b\tan(x)}{2\left(\tan^2(x)\right)+2}+\frac{(2a+3b)\arctan(\tan(x))}{2}}{b^2}+\frac{(a+b)^2\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}$
risch	$-\frac{ax}{b^2}-\frac{3x}{2b}+\frac{i e^{2ix}}{8b}-\frac{i e^{-2ix}}{8b}+\frac{\sqrt{-a(a+b)}\ln\left(e^{2ix}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2b^2}+\frac{\sqrt{-a(a+b)}\ln\left(e^{2ix}+\frac{2i}{}\right.}{2ab}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(1/2*b*tan(x)/(tan(x)^2+1)+1/2*(2*a+3*b)*arctan(tan(x)))+(a+b)^2/b^2
/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [A]
time = 0.50, size = 64, normalized size = 1.08

$$-\frac{(2a+3b)x}{2b^2}-\frac{\tan(x)}{2\left(b\tan(x)^2+b\right)}+\frac{\left(a^2+2ab+b^2\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 - 1/2*tan(x)/(b*tan(x)^2 + b) + (a^2 + 2*a*b + b^2)*
arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^2)
```

Fricas [A]
time = 0.42, size = 239, normalized size = 4.05

$$\left[-\frac{2b\cos(x)\sin(x)-(a+b)\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+5ab+b^2\right)\cos(x)^2-4\left(\left(2a^2+ab\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2+2ab+b^2}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2ab+b^2}\right)+2(2a+3b)x}{4b^2},\ -\frac{b\cos(x)\sin(x)+(a+b)\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2a+b)\cos(x)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2(a+b)\cos(x)\sin(x)}\right)+(2a+3b)x}{2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b*cos(x)*sin(x) - (a + b)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b
^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3
 - (a^2 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*co
s(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) + 2*(2*a + 3*b)*x)/b^
```

```
2, -1/2*(b*cos(x)*sin(x) + (a + b)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*co
s(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + (2*a + 3*b)*x)/b
^2]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 92, normalized size = 1.56

$$-\frac{(2\,a+3\,b)x}{2\,b^2}+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a^2+2\,ab+b^2)}{\sqrt{a^2+ab}\,b^2}-\frac{\tan\left(x\right)}{2\left(\tan\left(x\right)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 + (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*t
an(x) + b*tan(x))/sqrt(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*b^
2) - 1/2*tan(x)/((tan(x)^2 + 1)*b)
```

Mupad [**B**]
time = 14.67, size = 119, normalized size = 2.02

$$-\frac{3\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{2\,b}-\frac{a\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{b^2}-\frac{\cos\left(x\right)\,\sin\left(x\right)}{2\,b}-\frac{\mathrm{atanh}\left(\frac{\sin(x)\,\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-a\,b^3}}{\cos(x)\,a^2+b\,\cos(x)\,a}\right)\,\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-a\,b^3}}{a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a + b*sin(x)^2),x)
```

```
[Out] - (3*atan(sin(x)/cos(x)))/(2*b) - (a*atan(sin(x)/cos(x)))/b^2 - (cos(x)*sin
(x))/(2*b) - (atanh((sin(x)*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)^(1/2))/(a
^2*cos(x) + a*b*cos(x)))*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)^(1/2))/(a*b^
2)
```

3.305 $\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=36

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sin(x)}{b}$$

```
[Out] -sin(x)/b+(a+b)*arctan(sin(x)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 396, 211}

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sin(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Sin[x]/b
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1-x^2}{a+bx^2}\,dx, x, \sin(x)\right)$$
$$= -\frac{\sin(x)}{b} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{b}$$
$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} - \frac{\sin(x)}{b}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 1.00

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} - \frac{\sin(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a + b*Sin[x]^2),x]
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Sin[x]/b
```

Maple [A]
time = 0.21, size = 31, normalized size = 0.86

method	result
default	$-\frac{\sin(x)}{b}+\frac{(a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{b\sqrt{ab}}$
risch	$\frac{i e^{ix}}{2b}-\frac{i e^{-ix}}{2b}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)a}{2\sqrt{-ab}\,b}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}}+\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)a}{2\sqrt{-ab}\,b}+\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-\right.}{2\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] -sin(x)/b+(a+b)/b/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 30, normalized size = 0.83

$$\frac{(a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} - \frac{\sin\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] (a + b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b) - sin(x)/b
```

Fricas [A]
time = 0.43, size = 101, normalized size = 2.81

$$\left[-\frac{2\,ab\sin\left(x\right)+\sqrt{-ab}\,\left(a+b\right)\log\left(-\frac{b\cos\left(x\right)^{2}+2\,\sqrt{-ab}\,\sin\left(x\right)+a-b}{b\cos\left(x\right)^{2}-a-b}\right)}{2\,ab^{2}},-\frac{ab\sin\left(x\right)-\sqrt{ab}\,\left(a+b\right)\arctan\left(\frac{\sqrt{ab}\,\sin\left(x\right)}{a}\right)}{ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sin(x) + sqrt(-a*b)*(a + b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*si
n(x) + a - b)/(b*cos(x)^2 - a - b)))/(a*b^2), -(a*b*sin(x) - sqrt(a*b)*(a +
 b)*arctan(sqrt(a*b)*sin(x)/a))/(a*b^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 30, normalized size = 0.83

$$\frac{\left(a+b\right)\arctan\left(\frac{b\sin\left(x\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{\sin\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (a + b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b) - sin(x)/b
```

Mupad [B]
time = 0.09, size = 28, normalized size = 0.78

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin\left(x\right)}{\sqrt{a}}\right)\,\left(a+b\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sin\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a + b*sin(x)^2),x)
[Out] (atan((b^(1/2)*sin(x))/a^(1/2))*(a + b))/(a^(1/2)*b^(3/2)) - sin(x)/b
```

3.306 $\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=39

$$-\frac{x}{b}+\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}$$

```
[Out] -x/b+arctan((a+b)^(1/2)*tan(x)/a^(1/2))*(a+b)^(1/2)/b/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3270, 400, 209, 211}

$$\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] -(x/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
```

```
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{(1+x^2)\,(a+(a+b)x^2)}\,dx, x, \tan(x)\right)\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{b}+\frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{b}\\ &= -\frac{x}{b}+\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 1.00

$$-\frac{x}{b}+\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Sin[x]^2),x]
[Out] -(x/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b)
```

Maple [A]
time = 0.17, size = 38, normalized size = 0.97

method	result
default	$\frac{(a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}-\frac{\arctan(\tan(x))}{b}$
risch	$-\frac{x}{b}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2ab}-\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-a(a+b)}+2a+b}{b}\right)}{2ab}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] (a+b)/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))-1/b*arctan(tan
(x))
```

Maxima [A]
time = 0.49, size = 35, normalized size = 0.90

$$\frac{(a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] (a + b)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) - x/b
```

Fricas [A]
time = 0.44, size = 206, normalized size = 5.28

$$\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2-4\left((2a^2+ab)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2+2ab+b^2}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right)-4x}{4b},-\frac{\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2a+b)\cos(x)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2(a+b)\cos(x)\sin(x)}\right)+2x}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a
*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2 + a*b)*cos(x))*sqrt(-
(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)
^2 + a^2 + 2*a*b + b^2)) - 4*x)/b, -1/2*(sqrt((a + b)/a)*arctan(1/2*((2*a +
 b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + 2*x)/b]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 62, normalized size = 1.59

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^
2 + a*b)))*(a + b)/(sqrt(a^2 + a*b)*b) - x/b
```

Mupad [B]
time = 14.71, size = 272, normalized size = 6.97

$$-\frac{\mathrm{atan}\left(\frac{2\,a^2\,\tan(x)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{2\,b^2\,\tan(x)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{4\,a\,b\,\tan(x)}{2\,a^2+4\,a\,b+2\,b^2}\right)}{b}-\frac{\mathrm{atanh}\left(\frac{6\,b^2\,\tan(x)\,\sqrt{-a^2-b\,a}}{2\,a^3+6\,a^2\,b+6\,a\,b^2+2\,b^3}+\frac{2\,a\,\tan(x)\,\sqrt{-a^2-b\,a}}{6\,a\,b+2\,a^2+6\,b^2+\frac{2\,b^3}{a}}+\frac{6\,b\,\tan(x)\,\sqrt{-a^2-b\,a}}{6\,a\,b+2\,a^2+6\,b^2+\frac{2\,b^3}{a}}+\frac{2\,b^3\,\tan(x)\,\sqrt{-a^2-b\,a}}{2\,a^4+6\,a^3\,b+6\,a^2\,b^2+2\,a\,b^3}\right)\,\sqrt{-a\,(a+b)}}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*sin(x)^2),x)
```

```
[Out] - atan((2*a^2*tan(x))/(4*a*b + 2*a^2 + 2*b^2) + (2*b^2*tan(x))/(4*a*b + 2*a
^2 + 2*b^2) + (4*a*b*tan(x))/(4*a*b + 2*a^2 + 2*b^2))/b - (atanh((6*b^2*tan
(x)*(- a*b - a^2)^(1/2))/(6*a*b^2 + 6*a^2*b + 2*a^3 + 2*b^3) + (2*a*tan(x)*
(- a*b - a^2)^(1/2))/(6*a*b + 2*a^2 + 6*b^2 + (2*b^3)/a) + (6*b*tan(x)*(- a
*b - a^2)^(1/2))/(6*a*b + 2*a^2 + 6*b^2 + (2*b^3)/a) + (2*b^3*tan(x)*(- a*b
 - a^2)^(1/2))/(2*a*b^3 + 6*a^3*b + 2*a^4 + 6*a^2*b^2))*(-a*(a + b))^(1/2))
/(a*b)
```

3.307 $\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] arctan(sin(x)*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3269, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Sin[x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Sin[x]^2),x]
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.11, size = 17, normalized size = 0.68

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	17
default	$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	17
risch	$-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}}+\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.47, size = 16, normalized size = 0.64

$$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="maxima")
[Out] arctan(b*sin(x)/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.39, size = 78, normalized size = 3.12

$$\left[-\frac{\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^
2 - a - b))/(a*b), sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a)/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.
time = 0.41, size = 66, normalized size = 2.64

$$\begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{\sin(x)}{a} & \text{for } b = 0 \\ -\frac{1}{b\sin(x)} & \text{for } a = 0 \\ \frac{\log\left(-\sqrt{-\frac{a}{b}}+\sin(x)\right)}{2b\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\sin(x)\right)}{2b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)**2),x)
```

```
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (sin(x)/a, Eq(b, 0)), (-1/(b*s
in(x)), Eq(a, 0)), (log(-sqrt(-a/b) + sin(x))/(2*b*sqrt(-a/b)) - log(sqrt(-
a/b) + sin(x))/(2*b*sqrt(-a/b)), True))
```

Giac [A]
time = 0.44, size = 16, normalized size = 0.64

$$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] arctan(b*sin(x)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B]
time = 14.64, size = 17, normalized size = 0.68

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a + b*sin(x)^2),x)
```

```
[Out] atan((b^(1/2)*sin(x))/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.308 $\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}+\frac{\tanh^{-1}(\sin(x))}{a+b}$$

```
[Out] arctanh(sin(x))/(a+b)+arctan(sin(x)*b^(1/2)/a^(1/2))*b^(1/2)/(a+b)/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3269, 400, 212, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}+\frac{\tanh^{-1}(\sin(x))}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + b*Sin[x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ArcTanh[Sin[
x]]/(a + b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
```

```
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x^2)\,(a+bx^2)}\,dx, x, \sin(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{a+b} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{a+b}\\ &= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)} + \frac{\tanh^{-1}(\sin(x))}{a+b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(40) = 80.
time = 0.10, size = 96, normalized size = 2.40

$$\frac{-\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\csc(x)}{\sqrt{b}}\right)+\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)+2\sqrt{a}\,\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)}{2\sqrt{a}\,(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + b*Sin[x]^2),x]
```

```
[Out] (-(Sqrt[b]*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]]) + Sqrt[b]*ArcTan[(Sqrt[b]*Sin[
x])/Sqrt[a]] + 2*Sqrt[a]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/
2]]))/(2*Sqrt[a]*(a + b))
```

Maple [A]
time = 0.20, size = 55, normalized size = 1.38

method	result	size
default	$\frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(1+\sin(x))}{2a+2b}-\frac{\ln(\sin(x)-1)}{2a+2b}$	55
risch	$-\frac{\ln(e^{ix}-i)}{a+b}+\frac{\ln(e^{ix}+i)}{a+b}+\frac{\sqrt{-ab}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-ab}\,e^{ix}}{b}-1\right)}{2a(a+b)}-\frac{\sqrt{-ab}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-ab}\,e^{ix}}{b}-1\right)}{2a(a+b)}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] b/(a+b)/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))+1/(2*a+2*b)*ln(1+sin(x))-1
/(2*a+2*b)*ln(sin(x)-1)
```

Maxima [A]
time = 0.50, size = 47, normalized size = 1.18

$$\frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}+\frac{\log\left(\sin\left(x\right)+1\right)}{2\,(a+b)}-\frac{\log\left(\sin\left(x\right)-1\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) + 1/2*log(sin(x) + 1)/(a +
 b) - 1/2*log(sin(x) - 1)/(a + b)
```

Fricas [A]
time = 0.42, size = 116, normalized size = 2.90

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+\log\left(\sin\left(x\right)+1\right)-\log\left(-\sin\left(x\right)+1\right)}{2\,(a+b)},\ \frac{2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\sin\left(x\right)\right)+\log\left(\sin\left(x\right)+1\right)-\log\left(-\sin\left(x\right)+1\right)}{2\,(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x
)^2 - a - b)) + log(sin(x) + 1) - log(-sin(x) + 1))/(a + b), 1/2*(2*sqrt(b/
a)*arctan(sqrt(b/a)*sin(x)) + log(sin(x) + 1) - log(-sin(x) + 1))/(a + b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(x\right)}{a+b\sin^2\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)/(a + b*sin(x)**2), x)
```

Giac [A]
time = 0.45, size = 49, normalized size = 1.22

$$\frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}+\frac{\log\left(\sin\left(x\right)+1\right)}{2\,(a+b)}-\frac{\log\left(-\sin\left(x\right)+1\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) + 1/2*log(sin(x) + 1)/(a +
 b) - 1/2*log(-sin(x) + 1)/(a + b)
```

Mupad [B]
time = 14.69, size = 856, normalized size = 21.40

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(4\,b^3\,\sin(x)+\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}\right)\,1\mathrm{i}}{2\,(a+b)}+\frac{\left(4\,b^3\,\sin(x)-\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}\right)\,1\mathrm{i}}{2\,(a+b)}}{\frac{4\,b^3\,\sin(x)+\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}}{2\,(a+b)}-\frac{4\,b^3\,\sin(x)-\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}}{2\,(a+b)}}\right)\,1\mathrm{i}}{a+b}-\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,b^3\,\sin(x)+\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2-\frac{\sin(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-a\,b}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\,\sin(x)-\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2+\frac{\sin(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-a\,b}\,1\mathrm{i}}{a^2+b\,a}}{\frac{\left(2\,b^3\,\sin(x)+\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2-\frac{\sin(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-a\,b}}{a^2+b\,a}-\frac{\left(2\,b^3\,\sin(x)-\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2+\frac{\sin(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-a\,b}}{a^2+b\,a}}\right)\,\sqrt{-a\,b}\,1\mathrm{i}}{a\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a + b*sin(x)^2)),x)
```

```
[Out] - (atan((((4*b^3*sin(x) + (8*a*b^3 + 4*b^4 + 4*a^2*b^2 - (sin(x)*(8*a*b^4 +
 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))*1i)/(2*(a + b))
+ ((4*b^3*sin(x) - (8*a*b^3 + 4*b^4 + 4*a^2*b^2 + (sin(x)*(8*a*b^4 + 8*b^5
- 8*a^2*b^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))*1i)/(2*(a + b)))/((4*b^
3*sin(x) + (8*a*b^3 + 4*b^4 + 4*a^2*b^2 - (sin(x)*(8*a*b^4 + 8*b^5 - 8*a^2*
b^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))/(2*(a + b)) - (4*b^3*sin(x) - (
8*a*b^3 + 4*b^4 + 4*a^2*b^2 + (sin(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*
b^2))/(2*(a + b)))/(2*(a + b)))/(2*(a + b))))*1i)/(a + b) - (atan((((2*b^3*
sin(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (sin(x)*(-a*b)^(1/2)*
(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)
))*(-a*b)^(1/2)*1i)/(a*b + a^2) + ((2*b^3*sin(x) - ((-a*b)^(1/2)*(4*a*b^3 +
 2*b^4 + 2*a^2*b^2 + (sin(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*
a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)))*(-a*b)^(1/2)*1i)/(a*b + a^2))/
(((2*b^3*sin(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (sin(x)*(-a*
b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a
*b + a^2)))*(-a*b)^(1/2))/(a*b + a^2) - ((2*b^3*sin(x) - ((-a*b)^(1/2)*(4*a
*b^3 + 2*b^4 + 2*a^2*b^2 + (sin(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^
3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)))*(-a*b)^(1/2))/(a*b + a^2
)))*(-a*b)^(1/2)*1i)/(a*(a + b))
```

3.309 $\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=39

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}+\frac{\tan(x)}{a+b}$$

```
[Out] b*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/(a+b)^(3/2)/a^(1/2)+tan(x)/(a+b)
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3270, 396, 211}

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}+\frac{\tan(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) + Tan[x]/(
a + b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{a+(a+b)x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{a+b} + \frac{b\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{a+b}$$
$$= \frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}} + \frac{\tan(x)}{a+b}$$

Mathematica [A]
time = 0.06, size = 39, normalized size = 1.00

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}} + \frac{\tan(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Sin[x]^2),x]
[Out] (b*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) + Tan[x]/(
a + b)
```

Maple [A]
time = 0.23, size = 38, normalized size = 0.97

method	result
default	$\frac{\tan(x)}{a+b} + \frac{b\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{(a+b)\sqrt{a(a+b)}}$
risch	$\frac{2i}{(e^{2ix}+1)(a+b)} - \frac{b\ln\left(e^{2ix} - \frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)} + \frac{b\ln\left(e^{2ix} - \frac{-2ia^2-2iab+2a\sqrt{-a^2-ab}+b}{b\sqrt{-a^2-ab}}\right.}{2\sqrt{-a^2-ab}\,(a+b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] tan(x)/(a+b)+b/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [A]
time = 0.48, size = 37, normalized size = 0.95

$$\frac{b\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a+b)} + \frac{\tan(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b)) + tan(x)
/(a + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.
time = 0.45, size = 255, normalized size = 6.54

$$\left[-\frac{\sqrt{-a^2-ab}\,b\cos(x)\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4\left((2a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2+2ab+b^2}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right)-4(a^2+ab)\sin(x)}{4(a^3+2a^2b+ab^2)\cos(x)}, -\frac{\sqrt{a^2+ab}\,b\arctan\left(\frac{(2a+b)\cos(x)^2-a-b}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)\cos(x)-2(a^2+ab)\sin(x)}{2(a^3+2a^2b+ab^2)\cos(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*cos(x)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4
*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt
(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(
x)^2 + a^2 + 2*a*b + b^2)) - 4*(a^2 + a*b)*sin(x))/((a^3 + 2*a^2*b + a*b^2)
*cos(x)), -1/2*(sqrt(a^2 + a*b)*b*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)/(
sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x) - 2*(a^2 + a*b)*sin(x))/((a^3 + 2*a^
2*b + a*b^2)*cos(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(a + b*sin(x)**2), x)
```

Giac [A]
time = 0.48, size = 45, normalized size = 1.15

$$\frac{b\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}\,(a+b)}+\frac{\tan(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="giac")
```

[Out] b*arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b))/(sqrt(a^2 + a*b)*(a + b)) +
 tan(x)/(a + b)

Mupad [B]
time = 14.66, size = 39, normalized size = 1.00

$$\frac{\tan(x)}{a+b}+\frac{b\,\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(a + b*sin(x)^2)),x)

[Out] tan(x)/(a + b) + (b*atan((tan(x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2))))/(
a^(1/2)*(a + b)^(3/2))

3.310 $\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=61

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}+\frac{(a+3b)\tanh^{-1}(\sin(x))}{2(a+b)^2}+\frac{\sec(x)\tan(x)}{2(a+b)}$$

```
[Out] 1/2*(a+3*b)*arctanh(sin(x))/(a+b)^2+b^(3/2)*arctan(sin(x)*b^(1/2)/a^(1/2))/
(a+b)^2/a^(1/2)+1/2*sec(x)*tan(x)/(a+b)
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3269, 425, 536, 212, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}+\frac{(a+3b)\tanh^{-1}(\sin(x))}{2(a+b)^2}+\frac{\tan(x)\sec(x)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^2) + ((a + 3*b)
*ArcTanh[Sin[x]])/(2*(a + b)^2) + (Sec[x]*Tan[x])/(2*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x^2)^2\,(a+bx^2)}\,dx, x, \sin(x)\right)\\ &= \frac{\sec(x)\tan(x)}{2(a+b)} + \frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \sin(x)\right)}{2(a+b)}\\ &= \frac{\sec(x)\tan(x)}{2(a+b)} + \frac{b^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{(a+b)^2} + \frac{(a+3b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{2(a+b)^2}\\ &= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2} + \frac{(a+3b)\tanh^{-1}(\sin(x))}{2(a+b)^2} + \frac{\sec(x)\tan(x)}{2(a+b)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(61) = 122.
time = 0.24, size = 147, normalized size = 2.41

$$\frac{-\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\csc(x)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}} - 2(a+3b)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + 2(a+3b)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) + \frac{a+b}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2} - \frac{a+b}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}}{4(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] ((-2*b^(3/2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/Sqrt[a] + (2*b^(3/2)*ArcTan[
(Sqrt[b]*Sin[x])/Sqrt[a]])/Sqrt[a] - 2*(a + 3*b)*Log[Cos[x/2] - Sin[x/2]] +
```

```
2*(a + 3*b)*Log[Cos[x/2] + Sin[x/2]] + (a + b)/(Cos[x/2] - Sin[x/2])^2 - (
a + b)/(Cos[x/2] + Sin[x/2])^2)/(4*(a + b)^2)
```

Maple [A]
time = 0.31, size = 96, normalized size = 1.57

method	result
default	$\frac{b^2 \arctan\left(\frac{b \sin(x)}{\sqrt{ab}}\right)}{(a+b)^2 \sqrt{ab}} - \frac{1}{(4a+4b)(\sin(x)-1)} + \frac{(-a-3b) \ln(\sin(x)-1)}{4(a+b)^2} - \frac{1}{(4a+4b)(1+\sin(x))} + \frac{(a+3b) \ln(1+\sin(x))}{4(a+b)^2}$
risch	$-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2(a+b)} - \frac{\ln(e^{ix}-i)a}{2(a^2+2ab+b^2)} - \frac{3\ln(e^{ix}-i)b}{2(a^2+2ab+b^2)} + \frac{\ln(e^{ix}+i)a}{2a^2+4ab+2b^2} + \frac{3\ln(e^{ix}+i)b}{2(a^2+2ab+b^2)} + \frac{\sqrt{-ab}\, b \ln\left(e^{2ix}+\frac{2i\sqrt{-ab}\, e}{b}\right.}{2a(a+b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a+b)^2/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))-1/(4*a+4*b)/(sin(x)-1)
+1/4/(a+b)^2*(-a-3*b)*ln(sin(x)-1)-1/(4*a+4*b)/(1+sin(x))+1/4*(a+3*b)/(a+b)
^2*ln(1+sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(49) = 98.
time = 0.48, size = 104, normalized size = 1.70

$$\frac{b^2 \arctan\left(\frac{b \sin(x)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} + \frac{(a+3\,b)\log(\sin(x)+1)}{4\,(a^2+2\,ab+b^2)} - \frac{(a+3\,b)\log(\sin(x)-1)}{4\,(a^2+2\,ab+b^2)} - \frac{\sin(x)}{2\,((a+b)\sin(x)^2-a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*sin(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 1/4*(a + 3
*b)*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/4*(a + 3*b)*log(sin(x) - 1)/(a^
2 + 2*a*b + b^2) - 1/2*sin(x)/((a + b)*sin(x)^2 - a - b)
```

Fricas [A]
time = 0.46, size = 203, normalized size = 3.33

$$\left[\frac{2\,b\sqrt{-\frac{b}{a}}\cos(x)^2\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+(a+3\,b)\cos(x)^2\log(\sin(x)+1)-(a+3\,b)\cos(x)^2\log(-\sin(x)+1)+2\,(a+b)\sin(x)}{4\,(a^2+2\,ab+b^2)\cos(x)^2}, \frac{4\,b\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\sin(x)\right)\cos(x)^2+(a+3\,b)\cos(x)^2\log(\sin(x)+1)-(a+3\,b)\cos(x)^2\log(-\sin(x)+1)+2\,(a+b)\sin(x)}{4\,(a^2+2\,ab+b^2)\cos(x)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(-b/a)*cos(x)^2*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a
- b)/(b*cos(x)^2 - a - b)) + (a + 3*b)*cos(x)^2*log(sin(x) + 1) - (a + 3*b)
*cos(x)^2*log(-sin(x) + 1) + 2*(a + b)*sin(x))/((a^2 + 2*a*b + b^2)*cos(x)^
2), 1/4*(4*b*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))*cos(x)^2 + (a + 3*b)*cos(x)
^2*log(sin(x) + 1) - (a + 3*b)*cos(x)^2*log(-sin(x) + 1) + 2*(a + b)*sin(x)
)/((a^2 + 2*a*b + b^2)*cos(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(x)}{a + b\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**3/(a + b*sin(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(49) = 98.
time = 0.42, size = 102, normalized size = 1.67

$$\frac{b^2\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} + \frac{(a+3\,b)\log(\sin(x)+1)}{4\,(a^2+2\,ab+b^2)} - \frac{(a+3\,b)\log(-\sin(x)+1)}{4\,(a^2+2\,ab+b^2)} - \frac{\sin(x)}{2\left(\sin(x)^2-1\right)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*sin(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 1/4*(a + 3
*b)*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/4*(a + 3*b)*log(-sin(x) + 1)/(a
^2 + 2*a*b + b^2) - 1/2*sin(x)/((sin(x)^2 - 1)*(a + b))
```

Mupad [B]
time = 15.36, size = 1139, normalized size = 18.67

$$\frac{\sin(x)}{2\cos(x)^2\,(a+b)} - \ln(\sin(x)-1)\left(\frac{b}{2\,(a+b)^2}+\frac{1}{4\,(a+b)}\right) + \frac{\ln(\sin(x)+1)\,(a+3\,b)}{4\,(a+b)^2} + \frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-a\,b^3}\left(\frac{\sin(x)\left(a^2\,b^3+6\,a\,b^4+13\,b^5\right)}{4\left(a^2+2\,a\,b+b^2\right)}+\frac{\left(\frac{2\,a^5\,b^2+12\,a^4\,b^3+28\,a^3\,b^4+32\,a^2\,b^5+18\,a\,b^6+4\,b^7}{2\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{\sin(x)\sqrt{-a\,b^3}\left(-16\,a^5\,b^2-48\,a^4\,b^3-32\,a^3\,b^4+32\,a^2\,b^5+48\,a\,b^6+16\,b^7\right)}{8\left(a^2+2\,a\,b+b^2\right)\left(a^3+2\,a^2\,b+a\,b^2\right)}\right)\sqrt{-a\,b^3}}{2\left(a^3+2\,a^2\,b+a\,b^2\right)}\right)1\mathrm{i}}{a^3+2\,a^2\,b+a\,b^2}+\frac{\sqrt{-a\,b^3}\left(\frac{\sin(x)\left(a^2\,b^3+6\,a\,b^4+13\,b^5\right)}{4\left(a^2+2\,a\,b+b^2\right)}-\frac{\left(\frac{2\,a^5\,b^2+12\,a^4\,b^3+28\,a^3\,b^4+32\,a^2\,b^5+18\,a\,b^6+4\,b^7}{2\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}+\frac{\sin(x)\sqrt{-a\,b^3}\left(-16\,a^5\,b^2-48\,a^4\,b^3-32\,a^3\,b^4+32\,a^2\,b^5+48\,a\,b^6+16\,b^7\right)}{8\left(a^2+2\,a\,b+b^2\right)\left(a^3+2\,a^2\,b+a\,b^2\right)}\right)\sqrt{-a\,b^3}}{2\left(a^3+2\,a^2\,b+a\,b^2\right)}\right)1\mathrm{i}}{a^3+2\,a^2\,b+a\,b^2}}{\frac{\frac{3\,b^5}{2}+\frac{a\,b^4}{2}}{a^3+3\,a^2\,b+3\,a\,b^2+b^3}-\frac{\sqrt{-a\,b^3}\left(\frac{\sin(x)\left(a^2\,b^3+6\,a\,b^4+13\,b^5\right)}{4\left(a^2+2\,a\,b+b^2\right)}+\frac{\left(\frac{2\,a^5\,b^2+12\,a^4\,b^3+28\,a^3\,b^4+32\,a^2\,b^5+18\,a\,b^6+4\,b^7}{2\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{\sin(x)\sqrt{-a\,b^3}\left(-16\,a^5\,b^2-48\,a^4\,b^3-32\,a^3\,b^4+32\,a^2\,b^5+48\,a\,b^6+16\,b^7\right)}{8\left(a^2+2\,a\,b+b^2\right)\left(a^3+2\,a^2\,b+a\,b^2\right)}\right)\sqrt{-a\,b^3}}{2\left(a^3+2\,a^2\,b+a\,b^2\right)}\right)}{a^3+2\,a^2\,b+a\,b^2}+\frac{\sqrt{-a\,b^3}\left(\frac{\sin(x)\left(a^2\,b^3+6\,a\,b^4+13\,b^5\right)}{4\left(a^2+2\,a\,b+b^2\right)}-\frac{\left(\frac{2\,a^5\,b^2+12\,a^4\,b^3+28\,a^3\,b^4+32\,a^2\,b^5+18\,a\,b^6+4\,b^7}{2\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}+\frac{\sin(x)\sqrt{-a\,b^3}\left(-16\,a^5\,b^2-48\,a^4\,b^3-32\,a^3\,b^4+32\,a^2\,b^5+48\,a\,b^6+16\,b^7\right)}{8\left(a^2+2\,a\,b+b^2\right)\left(a^3+2\,a^2\,b+a\,b^2\right)}\right)\sqrt{-a\,b^3}}{2\left(a^3+2\,a^2\,b+a\,b^2\right)}\right)}{a^3+2\,a^2\,b+a\,b^2}}\right)\sqrt{-a\,b^3}\,1\mathrm{i}}{a^3+2\,a^2\,b+a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^3*(a + b*sin(x)^2)),x)
```

```
[Out] sin(x)/(2*cos(x)^2*(a + b)) - log(sin(x) - 1)*(b/(2*(a + b)^2) + 1/(4*(a +
b))) + (log(sin(x) + 1)*(a + 3*b))/(4*(a + b)^2) + (atan((((-a*b^3)^(1/2)*(
(sin(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6
 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 +
```

```
3*a^2*b + a^3 + b^3)) - (sin(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*
b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2
+ 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2
+ 2*a^2*b + a^3) + ((-a*b^3)^(1/2)*((sin(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(
4*(2*a*b + a^2 + b^2)) - (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12
*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (sin(x)*(-a*b^3
)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*
b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(
a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2 + 2*a^2*b + a^3))/(((a*b^4)/2 + (3*b^5)
/2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - ((-a*b^3)^(1/2)*((sin(x)*(6*a*b^4 + 1
3*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6 + 4*b^7 + 32*a^2*b^
5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)
) - (sin(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 4
8*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(
-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))))/(a*b^2 + 2*a^2*b + a^3) + ((-a
*b^3)^(1/2)*((sin(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2))
- (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(
2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (sin(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*
b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 +
 b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))
))/(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2)*1i)/(a*b^2 + 2*a^2*b + a^3)
```

3.311 $\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=59

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{(a+2b)\tan(x)}{(a+b)^2}+\frac{\tan^3(x)}{3(a+b)}$$

```
[Out] b^2*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/(a+b)^(5/2)/a^(1/2)+(a+2*b)*tan(x)/(
a+b)^2+1/3*tan(x)^3/(a+b)
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3270, 398, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{\tan^3(x)}{3(a+b)}+\frac{(a+2b)\tan(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((a +
2*b)*Tan[x])/(a + b)^2 + Tan[x]^3/(3*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{(1+x^2)^2}{a+(a+b)x^2}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2}+\frac{x^2}{a+b}+\frac{b^2}{(a+b)^2\,(a+(a+b)x^2)}\right)dx, x, \tan(x)\right)\\ &= \frac{(a+2b)\tan(x)}{(a+b)^2}+\frac{\tan^3(x)}{3(a+b)}+\frac{b^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{(a+b)^2}\\ &= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{(a+2b)\tan(x)}{(a+b)^2}+\frac{\tan^3(x)}{3(a+b)}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 59, normalized size = 1.00

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{(2a+5b+(a+b)\sec^2(x))\tan(x)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((2*a
+ 5*b + (a + b)*Sec[x]^2)*Tan[x])/(3*(a + b)^2)
```

Maple [A]
time = 0.27, size = 62, normalized size = 1.05

method	result
default	$\frac{\frac{a\left(\tan^3(x)\right)}{3}+\frac{b\left(\tan^3(x)\right)}{3}+\tan(x)a+2\tan(x)b}{(a+b)^2}+\frac{b^2\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{(a+b)^2\sqrt{a(a+b)}}$
risch	$\frac{2i\left(3b\,\mathrm{e}^{4ix}+6a\,\mathrm{e}^{2ix}+12b\,\mathrm{e}^{2ix}+2a+5b\right)}{3\left(\mathrm{e}^{2ix}+1\right)^3(a+b)^2}-\frac{b^2\ln\left(\mathrm{e}^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)^2}+\frac{b^2\ln\left(\mathrm{e}^{2ix}-\frac{-2ia^2-2iab}{}\right)}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a+b)^2*(1/3*a*tan(x)^3+1/3*b*tan(x)^3+tan(x)*a+2*tan(x)*b)+b^2/(a+b)^2/(
a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [A]
time = 0.48, size = 72, normalized size = 1.22

$$\frac{b^2\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+2\,ab+b^2)}+\frac{(a+b)\tan{(x)}^3+3\,(a+2\,b)\tan{(x)}}{3\,(a^2+2\,ab+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + 2*a*b +
b^2)) + 1/3*((a + b)*tan(x)^3 + 3*(a + 2*b)*tan(x))/(a^2 + 2*a*b + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(49) = 98.
time = 0.42, size = 343, normalized size = 5.81

$$\left[-\frac{3\,\sqrt{-a^2-ab}\,b^2\cos{(x)}^3\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+5\,ab+b^2)\cos(x)^2+4\,\left((2\,a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2+2\,ab+b^2}{b^2\cos(x)^4-2\,(ab+b^2)\cos(x)^2+a^2+2\,ab+b^2}\right)-4\,\left(a^3+2\,a^2b+ab^2+(2\,a^3+7\,a^2b+5\,ab^2)\cos{(x)}^2\right)\sin{(x)}}{12\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\cos{(x)}^3},\ -\frac{3\,\sqrt{a^2+ab}\,b^2\arctan\left(\frac{(2\,a+b)\cos(x)^2-a-b}{2\,\sqrt{a^2+ab}\,\cos(x)\sin(x)}\right)\cos{(x)}^3-2\,\left(a^3+2\,a^2b+ab^2+(2\,a^3+7\,a^2b+5\,ab^2)\cos{(x)}^2\right)\sin{(x)}}{6\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\cos{(x)}^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a^2 - a*b)*b^2*cos(x)^3*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4
 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x
))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^
2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*(a^3 + 2*a^2*b + a*b^2 + (2*a^3 + 7*a
^2*b + 5*a*b^2)*cos(x)^2)*sin(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(
x)^3), -1/6*(3*sqrt(a^2 + a*b)*b^2*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)/
(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x)^3 - 2*(a^3 + 2*a^2*b + a*b^2 + (2*a
^3 + 7*a^2*b + 5*a*b^2)*cos(x)^2)*sin(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*cos(x)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^4{(x)}}{a+b\sin^2{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**4/(a + b*sin(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(49) = 98.
time = 0.45, size = 134, normalized size = 2.27

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{(a^2+2\,ab+b^2)\sqrt{a^2+ab}}+\frac{a^2\tan{(x)}^3+2\,ab\tan{(x)}^3+b^2\tan{(x)}^3+3\,a^2\tan{(x)}+9\,ab\tan{(x)}+6\,b^2\tan{(x)}}{3\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^
2 + a*b)))*b^2/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)) + 1/3*(a^2*tan(x)^3 +
2*a*b*tan(x)^3 + b^2*tan(x)^3 + 3*a^2*tan(x) + 9*a*b*tan(x) + 6*b^2*tan(x))
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B]
time = 15.02, size = 77, normalized size = 1.31

$$\frac{\tan(x)^3}{3\,(a+b)}-\tan(x)\,\left(\frac{a}{{(a+b)}^2}-\frac{2}{a+b}\right)+\frac{b^2\,\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)\,\left(a^2+2\,a\,b+b^2\right)}{2\,\sqrt{a}\,{(a+b)}^{5/2}}\right)}{\sqrt{a}\,{(a+b)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a + b*sin(x)^2)),x)
```

```
[Out] tan(x)^3/(3*(a + b)) - tan(x)*(a/(a + b)^2 - 2/(a + b)) + (b^2*atan((tan(x)
*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2))))/(a^(1/2)*(a +
 b)^(5/2))
```

3.312 $\int \frac{\sec^5(x)}{a+b\sin^2(x)} dx$

Optimal. Leaf size=93

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3}+\frac{(3a^2+10ab+15b^2)\tanh^{-1}(\sin(x))}{8(a+b)^3}+\frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2}+\frac{\sec^3(x)\tan(x)}{4(a+b)}$$

```
[Out] 1/8*(3*a^2+10*a*b+15*b^2)*arctanh(sin(x))/(a+b)^3+b^(5/2)*arctan(sin(x)*b^(
1/2)/a^(1/2))/(a+b)^3/a^(1/2)+1/8*(3*a+7*b)*sec(x)*tan(x)/(a+b)^2+1/4*sec(x
)^3*tan(x)/(a+b)
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3269, 425, 541, 536, 212, 211}

$$\frac{(3a^2+10ab+15b^2)\tanh^{-1}(\sin(x))}{8(a+b)^3}+\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3}+\frac{\tan(x)\sec^3(x)}{4(a+b)}+\frac{(3a+7b)\tan(x)\sec(x)}{8(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] (b^(5/2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3) + ((3*a^2 +
10*a*b + 15*b^2)*ArcTanh[Sin[x]])/(8*(a + b)^3) + ((3*a + 7*b)*Sec[x]*Tan[x
])/(8*(a + b)^2) + (Sec[x]^3*Tan[x])/(4*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x^2)^3\,(a+bx^2)}\,dx, x, \sin(x)\right)\\
&= \frac{\sec^3(x)\tan(x)}{4(a+b)} + \frac{\text{Subst}\left(\int \frac{3a+4b+3bx^2}{(1-x^2)^2(a+bx^2)}\,dx, x, \sin(x)\right)}{4(a+b)}\\
&= \frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2} + \frac{\sec^3(x)\tan(x)}{4(a+b)} + \frac{\text{Subst}\left(\int \frac{3a^2+7ab+8b^2+b(3a+7b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \sin(x)\right)}{8(a+b)^2}\\
&= \frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2} + \frac{\sec^3(x)\tan(x)}{4(a+b)} + \frac{b^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{(a+b)^3} + \frac{(3a^2+10\cdots}{\cdots}\\
&= \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3} + \frac{(3a^2+10ab+15b^2)\tanh^{-1}(\sin(x))}{8(a+b)^3} + \frac{(3a+7b)\sec(x)\tan(x\cdots}{8(a+b)^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 214 vs. 2(93) = 186.
time = 0.93, size = 214, normalized size = 2.30

$$-\frac{\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{\sqrt{a}}-\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}}+2(3a^2+10ab+15b^2)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-2(3a^2+10ab+15b^2)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\frac{(a+b)^2}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^4}+\frac{(a+b)^2}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4}+\frac{(a+b)(3a+7b)}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}+\frac{(a+b)(3a+7b)}{-1+\sin(x)}}{16(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] -1/16*((8*b^(5/2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/Sqrt[a] - (8*b^(5/2)*Ar
cTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/Sqrt[a] + 2*(3*a^2 + 10*a*b + 15*b^2)*Log[C
os[x/2] - Sin[x/2]] - 2*(3*a^2 + 10*a*b + 15*b^2)*Log[Cos[x/2] + Sin[x/2]]
- (a + b)^2/(Cos[x/2] - Sin[x/2])^4 + (a + b)^2/(Cos[x/2] + Sin[x/2])^4 + (
(a + b)*(3*a + 7*b))/(Cos[x/2] + Sin[x/2])^2 + ((a + b)*(3*a + 7*b))/(-1 +
Sin[x]))/(a + b)^3
```

Maple [A]
time = 0.39, size = 154, normalized size = 1.66

method	result
default	$-\frac{1}{2(8a+8b)(1+\sin(x))^2}-\frac{3a+7b}{16(a+b)^2(1+\sin(x))}+\frac{(3a^2+10ab+15b^2)\ln(1+\sin(x))}{16(a+b)^3}+\frac{b^3\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}+\frac{1}{2(8a+8b)(\sin(x)}$
risch	$-\frac{i(3a\,e^{7ix}+7b\,e^{7ix}+11a\,e^{5ix}+15b\,e^{5ix}-11a\,e^{3ix}-15b\,e^{3ix}-3\,e^{ix}a-7\,e^{ix}b)}{4(e^{2ix}+1)^4(a+b)^2}+\frac{3\ln(e^{ix}+i)a^2}{8(a^3+3a^2b+3a\,b^2+b^3)}+\frac{5\ln(e^{ix}+i)ab}{4(a^3+3a^2b+3a\,b^2+b^3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^5/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(8*a+8*b)/(1+sin(x))^2-1/16*(3*a+7*b)/(a+b)^2/(1+sin(x))+1/16*(3*a^2+1
0*a*b+15*b^2)/(a+b)^3*ln(1+sin(x))+b^3/(a+b)^3/(a*b)^(1/2)*arctan(b*sin(x)/
(a*b)^(1/2))+1/2/(8*a+8*b)/(sin(x)-1)^2-1/16*(3*a+7*b)/(a+b)^2/(sin(x)-1)+1
/16/(a+b)^3*(-3*a^2-10*a*b-15*b^2)*ln(sin(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(79) = 158.
time = 0.50, size = 199, normalized size = 2.14

$$\frac{b^3\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}}+\frac{(3\,a^2+10\,ab+15\,b^2)\log(\sin(x)+1)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)}-\frac{(3\,a^2+10\,ab+15\,b^2)\log(\sin(x)-1)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)}-\frac{(3\,a+7\,b)\sin(x)^3-(5\,a+9\,b)\sin(x)}{8\left((a^2+2\,ab+b^2)\sin(x)^4-2\,(a^2+2\,ab+b^2)\sin(x)^2+a^2+2\,ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b^3*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
+ 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3) - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) - 1)/(a^3 + 3*a^2*b + 3*a
*b^2 + b^3) - 1/8*((3*a + 7*b)*sin(x)^3 - (5*a + 9*b)*sin(x))/((a^2 + 2*a*b
 + b^2)*sin(x)^4 - 2*(a^2 + 2*a*b + b^2)*sin(x)^2 + a^2 + 2*a*b + b^2)
```

Fricas [A]
time = 0.48, size = 327, normalized size = 3.52

$$\left[\frac{8\,b^2\sqrt{-\frac{b}{a}}\cos\left(x\right)^4\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+\left(3\,a^2+10\,ab+15\,b^2\right)\cos\left(x\right)^4\log\left(\sin\left(x\right)+1\right)-\left(3\,a^2+10\,ab+15\,b^2\right)\cos\left(x\right)^4\log\left(-\sin\left(x\right)+1\right)+2\,\left(\left(3\,a^2+10\,ab+7\,b^2\right)\cos\left(x\right)^2+2\,a^2+4\,ab+2\,b^2\right)\sin\left(x\right)}{16\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\cos\left(x\right)^4}, \frac{16\,b^2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\sin\left(x\right)\right)\cos\left(x\right)^4+\left(3\,a^2+10\,ab+15\,b^2\right)\cos\left(x\right)^4\log\left(\sin\left(x\right)+1\right)-\left(3\,a^2+10\,ab+15\,b^2\right)\cos\left(x\right)^4\log\left(-\sin\left(x\right)+1\right)+2\,\left(\left(3\,a^2+10\,ab+7\,b^2\right)\cos\left(x\right)^2+2\,a^2+4\,ab+2\,b^2\right)\sin\left(x\right)}{16\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\cos\left(x\right)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*b^2*sqrt(-b/a)*cos(x)^4*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) +
 a - b)/(b*cos(x)^2 - a - b)) + (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(sin(
x) + 1) - (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(-sin(x) + 1) + 2*((3*a^2 +
 10*a*b + 7*b^2)*cos(x)^2 + 2*a^2 + 4*a*b + 2*b^2)*sin(x))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cos(x)^4), 1/16*(16*b^2*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))
*cos(x)^4 + (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(sin(x) + 1) - (3*a^2 + 1
0*a*b + 15*b^2)*cos(x)^4*log(-sin(x) + 1) + 2*((3*a^2 + 10*a*b + 7*b^2)*cos
(x)^2 + 2*a^2 + 4*a*b + 2*b^2)*sin(x))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
(x)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5\left(x\right)}{a+b\sin^2\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**5/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**5/(a + b*sin(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(79) = 158.
time = 0.44, size = 177, normalized size = 1.90

$$\frac{b^3\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{ab}}+\frac{\left(3\,a^2+10\,ab+15\,b^2\right)\log\left(\sin\left(x\right)+1\right)}{16\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}-\frac{\left(3\,a^2+10\,ab+15\,b^2\right)\log\left(-\sin\left(x\right)+1\right)}{16\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}-\frac{3\,a\sin\left(x\right)^3+7\,b\sin\left(x\right)^3-5\,a\sin\left(x\right)-9\,b\sin\left(x\right)}{8\,\left(a^2+2\,ab+b^2\right)\left(\sin\left(x\right)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b^3*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
+ 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3) - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(-sin(x) + 1)/(a^3 + 3*a^2*b + 3*
a*b^2 + b^3) - 1/8*(3*a*sin(x)^3 + 7*b*sin(x)^3 - 5*a*sin(x) - 9*b*sin(x))/
((a^2 + 2*a*b + b^2)*(sin(x)^2 - 1)^2)
```

Mupad [B]
time = 17.45, size = 832, normalized size = 8.95

$$\frac{5a^3\sin(x)-3a^3\sin(x)^3+3a^3\operatorname{atanh}(\sin(x))+9ab^2\sin(x)+14a^2b\sin(x)-6a^3\operatorname{atanh}(\sin(x))\sin(x)^2+3a^3\operatorname{atanh}(\sin(x))\sin(x)^4-7ab^2\sin(x)^3-10a^2b\sin(x)^3+15ab^2\operatorname{atanh}(\sin(x))+10a^2b\operatorname{atanh}(\sin(x))-30ab^2\operatorname{atanh}(\sin(x))\sin(x)^2-20a^2b\operatorname{atanh}(\sin(x))\sin(x)^2+15ab^2\operatorname{atanh}(\sin(x))\sin(x)^4+10a^2b\operatorname{atanh}(\sin(x))\sin(x)^4+\operatorname{atan}\left(\frac{a\sin(x)(-ab^5)^{3/2}64i-b\sin(x)(-ab^5)^{3/2}64i+a^6b\sin(x)\sqrt{-ab^5}9i+a^2b^5\sin(x)\sqrt{-ab^5}289i+a^3b^4\sin(x)\sqrt{-ab^5}300i+a^4b^3\sin(x)\sqrt{-ab^5}190i+a^5b^2\sin(x)\sqrt{-ab^5}60i}{9a^7b^3+60a^6b^4+190a^5b^5+300a^4b^6+225a^3b^7+64a^2b^8}\right)\sqrt{-ab^5}\,8i-\operatorname{atan}\left(\frac{a\sin(x)(-ab^5)^{3/2}64i-b\sin(x)(-ab^5)^{3/2}64i+a^6b\sin(x)\sqrt{-ab^5}9i+a^2b^5\sin(x)\sqrt{-ab^5}289i+a^3b^4\sin(x)\sqrt{-ab^5}300i+a^4b^3\sin(x)\sqrt{-ab^5}190i+a^5b^2\sin(x)\sqrt{-ab^5}60i}{9a^7b^3+60a^6b^4+190a^5b^5+300a^4b^6+225a^3b^7+64a^2b^8}\right)\sin(x)^2\sqrt{-ab^5}\,16i+\operatorname{atan}\left(\frac{a\sin(x)(-ab^5)^{3/2}64i-b\sin(x)(-ab^5)^{3/2}64i+a^6b\sin(x)\sqrt{-ab^5}9i+a^2b^5\sin(x)\sqrt{-ab^5}289i+a^3b^4\sin(x)\sqrt{-ab^5}300i+a^4b^3\sin(x)\sqrt{-ab^5}190i+a^5b^2\sin(x)\sqrt{-ab^5}60i}{9a^7b^3+60a^6b^4+190a^5b^5+300a^4b^6+225a^3b^7+64a^2b^8}\right)\sin(x)^4\sqrt{-ab^5}\,8i}{8a^4\sin(x)^4-16a^4\sin(x)^2+8a^4+24a^3b\sin(x)^4-48a^3b\sin(x)^2+24a^3b+24a^2b^2\sin(x)^4-48a^2b^2\sin(x)^2+24a^2b^2+8ab^3\sin(x)^4-16ab^3\sin(x)^2+8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^5*(a + b*sin(x)^2)),x)
```

```
[Out] (5*a^3*sin(x) - 3*a^3*sin(x)^3 + 3*a^3*atanh(sin(x)) + atan((a*sin(x)*(-a*b
^5)^(3/2)*64i - b*sin(x)*(-a*b^5)^(3/2)*64i + a^6*b*sin(x)*(-a*b^5)^(1/2)*9
i + a^2*b^5*sin(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*sin(x)*(-a*b^5)^(1/2)*300i
 + a^4*b^3*sin(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*sin(x)*(-a*b^5)^(1/2)*60i)/
(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9*a^7*
b^3))*(-a*b^5)^(1/2)*8i + 9*a*b^2*sin(x) + 14*a^2*b*sin(x) - 6*a^3*atanh(si
n(x))*sin(x)^2 + 3*a^3*atanh(sin(x))*sin(x)^4 - 7*a*b^2*sin(x)^3 - 10*a^2*b
*sin(x)^3 + 15*a*b^2*atanh(sin(x)) + 10*a^2*b*atanh(sin(x)) - atan((a*sin(x
)*(-a*b^5)^(3/2)*64i - b*sin(x)*(-a*b^5)^(3/2)*64i + a^6*b*sin(x)*(-a*b^5)^
(1/2)*9i + a^2*b^5*sin(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*sin(x)*(-a*b^5)^(1/
2)*300i + a^4*b^3*sin(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*sin(x)*(-a*b^5)^(1/2
)*60i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 +
 9*a^7*b^3))*sin(x)^2*(-a*b^5)^(1/2)*16i + atan((a*sin(x)*(-a*b^5)^(3/2)*64
i - b*sin(x)*(-a*b^5)^(3/2)*64i + a^6*b*sin(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*
sin(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*sin(x)*(-a*b^5)^(1/2)*300i + a^4*b^3*s
in(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*sin(x)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8
+ 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9*a^7*b^3))*sin(x)
^4*(-a*b^5)^(1/2)*8i - 30*a*b^2*atanh(sin(x))*sin(x)^2 - 20*a^2*b*atanh(sin
(x))*sin(x)^2 + 15*a*b^2*atanh(sin(x))*sin(x)^4 + 10*a^2*b*atanh(sin(x))*si
n(x)^4)/(8*a^4*sin(x)^4 - 16*a^4*sin(x)^2 + 8*a*b^3 + 24*a^3*b + 8*a^4 + 24
*a^2*b^2 - 48*a^2*b^2*sin(x)^2 + 24*a^2*b^2*sin(x)^4 - 16*a*b^3*sin(x)^2 -
48*a^3*b*sin(x)^2 + 8*a*b^3*sin(x)^4 + 24*a^3*b*sin(x)^4)
```

3.313 $\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=87

$$\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}+\frac{(a^2+3ab+3b^2)\tan(x)}{(a+b)^3}+\frac{(2a+3b)\tan^3(x)}{3(a+b)^2}+\frac{\tan^5(x)}{5(a+b)}$$

```
[Out] b^3*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/(a+b)^(7/2)/a^(1/2)+(a^2+3*a*b+3*b^2
)*tan(x)/(a+b)^3+1/3*(2*a+3*b)*tan(x)^3/(a+b)^2+1/5*tan(x)^5/(a+b)
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3270, 398, 211}

$$\frac{(a^2+3ab+3b^2)\tan(x)}{(a+b)^3}+\frac{b^3\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}+\frac{\tan^5(x)}{5(a+b)}+\frac{(2a+3b)\tan^3(x)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2)) + ((a^2
+ 3*a*b + 3*b^2)*Tan[x])/(a + b)^3 + ((2*a + 3*b)*Tan[x]^3)/(3*(a + b)^2) +
 Tan[x]^5/(5*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{(1+x^2)^3}{a+(a+b)x^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a^2+3ab+3b^2}{(a+b)^3}+\frac{(2a+3b)x^2}{(a+b)^2}+\frac{x^4}{a+b}+\frac{b^3}{(a+b)^3\,(a+(a+b)x^2)}\right)dx, x$$
$$= \frac{(a^2+3ab+3b^2)\tan(x)}{(a+b)^3}+\frac{(2a+3b)\tan^3(x)}{3(a+b)^2}+\frac{\tan^5(x)}{5(a+b)}+\frac{b^3\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x,\right.}{(a+b)^3}$$
$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}+\frac{(a^2+3ab+3b^2)\tan(x)}{(a+b)^3}+\frac{(2a+3b)\tan^3(x)}{3(a+b)^2}+\frac{\tan^5(x)}{5(a+b)}$$

Mathematica [A]
time = 0.28, size = 90, normalized size = 1.03

$$\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}+\frac{(8a^2+26ab+33b^2+(4a^2+13ab+9b^2)\sec^2(x)+3(a+b)^2\sec^4(x))\tan(x)}{15(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2)) + ((8*a^
2 + 26*a*b + 33*b^2 + (4*a^2 + 13*a*b + 9*b^2)*Sec[x]^2 + 3*(a + b)^2*Sec[x
]^4)*Tan[x])/(15*(a + b)^3)
```

Maple [A]
time = 0.35, size = 109, normalized size = 1.25

method	result
default	$\frac{\frac{a^2\left(\tan^5(x)\right)}{5}+\frac{2ab\left(\tan^5(x)\right)}{5}+\frac{b^2\left(\tan^5(x)\right)}{5}+\frac{2a^2\left(\tan^3(x)\right)}{3}+\frac{5ab\left(\tan^3(x)\right)}{3}+b^2\left(\tan^3(x)\right)+a^2\tan(x)+3ab\tan(x)+3b^2\tan(x)}{(a+b)^3}+\frac{b^3\,\text{arcta}}{(a+}$
risch	$\frac{2i\left(15b^2e^{8ix}+30ab\,e^{6ix}+90b^2e^{6ix}+80a^2e^{4ix}+230ab\,e^{4ix}+240b^2e^{4ix}+40\,e^{2ix}a^2+130b\,e^{2ix}a+150b^2e^{2ix}+8a^2+26ab+33b^2\right)}{15(a+b)^3\left(e^{2ix}+1\right)^5}-\frac{b^3\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a+b)^3*(1/5*a^2*tan(x)^5+2/5*a*b*tan(x)^5+1/5*b^2*tan(x)^5+2/3*a^2*tan(x
)^3+5/3*a*b*tan(x)^3+b^2*tan(x)^3+a^2*tan(x)+3*a*b*tan(x)+3*b^2*tan(x))+b^3
/(a+b)^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [A]
time = 0.49, size = 126, normalized size = 1.45

$$\frac{b^3 \arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)a}}+\frac{3\,(a^2+2\,ab+b^2)\tan(x)^5+5\,(2\,a^2+5\,ab+3\,b^2)\tan(x)^3+15\,(a^2+3\,ab+3\,b^2)\tan(x)}{15\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b^3*arctan((a + b)*tan(x)/sqrt((a + b)*a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*sqrt((a + b)*a)) + 1/15*(3*(a^2 + 2*a*b + b^2)*tan(x)^5 + 5*(2*a^2 + 5*a*b
 + 3*b^2)*tan(x)^3 + 15*(a^2 + 3*a*b + 3*b^2)*tan(x))/(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(75) = 150.
time = 0.47, size = 459, normalized size = 5.28

$$\left[-\frac{15\,\sqrt{-a^2-ab}\,b^3\cos(x)^5\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+5\,ab+b^2)\cos(x)^2+4\left((2\,a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2+2\,ab+b^2}{b^2\cos(x)^4-2\,(ab+b^2)\cos(x)^2+a^2+2\,ab+b^2}\right)-4\left((8\,a^4+34\,a^3b+59\,a^2b^2+33\,ab^3)\cos(x)^4+3\,a^4+9\,a^3b+9\,a^2b^2+3\,ab^3+(4\,a^4+17\,a^3b+22\,a^2b^2+9\,ab^3)\cos(x)^2\right)\sin(x)}{60\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\cos(x)^5},\ -\frac{15\,\sqrt{a^2+ab}\,b^3\arctan\left(\frac{(2\,a+b)\cos(x)^2-a-b}{2\,\sqrt{a^2+ab}\,\cos(x)\sin(x)}\right)\cos(x)^5-2\left((8\,a^4+34\,a^3b+59\,a^2b^2+33\,ab^3)\cos(x)^4+3\,a^4+9\,a^3b+9\,a^2b^2+3\,ab^3+(4\,a^4+17\,a^3b+22\,a^2b^2+9\,ab^3)\cos(x)^2\right)\sin(x)}{30\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\cos(x)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*sqrt(-a^2 - a*b)*b^3*cos(x)^5*log(((8*a^2 + 8*a*b + b^2)*cos(x)^
4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(
x))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b
^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*((8*a^4 + 34*a^3*b + 59*a^2*b^2 + 33
*a*b^3)*cos(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 + (4*a^4 + 17*a^3*
b + 22*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4
*a^2*b^3 + a*b^4)*cos(x)^5), -1/30*(15*sqrt(a^2 + a*b)*b^3*arctan(1/2*((2*a
 + b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x)^5 - 2*((8*a
^4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b
^2 + 3*a*b^3 + (4*a^4 + 17*a^3*b + 22*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/
((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**6/(a + b*sin(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(75) = 150$.

time = 0.44, size = 254, normalized size = 2.92

$$\frac{\left(\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2\,a + 2\,b) + \arctan\left(\frac{a\tan(x) + b\tan(x)}{\sqrt{a^2 + ab}}\right)\right) b^3}{(a^3 + 3\,a^2b + 3\,ab^2 + b^3)\sqrt{a^2 + ab}} + \frac{3\,a^4\tan(x)^5 + 12\,a^3b\tan(x)^5 + 18\,a^2b^2\tan(x)^5 + 12\,ab^3\tan(x)^5 + 3\,b^4\tan(x)^5 + 10\,a^4\tan(x)^3 + 45\,a^3b\tan(x)^3 + 75\,a^2b^2\tan(x)^3 + 55\,ab^3\tan(x)^3 + 15\,b^4\tan(x)^3 + 15\,a^4\tan(x) + 75\,a^3b\tan(x) + 150\,a^2b^2\tan(x) + 135\,ab^3\tan(x) + 45\,b^4\tan(x)}{15\,(a^5 + 5\,a^4b + 10\,a^3b^2 + 10\,a^2b^3 + 5\,ab^4 + b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^
2 + a*b)))*b^3/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a^2 + a*b)) + 1/15*(3*
a^4*tan(x)^5 + 12*a^3*b*tan(x)^5 + 18*a^2*b^2*tan(x)^5 + 12*a*b^3*tan(x)^5
+ 3*b^4*tan(x)^5 + 10*a^4*tan(x)^3 + 45*a^3*b*tan(x)^3 + 75*a^2*b^2*tan(x)^
3 + 55*a*b^3*tan(x)^3 + 15*b^4*tan(x)^3 + 15*a^4*tan(x) + 75*a^3*b*tan(x) +
 150*a^2*b^2*tan(x) + 135*a*b^3*tan(x) + 45*b^4*tan(x))/(a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
```

Mupad [B]

time = 13.99, size = 121, normalized size = 1.39

$$\frac{\tan(x)^5}{5\,(a+b)} - \tan(x)^3\left(\frac{a}{3\,(a+b)^2} - \frac{1}{a+b}\right) + \tan(x)\left(\frac{3}{a+b} + \frac{a\left(\frac{a}{(a+b)^2} - \frac{3}{a+b}\right)}{a+b}\right) + \frac{b^3\,\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{2\,\sqrt{a}\,(a+b)^{7/2}}\right)}{\sqrt{a}\,(a+b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^6*(a + b*sin(x)^2)),x)
```

```
[Out] tan(x)^5/(5*(a + b)) - tan(x)^3*(a/(3*(a + b)^2) - 1/(a + b)) + tan(x)*(3/(
a + b) + (a*(a/(a + b)^2 - 3/(a + b)))/(a + b)) + (b^3*atan((tan(x)*(2*a +
2*b)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2))))/(a^(1/2)*
(a + b)^(7/2))
```

3.314 $\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx$

Optimal. Leaf size=113

$$\frac{(4a+5b)x}{2b^3}-\frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3}-\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)}+\frac{(a+b)(2a+b)\tan(x)}{2ab^2\left(a+(a+b)\tan^2(x)\right)}$$

```
[Out] 1/2*(4*a+5*b)*x/b^3-1/2*(4*a-b)*(a+b)^(3/2)*arctan((a+b)^(1/2)*tan(x)/a^(1/
2))/a^(3/2)/b^3-1/2*cos(x)*sin(x)/b/(a+(a+b)*tan(x)^2)+1/2*(a+b)*(2*a+b)*ta
n(x)/a/b^2/(a+(a+b)*tan(x)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3270, 425, 541, 536, 209, 211}

$$-\frac{(4a-b)(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3}+\frac{x(4a+5b)}{2b^3}+\frac{(2a+b)(a+b)\tan(x)}{2ab^2\left((a+b)\tan^2(x)+a\right)}-\frac{\sin(x)\cos(x)}{2b\left((a+b)\tan^2(x)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((4*a + 5*b)*x)/(2*b^3) - ((4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[
x])/Sqrt[a]])/(2*a^(3/2)*b^3) - (Cos[x]*Sin[x])/(2*b*(a + (a + b)*Tan[x]^2)
) + ((a + b)*(2*a + b)*Tan[x])/(2*a*b^2*(a + (a + b)*Tan[x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)$$

$$= -\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b-3(a+b)x^2}{(1+x^2)(a+(a+b)x^2)^2}\,dx, x, \tan(x)\right)}{2b}$$

$$= -\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{(a+b)(2a+b)\tan(x)}{2ab^2\left(a+(a+b)\tan^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{2(2a^2+2ab-b^2)-2(a}{(1+x^2)(a+(a}\right.}{4a}$$

$$= -\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{(a+b)(2a+b)\tan(x)}{2ab^2\left(a+(a+b)\tan^2(x)\right)} - \frac{((4a-b)(a+b)^2)\,\text{Subst}}{2}$$

$$= \frac{(4a+5b)x}{2b^3} - \frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3} - \frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)}$$

Mathematica [A]
time = 0.22, size = 90, normalized size = 0.80

$$\frac{2(4a+5b)x-\frac{2(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}}+b\sin(2x)+\frac{2b(a+b)^2\sin(2x)}{a(2a+b-b\cos(2x))}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (2*(4*a + 5*b)*x - (2*(4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/S
qrt[a]])/a^(3/2) + b*Sin[2*x] + (2*b*(a + b)^2*Sin[2*x])/(a*(2*a + b - b*Co
s[2*x])))/(4*b^3)
```

Maple [A]
time = 0.25, size = 100, normalized size = 0.88

method	result
default	$\frac{\frac{b\tan(x)}{2\left(\tan^2(x)\right)+2}+\frac{(4a+5b)\arctan(\tan(x))}{2}}{b^3}-\frac{(a+b)^2\left(-\frac{b\tan(x)}{2a\left(a\left(\tan^2(x)\right)+b\left(\tan^2(x)\right)+a\right)}+\frac{(4a-b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}\right)}{b^3}$
risch	$\frac{2ax}{b^3}+\frac{5x}{2b^2}-\frac{i\mathrm{e}^{2ix}}{8b^2}+\frac{i\mathrm{e}^{-2ix}}{8b^2}-\frac{i(2a^3\mathrm{e}^{2ix}+5a^2b\,\mathrm{e}^{2ix}+4a\,b^2\mathrm{e}^{2ix}+b^3\mathrm{e}^{2ix}-a^2b-2a\,b^2-b^3)}{a\,b^3(-b\,\mathrm{e}^{4ix}+4a\,\mathrm{e}^{2ix}+2b\,\mathrm{e}^{2ix}-b)}+\frac{\sqrt{-a(a+b)}\ \ln\left(\mathrm{e}^{2ix}-\frac{2i\sqrt{}}{}\right.}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/2*b*tan(x)/(tan(x)^2+1)+1/2*(4*a+5*b)*arctan(tan(x)))-(a+b)^2/b^3*
(-1/2/a*b*tan(x)/(a*tan(x)^2+b*tan(x)^2+a)+1/2*(4*a-b)/a/(a*(a+b))^(1/2)*ar
ctan((a+b)*tan(x)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.50, size = 150, normalized size = 1.33

$$\frac{(2\,a^2+3\,ab+b^2)\tan(x)^3+(2\,a^2+2\,ab+b^2)\tan(x)}{2\left((a^2b^2+ab^3)\tan(x)^4+a^2b^2+(2\,a^2b^2+ab^3)\tan(x)^2\right)}+\frac{(4\,a+5\,b)x}{2\,b^3}-\frac{(4\,a^3+7\,a^2b+2\,ab^2-b^3)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\,\sqrt{(a+b)a}\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((2*a^2 + 3*a*b + b^2)*tan(x)^3 + (2*a^2 + 2*a*b + b^2)*tan(x))/((a^2*b
^2 + a*b^3)*tan(x)^4 + a^2*b^2 + (2*a^2*b^2 + a*b^3)*tan(x)^2) + 1/2*(4*a +
```

```
5*b)*x/b^3 - 1/2*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*arctan((a + b)*tan(x)/s
qrt((a + b)*a))/(sqrt((a + b)*a)*a*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(97) = 194$.
time = 0.46, size = 491, normalized size = 4.35

$$\left[\frac{4\,(4\,a^2b+5\,ab^2)x\cos(x)^2+\left(4\,a^3+7\,a^2b+2\,ab^2-b^3-(4\,a^2b+3\,ab^2-b^3)\cos(x)^2\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+5\,ab+b^2)\cos(x)^2-4\left((2\,a^2+ab)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2+2\,ab+b^2}{b^2\cos(x)^4-2\,(ab+b^2)\cos(x)^2+a^2+2\,ab+b^2}\right)-4\,(4\,a^3+9\,a^2b+5\,ab^2)x+4\left(ab^2\cos(x)^3-(2\,a^2b+3\,ab^2+b^3)\cos(x)\right)\sin(x)}{8\left(ab^4\cos(x)^2-a^2b^3-ab^4\right)}, \frac{2\,(4\,a^2b+5\,ab^2)x\cos(x)^2-\left(4\,a^3+7\,a^2b+2\,ab^2-b^3-(4\,a^2b+3\,ab^2-b^3)\cos(x)^2\right)\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2\,(a+b)\cos(x)\sin(x)}\right)-2\,(4\,a^3+9\,a^2b+5\,ab^2)x+2\left(ab^2\cos(x)^3-(2\,a^2b+3\,ab^2+b^3)\cos(x)\right)\sin(x)}{4\left(ab^4\cos(x)^2-a^2b^3-ab^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(4*a^2*b + 5*a*b^2)*x*cos(x)^2 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 -
 (4*a^2*b + 3*a*b^2 - b^3)*cos(x)^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b +
 b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)
^3 - (a^2 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*
cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*(4*a^3 + 9*a^2*
b + 5*a*b^2)*x + 4*(a*b^2*cos(x)^3 - (2*a^2*b + 3*a*b^2 + b^3)*cos(x))*sin(
x))/(a*b^4*cos(x)^2 - a^2*b^3 - a*b^4), 1/4*(2*(4*a^2*b + 5*a*b^2)*x*cos(x)
^2 - (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 - (4*a^2*b + 3*a*b^2 - b^3)*cos(x)^2)
*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((
a + b)*cos(x)*sin(x))) - 2*(4*a^3 + 9*a^2*b + 5*a*b^2)*x + 2*(a*b^2*cos(x)^
3 - (2*a^2*b + 3*a*b^2 + b^3)*cos(x))*sin(x))/(a*b^4*cos(x)^2 - a^2*b^3 - a
*b^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 175, normalized size = 1.55

$$\frac{(4\,a+5\,b)x}{2\,b^3}-\frac{(4\,a^3+7\,a^2b+2\,ab^2-b^3)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{2\,\sqrt{a^2+ab}\,ab^3}+\frac{2\,a^2\tan(x)^3+3\,ab\tan(x)^3+b^2\tan(x)^3+2\,a^2\tan(x)+2\,ab\tan(x)+b^2\tan(x)}{2\left(a\tan(x)^4+b\tan(x)^4+2\,a\tan(x)^2+b\tan(x)^2+a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a + 5*b)*x/b^3 - 1/2*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*(pi*floor(x/p
i + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b)))/(s
```

```
qrt(a^2 + a*b)*a*b^3) + 1/2*(2*a^2*tan(x)^3 + 3*a*b*tan(x)^3 + b^2*tan(x)^3
 + 2*a^2*tan(x) + 2*a*b*tan(x) + b^2*tan(x))/((a*tan(x)^4 + b*tan(x)^4 + 2*
a*tan(x)^2 + b*tan(x)^2 + a)*a*b^2)
```

Mupad [B]
time = 14.70, size = 463, normalized size = 4.10

$$\frac{\frac{\tan(x)\,(2\,a^2+2\,a\,b+b^2)}{2\,a\,b^2}+\frac{\tan(x)^3\,(a+b)\,(2\,a+b)}{2\,a\,b^2}}{(a+b)\,\tan(x)^4+(2\,a+b)\,\tan(x)^2+a}-\frac{\ln\left(a^2\,b-\tan(x)\,\sqrt{-a^3\,{(a+b)}^3}+a^3\right)\,\sqrt{-a^3\,{(a+b)}^3}\,(4\,a-b)}{4\,a^3\,b^3}+\frac{\ln\left(\tan(x)\,\sqrt{-a^3\,{(a+b)}^3}+a^2\,b+a^3\right)\,\left(a-\frac{b}{4}\right)\,\sqrt{-a^3\,{(a+b)}^3}}{a^3\,b^3}-\frac{\mathrm{atan}\left(\frac{41\,\tan(x)}{2\,\left(\frac{131\,a}{4\,b}+\frac{11\,b}{4\,a}-\frac{5\,b^2}{4\,a^2}+\frac{85\,a^2}{4\,b^2}+\frac{5\,a^3}{b^3}+\frac{41}{2}\right)}+\frac{11\,\tan(x)}{4\,\left(\frac{41\,a}{2\,b}-\frac{5\,b}{4\,a}+\frac{131\,a^2}{4\,b^2}+\frac{85\,a^3}{4\,b^3}+\frac{5\,a^4}{b^4}+\frac{11}{4}\right)}+\frac{131\,a\,\tan(x)}{4\,\left(\frac{131\,a}{4}+\frac{41\,b}{2}+\frac{11\,b^2}{4\,a}+\frac{85\,a^2}{4\,b}-\frac{5\,b^3}{4\,a^2}+\frac{5\,a^3}{b^2}\right)}-\frac{5\,b\,\tan(x)}{4\,\left(\frac{11\,a}{4}-\frac{5\,b}{4}+\frac{41\,a^2}{2\,b}+\frac{131\,a^3}{4\,b^2}+\frac{85\,a^4}{4\,b^3}+\frac{5\,a^5}{b^4}\right)}+\frac{85\,a^2\,\tan(x)}{4\,\left(\frac{131\,a\,b}{4}+\frac{85\,a^2}{4}+\frac{41\,b^2}{2}+\frac{11\,b^3}{4\,a}+\frac{5\,a^3}{b}-\frac{5\,b^4}{4\,a^2}\right)}+\frac{5\,a^3\,\tan(x)}{\frac{131\,a\,b^2}{4}+\frac{85\,a^2\,b}{4}+5\,a^3+\frac{41\,b^3}{2}+\frac{11\,b^4}{4\,a}-\frac{5\,b^5}{4\,a^2}}\right)\,\left(a\,1\mathrm{i}+\frac{b\,5\mathrm{i}}{4}\right)\,2\mathrm{i}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a + b*sin(x)^2)^2,x)
```

```
[Out] ((tan(x)*(2*a*b + 2*a^2 + b^2))/(2*a*b^2) + (tan(x)^3*(a + b)*(2*a + b))/(2
*a*b^2))/(a + tan(x)^2*(2*a + b) + tan(x)^4*(a + b)) - (atan((41*tan(x))/(2
*((131*a)/(4*b) + (11*b)/(4*a) - (5*b^2)/(4*a^2) + (85*a^2)/(4*b^2) + (5*a^
3)/b^3 + 41/2)) + (11*tan(x))/(4*((41*a)/(2*b) - (5*b)/(4*a) + (131*a^2)/(4
*b^2) + (85*a^3)/(4*b^3) + (5*a^4)/b^4 + 11/4)) + (131*a*tan(x))/(4*((131*a
)/4 + (41*b)/2 + (11*b^2)/(4*a) + (85*a^2)/(4*b) - (5*b^3)/(4*a^2) + (5*a^3
)/b^2)) - (5*b*tan(x))/(4*((11*a)/4 - (5*b)/4 + (41*a^2)/(2*b) + (131*a^3)/
(4*b^2) + (85*a^4)/(4*b^3) + (5*a^5)/b^4)) + (85*a^2*tan(x))/(4*((131*a*b)/
4 + (85*a^2)/4 + (41*b^2)/2 + (11*b^3)/(4*a) + (5*a^3)/b - (5*b^4)/(4*a^2))
) + (5*a^3*tan(x))/((131*a*b^2)/4 + (85*a^2*b)/4 + 5*a^3 + (41*b^3)/2 + (11
*b^4)/(4*a) - (5*b^5)/(4*a^2)))*(a*1i + (b*5i)/4)*2i)/b^3 - (log(a^2*b - ta
n(x)*(-a^3*(a + b)^3)^(1/2) + a^3)*(-a^3*(a + b)^3)^(1/2)*(4*a - b))/(4*a^3
*b^3) + (log(tan(x)*(-a^3*(a + b)^3)^(1/2) + a^2*b + a^3)*(a - b/4)*(-a^3*(
a + b)^3)^(1/2))/(a^3*b^3)
```

3.315 $\int \frac{\cos^5(x)}{\left(a+b\sin^2(x)\right)^2} dx$

Optimal. Leaf size=72

$$-\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{\sin(x)}{b^2}+\frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}$$

```
[Out] -1/2*(3*a-b)*(a+b)*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)+sin(x)/b^
2+1/2*(a+b)^2*sin(x)/a/b^2/(a+b*sin(x)^2)
```

Rubi [A]

time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3269, 398, 393, 211}

$$-\frac{(3a-b)(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}+\frac{\sin(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a + b*Sin[x]^2)^2,x]
```

```
[Out] -1/2*((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(a^(3/2)*b^(5/2))
 + Sin[x]/b^2 + ((a + b)^2*Sin[x])/(2*a*b^2*(a + b*Sin[x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2b(a+b)x^2}{b^2\left(a+bx^2\right)^2}\right)dx, x, \sin(x)\right)\\ &= \frac{\sin(x)}{b^2}-\frac{\text{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)}{b^2}\\ &= \frac{\sin(x)}{b^2}+\frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}-\frac{((3a-b)(a+b))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{2ab^2}\\ &= -\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{\sin(x)}{b^2}+\frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 118, normalized size = 1.64

$$\frac{\frac{\left(3a^2+2ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}}+\frac{\left(-3a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}}+4\sqrt{b}\sin(x)+\frac{4\sqrt{b}(a+b)^2\sin(x)}{a(2a+b-b\cos(2x))}}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (((3*a^2 + 2*a*b - b^2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2) + ((-3*a^
2 - 2*a*b + b^2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) + 4*Sqrt[b]*Sin[
x] + (4*Sqrt[b]*(a + b)^2*Sin[x])/(a*(2*a + b - b*Cos[2*x])))/(4*b^(5/2))
```

Maple [A]
time = 0.32, size = 77, normalized size = 1.07

method	result

default	$\frac{\sin(x)}{b^2}-\frac{-\frac{\left(a^2+2ab+b^2\right)\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)}+\frac{\left(3a^2+2ab-b^2\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}$
risch	$-\frac{i e^{ix}}{2b^2}+\frac{i e^{-ix}}{2b^2}-\frac{i(a^2+2ab+b^2)\left(e^{3ix}-e^{ix}\right)}{a\,b^2(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b)}-\frac{3a\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,b^2}-\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}\,b}+\frac{\ln\left(e^{2ix}+\right.}{4\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)/b^2-1/b^2*(-1/2*(a^2+2*a*b+b^2)/a*sin(x)/(a+b*sin(x)^2)+1/2*(3*a^2+2
*a*b-b^2)/a/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.48, size = 79, normalized size = 1.10

$$\frac{(a^2+2\,ab+b^2)\sin(x)}{2\left(ab^3\sin(x)^2+a^2b^2\right)}+\frac{\sin(x)}{b^2}-\frac{(3\,a^2+2\,ab-b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2 + 2*a*b + b^2)*sin(x)/(a*b^3*sin(x)^2 + a^2*b^2) + sin(x)/b^2 - 1/
2*(3*a^2 + 2*a*b - b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(60) = 120.
time = 0.42, size = 296, normalized size = 4.11

$$\left[-\frac{(3\,a^3+5\,a^2b+ab^2-b^3-(3\,a^2b+2\,ab^2-b^3)\cos(x)^2)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)-2\left(2\,a^2b^2\cos(x)^2-3\,a^3b-4\,a^2b^2-ab^3\right)\sin(x)}{4\left(a^2b^4\cos(x)^2-a^3b^3-a^2b^4\right)},\ \frac{(3\,a^3+5\,a^2b+ab^2-b^3-(3\,a^2b+2\,ab^2-b^3)\cos(x)^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)+\left(2\,a^2b^2\cos(x)^2-3\,a^3b-4\,a^2b^2-ab^3\right)\sin(x)}{2\left(a^2b^4\cos(x)^2-a^3b^3-a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a^3 + 5*a^2*b + a*b^2 - b^3 - (3*a^2*b + 2*a*b^2 - b^3)*cos(x)^2)
*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a
 - b)) - 2*(2*a^2*b^2*cos(x)^2 - 3*a^3*b - 4*a^2*b^2 - a*b^3)*sin(x))/(a^2*
b^4*cos(x)^2 - a^3*b^3 - a^2*b^4), 1/2*((3*a^3 + 5*a^2*b + a*b^2 - b^3 - (3
*a^2*b + 2*a*b^2 - b^3)*cos(x)^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a) + (2
*a^2*b^2*cos(x)^2 - 3*a^3*b - 4*a^2*b^2 - a*b^3)*sin(x))/(a^2*b^4*cos(x)^2
- a^3*b^3 - a^2*b^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 82, normalized size = 1.14

$$\frac{\sin(x)}{b^2} - \frac{(3a^2 + 2ab - b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab^2} + \frac{a^2\sin(x) + 2ab\sin(x) + b^2\sin(x)}{2\left(b\sin(x)^2 + a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] sin(x)/b^2 - 1/2*(3*a^2 + 2*a*b - b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b
)*a*b^2) + 1/2*(a^2*sin(x) + 2*a*b*sin(x) + b^2*sin(x))/((b*sin(x)^2 + a)*a
*b^2)
```

Mupad [B]
time = 14.40, size = 96, normalized size = 1.33

$$\frac{\sin(x)}{b^2} + \frac{\sin(x)\,\left(a^2 + 2\,a\,b + b^2\right)}{2\,a\,\left(b^3\sin(x)^2 + a\,b^2\right)} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,(a+b)\,(3\,a-b)}{\sqrt{a}\,(3\,a^2+2\,a\,b-b^2)}\right)\,(a+b)\,(3\,a-b)}{2\,a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a + b*sin(x)^2)^2,x)
```

```
[Out] sin(x)/b^2 + (sin(x)*(2*a*b + a^2 + b^2))/(2*a*(b^3*sin(x)^2 + a*b^2)) - (a
tan((b^(1/2)*sin(x)*(a + b)*(3*a - b))/(a^(1/2)*(2*a*b + 3*a^2 - b^2)))*(a
+ b)*(3*a - b))/(2*a^(3/2)*b^(5/2))
```

3.316 $\int \frac{\cos^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=75

$$\frac{x}{b^2}-\frac{(2a-b)\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2}+\frac{(a+b)\tan(x)}{2ab\,(a+(a+b)\tan^2(x))}$$

```
[Out] x/b^2-1/2*(2*a-b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))*(a+b)^(1/2)/a^(3/2)/b^
2+1/2*(a+b)*tan(x)/a/b/(a+(a+b)*tan(x)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3270, 425, 536, 209, 211}

$$-\frac{(2a-b)\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2}+\frac{(a+b)\tan(x)}{2ab\,((a+b)\tan^2(x)+a)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] x/b^2 - ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(
3/2)*b^2) + ((a + b)*Tan[x])/(2*a*b*(a + (a + b)*Tan[x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \frac{(a+b)\tan(x)}{2ab\left(a+(a+b)\tan^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{a-b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \tan(x)\right)}{2ab}\\
&= \frac{(a+b)\tan(x)}{2ab\left(a+(a+b)\tan^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{b^2} - \frac{((2a-b)(a+b))\text{Subst}\left(\cdots\right.}{2a\cdots}\\
&= \frac{x}{b^2} - \frac{(2a-b)\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2} + \frac{(a+b)\tan(x)}{2ab\left(a+(a+b)\tan^2(x)\right)}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 79, normalized size = 1.05

$$\frac{2x+\frac{(-2a^2-ab+b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{b(a+b)\sin(2x)}{a(2a+b-b\cos(2x))}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (2*x + ((-2*a^2 - a*b + b^2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)
*Sqrt[a + b]) + (b*(a + b)*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])))/(2*b^2)
```

Maple [A]
time = 0.28, size = 75, normalized size = 1.00

method	result
default	$-\frac{(a+b)\left(-\frac{b\tan(x)}{2a\left(a\left(\tan^2(x)\right)+b\left(\tan^2(x)\right)+a\right)}+\frac{(2a-b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}\right)}{b^2}+\frac{\arctan(\tan(x))}{b^2}$
risch	$\frac{x}{b^2}-\frac{i(2\,e^{2ix}a^2+3b\,e^{2ix}a+b^2e^{2ix}-ab-b^2)}{a\,b^2(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b)}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-a(a+b)}+2a+b}{b}\right)}{2a\,b^2}-\frac{\sqrt{-a(a+b)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(a+b)/b^2*(-1/2/a*b*tan(x)/(a*tan(x)^2+b*tan(x)^2+a)+1/2*(2*a-b)/a/(a*(a+b
))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2)))+1/b^2*arctan(tan(x))
```

Maxima [A]
time = 0.48, size = 80, normalized size = 1.07

$$\frac{(a+b)\tan(x)}{2\left(a^2b+(a^2b+ab^2)\tan(x)^2\right)}+\frac{x}{b^2}-\frac{(2\,a^2+ab-b^2)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a + b)*tan(x)/(a^2*b + (a^2*b + a*b^2)*tan(x)^2) + x/b^2 - 1/2*(2*a^2
 + a*b - b^2)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(63) = 126.
time = 0.45, size = 367, normalized size = 4.89

$$\left[\frac{8\,abx\cos(x)^2-4\,(ab+b^2)\cos(x)\sin(x)-\left((2\,ab-b^2)\cos(x)^2-2\,a^2-ab+b^2\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+5\,ab+b^2)\cos(x)^2-4\left((2\,a^2+ab)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2+2\,ab+b^2}{b^2\cos(x)^4-2\,(ab+b^2)\cos(x)^2+a^2+2\,ab+b^2}\right)-8\,(a^2+ab)x}{8\left(ab^3\cos(x)^2-a^2b^2-ab^3\right)},\ \frac{4\,abx\cos(x)^2-2\,(ab+b^2)\cos(x)\sin(x)+\left((2\,ab-b^2)\cos(x)^2-2\,a^2-ab+b^2\right)\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2\,(a+b)\cos(x)\sin(x)}\right)-4\,(a^2+ab)x}{4\left(ab^3\cos(x)^2-a^2b^2-ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*x*cos(x)^2 - 4*(a*b + b^2)*cos(x)*sin(x) - ((2*a*b - b^2)*cos(x
)^2 - 2*a^2 - a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)
^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2 +
```

```
a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2
*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 8*(a^2 + a*b)*x)/(a*b^3*cos(x
)^2 - a^2*b^2 - a*b^3), 1/4*(4*a*b*x*cos(x)^2 - 2*(a*b + b^2)*cos(x)*sin(x)
 + ((2*a*b - b^2)*cos(x)^2 - 2*a^2 - a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*
((2*a + b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) - 4*(
a^2 + a*b)*x)/(a*b^3*cos(x)^2 - a^2*b^2 - a*b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 109, normalized size = 1.45

$$\frac{x}{b^2} - \frac{\left(\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2\,a + 2\,b) + \arctan\left(\frac{a\tan(x) + b\tan(x)}{\sqrt{a^2 + ab}}\right)\right)(2\,a^2 + ab - b^2)}{2\,\sqrt{a^2 + ab}\,ab^2} + \frac{a\tan(x) + b\tan(x)}{2\,\left(a\tan(x)^2 + b\tan(x)^2 + a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] x/b^2 - 1/2*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan
(x))/sqrt(a^2 + a*b)))*(2*a^2 + a*b - b^2)/(sqrt(a^2 + a*b)*a*b^2) + 1/2*(a
*tan(x) + b*tan(x))/((a*tan(x)^2 + b*tan(x)^2 + a)*a*b)
```

Mupad [B]
time = 14.33, size = 533, normalized size = 7.11

$$\frac{\mathrm{atan}\left(\frac{5\tan(x)}{2\left(\frac{3a}{2b}+\frac{b}{2a}-\frac{b^2}{2a^2}+\frac{5}{2}\right)}+\frac{\tan(x)}{2\left(\frac{5a}{2b}-\frac{b}{2a}+\frac{3a^2}{2b^2}+\frac{1}{2}\right)}+\frac{3a\tan(x)}{2\left(\frac{3a}{2}+\frac{5b}{2}+\frac{b^2}{2a}-\frac{b^3}{2a^2}\right)}-\frac{b\tan(x)}{2\left(\frac{a}{2}-\frac{b}{2}+\frac{5a^2}{2b}+\frac{3a^3}{2b^2}\right)}\right)}{b^2}+\frac{\mathrm{atanh}\left(\frac{\tan(x)\sqrt{-a^4-b\,a^3}}{a^2-\frac{3ab}{2}-\frac{b^2}{2}+\frac{b^3}{4a}+\frac{13a^3}{4b}+\frac{3a^4}{2b^2}}+\frac{3\tan(x)\sqrt{-a^4-b\,a^3}}{2\left(\frac{13ab}{4}+\frac{3a^2}{2}+b^2-\frac{3b^3}{2a}-\frac{b^4}{2a^2}+\frac{b^5}{4a^3}\right)}+\frac{13\tan(x)\sqrt{-a^4-b\,a^3}}{4\left(ab+\frac{13a^2}{4}-\frac{3b^2}{2}-\frac{b^3}{2a}+\frac{3a^3}{2b}+\frac{b^4}{4a^2}\right)}-\frac{3b\tan(x)\sqrt{-a^4-b\,a^3}}{2\left(a^3-\frac{3a^2b}{2}-\frac{ab^2}{2}+\frac{b^3}{4}+\frac{13a^4}{4b}+\frac{3a^5}{2b^2}\right)}-\frac{b^2\tan(x)\sqrt{-a^4-b\,a^3}}{2\left(\frac{ab^3}{4}-\frac{3a^3b}{2}+a^4-\frac{a^2b^2}{2}+\frac{13a^5}{4b}+\frac{3a^6}{2b^2}\right)}+\frac{b^3\tan(x)\sqrt{-a^4-b\,a^3}}{4\left(a^5-\frac{3a^4b}{2}+\frac{a^2b^3}{4}-\frac{a^3b^2}{2}+\frac{13a^6}{4b}+\frac{3a^7}{2b^2}\right)}\right)\sqrt{-a^3\,(a+b)}\,(2a-b)}{2a^3b^2}+\frac{\tan(x)\,(a+b)}{2ab\,\left((a+b)\tan(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a + b*sin(x)^2)^2,x)
```

```
[Out] atan((5*tan(x))/(2*((3*a)/(2*b) + b/(2*a) - b^2/(2*a^2) + 5/2)) + tan(x)/(2
*((5*a)/(2*b) - b/(2*a) + (3*a^2)/(2*b^2) + 1/2)) + (3*a*tan(x))/(2*((3*a)/
2 + (5*b)/2 + b^2/(2*a) - b^3/(2*a^2))) - (b*tan(x))/(2*(a/2 - b/2 + (5*a^2
)/(2*b) + (3*a^3)/(2*b^2))))/b^2 + (atanh((tan(x)*(- a^3*b - a^4)^(1/2))/(a
^2 - (3*a*b)/2 - b^2/2 + b^3/(4*a) + (13*a^3)/(4*b) + (3*a^4)/(2*b^2)) + (3
*tan(x)*(- a^3*b - a^4)^(1/2))/(2*((13*a*b)/4 + (3*a^2)/2 + b^2 - (3*b^3)/(
2*a) - b^4/(2*a^2) + b^5/(4*a^3))) + (13*tan(x)*(- a^3*b - a^4)^(1/2))/(4*(
```

```
a*b + (13*a^2)/4 - (3*b^2)/2 - b^3/(2*a) + (3*a^3)/(2*b) + b^4/(4*a^2))) -
(3*b*tan(x)*(- a^3*b - a^4)^(1/2))/(2*(a^3 - (3*a^2*b)/2 - (a*b^2)/2 + b^3/
4 + (13*a^4)/(4*b) + (3*a^5)/(2*b^2))) - (b^2*tan(x)*(- a^3*b - a^4)^(1/2))
/(2*((a*b^3)/4 - (3*a^3*b)/2 + a^4 - (a^2*b^2)/2 + (13*a^5)/(4*b) + (3*a^6)
/(2*b^2))) + (b^3*tan(x)*(- a^3*b - a^4)^(1/2))/(4*(a^5 - (3*a^4*b)/2 + (a^
2*b^3)/4 - (a^3*b^2)/2 + (13*a^6)/(4*b) + (3*a^7)/(2*b^2))))*(-a^3*(a + b))
^(1/2)*(2*a - b))/(2*a^3*b^2) + (tan(x)*(a + b))/(2*a*b*(a + tan(x)^2*(a +
b)))
```

3.317 $\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2} \, dx$

Optimal. Leaf size=59

$$-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}$$

```
[Out] -1/2*(a-b)*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/2*(a+b)*sin(x)/
a/b/(a+b*sin(x)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 393, 211}

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] -1/2*((a - b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(a^(3/2)*b^(3/2)) + ((a + b
)*Sin[x])/(2*a*b*(a + b*Sin[x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)\\
&= \frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{2ab}\\
&= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} + \frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 1.00

$$-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} + \frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] -1/2*((a - b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(a^(3/2)*b^(3/2)) + ((a + b
)*Sin[x])/(2*a*b*(a + b*Sin[x]^2))
```

Maple [A]
time = 0.26, size = 53, normalized size = 0.90

method	result
default	$\frac{(a+b)\sin(x)}{2ab\left(a+b\left(\sin^2(x)\right)\right)} - \frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$
risch	$\frac{i(a+b)\left(e^{3ix}-e^{ix}\right)}{ab\left(b\,e^{4ix}-4a\,e^{2ix}-2b\,e^{2ix}+b\right)} - \frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,b} + \frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,a} + \frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,b} - \frac{\ln\left(e^{2ix}-\right.}{4\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a+b)*sin(x)/a/b/(a+b*sin(x)^2)-1/2*(a-b)/a/b/(a*b)^(1/2)*arctan(b*sin(
x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 53, normalized size = 0.90

$$\frac{(a+b)\sin(x)}{2\left(ab^2\sin(x)^2+a^2b\right)}-\frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a + b)*sin(x)/(a*b^2*sin(x)^2 + a^2*b) - 1/2*(a - b)*arctan(b*sin(x)/s
qrt(a*b))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.41, size = 206, normalized size = 3.49

$$\left[\frac{\left((ab-b^2)\cos(x)^2-a^2+b^2\right)\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\,\sin(x)+a-b}{b\cos(x)^2-a-b}\right)-2\left(a^2b+ab^2\right)\sin(x)}{4\left(a^2b^3\cos(x)^2-a^3b^2-a^2b^3\right)},-\frac{\left((ab-b^2)\cos(x)^2-a^2+b^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,\sin(x)}{a}\right)+\left(a^2b+ab^2\right)\sin(x)}{2\left(a^2b^3\cos(x)^2-a^3b^2-a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b - b^2)*cos(x)^2 - a^2 + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sq
rt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) - 2*(a^2*b + a*b^2)*sin(x))/
(a^2*b^3*cos(x)^2 - a^3*b^2 - a^2*b^3), -1/2*(((a*b - b^2)*cos(x)^2 - a^2 +
 b^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a) + (a^2*b + a*b^2)*sin(x))/(a^2*b
^3*cos(x)^2 - a^3*b^2 - a^2*b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 56, normalized size = 0.95

$$-\frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}+\frac{a\sin(x)+b\sin(x)}{2\left(b\sin(x)^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a - b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(a*sin(x) + b
*sin(x))/((b*sin(x)^2 + a)*a*b)
```

Mupad [B]
time = 0.14, size = 47, normalized size = 0.80

$$\frac{\sin(x)\ (a+b)}{2\,a\,b\ \left(b\sin(x)^2+a\right)}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\ \sin(x)}{\sqrt{a}}\right)\ (a-b)}{2\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a + b*sin(x)^2)^2,x)
```

```
[Out] (sin(x)*(a + b))/(2*a*b*(a + b*sin(x)^2)) - (atan((b^(1/2)*sin(x))/a^(1/2))
*(a - b))/(2*a^(3/2)*b^(3/2))
```

3.318 $\int \frac{\cos^2(x)}{(a+b\sin^2(x))^2}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\left(a+(a+b)\tan^2(x)\right)}$$

```
[Out] 1/2*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(1/2)+1/2*tan(x)/a/(a+
(a+b)*tan(x)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3270, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\left((a+b)\tan^2(x)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) + Tan[x]/(2*a*
(a + (a + b)*Tan[x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{2a\left(a+(a+b)\tan^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{2a} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}} + \frac{\tan(x)}{2a\left(a+(a+b)\tan^2(x)\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 1.09

$$
\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}} - \frac{\sin(2x)}{2a(-2a-b+b\cos(2x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) - Sin[2*x]/(2*
a*(-2*a - b + b*Cos[2*x]))
```

Maple [A]
time = 0.20, size = 51, normalized size = 0.94

method	result
default	$\frac{\tan(x)}{2a(a(\tan^2(x))+b(\tan^2(x))+a)} + \frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}$
risch	$-\frac{i(2a\,e^{2ix}+b\,e^{2ix}-b)}{ab(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b)} - \frac{\ln\left(e^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{4\sqrt{-a^2-ab}\,a} + \frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab-2a\sqrt{-\cdots}}{b\cdots}\right)}{4\sqrt{-\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tan(x)/a/(a*tan(x)^2+b*tan(x)^2+a)+1/2/a/(a*(a+b))^(1/2)*arctan((a+b)*t
an(x)/(a*(a+b))^(1/2))
```

Maxima [A]
time = 0.48, size = 49, normalized size = 0.91

$$\frac{\tan(x)}{2\left((a^2+ab)\tan(x)^2+a^2\right)}+\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)/((a^2 + a*b)*tan(x)^2 + a^2) + 1/2*arctan((a + b)*tan(x)/sqrt((a
 + b)*a))/(sqrt((a + b)*a)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(42) = 84.
time = 0.41, size = 313, normalized size = 5.80

$$\left[\frac{4\,(a^2+ab)\cos(x)\sin(x)+\left(b\cos(x)^2-a-b\right)\sqrt{-a^2-ab}\,\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+5\,ab+b^2)\cos(x)^2+4\left((2\,a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2+2\,ab+b^2}{b^2\cos(x)^4-2\,(ab+b^2)\cos(x)^2+a^2+2\,ab+b^2}\right)}{8\left(a^4+2\,a^3b+a^2b^2-(a^3b+a^2b^2)\cos(x)^2\right)},\ \frac{2\,(a^2+ab)\cos(x)\sin(x)+\left(b\cos(x)^2-a-b\right)\sqrt{a^2+ab}\,\arctan\left(\frac{(2\,a+b)\cos(x)^2-a-b}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{4\left(a^4+2\,a^3b+a^2b^2-(a^3b+a^2b^2)\cos(x)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + a*b)*cos(x)*sin(x) + (b*cos(x)^2 - a - b)*sqrt(-a^2 - a*b)*l
og(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*(
(2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b
+ b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)))/(a^4 +
 2*a^3*b + a^2*b^2 - (a^3*b + a^2*b^2)*cos(x)^2), 1/4*(2*(a^2 + a*b)*cos(x)
*sin(x) + (b*cos(x)^2 - a - b)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)
^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))))/(a^4 + 2*a^3*b + a^2*b^2 - (a
^3*b + a^2*b^2)*cos(x)^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 77, normalized size = 1.43

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{2\sqrt{a^2+ab}\,a}+\frac{\tan(x)}{2\left(a\tan(x)^2+b\tan(x)^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqr
t(a^2 + a*b)))/(sqrt(a^2 + a*b)*a) + 1/2*tan(x)/((a*tan(x)^2 + b*tan(x)^2 +
 a)*a)
```

Mupad [B]
time = 14.31, size = 50, normalized size = 0.93

$$\frac{\operatorname{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)}{2\,a^{3/2}\,\sqrt{a+b}}+\frac{\tan(x)}{2\,a\,\left((a+b)\,\tan\left(x\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*sin(x)^2)^2,x)
```

```
[Out] atan((tan(x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2)))/(2*a^(3/2)*(a + b)^(1/
2)) + tan(x)/(2*a*(a + tan(x)^2*(a + b)))
```

3.319 $\int \frac{\cos(x)}{(a+b\sin^2(x))^2}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

```
[Out] 1/2*sin(x)/a/(a+b*sin(x)^2)+1/2*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/b^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3269, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sin[x]/(2*a*(a + b*S
in[x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)\\
&= \frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{2a}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 1.00

$$
\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sin[x]/(2*a*(a + b*S
in[x]^2))
```

Maple [A]
time = 0.15, size = 39, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)} + \frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	39
default	$\frac{\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)} + \frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	39
risch	$\frac{i\left(e^{3ix}-e^{ix}\right)}{a\left(b\,e^{4ix}-4a\,e^{2ix}-2b\,e^{2ix}+b\right)} - \frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,a} + \frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x)/a/(a+b*sin(x)^2)+1/2/a/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 38, normalized size = 0.79

$$\frac{\sin(x)}{2\left(ab\sin(x)^2+a^2\right)}+\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)/(a*b*sin(x)^2 + a^2) + 1/2*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)
*a)
```

Fricas [A]
time = 0.39, size = 165, normalized size = 3.44

$$\left[-\frac{2\,ab\sin(x)+\left(b\cos(x)^2-a-b\right)\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\,\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{4\left(a^2b^2\cos(x)^2-a^3b-a^2b^2\right)},\,-\frac{ab\sin(x)-\left(b\cos(x)^2-a-b\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,\sin(x)}{a}\right)}{2\left(a^2b^2\cos(x)^2-a^3b-a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*sin(x) + (b*cos(x)^2 - a - b)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*
sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)))/(a^2*b^2*cos(x)^2 - a^3*b
 - a^2*b^2), -1/2*(a*b*sin(x) - (b*cos(x)^2 - a - b)*sqrt(a*b)*arctan(sqrt(
a*b)*sin(x)/a))/(a^2*b^2*cos(x)^2 - a^3*b - a^2*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(41) = 82.
time = 3.44, size = 289, normalized size = 6.02

$$\begin{cases}\frac{\tilde{\infty}}{\sin^3(x)} & \text{for } a=0 \wedge b=0\\ \frac{\sin(x)}{a^2} & \text{for } b=0\\ -\frac{1}{3b^2\sin^3(x)} & \text{for } a=0\\ \frac{a\log\left(-\sqrt{-\frac{a}{b}}+\sin(x)\right)}{4a^2b\sqrt{-\frac{a}{b}}+4ab^2\sqrt{-\frac{a}{b}}\sin^2(x)}-\frac{a\log\left(\sqrt{-\frac{a}{b}}+\sin(x)\right)}{4a^2b\sqrt{-\frac{a}{b}}+4ab^2\sqrt{-\frac{a}{b}}\sin^2(x)}+\frac{2b\sqrt{-\frac{a}{b}}\sin(x)}{4a^2b\sqrt{-\frac{a}{b}}+4ab^2\sqrt{-\frac{a}{b}}\sin^2(x)}+\frac{b\log\left(-\sqrt{-\frac{a}{b}}+\sin(x)\right)\sin^2(x)}{4a^2b\sqrt{-\frac{a}{b}}+4ab^2\sqrt{-\frac{a}{b}}\sin^2(x)}-\frac{b\log\left(\sqrt{-\frac{a}{b}}+\sin(x)\right)\sin^2(x)}{4a^2b\sqrt{-\frac{a}{b}}+4ab^2\sqrt{-\frac{a}{b}}\sin^2(x)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)**2)**2,x)
```

```
[Out] Piecewise((zoo/sin(x)**3, Eq(a, 0) & Eq(b, 0)), (sin(x)/a**2, Eq(b, 0)), (-
1/(3*b**2*sin(x)**3), Eq(a, 0)), (a*log(-sqrt(-a/b) + sin(x))/(4*a**2*b*sqr
t(-a/b) + 4*a*b**2*sqrt(-a/b)*sin(x)**2) - a*log(sqrt(-a/b) + sin(x))/(4*a*
*2*b*sqrt(-a/b) + 4*a*b**2*sqrt(-a/b)*sin(x)**2) + 2*b*sqrt(-a/b)*sin(x)/(4
*a**2*b*sqrt(-a/b) + 4*a*b**2*sqrt(-a/b)*sin(x)**2) + b*log(-sqrt(-a/b) + s
in(x))*sin(x)**2/(4*a**2*b*sqrt(-a/b) + 4*a*b**2*sqrt(-a/b)*sin(x)**2) - b*
```

```
log(sqrt(-a/b) + sin(x))*sin(x)**2/(4*a**2*b*sqrt(-a/b) + 4*a*b**2*sqrt(-a/
b)*sin(x)**2), True))
```

Giac [A]
time = 0.44, size = 38, normalized size = 0.79

$$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}+\frac{\sin(x)}{2\left(b\sin(x)^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*sin(x)/((b*sin(x)^2 + a)
*a)
```

Mupad [B]
time = 15.38, size = 36, normalized size = 0.75

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}+\frac{\sin(x)}{2\,a\,\left(b\sin(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a + b*sin(x)^2)^2,x)
```

```
[Out] atan((b^(1/2)*sin(x))/a^(1/2))/(2*a^(3/2)*b^(1/2)) + sin(x)/(2*a*(a + b*sin
(x)^2))
```

3.320 $\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2}+\frac{\tanh^{-1}(\sin(x))}{(a+b)^2}+\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}$$

```
[Out] arctanh(sin(x))/(a+b)^2+1/2*b*sin(x)/a/(a+b)/(a+b*sin(x)^2)+1/2*(3*a+b)*arc
tan(sin(x)*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(a+b)^2
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3269, 425, 536, 212, 211}

$$\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2}+\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}+\frac{\tanh^{-1}(\sin(x))}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^2)
+ ArcTanh[Sin[x]]/(a + b)^2 + (b*Sin[x])/(2*a*(a + b)*(a + b*Sin[x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+bx^2\right)^2}\,dx,x,\sin(x)\right)\\
&= \frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}-\frac{\text{Subst}\left(\int \frac{b-2(a+b)+bx^2}{(1-x^2)(a+bx^2)}\,dx,x,\sin(x)\right)}{2a(a+b)}\\
&= \frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(x)\right)}{(a+b)^2}+\frac{(b(3a+b))\text{Subst}\left(\int \frac{1}{a+bx^2}\right.}{2a(a+b)^2}\\
&= \frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2}+\frac{\tanh^{-1}(\sin(x))}{(a+b)^2}+\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 130, normalized size = 1.78

$$
\frac{-\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\,\csc(x)}{\sqrt{b}}\right)}{a^{3/2}}+\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{a^{3/2}}+4\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{b(a+b)\sin(x)}{a(2a+b-b\cos(2x))}\right)}{4(a+b)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (-((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2)) + (Sqrt[b]
*(3*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) + 4*(-Log[Cos[x/2] - S
```

```
in[x/2]] + Log[Cos[x/2] + Sin[x/2]] + (b*(a + b)*Sin[x])/(a*(2*a + b - b*Co
s[2*x]))))/(4*(a + b)^2)
```

Maple [A]
time = 0.32, size = 79, normalized size = 1.08

method	result
default	$\frac{b\left(\frac{(a+b)\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)}+\frac{(3a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(1+\sin(x))}{2(a+b)^2}-\frac{\ln(\sin(x)-1)}{2(a+b)^2}$
risch	$\frac{ib\left(e^{3ix}-e^{ix}\right)}{a(a+b)\left(b\,e^{4ix}-4a\,e^{2ix}-2b\,e^{2ix}+b\right)}+\frac{\ln\left(e^{ix}+i\right)}{a^2+2ab+b^2}-\frac{\ln\left(e^{ix}-i\right)}{a^2+2ab+b^2}+\frac{3\sqrt{-ab}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-ab}\,e^{ix}}{b}-1\right)}{4a(a+b)^2}+\frac{\sqrt{-ab}\,\ln\left(e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b/(a+b)^2*(1/2*(a+b)/a*sin(x)/(a+b*sin(x)^2)+1/2*(3*a+b)/a/(a*b)^(1/2)*arct
an(b*sin(x)/(a*b)^(1/2)))+1/2/(a+b)^2*ln(1+sin(x))-1/2/(a+b)^2*ln(sin(x)-1)
```

Maxima [A]
time = 0.48, size = 115, normalized size = 1.58

$$\frac{b\sin(x)}{2\left(a^3+a^2b+(a^2b+ab^2)\sin(x)^2\right)}+\frac{(3\,ab+b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\left(a^3+2\,a^2b+ab^2\right)\sqrt{ab}}+\frac{\log(\sin(x)+1)}{2\left(a^2+2\,ab+b^2\right)}-\frac{\log(\sin(x)-1)}{2\left(a^2+2\,ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*sin(x)/(a^3 + a^2*b + (a^2*b + a*b^2)*sin(x)^2) + 1/2*(3*a*b + b^2)*a
rctan(b*sin(x)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) + 1/2*log(sin
(x) + 1)/(a^2 + 2*a*b + b^2) - 1/2*log(sin(x) - 1)/(a^2 + 2*a*b + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(61) = 122.
time = 0.49, size = 354, normalized size = 4.85

$$\left[-\frac{\left((3\,ab+b^2)\cos(x)^2-3\,a^2-4\,ab-b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\left(ab\cos(x)^2-a^2-ab\right)\log(\sin(x)+1)-2\left(ab\cos(x)^2-a^2-ab\right)\log(-\sin(x)+1)-2\left(ab+b^2\right)\sin(x)}{4\left(a^4+3\,a^3b+3\,a^2b^2+ab^3-(a^3b+2\,a^2b^2+ab^3)\cos(x)^2\right)},\ -\frac{\left((3\,ab+b^2)\cos(x)^2-3\,a^2-4\,ab-b^2\right)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\sin(x)\right)+\left(ab\cos(x)^2-a^2-ab\right)\log(\sin(x)+1)-\left(ab\cos(x)^2-a^2-ab\right)\log(-\sin(x)+1)-\left(ab+b^2\right)\sin(x)}{2\left(a^4+3\,a^3b+3\,a^2b^2+ab^3-(a^3b+2\,a^2b^2+ab^3)\cos(x)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*a*b + b^2)*cos(x)^2 - 3*a^2 - 4*a*b - b^2)*sqrt(-b/a)*log(-(b*co
s(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(a*b*cos(
x)^2 - a^2 - a*b)*log(sin(x) + 1) - 2*(a*b*cos(x)^2 - a^2 - a*b)*log(-sin(x
) + 1) - 2*(a*b + b^2)*sin(x))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^3*b
+ 2*a^2*b^2 + a*b^3)*cos(x)^2), -1/2*(((3*a*b + b^2)*cos(x)^2 - 3*a^2 - 4*a
*b - b^2)*sqrt(b/a)*arctan(sqrt(b/a)*sin(x)) + (a*b*cos(x)^2 - a^2 - a*b)*l
og(sin(x) + 1) - (a*b*cos(x)^2 - a^2 - a*b)*log(-sin(x) + 1) - (a*b + b^2)*
sin(x))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^3*b + 2*a^2*b^2 + a*b^3)*co
s(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)/(a + b*sin(x)**2)**2, x)
```

Giac [A]
time = 0.48, size = 109, normalized size = 1.49

$$\frac{(3\,ab+b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\,(a^3+2\,a^2b+ab^2)\sqrt{ab}}+\frac{\log(\sin(x)+1)}{2\,(a^2+2\,ab+b^2)}-\frac{\log(-\sin(x)+1)}{2\,(a^2+2\,ab+b^2)}+\frac{b\sin(x)}{2\,\left(b\sin(x)^2+a\right)(a^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(
a*b)) + 1/2*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/2*log(-sin(x) + 1)/(a^2
 + 2*a*b + b^2) + 1/2*b*sin(x)/((b*sin(x)^2 + a)*(a^2 + a*b))
```

Mupad [B]
time = 15.84, size = 2213, normalized size = 30.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a + b*sin(x)^2)^2),x)
```

```
[Out] (b*sin(x))/(2*a*(a + b)*(a + b*sin(x)^2)) - (atan((((3*a + b)*(-a^3*b)^(1/2
)*((sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) + ((
3*a + b)*(-a^3*b)^(1/2)*((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 +
```

18*a^5*b^3 + 4*a^6*b^2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) - (sin(x)*(3*a + b)*(-a^3*b)^(1/2)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4*(2*a^4*b + a^5 + a^3*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)*((sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((3*a + b)*(-a^3*b)^(1/2)*((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) + (sin(x)*(3*a + b)*(-a^3*b)^(1/2)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4*(2*a^4*b + a^5 + a^3*b^2)))/(((3*a*b^3)/2 + b^4/2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) + ((3*a + b)*(-a^3*b)^(1/2)*((sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)*((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) - (sin(x)*(3*a + b)*(-a^3*b)^(1/2)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)) - ((3*a + b)*(-a^3*b)^(1/2)*((sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((3*a + b)*(-a^3*b)^(1/2)*((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) + (sin(x)*(3*a + b)*(-a^3*b)^(1/2)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2))))*(3*a + b)*(-a^3*b)^(1/2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) - (atan((((((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(2*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (sin(x)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(a + b)^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*(a + b)^2) + (sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/(a + b)^2 - ((((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(2*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (sin(x)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(a + b)^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*(a + b)^2) - (sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/(a + b)^2)/(((3*a*b^3)/2 + b^4/2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) + (((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(2*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (sin(x)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(a + b)^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*(a + b)^2) + (sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/(a + b)^2 + (((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(2*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (sin(x)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(a + b)^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*(a + b)^2) - (sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/(a + b)^2))*1i)/(a + b)^2

3.321 $\int \frac{\sec^2(x)}{(a+b\sin^2(x))^2}\,dx$

Optimal. Leaf size=76

$$\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{\tan(x)}{(a+b)^2}+\frac{b^2\tan(x)}{2a(a+b)^2\,(a+(a+b)\tan^2(x))}$$

```
[Out] 1/2*b*(4*a+b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(5/2)+tan(x)
/(a+b)^2+1/2*b^2*tan(x)/a/(a+b)^2/(a+(a+b)*tan(x)^2)
```

Rubi [A]

time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3270, 398, 393, 211}

$$\frac{b(4a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{b^2\tan(x)}{2a(a+b)^2\,((a+b)\tan^2(x)+a)}+\frac{\tan(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(5/2)
) + Tan[x]/(a + b)^2 + (b^2*Tan[x])/(2*a*(a + b)^2*(a + (a + b)*Tan[x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{(a+b)^2}+\frac{b(2a+b)+2b(a+b)x^2}{(a+b)^2\left(a+(a+b)x^2\right)^2}\right) dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{(a+b)^2}+\frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)}{(a+b)^2}\\
&= \frac{\tan(x)}{(a+b)^2}+\frac{b^2\tan(x)}{2a(a+b)^2\left(a+(a+b)\tan^2(x)\right)}+\frac{(b(4a+b))\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \text{ta}\right.}{2a(a+b)^2}\\
&= \frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{\tan(x)}{(a+b)^2}+\frac{b^2\tan(x)}{2a(a+b)^2\left(a+(a+b)\tan^2(x)\right)}
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 76, normalized size = 1.00

$$\frac{1}{2}\left(\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}}+\frac{\frac{b^2\sin(2x)}{a(2a+b-b\cos(2x))}+2\tan(x)}{(a+b)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(5/2))
 + ((b^2*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])) + 2*Tan[x])/(a + b)^2)/2
```

Maple [A]
time = 0.30, size = 81, normalized size = 1.07

method	result

default	$\frac{\tan(x)}{a^2+2ab+b^2}+\frac{b\left(\frac{b\tan(x)}{2a\left(a\left(\tan^2(x)\right)+b\left(\tan^2(x)\right)+a\right)}+\frac{(4a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}\right)}{(a+b)^2}$
risch	$\frac{i(-4ab\,e^{4ix}-b^2e^{4ix}+8\,e^{2ix}a^2+2b\,e^{2ix}a-2ab+b^2)}{a(a+b)^2(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b)(e^{2ix}+1)}-\frac{b\ln\left(e^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,(a+b)^2}-\frac{b^2\ln\left(e^{2ix}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2+2*a*b+b^2)*tan(x)+b/(a+b)^2*(1/2/a*b*tan(x)/(a*tan(x)^2+b*tan(x)^2+a
)+1/2*(4*a+b)/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.48, size = 119, normalized size = 1.57

$$\frac{b^2\tan(x)}{2\left(a^4+2a^3b+a^2b^2+(a^4+3a^3b+3a^2b^2+ab^3)\tan(x)^2\right)}+\frac{(4ab+b^2)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2(a^3+2a^2b+ab^2)\sqrt{(a+b)a}}+\frac{\tan(x)}{a^2+2ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*tan(x)/(a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*tan(x)^2) + 1/2*(4*a*b + b^2)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/((a
^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*a)) + tan(x)/(a^2 + 2*a*b + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(64) = 128.
time = 0.44, size = 505, normalized size = 6.64

$$\left[-\frac{\left((4ab^2+b^3)\cos(x)^3-(4a^2b+5ab^2+b^3)\cos(x)\right)\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4\left((2a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2+2ab+b^2}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right)+4\left(2a^4+4a^3b+2a^2b^2-(2a^3b+a^2b^2-ab^3)\cos(x)^2\right)\sin(x)}{8\left((a^5b+3a^4b^2+3a^3b^3+a^2b^4)\cos(x)^3-(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4)\cos(x)\right)},-\frac{\left((4ab^2+b^3)\cos(x)^3-(4a^2b+5ab^2+b^3)\cos(x)\right)\sqrt{a^2+ab}\arctan\left(\frac{(2a+b)\cos(x)^2-a-b}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)+2\left(2a^4+4a^3b+2a^2b^2-(2a^3b+a^2b^2-ab^3)\cos(x)^2\right)\sin(x)}{4\left((a^5b+3a^4b^2+3a^3b^3+a^2b^4)\cos(x)^3-(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4)\cos(x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*a*b^2 + b^3)*cos(x)^3 - (4*a^2*b + 5*a*b^2 + b^3)*cos(x))*sqrt(-
a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*co
s(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*sin(x) +
a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b
^2)) + 4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - (2*a^3*b + a^2*b^2 - a*b^3)*cos(x)^
2)*sin(x))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*cos(x)^3 - (a^6 + 4*a
```

^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(x)), -1/4*(((4*a*b^2 + b^3)*cos
(x)^3 - (4*a^2*b + 5*a*b^2 + b^3)*cos(x))*sqrt(a^2 + a*b)*arctan(1/2*((2*a
+ b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))) + 2*(2*a^4 + 4*a^3*
b + 2*a^2*b^2 - (2*a^3*b + a^2*b^2 - a*b^3)*cos(x)^2)*sin(x))/((a^5*b + 3*a
^4*b^2 + 3*a^3*b^3 + a^2*b^4)*cos(x)^3 - (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3
*b^3 + a^2*b^4)*cos(x))]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{(a+b\sin^2(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(a+b*sin(x)**2)**2,x)

[Out] Integral(sec(x)**2/(a + b*sin(x)**2)**2, x)

Giac [A]
time = 0.45, size = 113, normalized size = 1.49

$$\frac{b^2\tan(x)}{2\,(a^3+2\,a^2b+ab^2)\left(a\tan(x)^2+b\tan(x)^2+a\right)}+\frac{(4\,ab+b^2)\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{2\,(a^3+2\,a^2b+ab^2)\sqrt{a^2+ab}}+\frac{\tan(x)}{a^2+2\,ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="giac")

[Out] 1/2*b^2*tan(x)/((a^3 + 2*a^2*b + a*b^2)*(a*tan(x)^2 + b*tan(x)^2 + a)) + 1/
2*(4*a*b + b^2)*arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b))/((a^3 + 2*a^2
*b + a*b^2)*sqrt(a^2 + a*b)) + tan(x)/(a^2 + 2*a*b + b^2)

Mupad [B]
time = 14.78, size = 123, normalized size = 1.62

$$\frac{\tan(x)}{(a+b)^2}+\frac{b^2\tan(x)}{2\,a\,\left(a\,b^2+2\,a^2\,b+\tan(x)^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)+a^3\right)}+\frac{b\,\mathrm{atan}\left(\frac{b\tan(x)\,(4\,a+b)\,(2\,a+2\,b)\,(a^2+2\,a\,b+b^2)}{2\,\sqrt{a}\,\,(a+b)^{5/2}\,(b^2+4\,a\,b)}\right)\,(4\,a+b)}{2\,a^{3/2}\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(a + b*sin(x)^2)^2),x)

[Out] tan(x)/(a + b)^2 + (b^2*tan(x))/(2*a*(a*b^2 + 2*a^2*b + tan(x)^2*(3*a*b^2 +
3*a^2*b + a^3 + b^3) + a^3)) + (b*atan((b*tan(x)*(4*a + b)*(2*a + 2*b)*(2*
a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2)*(4*a*b + b^2)))*(4*a + b))/(2*a^
(3/2)*(a + b)^(5/2))

3.322 $\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx$

Optimal. Leaf size=109

$$\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3}+\frac{(a+5b)\tanh^{-1}(\sin(x))}{2(a+b)^3}-\frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)}+\frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)}$$

```
[Out] 1/2*b^(3/2)*(5*a+b)*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/(a+b)^3+1/2*(a+5
*b)*arctanh(sin(x))/(a+b)^3-1/2*(a-b)*b*sin(x)/a/(a+b)^2/(a+b*sin(x)^2)+1/2
*sec(x)*tan(x)/(a+b)/(a+b*sin(x)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3269, 425, 541, 536, 212, 211}

$$\frac{b^{3/2}(5a+b)\text{ArcTan}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3}-\frac{b(a-b)\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)}+\frac{(a+5b)\tanh^{-1}(\sin(x))}{2(a+b)^3}+\frac{\tan(x)\sec(x)}{2(a+b)\left(a+b\sin^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^3)
+ ((a + 5*b)*ArcTanh[Sin[x]])/(2*(a + b)^3) - ((a - b)*b*Sin[x])/(2*a*(a +
b)^2*(a + b*Sin[x]^2)) + (Sec[x]*Tan[x])/(2*(a + b)*(a + b*Sin[x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)\\
&= \frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b+3bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)}{2(a+b)}\\
&= -\frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)} + \frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(a^2+4ab+b^2\right)-2(a-}{\left(1-x^2\right)\left(a+bx^2\right)}\right.}{4a(a+b)}\\
&= -\frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)} + \frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)} + \frac{\left(b^2(5a+b)\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\right.}{2a(a+b)^3}\\
&= \frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3} + \frac{(a+5b)\tanh^{-1}(\sin(x))}{2(a+b)^3} - \frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(\right.}
\end{aligned}
$$

Mathematica [A]
time = 0.76, size = 183, normalized size = 1.68

$$\frac{-\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}}+\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}}-2(a+5b)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2(a+5b)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{a+b}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2}-\frac{a+b}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}+\frac{4b^2(a+b)\sin(x)}{a(2a+b-b\cos(2x))}}{4(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (-((b^(3/2)*(5*a + b)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2)) + (b^(3/2)
*(5*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) - 2*(a + 5*b)*Log[Cos[
x/2] - Sin[x/2]] + 2*(a + 5*b)*Log[Cos[x/2] + Sin[x/2]] + (a + b)/(Cos[x/2]
 - Sin[x/2])^2 - (a + b)/(Cos[x/2] + Sin[x/2])^2 + (4*b^2*(a + b)*Sin[x])/(
a*(2*a + b - b*Cos[2*x])))/(4*(a + b)^3)
```

Maple [A]
time = 0.41, size = 119, normalized size = 1.09

method	result
default	$\frac{b^2\left(\frac{(a+b)\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)}+\frac{(5a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^3}-\frac{1}{4(a+b)^2(\sin(x)-1)}+\frac{(-a-5b)\ln(\sin(x)-1)}{4(a+b)^3}-\frac{1}{4(a+b)^2(1+\sin(x))}+$ (a+
risch	$-\frac{i\left(ab\,e^{7ix}-b^2e^{7ix}-4a^2e^{5ix}-3ab\,e^{5ix}-b^2e^{5ix}+4\,e^{3ix}a^2+3b\,e^{3ix}a+b^2e^{3ix}-e^{ix}ab+e^{ix}b^2\right)}{(a+b)^2\left(e^{2ix}+1\right)^2a\left(b\,e^{4ix}-4a\,e^{2ix}-2b\,e^{2ix}+b\right)}+\frac{\ln\left(e^{ix}+i\right)a}{2a^3+6a^2b+6a\,b^2+2b^3}+$ 5 ln(e … 2(a^3+3a^2b

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a+b)^3*b^2*(1/2*(a+b)/a*sin(x)/(a+b*sin(x)^2)+1/2*(5*a+b)/a/(a*b)^(1/2)*
arctan(b*sin(x)/(a*b)^(1/2)))-1/4/(a+b)^2/(sin(x)-1)+1/4/(a+b)^3*(-a-5*b)*l
n(sin(x)-1)-1/4/(a+b)^2/(1+sin(x))+1/4*(a+5*b)/(a+b)^3*ln(1+sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(93) = 186.
time = 0.48, size = 220, normalized size = 2.02

$$\frac{(a+5\,b)\log(\sin(x)+1)}{4\,(a^3+3\,a^2b+3\,ab^2+b^3)}-\frac{(a+5\,b)\log(\sin(x)-1)}{4\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{(5\,ab^2+b^3)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}}-\frac{(ab-b^2)\sin(x)^3+(a^2+b^2)\sin(x)}{2\,\left((a^3b+2\,a^2b^2+ab^3)\sin(x)^4-a^4-2\,a^3b-a^2b^2+(a^4+a^3b-a^2b^2-ab^3)\sin(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a + 5*b)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/4*(a + 5*
b)*log(sin(x) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*(5*a*b^2 + b^3)*ar
```

```
ctan(b*sin(x)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) -
1/2*((a*b - b^2)*sin(x)^3 + (a^2 + b^2)*sin(x))/((a^3*b + 2*a^2*b^2 + a*b^3
)*sin(x)^4 - a^4 - 2*a^3*b - a^2*b^2 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*sin(
x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(93) = 186$.
time = 0.49, size = 560, normalized size = 5.14

$$\left[\frac{\left(\left(5\,ab^2+b^3\right)\cos\left(x\right)^4-\left(5\,a^2b+6\,ab^2+b^3\right)\cos\left(x\right)^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos\left(x\right)^2-2\,a\sqrt{-\frac{b}{a}}\sin\left(x\right)+a-b}{b\cos\left(x\right)^2-a-b}\right)+\left(\left(a^2b+5\,ab^2\right)\cos\left(x\right)^4-\left(a^3+6\,a^2b+5\,ab^2\right)\cos\left(x\right)^2\right)\log\left(\sin\left(x\right)+1\right)-\left(\left(a^2b+5\,ab^2\right)\cos\left(x\right)^4-\left(a^3+6\,a^2b+5\,ab^2\right)\cos\left(x\right)^2\right)\log\left(-\sin\left(x\right)+1\right)-2\left(a^3+2\,a^2b+ab^2-\left(a^2b-b^3\right)\cos\left(x\right)^2\right)\sin\left(x\right)}{4\left(\left(a^4b+3\,a^3b^2+3\,a^2b^3+ab^4\right)\cos\left(x\right)^4-\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\cos\left(x\right)^2\right)}, \frac{2\left(\left(5\,ab^2+b^3\right)\cos\left(x\right)^4-\left(5\,a^2b+6\,ab^2+b^3\right)\cos\left(x\right)^2\right)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\sin\left(x\right)\right)+\left(\left(a^2b+5\,ab^2\right)\cos\left(x\right)^4-\left(a^3+6\,a^2b+5\,ab^2\right)\cos\left(x\right)^2\right)\log\left(\sin\left(x\right)+1\right)-\left(\left(a^2b+5\,ab^2\right)\cos\left(x\right)^4-\left(a^3+6\,a^2b+5\,ab^2\right)\cos\left(x\right)^2\right)\log\left(-\sin\left(x\right)+1\right)-2\left(a^3+2\,a^2b+ab^2-\left(a^2b-b^3\right)\cos\left(x\right)^2\right)\sin\left(x\right)}{4\left(\left(a^4b+3\,a^3b^2+3\,a^2b^3+ab^4\right)\cos\left(x\right)^4-\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\cos\left(x\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((5*a*b^2 + b^3)*cos(x)^4 - (5*a^2*b + 6*a*b^2 + b^3)*cos(x)^2)*sqrt(
-b/a)*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x)^2 - a - b
)) + ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2)*log(
sin(x) + 1) - ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x
)^2)*log(-sin(x) + 1) - 2*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - b^3)*cos(x)^2)*
sin(x))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^4 - (a^5 + 4*a^4*b
+ 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2), 1/4*(2*((5*a*b^2 + b^3)*cos(x)^
4 - (5*a^2*b + 6*a*b^2 + b^3)*cos(x)^2)*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))
+ ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2)*log(sin
(x) + 1) - ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2
)*log(-sin(x) + 1) - 2*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - b^3)*cos(x)^2)*sin
(x))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^4 - (a^5 + 4*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(x\right)}{\left(a+b\sin^2\left(x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a+b*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)**3/(a + b*sin(x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(93) = 186$.
time = 0.48, size = 194, normalized size = 1.78

$$\frac{\left(a+5\,b\right)\log\left(\sin\left(x\right)+1\right)}{4\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}-\frac{\left(a+5\,b\right)\log\left(-\sin\left(x\right)+1\right)}{4\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\frac{\left(5\,ab^2+b^3\right)\arctan\left(\frac{b\sin\left(x\right)}{\sqrt{ab}}\right)}{2\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\sqrt{ab}}-\frac{ab\sin\left(x\right)^3-b^2\sin\left(x\right)^3+a^2\sin\left(x\right)+b^2\sin\left(x\right)}{2\left(b\sin\left(x\right)^4+a\sin\left(x\right)^2-b\sin\left(x\right)^2-a\right)\left(a^3+2\,a^2b+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(a + 5*b)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/4*(a + 5*
b)*log(-sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*(5*a*b^2 + b^3)*a
rctan(b*sin(x)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) -
 1/2*(a*b*sin(x)^3 - b^2*sin(x)^3 + a^2*sin(x) + b^2*sin(x))/((b*sin(x)^4 +
 a*sin(x)^2 - b*sin(x)^2 - a)*(a^3 + 2*a^2*b + a*b^2))
```

Mupad [B]
time = 15.06, size = 2009, normalized size = 18.43

$$\frac{\ln(\sin(x)+1)\,(a+5\,b)}{4\,(a+b)^3}-\ln(\sin(x)-1)\,\left(\frac{b}{(a+b)^3}+\frac{1}{4\,(a+b)^2}\right)-\frac{\frac{\sin(x)\,(a^2+b^2)}{2\,a\,(a^2+2\,a\,b+b^2)}+\frac{b\sin(x)^3\,(a-b)}{2\,a\,(a^2+2\,a\,b+b^2)}}{b\sin(x)^4+(a-b)\,\sin(x)^2-a}-\ldots$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^3*(a + b*sin(x)^2)^2),x)
```

```
[Out] (log(sin(x) + 1)*(a + 5*b))/(4*(a + b)^3) - log(sin(x) - 1)*(b/(a + b)^3 +
1/(4*(a + b)^2)) - ((sin(x)*(a^2 + b^2))/(2*a*(2*a*b + a^2 + b^2)) + (b*sin
(x)^3*(a - b))/(2*a*(2*a*b + a^2 + b^2)))/(b*sin(x)^4 - a + sin(x)^2*(a - b
)) - (atan(((((sin(x)*(10*a*b^6 + b^7 + 50*a^2*b^5 + 10*a^3*b^4 + a^4*b^3))
/(2*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2)) + ((5*a + b)*(-a^3*b
^3)^(1/2)*((2*a*b^10 + 20*a^2*b^9 + 80*a^3*b^8 + 172*a^4*b^7 + 220*a^5*b^6
+ 172*a^6*b^5 + 80*a^7*b^4 + 20*a^8*b^3 + 2*a^9*b^2)/(6*a^7*b + a^8 + a^2*b
^6 + 6*a^3*b^5 + 15*a^4*b^4 + 20*a^5*b^3 + 15*a^6*b^2) - (sin(x)*(5*a + b)*
(-a^3*b^3)^(1/2)*(16*a^2*b^9 + 80*a^3*b^8 + 144*a^4*b^7 + 80*a^5*b^6 - 80*a
^6*b^5 - 144*a^7*b^4 - 80*a^8*b^3 - 16*a^9*b^2))/(8*(3*a^5*b + a^6 + a^3*b^
3 + 3*a^4*b^2)*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2))))/(4*(3*a
^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*(5*a + b)*(-a^3*b^3)^(1/2)*1i)/(4*(3*a^
5*b + a^6 + a^3*b^3 + 3*a^4*b^2)) + (((sin(x)*(10*a*b^6 + b^7 + 50*a^2*b^5
+ 10*a^3*b^4 + a^4*b^3))/(2*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^
2)) - ((5*a + b)*(-a^3*b^3)^(1/2)*((2*a*b^10 + 20*a^2*b^9 + 80*a^3*b^8 + 17
2*a^4*b^7 + 220*a^5*b^6 + 172*a^6*b^5 + 80*a^7*b^4 + 20*a^8*b^3 + 2*a^9*b^2
)/(6*a^7*b + a^8 + a^2*b^6 + 6*a^3*b^5 + 15*a^4*b^4 + 20*a^5*b^3 + 15*a^6*b
^2) + (sin(x)*(5*a + b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 + 80*a^3*b^8 + 144*a^4
*b^7 + 80*a^5*b^6 - 80*a^6*b^5 - 144*a^7*b^4 - 80*a^8*b^3 - 16*a^9*b^2))/(8
*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3
 + 6*a^4*b^2))))/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*(5*a + b)*(-a^3
*b^3)^(1/2)*1i)/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))/((((21*a*b^6)/4 +
 (5*b^7)/4 - (21*a^2*b^5)/4 - (5*a^3*b^4)/4)/(6*a^7*b + a^8 + a^2*b^6 + 6*a
^3*b^5 + 15*a^4*b^4 + 20*a^5*b^3 + 15*a^6*b^2) + (((sin(x)*(10*a*b^6 + b^7
+ 50*a^2*b^5 + 10*a^3*b^4 + a^4*b^3))/(2*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b
^3 + 6*a^4*b^2)) + ((5*a + b)*(-a^3*b^3)^(1/2)*((2*a*b^10 + 20*a^2*b^9 + 80
*a^3*b^8 + 172*a^4*b^7 + 220*a^5*b^6 + 172*a^6*b^5 + 80*a^7*b^4 + 20*a^8*b^
3 + 2*a^9*b^2)/(6*a^7*b + a^8 + a^2*b^6 + 6*a^3*b^5 + 15*a^4*b^4 + 20*a^5*b
^3 + 15*a^6*b^2) - (sin(x)*(5*a + b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 + 80*a^3*
```

```
b^8 + 144*a^4*b^7 + 80*a^5*b^6 - 80*a^6*b^5 - 144*a^7*b^4 - 80*a^8*b^3 - 16
*a^9*b^2))/(8*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)*(4*a^5*b + a^6 + a^2*b^
4 + 4*a^3*b^3 + 6*a^4*b^2))))/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*(5
*a + b)*(-a^3*b^3)^(1/2))/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)) - (((si
n(x)*(10*a*b^6 + b^7 + 50*a^2*b^5 + 10*a^3*b^4 + a^4*b^3))/(2*(4*a^5*b + a^
6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2)) - ((5*a + b)*(-a^3*b^3)^(1/2)*((2*a*b
^10 + 20*a^2*b^9 + 80*a^3*b^8 + 172*a^4*b^7 + 220*a^5*b^6 + 172*a^6*b^5 + 8
0*a^7*b^4 + 20*a^8*b^3 + 2*a^9*b^2)/(6*a^7*b + a^8 + a^2*b^6 + 6*a^3*b^5 +
15*a^4*b^4 + 20*a^5*b^3 + 15*a^6*b^2) + (sin(x)*(5*a + b)*(-a^3*b^3)^(1/2)*
(16*a^2*b^9 + 80*a^3*b^8 + 144*a^4*b^7 + 80*a^5*b^6 - 80*a^6*b^5 - 144*a^7*
b^4 - 80*a^8*b^3 - 16*a^9*b^2))/(8*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)*(4
*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2))))/(4*(3*a^5*b + a^6 + a^3*
b^3 + 3*a^4*b^2)))*(5*a + b)*(-a^3*b^3)^(1/2))/(4*(3*a^5*b + a^6 + a^3*b^3
+ 3*a^4*b^2))))*(5*a + b)*(-a^3*b^3)^(1/2)*1i)/(2*(3*a^5*b + a^6 + a^3*b^3
+ 3*a^4*b^2))
```

3.323 $\int \frac{\sec^4(x)}{(a+b\sin^2(x))^2} dx$

Optimal. Leaf size=96

$$\frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{b^3\tan(x)}{2a(a+b)^3\left(a+(a+b)\tan^2(x)\right)}$$

```
[Out] 1/2*b^2*(6*a+b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(7/2)+(a+3
*b)*tan(x)/(a+b)^3+1/3*tan(x)^3/(a+b)^2+1/2*b^3*tan(x)/a/(a+b)^3/(a+(a+b)*t
an(x)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3270, 398, 393, 211}

$$\frac{b^2(6a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{b^3\tan(x)}{2a(a+b)^3\left((a+b)\tan^2(x)+a\right)}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{(a+3b)\tan(x)}{(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(7/
2)) + ((a + 3*b)*Tan[x])/(a + b)^3 + Tan[x]^3/(3*(a + b)^2) + (b^3*Tan[x])/
(2*a*(a + b)^3*(a + (a + b)*Tan[x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a+3b}{(a+b)^3}+\frac{x^2}{(a+b)^2}+\frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+b)^3\left(a+(a+b)x^2\right)^2}\right)\,dx, x, \tan(x)\right)$$
$$= \frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{\text{Subst}\left(\int \frac{b^2(3a+b)+3b^2(a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)}{(a+b)^3}$$
$$= \frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{b^3\tan(x)}{2a(a+b)^3\left(a+(a+b)\tan^2(x)\right)}+\frac{\left(b^2(6a+b)\right)\text{Subst}}{2}$$
$$= \frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{b^3\,\text{ta}}{2a(a+b)^3\,(a+}$$

Mathematica [A]
time = 0.70, size = 97, normalized size = 1.01

$$\frac{1}{6}\left(\frac{3b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{7/2}}+\frac{\frac{3b^3\sin(2x)}{a(2a+b-b\cos(2x))}+4a\tan(x)+16b\tan(x)+2(a+b)\sec^2(x)\tan(x)}{(a+b)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((3*b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(7
/2)) + ((3*b^3*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])) + 4*a*Tan[x] + 16*b*Tan
[x] + 2*(a + b)*Sec[x]^2*Tan[x])/(a + b)^3)/6
```

Maple [A]
time = 0.37, size = 110, normalized size = 1.15

method	result
default	$\frac{\frac{a\left(\tan^{3}(x)\right)}{3}+\frac{b\left(\tan^{3}(x)\right)}{3}+\tan(x)a+3\tan(x)b}{(a^{2}+2ab+b^{2})(a+b)}+\frac{b^{2}\left(\frac{b\tan(x)}{2a\left(a\left(\tan^{2}(x)\right)+b\left(\tan^{2}(x)\right)+a\right)}+\frac{(6a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}\right)}{(a+b)^{3}}$
risch	$\frac{i(-18a\,b^{2}e^{8ix}-3b^{3}e^{8ix}+36a^{2}b\,e^{6ix}-30a\,b^{2}e^{6ix}-6b^{3}e^{6ix}+48a^{3}e^{4ix}+164a^{2}b\,e^{4ix}+26a\,b^{2}e^{4ix}+16a^{3}e^{2ix}+60a^{2}b\,e^{2ix}-10a\,b^{2}e^{2ix}+6b}{3(e^{2ix}+1)^{3}(a+b)^{3}a(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2+2*a*b+b^2)/(a+b)*(1/3*a*tan(x)^3+1/3*b*tan(x)^3+tan(x)*a+3*tan(x)*b)
+b^2/(a+b)^3*(1/2/a*b*tan(x)/(a*tan(x)^2+b*tan(x)^2+a)+1/2*(6*a+b)/a/(a*(a+
b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(82) = 164$.

time = 0.49, size = 170, normalized size = 1.77

$$\frac{b^{3}\tan(x)}{2\left(a^{5}+3\,a^{4}b+3\,a^{3}b^{2}+a^{2}b^{3}+\left(a^{5}+4\,a^{4}b+6\,a^{3}b^{2}+4\,a^{2}b^{3}+ab^{4}\right)\tan(x)^{2}\right)}+\frac{\left(6\,ab^{2}+b^{3}\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}\right)\sqrt{(a+b)a}}+\frac{(a+b)\tan(x)^{3}+3\,(a+3\,b)\tan(x)}{3\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*tan(x)/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 + 4*a^4*b + 6*a^
3*b^2 + 4*a^2*b^3 + a*b^4)*tan(x)^2) + 1/2*(6*a*b^2 + b^3)*arctan((a + b)*t
an(x)/sqrt((a + b)*a))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt((a + b)*a)
) + 1/3*((a + b)*tan(x)^3 + 3*(a + 3*b)*tan(x))/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(82) = 164$.

time = 0.44, size = 653, normalized size = 6.80

$$\left[-\frac{3\left(\left(6\,ab^{3}+b^{4}\right)\cos(x)^{5}-\left(6\,a^{2}b^{2}+7\,ab^{3}+b^{4}\right)\cos(x)^{3}\right)\sqrt{-a^{2}-ab}\log\left(\frac{\left(8\,a^{2}+8\,ab+b^{2}\right)\cos(x)^{4}-2\left(4\,a^{2}+5\,ab+b^{2}\right)\cos(x)^{2}+4\left((2\,a+b)\cos(x)^{3}-(a+b)\cos(x)\right)\sqrt{-a^{2}-ab}\sin(x)+a^{2}+2\,ab+b^{2}}{b^{2}\cos(x)^{4}-2\left(ab+b^{2}\right)\cos(x)^{2}+a^{2}+2\,ab+b^{2}}\right)+4\left(2\,a^{5}+6\,a^{4}b+6\,a^{3}b^{2}+2\,a^{2}b^{3}-\left(4\,a^{4}b+20\,a^{3}b^{2}+13\,a^{2}b^{3}-3\,ab^{4}\right)\cos(x)^{4}+2\left(2\,a^{5}+11\,a^{4}b+16\,a^{3}b^{2}+7\,a^{2}b^{3}\right)\cos(x)^{2}\right)\sin(x)}{24\left(\left(a^{6}b+4\,a^{5}b^{2}+6\,a^{4}b^{3}+4\,a^{3}b^{4}+a^{2}b^{5}\right)\cos(x)^{5}-\left(a^{7}+5\,a^{6}b+10\,a^{5}b^{2}+10\,a^{4}b^{3}+5\,a^{3}b^{4}+a^{2}b^{5}\right)\cos(x)^{3}\right)},-\frac{3\left(\left(6\,ab^{3}+b^{4}\right)\cos(x)^{5}-\left(6\,a^{2}b^{2}+7\,ab^{3}+b^{4}\right)\cos(x)^{3}\right)\sqrt{a^{2}+ab}\arctan\left(\frac{(2\,a+b)\cos(x)^{2}-a-b}{2\sqrt{a^{2}+ab}\cos(x)\sin(x)}\right)+2\left(2\,a^{5}+6\,a^{4}b+6\,a^{3}b^{2}+2\,a^{2}b^{3}-\left(4\,a^{4}b+20\,a^{3}b^{2}+13\,a^{2}b^{3}-3\,ab^{4}\right)\cos(x)^{4}+2\left(2\,a^{5}+11\,a^{4}b+16\,a^{3}b^{2}+7\,a^{2}b^{3}\right)\cos(x)^{2}\right)\sin(x)}{12\left(\left(a^{6}b+4\,a^{5}b^{2}+6\,a^{4}b^{3}+4\,a^{3}b^{4}+a^{2}b^{5}\right)\cos(x)^{5}-\left(a^{7}+5\,a^{6}b+10\,a^{5}b^{2}+10\,a^{4}b^{3}+5\,a^{3}b^{4}+a^{2}b^{5}\right)\cos(x)^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((6*a*b^3 + b^4)*cos(x)^5 - (6*a^2*b^2 + 7*a*b^3 + b^4)*cos(x)^3)
*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b +
```

```
b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*si
n(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*
a*b + b^2)) + 4*(2*a^5 + 6*a^4*b + 6*a^3*b^2 + 2*a^2*b^3 - (4*a^4*b + 20*a^
3*b^2 + 13*a^2*b^3 - 3*a*b^4)*cos(x)^4 + 2*(2*a^5 + 11*a^4*b + 16*a^3*b^2 +
 7*a^2*b^3)*cos(x)^2)*sin(x))/((a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 +
 a^2*b^5)*cos(x)^5 - (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 +
 a^2*b^5)*cos(x)^3), -1/12*(3*((6*a*b^3 + b^4)*cos(x)^5 - (6*a^2*b^2 + 7*a*
b^3 + b^4)*cos(x)^3)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b
)/(sqrt(a^2 + a*b)*cos(x)*sin(x))) + 2*(2*a^5 + 6*a^4*b + 6*a^3*b^2 + 2*a^2
*b^3 - (4*a^4*b + 20*a^3*b^2 + 13*a^2*b^3 - 3*a*b^4)*cos(x)^4 + 2*(2*a^5 +
11*a^4*b + 16*a^3*b^2 + 7*a^2*b^3)*cos(x)^2)*sin(x))/((a^6*b + 4*a^5*b^2 +
6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(x)^5 - (a^7 + 5*a^6*b + 10*a^5*b^2 + 1
0*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(x)^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a+b*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)**4/(a + b*sin(x)**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(82) = 164.
time = 0.48, size = 270, normalized size = 2.81

$$\frac{b^3\tan(x)}{2\left(a^4+3a^3b+3a^2b^2+ab^3\right)\left(a\tan(x)^2+b\tan(x)^2+a\right)}+\frac{\left(6ab^2+b^3\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{2\left(a^4+3a^3b+3a^2b^2+ab^3\right)\sqrt{a^2+ab}}+\frac{a^4\tan(x)^3+4a^3b\tan(x)^3+6a^2b^2\tan(x)^3+4ab^3\tan(x)^3+b^4\tan(x)^3+3a^4\tan(x)+18a^3b\tan(x)+36a^2b^2\tan(x)+30ab^3\tan(x)+9b^4\tan(x)}{3\left(a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*tan(x)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*tan(x)^2 + b*tan(x)^
2 + a)) + 1/2*(6*a*b^2 + b^3)*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan
((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b)))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*sqrt(a^2 + a*b)) + 1/3*(a^4*tan(x)^3 + 4*a^3*b*tan(x)^3 + 6*a^2*b^2*tan(
x)^3 + 4*a*b^3*tan(x)^3 + b^4*tan(x)^3 + 3*a^4*tan(x) + 18*a^3*b*tan(x) + 3
6*a^2*b^2*tan(x) + 30*a*b^3*tan(x) + 9*b^4*tan(x))/(a^6 + 6*a^5*b + 15*a^4*
b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

Mupad [**B**]
time = 14.30, size = 176, normalized size = 1.83

$$\frac{\tan(x)^3}{3\,(a+b)^2}-\tan(x)\,\left(\frac{2\,a}{(a+b)^3}-\frac{3}{(a+b)^2}\right)+\frac{b^3\tan(x)}{2\,a\,\left(\tan(x)^2\,\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)+a\,b^3+3\,a^3\,b+a^4+3\,a^2\,b^2\right)}+\frac{b^2\,\mathrm{atan}\left(\frac{b^2\tan(x)\,(6\,a+b)\,(2\,a+2\,b)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{2\,\sqrt{a}\,(a+b)^{7/2}\,\left(b^3+6\,a\,b^2\right)}\right)\,(6\,a+b)}{2\,a^{3/2}\,(a+b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a + b*sin(x)^2)^2),x)
```

```
[Out] tan(x)^3/(3*(a + b)^2) - tan(x)*((2*a)/(a + b)^3 - 3/(a + b)^2) + (b^3*tan(
x))/(2*a*(tan(x)^2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2) + a*b^3 + 3*
a^3*b + a^4 + 3*a^2*b^2)) + (b^2*atan((b^2*tan(x)*(6*a + b)*(2*a + 2*b)*(3*
a*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2)*(6*a*b^2 + b^3)))*(6
*a + b))/(2*a^(3/2)*(a + b)^(7/2))
```

3.324 $\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{a(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf}-\frac{\sin(e+fx)\,(a+b\,\mathrm{s}}{4bf}$$

```
[Out] 1/8*a*(a+4*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/2)/
f-1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b/f+1/8*(a+4*b)*sin(f*x+e)*(a+b*s
in(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]
time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 396, 201, 223, 212}

$$\frac{a(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f}-\frac{\sin(e+fx)\,(a+b\sin^2(e+fx))^{3/2}}{4bf}+\frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*(a + 4*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8
*b^(3/2)*f) + ((a + 4*b)*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(8*b*f) -
 (Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int (1-x^2)\sqrt{a+bx^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4bf} + \frac{(a+4b)\text{Subst}\left(\int \sqrt{a+b\ldots\right.}{4b\ldots}$$

$$= \frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf} - \frac{\sin(e+fx)\left(a+b\,\text{s}\ldots\right.}{4bf}$$

$$= \frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf} - \frac{\sin(e+fx)\left(a+b\,\text{s}\ldots\right.}{4bf}$$

$$= \frac{a(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a+4b)\sin(e+f\ldots}{}$$

Mathematica [A]
time = 0.32, size = 125, normalized size = 1.07

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(\sqrt{a}\,(a+4b)\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)-\sqrt{b}\sin(e+fx)\left(a-4b+2b\sin^2(e+fx)\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\right)}{8b^{3/2}f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]^2]*(Sqrt[a]*(a + 4*b)*ArcSinh[(Sqrt[b]*Sin[e + f*x
])/Sqrt[a]] - Sqrt[b]*Sin[e + f*x]*(a - 4*b + 2*b*Sin[e + f*x]^2)*Sqrt[1 +
(b*Sin[e + f*x]^2)/a]))/(8*b^(3/2)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

Maple [A]
time = 7.82, size = 144, normalized size = 1.23

method	result
default	$\frac{-\frac{\left(\sin^{3}(fx+e)\right)\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{4}-\frac{a\sin(fx+e)\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{8b}+\frac{a^{2}\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin^{2}\right.}\right.}{8b^{\frac{3}{2}}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2)-1/8/b*a*sin(f*x+e)*(a+b*sin(f*x
+e)^2)^(1/2)+1/8/b^(3/2)*a^2*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)
)+1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)+1/2*a*ln(sin(f*x+e)*b^(1/2)+(a+b*
sin(f*x+e)^2)^(1/2))/b^(1/2))/f
```

Maxima [A]
time = 0.27, size = 127, normalized size = 1.09

$$\frac{\frac{a^{2}\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{4\,a\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}}+4\,\sqrt{b\sin\left(fx+e\right)^{2}+a}\,\sin\left(fx+e\right)-\frac{2\left(b\sin(fx+e)^{2}+a\right)^{\frac{3}{2}}\sin(fx+e)}{b}+\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a\sin(fx+e)}{b}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) + 4*a*arcsinh(b*sin(f*x
+ e)/sqrt(a*b))/sqrt(b) + 4*sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e) - 2*(b*
sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)/b + sqrt(b*sin(f*x + e)^2 + a)*a*sin
(f*x + e)/b)/f
```

Fricas [A]
time = 0.65, size = 511, normalized size = 4.37

$$\left[\frac{\left(a^{2}+4\,ab\right)\sqrt{b}\log\left(128\,b^{4}\cos\left(fx+e\right)^{8}-256\left(ab^{3}+2\,b^{4}\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{2}b^{2}+24\,ab^{3}+24\,b^{4}\right)\cos\left(fx+e\right)^{4}+a^{4}+32\,a^{3}b+160\,a^{2}b^{2}+256\,ab^{3}+128\,b^{4}-32\left(a^{3}b+10\,a^{2}b^{2}+24\,ab^{3}+16\,b^{4}\right)\cos\left(fx+e\right)^{2}-8\left(16\,b^{3}\cos\left(fx+e\right)^{6}-24\left(ab^{2}+2\,b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-10\,a^{2}b-24\,ab^{2}-16\,b^{3}+2\left(5\,a^{2}b+24\,ab^{2}+24\,b^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{b}\sin\left(fx+e\right)\right)+8\left(2\,b^{2}\cos\left(fx+e\right)^{2}-ab+2\,b^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sin\left(fx+e\right)}{64\,b^{2}f},-\frac{\left(a^{2}+4\,ab\right)\sqrt{-b}\arctan\left(\frac{\left(8\,b^{2}\cos(fx+e)^{4}-8\left(ab+2\,b^{2}\right)\cos(fx+e)^{2}+a^{2}+8\,ab+8\,b^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-b}}{4\left(2\,b^{3}\cos(fx+e)^{4}+a^{2}b+3\,ab^{2}+2\,b^{3}-\left(3\,ab^{2}+4\,b^{3}\right)\cos(fx+e)^{2}\right)\sin(fx+e)}\right)-4\left(2\,b^{2}\cos\left(fx+e\right)^{2}-ab+2\,b^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sin\left(fx+e\right)}{32\,b^{2}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 4*a*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^
4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4
```

```
 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 2
4*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*s
in(f*x + e)) + 8*(2*b^2*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^
2 + a + b)*sin(f*x + e))/(b^2*f), -1/32*((a^2 + 4*a*b)*sqrt(-b)*arctan(1/4*
(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^
2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b
+ 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2
*b^2*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
+ e))/(b^2*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 96, normalized size = 0.82

$$-\frac{\sqrt{b\sin\left(fx+e\right)^2+a}\left(2\,\sin\left(fx+e\right)^2+\frac{ab-4\,b^2}{b^2}\right)\sin\left(fx+e\right)+\frac{\left(a^2+4\,ab\right)\log\left(\left|-\sqrt{b}\,\sin\left(fx+e\right)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{b^{\frac{3}{2}}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(b*sin(f*x + e)^2 + a)*(2*sin(f*x + e)^2 + (a*b - 4*b^2)/b^2)*sin
(f*x + e) + (a^2 + 4*a*b)*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x +
e)^2 + a)))/b^(3/2))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.325 $\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}\,f}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

```
[Out] 1/2*a*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*si
n(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 201, 223, 212}

$$\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[b]*f
) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \text{s}\right.}{2f}$$

$$= \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a+}}\right.}{2f}$$

$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}\,f} + \frac{\sin(e+fx)\sqrt{a+b\sin^2(e-}}{2f}$$

Mathematica [A]
time = 0.19, size = 96, normalized size = 1.33

$$\frac{\sqrt{b}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)+a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{2\sqrt{b}\,f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2) + a^(3/2)*ArcSinh[(Sqrt[b]*Sin
[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(2*Sqrt[b]*f*Sqrt[a + b
*Sin[e + f*x]^2])
```

Maple [A]
time = 0.13, size = 60, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{2\sqrt{b}}}{f}$	60
default	$\frac{\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{2\sqrt{b}}}{f}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)+1/2*a*ln(sin(f*x+e)*b^(1/2)+(a
+b*sin(f*x+e)^2)^(1/2))/b^(1/2))
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.68

$$\frac{\frac{a\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}}+\sqrt{b\sin(fx+e)^2+a}\,\sin(fx+e)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*arcsinh(b*sin(f*x + e)/sqrt(a*b))/sqrt(b) + sqrt(b*sin(f*x + e)^2 +
a)*sin(f*x + e))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(60) = 120$.
time = 0.48, size = 453, normalized size = 6.29

$$\left[\frac{a\sqrt{b}\log\left(128\,b^4\cos(fx+e)^8-256\,(ab^3+2\,b^4)\cos(fx+e)^6+32\,(5\,a^2b^2+24\,ab^3+24\,b^4)\cos(fx+e)^4+a^4+32\,a^3b+160\,a^2b^2+256\,ab^3+128\,b^4-32\,(a^3b+10\,a^2b^2+24\,ab^3+16\,b^4)\cos(fx+e)^2-8\,(16\,b^3\cos(fx+e)^6-24\,(ab^2+2\,b^3)\cos(fx+e)^4-a^3-10\,a^2b-24\,ab^2-16\,b^3+2\,(5\,a^2b+24\,ab^2+24\,b^3)\cos(fx+e)^2)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{b}\,\sin(fx+e)\right)+8\sqrt{-b\cos(fx+e)^2+a+b}\,b\sin(fx+e)}{16\,bf},\ -\frac{a\sqrt{-b}\,\arctan\left(\frac{(8\,b^2\cos(fx+e)^4-8\,(ab+2\,b^2)\cos(fx+e)^2+a^2+8\,ab+8\,b^2)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-b}}{4\,(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-(3\,ab^2+4\,b^3)\cos(fx+e)^2)\sin(fx+e)}\right)-4\sqrt{-b\cos(fx+e)^2+a+b}\,b\sin(fx+e)}{8\,bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(a*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x +
 e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b
+ 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 1
6*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f
*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24
*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e))
 + 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b*f), -1/8*(a*sqrt(-b
```

```
)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 +
 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x +
e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x
+ e))) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\,\cos\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cos(e + f*x), x)
```

Giac [A]
time = 0.52, size = 65, normalized size = 0.90

$$-\frac{\frac{a\log\left(\left|-\sqrt{b}\sin(fx+e)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{\sqrt{b}}-\sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/sqrt(b
) - sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e))/f
```

Mupad [B]
time = 14.48, size = 61, normalized size = 0.85

$$\frac{\sin\left(e+f\,x\right)\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}{2\,f}+\frac{a\,\ln\left(\sqrt{b}\,\sin\left(e+f\,x\right)+\sqrt{b\sin\left(e+f\,x\right)^2+a}\right)}{2\,\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2))/(2*f) + (a*log(b^(1/2)*sin(e +
f*x) + (a + b*sin(e + f*x)^2)^(1/2)))/(2*b^(1/2)*f)
```

3.326 $\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

```
[Out] -arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*b^(1/2)/f+arctanh(sin
(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3269, 399, 223, 212, 385}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/f) +
 (Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]
])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f} + \frac{(a+b)\text{Subst}\left(\int \frac{}{(1-}\right.}{}$$

$$= -\frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f} + \frac{(a+b)\text{Subst}}{}$$

$$= -\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f} + \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{f}$$

Mathematica [A]
time = 0.19, size = 129, normalized size = 1.57

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{2a+2b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right) + \frac{\sqrt{a}\sqrt{-b}\sin^{-1}\left(\frac{\sqrt{-b}\sin(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}}{\sqrt{2a+b-b\cos(2(e+fx))}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

[Out] (Sqrt[a + b]*ArcTanh[(Sqrt[2*a + 2*b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]] + (Sqrt[a]*Sqrt[-b]*ArcSin[(Sqrt[-b]*Sin[e + f*x])/Sqrt[a]]*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]])/f

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(70) = 140$.
time = 24.25, size = 150, normalized size = 1.83

method	result
default	$\frac{-\sqrt{b}\,\ln\left(\frac{\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,\sqrt{b}+b\sin\left(fx+e\right)}{\sqrt{b}}\right)+\frac{\sqrt{a+b}\,\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+\right.\right.}}{\sin\left(fx+e\right)-1}\right.}{2}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-b^(1/2)*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))+1/2*(a+b)^(1/2)*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))-1/2*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)))/f

Maxima [A]
time = 0.55, size = 133, normalized size = 1.62

$$-\frac{2\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)-\sqrt{a+b}\,\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}\right)-\sqrt{a+b}\,\operatorname{arsinh}\left(-\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*(2*sqrt(b)*arcsinh(b*sin(f*x + e)/sqrt(a*b)) - sqrt(a + b)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) + 1))) - sqrt(a + b)*arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1))))/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(70) = 140$.
time = 0.60, size = 1246, normalized size = 15.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

```
[Out] [1/8*(sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)
^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 1
60*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b
^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x
+ e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^
3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) +
2*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b +
2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/co
s(f*x + e)^4))/f, -1/8*(4*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - sqrt(b)*log(128*b^4*cos(
f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3
+ 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b
^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3
*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a
*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos
(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)))/f, 1/4*(sqrt(-b)*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3
*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + sqrt(a
+ b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*co
s(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e
)^4))/f, -1/4*(2*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a -
2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e
)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - sqrt(-b)*arctan(1/4*(8*b^2*cos(f*
x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2
*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2(e + fx)}\,\sec(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x), x)
```

3.327 $\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{a+b}\,f}+\frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f}$$

```
[Out] 1/2*a*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/(a+b)^(1/2
)+1/2*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 386, 385, 212}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f\sqrt{a+b}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[
a + b]*f) + (Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
```

```
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f} + \frac{a\operatorname{Subst}\left(\int \frac{}{(1-x^2)\cdots}\right.}{}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f} + \frac{a\operatorname{Subst}\left(\int \frac{1}{1-(a+b\cdots}\right.}{}$$

$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{a+b}\,f} + \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+\cdots}}{2f}$$

Mathematica [A]
time = 1.52, size = 164, normalized size = 2.00

$$\frac{\sin(e+fx)\left(\sqrt{2}\,a\tanh^{-1}\left(\frac{\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\right)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}+(2a+b-b\cos(2(e+fx)))\sec^2(e+fx)\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}\right)}{4f\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sin[e + f*x]*(Sqrt[2]*a*ArcTanh[Sqrt[((a + b)*Sin[e + f*x]^2)/a]/Sqrt[1 +
(b*Sin[e + f*x]^2)/a]]*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a] + (2*a + b -
b*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Sqrt[((a + b)*Sin[e + f*x]^2)/a]))/(4*f*
Sqrt[((a + b)*Sin[e + f*x]^2)/a]*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(70) = 140$.
time = 24.38, size = 290, normalized size = 3.54

method	result
default	$\frac{2\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\,\sqrt{a+b}\,b\sin(fx+e)\left(\cos^{2}(fx+e)\right)+2\left(a+b-b\left(\cos^{2}(fx+e)\right)\right)^{\frac{3}{2}}\sqrt{a+b}\,\sin(fx+e)+a\Big(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+2*(
a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2)*sin(f*x+e)+a*(ln(2/(sin(f*x+e)-1)*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a+ln(2/(sin(f*x+e)-1)
*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b-ln(2/(1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a-ln(2/(1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b)*cos(f*
x+e)^2)/(a+b)^(3/2)/cos(f*x+e)^2/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(70) = 140$.
time = 0.49, size = 337, normalized size = 4.11

$$\left[\frac{\sqrt{a+b}\,a\cos\left(fx+e\right)^{2}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos(fx+e)^{4}-8\left(a^{2}+3\,ab+2\,b^{2}\right)\cos(fx+e)^{2}-4\left(\left(a+2\,b\right)\cos(fx+e)^{2}-2\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a+b}\,\sin(fx+e)+8\,a^{2}+16\,ab+8\,b^{2}}{\cos(fx+e)^{4}}\right)+4\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)\sin\left(fx+e\right)}{8\,(a+b)f\cos\left(fx+e\right)^{2}},\ -\frac{a\sqrt{-a-b}\,\arctan\left(\frac{\left(\left(a+2\,b\right)\cos(fx+e)^{2}-2\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-a-b}}{2\left(\left(ab+b^{2}\right)\cos(fx+e)^{2}-a^{2}-2\,ab-b^{2}\right)\sin(fx+e)}\right)\cos\left(fx+e\right)^{2}-2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)\sin\left(fx+e\right)}{4\,(a+b)f\cos\left(fx+e\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a + b)*a*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^
4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 -
2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2
 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*(a +
 b)*sin(f*x + e))/((a + b)*f*cos(f*x + e)^2), -1/4*(a*sqrt(-a - b)*arctan(1
```

```
/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))
)*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*sin(f*x + e))/
((a + b)*f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2(e+fx)}\,\sec^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin(e+f\,x)^2+a}}{\cos(e+f\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^3, x)
```

3.328 $\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=143

$$\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8(a+b)^{3/2}f}+\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{8(a+b)f}+\frac{\sec}{}$$

```
[Out] 1/8*a*(3*a+4*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a
+b)^(3/2)/f+1/4*sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)/(a+b)/f+1/
8*(3*a+4*b)*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 390, 386, 385, 212}

$$\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f(a+b)}+\frac{(3a+4b)\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*(3*a + 4*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2
]])/(8*(a + b)^(3/2)*f) + ((3*a + 4*b)*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]
^2]*Tan[e + f*x])/(8*(a + b)*f) + (Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3
/2)*Tan[e + f*x])/(4*(a + b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{4(a+b)f} + \frac{(3a+4b)\text{Subs}}{}$$

$$= \frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{8(a+b)f} + \frac{\sec^3(e}{}$$

$$= \frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{8(a+b)f} + \frac{\sec^3(e}{}$$

$$= \frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8(a+b)^{3/2}f} + \frac{(3a+4b)\sec(e+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.75, size = 669, normalized size = 4.68

$$-\frac{\sec^3(e+fx)\left(1+\frac{b\sin^2(e+fx)}{a}\right)\tan(e+fx)\left(-15a\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)-10b\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)\sin^2(e+fx)-30a\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{3/2}-20b\sin^2(e+fx)\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{3/2}-32a\,{}_2F_1\left(2,4;\frac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{5/2}-32b\,{}_2F_1\left(2,4;\frac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sin^2(e+fx)\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{5/2}+32a\,{}_2F_1\left(2,4;\frac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{7/2}+32b\,{}_2F_1\left(2,4;\frac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sin^2(e+fx)\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{7/2}+15a\sqrt{-\frac{(a+b)\sec^2(e+fx)(a+b\sin^2(e+fx))\tan^2(e+fx)}{a^2}}+10b\sin^2(e+fx)\sqrt{-\frac{(a+b)\sec^2(e+fx)(a+b\sin^2(e+fx))\tan^2(e+fx)}{a^2}}\right)}{40f\sqrt{a+b\sin^2(e+fx)}\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/40*(Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)*Tan[e + f*x]*(-15*a*ArcSin
[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]] - 10*b*ArcSin[Sqrt[-(((a + b)*Tan[e +
 f*x]^2)/a)]]*Sin[e + f*x]^2 - 30*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) - 20*b*Sin[e + f*x]^2*Sqrt[(
Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(
3/2) - 32*a*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqr
t[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a)
)^(5/2) - 32*b*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*
Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*
Tan[e + f*x]^2)/a))^(5/2) + 32*a*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Ta
n[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a +
b)*Tan[e + f*x]^2)/a))^(7/2) + 32*b*Hypergeometric2F1[2, 4, 7/2, -(((a + b)
*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 15*a*Sqrt[-(((a + b)*Sec[e
 + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] + 10*b*Sin[e + f*x]^
2*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2
)]))/(f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]
^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(127) = 254$.
time = 26.53, size = 570, normalized size = 3.99

method	result
default	$\frac{2\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(a+b)^{\frac{3}{2}}b(3a+4b)\sin(fx+e)\left(\cos^4(fx+e)\right)+2\left(a+b-b\left(\cos^2(fx+e)\right)\right)^{\frac{3}{2}}(a+b)^{\frac{3}{2}}(3a+4b)\left(\cos^2(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*b*(3*a+4*b)*sin(f*x+e)*cos(f
*x+e)^4+2*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*(3*a+4*b)*cos(f*x+e)^2*sin
(f*x+e)+4*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(5/2)*sin(f*x+e)+a*(3*ln(2/(sin(
f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3+10*l
n(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*a^2*b+11*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin
```

```
(f*x+e)+a))*a*b^2+4*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)+b*sin(f*x+e)+a))*b^3-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3-10*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b-11*ln(2/(1+sin(f*x+e))*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^2-4*ln(2/(1+sin(f*x+e
))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^3)*cos(f*x+e)
^4)/(a+b)^(3/2)/cos(f*x+e)^4/(a^2+2*a*b+b^2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Fricas [A]
time = 0.69, size = 443, normalized size = 3.10

$$\left[\frac{(3a^2+4ab)\sqrt{a+b}\cos(fx+e)^4\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-8(a^2+3ab+2b^2)\cos(fx+e)^2-4((a+2b)\cos(fx+e)^2-2a-2b)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}\sin(fx+e)+8a^2+16ab+8b^2}{\cos(fx+e)^4}\right)+4((3a^2+5ab+2b^2)\cos(fx+e)^2+2a^2+4ab+2b^2)\sqrt{-b\cos(fx+e)^2+a+b}\sin(fx+e)}{32(a^2+2ab+b^2)f\cos(fx+e)^4}, -\frac{(3a^2+4ab)\sqrt{-a-b}\arctan\left(\frac{((a+2b)\cos(fx+e)^2-2a-2b)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{2((ab+b^2)\cos(fx+e)^2-a^2-2ab-b^2)\sin(fx+e)}\right)\cos(fx+e)^4-2((3a^2+5ab+2b^2)\cos(fx+e)^2+2a^2+4ab+2b^2)\sqrt{-b\cos(fx+e)^2+a+b}\sin(fx+e)}{16(a^2+2ab+b^2)f\cos(fx+e)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*a^2 + 4*a*b)*sqrt(a + b)*cos(f*x + e)^4*log(((a^2 + 8*a*b + 8*b^2
)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*co
s(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f
*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((3*a^2 + 5*a*b + 2*b
^2)*cos(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)
*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4), -1/16*((3*a^2 + 4*a*
b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*
a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^4 - 2*((3*a^2 + 5*a*b + 2*b^2)*cos(f
*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
+ e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^2(e+fx)}\,\sec^5(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\cos\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^5,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^5, x)
```

3.329 $\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=220

$$\frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf}-\frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5bf}-\frac{(2a$$

```
[Out] -1/5*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b/f+2/15*(a+3*b)*cos(f*
x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-1/15*(2*a^2+7*a*b-3*b^2)*(cos(
f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+
e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+2/15*a*(a+b)*(a+3*b)*(cos(f*x+
e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2
/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 260, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 427, 542, 538, 437, 435, 432, 430}

$$-\frac{(2a^2+7ab-3b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a+b)(a+3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{15b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5bf}+\frac{2(a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (2*(a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f)
 - (Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(5*b*f) - ((2*a
^2 + 7*a*b - 3*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b^2*f*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a]) + (2*a*(a + b)*(a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSi
n[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b
^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
```

```
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \left(1-x^2\right)^{3/2}\sqrt{a+bx^2}\,dx\right.}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5bf} + \frac{\left(\sqrt{\cos^2(e+\right.}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+}{}$$

Mathematica [A]
time = 1.02, size = 199, normalized size = 0.90

$$\frac{-16a(2a^2+7ab-3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+32a(a^2+4ab+3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b(8a^2-32ab-15b^2-4(4a-3b)b\cos(2(e+fx))+3b^2\cos(4(e+fx)))\sin(2(e+fx))}{240b^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-16*a*(2*a^2 + 7*a*b - 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] + 32*a*(a^2 + 4*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(8*a^2 - 32*a*b - 15*
```

```
b^2 - 4*(4*a - 3*b)*b*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e +
 f*x)])/(240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 8.16, size = 432, normalized size = 1.96

method	result
default	$\frac{-3b^3\sin(fx+e)(\cos^6(fx+e))+4a\,b^2\sin(fx+e)(\cos^4(fx+e))+(-a^2b+2a\,b^2+3b^3)(\cos^2(fx+e))\sin(fx+e)+2\sqrt{\frac{\cos(2fx+2e)}{2}+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(-3*b^3*sin(f*x+e)*cos(f*x+e)^6+4*a*b^2*sin(f*x+e)*cos(f*x+e)^4+(-a^2*
b+2*a*b^2+3*b^3)*cos(f*x+e)^2*sin(f*x+e)+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f
*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+8*a^2*(cos(
f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*b+6*a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*El
lipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x
+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-7*(cos(f*x+e)
^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(
1/2))*a^2*b+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/
2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

Fricas [F]
time = 0.13, size = 27, normalized size = 0.12

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^4\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.330 $\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=159

$$\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{a(a+b)F(e-}{3bf}$$

```
[Out] 1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*(a-b)*(cos(f*x+e)^
2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(
1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*
x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a+b
*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 199, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 428, 538, 437, 435, 432, 430}

$$\frac{a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3bf\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((a - b)*Sqr
t[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt
[a + b*Sin[e + f*x]^2])/(3*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)
*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*
Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 428

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^p*((c + d*x^n)^q/(n*(p + q) + 1)), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \sqrt{1-x^2}\,\sqrt{a+bx^2}\,dx,\right.}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cos^2(e+f\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\left((a-b)\sqrt{\cos^2(\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\left((a-b)\sqrt{\cos^2\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a-b)\sqrt{\cos^2(e}{}$$

Mathematica [A]
time = 0.68, size = 158, normalized size = 0.99

$$\frac{-2\sqrt{2}\,a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}\,a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+b(2a+b-b\cos(2(e+fx)))\sin(2(e+fx))}{6\sqrt{2}\,bf\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
f*x, -(b/a)] + 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*E
llipticF[e + f*x, -(b/a)] + b*(2*a + b - b*Cos[2*(e + f*x)])*Sin[2*(e + f*x
)])/(6*Sqrt[2]*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 6.22, size = 265, normalized size = 1.67

method	result

default	$\frac{-b^2(\sin^5(fx+e))+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\ \mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\ \sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-b^2*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*El
lipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^2)^(1/2)
*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+(cos
(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)
^(1/2))*a*b-a*b*sin(f*x+e)^3+b^2*sin(f*x+e)^3+sin(f*x+e)*a*b)/b/cos(f*x+e)/
(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.10, size = 27, normalized size = 0.17

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^2\left(e+fx\right)}\,\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cos(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.331 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=51

$$\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3257, 3256}

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$
$$= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [**A**]
time = 0.06, size = 61, normalized size = 1.20

$$\frac{a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [**A**]
time = 4.40, size = 71, normalized size = 1.39

method	result	size
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)}{\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.09, size = 18, normalized size = 0.35

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases}\dfrac{\sqrt{a}\,\mathrm{E}\left(e+f\,x\middle|-\frac{b}{a}\right)}{f} & \text{if } 0<a\\ \int\sqrt{b\sin\left(e+f\,x\right)^{2}+a}\,dx & \text{if } \neg 0<a\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] piecewise(0 < a, (a^(1/2)*ellipticE(e + f*x, -b/a))/f, ~0 < a, int((a + b*s
in(e + f*x)^2)^(1/2), x))
```

3.332 $\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=131

$$-\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{aF\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\tan}{f}$$

```
[Out] -(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*si
n(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+a*(cos(f*x+e)^2)^(1/2)/cos(f
*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*
sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]

time = 0.10, antiderivative size = 171, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 423, 12, 507, 437, 435, 432, 430}

$$\frac{a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (a*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a + b*Sin
[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1-x^2)^{3/2}}\,dx, x, \sin(e\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+f\right.}{}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(b\sqrt{\cos^2(e+fx)}\sec(e+\right.}{}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+f\right.}{}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+f\right.}{}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.36, size = 134, normalized size = 1.02

$$\frac{-2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,(2a+b-b\cos(2(e+fx)))\tan(e+fx)}{2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2
*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt
[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*f*Sqrt[2*a + b - b*Cos[
2*(e + f*x)]])
```

Maple [A]
time = 11.38, size = 294, normalized size = 2.24

method	result
default	$-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b\sin(fx+e)\left(\cos^2(fx+e)\right)+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)+a*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos
(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*Ellipti
cE(sin(f*x+e),(-1/a*b)^(1/2)))/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f
*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.14, size = 640, normalized size = 4.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(-4*I*sqrt(-b)*b*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((a
^2 + a*b)/b^2)*cos(f*x + e)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^
2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4
*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 4*I*sqrt(-b)*b*sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)*elliptic_
f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*si
n(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))
/b^2) + (2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (2*I*a + I*b)*
sqrt(-b)*cos(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellipt
ic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I
```

```
*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^
2))/b^2) + (-2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (-2*I*a -
I*b)*sqrt(-b)*cos(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*e
lliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e
) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*
b)/b^2))/b^2) - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b*f*cos(
f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin^2(e + fx)} \sec^2(e + fx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b \sin(e + f\,x)^2 + a}}{\cos(e + f\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^2,x)
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^2, x)
```

3.333 $\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=196

$$-\frac{(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{2aF\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{a+b\mathrm{s}\cdots}}{3\cdots}$$

```
[Out] -1/3*(2*a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+2/3*a*(cos
(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x
+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(2*a+b)*(a+b*sin(f*x+e)^2)^(1
/2)*tan(f*x+e)/(a+b)/f+1/3*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)
/f
```

Rubi [A]
time = 0.14, antiderivative size = 236, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 423, 541, 538, 437, 435, 432, 430}

$$\frac{2a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\mathrm{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\mathrm{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/3*((2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)
]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*f*Sqrt[1 + (b*Sin[e + f
*x]^2)/a]) + (2*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/
a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*
x]^2]) + ((2*a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f)
+ (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f)
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1-x^2)^{5/2}}\,dx, x, \sin(e+\right.}{f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx\right.}{}$$

$$= \frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sec^2(e+fx)\sqrt{a+b}}{}$$

$$= \frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sec^2(e+fx)\sqrt{a+b}}{}$$

$$= \frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sec^2(e+fx)\sqrt{a+b}}{}$$

$$= -\frac{(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 1.32, size = 187, normalized size = 0.95

$$\frac{-2a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+4a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\frac{((8a^2-4b^2)\cos(2(e+fx))+(2a+b)(8a+5b-b\cos(4(e+fx))))\sec^2(e+fx)\tan(e+fx)}{2\sqrt{2}}}{6(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] + 4*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] + (((8*a^2 - 4*b^2)*Cos[2*(e + f*x)] + (2*a + b)*(8*a + 5*b - b
*Cos[4*(e + f*x)]))*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqrt[2]))/(6*(a + b)*f*
Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 13.52, size = 367, normalized size = 1.87

method	result
default	$-\frac{-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b(2a+b)\sin(fx+e)(\cos^4(fx+e))+2\sqrt{-b\left(\cos^4\left(fx+e\right)\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(2*a+b)*sin(f*x+e)*cos(
f*x+e)^4+2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*cos(f*x+e)^2*
sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f
*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2
)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))
*a-2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(sin(f*x+e)-1)/(
a+b)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(1+sin(f*x+e
))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.18, size = 789, normalized size = 4.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*(-2*I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 -
(4*I*a^2 + 4*I*a*b + I*b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*
a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(2*I*a*b + I*b^2)*sqrt(-b)*sqrt
((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (-4*I*a^2 - 4*I*a*b - I*b^2)*sqrt(-b)*co
```

```
s(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcs
in(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x
+ e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)
- 2*(2*(-I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 + (-2
*I*a^2 - I*a*b)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)
*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*s
qrt((a^2 + a*b)/b^2))/b^2) - 2*(2*(I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)
/b^2)*cos(f*x + e)^3 + (2*I*a^2 + I*a*b)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b
*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b
+ b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*((2*a*b + b^2)*cos(
f*x + e)^2 + a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a*b
 + b^2)*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2{(e + fx)}}\,\sec^4{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}}{{\cos\left(e+f\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^4,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^4, x)
```

3.334 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=157

$$\frac{a^2(a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16bf}+\frac{(a+6b)\sin(e+$$

```
[Out] 1/16*a^2*(a+6*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/
2)/f+1/24*(a+6*b)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b/f-1/6*sin(f*x+e)*(a
+b*sin(f*x+e)^2)^(5/2)/b/f+1/16*a*(a+6*b)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/
2)/b/f
```

Rubi [A]
time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 396, 201, 223, 212}

$$\frac{a^2(a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6bf}+\frac{(a+6b)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24bf}+\frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*(a + 6*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(16*b^(3/2)*f) + (a*(a + 6*b)*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(16*
b*f) + ((a + 6*b)*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(24*b*f) - (Si
n[e + f*x]*(a + b*Sin[e + f*x]^2)^(5/2))/(6*b*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+bx^2\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6bf} + \frac{(a+6b)\text{Subst}\left(\int (a+b\right.}{6}$$

$$= \frac{(a+6b)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24bf} - \frac{\sin(e+fx)(a+}{6}$$

$$= \frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16bf} + \frac{(a+6b)\sin(e+f}{}$$

$$= \frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16bf} + \frac{(a+6b)\sin(e+f}{}$$

$$= \frac{a^2(a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{a(a+6b)\sin(e}{}$$

Mathematica [A]

time = 0.58, size = 149, normalized size = 0.95

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(3a^{3/2}(a+6b)\sinh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\,\sin(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,(-3a(a-10b)-2(7a-6b)b\sin^2(e+fx)-8b^2\sin^4(e+fx))\right)}{48b^{3/2}f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]^2]*(3*a^(3/2)*(a + 6*b)*ArcSinh[(Sqrt[b]*Sin[e + f
*x])/Sqrt[a]] + Sqrt[b]*Sin[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a]*(-3*a*(
a - 10*b) - 2*(7*a - 6*b)*b*Sin[e + f*x]^2 - 8*b^2*Sin[e + f*x]^4)))/(48*b^
(3/2)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

Maple [A]

time = 7.60, size = 198, normalized size = 1.26

method	result
default	$\frac{-8\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,b^{\frac{7}{2}}\sin(fx+e)(\cos^4(fx+e))+2\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,b^{\frac{5}{2}}(2b+7a)(\cos^2(fx+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(-8*(a+b-b*cos(f*x+e)^2)^(1/2)*b^(7/2)*sin(f*x+e)*cos(f*x+e)^4+2*(a+b-
b*cos(f*x+e)^2)^(1/2)*b^(5/2)*(2*b+7*a)*cos(f*x+e)^2*sin(f*x+e)+(a+b-b*cos(
f*x+e)^2)^(1/2)*b^(3/2)*(-3*a^2+16*a*b+4*b^2)*sin(f*x+e)+3*a^3*ln(sin(f*x+e
)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*b+18*a^2*ln(sin(f*x+e)*b^(1/2)+(a+b-b
*cos(f*x+e)^2)^(1/2))*b^2)/b^(5/2)/f
```

Maxima [A]

time = 0.29, size = 186, normalized size = 1.18

$$\frac{\frac{3a^3\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{18a^2\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}}+12\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)+18\sqrt{b\sin\left(fx+e\right)^2+a}\,a\sin\left(fx+e\right)-\frac{8\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}\sin(fx+e)}{b}+\frac{2\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a\sin(fx+e)}{b}+\frac{3\sqrt{b\sin\left(fx+e\right)^2+a}\,a^2\sin(fx+e)}{b}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^3*arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) + 18*a^2*arcsinh(b*si
n(f*x + e)/sqrt(a*b))/sqrt(b) + 12*(b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e
) + 18*sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*x + e) - 8*(b*sin(f*x + e)^2 + a)
^(5/2)*sin(f*x + e)/b + 2*(b*sin(f*x + e)^2 + a)^(3/2)*a*sin(f*x + e)/b + 3
*sqrt(b*sin(f*x + e)^2 + a)*a^2*sin(f*x + e)/b)/f
```

Fricas [**A**]
time = 1.45, size = 577, normalized size = 3.68

$$\left[\frac{3\,(a^3+6\,a^2b)\sqrt{b}\log\left(128\,b^4\cos(fx+e)^8-256\,(ab^3+2\,b^4)\cos(fx+e)^6+32\,(5\,a^2b^2+24\,ab^3+24\,b^4)\cos(fx+e)^4+a^4+32\,a^3b+160\,a^2b^2+256\,ab^3+128\,b^4-32\,(a^3b+10\,a^2b^2+24\,ab^3+16\,b^4)\cos(fx+e)^2-8\,(16\,b^3\cos(fx+e)^6-24\,(ab^2+2\,b^3)\cos(fx+e)^4-a^3-10\,a^2b-24\,ab^2-16\,b^3+2\,(5\,a^2b+24\,ab^2+24\,b^3)\cos(fx+e)^2)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{b}\sin(fx+e)\right)-8\,(8\,b^3\cos(fx+e)^4+3\,a^2b-16\,ab^2-4\,b^3-2\,(7\,ab^2+2\,b^3)\cos(fx+e)^2)\sqrt{-b\cos(fx+e)^2+a+b}\sin(fx+e)}{384\,b^2f},\ -\frac{3\,(a^3+6\,a^2b)\sqrt{-b}\arctan\left(\frac{(8\,b^2\cos(fx+e)^4-8\,(ab+2\,b^2)\cos(fx+e)^2+a^2+8\,ab+8\,b^2)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}}{4\,(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-(3\,ab^2+4\,b^3)\cos(fx+e)^2)\sin(fx+e)}\right)+4\,(8\,b^3\cos(fx+e)^4+3\,a^2b-16\,ab^2-4\,b^3-2\,(7\,ab^2+2\,b^3)\cos(fx+e)^2)\sqrt{-b\cos(fx+e)^2+a+b}\sin(fx+e)}{192\,b^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(a^3 + 6*a^2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 +
 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4
+ a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b
^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b
^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*
b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt
(b)*sin(f*x + e)) - 8*(8*b^3*cos(f*x + e)^4 + 3*a^2*b - 16*a*b^2 - 4*b^3 -
2*(7*a*b^2 + 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
 + e))/(b^2*f), -1/192*(3*(a^3 + 6*a^2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*
x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2
*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*b^3*cos(f*x
+ e)^4 + 3*a^2*b - 16*a*b^2 - 4*b^3 - 2*(7*a*b^2 + 2*b^3)*cos(f*x + e)^2)*s
qrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(b^2*f)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Giac [**A**]
time = 0.62, size = 134, normalized size = 0.85

$$-\frac{\left(2\left(4\,b\sin(fx+e)^2+\frac{7\,ab^4-6\,b^5}{b^4}\right)\sin(fx+e)^2+\frac{3\,(a^2b^3-10\,ab^4)}{b^4}\right)\sqrt{b\sin(fx+e)^2+a}\,\sin(fx+e)+\frac{3\,(a^3+6\,a^2b)\log\left(\left|-\sqrt{b}\sin(fx+e)+\sqrt{b\sin(fx+e)^2+a}\right|\right)}{b^{\frac{3}{2}}}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/48*((2*(4*b*sin(f*x + e)^2 + (7*a*b^4 - 6*b^5)/b^4)*sin(f*x + e)^2 + 3*(
a^2*b^3 - 10*a*b^4)/b^4)*sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e) + 3*(a^3 +
 6*a^2*b)*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/b^(3
/2))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (e+f\,x)^3\,{\left(b\,\sin (e+f\,x)^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.335 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}\,f}+\frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)}{4f}$$

```
[Out] 1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+3/8*a^2*arctanh(sin(f*x+e)*b^(1/2
)/(a+b*sin(f*x+e)^2)^(1/2))/f/b^(1/2)+3/8*a*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(
1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 201, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}\,f}+\frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8*Sqrt[
b]*f) + (3*a*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(8*f) + (Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(3/2))/(4*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f} + \frac{(3a)\text{Subst}\left(\int \sqrt{a+bx^2}\right.}{4f}$$

$$= \frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f} + \frac{\sin(e+fx)\left(a+b\sin^2(e\right.}{4f}$$

$$= \frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f} + \frac{\sin(e+fx)\left(a+b\sin^2(e\right.}{4f}$$

$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}\,f} + \frac{3a\sin(e+fx)\sqrt{a+b}}{8f}$$

Mathematica [A]
time = 0.36, size = 93, normalized size = 0.89

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(5a\sin(e+fx)+2b\sin^3(e+fx)+\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]^2]*(5*a*Sin[e + f*x] + 2*b*Sin[e + f*x]^3 + (3*a^(
3/2)*ArcSinh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*Sin[e +
f*x]^2)/a])))/(8*f)
```

Maple [A]
time = 0.13, size = 86, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin^2(\right.}\right.}{2\sqrt{b}}\right.}{4}}{f}$
default	$\frac{\frac{\sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin^2(\right.}\right.}{2\sqrt{b}}\right.}{4}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)+3/4*a*(1/2*sin(f*x+e)*(a+b*sin
(f*x+e)^2)^(1/2)+1/2*a*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1
/2)))
```

Maxima [A]
time = 0.28, size = 78, normalized size = 0.75

$$\frac{\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}}+2\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)+3\sqrt{b\sin\left(fx+e\right)^2+a}\;a\sin\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*a^2*arcsinh(b*sin(f*x + e)/sqrt(a*b))/sqrt(b) + 2*(b*sin(f*x + e)^2
+ a)^(3/2)*sin(f*x + e) + 3*sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*x + e))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.
time = 0.66, size = 503, normalized size = 4.84

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)^4+a^4+32\,a^3b+160\,a^2b^2+256\,ab^3+128\,b^4-32\left(a^3b+10\,a^2b^2+24\,ab^3+16\,b^4\right)\cos\left(fx+e\right)^2-8\left(16\,b^3\cos\left(fx+e\right)^6-24\left(ab^2+2\,b^3\right)\cos\left(fx+e\right)^4-a^3-10\,a^2b-24\,ab^2-16\,b^3+2\left(5\,a^2b+24\,ab^2+24\,b^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{b}\sin\left(fx+e\right)\right)-8\left(2\,b^2\cos\left(fx+e\right)^2-5\,ab-2\,b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sin\left(fx+e\right)}{64\,bf},-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\left(ab+2\,b^2\right)\cos(fx+e)^2+a^2+8\,ab+8\,b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-b}}{4\left(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-\left(3\,ab^2+4\,b^3\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)+4\left(2\,b^2\cos\left(fx+e\right)^2-5\,ab-2\,b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sin\left(fx+e\right)}{32\,bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*a^2*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f
*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^
3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3
 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*c
os(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2
+ 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x +
 e)) - 8*(2*b^2*cos(f*x + e)^2 - 5*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sin(f*x + e))/(b*f), -1/32*(3*a^2*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x +
 e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^
3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*b^2*cos(f*x + e
)^2 - 5*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(b*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 84, normalized size = 0.81

$$-\frac{\frac{3\,a^2\log\left(\left|-\sqrt{b}\,\sin(fx+e)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{\sqrt{b}}-\left(2\,b\sin\left(fx+e\right)^2+5\,a\right)\sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*a^2*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/sq
rt(b) - (2*b*sin(f*x + e)^2 + 5*a)*sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e))
/f
```

Mupad [B]
time = 14.54, size = 60, normalized size = 0.58

$$\frac{\sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\sin(e+f\,x)^2}{a}\right)}{f\,\left(\frac{b\sin(e+f\,x)^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*
sin(e + f*x)^2)/a))/(f*((b*sin(e + f*x)^2)/a + 1)^(3/2))
```

3.336 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{b\sin(e+fx)}{}$$

```
[Out] (a+b)^(3/2)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f-1/2*
(3*a+2*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*b^(1/2)/f-1/
2*b*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3269, 427, 537, 223, 212, 385}

$$-\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e +
 f*x]^2]])/f + ((a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b
*Sin[e + f*x]^2]])/f - (b*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$=-\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{\text{Subst}\left(\int\frac{-a(2a+b)-b(3a+}{(1-x^2)\sqrt{a+b}}\right.}{2}$$

$$=-\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(a+b)^2\text{Subst}\left(\int\frac{}{(1-x^2)}\right.}{}$$

$$=-\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(a+b)^2\text{Subst}\left(\int\frac{1}{1-(a+}\right.}{}$$

$$=-\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)^{3/2}}{}$$

Mathematica [A]
time = 0.48, size = 233, normalized size = 1.93

$$\frac{\sqrt{2}\,b(3a+2b)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a+b}\,\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\sqrt{2}\,(4a^2+5ab+2b^2)\tanh^{-1}\left(\frac{\sqrt{2a+2b}\,\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)-2\sqrt{b}\,\sqrt{a+b}\left(\sqrt{2}\,(3a+2b)\log\left(\sqrt{2a+b-b\cos(2(e+fx))}+\sqrt{2}\,\sqrt{b}\,\sin(e+fx)\right)+\sqrt{b}\,\sqrt{2a+b-b\cos(2(e+fx))}\,\sin(e+fx)\right)}{4\sqrt{2}\,\sqrt{a+b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*b*(3*a + 2*b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Sin[e + f*x])/Sqrt[2*a
+ b - b*Cos[2*(e + f*x)]]] + Sqrt[2]*(4*a^2 + 5*a*b + 2*b^2)*ArcTanh[(Sqrt[
2*a + 2*b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]] - 2*Sqrt[b]*Sq
rt[a + b]*(Sqrt[2]*(3*a + 2*b)*Log[Sqrt[2*a + b - b*Cos[2*(e + f*x)]] + Sqr
t[2]*Sqrt[b]*Sin[e + f*x]] + Sqrt[b]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Sin
[e + f*x]))/(4*Sqrt[2]*Sqrt[a + b]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(103) = 206.
time = 19.57, size = 417, normalized size = 3.45

method	result
default	$-\frac{2b^{\frac{3}{2}}\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\right)\sqrt{a+b}+3a\sqrt{b}\,\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b-b\left(\cos^2\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a+b)^(1/2)*(2*b^(3/2)*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2
))*(a+b)^(1/2)+3*a*b^(1/2)*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2)
)*(a+b)^(1/2)+b*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)+ln(2/(1+s
in(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+2*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
*a*b+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+
e)+a))*b^2-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*si
n(f*x+e)+a))*a^2-2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))*a*b-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)+b*sin(f*x+e)+a))*b^2)/f
```

Maxima [A]
time = 0.53, size = 178, normalized size = 1.47

$$-\frac{3\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)+2\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)-(a+b)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)+1)}\right)-(a+b)^{\frac{3}{2}}\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)-1)}\right)+\sqrt{b\sin\left(fx+e\right)^2+a}\,b\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*a*sqrt(b)*arcsinh(b*sin(f*x + e)/sqrt(a*b)) + 2*b^(3/2)*arcsinh(b*s
in(f*x + e)/sqrt(a*b)) - (a + b)^(3/2)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(s
in(f*x + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) + 1))) - (a + b)^(3/2)*arcsi
nh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x +
 e) - 1))) + sqrt(b*sin(f*x + e)^2 + a)*b*sin(f*x + e))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(103) = 206$.
time = 0.79, size = 1381, normalized size = 11.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*a + 2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)
*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 +
 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24
*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*
b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*
a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin
(f*x + e)) + 4*(a + b)^(3/2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*
(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a -
2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*
a*b + 8*b^2)/cos(f*x + e)^4) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x
+ e))/f, -1/16*(8*(a + b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - (3*a + 2*b)*sqrt(b)*log(
128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2
 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a
*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2
 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*
a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) + 8*sqrt(-b*cos(f*x
+ e)^2 + a + b)*b*sin(f*x + e))/f, 1/8*((3*a + 2*b)*sqrt(-b)*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3
*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 2*(a +
b)^(3/2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2
)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x
 + e)^4) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/f, -1/8*(4*(a
+ b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b
*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 -
```

```
2*a*b - b^2)*sin(f*x + e))) - (3*a + 2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*
x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2
*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*sqrt(-b*cos(f*x
 + e)^2 + a + b)*b*sin(f*x + e))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}} \sec\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2)*sec(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e + f\,x\right)^2 + a\right)^{3/2}}{\cos\left(e + f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x), x)
```

3.337 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{(a-2b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)\sec(e+$$

[Out] b^(3/2)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f+1/2*(a-2*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f+1/2*(a+b)*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3269, 424, 537, 223, 212, 385}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{(a-2b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]

[Out] (b^(3/2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/f + ((a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*f) + ((a + b)*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(2*f)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx^2)^{3/2}}{(1-x^2)^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f}-\frac{\text{Subst}\Big(}{}$$

$$=\frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f}+\frac{b^2\text{Subst}}{}$$

$$=\frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f}+\frac{b^2\text{Subst}}{}$$

$$=\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{(a-2b)\sqrt{a+b}\,\tanh^{-}}{}$$

Mathematica [A]
time = 0.67, size = 210, normalized size = 1.65

$$\frac{-2b^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+2(a^2-ab-b^2)\tanh^{-1}\left(\frac{\sqrt{2a+2b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\sqrt{a+b}\left(4b^{3/2}\log\left(\sqrt{2a+b-b\cos(2(e+fx))}+\sqrt{2}\sqrt{b}\sin(e+fx)\right)+\sqrt{2}(a+b)\sqrt{2a+b-b\cos(2(e+fx))}\sec(e+fx)\tan(e+fx)\right)}{4\sqrt{a+b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*b^2*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] + 2*(a^2 - a*b - b^2)*ArcTanh[(Sqrt[2*a + 2*b]*Sin[e + f*x])/Sq
rt[2*a + b - b*Cos[2*(e + f*x)]]] + Sqrt[a + b]*(4*b^(3/2)*Log[Sqrt[2*a + b
 - b*Cos[2*(e + f*x)]] + Sqrt[2]*Sqrt[b]*Sin[e + f*x]] + Sqrt[2]*(a + b)*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]]*Sec[e + f*x]*Tan[e + f*x]))/(4*Sqrt[a + b]
*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(109) = 218$.
time = 21.86, size = 402, normalized size = 3.17

method	result
default	$\frac{2\sin(fx+e)\sqrt{a+b-b\left(\cos^2(fx+e)\right)}\,(a+b)^{\frac{5}{2}}-\left(-4b^{\frac{3}{2}}\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b-b\left(\cos^2(fx+e)\right)}\right)(a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)-(-4*b^(3/2)*ln(sin
(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*(a+b)^(3/2)+ln(2/(1+sin(f*x+e))
*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3-3*a*b^2*ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))-2*b
^3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)
+a))-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+
e)+a))*a^3+3*a*b^2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))+2*b^3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)+b*sin(f*x+e)+a)))*cos(f*x+e)^2)/(a+b)^(3/2)/cos(f*x+e)^2/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(109) = 218$.
time = 0.80, size = 1471, normalized size = 11.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b^(3/2)*cos(f*x + e)^2*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^
4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4
 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 2
4*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*s
in(f*x + e)) - sqrt(a + b)*(a - 2*b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b
^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*
cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin
(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e
)^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/8*(2*(a - 2*b)*sq
rt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*
x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b -
 b^2)*sin(f*x + e)))*cos(f*x + e)^2 - b^(3/2)*cos(f*x + e)^2*log(128*b^4*co
s(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^
3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128
*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b
^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24
*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) - 4*sqrt(-b*cos(f*x + e)^2 + a
 + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/8*(2*sqrt(-b)*b*arctan(1
/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8
*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2
*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos
(f*x + e)^2 + sqrt(a + b)*(a - 2*b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^
2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*c
os(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(
f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) - 4*sqrt(-b*cos(f*x + e)
^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/4*((a - 2*b)*sqrt(
-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^
2)*sin(f*x + e)))*cos(f*x + e)^2 + sqrt(-b)*b*arctan(1/4*(8*b^2*cos(f*x + e
)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3
```

```
- (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^2 - 2*sqrt(
-b*cos(f*x + e)^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^3,x)
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^3, x)
```

3.338 $\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{a+b}\,f}+\frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{8f}+\frac{\sec^3(e+fx)\,(a+b}{}$$

```
[Out] 3/8*a^2*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/(a+b)^(1
/2)+1/4*sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)/f+3/8*a*sec(f*x+e)
*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 386, 385, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8*S
qrt[a + b]*f) + (3*a*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/
(8*f) + (Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{4f} + \frac{(3a)\text{Subst}\left(\right.}{}$$

$$= \frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{8f} + \frac{\sec^3(e+f}{}$$

$$= \frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{8f} + \frac{\sec^3(e+f}{}$$

$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{a+b}\,f} + \frac{3a\sec(e+fx)\sqrt{a+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 63, normalized size = 0.52

$$\frac{a^2\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{(a+b)\sin^2(e+fx)}{a+b\sin^2(e+fx)}\right)\sin(e+fx)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

[Out] (a^2*Hypergeometric2F1[1/2, 3, 3/2, ((a + b)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]^2)]*Sin[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(106) = 212$.
time = 17.54, size = 406, normalized size = 3.33

method	result
default	$\frac{2\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(a+b)^{\frac{5}{2}}(3a-2b)\sin(fx+e)(\cos^2(fx+e))+4\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(a+b)^{\frac{7}{2}}\sin(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*(3*a-2*b)*sin(f*x+e)*cos(f*x
+e)^2+4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*sin(f*x+e)+3*a^2*(ln(2/(sin(
f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+2*ln
(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*
a*b+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e
)+a))*b^2-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin
(f*x+e)+a))*a^2-2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))*a*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^4)/(a+b)^(5/2)/cos(f*x+e)^4/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Fricas [A]
time = 0.74, size = 413, normalized size = 3.39

$$\left[\frac{3\sqrt{a+b}\,a^2\cos(fx+e)^4\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-8(a^2+3ab+2b^2)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a+b}\,\sin(fx+e)+8a^2+16ab+8b^2}{\cos(fx+e)^4}\right)+4\left((3a^2+ab-2b^2)\cos(fx+e)^2+2a^2+4ab+2b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sin(fx+e)}{32(a+b)f\cos(fx+e)^4}, -\frac{3a^2\sqrt{-a-b}\,\arctan\left(\frac{\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-a-b}}{2\left((ab+b^2)\cos(fx+e)^2-a^2-2ab-b^2\right)\sin(fx+e)}\right)\cos(fx+e)^4-2\left((3a^2+ab-2b^2)\cos(fx+e)^2+2a^2+4ab+2b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sin(fx+e)}{16(a+b)f\cos(fx+e)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*sqrt(a + b)*a^2*cos(f*x + e)^4*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
+ e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)
```

```
^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) +
8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((3*a^2 + a*b - 2*b^2)*cos(f*x
+ e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e
))/((a + b)*f*cos(f*x + e)^4), -1/16*(3*a^2*sqrt(-a - b)*arctan(1/2*((a + 2
*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b
)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x
+ e)^4 - 2*((3*a^2 + a*b - 2*b^2)*cos(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*s
qrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a + b)*f*cos(f*x + e)^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^5,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^5, x)
```

3.339 $\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=195

$$\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16(a+b)^{3/2}f}+\frac{a(5a+6b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{16(a+b)f}+\frac{(5$$

```
[Out] 1/16*a^2*(5*a+6*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))
/(a+b)^(3/2)/f+1/24*(5*a+6*b)*sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x
+e)/(a+b)/f+1/6*sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(5/2)*tan(f*x+e)/(a+b)/f+1/
16*a*(5*a+6*b)*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A]
time = 0.11, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 390, 386, 385, 212}

$$\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6f(a+b)}+\frac{(5a+6b)\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24f(a+b)}+\frac{a(5a+6b)\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*(5*a + 6*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]
^2]])/(16*(a + b)^(3/2)*f) + (a*(5*a + 6*b)*Sec[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]^2]*Tan[e + f*x])/(16*(a + b)*f) + ((5*a + 6*b)*Sec[e + f*x]^3*(a + b*
Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/(24*(a + b)*f) + (Sec[e + f*x]^5*(a + b
*Sin[e + f*x]^2)^(5/2)*Tan[e + f*x])/(6*(a + b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
```

```
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1-x^2)^4}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}\tan(e+fx)}{6(a+b)f} + \frac{(5a+6b)\text{Su}}{}$$

$$= \frac{(5a+6b)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{24(a+b)f} + \frac{\text{sec}}{}$$

$$= \frac{a(5a+6b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{16(a+b)f} + \frac{(5a}{}$$

$$= \frac{a(5a+6b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{16(a+b)f} + \frac{(5a}{}$$

$$= \frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16(a+b)^{3/2}f} + \frac{a(5a+6b)\sec(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 11.69, size = 938, normalized size = 4.81

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)^2*Tan[e + f*x]*(45*a*ArcSin[
Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]] + 30*b*ArcSin[Sqrt[-(((a + b)*Tan[e +
f*x]^2)/a)]]*Sin[e + f*x]^2 + 210*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) + 140*b*Sin[e + f*x]^2*Sqrt[
(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^
(3/2) - 120*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*T
an[e + f*x]^2)/a))^(5/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Ta
n[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a +
b)*Tan[e + f*x]^2)/a))^(5/2) - 80*b*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a
+ b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/2) + 256*b*Hyper
```

```
geometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[
(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^
(5/2) - 512*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*S
qrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/
a))^(7/2) - 512*b*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a
)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a +
b)*Tan[e + f*x]^2)/a))^(7/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b
)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((
a + b)*Tan[e + f*x]^2)/a))^(9/2) + 256*b*Hypergeometric2F1[2, 5, 7/2, -(((a
 + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e
 + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(9/2) - 45*a*Sqrt[-(((a + b)*
Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] - 30*b*Sin[e +
f*x]^2*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2
)/a^2)]))/(240*f*(a + b*Sin[e + f*x]^2)^(3/2)*Sqrt[(Sec[e + f*x]^2*(a + b*S
in[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(175) = 350$.

time = 21.06, size = 693, normalized size = 3.55

method	result
default	$\frac{2\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(a+b)^{\frac{7}{2}}\left(15a^2+8ab-4b^2\right)\sin(fx+e)\left(\cos^4(fx+e)\right)+4\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*(15*a^2+8*a*b-4*b^2)*sin(f*x
+e)*cos(f*x+e)^4+4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*(5*a^2+3*a*b-2*b^
2)*cos(f*x+e)^2*sin(f*x+e)+16*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*(a^2+2
*a*b+b^2)*sin(f*x+e)-3*a^2*(5*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4+21*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b+33*ln(2/(1+sin(f*x+e))*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2+23*ln(2/(1+sin(
f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+6*ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
b^4-5*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x
+e)+a))*a^4-21*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+
b*sin(f*x+e)+a))*a^3*b-33*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^2-23*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3-6*ln(2/(sin(f*x+e)-1)*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^6)/(a+b)
^(9/2)/cos(f*x+e)^6/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^7, x)
```

Fricas [A]
time = 2.32, size = 545, normalized size = 2.79

$$\left[\frac{3\,(5\,a^{3}+6\,a^{2}b)\sqrt{a+b}\cos\left(fx+e\right)^{6}\log\left(\frac{(a^{2}+8\,ab+8\,b^{2})\cos\left(fx+e\right)^{4}-8\,(a^{2}+3\,ab+2\,b^{2})\cos\left(fx+e\right)^{2}-4\,((a+2\,b)\cos\left(fx+e\right)^{2}-2\,a-2\,b)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{a+b}\sin\left(fx+e\right)+8\,a^{2}+16\,ab+8\,b^{2}}{\cos\left(fx+e\right)^{4}}\right)+4\,\left((15\,a^{3}+23\,a^{2}b+4\,ab^{2}-4\,b^{3})\cos\left(fx+e\right)^{4}+8\,a^{3}+24\,a^{2}b+24\,ab^{2}+8\,b^{3}+2\,(5\,a^{3}+8\,a^{2}b+ab^{2}-2\,b^{3})\cos\left(fx+e\right)^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sin\left(fx+e\right)}{192\,(a^{2}+2\,ab+b^{2})f\cos\left(fx+e\right)^{6}},\ -\frac{3\,(5\,a^{3}+6\,a^{2}b)\sqrt{-a-b}\arctan\left(\frac{((a+2\,b)\cos\left(fx+e\right)^{2}-2\,a-2\,b)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{-a-b}}{2\,((ab+b^{2})\cos\left(fx+e\right)^{2}-a^{2}-2\,ab-b^{2})\sin\left(fx+e\right)}\right)\cos\left(fx+e\right)^{6}-2\,\left((15\,a^{3}+23\,a^{2}b+4\,ab^{2}-4\,b^{3})\cos\left(fx+e\right)^{4}+8\,a^{3}+24\,a^{2}b+24\,ab^{2}+8\,b^{3}+2\,(5\,a^{3}+8\,a^{2}b+ab^{2}-2\,b^{3})\cos\left(fx+e\right)^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sin\left(fx+e\right)}{96\,(a^{2}+2\,ab+b^{2})f\cos\left(fx+e\right)^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*a^3 + 6*a^2*b)*sqrt(a + b)*cos(f*x + e)^6*log(((a^2 + 8*a*b +
8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*
b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*
sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((15*a^3 + 23*a^
2*b + 4*a*b^2 - 4*b^3)*cos(f*x + e)^4 + 8*a^3 + 24*a^2*b + 24*a*b^2 + 8*b^3
 + 2*(5*a^3 + 8*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)
^2 + a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^6), -1/96*(3*
(5*a^3 + 6*a^2*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a -
 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x +
e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^6 - 2*((15*a^3 + 23*a
^2*b + 4*a*b^2 - 4*b^3)*cos(f*x + e)^4 + 8*a^3 + 24*a^2*b + 24*a*b^2 + 8*b^
3 + 2*(5*a^3 + 8*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e
)^2 + a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^6)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**7*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^7, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^7,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^7, x)
```

3.340 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=321

$$-\frac{(a^2-9ab-2b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}+\frac{2(4a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b}}{35f}$$

```
[Out] -1/35*(a^2-9*a*b-2*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f+
2/35*(4*a+b)*cos(f*x+e)^3*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/7*b*cos(f
*x+e)^5*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-2/35*(a-b)*(a^2+6*a*b+b^2)*El
lipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f
*x+e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/35*a*(a+b)*(2*a^2+9*a*b-b
^2)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b
*sin(f*x+e)^2/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 427, 542, 538, 437, 435, 432, 430}

$$\frac{a(a+b)(2a^2+9ab-b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{35b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)(a^2+6ab+b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{35b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(a^2-9ab-2b^2)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}-\frac{b\sin(e+fx)\cos^5(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f}+\frac{2(4a+b)\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/35*((a^2 - 9*a*b - 2*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f
*x]^2])/(b*f) + (2*(4*a + b)*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]^2])/(35*f) - (b*Cos[e + f*x]^5*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2
])/(7*f) - (2*(a - b)*(a^2 + 6*a*b + b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[Ar
cSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(35*b^
2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*(2*a^2 + 9*a*b - b^2)*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(35*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}\right.}{f}$$

$$= -\frac{b\cos^5(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f} - \frac{\left(\sqrt{\cos^2(e\right.}{}$$

$$= \frac{2(4a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f} - \frac{b\cos}{}$$

$$= -\frac{(a^2-9ab-2b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}$$

$$= -\frac{(a^2-9ab-2b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}$$

$$= -\frac{(a^2-9ab-2b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}$$

$$= -\frac{(a^2-9ab-2b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}$$

Mathematica [A]
time = 1.85, size = 247, normalized size = 0.77

$$\frac{-128a(a^3+5a^2b-5ab^2-b^3)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+64a(2a^3+11a^2b+8ab^2-b^3)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b(-32a^3+400a^2b+212ab^2+30b^3+b(144a^2-192ab-37b^2)\cos(2(e+fx))+2b^2(-26a+b)\cos(4(e+fx))+5b^3\cos(6(e+fx)))\sin(2(e+fx))}{2240b^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]

[Out] (-128*a*(a^3 + 5*a^2*b - 5*a*b^2 - b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 64*a*(2*a^3 + 11*a^2*b + 8*a*b^2 - b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(-32*a^3 + 400*a^2*b + 212*a*b^2 + 30*b^3 + b*(144*a^2 - 192*a*b - 37*b^2)*Cos[2*(e + f*x)] + 2*b^2*(-26*a + b)*Cos[4*(e + f*x)] + 5*b^3*Cos[6*(e + f*x)])*Sin[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])

Maple [A]
time = 8.43, size = 590, normalized size = 1.84

method	result
default	$\frac{5b^4\sin(fx+e)(\cos^8(fx+e))+(-13ab^3-7b^4)(\cos^6(fx+e))\sin(fx+e)+(9a^2b^2+ab^3)(\cos^4(fx+e))\sin(fx+e)+(-a^3b+8a^2b^2+11a b}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/35*(5*b^4*sin(f*x+e)*cos(f*x+e)^8+(-13*a*b^3-7*b^4)*cos(f*x+e)^6*sin(f*x+e)+(9*a^2*b^2+a*b^3)*cos(f*x+e)^4*sin(f*x+e)+(-a^3*b+8*a^2*b^2+11*a*b^3+2*b^4)*cos(f*x+e)^2*sin(f*x+e)+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4+11*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b+8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^4-10*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b+10*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3)/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)

Fricas [F]
time = 0.14, size = 44, normalized size = 0.14

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^{6}-(a+b)\cos\left(fx+e\right)^{4}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^6 - (a + b)*cos(f*x + e)^4)*sqrt(-b*cos(f*x + e)^
2 + a + b), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^4\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.341 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=259

$$\frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{b\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f}-$$

```
[Out] 2/15*(3*a+b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/5*b*cos(f*x
+e)^3*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/15*(3*a^2-7*a*b-2*b^2)*Ellipt
icE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/15*a*(3*a-b)*(a+b)*EllipticF(si
n(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)
^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 427, 542, 538, 437, 435, 432, 430}

$$-\frac{(3a^2-7ab-2b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{a(3a-b)(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{15bf\sqrt{a+b\sin^2(e+fx)}}-\frac{b\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f}+\frac{2(3a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(3*a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*f) -
 (b*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(5*f) - ((3*a^2
 - 7*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/
a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f*Sqrt[1 + (b*Sin[e + f*
x]^2)/a]) + (a*(3*a - b)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[
e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
```

```
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\sqrt{1-x^2}\,(a+bx^2)^{3/2}\right.}{f}$$

$$= -\frac{b\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f} - \frac{\left(\sqrt{\cos^2(\right.}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\,\text{co}}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\,\text{co}}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\,\text{co}}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\,\text{co}}{}$$

Mathematica [A]
time = 0.94, size = 200, normalized size = 0.77

$$\frac{-16a(3a^2-7ab-2b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+16a(3a^2+2ab-b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b(48a^2+28ab+5b^2-4b(9a+2b)\cos(2(e+fx))+3b^2\cos(4(e+fx)))\sin(2(e+fx))}{240bf\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-16*a*(3*a^2 - 7*a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] + 16*a*(3*a^2 + 2*a*b - b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(48*a^2 + 28*a*b + 5*
```

```
b^2 - 4*b*(9*a + 2*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e +
 f*x)])/(240*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 8.31, size = 429, normalized size = 1.66

method	result
default	$\frac{-3b^3(\sin^7(fx+e))-9a\,b^2(\sin^5(fx+e))+4b^3(\sin^5(fx+e))+3\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(-3*b^3*sin(f*x+e)^7-9*a*b^2*sin(f*x+e)^5+4*b^3*sin(f*x+e)^5+3*(cos(f*
x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*a^3+2*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(
sin(f*x+e),(-1/a*b)^(1/2))*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*(cos(f*x+e)^2)^(1/2)*((a+b
*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/
2))*a^2*b+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin
(f*x+e),(-1/a*b)^(1/2))*a*b^2-6*a^2*b*sin(f*x+e)^3+10*a*b^2*sin(f*x+e)^3-b^
3*sin(f*x+e)^3+6*a^2*b*sin(f*x+e)-a*b^2*sin(f*x+e))/b/cos(f*x+e)/(a+b*sin(f
*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.13, size = 44, normalized size = 0.17

$$\mathrm{integral}\left(-\left(b\cos\left(fx+e\right)^4-(a+b)\cos\left(fx+e\right)^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^4 - (a + b)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^
2 + a + b), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(e + f\,x)^2\,\left(b\sin(e + f\,x)^2 + a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.342 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=154

$$-\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{a(a+b)F}{}$$

[Out] -1/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+2/3*(2*a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[e + f*x]^2)^(3/2),x]

[Out] -1/3*(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int\frac{a(3a+b)+2b(2a}{\sqrt{a+b\sin^2}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{1}{3}(a(a+b))\int\frac{}{\sqrt{a+b}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\Big(2(2a+b)\sqrt{a+b\sin^2}}{3\sqrt{}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right.}{3f\sqrt{1+\frac{b\sin}{}}}$$

Mathematica [A]
time = 0.05, size = 156, normalized size = 1.01

$$\frac{4\sqrt{2}\,a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}\,a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+b(-2a-b+b\cos(2(e+fx)))\sin(2(e+fx))}{6\sqrt{2}\,f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 5.86, size = 266, normalized size = 1.73

method	result
default	$-\frac{-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}}{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3-1/3*s
in(f*x+e)*a*b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.10, size = 18, normalized size = 0.12

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^{2}\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2), x)
```

3.343 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=182

$$-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{a(a+b)\sqrt{\cos^2(e+\dots}}{}$$

```
[Out] -(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)
*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+a*(a+b)*EllipticF(si
n(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)
^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)+(a+b)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)
/f
```

Rubi [A]
time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 424, 538, 437, 435, 432, 430}

$$\frac{a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]))
 + (a*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
 + ((a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\frac{(a+bx^2)^{3/2}}{(1-x^2)^{3/2}}dx,x,\sin(e+\right.}{f}$$

$$= \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\text{s}\right.}{}$$

$$= \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} + \frac{\left(a(a+b)\sqrt{\cos^2(e\right.}{}$$

$$= \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\left((a+2b)\sqrt{\cos^2(e\right.\right.}{}$$

$$= -\frac{(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.63, size = 144, normalized size = 0.79

$$\frac{-2a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+(a+b)\left(2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,(2a+b-b\cos(2(e+fx)))\tan(e+fx)\right)}{2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] + (a + b)*(2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
f*x, -(b/a)] + Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x]))/(2*f*S
qrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(170) = 340$.
time = 14.70, size = 466, normalized size = 2.56

method	result

default	$-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b(a+b)\sin(fx+e)(\cos^2(fx+e))+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+b)*sin(f*x+e)*cos(f*x+e)^
2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e)+(co
s(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2+a*b*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-(cos(f*x+e)^2)^(1/2)*(-b/
a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(
f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x
+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+
(a+b)*cos(f*x+e)^2)^(1/2)*a*b)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f
*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

Fricas [F]
time = 0.11, size = 45, normalized size = 0.25

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*
x + e)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^2, x)
```

3.344 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=236

$$-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{a(2a-b)\sqrt{\cos^2(e}{}$$

```
[Out] -2/3*(a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/
2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(2*a-b)*Elli
pticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x
+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(a-b)*(a+b*sin(f*x+e)^2)^(1/2
)*tan(f*x+e)/f+1/3*(a+b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.17, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 424, 541, 538, 437, 435, 432, 430}

$$\frac{a(2a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2(a-b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+b)\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
 + (a*(2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)
]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]
^2]) + (2*(a - b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f) + ((a + b)
*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f)
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1-x^2)^{5/2}}\,dx, x, \sin(e \right.}{f}$$

$$= \frac{(a+b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3f} - \frac{\left(\sqrt{c\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+}{}$$

$$= \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+}{}$$

$$= \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+}{}$$

$$= -\frac{2(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+f}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 1.47, size = 190, normalized size = 0.81

$$\frac{-4a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+2a(2a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\frac{\left(8a^2+3ab+b^2+\left(4a^2-6ab-2b^2\right)\cos(2(e+fx))+b(-a+b)\cos(4(e+fx))\right)\sec^2(e+fx)\tan(e+fx)}{\sqrt{2}}}{6f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-4*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)] + 2*a*(2*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] + ((8*a^2 + 3*a*b + b^2 + (4*a^2 - 6*a*b - 2*b^2)*Cos[2*(e + f*
x)] + b*(-a + b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[2])/(6
*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 12.94, size = 375, normalized size = 1.59

method	result
default	$\frac{2\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}\, b(a-b)\sin(fx+e)\left(\cos^4(fx+e)\right)-\sqrt{-b\left(\cos^4(fx+e)\right)+(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a-b)*sin(f*x+e)*cos(f*
x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2-a*b-3*b^2)*cos(f*x
+e)^2*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)
*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*a-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(sin(f*x+e
)-1)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(1+sin(f*x+e
))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.17, size = 784, normalized size = 3.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*(-I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (2
*I*a^2 - I*a*b - I*b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b
+ b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2
+ a*b)/b^2)*cos(f*x + e)^3 - (-2*I*a^2 + I*a*b + I*b^2)*sqrt(-b)*cos(f*x +
e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))),
```

```
(8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*
a*b - 2*I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (-2*I*a^2 -
I*a*b)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/
b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x
 + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2
+ a*b)/b^2))/b^2) + (2*(-I*a*b + 2*I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*co
s(f*x + e)^3 - (2*I*a^2 + I*a*b)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(a*b - b^2)*cos(f*x + e)^2
 + a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(b*f*cos(f*x +
e)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^4,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^4, x)
```

3.345 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}} dx$

Optimal. Leaf size=79

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f}-\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf}$$

```
[Out] 1/2*(a+2*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/2)/f-
1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 396, 223, 212}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f}-\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*b
^(3/2)*f) - (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*b*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf} + \frac{(a+2b)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin\right.}{2bf}$$

$$= -\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf} + \frac{(a+2b)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin}{\sqrt{a+b}}\right.}{2bf}$$

$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f} - \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx}}{2bf}$$

Mathematica [A]
time = 0.08, size = 79, normalized size = 1.00

$$\frac{-\frac{(-a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}} - \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2b}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-1/2*((-a - 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]
])/b^(3/2) - (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*b))/f
```

Maple [A]
time = 6.12, size = 93, normalized size = 1.18

method	result
default	$\frac{-\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2b}+\frac{a\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}+\frac{\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\right.\right.}}{\sqrt{b}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*sin(f*x+e)/b*(a+b*sin(f*x+e)^2)^(1/2)+1/2*a/b^(3/2)*ln(sin(f*x+e)*b^(
1/2)+(a+b*sin(f*x+e)^2)^(1/2))+ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^(1/
2))/b^(1/2))/f
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.92

$$\frac{\frac{a\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{2\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{\sqrt{b\sin\left(fx+e\right)^2+a}\,\sin(fx+e)}{b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) + 2*arcsinh(b*sin(f*x + e)
/sqrt(a*b))/sqrt(b) - sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)/b)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(67) = 134$.
time = 0.48, size = 461, normalized size = 5.84

$$\left[\frac{(a+2b)\sqrt{b}\log\left(128b^4\cos(fx+e)^8-256(ab^3+2b^4)\cos(fx+e)^6+32(5a^2b^2+24ab^3+24b^4)\cos(fx+e)^4+a^4+32a^3b+160a^2b^2+256ab^3+128b^4-32(a^3b+10a^2b^2+24ab^3+16b^4)\cos(fx+e)^2-8\left(16b^3\cos(fx+e)^6-24(ab^2+2b^3)\cos(fx+e)^4-a^3-10a^2b-24ab^2-16b^3+2(5a^2b+24ab^2+24b^3)\cos(fx+e)^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{b}\sin(fx+e)\right)-8\sqrt{-b\cos(fx+e)^2+a+b}\,b\sin(fx+e)}{16b^2f},-\frac{(a+2b)\sqrt{-b}\arctan\left(\frac{\left(8b^2\cos(fx+e)^4-8(ab+2b^2)\cos(fx+e)^2+a^2+8ab+8b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}}{4\left(2b^3\cos(fx+e)^4+a^2b+3ab^2+2b^3-(3ab^2+4b^3)\cos(fx+e)^2\right)\sin(fx+e)}\right)+4\sqrt{-b\cos(fx+e)^2+a+b}\,b\sin(fx+e)}{8b^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((a + 2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*c
os(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 3
2*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a
*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^
3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*
b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f
*x + e)) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b^2*f), -1/8*
((a + 2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(
f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/
```

```
((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*
x + e)^2)*sin(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e)
)/(b^2*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.65, size = 71, normalized size = 0.90

$$-\frac{\frac{(a+2\,b)\log\left(\left|-\sqrt{b}\,\sin(fx+e)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{b\sin\left(fx+e\right)^2+a}\,\sin(fx+e)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((a + 2*b)*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a))
)/b^(3/2) + sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)/b)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.346 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

```
[Out] arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[b]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[b]*f)
```

Maple [A]
time = 0.12, size = 34, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\right)}{f\sqrt{b}}$	34
default	$\frac{\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\right)}{f\sqrt{b}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/f*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.29, size = 22, normalized size = 0.58

$$\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*sin(f*x + e)/sqrt(a*b))/(sqrt(b)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(32) = 64.
time = 0.46, size = 394, normalized size = 10.37

$$\left[\frac{\log\left(128\,b^4\cos(fx+e)^8-256\,(ab^3+2\,b^4)\cos(fx+e)^6+32\,(5\,a^2b^2+24\,ab^3+24\,b^4)\cos(fx+e)^4+a^4+32\,a^3b+160\,a^2b^2+256\,ab^3+128\,b^4-32\,(a^3b+10\,a^2b^2+24\,ab^3+16\,b^4)\cos(fx+e)^2-8\,(16\,b^3\cos(fx+e)^6-24\,(ab^2+2\,b^3)\cos(fx+e)^4-a^3-10\,a^2b-24\,ab^2-16\,b^3+2\,(5\,a^2b+24\,ab^2+24\,b^3)\cos(fx+e)^2)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{b}\sin(fx+e)\right)}{8\,\sqrt{b}\,f},\ -\frac{\sqrt{-b}\arctan\left(\frac{(8\,b^2\cos(fx+e)^4-8\,(ab+2\,b^2)\cos(fx+e)^2+a^2+8\,ab+8\,b^2)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-b}}{4\,(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-(3\,ab^2+4\,b^3)\cos(fx+e)^2)\sin(fx+e)}\right)}{4\,bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(
5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^
2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f
*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 -
a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*
x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e))/(sqrt(b)*f)
, -1/4*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x
+ e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*
b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x +
e)^2)*sin(f*x + e)))/(b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [A]
time = 0.68, size = 65, normalized size = 1.71

$$-\frac{\frac{a\log\left(\left|-\sqrt{b}\sin(fx+e)+\sqrt{b\sin\left(fx+e\right)^{2}+a}\right|\right)}{\sqrt{b}}-\sqrt{b\sin\left(fx+e\right)^{2}+a}\,\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/sqrt(b
) - sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e))/f
```

Mupad [B]
time = 14.74, size = 33, normalized size = 0.87

$$\frac{\ln\left(\sqrt{b}\,\sin\left(e+f\,x\right)+\sqrt{b\sin\left(e+f\,x\right)^{2}+a}\right)}{\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*sin(e + f*x) + (a + b*sin(e + f*x)^2)^(1/2))/(b^(1/2)*f)
```

3.347 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{a+b}\,f}$$

[Out] arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3269, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]

[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[a + b]*f)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{a+b}\,f}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{a+b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[a + b]
*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(36) = 72.
time = 16.03, size = 105, normalized size = 2.50

method	result
default	$-\frac{-\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)+\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right.\right.}}{1+\sin(fx+e)}\right.}{2\sqrt{a+b}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a+b)^(1/2)*(-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)-b*sin(f*x+e)+a)))/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(38) = 76$.
time = 0.49, size = 111, normalized size = 2.64

$$\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)+1)}\right)}{\sqrt{a+b}}+\frac{\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)-1)}\right)}{\sqrt{a+b}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*(
sin(f*x + e) + 1)))/sqrt(a + b) + arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f
*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/sqrt(a + b))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(36) = 72$.
time = 0.46, size = 240, normalized size = 5.71

$$\left[\frac{\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-8(a^2+3ab+2b^2)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a+b}\,\sin(fx+e)+8a^2+16ab+8b^2}{\cos(fx+e)^4}\right)}{4\sqrt{a+b}\,f},\ -\frac{\sqrt{-a-b}\,\arctan\left(\frac{\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-a-b}}{2\left((ab+b^2)\cos(fx+e)^2-a^2-2ab-b^2\right)\sin(fx+e)}\right)}{2(a+b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*co
s(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e
)^4)/(sqrt(a + b)*f), -1/2*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*c
os(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))/((a + b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.348 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{3/2}f}+\frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2(a+b)f}$$

```
[Out] 1/2*(a+2*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^
(3/2)/f+1/2*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 390, 385, 212}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(2*(a + b)^(3/2)*f) + (Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]
)/(2*(a + b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2(a+b)f} + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+}}\right.}{2(a+}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2(a+b)f} + \frac{(a+2b)\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\right.}{2(a}$$

$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{3/2}f} + \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx}}{2(a+b)f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.87, size = 436, normalized size = 4.79

$$\frac{\sec^3(e+fx)\left(1+\frac{b\sin^2(e+fx)}{a}\right)\tan(e+fx)\left(45a\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)+30b\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)\sin^2(e+fx)+16a\,{}_2F_1\left(2,3;\tfrac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{5/2}+16b\,{}_2F_1\left(2,3;\tfrac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sin^2(e+fx)\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{5/2}-45a\sqrt{-\frac{\sec^2(e+fx)(a^2+b^2\sin^2(e+fx)+ab(1+\sin^2(e+fx)))\tan^2(e+fx)}{a^2}}-30b\sin^2(e+fx)\sqrt{-\frac{\sec^2(e+fx)(a^2+b^2\sin^2(e+fx)+ab(1+\sin^2(e+fx)))\tan^2(e+fx)}{a^2}}\right)}{30af\sqrt{a+b\sin^2(e+fx)}\sqrt{\frac{\sec^2(e+fx)(a+b\sin^2(e+fx))}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)*Tan[e + f*x]*(45*a*ArcSin[Sqrt[-
(((a + b)*Tan[e + f*x]^2)/a)]] + 30*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2
```

```
)/a)]]*Sin[e + f*x]^2 + 16*a*Hypergeometric2F1[2, 3, 7/2, -(((a + b)*Tan[e
+ f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*T
an[e + f*x]^2)/a))^(5/2) + 16*b*Hypergeometric2F1[2, 3, 7/2, -(((a + b)*Tan
[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)
)/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/2) - 45*a*Sqrt[-((Sec[e + f*x]^2*(a
^2 + b^2*Sin[e + f*x]^2 + a*b*(1 + Sin[e + f*x]^2))*Tan[e + f*x]^2)/a^2)] -
 30*b*Sin[e + f*x]^2*Sqrt[-((Sec[e + f*x]^2*(a^2 + b^2*Sin[e + f*x]^2 + a*b
*(1 + Sin[e + f*x]^2))*Tan[e + f*x]^2)/a^2)]))/(30*a*f*Sqrt[a + b*Sin[e + f
*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e +
f*x]^2)/a))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(79) = 158$.
time = 17.96, size = 360, normalized size = 3.96

method	result
default	$\frac{2\sin(fx+e)\sqrt{a+b-b\left(\cos^2(fx+e)\right)}\,(a+b)^{\frac{3}{2}}-\left(\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)-(ln(2/(1+sin(f*x+e
))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+3*ln(2/(1+s
in(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b+2*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
*b^2-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+
e)+a))*a^2-3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*
sin(f*x+e)+a))*a*b-2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)+b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^2)/(a+b)^(5/2)/cos(f*x+e)^2/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(79) = 158$.
time = 0.50, size = 361, normalized size = 3.97

$$\left[\frac{(a+2b)\sqrt{a+b}\cos(fx+e)^2\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-8(a^2+3ab+2b^2)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}\sin(fx+e)+8a^2+16ab+8b^2}{\cos(fx+e)^4}\right)+4\sqrt{-b\cos(fx+e)^2+a+b}(a+b)\sin(fx+e)}{8(a^2+2ab+b^2)f\cos(fx+e)^2}, -\frac{(a+2b)\sqrt{-a-b}\arctan\left(\frac{\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{2\left((ab+b^2)\cos(fx+e)^2-a^2-2ab-b^2\right)\sin(fx+e)}\right)\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}(a+b)\sin(fx+e)}{4(a^2+2ab+b^2)f\cos(fx+e)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + 2*b)*sqrt(a + b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^2)*cos(f
*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x +
 e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e)
 + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e)^2 + a +
 b)*(a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2), -1/4*((a
+ 2*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2
- 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a
 + b)*(a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)^3\,\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.349 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=168

$$-\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf}-\frac{2(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3b^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{(a+b)(2a$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-2/3*(a+2*b)*(cos(f*
x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)
^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(a+b)*(2*a+3*b)*(cos(f*x+e)^
2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)
^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 208, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 427, 538, 437, 435, 432, 430}

$$\frac{(a+b)(2a+3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/3*(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f) - (2*(a +
 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e +
f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) +
 ((a + b)*(2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]],
-(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b^2*f*Sqrt[a + b*Si
n[e + f*x]^2])
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+f\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a+2b)\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a+2b)\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{2(a+2b)\sqrt{\cos^2(e+fx)}\,E}{}$$

Mathematica [A]
time = 0.63, size = 170, normalized size = 1.01

$$\frac{-4\sqrt{2}\,a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}\,(2a^2+5ab+3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+b(-2a-b+b\cos(2(e+fx)))\sin(2(e+fx))}{6\sqrt{2}\,b^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-4*Sqrt[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e
+ f*x, -(b/a)] + 2*Sqrt[2]*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)
])*Sin[2*(e + f*x)])/(6*Sqrt[2]*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 8.47, size = 316, normalized size = 1.88

method	result

default	$\frac{b^2(\sin^5(fx+e))+2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\text{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+5a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b^2*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+3*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2-4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elli
pticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b+a*b*sin(f*x+e)^3-b^2*sin(f*x+e)^3-sin(
f*x+e)*a*b)/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.11, size = 47, normalized size = 0.28

$$\text{integral}\left(-\frac{\sqrt{-b\cos(fx+e)^2+a+b}\,\cos(fx+e)^4}{b\cos(fx+e)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+f\,x)^4}{\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.350 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=114

$$-\frac{aE\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+b)F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -a*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(1+b*
sin(f*x+e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)+(a+b)*(cos(f*x+e)^2)^(1/
2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)
/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3271, 434, 437, 435, 432, 430}

$$\frac{(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + ((a +
 b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{bf} +$$

$$= -\frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-x^2}}\right.}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx}}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.15, size = 83, normalized size = 0.73

$$\frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\left(-aE\left(e+fx\middle|-\frac{b}{a}\right)+(a+b)F\left(e+fx\middle|-\frac{b}{a}\right)\right)}{bf\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*(-(a*EllipticE[e + f*x, -(b/a)]) +
(a + b)*EllipticF[e + f*x, -(b/a)]))/(b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
])
```

Maple [A]
time = 6.19, size = 111, normalized size = 0.97

method	result
default	$\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\left(\text{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a+\text{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b-\text{EllipticE}\left(\right.\right.}{b\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*(a*EllipticF(sin(f*x+e),(
-1/a*b)^(1/2))+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-a*EllipticE(sin(f*x+e
),(-1/a*b)^(1/2)))/b/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.11, size = 47, normalized size = 0.41

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^2}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^2\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.351 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin
(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3262, 3261}

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sin^2(e+fx)}}$$
$$= \frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [**A**]
time = 0.06, size = 60, normalized size = 1.18

$$\frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 60, normalized size = 1.18

method	result	size
default	$\frac{\sqrt{-\frac{b(\cos^2(fx+e))-a-b}{a}}\,\mathrm{am}^{-1}\left(fx+e\middle|\frac{i\sqrt{b}}{\sqrt{a}}\right)}{f\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/(a+b-b*cos(f*x+e)^2)^(1/2)*(-(b*cos(f*x+e)^2-a-b)/a)^(1/2)*InverseJacob
iAM(f*x+e,I/a^(1/2)*b^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.12, size = 305, normalized size = 5.98

$$-\frac{\left(2i\sqrt{-b}\,b\sqrt{\frac{a^2+ab}{b^2}}+(-2i\,a-i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,(\cos(fx+e)+i\,\sin(fx+e))\right)\mid\frac{8\,a^2+8\,ab+b^2-4\,(2\,ab+b^2)\sqrt{\frac{a^2+ab}{b^2}}}{b^2})+\left(-2i\sqrt{-b}\,b\sqrt{\frac{a^2+ab}{b^2}}+(2i\,a+i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,(\cos(fx+e)-i\,\sin(fx+e))\right)\mid\frac{8\,a^2+8\,ab+b^2-4\,(2\,ab+b^2)\sqrt{\frac{a^2+ab}{b^2}}}{b^2})}{b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -((2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2) + (-2*I*a - I*b)*sqrt(-b))*sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2
 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b
 + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b*sqr
t((a^2 + a*b)/b^2) + (2*I*a + I*b)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2
) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^
2)*sqrt((a^2 + a*b)/b^2))/b^2))/(b^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\,\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.352 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=140

$$-\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e}{(a+b)f}$$

```
[Out] -(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*si
n(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(cos(f*x+e)^2)^(1/2)/c
os(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(
a+b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A]
time = 0.11, antiderivative size = 180, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 425, 21, 434, 437, 435, 432, 430}

$$\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) +
 (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]
*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a +
 b*Sin[e + f*x]^2]*Tan[e + f*x])/((a + b)*f)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\Big(\int}{(a+}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} + \frac{\left(b\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\Big(}{(a+b)f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\Big(\int}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} - \frac{\Big(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin}}{(a+b)f}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.44, size = 141, normalized size = 1.01

$$\frac{-2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+2(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,(2a+b-b\cos(2(e+fx)))\tan(e+fx)}{2(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2
*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)]
+ Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*(a + b)*f*Sqrt[2*
a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 13.32, size = 278, normalized size = 1.99

method	result
default	$\frac{\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}\left(a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\ \mathrm{Elliptic}\right.}{(a\text{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))+b*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))-a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))-sin(f*x+e)*cos(f*x+e)^2*b+a*sin(f*x+e)+b*sin(f*x+
e))/(a+b)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/cos(f*x
+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.15, size = 632, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(-2*I*a - I*b)*sqrt(-b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/
b)*cos(f*x + e)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^
2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(2*I*a + I*b)*sqrt(-b)*sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*cos(f*x + e)*elliptic_f(arcsin(sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8
*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*I*sqrt(-b)*b*
sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (2*I*a + I*b)*sqrt(-b)*cos(f*x + e))*s
qrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sq
rt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2
```

```
+ 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b
)*b*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (-2*I*a - I*b)*sqrt(-b)*cos(f*x +
e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8
*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*sqrt(-
b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/((a*b + b^2)*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sec(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)^2\,\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.353 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=212

$$-\frac{2(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{(2a+3b)F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(a+2
$$

```
[Out] -2/3*(a+2*b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(2*a
+3*b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1
+b*sin(f*x+e)^2/a)^(1/2)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(a+2*b)*(a+b*
sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)^2/f+1/3*sec(f*x+e)^2*(a+b*sin(f*x+e)^2
)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A]
time = 0.16, antiderivative size = 252, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 425, 541, 538, 437, 435, 432, 430}

$$\frac{(2a+3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2(a+2b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^2*f*Sqrt[1 + (b*Sin[e +
 f*x]^2)/a]) + ((2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f
*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)*f*Sqr
t[a + b*Sin[e + f*x]^2]) + (2*(a + 2*b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e +
f*x])/(3*(a + b)^2*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e +
f*x])/(3*(a + b)*f)
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx\right.}{f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+}}{3(a+b)f}$$

$$= \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+}}{3(a+b)f}$$

$$= \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+}}{3(a+b)f}$$

$$= -\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]

time = 1.53, size = 205, normalized size = 0.97

$$\frac{-4a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+2(2a^2+5ab+3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\frac{(8a^2+15ab+4b^2+(4a^2+6ab-2b^2)\cos(2(e+fx))-b(a+2b)\cos(4(e+fx)))\sec^2(e+fx)\tan(e+fx)}{\sqrt{2}}}{6(a+b)^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

[Out] (-4*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + ((8*a^2 + 15*a*b + 4*b^2 + (4*a^2 + 6*a*b - 2*b^2)*Cos[2*(e + f*x)] - b*(a + 2*b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[2])/(6*(a + b)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])

Maple [A]
time = 15.49, size = 405, normalized size = 1.91

method	result
default	$\frac{2\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^{2}\left(fx+e\right)\right)}\, b(a+2b)\sin(fx+e)(\cos^{4}(fx+e))-\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)+(}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+2*b)*sin(f*x+e)*cos(f*x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2+5*a*b+3*b^2)*cos(f*x+e)^2*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b+3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-4*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e)^2)/(1+sin(f*x+e))/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(a+b)^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sec(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)

Fricas [C] Result contains complex when optimal does not.
time = 0.17, size = 808, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

```
[Out] 1/3*((2*(-I*a*b - 2*I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 -
(2*I*a^2 + 5*I*a*b + 2*I*b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2)
 + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(
2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*a*b + 2*I*b^2)*sqrt(-b)*sq
rt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (-2*I*a^2 - 5*I*a*b - 2*I*b^2)*sqrt(-b
)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(
arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(
f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b
^2) + (2*(I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (-
2*I*a^2 - 7*I*a*b - 3*I*b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 +
 a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2)
+ 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2
*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-I*a*b - I*b^2)*sqrt(-b)*sqrt
((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (2*I*a^2 + 7*I*a*b + 3*I*b^2)*sqrt(-b)*c
os(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arc
sin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x
 + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)
 + (2*(a*b + 2*b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sin(f*x + e))/((a^2*b + 2*a*b^2 + b^3)*f*cos(f*x + e)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(e+f\,x)^4\,\sqrt{b\,\sin(e+f\,x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.354 $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f}+\frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/2)/f+(a+b)*sin(f
*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3269, 393, 223, 212}

$$\frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(b^(3/2)*f)) +
 ((a + b)*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{bf}\\ &= \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{bf}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f} + \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 88, normalized size = 1.17

$$\frac{\sqrt{b}\,(a+b)\sin(e+fx) - a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{ab^{3/2}f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(a + b)*Sin[e + f*x] - a^(3/2)*ArcSinh[(Sqrt[b]*Sin[e + f*x])/Sqrt
[a]]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(a*b^(3/2)*f*Sqrt[a + b*Sin[e + f*x]^2
])
```

Maple [A]
time = 7.85, size = 85, normalized size = 1.13

method	result	si
default	$\frac{\frac{\sin(fx+e)}{b\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{b^{\frac{3}{2}}}+\frac{\sin(fx+e)}{a\sqrt{a+b\left(\sin^2(fx+e)\right)}}}{f}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (sin(f*x+e)/b/(a+b*sin(f*x+e)^2)^(1/2)-1/b^(3/2)*ln(sin(f*x+e)*b^(1/2)+(a+b
*sin(f*x+e)^2)^(1/2))+sin(f*x+e)/a/(a+b*sin(f*x+e)^2)^(1/2))/f
```

Maxima [A]
time = 0.28, size = 79, normalized size = 1.05

$$-\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}-\frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,a}-\frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) - sin(f*x + e)/(sqrt(b*sin(f*x
+ e)^2 + a)*a) - sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*b))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(67) = 134.
time = 0.53, size = 559, normalized size = 7.45

$$\left[\frac{\left(ab\cos(fx+e)^2-a^2-ab\right)\sqrt{b}\log\left(128\,b^4\cos(fx+e)^8-256\left(ab^3+2\,b^4\right)\cos(fx+e)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos(fx+e)^4+a^4+32\,a^3b+160\,a^2b^2+256\,ab^3+128\,b^4-32\left(a^3b+10\,a^2b^2+24\,ab^3+16\,b^4\right)\cos(fx+e)^2+8\left(16\,b^3\cos(fx+e)^6-24\left(ab^2+2\,b^3\right)\cos(fx+e)^4-a^3-10\,a^2b-24\,ab^2-16\,b^3+2\left(5\,a^2b+24\,ab^2+24\,b^3\right)\cos(fx+e)^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{b}\sin(fx+e)\right)-8\sqrt{-b\cos(fx+e)^2+a+b}\left(ab+b^2\right)\sin(fx+e)}{8\left(ab^3f\cos(fx+e)^2-\left(a^2b^2+ab^3\right)f\right)},\ \frac{\left(ab\cos(fx+e)^2-a^2-ab\right)\sqrt{-b}\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\left(ab+2\,b^2\right)\cos(fx+e)^2+a^2+8\,ab+8\,b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-b}}{4\left(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-\left(3\,ab^2+4\,b^3\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4\sqrt{-b\cos(fx+e)^2+a+b}\left(ab+b^2\right)\sin(fx+e)}{4\left(ab^3f\cos(fx+e)^2-\left(a^2b^2+ab^3\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b*cos(f*x + e)^2 - a^2 - a*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 -
 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*co
s(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3
*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e
)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^
3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(b)*sin(f*x + e)) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*(a*b + b
```

```
^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b^2 + a*b^3)*f), 1/4*((a*b
*cos(f*x + e)^2 - a^2 - a*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*
(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b
^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*sqrt(-b*cos(f*x + e)^2 + a +
 b)*(a*b + b^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b^2 + a*b^3)*f
)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 0.69, size = 71, normalized size = 0.95

$$\frac{\frac{\log\left(\left|-\sqrt{b}\sin(fx+e)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{(a+b)\sin(fx+e)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,ab}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/b^(3/2) + (a
+ b)*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*b))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^3}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.355 $\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3269, 197}

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] Sin[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
[Out] Sin[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [A]
time = 0.14, size = 28, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\sin(fx+e)}{af\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}$	28
default	$\frac{\sin(fx+e)}{af\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 29, normalized size = 1.00

$$\frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*f)
```

Fricas [A]
time = 0.43, size = 49, normalized size = 1.69

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sin\left(fx+e\right)}{abf\cos\left(fx+e\right)^2-(a^2+ab)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(a*b*f*cos(f*x + e)^2 - (a^2
+ a*b)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (e + fx)}{(a + b \sin^2 (e + fx))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [A]
time = 0.66, size = 29, normalized size = 1.00

$$\frac{\sin (fx + e)}{\sqrt{b \sin (fx + e)^2 + a} \, af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*f)
```

Mupad [B]
time = 15.61, size = 117, normalized size = 4.03

$$\frac{\sqrt{2} \, \sqrt{2 \, a + b - b \, \cos (2 \, e + 2 \, f \, x)} \, (4 \, a \, \sin (e + f \, x) + 3 \, b \, \sin (e + f \, x) - b \, \sin (3 \, e + 3 \, f \, x))}{a \, f \, (8 \, a \, b + 8 \, a^2 + 3 \, b^2 - 4 \, b^2 \, \cos (2 \, e + 2 \, f \, x) + b^2 \, \cos (4 \, e + 4 \, f \, x) - 8 \, a \, b \, \cos (2 \, e + 2 \, f \, x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (2^(1/2)*(2*a + b - b*cos(2*e + 2*f*x))^(1/2)*(4*a*sin(e + f*x) + 3*b*sin(e
 + f*x) - b*sin(3*e + 3*f*x)))/(a*f*(8*a*b + 8*a^2 + 3*b^2 - 4*b^2*cos(2*e
+ 2*f*x) + b^2*cos(4*e + 4*f*x) - 8*a*b*cos(2*e + 2*f*x)))
```

3.356 $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{3/2}f}+\frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^(3/2)/f+b*si
n(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 390, 385, 212}

$$\frac{b\sin(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/((a + b)^(3/
2)*f) + (b*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{(a+b)f}\\
&= \frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{3/2}f} + \frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.07, size = 480, normalized size = 6.15

$$
\frac{\sec(e+fx)\tan(e+fx)\left(-45\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)-\frac{30b\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)\sin^2(e+fx)}{a}-\frac{45(a+b)\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)\tan^2(e+fx)}{a}-\frac{30b(a+b)\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)\sin^2(e+fx)\tan^2(e+fx)}{a^2}+4\,{}_2F_1\left(2,2;\frac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sqrt{\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{5/2}+\frac{4b\,{}_2F_1\left(2,2;\frac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sin^2(e+fx)\sqrt{\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{5/2}}{a}+45\sqrt{-\frac{(a+b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\tan^2(e+fx)}{a^2}}+\frac{30b\sin^2(e+fx)\sqrt{-\frac{(a+b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\tan^2(e+fx)}{a^2}}}{a}\right)}{15af\sqrt{a+b\sin^2(e+fx)}\sqrt{\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)}{a}}\left(-\frac{(a+b)\tan^2(e+fx)}{a}\right)^{3/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Tan[e + f*x]*(-45*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]
 - (30*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2)/a - (4
```

```
5*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^2)/a - (
30*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2*Tan
[e + f*x]^2)/a^2 + 4*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Tan[e + f*x]^2
)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f
*x]^2)/a))^(5/2) + (4*b*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Tan[e + f*x
]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(
((a + b)*Tan[e + f*x]^2)/a))^(5/2))/a + 45*Sqrt[-(((a + b)*Sec[e + f*x]^2*(
a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] + (30*b*Sin[e + f*x]^2*Sqrt[-((
(a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)])/a))/(1
5*a*f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2
))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(70) = 140$.
time = 28.67, size = 397, normalized size = 5.09

method	result
default	$\frac{2\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{a\,b^2+b^3}{b^2}}\,\sqrt{a+b}\,b\sin(fx+e)+ab\left(-\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b}{\sin(fx+e)-1}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^(1/2)/a/(-a*b*cos(f*x+e)^2-b^2*cos(f*x+e)^2+a^2+2*a*b+b^2)*(2*(-b
*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*b*sin(f*x+e)+a*b*(-ln(2/(s
in(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)))*co
s(f*x+e)^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*si
n(f*x+e)+a))*a^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)+b*sin(f*x+e)+a))*a*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)-b*sin(f*x+e)+a))*a^2-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(74) = 148$.
time = 0.53, size = 161, normalized size = 2.06

$$\frac{\frac{2\,b\sin(fx+e)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a^2+\sqrt{b\sin\left(fx+e\right)^2+a}\,ab}+\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)+1)}\right)}{(a+b)^{\frac{3}{2}}}+\frac{\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)-1)}\right)}{(a+b)^{\frac{3}{2}}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2 + sqrt(b*sin(f*x + e)
^2 + a)*a*b) + arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(s
qrt(a*b)*(sin(f*x + e) + 1)))/(a + b)^(3/2) + arcsinh(-b*sin(f*x + e)/(sqrt
(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/(a + b)^(3/2)
)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(70) = 140$.
time = 0.49, size = 453, normalized size = 5.81

$$\left[\frac{\left(ab\cos\left(fx+e\right)^{2}-a^{2}-ab\right)\sqrt{a+b}\,\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-8\,\left(a^{2}+3\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{2}-4\,\left(\left(a+2\,b\right)\cos\left(fx+e\right)^{2}-2\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a+b}\,\sin\left(fx+e\right)+8\,a^{2}+16\,ab+8\,b^{2}}{\cos\left(fx+e\right)^{4}}\right)-4\,\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\left(ab+b^{2}\right)\sin\left(fx+e\right)}{4\,\left(\left(a^{3}b+2\,a^{2}b^{2}+ab^{3}\right)f\cos\left(fx+e\right)^{2}-\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}\right)f\right)},\ -\frac{\left(ab\cos\left(fx+e\right)^{2}-a^{2}-ab\right)\sqrt{-a-b}\,\arctan\left(\frac{\left(\left(a+2\,b\right)\cos\left(fx+e\right)^{2}-2\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-a-b}}{2\,\left(\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}-a^{2}-2\,ab-b^{2}\right)\sin\left(fx+e\right)}\right)+2\,\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\left(ab+b^{2}\right)\sin\left(fx+e\right)}{2\,\left(\left(a^{3}b+2\,a^{2}b^{2}+ab^{3}\right)f\cos\left(fx+e\right)^{2}-\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cos(f*x + e)^2 - a^2 - a*b)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^
2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*c
os(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(
f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) - 4*sqrt(-b*cos(f*x + e)
^2 + a + b)*(a*b + b^2)*sin(f*x + e))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*
x + e)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/2*((a*b*cos(f*x + e)^
2 - a^2 - a*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*
b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^
2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*(
a*b + b^2)*sin(f*x + e))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 - (a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.357 $\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{5/2}f}-\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+f}$$

```
[Out] 1/2*(a+4*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^
(5/2)/f-1/2*(a-2*b)*b*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/2*s
ec(f*x+e)*tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3269, 425, 541, 12, 385, 212}

$$-\frac{b(a-2b)\sin(e+fx)}{2af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{5/2}}+\frac{\tan(e+fx)\sec(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 4*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(2*(a + b)^(5/2)*f) - ((a - 2*b)*b*Sin[e + f*x])/(2*a*(a + b)^2*f*Sqrt[a +
b*Sin[e + f*x]^2]) + (Sec[e + f*x]*Tan[e + f*x])/(2*(a + b)*f*Sqrt[a + b*Si
n[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+2b+2bx^2}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx\right)}{2(a+b)f}$$

$$= -\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Su}}{}$$

$$= -\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a}{}$$

$$= -\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a}{}$$

$$= \frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+f}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.13, size = 224, normalized size = 1.67

$$-\frac{\sec^5(e+fx)\left(16(a+b)\,{}_3F_2\left(2,2,3;1,\frac{9}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2+16(a+b)\,{}_2F_1\left(2,3;\frac{9}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sin^2(e+fx)\left(4a^2+7ab\sin^2(e+fx)+3b^2\sin^4(e+fx)\right)-7a\cos^2(e+fx)\,{}_2F_1\left(1,2;\frac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\left(15a^2+20ab\sin^2(e+fx)+8b^2\sin^4(e+fx)\right)\right)\tan(e+fx)}{105a^4f\sqrt{a+b\sin^2(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/105*(Sec[e + f*x]^5*(16*(a + b)*HypergeometricPFQ[{2, 2, 3}, {1, 9/2}, -
(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2 + 16*
(a + b)*Hypergeometric2F1[2, 3, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e +
 f*x]^2*(4*a^2 + 7*a*b*Sin[e + f*x]^2 + 3*b^2*Sin[e + f*x]^4) - 7*a*Cos[e +
 f*x]^2*Hypergeometric2F1[1, 2, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*(15*a^2
 + 20*a*b*Sin[e + f*x]^2 + 8*b^2*Sin[e + f*x]^4))*Tan[e + f*x])/(a^4*f*Sqrt
[a + b*Sin[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3216 vs. $2(118) = 236$.
time = 68.90, size = 3217, normalized size = 24.01

method	result	size
default	Expression too large to display	3217

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a+b)^(1/2)/a/b^5/cos(f*x+e)^2/(a^4*b^2*cos(f*x+e)^4+4*a^3*b^3*cos(f*x+
e)^4+6*a^2*b^4*cos(f*x+e)^4+4*a*b^5*cos(f*x+e)^4+b^6*cos(f*x+e)^4-2*a^5*b*c
os(f*x+e)^2-10*a^4*b^2*cos(f*x+e)^2-20*a^3*b^3*cos(f*x+e)^2-20*a^2*b^4*cos(
f*x+e)^2-10*a*b^5*cos(f*x+e)^2-2*b^6*cos(f*x+e)^2+a^6+6*a^5*b+15*a^4*b^2+20
*a^3*b^3+15*a^2*b^4+6*a*b^5+b^6)*(-2*cos(f*x+e)^4*sin(f*x+e)*(a+b-b*cos(f*x
+e)^2)^(1/2)*(a+b)^(1/2)*a*b^7*(a*b*cos(f*x+e)^2+b^2*cos(f*x+e)^2+a^2+2*a*b
+b^2)+2*sin(f*x+e)*cos(f*x+e)^6*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a*b^
8+a*(8*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/
b^(3/2))*(a+b)^(1/2)*b^(19/2)-8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*si
n(f*x+e))/b^(1/2))*(a+b)^(1/2)*b^(19/2)+24*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)
/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*(a+b)^(1/2)*b^(17/2)*a-24*ln((
(a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*(a+b)^(1/2)*b^(17
/2)*a+24*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2
)/b^(3/2))*(a+b)^(1/2)*b^(15/2)*a^2-24*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/
2)+b*sin(f*x+e))/b^(1/2))*(a+b)^(1/2)*b^(15/2)*a^2+8*ln(((-b*cos(f*x+e)^2+(
a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*(a+b)^(1/2)*b^(13/2)
*a^3-8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*(a+b)^
(1/2)*b^(13/2)*a^3+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))*a^5*b^5+8*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^6+22*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^7+28*ln(2/(sin(f*x+e)-1
)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^8+17*ln(2/
(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b
^9+4*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+
e)+a))*b^10-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*s
in(f*x+e)+a))*a^5*b^5-8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^6-22*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^7-28*ln(2/(1+sin(f*x+e))*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^8-17*ln(2/(1+sin(
f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^9-4*ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
b^10)*cos(f*x+e)^2+2*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2)*a*b^
5*(a^3+3*a^2*b+3*a*b^2+b^3)-2*cos(f*x+e)^2*sin(f*x+e)*(a+b)^(1/2)*b^6*(2*(a
+b-b*cos(f*x+e)^2)^(3/2)*a^3+4*(a+b-b*cos(f*x+e)^2)^(3/2)*a^2*b+2*(a+b-b*co
```

```
s(f*x+e)^2)^(3/2)*a*b^2-2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a^2*b-4*(
-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a*b^2-2*(-b*cos(f*x+e)^2+(a*b^2+b^3)
/b^2)^(3/2)*b^3-(a+b-b*cos(f*x+e)^2)^(1/2)*a^4-3*(a+b-b*cos(f*x+e)^2)^(1/2)
*a^3*b-3*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2*b^2-(a+b-b*cos(f*x+e)^2)^(1/2)*a*b^
3)-2*a*(8*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^
2)/b^(3/2))*(a+b)^(1/2)*b^(19/2)-8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b
*sin(f*x+e))/b^(1/2))*(a+b)^(1/2)*b^(19/2)+16*ln(((-b*cos(f*x+e)^2+(a*b^2+b
^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*(a+b)^(1/2)*b^(17/2)*a-16*l
n(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*(a+b)^(1/2)*b^
(17/2)*a+8*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b
^2)/b^(3/2))*(a+b)^(1/2)*b^(15/2)*a^2-8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1
/2)+b*sin(f*x+e))/b^(1/2))*(a+b)^(1/2)*b^(15/2)*a^2+ln(2/(sin(f*x+e)-1)*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^6+7*ln(2/(sin(f
*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^7+1
5*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+
a))*a^2*b^8+13*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+
b*sin(f*x+e)+a))*a*b^9+4*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)+b*sin(f*x+e)+a))*b^10-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^6-7*ln(2/(1+sin(f*x+e))*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^7-15*ln(2/(1+sin(f*x+e)
)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^8-13*ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b
^9-4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+
e)+a))*b^10)*cos(f*x+e)^4+a*(8*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*
b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*(a+b)^(1/2)*b^(19/2)-8*ln(((a+b-b*cos(f*x+
e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*(a+b)^(1/2)*b^(19/2)+8*ln(((-b*c
os(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*(a+b)^(
1/2)*b^(17/2)*a-8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1
/2))*(a+b)^(1/2)*b^(17/2)*a+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^7+6*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a
+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^8+9*ln(2/(sin(f*x+e)-1)*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^9+4*ln(2/(sin(f*x
+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(118) = 236$.

time = 0.73, size = 625, normalized size = 4.66

$$\left[\frac{\left(\left(a^{2} b+4 a b^{2}\right) \cos \left(f x+e\right)^{4}-\left(a^{3}+5 a^{2} b+4 a b^{2}\right) \cos \left(f x+e\right)^{2}\right) \sqrt{a+b} \log \left(\frac{\left(a^{2}+8 a b+8 b^{2}\right) \cos\left(f x+e\right)^{4}-8 \left(a^{2}+3 a b+2 b^{2}\right) \cos\left(f x+e\right)^{2}-4 \left(\left(a+2 b\right) \cos\left(f x+e\right)^{2}-2 a-2 b\right) \sqrt{-b \cos \left(f x+e\right)^{2}+a+b} \sqrt{a+b} \sin\left(f x+e\right)+8 a^{2}+16 a b+8 b^{2}}{\cos\left(f x+e\right)^{4}}\right)-4 \left(a^{3}+2 a^{2} b+a b^{2}-\left(a^{2} b-a b^{2}-2 b^{3}\right) \cos \left(f x+e\right)^{2}\right) \sqrt{-b \cos \left(f x+e\right)^{2}+a+b} \sin \left(f x+e\right)}{8 \left(\left(a^{4} b+3 a^{3} b^{2}+3 a^{2} b^{3}+a b^{4}\right) f \cos \left(f x+e\right)^{4}-\left(a^{5}+4 a^{4} b+6 a^{3} b^{2}+4 a^{2} b^{3}+a b^{4}\right) f \cos \left(f x+e\right)^{2}\right)}, -\frac{\left(\left(a^{2} b+4 a b^{2}\right) \cos \left(f x+e\right)^{4}-\left(a^{3}+5 a^{2} b+4 a b^{2}\right) \cos \left(f x+e\right)^{2}\right) \sqrt{-a-b} \arctan \left(\frac{\left(\left(a+2 b\right) \cos\left(f x+e\right)^{2}-2 a-2 b\right) \sqrt{-b \cos \left(f x+e\right)^{2}+a+b} \sqrt{-a-b}}{2 \left(\left(a b+b^{2}\right) \cos\left(f x+e\right)^{2}-a^{2}-2 a b-b^{2}\right) \sin\left(f x+e\right)}\right)+2 \left(a^{3}+2 a^{2} b+a b^{2}-\left(a^{2} b-a b^{2}-2 b^{3}\right) \cos \left(f x+e\right)^{2}\right) \sqrt{-b \cos \left(f x+e\right)^{2}+a+b} \sin \left(f x+e\right)}{4 \left(\left(a^{4} b+3 a^{3} b^{2}+3 a^{2} b^{3}+a b^{4}\right) f \cos \left(f x+e\right)^{4}-\left(a^{5}+4 a^{4} b+6 a^{3} b^{2}+4 a^{2} b^{3}+a b^{4}\right) f \cos \left(f x+e\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((a^2*b + 4*a*b^2)*cos(f*x + e)^4 - (a^3 + 5*a^2*b + 4*a*b^2)*cos(f*x
 + e)^2)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3
*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*
b^2)/cos(f*x + e)^4) - 4*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - a*b^2 - 2*b^3)*c
os(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^4*b + 3*a^
3*b^2 + 3*a^2*b^3 + a*b^4)*f*cos(f*x + e)^4 - (a^5 + 4*a^4*b + 6*a^3*b^2 +
4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2), -1/4*(((a^2*b + 4*a*b^2)*cos(f*x + e)
^4 - (a^3 + 5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((a
+ 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a
- b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) + 2*(
a^3 + 2*a^2*b + a*b^2 - (a^2*b - a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos
(f*x + e)^2 + a + b)*sin(f*x + e))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*f*cos(f*x + e)^4 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f
*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(e + f x\right)}{\left(a + b \sin^{2}\left(e + f x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^3\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.358 $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$\frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}+\frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3ab^2f}+\frac{(8a^2+13ab+3b^2)}{}$$

```
[Out] (a+b)*cos(f*x+e)^3*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(4*a+3*b)*
cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/b^2/f+1/3*(8*a^2+13*a*b+3*
b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+
b*sin(f*x+e)^2)^(1/2)/a/b^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(a+b)*(8*a+9*b
)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*s
in(f*x+e)^2/a)^(1/2)/b^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 424, 542, 538, 437, 435, 432, 430}

$$\frac{(8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3ab^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(a+b)(8a+9b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^3f\sqrt{a+b\sin^2(e+fx)}}+\frac{(4a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3ab^2f}+\frac{(a+b)\sin(e+fx)\cos^3(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((4*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*b^2
*f) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*b^3*f*Sqrt[
1 + (b*Sin[e + f*x]^2)/a]) - ((a + b)*(8*a + 9*b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2
)/a])/(3*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
```

```
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \cdots\right.}{a\cdots}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\cdots}}{3ab^2f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\cdots}}{3ab^2f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\cdots}}{3ab^2f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\cdots}}{3ab^2f}$$

Mathematica [A]
time = 0.83, size = 184, normalized size = 0.67

$$\frac{4a(8a^2+13ab+3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-4a(8a^2+17ab+9b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b(8a^2+13ab+6b^2-ab\cos(2(e+fx)))\sin(2(e+fx))}{12ab^3f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*a*(8*a^2 + 13*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellipt
icE[e + f*x, -(b/a)] - 4*a*(8*a^2 + 17*a*b + 9*b^2)*Sqrt[(2*a + b - b*Cos[2
*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(8*a^2 + 13*a*b + 6*
b^2 - a*b*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(12*a*b^3*f*Sqrt[2*a + b - b*
Cos[2*(e + f*x)]])
```

Maple [A]
time = 7.96, size = 415, normalized size = 1.51

method	result
default	$-\frac{a\,b^2 \sin(fx+e)(\cos^4(fx+e))+(-4a^2b-7a\,b^2-3b^3)(\cos^2(fx+e))\sin(fx+e)+8\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\cos^2(fx+e))}{a}+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a*b^2*sin(f*x+e)*cos(f*x+e)^4+(-4*a^2*b-7*a*b^2-3*b^3)*cos(f*x+e)^2*s
in(f*x+e)+8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*a^3+17*a^2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+
e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+9*a*(cos(f*x+e)^
2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*b^2-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE
(sin(f*x+e),(-1/a*b)^(1/2))*a^3-13*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+
(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-3*(cos(f*x+e)^2)^
(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*a*b^2)/a/b^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.18, size = 69, normalized size = 0.25

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cos\left(fx+e\right)^{6}}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^6/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^6}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.359 $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$\frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin}}{ab^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] (a+b)*cos(f*x+e)*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)+(2*a+b)*Elliptic
E(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^
2)^(1/2)/a/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-2*(a+b)*EllipticF(sin(f*x+e),(-
b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^2/
f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 424, 538, 437, 435, 432, 430}

$$-\frac{2(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{ab^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(a+b)\sin(e+fx)\cos(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((
2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]
) - (2*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b^2*f*Sqrt[a + b*Sin[e + f*x
]^2])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\Big(\int}{ab}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{S}}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{\Big((2a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{}}{ab}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(2a+b)\sqrt{\cos^2(e+fx)}\,E\big(\sin^{-1}(\sin(e+}{ab^2f\sqrt{1+}}$$

Mathematica [A]
time = 0.41, size = 139, normalized size = 0.69

$$\frac{2a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-(a+b)\left(4a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b\sin(2(e+fx))\right)}{2ab^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] - (a + b)*(4*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] - Sqrt[2]*b*Sin[2*(e + f*x)]))/(2*a*b^2*f*Sqrt[2*a + b - b*Cos[
2*(e + f*x)]])
```

Maple [A]
time = 8.31, size = 269, normalized size = 1.33

method	result

default	$\frac{(ab+b^2)\sin(fx+e)(\cos^2(fx+e))+2\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,a^2+\mathrm{Ellip}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a*b+b^2)*sin(f*x+e)*cos(f*x+e)^2+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(
cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a^2+EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*
a*b-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(
f*x+e),(-1/a*b)^(1/2))*a^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b)/a/b^2/cos(f*x+e)/(a+b*si
n(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.11, size = 69, normalized size = 0.34

$$\text{integral}\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\,\cos(fx+e)^4}{b^2\cos(fx+e)^4-2(ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^4}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.360 $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=188

$$\frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{abf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{\sqrt{\mathrm{c}}}{}$$

```
[Out] cos(f*x+e)*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)+EllipticE(sin(f*x+e),(-b
/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/b/f/(
1+b*sin(f*x+e)^2/a)^(1/2)-EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(co
s(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3271, 423, 507, 437, 435, 432, 430}

$$-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{abf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\sin(e+fx)\cos(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x])/(a*f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[Cos[e
+ f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*S
in[e + f*x]^2])/(a*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (Sqrt[Cos[e + f*x]
^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}}\right.}{af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}}\right.}{bf}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx}\right.}{abf\sqrt{1+\frac{b\sin}{}}}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e-}{abf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.22, size = 133, normalized size = 0.71

$$\frac{\sqrt{2}\,a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+b\sin(2(e+fx))}{\sqrt{2}\,abf\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)
] - Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b
/a)] + b*Sin[2*(e + f*x)])/(Sqrt[2]*a*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
])
```

Maple [A]
time = 5.82, size = 145, normalized size = 0.77

method	result

default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a-\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e)}{a}}}{ab\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a-(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))+b*sin(f*x+e)^3-b*sin(f*x+e))/a/b/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)

Fricas [C] Result contains complex when optimal does not.
time = 0.16, size = 777, normalized size = 4.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] -1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^2*cos(f*x + e)*sin(f*x + e) + 4*(I*b^2*cos(f*x + e)^2 - I*a*b - I*b^2)*sqrt(-b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((a^2 + a*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 4*(-I*b^2*cos(f*x + e)^2 + I*a*b + I*b^2)*sqrt(-b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((a^2 + a*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(I*b^2*cos(f*x + e)^2 - I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a*b - I*b^2)*cos(f*x + e)^2 + 2*I*a^2 + 3*I*a*b + I*b^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(-I*b^2*cos(f*x + e)^2 + I*a*b + I*b^2)*sq

```
rt(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a*b + I*b^2)*cos(f*x + e)^2 - 2*I*a^2
- 3*I*a*b - I*b^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*
elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x +
e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a
*b)/b^2))/b^2))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b^2 + a*b^3)*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.361 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(cos(f*x+e)^2)^(
1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)
/a/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3263, 21, 3257, 3256}

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
```

```
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a-b\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)} \\
&= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{a(a+b)} \\
&= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+f}{a}}}{a(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\
&= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 90, normalized size = 0.89

$$\frac{2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b\sin(2(e+fx))}{2a(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
[Out] (2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sq
rt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)
```

Maple [A]
time = 8.12, size = 103, normalized size = 1.02

method	result	size
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+\sin(fx+e)\left(\cos^2(fx+e)\right)b}{a(a+b)\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))+sin(f*x+e)*cos(f*x+e)^2*b)/a/(a+b)/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.17, size = 938, normalized size = 9.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^3*cos(f*x + e)*sin(f*x + e) - (2*
(I*b^3*cos(f*x + e)^2 - I*a*b^2 - I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (
2*I*a^2*b + 3*I*a*b^2 + I*b^3 + (-2*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqrt(-
b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8
*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(-I*b
^3*cos(f*x + e)^2 + I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-2*I
*a^2*b - 3*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b))*
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2
 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(2*(-I*a^2
*b - 2*I*a*b^2 - I*b^3 + (I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a
```

```
^2 + a*b)/b^2) + (2*I*a^3 + 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b - I*a*b^2)*co
s(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellip
tic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) +
I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b
^2))/b^2) + 2*(2*(I*a^2*b + 2*I*a*b^2 + I*b^3 + (-I*a*b^2 - I*b^3)*cos(f*x
+ e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + (-2*I*a^3 - 3*I*a^2*b - I*a*b^2 +
(2*I*a^2*b + I*a*b^2)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
+ b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
 b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/((a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 - (a
^3*b^2 + 2*a^2*b^3 + a*b^4)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sin(e + f*x)**2)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(1/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.362 $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin}}{a(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] -(a-b)*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-(a-b)*E
llipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(
f*x+e)^2)^(1/2)/a/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+EllipticF(sin(f*x+e)
,(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/(
a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 425, 541, 538, 437, 435, 432, 430}

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{af(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{b(a-b)\sin(e+fx)\cos(e+fx)}{af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2])) - ((a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)^2*f*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]],
-(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/((a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2]) + Tan[e + f*x]/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
```

```
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}(a+bx^2)^{3/2}}\,dx,x,\sin(e+fx)\right.}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \right.}{(a+b)}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{c\right.}{}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{((a\,-}{}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{((a}{}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e\right.}{a(a+b)^2f\sqrt{}}$$

Mathematica [A]
time = 0.87, size = 167, normalized size = 0.70

$$\frac{-\sqrt{2}\,a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\left(2a^2+ab+b^2+b(-a+b)\cos(2(e+fx))\right)\tan(e+fx)}{\sqrt{2}\,a(a+b)^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f
*x, -(b/a)]) + Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ell
ipticF[e + f*x, -(b/a)] + (2*a^2 + a*b + b^2 + b*(-a + b)*Cos[2*(e + f*x)])
*Tan[e + f*x])/(Sqrt[2]*a*(a + b)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(226) = 452$.
time = 14.68, size = 468, normalized size = 1.95

method	result
default	$-\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^{2}\left(fx+e\right)\right)}\, b(a-b)\sin(fx+e)\left(\cos^{2}(fx+e)\right)+\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)+(\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a-b)*sin(f*x+e)*cos(f*x+e)^
2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*sin(f*x+e)+(-b*cos(f*x
+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+
b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+(-b*cos(f*x+e)^4+(a+b)
*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)
*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)
^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(
sin(f*x+e),(-1/a*b)^(1/2))*a^2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(
cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),
(-1/a*b)^(1/2))*a*b)/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x
+e)))^(1/2)/a/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.18, size = 1079, normalized size = 4.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((2*((-I*a*b^2 + I*b^3)*cos(f*x + e)^3 + (I*a^2*b - I*b^3)*cos(f*x + e)
)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a^2*b - I*a*b^2 - I*b^3)*cos(f*x +
 e)^3 + (-2*I*a^3 - I*a^2*b + 2*I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b))*sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqr
t((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 +
 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^2 -
 I*b^3)*cos(f*x + e)^3 + (-I*a^2*b + I*b^3)*cos(f*x + e))*sqrt(-b)*sqrt((a^
2 + a*b)/b^2) - ((-2*I*a^2*b + I*a*b^2 + I*b^3)*cos(f*x + e)^3 + (2*I*a^3 +
 I*a^2*b - 2*I*a*b^2 - I*b^3)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 +
 a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2)
+ 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2
*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 4*(((I*a*b^2 + I*b^3)*cos(f*x + e
)^3 + (-I*a^2*b - 2*I*a*b^2 - I*b^3)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b
)/b^2) + ((-2*I*a^2*b - I*a*b^2)*cos(f*x + e)^3 + (2*I*a^3 + 3*I*a^2*b + I*
a*b^2)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b
)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x
+ e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 +
 a*b)/b^2))/b^2) - 4*(((-I*a*b^2 - I*b^3)*cos(f*x + e)^3 + (I*a^2*b + 2*I*a
*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((2*I*a^2*b +
I*a*b^2)*cos(f*x + e)^3 + (-2*I*a^3 - 3*I*a^2*b - I*a*b^2)*cos(f*x + e))*sq
rt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqr
t((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e)))
, (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(a
^2*b + a*b^2 - (a*b^2 - b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b
)*sin(f*x + e))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^3 - (a^4*b +
3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.363 $\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=130

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f}+\frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(5/2)/f+1/3*(a+b)*co
s(f*x+e)^2*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)-1/3*(3*a-2*b)*(a+b)*si
n(f*x+e)/a^2/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 424, 393, 223, 212}

$$-\frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f}+\frac{(a+b)\sin(e+fx)\cos^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(b^(5/2)*f) + ((
a + b)*Cos[e + f*x]^2*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2))
- ((3*a - 2*b)*(a + b)*Sin[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sin[e + f*x]^2
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a+2b+3ax^2}{(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3abf}$$

$$= \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\Big(}{}$$

$$= \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\Bigg(}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f} + \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a}{3a^2}$$

Mathematica [**A**]
time = 0.58, size = 128, normalized size = 0.98

$$\frac{\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{\sqrt{-b}} + \frac{2\sqrt{2}(a+b)(-3a^2+ab+b^2+(2a-b)b\cos(2(e+fx)))\sin(e+fx)}{a^2(2a+b-b\cos(2(e+fx)))^{3/2}}}{3b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*ArcTan[(Sqrt[2]*Sqrt[-b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)
]]])/Sqrt[-b] + (2*Sqrt[2]*(a + b)*(-3*a^2 + a*b + b^2 + (2*a - b)*b*Cos[2*
(e + f*x)])*Sin[e + f*x])/(a^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2)))/(3*b^
2*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(116) = 232$.
time = 23.86, size = 383, normalized size = 2.95

method	result
default	$\frac{3\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\right)a^4b^4+6\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\right)a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b^(13/2)/a^2/(b^2*cos(f*x+e)^4-2*a*b*cos(f*x+e)^2-2*b^2*cos(f*x+e)^2+a^
2+2*a*b+b^2)*(3*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*a^4*b^4+6
*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*a^3*b^5+3*ln(sin(f*x+e)*
b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*a^2*b^6+3*ln(sin(f*x+e)*b^(1/2)+(a+b-b*
cos(f*x+e)^2)^(1/2))*a^2*b^6*cos(f*x+e)^4-6*ln(sin(f*x+e)*b^(1/2)+(a+b-b*co
s(f*x+e)^2)^(1/2))*a^2*b^5*(a+b)*cos(f*x+e)^2+2*b^(11/2)*(-b*cos(f*x+e)^2+(
a*b^2+b^3)/b^2)^(1/2)*(2*a^2+a*b-b^2)*sin(f*x+e)*cos(f*x+e)^2-sin(f*x+e)*b^
(9/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(3*a^3+4*a^2*b-a*b^2-2*b^3))/
f
```

Maxima [A]
time = 0.29, size = 222, normalized size = 1.71

$$-\frac{\left(\frac{3\sin(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}b}+\frac{2a}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}b^2}\right)\sin(fx+e)-\frac{3\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{2\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,a^2}-\frac{\sin(fx+e)}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}a}+\frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,b^2}-\frac{2\sin(fx+e)}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}b}+\frac{2\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,ab}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*sin(f*x + e)^2/((b*sin(f*x + e)^2 + a)^(3/2)*b) + 2*a/((b*sin(f*x
+ e)^2 + a)^(3/2)*b^2))*sin(f*x + e) - 3*arcsinh(b*sin(f*x + e)/sqrt(a*b))/
b^(5/2) - 2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) - sin(f*x + e)/((
b*sin(f*x + e)^2 + a)^(3/2)*a) + sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*b
^2) - 2*sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*b) + 2*sin(f*x + e)/(sqr
t(b*sin(f*x + e)^2 + a)*a*b))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(116) = 232$.
time = 1.18, size = 799, normalized size = 6.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b + a^2
*b^2)*cos(f*x + e)^2)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b
```

```
^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^
4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b +
24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*
sin(f*x + e)) - 8*(3*a^3*b + 4*a^2*b^2 - a*b^3 - 2*b^4 - 2*(2*a^2*b^2 + a*b
^3 - b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(a^
2*b^5*f*cos(f*x + e)^4 - 2*(a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^4*b^3
+ 2*a^3*b^4 + a^2*b^5)*f), -1/12*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b
 + a^2*b^2 - 2*(a^3*b + a^2*b^2)*cos(f*x + e)^2)*sqrt(-b)*arctan(1/4*(8*b^2
*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*
b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(3*a^3*b
 + 4*a^2*b^2 - a*b^3 - 2*b^4 - 2*(2*a^2*b^2 + a*b^3 - b^4)*cos(f*x + e)^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(a^2*b^5*f*cos(f*x + e)^4 - 2
*(a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^5}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.364 $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\cos^2(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] 1/3*cos(f*x+e)^2*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*sin(f*x+e)/a^2
/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3269, 386, 197}

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)\cos^2(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x])/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*Sin
[e + f*x])/(3*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+bx^2)^{5/2}}\,dx,x,\sin(e+fx)\right)}{f}\\ &= \frac{\cos^2(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx,x,\sin(e+fx)\right)}{3af}\\ &= \frac{\cos^2(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 51, normalized size = 0.70

$$\frac{3a\sin(e+fx)-(a-2b)\sin^3(e+fx)}{3a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
[Out] (3*a*Sin[e + f*x] - (a - 2*b)*Sin[e + f*x]^3)/(3*a^2*f*(a + b*Sin[e + f*x]^
2)^(3/2))
```

Maple [A]
time = 24.44, size = 120, normalized size = 1.64

method	result	size
default	$\frac{\sin(fx+e)\sqrt{-b\left(\cos^2(fx+e)\right)+\frac{a\,b^2+b^3}{b^2}}\,\left(a(\cos^2(fx+e))-2b(\cos^2(fx+e))+2a+2b\right)}{3a^2(b^2(\cos^4(fx+e))-2ab(\cos^2(fx+e))-2b^2(\cos^2(fx+e))+a^2+2ab+b^2)f}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3/a^2/(b^2*cos(f*x+e)^4-2*a*b*cos(f*x+e)^2-2*b^2*cos(f*x+e)^2+a^2+2*a*b+b
^2)*sin(f*x+e)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a*cos(f*x+e)^2-2*b*
cos(f*x+e)^2+2*a+2*b)/f
```

Maxima [A]
time = 0.28, size = 115, normalized size = 1.58

$$\frac{\frac{2\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,a^2}+\frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a}+\frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b}-\frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,ab}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) + sin(f*x + e)/((b*sin
(f*x + e)^2 + a)^(3/2)*a) + sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*b) -
 sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*b))/f
```

Fricas [A]
time = 0.58, size = 107, normalized size = 1.47

$$\frac{\left((a-2\,b)\cos\left(fx+e\right)^{2}+2\,a+2\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sin\left(fx+e\right)}{3\left(a^{2}b^{2}f\cos\left(fx+e\right)^{4}-2\left(a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - 2*b)*cos(f*x + e)^2 + 2*a + 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*
sin(f*x + e)/(a^2*b^2*f*cos(f*x + e)^4 - 2*(a^3*b + a^2*b^2)*f*cos(f*x + e)
^2 + (a^4 + 2*a^3*b + a^2*b^2)*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [A]
time = 0.68, size = 58, normalized size = 0.79

$$-\frac{\left(\frac{\left(ab-2\,b^{2}\right)\sin\left(fx+e\right)^{2}}{a^{2}b}-\frac{3}{a}\right)\sin\left(fx+e\right)}{3\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((a*b - 2*b^2)*sin(f*x + e)^2/(a^2*b) - 3/a)*sin(f*x + e)/((b*sin(f*x
+ e)^2 + a)^(3/2)*f)
```

Mupad [B]
time = 23.81, size = 183, normalized size = 2.51

$$-\frac{2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,\sqrt{a+b\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,\left(a\,1\mathrm{i}-b\,2\mathrm{i}+a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,10\mathrm{i}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}+b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,4\mathrm{i}-b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,2\mathrm{i}\right)}{3\,a^2\,f\,{\left(b-4\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-2\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] -(2*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) - 1)*(a + b*((exp(- e*1i - f*x*1
i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(a*1i - b*2i + a*exp(e*2i +
f*x*2i)*10i + a*exp(e*4i + f*x*4i)*1i + b*exp(e*2i + f*x*2i)*4i - b*exp(e*4
i + f*x*4i)*2i))/(3*a^2*f*(b - 4*a*exp(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*
2i) + b*exp(e*4i + f*x*4i))^2)
```

3.365 $\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] 1/3*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*sin(f*x+e)/a^2/f/(a+b*sin(f
*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 198, 197}

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] Sin[e + f*x]/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*Sin[e + f*x])/(3*a^2
*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\sin(e+fx)}{3af\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3af}$$
$$= \frac{\sin(e+fx)}{3af\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 0.72

$$\frac{\sin(e+fx)\,(3a+2b\sin^2(e+fx))}{3a^2f\,(a+b\sin^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sin[e + f*x]*(3*a + 2*b*Sin[e + f*x]^2))/(3*a^2*f*(a + b*Sin[e + f*x]^2)^(
3/2))
```

Maple [A]
time = 0.16, size = 56, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sin(fx+e)}{3a\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}}{f}$	56
default	$\frac{\frac{\sin(fx+e)}{3a\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}}{f}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*sin(f*x+e)/a/(a+b*sin(f*x+e)^2)^(3/2)+2/3/a^2*sin(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2))
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.91

$$\frac{\frac{2\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,a^2}+\frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] 1/3*(2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) + sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*a))/f

Fricas [A]
time = 0.53, size = 104, normalized size = 1.60

$$-\frac{\left(2\,b\cos\left(fx+e\right)^{2}-3\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sin\left(fx+e\right)}{3\,\left(a^{2}b^{2}f\cos\left(fx+e\right)^{4}-2\,(a^{3}b+a^{2}b^{2})f\cos\left(fx+e\right)^{2}+(a^{4}+2\,a^{3}b+a^{2}b^{2})f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] -1/3*(2*b*cos(f*x + e)^2 - 3*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(a^2*b^2*f*cos(f*x + e)^4 - 2*(a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^4 + 2*a^3*b + a^2*b^2)*f)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [A]
time = 0.75, size = 48, normalized size = 0.74

$$\frac{\left(\frac{2\,b\sin(fx+e)^{2}}{a^{2}}+\frac{3}{a}\right)\sin\left(fx+e\right)}{3\,\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] 1/3*(2*b*sin(f*x + e)^2/a^2 + 3/a)*sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*f)

Mupad [B]
time = 21.84, size = 164, normalized size = 2.52

$$\frac{4\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,\sqrt{a+b\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^{2}}\,\left(b\,1\mathrm{i}-a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,6\mathrm{i}-b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,2\mathrm{i}+b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}\right)}{3\,a^{2}\,f\,{\left(b-4\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-2\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] (4*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) - 1)*(a + b*((exp(- e*1i - f*x*1i
)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(b*1i - a*exp(e*2i + f*x*2i)*
6i - b*exp(e*2i + f*x*2i)*2i + b*exp(e*4i + f*x*4i)*1i))/(3*a^2*f*(b - 4*a*
exp(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*2i) + b*exp(e*4i + f*x*4i))^2)
```

3.366 $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{5/2}f}+\frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^(5/2)/f+1/3*
b*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*b*(5*a+2*b)*sin(f*x+e)/
a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3269, 425, 541, 12, 385, 212}

$$\frac{b(5a+2b)\sin(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b\sin(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/((a + b)^(5/
2)*f) + (b*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (b*
(5*a + 2*b)*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{b-3(a+b)+2bx^2}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \sin(e+f\right.}{3a(a+b)f}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Su}}{}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Su}}{}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Su}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{5/2}f} + \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{3a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.44, size = 1291, normalized size = 10.25

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sec[e + f*x]*Tan[e + f*x]*(1575*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]
] + (2100*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2)/a +
 (840*b^2*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^4)/a^2 +
 (3150*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^2)/
a + (4200*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x
]^2*Tan[e + f*x]^2)/a^2 + (1680*b^2*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e +
f*x]^2)/a)]]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (1575*(a + b)^2*ArcSin[Sq
rt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^4)/a^2 + (2100*b*(a + b)^2*
```

```
ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/
a^3 + (840*b^2*(a + b)^2*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e
+ f*x]^4*Tan[e + f*x]^4)/a^4 + 2100*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) + (2800*b*Sin[e + f*x]^2*Sqr
t[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a)
)^(3/2))/a + (1120*b^2*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f
*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))/a^2 + 96*Hypergeometric2F
1[2, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin
[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 24*HypergeometricP
FQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2
*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + (168*b*
Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*
Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)
/a))^(7/2))/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan
[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)
)/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2))/a + (72*b^2*Hypergeometric2F1[2
, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^
2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2))/a^2 + (
24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan[e + f*x]^2)/a)
]*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b
)*Tan[e + f*x]^2)/a))^(7/2))/a^2 - 1575*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a +
 b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] - (2100*b*Sin[e + f*x]^2*Sqrt[-(((
a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)])/a - (84
0*b^2*Sin[e + f*x]^4*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*
Tan[e + f*x]^2)/a^2)])/a^2))/(315*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Se
c[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(1 + (b*Sin[e + f*x]^2)/a)*(-(((a +
 b)*Tan[e + f*x]^2)/a))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(112) = 224$.
time = 36.23, size = 899, normalized size = 7.13

method	result
default	$\frac{3a^4b^2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)-3a^4b^2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(f\right.\right.}}{1+\sin(fx+e)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b^2/a^2/(a+b)^(1/2)/(a^2*b^2*cos(f*x+e)^4+2*a*b^3*cos(f*x+e)^4+b^4*cos(
f*x+e)^4-2*a^3*b*cos(f*x+e)^2-6*a^2*b^2*cos(f*x+e)^2-6*a*b^3*cos(f*x+e)^2-2
*b^4*cos(f*x+e)^2+a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(3*a^4*b^2*ln(2/(sin(f
*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))-3*a^4*b^2
*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a
```

))+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^4-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^4+6*a^3*b^3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))-6*a^3*b^3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+3*a^2*b^4*(ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)))*cos(f*x+e)^4-2*sin(f*x+e)*cos(f*x+e)^2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*b^4*(5*a+2*b)+4*sin(f*x+e)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*b^3*(3*a^2+4*a*b+b^2)-6*a^2*b^3*(ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b)*cos(f*x+e)^2)/f

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(118) = 236$.
time = 0.52, size = 288, normalized size = 2.29

$$\frac{\frac{2\,b\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a^2+\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}ab}+\frac{6\,b\sin(fx+e)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a^3+2\sqrt{b\sin\left(fx+e\right)^2+a}\,a^2b+\sqrt{b\sin\left(fx+e\right)^2+a}\,ab^2}+\frac{4\,b\sin(fx+e)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a^3+\sqrt{b\sin\left(fx+e\right)^2+a}\,a^2b}+\frac{3\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)+1)}\right)}{(a+b)^{\frac{5}{2}}}+\frac{3\,\mathrm{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)-1)}\right)}{(a+b)^{\frac{5}{2}}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] 1/6*(2*b*sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*a^2 + (b*sin(f*x + e)^2 + a)^(3/2)*a*b) + 6*b*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^3 + 2*sqrt(b*sin(f*x + e)^2 + a)*a^2*b + sqrt(b*sin(f*x + e)^2 + a)*a*b^2) + 4*b*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^3 + sqrt(b*sin(f*x + e)^2 + a)*a^2*b) + 3*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) + 1)))/(a + b)^(5/2) + 3*arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/(a + b)^(5/2))/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(112) = 224$.
time = 0.72, size = 775, normalized size = 6.15

$$\left[\frac{3\left(a^2b^2\cos\left(fx+e\right)^4+a^4+2\,a^3b+a^2b^2-2\left(a^3b+a^2b^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a+b}\,\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-8\left(a^2+3\,ab+2\,b^2\right)\cos(fx+e)^2-4\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}\,\sin(fx+e)+8\,a^2+16\,ab+8\,b^2}{\cos(fx+e)^4}\right)+4\left(6\,a^3b+14\,a^2b^2+10\,ab^3+2\,b^4-\left(5\,a^2b^2+7\,ab^3+2\,b^4\right)\cos\left(fx+e\right)^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sin\left(fx+e\right)}{12\left(\left(a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5\right)f\cos\left(fx+e\right)^4-2\left(a^6b+4\,a^5b^2+6\,a^4b^3+4\,a^3b^4+a^2b^5\right)f\cos\left(fx+e\right)^2+\left(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5\right)f\right)},-\frac{3\left(a^2b^2\cos\left(fx+e\right)^4+a^4+2\,a^3b+a^2b^2-2\left(a^3b+a^2b^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a-b}\,\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{2\left(\left(ab+b^2\right)\cos(fx+e)^2-a^2-2\,ab-b^2\right)\sin(fx+e)}\right)-2\left(6\,a^3b+14\,a^2b^2+10\,ab^3+2\,b^4-\left(5\,a^2b^2+7\,ab^3+2\,b^4\right)\cos\left(fx+e\right)^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sin\left(fx+e\right)}{6\left(\left(a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5\right)f\cos\left(fx+e\right)^4-2\left(a^6b+4\,a^5b^2+6\,a^4b^3+4\,a^3b^4+a^2b^5\right)f\cos\left(fx+e\right)^2+\left(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b + a^2*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2

```
*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2
+ 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*(6*a^3*b + 14*a^2*b^2 + 10*a*b^3 + 2*
b^4 - (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f
*x + e)^4 - 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f*cos(f
*x + e)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)
*f), -1/6*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b +
 a^2*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - 2*(6*a^3*b + 14*a^2*b^2
+ 10*a*b^3 + 2*b^4 - (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x + e)^2)*sqrt(-b*
cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a
^2*b^5)*f*cos(f*x + e)^4 - 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a
^2*b^5)*f*cos(f*x + e)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3
*b^4 + a^2*b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral(sec(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\left(b\sin(e+f\,x)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)),x)
[Out] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

3.367 $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=243

$$\frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{(8a^2+3ab-2b^2)\,E\left(e+fx\middle|-\frac{b}{a}\right)\ldots}{3a^2b^3f\sqrt{1+\frac{b\sin^2\ldots}{\ldots}}}$$

```
[Out] 1/3*(a+b)*cos(f*x+e)^3*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)-2/3*(2*a-b
)*(a+b)*cos(f*x+e)*sin(f*x+e)/a^2/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(8*a^2
+3*a*b-2*b^2)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(
1/2))*(a+b*sin(f*x+e)^2)^(1/2)/a^2/b^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(8*
a-b)*(a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2
))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/b^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 283, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 424, 540, 538, 437, 435, 432, 430}

$$-\frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^2b^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2(2a-b)(a+b)\sin(e+fx)\cos(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(8a-b)(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3ab^3f\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+b)\sin(e+fx)\cos^3(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2)
) - (2*(2*a - b)*(a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b
*Sin[e + f*x]^2]) - ((8*a^2 + 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE
[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*
a^2*b^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((8*a - b)*(a + b)*Sqrt[Cos[e +
 f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*S
in[e + f*x]^2)/a])/(3*a*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
```

```
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \right.}{3}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A]
time = 1.50, size = 194, normalized size = 0.80

$$\frac{-2a^2\left(8a^2+3ab-2b^2\right)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+\frac{1}{2}(a+b)\left(4a^2(8a-b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}\,b\left(8a^2-ab-2b^2+b(-5a+2b)\cos(2(e+fx))\right)\sin(2(e+fx))\right)}{6a^2b^3f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

[Out] (-2*a^2*(8*a^2 + 3*a*b - 2*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x, -(b/a)] + ((a + b)*(4*a^2*(8*a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] - 2*Sqrt[2]*b*(8*a^2 - a*b - 2*b^2 + b*(-5*a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/2)/(6*a^2*b^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(265) = 530$.
time = 10.57, size = 712, normalized size = 2.93

method	result
default	$\frac{(5a^2b^2+3a\,b^3-2b^4)\sin(fx+e)(\cos^4(fx+e))+(-4a^3b-6a^2b^2+2b^4)(\cos^2(fx+e))\sin(fx+e)-\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\sqrt{\cos}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/3*((5*a^2*b^2+3*a*b^3-2*b^4)*sin(f*x+e)*cos(f*x+e)^4+(-4*a^3*b-6*a^2*b^2+2*b^4)*cos(f*x+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+7*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-3*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4+15*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b+6*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^4-11*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3)/a^2/(a+b*sin(f*x+e)^2)^(3/2)/b^3/cos(f*x+e)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F]
time = 0.17, size = 109, normalized size = 0.45

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cos\left(fx+e\right)^{6}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^6/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^{6}}{{\left(b\sin\left(e+f\,x\right)^{2}+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.368 $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e-}{a}}}{3ab^2f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] 1/3*(a+b)*cos(f*x+e)*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)-2/3*(a-b)*co
s(f*x+e)*sin(f*x+e)/a^2/b/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(a-b)*(cos(f*x+e)^
2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)
^(1/2)/a/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(2*a-b)*(cos(f*x+e)^2)^(1/2)/co
s(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/b^
2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 263, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 424, 541, 538, 437, 435, 432, 430}

$$-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^2b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2(a-b)\sin(e+fx)\cos(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3ab^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+b)\sin(e+fx)\cos(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2))
- (2*(a - b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*b*f*Sqrt[a + b*Sin[e + f*x]^
2]) - (2*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b^2*f*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a]) + ((2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*b^2*f*Sqrt[a
 + b*Sin[e + f*x]^2])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
```

```
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \right.}{3ab}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{c\right.}}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{((2a}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{(2(a}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{2(a-}{}$$

Mathematica [A]
time = 0.99, size = 171, normalized size = 0.77

$$\frac{-2a^2(a-b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+a^2(2a-b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b\left(a^2-2ab-b^2+b(-a+b)\cos(2(e+fx))\right)\sin(2(e+fx))}{3a^2b^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

[Out] (-2*a^2*(a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x, -(b/a)] + a^2*(2*a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(a^2 - 2*a*b - b^2 + b*(-a + b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(3*a^2*b^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))

Maple [A]
time = 10.00, size = 485, normalized size = 2.17

method	result
default	$\frac{(2a\,b^2-2b^3)\sin(fx+e)(\cos^4(fx+e))+(-a^2b+a\,b^2+2b^3)(\cos^2(fx+e))\sin(fx+e)-\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos(2fx+2e)}{2}\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/3*((2*a*b^2-2*b^3)*sin(f*x+e)*cos(f*x+e)^4+(-a^2*b+a*b^2+2*b^3)*cos(f*x+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/a^2/(a+b*sin(f*x+e)^2)^(3/2)/b^2/cos(f*x+e)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)

Fricas [C] Result contains complex when optimal does not.
time = 0.23, size = 1343, normalized size = 6.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*((-I*a*b^3 + I*b^4)*cos(f*x + e)^4 - I*a^3*b - I*a^2*b^2 + I*a*b^3
+ I*b^4 - 2*(-I*a^2*b^2 + I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/
b^2) - ((2*I*a^2*b^2 - I*a*b^3 - I*b^4)*cos(f*x + e)^4 + 2*I*a^4 + 3*I*a^3*
b - I*a^2*b^2 - 3*I*a*b^3 - I*b^4 + 2*(-2*I*a^3*b - I*a^2*b^2 + 2*I*a*b^3 +
 I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f
*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^
2 + a*b)/b^2))/b^2) + (2*((I*a*b^3 - I*b^4)*cos(f*x + e)^4 + I*a^3*b + I*a^
2*b^2 - I*a*b^3 - I*b^4 - 2*(I*a^2*b^2 - I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sq
rt((a^2 + a*b)/b^2) - ((-2*I*a^2*b^2 + I*a*b^3 + I*b^4)*cos(f*x + e)^4 - 2*
I*a^4 - 3*I*a^3*b + I*a^2*b^2 + 3*I*a*b^3 + I*b^4 + 2*(2*I*a^3*b + I*a^2*b^
2 - 2*I*a*b^3 - I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b
+ b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^3 - 2*I*b^4)*cos(f*x + e)^4
 + I*a^3*b - 3*I*a*b^3 - 2*I*b^4 - 2*(I*a^2*b^2 - I*a*b^3 - 2*I*b^4)*cos(f*
x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a^2*b^2 - I*a*b^3)*cos(f*
x + e)^4 - 2*I*a^4 - 5*I*a^3*b - 4*I*a^2*b^2 - I*a*b^3 + 2*(2*I*a^3*b + 3*I
*a^2*b^2 + I*a*b^3)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^
2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b
^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((-I*a*b^3 + 2*I*b^4)*cos(f*x + e)^4 -
 I*a^3*b + 3*I*a*b^3 + 2*I*b^4 - 2*(-I*a^2*b^2 + I*a*b^3 + 2*I*b^4)*cos(f*x
 + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a^2*b^2 + I*a*b^3)*cos(f*x
+ e)^4 + 2*I*a^4 + 5*I*a^3*b + 4*I*a^2*b^2 + I*a*b^3 + 2*(-2*I*a^3*b - 3*I*
a^2*b^2 - I*a*b^3)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2
) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^
2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(a*b^3 - b^4)*cos(f*x + e)^3 - (a^2*b^2
 - a*b^3 - 2*b^4)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e
))/(a^2*b^5*f*cos(f*x + e)^4 - 2*(a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^
4*b^3 + 2*a^3*b^4 + a^2*b^5)*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^4}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.369 $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3ab(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] 1/3*cos(f*x+e)*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*(a+2*b)*cos(f*x+
e)*sin(f*x+e)/a^2/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(a+2*b)*(cos(f*x+e)^
2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)
^(1/2)/a/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(cos(f*x+e)^2)^(1/2)/cos(f*
x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/b/f/(a
+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 257, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 423, 541, 538, 437, 435, 432, 430}

$$\frac{(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^2bf(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(a+2b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3abf\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)\cos(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x])/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + ((a + 2*
b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
+ ((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b*(a + b)*f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a]) - (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b
/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*b*f*Sqrt[a + b*Sin[e
 + f*x]^2])
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
```

```
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\quad}{\sqrt{1}}\right.}{3af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left((a+2b\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a+2b)}{}$$

Mathematica [A]
time = 1.02, size = 175, normalized size = 0.81

$$\frac{2a^2(a+2b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-2a^2(a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b\left(-4a^2-7ab-2b^2+b(a+2b)\cos(2(e+fx))\right)\sin(2(e+fx))}{6a^2b(a+b)f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*a^2*(a + 2*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - 2*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Elliptic
F[e + f*x, -(b/a)] - Sqrt[2]*b*(-4*a^2 - 7*a*b - 2*b^2 + b*(a + 2*b)*Cos[2*
(e + f*x)])*Sin[2*(e + f*x)])/(6*a^2*b*(a + b)*f*(2*a + b - b*Cos[2*(e + f*
x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(239) = 478$.
time = 10.24, size = 549, normalized size = 2.53

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b(\sin^2(fx+e))+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-(cos(f*
x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1
/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)
*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+a*b^2*sin(f*x+e)^5
+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2
)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-(cos(f*x+e)^2)^(1/2)*
((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-2*(co
s(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b
)^(1/2))*a^2*b+2*a^2*b*sin(f*x+e)^3+2*a*b^2*sin(f*x+e)^3-2*b^3*sin(f*x+e)^3
-2*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/a^2/(a+b)/(a+b*sin(f*x+e)^2)^(3/2)/
b/cos(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.22, size = 1394, normalized size = 6.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*((I*a*b^3 + 2*I*b^4)*cos(f*x + e)^4 + I*a^3*b + 4*I*a^2*b^2 + 5*I*a
*b^3 + 2*I*b^4 - 2*(I*a^2*b^2 + 3*I*a*b^3 + 2*I*b^4)*cos(f*x + e)^2)*sqrt(-
b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a^2*b^2 - 5*I*a*b^3 - 2*I*b^4)*cos(f*x +
e)^4 - 2*I*a^4 - 9*I*a^3*b - 14*I*a^2*b^2 - 9*I*a*b^3 - 2*I*b^4 + 2*(2*I*a^
3*b + 7*I*a^2*b^2 + 7*I*a*b^3 + 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2
 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b
 + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((-I*a*b^3 - 2*I*
b^4)*cos(f*x + e)^4 - I*a^3*b - 4*I*a^2*b^2 - 5*I*a*b^3 - 2*I*b^4 - 2*(-I*a
^2*b^2 - 3*I*a*b^3 - 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2
) - ((2*I*a^2*b^2 + 5*I*a*b^3 + 2*I*b^4)*cos(f*x + e)^4 + 2*I*a^4 + 9*I*a^3
*b + 14*I*a^2*b^2 + 9*I*a*b^3 + 2*I*b^4 + 2*(-2*I*a^3*b - 7*I*a^2*b^2 - 7*I
*a*b^3 - 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2)
 + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2
)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(4*((I*a*b^3 + I*b^4)*cos(f*x + e)^4 + I*
a^3*b + 3*I*a^2*b^2 + 3*I*a*b^3 + I*b^4 + 2*(-I*a^2*b^2 - 2*I*a*b^3 - I*b^4
)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((-2*I*a^2*b^2 - I*a*b^3
)*cos(f*x + e)^4 - 2*I*a^4 - 5*I*a^3*b - 4*I*a^2*b^2 - I*a*b^3 + 2*(2*I*a^3
*b + 3*I*a^2*b^2 + I*a*b^3)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 +
 a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2)
+ 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2
*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(4*((-I*a*b^3 - I*b^4)*cos(f*x
+ e)^4 - I*a^3*b - 3*I*a^2*b^2 - 3*I*a*b^3 - I*b^4 + 2*(I*a^2*b^2 + 2*I*a*b
^3 + I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((2*I*a^2*b^2
+ I*a*b^3)*cos(f*x + e)^4 + 2*I*a^4 + 5*I*a^3*b + 4*I*a^2*b^2 + I*a*b^3 + 2
*(-2*I*a^3*b - 3*I*a^2*b^2 - I*a*b^3)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b +
b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*((a*b^3 + 2*b^4)*cos(
f*x + e)^3 - 2*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e))*sqrt(-b*cos(f*x + e)
^2 + a + b)*sin(f*x + e))/((a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 - 2*(a^4*b^
3 + 2*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^
4 + a^2*b^5)*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.370 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3a^2(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] 1/3*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(2*a+b
)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(2*a+b)*
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/a^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(cos(f*x+e)^2
)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^
(1/2)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3263, 3252, 3251, 3257, 3256, 3262, 3261}

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
```

```
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a-2b+b\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} -$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} -$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} +$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} +$$

Mathematica [A]
time = 0.87, size = 172, normalized size = 0.77

$$\frac{2a^2(2a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-a^2(a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b\left(-5a^2-5ab-b^2+b(2a+b)\cos(2(e+fx))\right)\sin(2(e+fx))}{3a^2(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[
e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])
^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(245) = 490$.
time = 10.77, size = 547, normalized size = 2.45

method	result

default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\;\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b(\sin^2(fx+e))+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*sin(f*x+
e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-4*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*b^3*sin(f*x+
e)^3-5*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a
+b)^2/cos(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.23, size = 1531, normalized size = 6.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*(2*I*a^3*b^2 + 5*I*a^2*b^3 + 4*I*a*b^4 + I*b^5 + (2*I*a*b^4 + I*b^5
)*cos(f*x + e)^4 - 2*(2*I*a^2*b^3 + 3*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) - (-4*I*a^4*b - 12*I*a^3*b^2 - 13*I*a^2*b^3 - 6*
I*a*b^4 - I*b^5 + (-4*I*a^2*b^3 - 4*I*a*b^4 - I*b^5)*cos(f*x + e)^4 + 2*(4*
```

```
I*a^3*b^2 + 8*I*a^2*b^3 + 5*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sqrt(-b))*sqrt
((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8
*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-2*I*a^3*b^2
 - 5*I*a^2*b^3 - 4*I*a*b^4 - I*b^5 + (-2*I*a*b^4 - I*b^5)*cos(f*x + e)^4 -
2*(-2*I*a^2*b^3 - 3*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a
*b)/b^2) - (4*I*a^4*b + 12*I*a^3*b^2 + 13*I*a^2*b^3 + 6*I*a*b^4 + I*b^5 + (
4*I*a^2*b^3 + 4*I*a*b^4 + I*b^5)*cos(f*x + e)^4 + 2*(-4*I*a^3*b^2 - 8*I*a^2
*b^3 - 5*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a
*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a
*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^4*b - 11*I*a^3*b^2 - 15*
I*a^2*b^3 - 9*I*a*b^4 - 2*I*b^5 + (-3*I*a^2*b^3 - 5*I*a*b^4 - 2*I*b^5)*cos(
f*x + e)^4 - 2*(-3*I*a^3*b^2 - 8*I*a^2*b^3 - 7*I*a*b^4 - 2*I*b^5)*cos(f*x +
 e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-6*I*a^5 - 17*I*a^4*b - 17*I*a^3*b
^2 - 7*I*a^2*b^3 - I*a*b^4 + (-6*I*a^3*b^2 - 5*I*a^2*b^3 - I*a*b^4)*cos(f*x
 + e)^4 + 2*(6*I*a^4*b + 11*I*a^3*b^2 + 6*I*a^2*b^3 + I*a*b^4)*cos(f*x + e)
^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcs
in(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x
+ e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)
+ (2*(3*I*a^4*b + 11*I*a^3*b^2 + 15*I*a^2*b^3 + 9*I*a*b^4 + 2*I*b^5 + (3*I*
a^2*b^3 + 5*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^4 - 2*(3*I*a^3*b^2 + 8*I*a^2*b^
3 + 7*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (
6*I*a^5 + 17*I*a^4*b + 17*I*a^3*b^2 + 7*I*a^2*b^3 + I*a*b^4 + (6*I*a^3*b^2
+ 5*I*a^2*b^3 + I*a*b^4)*cos(f*x + e)^4 + 2*(-6*I*a^4*b - 11*I*a^3*b^2 - 6*
I*a^2*b^3 - I*a*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
 b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(2*a*b^4 + b^5)*cos(f*x + e)^3 - (5*a
^2*b^3 + 7*a*b^4 + 2*b^5)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sin
(f*x + e))/((a^4*b^4 + 2*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^4 - 2*(a^5*b^3 +
 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^2 + (a^6*b^2 + 4*a^5*b^3 +
 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.371 $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=288

$$-\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{b(3a^2-7ab-2b^2)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(3a^2-7ab-2b^2)\,E(e}{3a(a+b)^3f}$$

```
[Out] -1/3*(3*a-b)*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1
/3*b*(3*a^2-7*a*b-2*b^2)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^3/f/(a+b*sin(f*x+e
)^2)^(1/2)-1/3*(3*a^2-7*a*b-2*b^2)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*Elliptic
E(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/(a+b)^3/f/(a+b*sin(
f*x+e)^2)^(1/2)+1/3*(3*a-b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f
*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/(a+b)^2/f/(a+b*sin(f*x+e)^
2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.23, antiderivative size = 328, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3271, 425, 541, 538, 437, 435, 432, 430}

$$-\frac{(3a^2-7ab-2b^2)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{3a^2f(a+b)^3\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{b(3a^2-7ab-2b^2)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{(3a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{3af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{\tan(e+fx)}{f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{b(3a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*((3*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)^2*f*(a + b*Sin[e +
f*x]^2)^(3/2)) - (b*(3*a^2 - 7*a*b - 2*b^2)*Cos[e + f*x]*Sin[e + f*x])/(3*a
^2*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((3*a^2 - 7*a*b - 2*b^2)*Sqrt[
Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a
 + b*Sin[e + f*x]^2])/(3*a^2*(a + b)^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) +
((3*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec
[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)^2*f*Sqrt[a + b*Sin[e
 + f*x]^2]) + Tan[e + f*x]/((a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right.}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\right.}{(a+}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b(3a^2-7ab-2b^2)\cos(e+fx)\sin(}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+f}}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b(3a^2-7ab-2b^2)\cos(e+fx)\sin(}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+f}}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b(3a^2-7ab-2b^2)\cos(e+fx)\sin(}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+f}}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b(3a^2-7ab-2b^2)\cos(e+fx)\sin(}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+f}}$$

Mathematica [A]
time = 2.33, size = 245, normalized size = 0.85

$$\frac{-2a^2(3a^2-7ab-2b^2)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+2a^2(3a^2+2ab-b^2)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)+\frac{(24a^4+24a^3b+41a^2b^2+19ab^3+2b^4-4ab(6a^2-5ab-3b^2)\cos(2(e+fx))+b^2(3a^2-7ab-2b^2)\cos(4(e+fx)))\tan(e+fx)}{\sqrt{2}}}{6a^2(a+b)^3f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(3*a^2 - 7*a*b - 2*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*El
lipticE[e + f*x, -(b/a)] + 2*a^2*(3*a^2 + 2*a*b - b^2)*((2*a + b - b*Cos[2*
(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] + ((24*a^4 + 24*a^3*b + 41*
a^2*b^2 + 19*a*b^3 + 2*b^4 - 4*a*b*(6*a^2 - 5*a*b - 3*b^2)*Cos[2*(e + f*x)]
 + b^2*(3*a^2 - 7*a*b - 2*b^2)*Cos[4*(e + f*x)])*Tan[e + f*x])/Sqrt[2])/(6*
a^2*(a + b)^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(308) = 616$.
time = 17.94, size = 1082, normalized size = 3.76

method	result	size
default	Expression too large to display	1082

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(3*a^2-7*a*b-2*b^2)*sin
(f*x+e)*cos(f*x+e)^4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(3*a^3-
a^2*b-5*a*b^2-b^3)*cos(f*x+e)^2*sin(f*x+e)+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x
+e)^2)^(1/2)*a^2*(a^2+2*a*b+b^2)*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/
2)*(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*b*(3*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a^2+7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b+2*EllipticE(sin
(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(
f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)
^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^4+5*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+
(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*
cos(f*x+e)^2)^(1/2)*a^3*b+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e
)^2)^(1/2)*a^2*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1
/2)*a*b^3-3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
E(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^4
+4*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^3*b+9*(cos
(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2*b^2+2*(cos(f*x+
e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*
x+e)^2)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3)/(-(a+b*sin(f*x+e)
```

```
^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a+b)
^3/cos(f*x+e)/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 0.27, size = 1746, normalized size = 6.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] 1/6*((2*((-3*I*a^2*b^3 + 7*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^5 - 2*(-3*I*a^3*
b^2 + 4*I*a^2*b^3 + 9*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^3 + (-3*I*a^4*b + I*a
^3*b^2 + 13*I*a^2*b^3 + 11*I*a*b^4 + 2*I*b^5)*cos(f*x + e))*sqrt(-b)*sqrt((
a^2 + a*b)/b^2) - ((6*I*a^3*b^2 - 11*I*a^2*b^3 - 11*I*a*b^4 - 2*I*b^5)*cos(
f*x + e)^5 + 2*(-6*I*a^4*b + 5*I*a^3*b^2 + 22*I*a^2*b^3 + 13*I*a*b^4 + 2*I*
b^5)*cos(f*x + e)^3 + (6*I*a^5 + I*a^4*b - 27*I*a^3*b^2 - 35*I*a^2*b^3 - 15
*I*a*b^4 - 2*I*b^5)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2)
 + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2
)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((3*I*a^2*b^3 - 7*I*a*b^4 - 2*I*b^5)*cos
(f*x + e)^5 - 2*(3*I*a^3*b^2 - 4*I*a^2*b^3 - 9*I*a*b^4 - 2*I*b^5)*cos(f*x +
 e)^3 + (3*I*a^4*b - I*a^3*b^2 - 13*I*a^2*b^3 - 11*I*a*b^4 - 2*I*b^5)*cos(f
*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-6*I*a^3*b^2 + 11*I*a^2*b^3 + 1
1*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^5 + 2*(6*I*a^4*b - 5*I*a^3*b^2 - 22*I*a^2
*b^3 - 13*I*a*b^4 - 2*I*b^5)*cos(f*x + e)^3 + (-6*I*a^5 - I*a^4*b + 27*I*a^
3*b^2 + 35*I*a^2*b^3 + 15*I*a*b^4 + 2*I*b^5)*cos(f*x + e))*sqrt(-b))*sqrt((
2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a
*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(4*((3*I*a^2*b^3
 + 4*I*a*b^4 + I*b^5)*cos(f*x + e)^5 + 2*(-3*I*a^3*b^2 - 7*I*a^2*b^3 - 5*I*
a*b^4 - I*b^5)*cos(f*x + e)^3 + (3*I*a^4*b + 10*I*a^3*b^2 + 12*I*a^2*b^3 +
6*I*a*b^4 + I*b^5)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((-18*I*a
^3*b^2 - 11*I*a^2*b^3 - I*a*b^4)*cos(f*x + e)^5 + 2*(18*I*a^4*b + 29*I*a^3*
b^2 + 12*I*a^2*b^3 + I*a*b^4)*cos(f*x + e)^3 + (-18*I*a^5 - 47*I*a^4*b - 41
*I*a^3*b^2 - 13*I*a^2*b^3 - I*a*b^4)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt
```

```
((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b
)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2
 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(4*((-3*I*a^2*b^3 - 4*I*
a*b^4 - I*b^5)*cos(f*x + e)^5 + 2*(3*I*a^3*b^2 + 7*I*a^2*b^3 + 5*I*a*b^4 +
I*b^5)*cos(f*x + e)^3 + (-3*I*a^4*b - 10*I*a^3*b^2 - 12*I*a^2*b^3 - 6*I*a*b
^4 - I*b^5)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((18*I*a^3*b^2 +
 11*I*a^2*b^3 + I*a*b^4)*cos(f*x + e)^5 + 2*(-18*I*a^4*b - 29*I*a^3*b^2 - 1
2*I*a^2*b^3 - I*a*b^4)*cos(f*x + e)^3 + (18*I*a^5 + 47*I*a^4*b + 41*I*a^3*b
^2 + 13*I*a^2*b^3 + I*a*b^4)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*
a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(3*a^4*b + 6*a^3*b^2 + 3*a^2*b^3
 + (3*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^4 - 2*(3*a^3*b^2 - a^2*b^3 -
5*a*b^4 - b^5)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e)
)/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^5 - 2*(a^6*b^
2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^3 + (a^7*b
+ 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral(sec(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cos\left(e+f\,x\right)}^2\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)),x)
[Out] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

3.372 $\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=115

$$\frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(d\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}\sin(e+fx)\left(a+b\sin^2(e\right.}{f}$$

```
[Out] d*AppellF1(1/2,1/2-1/2*m,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(d*cos(f*x+
e))^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^p/f/((1
+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3272, 441, 440}

$$\frac{d\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{m-1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a
)]*(d*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3272

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[f
f*d^(2*IntPart[(m - 1)/2] + 1)*((d*Cos[e + f*x])^(2*FracPart[(m - 1)/2])/(f
*(Cos[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/
```

```
2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f
, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx = \frac{\left(d(d\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\operatorname{Subst}\left(\int (1-\right.}{f}$$

$$= \frac{\left(d(d\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\left(a+b\sin^2(e+\right.\right.}{}$$

$$= \frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(d\cos(e+fx}{}$$

Mathematica [A]
time = 0.64, size = 228, normalized size = 1.98

$$\frac{3aF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(d\cos(e+fx))^m\left(a+b\sin^2(e+fx)\right)^p\tan(e+fx)}{f\left(3aF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)+\left(2bpF_1\left(\frac{3}{2};\frac{1-m}{2},1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-a(-1+m)F_1\left(\frac{3}{2};\frac{3-m}{2},-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\right)\sin^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)
/a)]*(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*a*Appe
llF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2
*b*p*AppellF1[3/2, (1 - m)/2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]
^2)/a)] - a*(-1 + m)*AppellF1[3/2, (3 - m)/2, -p, 5/2, Sin[e + f*x]^2, -((b
*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int (d\cos(fx+e))^m \left(a+b\left(\sin^2(fx+e)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.46, size = 29, normalized size = 0.25

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\left(d\cos\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\cos\left(e+f\,x\right)\right)}^m\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int((d*cos(e + f*x))^m*(a + b*sin(e + f*x)^2)^p, x)
```

3.373 $\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=214

$$-\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}-\frac{\cos^2(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(5+2p)}+\cdots$$

```
[Out] -(3*a+b*(7+2*p))*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+15)-
cos(f*x+e)^2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1+p)/b/f/(5+2*p)+(3*a^2+2*a*b*(
5+2*p)+b^2*(4*p^2+16*p+15))*hypergeom([1/2, -p],[3/2],-b*sin(f*x+e)^2/a)*si
n(f*x+e)*(a+b*sin(f*x+e)^2)^p/b^2/f/(4*p^2+16*p+15)/((1+b*sin(f*x+e)^2/a)^p
)
```

Rubi [A]
time = 0.14, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3269, 427, 396, 252, 251}

$$\frac{(3a^2+2ab(2p+5)+b^2(4p^2+16p+15))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{b^2f(2p+3)(2p+5)}-\frac{(3a+b(2p+7))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{p+1}}{b^2f(2p+3)(2p+5)}-\frac{\sin(e+fx)\cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{p+1}}{bf(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a + b*(7 + 2*p))*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(1 + p))/(b^2*f*
(3 + 2*p)*(5 + 2*p))) - (Cos[e + f*x]^2*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)
^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 + 2*a*b*(5 + 2*p) + b^2*(15 + 16*p + 4*
p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sin[e + f*x]^
2)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a+bx^2\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{\cos^2(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{\text{Subst}\left(\int (a\right.}{}$$

$$= -\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos^2(e+}{}$$

$$= -\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos^2(e+}{}$$

$$= -\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos^2(e+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.32, size = 191, normalized size = 0.89

$$\frac{3aF_1\left(\frac{1}{2};-2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^4(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p}{f\left(3aF_1\left(\frac{1}{2};-2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)+2\left(bpF_1\left(\frac{3}{2};-2,1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-2aF_1\left(\frac{3}{2};-1,-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\right)\sin^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Co
s[e + f*x]^4*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(3*a*AppellF1[1/2, -
2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + 2*(b*p*AppellF1[3/2,
 -2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - 2*a*AppellF1[3/
2, -1, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

Maple [**F**]
time = 1.11, size = 0, normalized size = 0.00

$$\int \left(\cos^5\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Fricas [**F**]
time = 0.44, size = 27, normalized size = 0.13

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^5\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^2)^p, x)
```

3.374 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=124

$$-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)}+\frac{(a+b(3+2p))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right.}{bf(3+2p)}$$

```
[Out] -sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1+p)/b/f/(3+2*p)+(a+b*(3+2*p))*hypergeom([1
/2, -p],[3/2],-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^p/b/f/(3+2*p
)/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3269, 396, 252, 251}

$$\frac{\left(\frac{a}{2bp+3b}+1\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{p+1}}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p))) + ((1 + a/
(3*b + 2*b*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin
[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+bx^2\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$
$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1+\frac{a}{3b+2bp}\right)\text{Subst}\left(\int (a\right.}{}$$
$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(\left(1+\frac{a}{3b+2bp}\right)(a+b\sin^2\right.}{}$$
$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1+\frac{a}{3b+2bp}\right){}_2F_1\left(\frac{1}{2},-p;\right.}{}$$

Mathematica [A]
time = 0.15, size = 120, normalized size = 0.97

$$-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\left(-\left((a+b(3+2p))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)\right)+\left(a+b\sin^2(e+fx)\right)\left(1+\frac{b\sin^2(e+fx)}{a}\right)^p\right)}{bf(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p*(-((a + b*(3 + 2*p))*Hypergeometri
c2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]) + (a + b*Sin[e + f*x]^2)*(1 +
(b*Sin[e + f*x]^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*Sin[e + f*x]^2)/a)^p))
```

Maple [F]
time = 0.58, size = 0, normalized size = 0.00

$$\int \left(\cos^3(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Fricas [F]
time = 0.44, size = 27, normalized size = 0.22

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^p, x)
```

3.375 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] hypergeom([1/2, -p],[3/2],-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^
p/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3269, 252, 251}

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^p\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^2}{a}\right)^p$$
$$= \frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 1.00

$$\frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \cos(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Fricas [F]
time = 0.41, size = 25, normalized size = 0.37

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Mupad [B]
time = 15.22, size = 64, normalized size = 0.96

$$\frac{\sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{p}\,{}_{2}\mathrm{F}_{1}\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\sin(e+f\,x)^{2}}{a}\right)}{f\,\left(\frac{b\sin(e+f\,x)^{2}}{a}+1\right)^{p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^2)^p*hypergeom([1/2, -p], 3/2, -(b*sin(e
+ f*x)^2)/a))/(f*((b*sin(e + f*x)^2)/a + 1)^p)
```

3.376 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f
*x+e)^2)^p/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3269, 441, 440}

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1-x^2}\,dx,\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e\right.}{f}$$

Mathematica [F]
time = 4.80, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Fricas [**F**]
time = 0.38, size = 25, normalized size = 0.33

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x),x)
[Out] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x), x)
```

3.377 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f
*x+e)^2)^p/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3269, 441, 440}

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx^2)^p}{(1-x^2)^2}\,dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{\left(\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{(1-x^2)^2}\,d$$
$$=\frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin\right.}{f}$$

Mathematica [F]
time = 9.84, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\sec^3(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Fricas [F]
time = 0.41, size = 27, normalized size = 0.36

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{p}}{\cos\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^3,x)
[Out] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^3, x)
```

3.378 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+fx)}{f}$$

```
[Out] AppellF1(1/2,-3/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)
^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3271, 441, 440}

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \left(1-x^2\right)^{3/2}\left(a+bx^2\right)^p\,dx\right.}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+f}{a}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,(a+}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(90) = 180.
time = 0.42, size = 199, normalized size = 2.21

$$\frac{3aF_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^3(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p}{f\left(3aF_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)+\left(2bpF_1\left(\frac{3}{2};-\frac{3}{2},1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-3aF_1\left(\frac{3}{2};-\frac{1}{2},-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\right)\sin^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*
Cos[e + f*x]^3*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(3*a*AppellF1[1/2,
 -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2*b*p*AppellF1[
3/2, -3/2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - 3*a*Appel
lF1[3/2, -1/2, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f
*x]^2))
```

Maple [F]
time = 1.61, size = 0, normalized size = 0.00

$$\int \left(\cos^4(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Fricas [F]
time = 0.40, size = 27, normalized size = 0.30

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4\,\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^p, x)
```

3.379 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+}{f}$$

```
[Out] AppellF1(1/2,-1/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)
^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3271, 441, 440}

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \sqrt{1-x^2}\,(a+bx^2)^p\,dx\right.}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,(a-}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(90) = 180.
time = 0.39, size = 195, normalized size = 2.17

$$\frac{3aF_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\left(a+b\sin^2(e+fx)\right)^p\sin(2(e+fx))}{2f\left(3aF_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)+\left(2bpF_1\left(\frac{3}{2};-\frac{1}{2},1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-aF_1\left(\frac{3}{2};\frac{1}{2},-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\right)\sin^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*
(a + b*Sin[e + f*x]^2)^p*Sin[2*(e + f*x)])/(2*f*(3*a*AppellF1[1/2, -1/2, -p
, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, -1/2
, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - a*AppellF1[3/2, 1/
2, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

Maple [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.43, size = 27, normalized size = 0.30

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^p, x)
```

3.380 $\int (a+b\sin^2(e+fx))^p\,dx$

Optimal. Leaf size=90

$$\frac{F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,(a+b\sin^2(e+fx))^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+fx)}{f}$$

```
[Out] AppellF1(1/2,1/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)^
p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3264, 441, 440}

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,(a+b\sin^2(e+fx))^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3264

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*
x])), Subst[Int[(a + b*ff^2*x^2)^p/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (a+b\sin^2(e+fx))^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^p}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\,(a+b\sin^2(e+fx))^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,(a+b\sin^2(e+fx))}{f}$$

Mathematica [A]
time = 0.37, size = 145, normalized size = 1.61

$$\frac{2^{-1-p}F_1\left(1+p;\frac{1}{2},\frac{1}{2};2+p;\frac{2a+b-b\cos(2(e+fx))}{2(a+b)},\frac{2a+b-b\cos(2(e+fx))}{2a}\right)\sqrt{\frac{b\cos^2(e+fx)}{a+b}}\,(2a+b-b\cos(2(e+fx)))^{1+p}\csc(2(e+fx))\sqrt{-\frac{b\sin^2(e+fx)}{a}}}{bf(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (2^(-1 - p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a + b - b*Cos[2*(e + f*x)])
/(2*(a + b)), (2*a + b - b*Cos[2*(e + f*x)])/(2*a)]*Sqrt[(b*Cos[e + f*x]^2)
/(a + b)]*(2*a + b - b*Cos[2*(e + f*x)])^(1 + p)*Csc[2*(e + f*x)]*Sqrt[-((b
*Sin[e + f*x]^2)/a)])/(b*f*(1 + p))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p, x)
```

Fricas [F]
time = 0.40, size = 18, normalized size = 0.20

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p,x)
[Out] int((a + b*sin(e + f*x)^2)^p, x)
```

3.381 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+f}{f}$$

```
[Out] AppellF1(1/2,3/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)^
p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3271, 441, 440}

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^p}{(1-x^2)^{3/2}}\,dx,x,\sin(e+$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+}{a}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,(a+}{f}$$

Mathematica [F]
time = 4.99, size = 0, normalized size = 0.00

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(\sec^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Fricas [F]
time = 0.42, size = 27, normalized size = 0.30

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^2, x)
```

3.382 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+fx)}{f}$$

```
[Out] AppellF1(1/2,5/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)^
p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3271, 441, 440}

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 5/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3271

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
```

```
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{\left(1-x^2\right)^{5/2}}\,dx,x,\sin(e+fx\right.}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+f}{a}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,(a+b\sin}{f}$$

Mathematica [**F**]
time = 10.84, size = 0, normalized size = 0.00

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\sec^4(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Fricas [F]
time = 0.42, size = 27, normalized size = 0.30

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\sec\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p}{{\cos\left(e+f\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^4,x)
[Out] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^4, x)
```

3.383 $\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=219

$$\frac{\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d}-\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/}\right.}{}$$

```
[Out] 1/3*(a^(4/3)+b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/b^(5/3)/d-1/6*
(a^(4/3)+b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^
2)/a^(2/3)/b^(5/3)/d-2/3*ln(a+b*sin(d*x+c)^3)/b/d+1/2*sin(d*x+c)^2/b/d+1/3*
(a^(4/3)-b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2)
)/a^(2/3)/b^(5/3)/d*3^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3302, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\left(a^{4/3}-b^{4/3}\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{5/3}d}-\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d}-\frac{2\log\left(a+b\sin^3(c+dx)\right)}{3bd}+\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)*b^(5/3)*d) + ((a^(4/3) + b^(4/3))*Log[a^(1/3) + b^
(1/3)*Sin[c + d*x]])/(3*a^(2/3)*b^(5/3)*d) - ((a^(4/3) + b^(4/3))*Log[a^(2/
3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*b^(
5/3)*d) - (2*Log[a + b*Sin[c + d*x]^3])/(3*b*d) + Sin[c + d*x]^2/(2*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
```

```
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{x}{b}+\frac{b-ax-2bx^2}{b(a+bx^3)}\right)dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\sin^2(c+dx)}{2bd}+\frac{\text{Subst}\left(\int \frac{b-ax-2bx^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{bd}\\
&= \frac{\sin^2(c+dx)}{2bd}-\frac{2\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}+\frac{\text{Subst}\left(\int \frac{b-ax}{a+bx^3}\,dx, x, \sin(c+\right.}{bd}\\
&= -\frac{2\log\left(a+b\sin^3(c+dx)\right)}{3bd}+\frac{\sin^2(c+dx)}{2bd}+\frac{\left(\frac{1}{a^{2/3}}+\frac{a^{2/3}}{b^{4/3}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx\right.}{3d}\\
&= \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d}-\frac{2\log\left(a+b\sin^3(c+dx)\right)}{3bd}+\frac{\sin^2(c+}{2bd}\\
&= \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d}-\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin\right.}{6a^{2/3}b^{5/3}}\\
&= \frac{\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx\right.}{3a^{2/3}b^{5/3}d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.18, size = 203, normalized size = 0.93

$$
\frac{-2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)+2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)-4a^{2/3}\log\left(a+b\sin^3(c+dx)\right)+3a^{2/3}\sin^2(c+dx)-3a^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)\sin^2(c+dx)}{6a^{2/3}bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1
/3))] + 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - b^(2/3)*Log[a^(2/3)
```

```
 - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2] - 4*a^(2/3)*Log[a
 + b*Sin[c + d*x]^3] + 3*a^(2/3)*Sin[c + d*x]^2 - 3*a^(2/3)*Hypergeometric2
F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(6*a^(2/3)*b*d)
```

Maple [A]

time = 1.10, size = 264, normalized size = 1.21

method	result
derivativedivides	$\frac{\sin^2(dx+c)}{2b}+\frac{b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$
default	$\frac{\sin^2(dx+c)}{2b}+\frac{b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}$
risch	$\frac{2ix}{b}-\frac{e^{2i(dx+c)}}{8bd}-\frac{e^{-2i(dx+c)}}{8bd}+\frac{4ic}{bd}+\left(\sum_{\_R=\text{RootOf}\left(27a^2b^5d^3\_Z^3+54a^2b^4d^2\_Z^2+27a^2b^3d\_Z-a^4+2a^2b^2-b\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*sin(d*x+c)^2/b+(b*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3)
)-1/6/b/(1/b*a)^(2/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3
))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+
c)-1)))-a*(-1/3/b/(1/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^
(1/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b
/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))-2/3*ln(a
+b*sin(d*x+c)^3))/b)
```

Maxima [A]

time = 0.50, size = 210, normalized size = 0.96

$$\frac{\frac{9\sin(dx+c)^2}{b}-\frac{2\sqrt{3}\left(a\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\right)-b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{4a}{b}\right)\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{3\left(b\left(4\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)+a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{6\left(b\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)-a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{18d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(9*sin(d*x + c)^2/b - 2*sqrt(3)*(a*(3*(a/b)^(2/3) - 4) - b*(3*(a/b)^(1
/3) - 4*a/b))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3
))/(a*b) - 3*(b*(4*(a/b)^(2/3) + 1) + a*(a/b)^(1/3))*log(sin(d*x + c)^2 - (
a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 6*(b*(2*(a/b)^(2
/3) - 1) - a*(a/b)^(1/3))*log((a/b)^(1/3) + sin(d*x + c))/(b^2*(a/b)^(2/3))
)/d
```

Fricas [C] Result contains complex when optimal does not.
time = 1.20, size = 3216, normalized size = 14.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4
)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3
)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5
*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*b*d*log(1/4*((1/2)^(1/3)*(I*sqrt
(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)
/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(
2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*
d^3))^(1/3)))^2*a^3*b^3*d^2 + 2*a^3*b - 2*a*b^3 - 1/2*(4*a^3*b^2 - a*b^4)*(
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5
*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt
(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a
^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*d + (a^4 - b^4)*sin(d*x + c)) + 6*cos(d*x
+ c)^2 - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^
4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^
5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*b*d + 3*sqrt(1/3)*b*d*sqrt(-(((
1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*
d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^
4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*b*d - 8*(1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b
^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3)
)^(1/3) - 32/(b*d) - 16*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3)
+ (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3))
)/(b*d)) - 12)*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2
*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d)
+ 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 +
 b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*a^3*b^3*d^2 + 2*
```

```
a^3*b - 2*a*b^3 - 1/2*(4*a^3*b^2 - a*b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(
b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3
))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) +
 (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))
*d - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^
2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2
*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^
4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*a^3*b^3*d^2 - 2*(2*a^
3*b^2 + a*b^4)*d)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 -
 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d
) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2
 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*b*d - 8*(1/2)^
(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3)
- (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) - 32/(b*d) - 16*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 -
 b^4)/(a^2*b^5*d^3))^(1/3)))/(b*d)) - 2*(a^4 - b^4)*sin(d*x + c)) - (((1/2)
^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3)
 - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 -
b^4)/(a^2*b^5*d^3))^(1/3)))*b*d - 3*sqrt(1/3)*b*d*sqrt(-(((1/2)^(1/3)*(I*sq
rt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^
4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2
*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^
5*d^3))^(1/3)))^2*b*d - 8*(1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 -
 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) - 32/(b*
d) - 16*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b
^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))/(b*d)) - 12)*l
og(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)
/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*
d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*a^3*b^3*d^2 - 2*a^3*b + 2*a*b^3
 + 1/2*(4*a^3*b^2 - a*b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4
 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b
*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b
^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*d - 3/4*sqrt(1
/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a
^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-
I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3
) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*a^3*b^3*d^2 - 2*(2*a^3*b^2 + a*b^4)*
d)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 221, normalized size = 1.01

$$\frac{\frac{3\sin(dx+c)^2}{b} - \frac{4\log\left(\left|b\sin(dx+c)^3+a\right|\right)}{b} + \frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)^{\frac{2}{3}}a\right)\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^3} + \frac{2\left(ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{ab^5}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(3*sin(d*x + c)^2/b - 4*log(abs(b*sin(d*x + c)^3 + a))/b + 2*sqrt(3)*((
-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*
sin(d*x + c))/(-a/b)^(1/3))/(a*b^3) + ((-a*b^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*
a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a*b^3) +
 2*(a*b^4*(-a/b)^(1/3) - b^5)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x
+ c)))/(a*b^5))/d
```

Mupad [B]
time = 15.02, size = 229, normalized size = 1.05

$$\frac{\left(\sum_{k=1}^{3}\ln\left(3\,a+\mathrm{root}(27\,a^2\,b^5\,d^3+54\,a^2\,b^4\,d^2+27\,a^2\,b^3\,d+2\,a^2\,b^2-b^4-a^4,d,k)\,\left(12\,a\,b+3\,b^2\,\sin\left(c+d\,x\right)+\mathrm{root}(27\,a^2\,b^5\,d^3+54\,a^2\,b^4\,d^2+27\,a^2\,b^3\,d+2\,a^2\,b^2-b^4-a^4,d,k)\,a\,b^2\,9\right)+\frac{\sin(c+d\,x)\,\left(a^2+2\,b^2\right)}{b}\right)\,\mathrm{root}(27\,a^2\,b^5\,d^3+54\,a^2\,b^4\,d^2+27\,a^2\,b^3\,d+2\,a^2\,b^2-b^4-a^4,d,k)\right)+\frac{\sin(c+d\,x)^2}{2\,b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x)^3),x)
```

```
[Out] (symsum(log(3*a + root(27*a^2*b^5*d^3 + 54*a^2*b^4*d^2 + 27*a^2*b^3*d + 2*a
^2*b^2 - b^4 - a^4, d, k)*(12*a*b + 3*b^2*sin(c + d*x) + 9*root(27*a^2*b^5*
d^3 + 54*a^2*b^4*d^2 + 27*a^2*b^3*d + 2*a^2*b^2 - b^4 - a^4, d, k)*a*b^2) +
 (sin(c + d*x)*(a^2 + 2*b^2))/b)*root(27*a^2*b^5*d^3 + 54*a^2*b^4*d^2 + 27*
a^2*b^3*d + 2*a^2*b^2 - b^4 - a^4, d, k), k, 1, 3) + sin(c + d*x)^2/(2*b))/
d
```

3.384 $\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=167

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)
*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*ln(a+b*sin(
d*x+c)^3)/b/d-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2)
)/a^(2/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3302, 1885, 206, 31, 648, 631, 210, 642, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\log\left(a+b\sin^3(c+dx)\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(
2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)*d
) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(6
*a^(2/3)*b^(1/3)*d) - Log[a + b*Sin[c + d*x]^3]/(3*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(c+dx)\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+\right.}{6a^{2/3}\sqrt[3]{b}\,d}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{\right.}{}$$

Mathematica [A]
time = 0.11, size = 139, normalized size = 0.83

$$\frac{\left(-a^{2/3}+(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)+\left(-a^{2/3}+b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)-\left(a^{2/3}+\sqrt[3]{-1}\,b^{2/3}\right)\log\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((-a^(2/3) + (-1)^(2/3)*b^(2/3))*Log[-((-1)^(2/3)*a^(1/3)) - b^(1/3)*Sin[c
+ d*x]] + (-a^(2/3) + b^(2/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - (a^(2/
3) + (-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b^(1/3)*Sin[c + d*x]])/(3
*a^(2/3)*b*d)
```

Maple [A]
time = 0.70, size = 133, normalized size = 0.80

method	result
risch	$\frac{ix}{b}+\frac{2ic}{bd}+\left(\sum_{\_R=\text{RootOf}\left(27a^2\_Z^3d^3b^3+27a^2b^2d^2\_Z^2+9a^2d\_Zb+a^2-b^2\right)}\_R\ln\left(e^{2i(dx+c)}+\left(6iad\_R\right.\right.\right.$

derivativedivides	$\frac{\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(a+b\left(\sin^3(dx\cdots\right.\right.}{3b}}{d}$
default	$\frac{\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(a+b\left(\sin^3(dx\cdots\right.\right.}{3b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*l
n(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1))-1/3/b*ln(a+b*sin
(d*x+c)^3))
```

Maxima [A]
time = 0.51, size = 159, normalized size = 0.95

$$\frac{\frac{2\sqrt{3}\left(b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)+2a\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{3\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{6\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{18\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(2*sqrt(3)*(b*(3*(a/b)^(1/3) - 2*a/b) + 2*a)*arctan(-1/3*sqrt(3)*((a/b
)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/(a*b) - 3*(2*(a/b)^(2/3) + 1)*log(si
n(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 6*
((a/b)^(2/3) - 1)*log((a/b)^(1/3) + sin(d*x + c))/(b*(a/b)^(2/3)))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 40.62, size = 2298, normalized size = 13.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(1/3)*b*d*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(
a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/
```

2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2))*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*a^2*b^2*d^2 - 4*b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) + 2*(a*b^2*d*sin(d*x + c) - 2*a^2*b*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d)) + 4*a^2 + 4*b^2)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*a*b*d^2 - 2*a*d)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2)) + sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*a^2*b^2*d^3 - 8*a*b*d*sin(d*x + c) + 4*a^2*d + 4*(a*b^2*d^2*sin(d*x + c) - a^2*b*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d)))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2)))/b) - 6*sqrt(1/3)*b*d*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2))*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*a^2*b^2*d^2 - 4*b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) + 2*(a*b^2*d*sin(d*x + c) - 2*a^2*b*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d)) + 4*a^2 + 4*b^2)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*a*b*d^2 - 2*a*d)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2)) - sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*a^2*b^2*d^3 - 8*a*b*d*sin(d*x + c) + 4*a^2*d + 4*(a*b^2*d^2*sin(d*x + c) - a^2*b*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d)))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2)))/b) - ((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*a^2*b^2*d^2 - b^2*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) - (a*b^2*d*sin(d*x + c) + a^2*b*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d)) + a^

```
2 + b^2) + (((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^
2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d - 6)*log(((1/2)^(1/3)*(I*sqrt(
3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2
/(b*d))^2*a^2*b^2*d^2 - 4*b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) + 2*(a*b^
2*d*sin(d*x + c) - 2*a^2*b*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1
/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d)) + 4*a^2 + 4*b^2)
)/(b*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 156, normalized size = 0.93

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a}+\frac{2\log\left(\left|b\sin(dx+c)^{3}+a\right|\right)}{b}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a + 2*log(abs(b
*sin(d*x + c)^3 + a))/b - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/
b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1/3)*log(sin(d*x
 + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a*b))/d
```

Mupad [B]
time = 15.02, size = 153, normalized size = 0.92

$$\frac{\sum_{k=1}^{3}\ln\left(\left(\mathrm{root}(27\,a^2\,b^3\,d^3+27\,a^2\,b^2\,d^2+9\,a^2\,b\,d-b^2+a^2,d,k)\;b\,3+1\right)\,\left(a+b\,\sin\left(c+d\,x\right)+\mathrm{root}(27\,a^2\,b^3\,d^3+27\,a^2\,b^2\,d^2+9\,a^2\,b\,d-b^2+a^2,d,k)\;a\,b\,3\right)\right)\,\mathrm{root}(27\,a^2\,b^3\,d^3+27\,a^2\,b^2\,d^2+9\,a^2\,b\,d-b^2+a^2,d,k)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log((3*root(27*a^2*b^3*d^3 + 27*a^2*b^2*d^2 + 9*a^2*b*d - b^2 + a^2,
 d, k)*b + 1)*(a + b*sin(c + d*x) + 3*root(27*a^2*b^3*d^3 + 27*a^2*b^2*d^2
+ 9*a^2*b*d - b^2 + a^2, d, k)*a*b))*root(27*a^2*b^3*d^3 + 27*a^2*b^2*d^2 +
 9*a^2*b*d - b^2 + a^2, d, k), k, 1, 3)/d
```

3.385 $\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=144

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c\right.}{6a^{2/3}\sqrt[3]{b}\,d}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)
*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*arctan(1/3*
(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3302, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(
2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)*d
) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(6
*a^(2/3)*b^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(c+dx)\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \text{s}\right.}{3a^{2/3}d}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt[3]{a}\,d}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+\right.}{6a^{2/3}\sqrt[3]{b}\,d}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{}\right.}{}$$

Mathematica [A]
time = 0.04, size = 116, normalized size = 0.81

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))
] - 2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*S
in[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(a^(2/3)*b^(1/3)*d)
```

Maple [A]
time = 0.47, size = 115, normalized size = 0.80

method	result	size
risch	$\sum_{\_R=\text{RootOf}\left(27a^2b\,d^3\_Z^3-1\right)} \_R\ln\left(e^{2i(dx+c)}+6iad\_R\,e^{i(dx+c)}-1\right)$	48

derivativedivides	$\frac{\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{d}$	115
default	$\frac{\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{d}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*l
n(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))
```

Maxima [A]
time = 0.53, size = 121, normalized size = 0.84

$$\frac{\frac{2\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/
3))/(b*(a/b)^(2/3)) - log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)
^(2/3))/(b*(a/b)^(2/3)) + 2*log((a/b)^(1/3) + sin(d*x + c))/(b*(a/b)^(2/3))
)/d
```

Fricas [A]
time = 0.45, size = 401, normalized size = 2.78

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(-\frac{3\,(a^2b)^{\frac{1}{3}}a\sin(dx+c)+a^2+3\sqrt{\frac{1}{3}}\left(2\,ab\cos(dx+c)^2-2\,ab-(a^2b)^{\frac{2}{3}}\sin(dx+c)+(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}+2\left(ab\cos(dx+c)^2-ab\right)\sin(dx+c)}{\left(b\cos(dx+c)^2-b\right)\sin(dx+c)-a}\right)-(a^2b)^{\frac{2}{3}}\log\left(-ab\cos(dx+c)^2+ab-(a^2b)^{\frac{2}{3}}\sin(dx+c)+(a^2b)^{\frac{1}{3}}a\right)+2\,(a^2b)^{\frac{2}{3}}\log\left(ab\sin(dx+c)+(a^2b)^{\frac{2}{3}}\right)}{6\,a^2bd},\ \frac{6\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}\sin(dx+c)-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-(a^2b)^{\frac{2}{3}}\log\left(-ab\cos(dx+c)^2+ab-(a^2b)^{\frac{2}{3}}\sin(dx+c)+(a^2b)^{\frac{1}{3}}a\right)+2\,(a^2b)^{\frac{2}{3}}\log\left(ab\sin(dx+c)+(a^2b)^{\frac{2}{3}}\right)}{6\,a^2bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*sin(d*
x + c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2/3)*si
n(d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x + c)^
2 - a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) - (a^2*b)
^(2/3)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)
^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a^2*b*d
), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*sin(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(
2/3)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(
1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a^2*b*d)]
```

Sympy [A]
time = 2.71, size = 184, normalized size = 1.28

$$\begin{cases} \frac{\tilde{\infty} x \cos(c)}{\sin^3(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{ad} & \text{for } b = 0 \\ -\frac{1}{2bd\sin^2(c+dx)} & \text{for } a = 0 \\ \frac{x\cos(c)}{a+b\sin^3(c)} & \text{for } d = 0 \\ -\frac{\sqrt[3]{-\frac{a}{b}}\log\left(-\sqrt[3]{-\frac{a}{b}}+\sin(c+dx)\right)}{3ad} + \frac{\sqrt[3]{-\frac{a}{b}}\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\sin(c+dx)+4\sin^2(c+dx)\right)}{6ad} + \frac{\sqrt{3}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sin(c+dx)}{3\sqrt[3]{-\frac{a}{b}}}\right)}{3ad} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sin(c
+ d*x)/(a*d), Eq(b, 0)), (-1/(2*b*d*sin(c + d*x)**2), Eq(a, 0)), (x*cos(c)/
(a + b*sin(c)**3), Eq(d, 0)), (-(-a/b)**(1/3)*log(-(-a/b)**(1/3) + sin(c +
d*x))/(3*a*d) + (-a/b)**(1/3)*log(4*(-a/b)**(2/3) + 4*(-a/b)**(1/3)*sin(c +
 d*x) + 4*sin(c + d*x)**2)/(6*a*d) + sqrt(3)*(-a/b)**(1/3)*atan(sqrt(3)/3 +
 2*sqrt(3)*sin(c + d*x)/(3*(-a/b)**(1/3)))/(3*a*d), True))
```

Giac [A]
time = 0.47, size = 137, normalized size = 0.95

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a} - \frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a - 2*sqrt(3)*(
-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/
3))/(a*b) - (-a*b^2)^(1/3)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) +
 (-a/b)^(2/3))/(a*b))/d
```

Mupad [B]
time = 0.28, size = 123, normalized size = 0.85

$$\frac{\ln\left(b^{1/3}\sin\left(c+d\,x\right)+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}\,d}+\frac{\ln\left(3\,b^2\sin\left(c+d\,x\right)+\frac{3\,a^{1/3}\,b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}-\frac{\ln\left(3\,b^2\sin\left(c+d\,x\right)-\frac{3\,a^{1/3}\,b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x)^3),x)
```

```
[Out] log(b^(1/3)*sin(c + d*x) + a^(1/3))/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*sin(
c + d*x) + (3*a^(1/3)*b^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(
2/3)*b^(1/3)*d) - (log(3*b^2*sin(c + d*x) - (3*a^(1/3)*b^(5/3)*(3^(1/2)*1i
+ 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*d)
```

3.386 $\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a^2-b^2\right)d}-\frac{\log(1-\sin(c+dx))}{2(a+b)d}+\frac{\log(1+\sin(c+dx))}{2(a-b)d}-\frac{\sqrt[3]{b}\left(a^{4/3}+b\right.}{}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d+1/2*ln(1+sin(d*x+c))/(a-b)/d-1/3*b^(1/3)*(a^(
4/3)+b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/(a^2-b^2)/d+1/6*b^(1/3
)*(a^(4/3)+b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c
)^2)/a^(2/3)/(a^2-b^2)/d-1/3*b*ln(a+b*sin(d*x+c)^3)/(a^2-b^2)/d-1/3*b^(1/3)
*(a^(4/3)-b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2
))/a^(2/3)/(a^2-b^2)/d*3^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3302, 2099, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{b\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d\left(a^2-b^2\right)}+\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}d\left(a^2-b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -((b^(1/3)*(a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2)*d)) - Log[1 - Sin[c + d*x]]/
(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d) - (b^(1/3)*(a^(4/3) + b
^(4/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)*d) + (b
^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(
2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)*d) - (b*Log[a + b*Sin[c + d*x]
^3])/(3*(a^2 - b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^3)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)(-1+x)} + \frac{1}{2(a-b)(1+x)} + \frac{b(b-ax+bx^2)}{(-a^2+b^2)(a+bx^3)}\right) dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\text{Subst}\left(\int \frac{b-ax+bx^2}{a+bx^3}\,dx, x, \sin(c-\right.}{(a^2-b^2)\,d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\text{Subst}\left(\int \frac{b-ax}{a+bx^3}\,dx, x, \sin(c+d\right.}{(a^2-b^2)\,d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\log\left(a+b\sin^3(c+dx)\right)}{3\left(a^2-b^2\right)d} - \frac{b^{2/3}}{}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\right.}{3a^{2/3}\left(a^2-b^2\right)d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\right.}{3a^{2/3}\left(a^2-b^2\right)d}$$

$$= -\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\left(a^2-b^2\right)d} - \frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\text{si}}{2(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.15, size = 268, normalized size = 0.92

$$\frac{2\sqrt{3}\,b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 3a^{5/3}\log(1-\sin(c+dx)) + 3a^{2/3}b\log(1-\sin(c+dx)) + 3a^{5/3}\log(1+\sin(c+dx)) + 3a^{2/3}b\log(1+\sin(c+dx)) - 2b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right) + b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right) - 2a^{2/3}b\log\left(a+b\sin^3(c+dx)\right) + 3a^{2/3}b\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)\sin^2(c+dx)}{6a^{2/3}(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/
3))] - 3*a^(5/3)*Log[1 - Sin[c + d*x]] + 3*a^(2/3)*b*Log[1 - Sin[c + d*x]]
+ 3*a^(5/3)*Log[1 + Sin[c + d*x]] + 3*a^(2/3)*b*Log[1 + Sin[c + d*x]] - 2*b
^(5/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] + b^(5/3)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2] - 2*a^(2/3)*b*Log[a + b*Sin[
c + d*x]^3] + 3*a^(2/3)*b*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^
3)/a)]*Sin[c + d*x]^2)/(6*a^(2/3)*(a - b)*(a + b)*d)
```

Maple [A]
time = 1.01, size = 300, normalized size = 1.03

method	result
derivativedivides	$\left(-b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\right)+a\left(-\frac{\ln(}{}\right.$ … $(a$
default	$\left(-b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\right)+a\left(-\frac{\ln(}{}\right.$ … $(a$
risch	$\frac{ix}{a+b}+\frac{ic}{(a+b)d}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}+\frac{2ia^2b\,d^3x}{a^4d^3-a^2b^2d^3}+\frac{2ia^2b\,d^2c}{a^4d^3-a^2b^2d^3}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{d(a+b)}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{d(a-b)}+2\Big($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-b*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2
/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+a*(-1/3/b/
(1/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*x+c
)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arc
```

```
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))-1/3*ln(a+b*sin(d*x+c)^3))*
b/(a-b)/(a+b)-1/(2*a+2*b)*ln(sin(d*x+c)-1)+1/(2*a-2*b)*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.53, size = 288, normalized size = 0.99

$$\frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\right)-b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2a}{b}\right)\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{3\left(b\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)-a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{6\left(b\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)+a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{9\log(\sin(dx+c)+1)}{a-b}-\frac{9\log(\sin(dx+c)-1)}{a+b}}{18\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(2*sqrt(3)*(a*(3*(a/b)^(2/3) + 2) - b*(3*(a/b)^(1/3) + 2*a/b))*arctan(
-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/((a^2*(a/b)^(2/3)
- b^2*(a/b)^(2/3))*(a/b)^(1/3)) - 3*(b*(2*(a/b)^(2/3) - 1) - a*(a/b)^(1/3))
*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a^2*(a/b)^(2
/3) - b^2*(a/b)^(2/3)) - 6*(b*((a/b)^(2/3) + 1) + a*(a/b)^(1/3))*log((a/b)^
(1/3) + sin(d*x + c))/(a^2*(a/b)^(2/3) - b^2*(a/b)^(2/3)) + 9*log(sin(d*x +
 c) + 1)/(a - b) - 9*log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 1.30, size = 4396, normalized size = 15.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*(a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1
/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d
^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^
3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d*log(-1/36*(a^5 - a^3*b^2)*(9*(I*sqrt(3)
 + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*
(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d
 - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 +
 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))^
2*d^2 + a*b^2 + 1/6*(2*a^3*b + a*b^3)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3
- a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^
2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^
4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^
2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - (a^2*b + b^3)*sin(d*
x + c)) - ((a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3)
- 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^
(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2
```

*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*
d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - 3*sqrt(1/3)*(a^2 - b^2)*d*sqrt(-((a
^4 - 2*a^2*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(
1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*
d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d
^3))^(1/3)) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) +
 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a
^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d -
 b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1
/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d
 - 108*b^2)/((a^4 - 2*a^2*b^2 + b^4)*d^2)) - 18*b)*log(1/36*(a^5 - a^3*b^2)*
(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2
d)^3 + 1/54(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3)
 + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d
 - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2
*d - b^2*d))^2*d^2 - a*b^2 - 1/6*(2*a^3*b + a*b^3)*(9*(I*sqrt(3) + 1)*(-1/5
4*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)
*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2
*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2
+ b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d + 1/12*sq
rt(1/3)*((a^5 - a^3*b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3
) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3)
)^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b
^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^
2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d^2 - 6*(a^3*b - a*b^3)*d)*sqrt(-((a^
4 - 2*a^2*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1
/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d
^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^
3))^(1/3)) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) +
1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^
2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d -
b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/
54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d -
 108*b^2)/((a^4 - 2*a^2*b^2 + b^4)*d^2)) - 2*(a^2*b + b^3)*sin(d*x + c)) -
((a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3
/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^
2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/
27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3
)) + 6*b/(a^2*d - b^2*d))*d + 3*sqrt(1/3)*(a^2 - b^2)*d*sqrt(-((a^4 - 2*a^2
b^2 + b^4)(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/
(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2
*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/2
7*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)

```
) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) + 1)*(-1/54
*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*
b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*
(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 +
 b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

Giac [A]
time = 0.45, size = 309, normalized size = 1.07

$$-\frac{\frac{2\left(a^3b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3+b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^5b-2a^3b^3+ab^5}+\frac{2\left(\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}b^2+\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^3b-ab^3}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)^{\frac{2}{3}}a\right)\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^3b-ab^3}+\frac{2b\log\left(\left|b\sin(dx+c)^3+a\right|\right)}{a^2-b^2}-\frac{3\log(|\sin(dx+c)+1|)}{a-b}+\frac{3\log(|\sin(dx+c)-1|)}{a+b}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(2*(a^3*b^2*(-a/b)^(1/3) - a*b^4*(-a/b)^(1/3) - a^2*b^3 + b^5)*(-a/b)^
(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^5*b - 2*a^3*b^3 + a*b^5) +
2*(sqrt(3)*(-a*b^2)^(1/3)*b^2 + sqrt(3)*(-a*b^2)^(2/3)*a)*arctan(1/3*sqrt(3
)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^3*b - a*b^3) + ((-a*b^2)
^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x +
c) + (-a/b)^(2/3))/(a^3*b - a*b^3) + 2*b*log(abs(b*sin(d*x + c)^3 + a))/(a^
2 - b^2) - 3*log(abs(sin(d*x + c) + 1))/(a - b) + 3*log(abs(sin(d*x + c) -
1))/(a + b))/d
```

Mupad [B]
time = 0.27, size = 600, normalized size = 2.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] (symsum(log(- 13*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)
^2*a*b^4 - 36*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^3*
a*b^5 - 36*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^4*a*b
```

```
^6 - 16*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^2*b^5*si
n(c + d*x) - 12*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^
3*b^6*sin(c + d*x) - 27*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b
, z, k)^3*a^3*b^3 - 180*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b
, z, k)^4*a^3*b^4 - 5*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b,
z, k)*b^4*sin(c + d*x) - 69*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2
 - b, z, k)^3*a^2*b^4*sin(c + d*x) - 162*root(27*a^2*b^2*z^3 - 27*a^4*z^3 -
 27*a^2*b*z^2 - b, z, k)^4*a^2*b^5*sin(c + d*x) - 54*root(27*a^2*b^2*z^3 -
27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^4*a^4*b^3*sin(c + d*x))*root(27*a^2*b^
2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k), k, 1, 3) - log(sin(c + d*x) -
 1)/(2*a + 2*b) + log(sin(c + d*x) + 1)/(2*a - 2*b))/d
```

3.387 $\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=385

$$-\frac{b^{5/3}\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sin(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a^2-b^2\right)^2 d}-\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{(a-4b)\log(1+\sin}{4(a-b)^2}$$

```
[Out] -1/4*(a+4*b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(a-4*b)*ln(1+sin(d*x+c))/(a-b)^
2/d+1/3*b^(5/3)*(2*a^2+3*a^(4/3)*b^(2/3)+b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+c)
)/a^(2/3)/(a^2-b^2)^2/d-1/6*b^(5/3)*(2*a^2+3*a^(4/3)*b^(2/3)+b^2)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(2/3)/(a^2-b^2)^2/d+1/
3*b*(a^2+2*b^2)*ln(a+b*sin(d*x+c)^3)/(a^2-b^2)^2/d+1/4/(a+b)/d/(1-sin(d*x+c
))-1/4/(a-b)/d/(1+sin(d*x+c))-1/3*b^(5/3)*(2*a^2-3*a^(4/3)*b^(2/3)+b^2)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^2-b^2)^2
/d*3^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3302, 2099, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{b(a^2+2b^2)\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)^2}-\frac{b^{5/3}\left(-3a^{4/3}b^{2/3}+2a^2+b^2\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sin(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d\left(a^2-b^2\right)^2}-\frac{b^{5/3}\left(3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}d\left(a^2-b^2\right)^2}+\frac{b^{5/3}\left(3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sin(c+dx)\right)}{3a^{2/3}d\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(a+4b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(a-4b)\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -((b^(5/3)*(2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Si
n[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2)^2*d)) - ((a +
4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a - 4*b)*Log[1 + Sin[c + d*
x]])/(4*(a - b)^2*d) + (b^(5/3)*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(1/
3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)^2*d) - (b^(5/3)*(2*a^2 +
 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2
/3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)^2*d) + (b*(a^2 + 2*b^2)*Log[a +
 b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])
) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
```

```
PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^3)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{-a-4b}{4(a+b)^2(-1+x)} + \frac{1}{4(a-b)(1+x)^2} + \frac{a-4b}{4(a-b)^2(1+x)} + \frac{b^2(2a^2+b^2-3a}{(a^2-b^2)}\right.\right.}{d}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{1}{4(a+b)d(1-}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{1}{4(a+b)d(1-}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b(a^2+2b^2)\log}{3(a}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b^{5/3}(2a^2+3a}{}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b^{5/3}(2a^2+3a}{}$$

$$= -\frac{b^{5/3}\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\left(a^2-b^2\right)^2 d} - \frac{(a+4b)\log(1-\sin(c+}{4(a+b)^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.58, size = 333, normalized size = 0.86

$$-\frac{\frac{3(a+4b)\log(1-\sin(c+dx))}{(a+b)^2}-\frac{3(a-4b)\log(1+\sin(c+dx))}{(a-b)^2}-\frac{4b^{5/3}(2a^2+b^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{2/3}(a^2-b^2)^2}+\frac{2b^{5/3}(2a^2+b^2)\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)\right)}{a^{2/3}(a^2-b^2)^2}-\frac{4b(a^2+2b^2)\log(a+b\sin^3(c+dx))}{(a^2-b^2)^2}+\frac{3}{(a+b)(-1+\sin(c+dx))}+\frac{18b^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)\sin^2(c+dx)}{(a^2-b^2)^2}+\frac{3}{(a-b)(1+\sin(c+dx))}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/12*((3*(a + 4*b)*Log[1 - Sin[c + d*x]])/(a + b)^2 - (3*(a - 4*b)*Log[1 +
 Sin[c + d*x]])/(a - b)^2 - (4*b^(5/3)*(2*a^2 + b^2)*Log[a^(1/3) + b^(1/3)*
Sin[c + d*x]])/(a^(2/3)*(a^2 - b^2)^2) + (2*b^(5/3)*(2*a^2 + b^2)*(2*Sqrt[3
]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] + Log[a^(2/3
) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]))/(a^(2/3)*(a^2
- b^2)^2) - (4*b*(a^2 + 2*b^2)*Log[a + b*Sin[c + d*x]^3])/(a^2 - b^2)^2 + 3
/((a + b)*(-1 + Sin[c + d*x])) + (18*b^3*Hypergeometric2F1[2/3, 1, 5/3, -((
b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a^2 - b^2)^2 + 3/((a - b)*(1 + Sin[c
 + d*x])))/d
```

Maple [A]
time = 1.38, size = 372, normalized size = 0.97 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(4*a-4*b)/(1+sin(d*x+c))+1/4*(a-4*b)/(a-b)^2*ln(1+sin(d*x+c))+((2*a
^2+b^2)*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/
3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))-3*a*b*(-1/3
/b/(1/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*
x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*
arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+1/3*(a^2+2*b^2)/b*ln(a+
b*sin(d*x+c)^3))*b^2/(a-b)^2/(a+b)^2-1/(4*a+4*b)/(sin(d*x+c)-1)+1/4/(a+b)^2
*(-4*b-a)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.53, size = 470, normalized size = 1.22

$$-\frac{\frac{4\sqrt{3}\left(ab^2\left(9\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\right)-b^3\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{4a}{b}\right)-2a^2b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{a}{b}\right)+2a^3\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{6\left(b^3\left(4\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)+2a^2b\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)-3ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{12\left(b^3\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)+a^2b\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\right)+3ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{9(a-4b)\log(\sin(dx+c)+1)}{a^2-2ab+b^2}+\frac{9(a+4b)\log(\sin(dx+c)-1)}{a^2+2ab+b^2}+\frac{18(a\sin(dx+c)-b)}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2}}{36d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/36*(4*sqrt(3)*(a*b^2*(9*(a/b)^(2/3) + 4) - b^3*(3*(a/b)^(1/3) + 4*a/b) -
 2*a^2*b*(3*(a/b)^(1/3) + a/b) + 2*a^3)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) -
2*sin(d*x + c))/(a/b)^(1/3))/((a^4*(a/b)^(2/3) - 2*a^2*b^2*(a/b)^(2/3) + b^
4*(a/b)^(2/3))*(a/b)^(1/3)) - 6*(b^3*(4*(a/b)^(2/3) - 1) + 2*a^2*b*((a/b)^(
```

```
2/3) - 1) - 3*a*b^2*(a/b)^(1/3))*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x +
 c) + (a/b)^(2/3))/(a^4*(a/b)^(2/3) - 2*a^2*b^2*(a/b)^(2/3) + b^4*(a/b)^(2/
3)) - 12*(b^3*(2*(a/b)^(2/3) + 1) + a^2*b*((a/b)^(2/3) + 2) + 3*a*b^2*(a/b)
^(1/3))*log((a/b)^(1/3) + sin(d*x + c))/(a^4*(a/b)^(2/3) - 2*a^2*b^2*(a/b)^
(2/3) + b^4*(a/b)^(2/3)) - 9*(a - 4*b)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b +
 b^2) + 9*(a + 4*b)*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2) + 18*(a*sin(d
*x + c) - b)/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 2.17, size = 10135, normalized size = 26.32

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a^4 - 2*a^2*b^2 + b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^
2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(
3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^
2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(
a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2
- b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*
b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*
d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*
d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2
*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d*cos(d*x + c)^2*log(7*a^3*
b^2 + 2*a*b^4 + 3/4*(a^7 - 2*a^5*b^2 + a^3*b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^
2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4
*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a
^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d
 + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 +
 b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^
4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
+ b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*
a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*
sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 1/2
*(10*a^5*b + 16*a^3*b^3 + a*b^5)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d
^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt
(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b
^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*
(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2
 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2
*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4
*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4
*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) +
```

```
2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d - (8*a^2*b^3 + b^5)*sin(
d*x + c)) + 3*(a^3 - 2*a^2*b - 7*a*b^2 - 4*b^3)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) - 6*a^2*b + 6*b^3 - ((a^4 - 2*a^2*b^2 + b^4)*(2*(1/2)^(2/3)*(b^2
/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2
*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3
) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a
^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 +
(8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^
3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*
b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^
4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(
1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d*c
os(d*x + c)^2 - 3*sqrt(1/3)*(a^4 - 2*a^2*b^2 + b^4)*d*sqrt(-(4*a^4*b^2 - 80
*a^2*b^4 - 32*b^6 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(2*(1/2
)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d
 - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 +
 a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*
(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d +
 b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*
(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*
d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b +
2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4
*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d +
 b^4*d))^2*d^2 - 4*(a^6*b - 3*a^2*b^5 + 2*b^7)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2
 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*
d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^
2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d
+ b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 +
b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4
*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 +
 b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a
^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*s
qrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4
*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2))*cos(d*x + c)^2 - 6*(a^2*b + 2
*b^3)*cos(d*x + c)^2)*log(7*a^3*b^2 + 2*a*b^4 + 3/4*(a^7 - 2*a^5*b^2 + a^3*
b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b
^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c + dx)}{a + b\sin^3(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x)**3), x)
```

Giac [A]
time = 0.50, size = 510, normalized size = 1.32

$$\frac{\frac{4\left(3a^5b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-6a^3b^6\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3ab^8\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2a^6b^3+3a^4b^5-b^9\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^9b-4a^7b^3+6a^5b^5-4a^3b^7+ab^9}+\frac{4\left(3\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}ab+\left(2\sqrt{3}a^2b+\sqrt{3}b^3\right)\left(-ab^2\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^5-2a^3b^2+ab^4}-\frac{2\left(3\left(-ab^2\right)^{\frac{2}{3}}ab-\left(2a^2b+b^3\right)\left(-ab^2\right)^{\frac{1}{3}}\right)\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^5-2a^3b^2+ab^4}+\frac{4\left(a^2b+2b^3\right)\log\left(\left|b\sin(dx+c)^3+a\right|\right)}{a^4-2a^2b^2+b^4}+\frac{3(a-4b)\log(|\sin(dx+c)+1|)}{a^2-2ab+b^2}-\frac{3(a+4b)\log(|\sin(dx+c)-1|)}{a^2+2ab+b^2}+\frac{6\left(a^2b\sin(dx+c)^2+2b^3\sin(dx+c)^2-a^3\sin(dx+c)+ab^2\sin(dx+c)-3b^3\right)}{\left(a^4-2a^2b^2+b^4\right)\left(\sin(dx+c)^2-1\right)}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/12*(4*(3*a^5*b^4*(-a/b)^(1/3) - 6*a^3*b^6*(-a/b)^(1/3) + 3*a*b^8*(-a/b)^(
1/3) - 2*a^6*b^3 + 3*a^4*b^5 - b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + si
n(d*x + c)))/(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9) + 4*(3*sqr
t(3)*(-a*b^2)^(2/3)*a*b + (2*sqrt(3)*a^2*b + sqrt(3)*b^3)*(-a*b^2)^(1/3))*a
rctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^5 - 2*a^
3*b^2 + a*b^4) - 2*(3*(-a*b^2)^(2/3)*a*b - (2*a^2*b + b^3)*(-a*b^2)^(1/3))*
log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^5 - 2*a^3
*b^2 + a*b^4) + 4*(a^2*b + 2*b^3)*log(abs(b*sin(d*x + c)^3 + a))/(a^4 - 2*a
^2*b^2 + b^4) + 3*(a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2)
- 3*(a + 4*b)*log(abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 6*(a^2*b*sin
(d*x + c)^2 + 2*b^3*sin(d*x + c)^2 - a^3*sin(d*x + c) + a*b^2*sin(d*x + c)
- 3*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

Mupad [B]
time = 15.27, size = 898, normalized size = 2.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] (symsum(log((a*b^6)/(2*(a^4 + b^4 - 2*a^2*b^2)) - root(54*a^4*b^2*z^3 - 27*
a^2*b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^
3, z, k)*(root(54*a^4*b^2*z^3 - 27*a^2*b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3*z^
2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z, k)*(root(54*a^4*b^2*z^3 - 27*a^2*b
^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z,
 k)*((48*a*b^9 + (51*a^3*b^7)/2 - 87*a^5*b^5 + (27*a^7*b^3)/2)/(a^4 + b^4 -
 2*a^2*b^2) + root(54*a^4*b^2*z^3 - 27*a^2*b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^
3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z, k)*((36*a*b^10 + 108*a^3*b^8 -
 324*a^5*b^6 + 180*a^7*b^4)/(a^4 + b^4 - 2*a^2*b^2) + (sin(c + d*x)*(648*a^
2*b^9 - 1080*a^4*b^7 + 216*a^6*b^5 + 216*a^8*b^3))/(4*(a^4 + b^4 - 2*a^2*b^
2))) + (sin(c + d*x)*(48*b^10 + 552*a^2*b^8 - 600*a^4*b^6))/(4*(a^4 + b^4 -
 2*a^2*b^2))) - (12*a*b^8 - (219*a^3*b^6)/4 + 18*a^5*b^4)/(a^4 + b^4 - 2*a^
2*b^2) + (sin(c + d*x)*(96*b^9 + 120*a^2*b^7 - 171*a^4*b^5))/(4*(a^4 + b^4
```

```
- 2*a^2*b^2))) - (28*a*b^7 - 6*a^3*b^5)/(a^4 + b^4 - 2*a^2*b^2) + (sin(c +
d*x)*(40*b^8 + 61*a^2*b^6))/(4*(a^4 + b^4 - 2*a^2*b^2))) + (2*b^7*sin(c + d
*x))/(a^4 + b^4 - 2*a^2*b^2))*root(54*a^4*b^2*z^3 - 27*a^2*b^4*z^3 - 27*a^6
*z^3 + 54*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z, k), k, 1, 3) +
 (b/(2*(a^2 - b^2)) - (a*sin(c + d*x))/(2*(a^2 - b^2)))/(sin(c + d*x)^2 - 1
) - (log(sin(c + d*x) - 1)*(a + 4*b))/(8*a*b + 4*a^2 + 4*b^2) + (log(sin(c
+ d*x) + 1)*(a - 4*b))/(4*a^2 - 8*a*b + 4*b^2))/d
```

3.388 $\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=764

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\cdots}{\sqrt{\cdots}}\right.}{3\sqrt{a^{2/3}-b\cdots}}$$

```
[Out] -cos(d*x+c)/b/d+2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^
(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*a^(2/3)*arctan((b^(1/3)
+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)-b^
(2/3))^(1/2)-4/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/
3))^(1/2))/b^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+4/3*arctanh((b^(1/3)+(-1)^(2/3
)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d
/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+4/3*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)
*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d/(-(-1)^
(2/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2
*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+(-1)^(1
/3)*b^(2/3))^(1/2)-2/3*(-1)^(1/3)*a^(2/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3
)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3
)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-1)^(2/3)*a^(2/3)*arctan(((-1)^(1/3)*b^(1/
3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^(4/3)/
d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2
/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)
/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]
time = 1.05, antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3305, 3292, 2739, 632, 210, 3299, 212, 2718}

$$-\frac{2(-1)^{2/3}a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{4\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\sqrt[3]{-1}\,a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{4\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{4\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*(-1)^(2/3)*a^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*b^(4/3)*d) + (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
 - b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*a^(2/3)*ArcTan[(b^
(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3)
 - b^(2/3)]*b^(4/3)*d) - (4*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqr
t[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) + (2*ArcTan[((
```

```
-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(
2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*(-1)^(1/3)*a^
(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) +
 (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d) - (
2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[
a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*d) + (4*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((
-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^
(2/3)*d) + (4*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(2/
3)*d) - Cos[c + d*x]/(b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3305

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)
), x_Symbol] :> Int[Expand[(1 - Sin[e + f*x]^2)^(m/2)/(a + b*Sin[e + f*x]^n
), x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m/2, 0] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{a+b\sin^3(c+dx)} - \frac{2\sin^2(c+dx)}{a+b\sin^3(c+dx)} + \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\right)dx$$

$$= -\left(2\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx\right) + \int \frac{1}{a+b\sin^3(c+dx)}\,dx + \int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)} \cdots$$

$$= -\left(2\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx\right.} \cdots\right.\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,\mathrm{s}\cdots}}{3a^{2/3}}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\cdots\right.}\right.}{b}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}\,a^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+\cdots}}{3b^{4/3}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{4\,\mathrm{t}\cdots}{\cdots}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{4\,\mathrm{t}\cdots}{\cdots}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} - \frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\,\mathrm{t}\cdots}{\sqrt{a^{2/3}-(-1)\cdots}}\right.}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.12, size = 300, normalized size = 0.39

$$-\frac{3\cos(c+dx) + i\,\mathrm{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right) - ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right) - 2ia\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1 - a\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1 + 2ia\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^3 + a\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^3 + 2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4 - ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/3*(3*Cos[c + d*x] + I*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b
*#1^4 + I*b*#1^6 & , (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2] - (2*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1 - a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + (2*I)*a*ArcTan[Sin[c
+ d*x]/(Cos[c + d*x] - #1)]*#1^3 + a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3
 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(b*d
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.68, size = 121, normalized size = 0.16

method	result
derivativedivides	$\frac{\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4 b-2\_R^3 a-6\_R^2 b-2\_R a+b\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5 a+2\_R^3 a+4\_R^2 b+\_R a}}{3b}-\frac{}{b\left(1+\right.}}{d}$
default	$\frac{\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4 b-2\_R^3 a-6\_R^2 b-2\_R a+b\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5 a+2\_R^3 a+4\_R^2 b+\_R a}}{3b}-\frac{}{b\left(1+\right.}}{d}$
risch	$-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(729a^4b^8d^6\_Z^6-729a^4b^6d^4\_Z^4+\left(162a^4b^4d^2+81a^2b^6d^2\right)\_Z^2+a^6-3a^4b^2\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3/b*sum((_R^4*b-2*_R^3*a-6*_R^2*b-2*_R*a+b)/(_R^5*a+2*_R^3*a+4*_R^2*
b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2
*a+a))-2/b/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -(b*d*integrate(4*(3*a*b*cos(4*d*x + 4*c)^2 + 3*a*b*cos(2*d*x + 2*c)^2 + 3*
a*b*sin(4*d*x + 4*c)^2 + 3*b^2*cos(d*x + c)*sin(2*d*x + 2*c) + 3*a*b*sin(2*
d*x + 2*c)^2 + b^2*sin(d*x + c) - (a*b*cos(4*d*x + 4*c) - a*b*cos(2*d*x + 2
*c) + b^2*sin(5*d*x + 5*c) + b^2*sin(d*x + c))*cos(6*d*x + 6*c) - (8*a*b*co
```

```
s(3*d*x + 3*c) + 3*b^2*sin(4*d*x + 4*c) - 3*b^2*sin(2*d*x + 2*c))*cos(5*d*x
 + 5*c) - (6*a*b*cos(2*d*x + 2*c) + 8*a^2*sin(3*d*x + 3*c) - 3*b^2*sin(d*x
+ c) - a*b)*cos(4*d*x + 4*c) - 8*(a*b*cos(d*x + c) + a^2*sin(2*d*x + 2*c))*
cos(3*d*x + 3*c) - (3*b^2*sin(d*x + c) + a*b)*cos(2*d*x + 2*c) + (b^2*cos(5
*d*x + 5*c) + b^2*cos(d*x + c) - a*b*sin(4*d*x + 4*c) + a*b*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + (3*b^2*cos(4*d*x + 4*c) - 3*b^2*cos(2*d*x + 2*c) - 8*
a*b*sin(3*d*x + 3*c) + b^2)*sin(5*d*x + 5*c) + (8*a^2*cos(3*d*x + 3*c) - 3*
b^2*cos(d*x + c) - 6*a*b*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 8*(a^2*cos(2*
d*x + 2*c) - a*b*sin(d*x + c))*sin(3*d*x + 3*c))/(b^3*cos(6*d*x + 6*c)^2 +
9*b^3*cos(4*d*x + 4*c)^2 + 64*a^2*b*cos(3*d*x + 3*c)^2 + 9*b^3*cos(2*d*x +
2*c)^2 + b^3*sin(6*d*x + 6*c)^2 + 9*b^3*sin(4*d*x + 4*c)^2 + 64*a^2*b*sin(3
*d*x + 3*c)^2 - 48*a*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*b^3*sin(2*d*
x + 2*c)^2 - 6*b^3*cos(2*d*x + 2*c) + b^3 - 2*(3*b^3*cos(4*d*x + 4*c) - 3*b
^3*cos(2*d*x + 2*c) - 8*a*b^2*sin(3*d*x + 3*c) + b^3)*cos(6*d*x + 6*c) - 6*
(3*b^3*cos(2*d*x + 2*c) + 8*a*b^2*sin(3*d*x + 3*c) - b^3)*cos(4*d*x + 4*c)
- 2*(8*a*b^2*cos(3*d*x + 3*c) + 3*b^3*sin(4*d*x + 4*c) - 3*b^3*sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + 6*(8*a*b^2*cos(3*d*x + 3*c) - 3*b^3*sin(2*d*x + 2*
c))*sin(4*d*x + 4*c) + 16*(3*a*b^2*cos(2*d*x + 2*c) - a*b^2)*sin(3*d*x + 3*
c)), x) + cos(d*x + c))/(b*d)
```

Fricas [C] Result contains complex when optimal does not.
time = 4.64, size = 23437, normalized size = 30.68

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2/3)*sqrt(1/6)*b*d*sqrt(((6*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(3/(b^4
*d^4) - (2*a^2 + b^2)/(a^2*b^4*d^4))/(54/(b^6*d^6) - 27*(2*a^2 + b^2)/(a^2*
b^6*d^6) - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)/(a^4*b^8*d^6) - (a^6 - 3*a^4
*b^2 - 24*a^2*b^4 - b^6)/(a^4*b^8*d^6))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*(54/(b^6*d^6) - 27*(2*a^2 + b^2)/(a^2*b^6*d^6) - (a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)/(a^4*b^8*d^6) - (a^6 - 3*a^4*b^2 - 24*a^2*b^4 - b^6)/(a^4*b^8*d^
6))^(1/3) - 6/(b^2*d^2))*b^2*d^2 + 3*sqrt(1/3)*b^2*d^2*sqrt(-((6*(1/2)^(2/3
)*(-I*sqrt(3) + 1)*(3/(b^4*d^4) - (2*a^2 + b^2)/(a^2*b^4*d^4))/(54/(b^6*d^6
) - 27*(2*a^2 + b^2)/(a^2*b^6*d^6) - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)/(a
^4*b^8*d^6) - (a^6 - 3*a^4*b^2 - 24*a^2*b^4 - b^6)/(a^4*b^8*d^6))^(1/3) + (
1/2)^(1/3)*(I*sqrt(3) + 1)*(54/(b^6*d^6) - 27*(2*a^2 + b^2)/(a^2*b^6*d^6) -
 (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)/(a^4*b^8*d^6) - (a^6 - 3*a^4*b^2 - 24*
a^2*b^4 - b^6)/(a^4*b^8*d^6))^(1/3) - 6/(b^2*d^2))^2*a^2*b^4*d^4 + 12*(6*(1
/2)^(2/3)*(-I*sqrt(3) + 1 ...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(cos(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 18.04, size = 2338, normalized size = 3.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(172032*a^4*b^9 - 81920*a^2*b^11 - 98304*a^6*b^7 + 8192*a^8*b^5 -
 319488*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b
^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)*a^2*b^12 - 688128*root(72
9*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*
b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)*a^4*b^10 + 344064*root(729*a^4*b^8*d^6 -
 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4
 + a^6 - b^6, d, k)*a^6*b^8 - 98304*a*b^12*tan(c/2 + (d*x)/2) - 294912*root
(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a
^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^2*a^2*b^13 - 1400832*root(729*a^4*b^8
*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a
^2*b^4 + a^6 - b^6, d, k)^2*a^4*b^11 + 1695744*root(729*a^4*b^8*d^6 - 729*a
^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6
 - b^6, d, k)^2*a^6*b^9 + 5750784*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 +
162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)
^3*a^4*b^12 - 3760128*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*
d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^3*a^6*b^10
+ 3317760*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2
```

```
*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^4*b^13 - 3317760*ro
ot(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3
*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^6*b^11 - 7962624*root(729*a^4*b
^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3
*a^2*b^4 + a^6 - b^6, d, k)^5*a^4*b^14 + 5971968*root(729*a^4*b^8*d^6 - 729
*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a
^6 - b^6, d, k)^5*a^6*b^12 + 196608*a^3*b^10*tan(c/2 + (d*x)/2) - 98304*a^5
*b^8*tan(c/2 + (d*x)/2) - 1277952*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 +
162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)
*a^3*b^11*tan(c/2 + (d*x)/2) + 712704*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^
4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d
, k)*a^5*b^9*tan(c/2 + (d*x)/2) + 98304*root(729*a^4*b^8*d^6 - 729*a^4*b^6*
d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6,
 d, k)*a^7*b^7*tan(c/2 + (d*x)/2) + 589824*root(729*a^4*b^8*d^6 - 729*a^4*b
^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b
^6, d, k)^2*a^3*b^12*tan(c/2 + (d*x)/2) - 294912*root(729*a^4*b^8*d^6 - 729
*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a
^6 - b^6, d, k)^2*a^5*b^10*tan(c/2 + (d*x)/2) - 294912*root(729*a^4*b^8*d^6
 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b
^4 + a^6 - b^6, d, k)^2*a^7*b^8*tan(c/2 + (d*x)/2) + 5308416*root(729*a^4*b
^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3
*a^2*b^4 + a^6 - b^6, d, k)^3*a^3*b^13*tan(c/2 + (d*x)/2) - 3538944*root(72
9*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*
b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^3*a^5*b^11*tan(c/2 + (d*x)/2) + 221184*r
oot(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 -
3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^3*a^7*b^9*tan(c/2 + (d*x)/2) - 530
8416*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*
d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^3*b^14*tan(c/2 + (d*x)/2
) + 5308416*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a
^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^5*b^12*tan(c/2 +
(d*x)/2) - 1990656*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2
 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^5*a^5*b^13*tan
(c/2 + (d*x)/2) - 196608*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b
^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)*a*b^13*t
an(c/2 + (d*x)/2))*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2
 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k), k, 1, 6)/d -
2/(d*(b + b*tan(c/2 + (d*x)/2)^2))
```

3.389 $\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=484

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{2/3}d}+\frac{2\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}\,d}$$

```
[Out] 2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^
(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2
*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*arctanh
((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3
))^(1/2))/b^(2/3)/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*arctanh((b^(1/3)
-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)
)/b^(2/3)/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1
/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)
/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(
2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3
)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3305, 3292, 2739, 632, 210, 3299, 212}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\,\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c +
 d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) +
 (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (
-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*
ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^
(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]
*d) + (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1
)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(2
/3)*d) + (2*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-
1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(2/3)
*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3305

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)
), x_Symbol] :> Int[Expand[(1 - Sin[e + f*x]^2)^(m/2)/(a + b*Sin[e + f*x]^n
), x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m/2, 0] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{a+b\sin^3(c+dx)} - \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\right)dx$$

$$= \int \frac{1}{a+b\sin^3(c+dx)}\,dx - \int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= -\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}{3a^{2/3}}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d} - \frac{2\text{Subst}\left(\int \frac{}{-\sqrt[3]{a}+2\sqrt[3]{}}\right.}{}$$

$$= \frac{4\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d} + \frac{4\text{Subst}\left(\int \right.}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.07, size = 231, normalized size = 0.48

$$-\frac{i\,\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-2i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right]}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((-1/6*I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^
6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2] + 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (2*I)*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
#1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2
- 2*b*#1^3 + b*#1^5) & ])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.93, size = 83, normalized size = 0.17

method	result
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(729a^4b^4d^6\_Z^6+27a^2b^2d^2\_Z^2+a^2-b^2\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}-\frac{243a^4b^3d^5\_R^5}{a^2+b^2}+\frac{81id^4b^3a^3\_R^4}{a^2+b^2}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 17.40, size = 951, normalized size = 1.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(24576*a^4 - 24576*a^2*b^2 - 122880*root(729*a^4*b^4*d^6 + 27*a^2
*b^2*d^2 + a^2 - b^2, d, k)*a^2*b^3 - 24576*root(729*a^4*b^4*d^6 + 27*a^2*b
^2*d^2 + a^2 - b^2, d, k)*a^5*tan(c/2 + (d*x)/2) - 32768*a*b^3*tan(c/2 + (d
*x)/2) + 32768*a^3*b*tan(c/2 + (d*x)/2) - 294912*root(729*a^4*b^4*d^6 + 27*
a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^2*b^4 + 294912*root(729*a^4*b^4*d^6 + 27
*a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^4*b^2 + 663552*root(729*a^4*b^4*d^6 + 2
7*a^2*b^2*d^2 + a^2 - b^2, d, k)^4*a^4*b^4 - 663552*root(729*a^4*b^4*d^6 +
27*a^2*b^2*d^2 + a^2 - b^2, d, k)^4*a^6*b^2 - 7962624*root(729*a^4*b^4*d^6
+ 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^5*a^4*b^5 + 5971968*root(729*a^4*b^4*d^
6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^5*a^6*b^3 + 49152*root(729*a^4*b^4*d^
6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)*a^4*b + 147456*root(729*a^4*b^4*d^6 +
 27*a^2*b^2*d^2 + a^2 - b^2, d, k)*a^3*b^2*tan(c/2 + (d*x)/2) + 294912*root
(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^5*b*tan(c/2 + (d*x
)/2) - 294912*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^
3*b^3*tan(c/2 + (d*x)/2) + 1769472*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 +
a^2 - b^2, d, k)^3*a^3*b^4*tan(c/2 + (d*x)/2) - 1769472*root(729*a^4*b^4*d^
6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^3*a^5*b^2*tan(c/2 + (d*x)/2) - 530841
6*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^4*a^3*b^5*tan(c/
2 + (d*x)/2) + 5308416*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d
, k)^4*a^5*b^3*tan(c/2 + (d*x)/2) - 1990656*root(729*a^4*b^4*d^6 + 27*a^2*b
^2*d^2 + a^2 - b^2, d, k)^5*a^5*b^4*tan(c/2 + (d*x)/2) - 196608*root(729*a^
4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)*a*b^4*tan(c/2 + (d*x)/2))*roo
t(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k), k, 1, 6)/d
```

3.390 $\int \frac{1}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\,\tan\ldots\right.}{\sqrt{a^{2/3}-(-1)^{2/3}b\ldots}}\right.}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/}\ldots}}$$

```
[Out] 2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^
(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(
1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+(-1)
^(1/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan
(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)-(-1
)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3292, 2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\,\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/
3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a
^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/
3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/
3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx\right.}\right.$$

$$=-\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}{3a^{2/3}}$$

$$=-\frac{2\text{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}-\frac{2\text{Subst}\left(\int \frac{}{-\sqrt[3]{a}+2\sqrt[3]{}}\right.}{}$$

$$=\frac{4\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}+\frac{4\text{Subst}\left(\int \right.}{}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+-\frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 126, normalized size = 0.51

$$-\frac{2i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1}{b-4ia\#1-2b\#1^2+b\#1^4}\&\right]}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (((-2*I)/3)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#
1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.47, size = 83, normalized size = 0.34

method	result
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+8b\_Z^{3}+3a\_Z^{2}+a\right)}\frac{\left(\_R^{4}+2\_R^{2}+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}}{3d}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+8b\_Z^{3}+3a\_Z^{2}+a\right)}\frac{\left(\_R^{4}+2\_R^{2}+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}}{3d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+\left(729a^{6}d^{6}-729a^{4}b^{2}d^{6}\right)\_Z^{6}+243a^{4}d^{4}\_Z^{4}+27a^{2}d^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(-\frac{486d^{5}a^{6}}{b}+486b\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.49, size = 25429, normalized size = 103.79

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2/3)*sqrt(1/6)*sqrt(-((a^2 - b^2)*((-I*sqrt(3) + 1)*(1/(a^4*d^4 -
 a^2*b^2*d^4) - 1/(a^2*d^2 - b^2*d^2)^2)/(-1/1458/(a^6*d^6 - a^4*b^2*d^6) +
 1/486/((a^4*d^4 - a^2*b^2*d^4)*(a^2*d^2 - b^2*d^2)) - 1/729/(a^2*d^2 - b^2
*d^2)^3 + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) - 81*(I*sqrt(3) + 1)*(-
1/1458/(a^6*d^6 - a^4*b^2*d^6) + 1/486/((a^4*d^4 - a^2*b^2*d^4)*(a^2*d^2 -
b^2*d^2)) - 1/729/(a^2*d^2 - b^2*d^2)^3 + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6
))^(1/3) - 18/(a^2*d^2 - b^2*d^2))*d^2 - 3*sqrt(1/3)*(a^2 - b^2)*d^2*sqrt(-
((a^6 - 2*a^4*b^2 + a^2*b^4)*((-I*sqrt(3) + 1)*(1/(a^4*d^4 - a^2*b^2*d^4) -
 1/(a^2*d^2 - b^2*d^2)^2)/(-1/1458/(a^6*d^6 - a^4*b^2*d^6) + 1/486/((a^4*d^
4 - a^2*b^2*d^4)*(a^2*d^2 - b^2*d^2)) - 1/729/(a^2*d^2 - b^2*d^2)^3 + 1/145
8*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) - 81*(I*sqrt(3) + 1)*(-1/1458/(a^6*d^6
 - a^4*b^2*d^6) + 1/486/((a^4*d^4 - a^2*b^2*d^4)*(a^2*d^2 - b^2*d^2)) - 1/7
29/(a^2*d^2 - b^2*d^2)^3 + 1/1458*b^2/((a^2 - b^2)^2*a^4*d^6))^(1/3) - 18/(
a^2*d^2 - b^2*d^2))^2*d^4 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin^3(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 16.71, size = 609, normalized size = 2.49

$$\frac{\sum_{k=1}^{6}\ln\left(-\frac{8192\,a\,b^3\left(-729\,a^5+243\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,b-324\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)+972\,a^3\,b^2+a^3\,b\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)243-162\,a^3\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)^2+648\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b^2\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)+216\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)^2-72\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)^3+36\,a\,b\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)^3-9\,a\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)^4+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)b\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)^5\right)}{\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,(a^2-b^2),d,k\right)^5}\right)\mathrm{root}(729\,a^4\,b^2\,d^6-729\,a^6\,d^6-243\,a^4\,d^4-27\,a^2\,d^2-1,d,k)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(-(8192*a*b^3*(972*a^3*b^2 - 729*a^5 - 9*a*root(d^6 + 27*a^2*d^4
+ 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 162*a^3*root(d^6 + 27*a^2*d^
```

4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2 - 4*tan(c/2 + (d*x)/2)*root(
d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5 + 243*a^4*b*t
an(c/2 + (d*x)/2) - 324*tan(c/2 + (d*x)/2)*a^4*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 24*b*tan(c/2 + (d*x)/2)*root(d^6 + 2
7*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 72*a^2*tan(c/2 + (
d*x)/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3
+ 36*a*b*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3
 + 243*b*a^3*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d,
k) + 648*tan(c/2 + (d*x)/2)*a^2*b^2*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 7
29*a^4*(a^2 - b^2), d, k) + 216*a^2*b*tan(c/2 + (d*x)/2)*root(d^6 + 27*a^2*
d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^6 + 27*a^2*d^4 +
243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*b^2*d^6 - 729*a^6*
d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)/d

3.391 $\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=299

$$\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\left(a^{2/3}-(-1)^{2/3}b^{2/3}\right)^{3/2}d}-\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\left(a^{2/3}-b^{2/3}\right)^{3/2}d}+\frac{2\sqrt[3]{-1}\,b^{2/3}\tan^{-1}}{3a^{2/3}\,(a}$$

```
[Out] 2/3*(-1)^(2/3)*b^(2/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c
))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)-(-1)^(2/3)*b^(2/3))
^(3/2)/d-2/3*b^(2/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b
^(2/3))^(1/2))/a^(2/3)/(a^(2/3)-b^(2/3))^(3/2)/d+2/3*(-1)^(1/3)*b^(2/3)*arc
tan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(
2/3))^(1/2))/a^(2/3)/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(3/2)/d+sec(d*x+c)*(b-a*s
in(d*x+c))/(-a^2+b^2)/d
```

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx=\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.20, size = 432, normalized size = 1.44

$$\frac{-6b+6b\cos(c+dx)-ib\cos(c+dx)\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+4ia\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+2a\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1-12b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+6ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-4ia\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^3-2a\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^3+2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right]+6a\sin(c+dx)}{6(a-b)(a+b)d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-6*b + 6*b*Cos[c + d*x] - I*b*Cos[c + d*x]*RootSum[(-I)*b + (3*I)*b*#1^2 +
 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)] - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (4*I)*a*ArcTan[Sin[c +
 d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 -
12*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (6*I)*b*Log[1 - 2*Cos[
c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - 2*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2*b*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(
b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + 6*a*Sin[c + d*x])/(6*(a - b)
*(a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.12, size = 162, normalized size = 0.54

method	result
derivativedivides	$\frac{-\frac{2}{(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(-\_R^4 b+2\_R^3 a-6\_R^2 b+2\_R a-b\right)}{\_R^5 a+2\_R^3 a+4\_R^2 b\cdots}\right.}{3(a-b)(a+b)}}{d}$
default	$\frac{-\frac{2}{(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(-\_R^4 b+2\_R^3 a-6\_R^2 b+2\_R a-b\right)}{\_R^5 a+2\_R^3 a+4\_R^2 b\cdots}\right.}{3(a-b)(a+b)}}{d}$
risch	$\frac{-2ia+2b\,\mathrm{e}^{i(dx+c)}}{d(-a^2+b^2)\left(\mathrm{e}^{2i(dx+c)}+1\right)}+4\left(\sum_{\_R=\mathrm{RootOf}\left(\left(2985984a^{10}d^6-8957952a^8b^2d^6+8957952a^6b^4d^6-2985984a^4b^6d^6\right)\_Z^6+\cdots\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)+1/3*b/(a-b)/(a+b)*sum((-_R^4*b+2*_
R^3*a-6*_R^2*b+2*_R*a-b)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2
*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-2/(2*a-2*b)/(tan(1/
2*d*x+1/2*c)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -(2*b*cos(2*d*x + 2*c)*cos(d*x + c) + 2*b*cos(d*x + c) - ((a^2 - b^2)*d*cos
(2*d*x + 2*c)^2 + (a^2 - b^2)*d*sin(2*d*x + 2*c)^2 + 2*(a^2 - b^2)*d*cos(2*
d*x + 2*c) + (a^2 - b^2)*d)*integrate(2*(6*a*b^2*cos(4*d*x + 4*c)^2 - 48*a*
b^2*cos(3*d*x + 3*c)^2 + 6*a*b^2*cos(2*d*x + 2*c)^2 + 6*a*b^2*sin(4*d*x + 4
*c)^2 - 48*a*b^2*sin(3*d*x + 3*c)^2 - 3*b^3*cos(d*x + c)*sin(2*d*x + 2*c) +
 6*a*b^2*sin(2*d*x + 2*c)^2 - b^3*sin(d*x + c) - (2*a*b^2*cos(4*d*x + 4*c)
- 2*a*b^2*cos(2*d*x + 2*c) - b^3*sin(5*d*x + 5*c) + 6*b^3*sin(3*d*x + 3*c)
- b^3*sin(d*x + c))*cos(6*d*x + 6*c) + (8*a*b^2*cos(3*d*x + 3*c) + 3*b^3*si
n(4*d*x + 4*c) - 3*b^3*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - (12*a*b^2*cos(2
*d*x + 2*c) + 3*b^3*sin(d*x + c) - 2*a*b^2 + 2*(8*a^2*b - 9*b^3)*sin(3*d*x
+ 3*c))*cos(4*d*x + 4*c) + 2*(4*a*b^2*cos(d*x + c) - (8*a^2*b - 9*b^3)*sin(
2*d*x + 2*c))*cos(3*d*x + 3*c) + (3*b^3*sin(d*x + c) - 2*a*b^2)*cos(2*d*x +
 2*c) - (b^3*cos(5*d*x + 5*c) - 6*b^3*cos(3*d*x + 3*c) + b^3*cos(d*x + c) +
 2*a*b^2*sin(4*d*x + 4*c) - 2*a*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) - (3
*b^3*cos(4*d*x + 4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*sin(3*d*x + 3*c) +
 b^3)*sin(5*d*x + 5*c) + (3*b^3*cos(d*x + c) - 12*a*b^2*sin(2*d*x + 2*c) +
2*(8*a^2*b - 9*b^3)*cos(3*d*x + 3*c))*sin(4*d*x + 4*c) + 2*(4*a*b^2*sin(d*x
 + c) + 3*b^3 + (8*a^2*b - 9*b^3)*cos(2*d*x + 2*c))*sin(3*d*x + 3*c))/(a^2*
b^2 - b^4 + (a^2*b^2 - b^4)*cos(6*d*x + 6*c)^2 + 9*(a^2*b^2 - b^4)*cos(4*d*
x + 4*c)^2 + 64*(a^4 - a^2*b^2)*cos(3*d*x + 3*c)^2 + 9*(a^2*b^2 - b^4)*cos(
2*d*x + 2*c)^2 + (a^2*b^2 - b^4)*sin(6*d*x + 6*c)^2 + 9*(a^2*b^2 - b^4)*sin
(4*d*x + 4*c)^2 + 64*(a^4 - a^2*b^2)*sin(3*d*x + 3*c)^2 - 48*(a^3*b - a*b^3
)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*(a^2*b^2 - b^4)*sin(2*d*x + 2*c)^2
- 2*(a^2*b^2 - b^4 + 3*(a^2*b^2 - b^4)*cos(4*d*x + 4*c) - 3*(a^2*b^2 - b^4)
*cos(2*d*x + 2*c) - 8*(a^3*b - a*b^3)*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) +
6*(a^2*b^2 - b^4 - 3*(a^2*b^2 - b^4)*cos(2*d*x + 2*c) - 8*(a^3*b - a*b^3)*s
in(3*d*x + 3*c))*cos(4*d*x + 4*c) - 6*(a^2*b^2 - b^4)*cos(2*d*x + 2*c) - 2*
(8*(a^3*b - a*b^3)*cos(3*d*x + 3*c) + 3*(a^2*b^2 - b^4)*sin(4*d*x + 4*c) -
3*(a^2*b^2 - b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*(a^3*b - a*b^3)
*cos(3*d*x + 3*c) - 3*(a^2*b^2 - b^4)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) -
16*(a^3*b - a*b^3 - 3*(a^3*b - a*b^3)*cos(2*d*x + 2*c))*sin(3*d*x + 3*c)),
x) + 2*(b*sin(d*x + c) - a)*sin(2*d*x + 2*c))/((a^2 - b^2)*d*cos(2*d*x + 2*
c)^2 + (a^2 - b^2)*d*sin(2*d*x + 2*c)^2 + 2*(a^2 - b^2)*d*cos(2*d*x + 2*c)
+ (a^2 - b^2)*d)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.80, size = 59362, normalized size = 198.54

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2/3)*sqrt(1/2)*(a^2 - b^2)*d*sqrt(-(54*a^2*b^2 + 108*b^4 + (a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((b^4/(a^8*d^4 - 3*a^6*b^2*d^4 + 3*a^4*b^4*d
```

```
^4 - a^2*b^6*d^4) - 3*(a^2*b^2 + 2*b^4)^2/(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a^2*
b^4*d^2 - b^6*d^2)^2)*(-I*sqrt(3) + 1)/(-1/1458*b^4/(a^10*d^6 - 3*a^8*b^2*d
^6 + 3*a^6*b^4*d^6 - a^4*b^6*d^6) + 1/54*(a^2*b^2 + 2*b^4)*b^4/((a^8*d^4 -
3*a^6*b^2*d^4 + 3*a^4*b^4*d^4 - a^2*b^6*d^4)*(a^6*d^2 - 3*a^4*b^2*d^2 + 3*a
^2*b^4*d^2 - b^6*d^2)) - 1/27*(a^2*b^2 + 2*b^4)^3/(a^6*d^2 - 3*a^4*b^2*d^2
+ 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*(a^6 + 24*a^4*b^2 + 3*a^2*b^4 - b^6)*
b^4/((a^2 - b^2)^6*a^4*d^6))^(1/3) - 27*(-1/1458*b^4/(a^10*d^6 - 3*a^8*b^2*
d^6 + 3*a^6*b^4*d^6 - a^4*b^6*d^6) + 1/54*(a^2*b^2 + 2*b^4)*b^4/((a^8*d^4 -
 3*a^6*b^2*d^4 + 3*a^4*b^4*d^4 - a^2*b^6*d^4)*(a^6*d^2 - 3*a^4*b^2*d^2 + 3*
a^2*b^4*d^2 - b^6*d^2)) - 1/27*(a^2*b^2 + 2*b^4)^3/(a^6*d^2 - 3*a^4*b^2*d^2
 + 3*a^2*b^4*d^2 - b^6*d^2)^3 - 1/1458*(a^6 + 24*a^4*b^2 + 3*a^2*b^4 - b^6)
*b^4/((a^2 - b^2)^6*a^4*d ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 18.52, size = 2500, normalized size = 8.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] (a^2*symsum(log((8192*(a^2*b^20*cos(c/2 + (d*x)/2) - 12*a*b^21*sin(c/2 + (d
*x)/2) - 7*a^4*b^18*cos(c/2 + (d*x)/2) + 21*a^6*b^16*cos(c/2 + (d*x)/2) - 3
5*a^8*b^14*cos(c/2 + (d*x)/2) + 35*a^10*b^12*cos(c/2 + (d*x)/2) - 21*a^12*b
^10*cos(c/2 + (d*x)/2) + 7*a^14*b^8*cos(c/2 + (d*x)/2) - a^16*b^6*cos(c/2 +
```

(d*x)/2) + 84*a^3*b^19*sin(c/2 + (d*x)/2) - 252*a^5*b^17*sin(c/2 + (d*x)/2
) + 420*a^7*b^15*sin(c/2 + (d*x)/2) - 420*a^9*b^13*sin(c/2 + (d*x)/2) + 252
*a^11*b^11*sin(c/2 + (d*x)/2) - 84*a^13*b^9*sin(c/2 + (d*x)/2) + 12*a^15*b^
7*sin(c/2 + (d*x)/2) - 198*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a
^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4
*d^2 - b^4, d, k)*a^3*b^20*sin(c/2 + (d*x)/2) + 714*root(2187*a^8*b^2*d^6 -
2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729
*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^5*b^18*sin(c/2 + (d*x)/2) - 14
70*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^
6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^7*b^
16*sin(c/2 + (d*x)/2) + 1890*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729
*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b
^4*d^2 - b^4, d, k)*a^9*b^14*sin(c/2 + (d*x)/2) - 1554*root(2187*a^8*b^2*d^
6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 -
729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^11*b^12*sin(c/2 + (d*x)/2)
+ 798*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10
*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^1
3*b^10*sin(c/2 + (d*x)/2) - 234*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 +
729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^
2*b^4*d^2 - b^4, d, k)*a^15*b^8*sin(c/2 + (d*x)/2) + 30*root(2187*a^8*b^2*d
^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 -
729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^17*b^6*sin(c/2 + (d*x)/2)
+ 36*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*
d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^
2*b^22*cos(c/2 + (d*x)/2) - 369*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 +
729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^
2*b^4*d^2 - b^4, d, k)^2*a^4*b^20*cos(c/2 + (d*x)/2) + 1575*root(2187*a^8*b
^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d
^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^6*b^18*cos(c/2 + (d*
x)/2) - 3717*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 7
29*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d,
k)^2*a^8*b^16*cos(c/2 + (d*x)/2) + 5355*root(2187*a^8*b^2*d^6 - 2187*a^6*b
^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^
4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^10*b^14*cos(c/2 + (d*x)/2) - 4851*root(
2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458
*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^12*b^12*co
s(c/2 + (d*x)/2) + 2709*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*
b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^
2 - b^4, d, k)^2*a^14*b^10*cos(c/2 + (d*x)/2) - 855*root(2187*a^8*b^2*d^6 -
2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729
*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^16*b^8*cos(c/2 + (d*x)/2) +
117*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d
^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^1
8*b^6*cos(c/2 + (d*x)/2) - 1188*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 +
729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^

```
2*b^4*d^2 - b^4, d, k)^3*a^4*b^21*cos(c/2 + (d*x)/2) + 7803*root(2187*a^8*b
^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d
^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^6*b^19*cos(c/2 + (d*
x)/2) - 21357*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 -
729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d
, k)^3*a^8*b^17*cos(c/2 + (d*x)/2) + 30807*root(2187*a^8*b^2*d^6 - 2187*a^6
*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*
d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^10*b^15*cos(c/2 + (d*x)/2) - 23625*ro
ot(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1
458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^12*b^13
*cos(c/2 + (d*x)/2) + 6993*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a
^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4
*d^2 - b^4, d, k)^3*a^14*b^11*cos(c/2 + (d*x)/2) + 2457*root(2187*a^8*b^2*d
^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 -
 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^16*b^9*cos(c/2 + (d*x)/2
) - 2403*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a
^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^
3*a^18*b^7*cos(c/2 + (d*x)/2) + 513*root(2187*a...
```

3.392 $\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=1093

$$-\frac{2(-1)^{2/3}a^{2/3}b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,(a^2-b^2)^2\,d}-\frac{2b^2(2a^2+b^2)\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,(a^2-b^2)^2\,d}$$

```
[Out] 1/12*cos(d*x+c)/(a+b)/d/(1-sin(d*x+c))^2+1/12*cos(d*x+c)/(a+b)/d/(1-sin(d*x
+c))+1/4*(a+4*b)*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/12*cos(d*x+c)/(a-b)/
d/(1+sin(d*x+c))^2-1/4*(a-4*b)*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))-1/12*cos
(d*x+c)/(a-b)/d/(1+sin(d*x+c))+2*a^(2/3)*b^(8/3)*arctan((b^(1/3)+a^(1/3)*ta
n(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/(a^2-b^2)^2/d/(a^(2/3)-b^(2/3))^
(1/2)+2/3*b^2*(2*a^2+b^2)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2
/3)-b^(2/3))^(1/2))/a^(2/3)/(a^2-b^2)^2/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*b^(4/
3)*(a^2+2*b^2)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3)
)^(1/2))/(a^2-b^2)^2/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*b^(4/3)*(a^2+2*b^2)*arct
anh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(
2/3))^(1/2))/(a^2-b^2)^2/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)-2/3*b^(4/3)*(
a^2+2*b^2)*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(
2/3)*a^(2/3)+b^(2/3))^(1/2))/(a^2-b^2)^2/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1
/2)-2*(-1)^(1/3)*a^(2/3)*b^(8/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2
*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/(a^2-b^2)^2/d/(a^(2/3)+(-1
)^(1/3)*b^(2/3))^(1/2)+2/3*b^2*(2*a^2+b^2)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/
3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/(a^2-b^2
)^2/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2*(-1)^(2/3)*a^(2/3)*b^(8/3)*arcta
n(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/
3))^(1/2))/(a^2-b^2)^2/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)-2/3*b^2*(2*a^2+
b^2)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(
2/3)*b^(2/3))^(1/2))/a^(2/3)/(a^2-b^2)^2/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/
2)
```

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx=\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.22, size = 679, normalized size = 0.62

$$\frac{4ib^2\,\mathrm{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2a^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+4b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-ia^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)-2ib^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+12iab\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+6ab\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1-20a^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-16b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+10ia^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2+8ib^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-12iab\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^3-6ab\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^3+2a^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+4b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-ia^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4-2ib^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right]+\sec^3(c+dx)\left(4a^2b+32b^3-3b(5a^2+13b^2)\cos(c+dx)+12b(a^2+2b^2)\cos(2(c+dx))-5a^2b\cos(3(c+dx))-13b^3\cos(3(c+dx))+12a^3\sin(c+dx)-30ab^2\sin(c+dx)+4a^3\sin(3(c+dx))-22ab^2\sin(3(c+dx))\right)}{24(a-b)^2(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((4*I)*b^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1
^6 & , (2*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 4*b^2*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)] - I*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (2
*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (12*I)*a*b*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1 + 6*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 20*a
^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 16*b^2*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 + (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^2 + (8*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*a*b*ArcTa
n[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 6*a*b*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^3 + 2*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 4*b^2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*a^2*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1^4 - (2*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1
 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + Sec[c + d*x]^3*(4*a^2*b + 32*b^3
 - 3*b*(5*a^2 + 13*b^2)*Cos[c + d*x] + 12*b*(a^2 + 2*b^2)*Cos[2*(c + d*x)]
- 5*a^2*b*Cos[3*(c + d*x)] - 13*b^3*Cos[3*(c + d*x)] + 12*a^3*Sin[c + d*x]
- 30*a*b^2*Sin[c + d*x] + 4*a^3*Sin[3*(c + d*x)] - 22*a*b^2*Sin[3*(c + d*x)
]))/(24*(a - b)^2*(a + b)^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.96, size = 291, normalized size = 0.27

method	result
derivativedivides	$\frac{b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\left(2a^2+b^2\right)\_R^4-6ab\_R^3+2\left(4a^2+5b^2\right)\_R^2-6a\_Rb+2a^2+b^2\right)\ln(\cdots}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.\cdots}{3(a-b)^2(a+b)^2}\cdots$

default	$\frac{b^2\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\left(2a^2+b^2\right)\_R^4-6ab\_R^3+2\left(4a^2+5b^2\right)\_R^2-6a\_Rb+2a^2+b^2\right)\ln}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.}{3(a-b)^2(a+b)^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^2/(a-b)^2/(a+b)^2*sum(((2*a^2+b^2)*_R^4-6*a*b*_R^3+2*(4*a^2+5*b^
2)*_R^2-6*a*_R*b+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+
1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-2/3/(tan(1/2*d*x
+1/2*c)+1)^3/(2*a-2*b)+1/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(2*a-5*b)/(
a-b)^2/(tan(1/2*d*x+1/2*c)+1)-2/3/(tan(1/2*d*x+1/2*c)-1)^3/(2*a+2*b)-1/(2*a
+2*b)/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*a+5*b)/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)
)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**C**] Result contains complex when optimal does not.
time = 11.21, size = 85064, normalized size = 77.83

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(2/3)*sqrt(1/6)*(a^4 - 2*a^2*b^2 + b^4)*d*sqrt(-(810*a^4*b^4 + 2
754*a^2*b^6 + 810*b^8 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2
*b^8 - b^10)*((5*b^6/(a^10*d^4 - 4*a^8*b^2*d^4 + 6*a^6*b^4*d^4 - 4*a^4*b^6*
d^4 + a^2*b^8*d^4) + 9*(5*a^4*b^4 + 17*a^2*b^6 + 5*b^8)^2/(a^10*d^2 - 5*a^8
*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8*d^2 - b^10*d^2)^2)*(
-I*sqrt(3) + 1)/(-1/1458*b^8/(a^14*d^6 - 5*a^12*b^2*d^6 + 10*a^10*b^4*d^6 -
 10*a^8*b^6*d^6 + 5*a^6*b^8*d^6 - a^4*b^10*d^6) - 5/162*(5*a^4*b^4 + 17*a^2
*b^6 + 5*b^8)*b^6/((a^10*d^4 - 4*a^8*b^2*d^4 + 6*a^6*b^4*d^4 - 4*a^4*b^6*d^
```

```
4 + a^2*b^8*d^4)*(a^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^
2 + 5*a^2*b^8*d^2 - b^10*d^2)) - 1/27*(5*a^4*b^4 + 17*a^2*b^6 + 5*b^8)^3/(a
^10*d^2 - 5*a^8*b^2*d^2 + 10*a^6*b^4*d^2 - 10*a^4*b^6*d^2 + 5*a^2*b^8*d^2 -
 b^10*d^2)^3 + 1/1458*(a^10 - 30*a^8*b^2 - 700*a^6*b^4 - 700*a^4*b^6 - 30*a
^2*b^8 + b^10)*b^8/((a^2 - b^2)^10*a^4*d^6))^(1/3) + 81*(-1/1458*b^8/(a^14*
d^6 - 5*a^12*b^2*d^6 + 10 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

Mupad [B]
time = 25.85, size = 2500, normalized size = 2.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] (14*b^3*cos(c/2 + (d*x)/2)^6)/(3*(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/
2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 -
2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4
*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4
*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 -
 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2
+ (d*x)/2)^2)) - (4*a^3*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^3)/(3*(a^4*
d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)
/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a
```

^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x
)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/
2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2
+ (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^
2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)) + (6*b^3*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^4)/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 +
(d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^
2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b
^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d
*x)/2)^2) - (8*b^3*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)/(a^4*d*cos(c/
2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 -
b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d
*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 -
3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*
cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) + (4*a^2*b*cos(c/2 + (d*x)/2)^6)
/(3*(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/
2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2
)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c
/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4
*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4
*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)
^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)) + (2*a^3*cos(c
/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^5)/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos
(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6
- 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*
a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2
)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^
4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2
+ (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c
/2 + (d*x)/2)^2) + (2*a^3*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))/(a^4*d*c
os(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)
^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*
b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (
d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b
^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) + (a^4*cos(c/2 + (d*x)/2)^6
symsum(log(-(8192(6*a^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*
x)/2) - 84*a^4*b^36*cos(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) -
2184*a^8*b^32*cos(c/2 + (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 1201

2*a^12*b^28*cos(c/2 + (d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592
*a^16*b^24*cos(c/2 + (d*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*
a^20*b^20*cos(c/2 + (d*x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^2
4*b^16*cos(c/2 + (d*x)/2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12
*cos(c/2 + (d*x)/2) + 6*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2
 + (d*x)/2) - 1820*a^5*b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d
*x)/2) - 20020*a^9*b^31*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x
)/2) - 60060*a^13*b^27*sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)
/2) - 60060*a^17*b^23*sin(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/
2) - 20020*a^21*b^19*sin(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2)
 - 1820*a^25*b^15*sin(c/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 2
0*a^29*b^11*sin(c/2 + (d*x)/2) - 588*root(7290*...

3.393 $\int \frac{\cos^7(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

Optimal. Leaf size=288

$$-\frac{2\left(2a^2+3a^{4/3}b^{2/3}+b^2\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{7/3}d}+\frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{7/3}d}$$

```
[Out] 2/9*(2*a^2-3*a^(4/3)*b^(2/3)+b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^
(7/3)/d-1/9*(2*a^2-3*a^(4/3)*b^(2/3)+b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*
x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/b^(7/3)/d-sin(d*x+c)/b^2/d-1/3*sin(d*x+c
)*(a^2-b^2+3*a*b*sin(d*x+c)+3*b^2*sin(d*x+c)^2)/a/b^2/d/(a+b*sin(d*x+c)^3)-
2/9*(2*a^2+3*a^(4/3)*b^(2/3)+b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))
/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)/d*3^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3302, 1872, 1901, 1874, 31, 648, 631, 210, 642}

$$-\frac{\sin(c+dx)\left(a^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)-b^2\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)}-\frac{2\left(3a^{4/3}b^{2/3}+2a^2+b^2\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{7/3}d}-\frac{\left(-3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}b^{7/3}d}+\frac{2\left(-3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{7/3}d}-\frac{\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d
*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(7/3)*d) + (2*(2*a^2 - 3*a^(4
/3)*b^(2/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b^(7/3)*
d) - ((2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c
 + d*x] + b^(2/3)*Sin[c + d*x]^2])/(9*a^(5/3)*b^(7/3)*d) - Sin[c + d*x]/(b^
2*d) - (Sin[c + d*x]*(a^2 - b^2 + 3*a*b*Sin[c + d*x] + 3*b^2*Sin[c + d*x]^2
))/(3*a*b^2*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
```

```
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a^2-2}{} \right.}{}$$

$$= -\frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(3a - \right.\right.}{}$$

$$= -\frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} +$$

$$= -\frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} +$$

$$= \frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{7/3}d} - \frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+}{}$$

$$= \frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{7/3}d} - \frac{\left(2a^2-3a^{4/3}b^{2/3}+b^2\right.}{}$$

$$= -\frac{2\left(2a^2+3a^{4/3}b^{2/3}+b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}b^{7/3}d} + \frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)}{9a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.44, size = 402, normalized size = 1.40

$$\frac{\frac{6\sqrt[3]{-1}\left(2\sqrt[3]{-1}\,a^{2/3}+3b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}{\sqrt[3]{a}\,b^{7/3}}+\frac{6\left(2a^{2/3}-3b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{\sqrt[3]{a}\,b^{7/3}}-\frac{6\sqrt[3]{-1}\left(2a^{2/3}+3\sqrt[3]{-1}\,b^{2/3}\right)\log\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)\right)}{\sqrt[3]{a}\,b^{7/3}}+\frac{2(a^2-b^2)\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)\right)}{a^{5/3}b^{7/3}}-\frac{18\sin(c+dx)}{b^2}-\frac{27\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)\sin^2(c+dx)}{ab}+\frac{18}{b\left(a+b\sin^3(c+dx)\right)}+\frac{6\left(1-\frac{a^2}{b^2}\right)\sin(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}}{18d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((6*(-1)^(1/3)*(2*(-1)^(1/3)*a^(2/3) + 3*b^(2/3))*Log[-((-1)^(2/3)*a^(1/3))
 - b^(1/3)*Sin[c + d*x]])/(a^(1/3)*b^(7/3)) + (6*(2*a^(2/3) - 3*b^(2/3))*Lo
g[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(1/3)*b^(7/3)) - (6*(-1)^(1/3)*(2*a^(
2/3) + 3*(-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b^(1/3)*Sin[c + d*x]]
)/(a^(1/3)*b^(7/3)) + (2*(a^2 - b^2)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)
*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] - 2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]))/(a^
(5/3)*b^(7/3)) - (18*Sin[c + d*x])/b^2 - (27*Hypergeometric2F1[2/3, 2, 5/3,
 -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a*b) + 18/(b*(a + b*Sin[c + d*x]
^3)) + (6*(1 - a^2/b^2)*Sin[c + d*x])/(a*(a + b*Sin[c + d*x]^3)))/(18*d)
```

Maple [A]

time = 1.30, size = 310, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{\sin(dx+c)}{b^2}+\frac{-\left(\sin^2(dx+c)\right)b-\frac{\left(a^2-b^2\right)\sin(dx+c)}{3a}+b}{a+b\left(\sin^3(dx+c)\right)}+\frac{2\left(2a^2+b^2\right)\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\cdots\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots\right)}{3}\cdots}{\cdots}$
default	$\frac{-\frac{\sin(dx+c)}{b^2}+\frac{-\left(\sin^2(dx+c)\right)b-\frac{\left(a^2-b^2\right)\sin(dx+c)}{3a}+b}{a+b\left(\sin^3(dx+c)\right)}+\frac{2\left(2a^2+b^2\right)\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\cdots\right.\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots\right)}{3}\cdots}{\cdots}$

risch	$\frac{ie^{i(dx+c)}}{2b^2d}-\frac{ie^{-i(dx+c)}}{2b^2d}-\frac{2i\left(3b\,e^{5i(dx+c)}a+6ab\,e^{3i(dx+c)}+2ia^2e^{4i(dx+c)}-2ib^2e^{4i(dx+c)}+3b\,e^{i(dx+c)}a-2ia^2e^{2i(dx+c}\right.}{3b^2da\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2*sin(d*x+c)+1/b^2*((-sin(d*x+c)^2*b-1/3*(a^2-b^2)/a*sin(d*x+c)+b
)/(a+b*sin(d*x+c)^3)+2/3/a*((2*a^2+b^2)*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+
(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)
+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(
1/3)*sin(d*x+c)-1)))+3*a*b*(-1/3/b/(1/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3
))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/
3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x
+c)-1))))))
```

Maxima [A]
time = 0.57, size = 263, normalized size = 0.91

$$-\frac{\frac{3\left(3\,ab\sin(dx+c)^2-3\,ab+(a^2-b^2)\sin(dx+c)\right)}{ab^3\sin(dx+c)^3+a^2b^2}+\frac{9\sin(dx+c)}{b^2}-\frac{2\sqrt{3}\left(3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2+b^2\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2-b^2\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\left(3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2-b^2\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/9*(3*(3*a*b*sin(d*x + c)^2 - 3*a*b + (a^2 - b^2)*sin(d*x + c))/(a*b^3*si
n(d*x + c)^3 + a^2*b^2) + 9*sin(d*x + c)/b^2 - 2*sqrt(3)*(3*a*b*(a/b)^(1/3)
 + 2*a^2 + b^2)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1
/3))/(a*b^3*(a/b)^(2/3)) - (3*a*b*(a/b)^(1/3) - 2*a^2 - b^2)*log(sin(d*x +
c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 2*(3*a
*b*(a/b)^(1/3) - 2*a^2 - b^2)*log((a/b)^(1/3) + sin(d*x + c))/(a*b^3*(a/b)^
(2/3)))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 95.60, size = 6415, normalized size = 22.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(7*a^2*b - b^3)*cos(d*x + c)^4 + 84*a^2*b - 12*b^3 - 6*sqrt(1/3)*(
a*b^4*d*cos(d*x + c)^6 - 3*a*b^4*d*cos(d*x + c)^4 + 3*a*b^4*d*cos(d*x + c)^
2 - (a^3*b^2 + a*b^4)*d + 2*(a^2*b^3*d*cos(d*x + c)^2 - a^2*b^3*d)*sin(d*x
+ c))*sqrt(((4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6
```

)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/
3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4
*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b
^6)/(a^5*b^7*d^3))^(1/3)))^2*a^2*b^4*d^2 + 384*a^2 + 192*b^2)/(a^2*b^4*d^2)
)*arctan(1/64*sqrt(1/3)*((8*a^11*b^7 + 39*a^9*b^9 + 6*a^7*b^11 + a^5*b^13)*
(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d
^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/
3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2
*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7
*d^3))^(1/3)))^2*d^3 + 8*(16*a^11*b^5 + 86*a^9*b^7 + 51*a^7*b^9 + 8*a^5*b^1
1 + a^3*b^13)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b
^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(
1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a
^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 +
 b^6)/(a^5*b^7*d^3))^(1/3)))*d^2*sin(d*x + c) - 288*(8*a^10*b^4 + 39*a^8*b^
6 + 6*a^6*b^8 + a^4*b^10)*d*sin(d*x + c) + 48*(16*a^11*b^3 + 86*a^9*b^5 + 5
1*a^7*b^7 + 8*a^5*b^9 + a^3*b^11)*d + 2*sqrt(4096*a^12 + 28032*a^10*b^2 + 4
3920*a^8*b^4 + 14176*a^6*b^6 + 2784*a^4*b^8 + 288*a^2*b^10 + 16*b^12 - (3*(
8*a^11*b^5 + 39*a^9*b^7 + 6*a^7*b^9 + a^5*b^11)*d^2*sin(d*x + c) - (16*a^12
*b^4 + 86*a^10*b^6 + 51*a^8*b^8 + 8*a^6*b^10 + a^4*b^12)*d^2)*(4^(1/3)*(I*s
qrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6
- 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b
^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(
a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3))
)^2 - 16*(64*a^12 + 624*a^10*b^2 + 1617*a^8*b^4 + 484*a^6*b^6 + 114*a^4*b^8
 + 12*a^2*b^10 + b^12)*cos(d*x + c)^2 + 4*((32*a^12*b^2 + 188*a^10*b^4 + 18
8*a^8*b^6 + 67*a^6*b^8 + 10*a^4*b^10 + a^2*b^12)*d*sin(d*x + c) + 9*(8*a^11
*b^3 + 39*a^9*b^5 + 6*a^7*b^7 + a^5*b^9)*d)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^
6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a
^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) +
1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*
a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3))) - 288*(16*a^11*b
 + 86*a^9*b^3 + 51*a^7*b^5 + 8*a^5*b^7 + a^3*b^9)*sin(d*x + c))*(36*a^4*b^4
*d - (2*a^5*b^5 + a^3*b^7)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 +
6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^
5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*(
(8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2
+ 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))*d^2))*sqrt(((4^(1/3)*(I*sqrt(3) +
 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4
*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*
sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*
d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))^2*a^2*
b^4*d^2 + 384*a^2 + 192*b^2)/(a^2*b^4*d^2))/(64*a^12 + 624*a^10*b^2 + 1617*
a^8*b^4 + 484*a^6*b^6 + 114*a^4*b^8 + 12*a^2*b^10 + b^12)) + 6*sqrt(1/3)*(a
*b^4*d*cos(d*x + c)^6 - 3*a*b^4*d*cos(d*x + c)^4 + 3*a*b^4*d*cos(d*x + c)^2

```
 - (a^3*b^2 + a*b^4)*d + 2*(a^2*b^3*d*cos(d*x + c)^2 - a^2*b^3*d)*sin(d*x +
 c))*sqrt(((4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)
/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3
) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*
b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^
6)/(a^5*b^7*d^3))^(1/3)))^2*a^2*b^4*d^2 + 384*a^2 + 192*b^2)/(a^2*b^4*d^2))
*arctan(-1/64*sqrt(1/3)*((8*a^11*b^7 + 39*a^9*b^9 + 6*a^7*b^11 + a^5*b^13)*
(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d
^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/
3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2
*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7
*d^3))^(1/3)))^2*d^3 + 8*(16*a^11*b^5 + 86*a^9*b^7 + 51*a^7*b^9 + 8*a^5*b^1
1 + a^3*b^13)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b
^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(
1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a
^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 +
 b^6)/(a^5*b^7*d^3))^(1/3)))*d^2*sin(d*x + c) - 288*(8*a^10*b^4 + 39*a^8*b^
6 + 6*a^6*b^8 + a^4*b^10)*d*sin(d*x + c) + 48*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 277, normalized size = 0.96

$$-\frac{\frac{9\sin(dx+c)}{b^2}+\frac{2\left(3ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^2+b^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2b^2}+\frac{2\sqrt{3}\left(3\left(-ab^2\right)^{\frac{2}{3}}a-\left(-ab^2\right)^{\frac{1}{3}}\left(2a^2+b^2\right)\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b^3}+\frac{3\left(3ab\sin(dx+c)^2+a^2\sin(dx+c)-b^2\sin(dx+c)-3ab\right)}{\left(b\sin(dx+c)^3+a\right)ab^2}-\frac{\left(3\left(-ab^2\right)^{\frac{2}{3}}a+\left(-ab^2\right)^{\frac{1}{3}}\left(2a^2+b^2\right)\right)\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b^3}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(9*sin(d*x + c)/b^2 + 2*(3*a*b*(-a/b)^(1/3) + 2*a^2 + b^2)*(-a/b)^(1/3
)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^2*b^2) + 2*sqrt(3)*(3*(-a*b^2)^
(2/3)*a - (-a*b^2)^(1/3)*(2*a^2 + b^2))*arctan(1/3*sqrt(3)*((-a/b)^(1/3) +
2*sin(d*x + c))/(-a/b)^(1/3))/(a^2*b^3) + 3*(3*a*b*sin(d*x + c)^2 + a^2*sin
(d*x + c) - b^2*sin(d*x + c) - 3*a*b)/((b*sin(d*x + c)^3 + a)*a*b^2) - (3*(
-a*b^2)^(2/3)*a + (-a*b^2)^(1/3)*(2*a^2 + b^2))*log(sin(d*x + c)^2 + (-a/b)
^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^2*b^3))/d
```

Mupad [B]
time = 0.42, size = 384, normalized size = 1.33

$$\frac{\sum_{k=1}^{3}\ln\left(\frac{8\,a^2+4\,b^2+\mathrm{root}\left(729\,a^5\,b^7\,d^3+648\,a^5\,b^3\,d+324\,a^3\,b^5\,d+120\,a^4\,b^2-48\,a^2\,b^4-8\,b^6-64\,a^6,d,k\right)^2\,a^2\,b^4\,27+12\,a\,b\,\sin(c+d\,x)+\mathrm{root}\left(729\,a^5\,b^7\,d^3+648\,a^5\,b^3\,d+324\,a^3\,b^5\,d+120\,a^4\,b^2-48\,a^2\,b^4-8\,b^6-64\,a^6,d,k\right)\,b^4\,\sin(c+d\,x)\,6+\mathrm{root}\left(729\,a^5\,b^7\,d^3+648\,a^5\,b^3\,d+324\,a^3\,b^5\,d+120\,a^4\,b^2-48\,a^2\,b^4-8\,b^6-64\,a^6,d,k\right)\,a^2\,b^2\,\sin(c+d\,x)\,12}{a\,b^2\,3}\right)\,\mathrm{root}\left(729\,a^5\,b^7\,d^3+648\,a^5\,b^3\,d+324\,a^3\,b^5\,d+120\,a^4\,b^2-48\,a^2\,b^4-8\,b^6-64\,a^6,d,k\right)}{d}-\frac{\sin\left(c+d\,x\right)}{b^2\,d}-\frac{b\,\sin\left(c+d\,x\right)^2-b+\frac{\sin(c+d\,x)\,\left(a^2-b^2\right)}{3\,a}}{d\,\left(b^3\,\sin\left(c+d\,x\right)^3+a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] symsum(log((8*a^2 + 4*b^2 + 27*root(729*a^5*b^7*d^3 + 648*a^5*b^3*d + 324*a
^3*b^5*d + 120*a^4*b^2 - 48*a^2*b^4 - 8*b^6 - 64*a^6, d, k)^2*a^2*b^4 + 12*
a*b*sin(c + d*x) + 6*root(729*a^5*b^7*d^3 + 648*a^5*b^3*d + 324*a^3*b^5*d +
 120*a^4*b^2 - 48*a^2*b^4 - 8*b^6 - 64*a^6, d, k)*b^4*sin(c + d*x) + 12*roo
t(729*a^5*b^7*d^3 + 648*a^5*b^3*d + 324*a^3*b^5*d + 120*a^4*b^2 - 48*a^2*b^
4 - 8*b^6 - 64*a^6, d, k)*a^2*b^2*sin(c + d*x))/(3*a*b^2))*root(729*a^5*b^7
*d^3 + 648*a^5*b^3*d + 324*a^3*b^5*d + 120*a^4*b^2 - 48*a^2*b^4 - 8*b^6 - 6
4*a^6, d, k), k, 1, 3)/d - sin(c + d*x)/(b^2*d) - (b*sin(c + d*x)^2 - b + (
sin(c + d*x)*(a^2 - b^2))/(3*a))/(d*(a*b^2 + b^3*sin(c + d*x)^3))
```

3.394 $\int \frac{\cos^5(c+dx)}{(a+b\sin^3(c+dx))^2} dx$

Optimal. Leaf size=238

$$-\frac{2\left(a^{4/3}+b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d}+\frac{\left(a^{4/3}-b^{4/3}\right)\log}{}$$

```
[Out] -2/9*(a^(4/3)-b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^(5/3)/d+1/9
*(a^(4/3)-b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)
^2)/a^(5/3)/b^(5/3)/d+1/3*sin(d*x+c)*(b-a*sin(d*x+c)-2*b*sin(d*x+c)^2)/a/b/
d/(a+b*sin(d*x+c)^3)-2/9*(a^(4/3)+b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*si
n(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/b^(5/3)/d*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3302, 1872, 1874, 31, 648, 631, 210, 642}

$$-\frac{2\left(a^{4/3}+b^{4/3}\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}d}+\frac{\left(a^{4/3}-b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d}+\frac{\sin(c+dx)\left(-a\sin(c+dx)-2b\sin^2(c+dx)+b\right)}{3abd\left(a+b\sin^3(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*(a^(4/3) + b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*
a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)*d) - (2*(a^(4/3) - b^(4/3))*Log[a^(1/
3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b^(5/3)*d) + ((a^(4/3) - b^(4/3))*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(9*a^(5
/3)*b^(5/3)*d) + (Sin[c + d*x]*(b - a*Sin[c + d*x] - 2*b*Sin[c + d*x]^2))/(
3*a*b*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sin(c+dx)\,(b-a\sin(c+dx)-2b\sin^2(c+dx))}{3abd\,(a+b\sin^3(c+dx))} - \frac{\text{Subst}\left(\int \frac{-2b^2-2abx}{a+bx^3}\,dx, x, \text{s}\right.}{3ab^2d}$$

$$= \frac{\sin(c+dx)\,(b-a\sin(c+dx)-2b\sin^2(c+dx))}{3abd\,(a+b\sin^3(c+dx))} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\,(-2a^{4/3}b-4b^{7/3}}{a^{2/3}-\sqrt[3]{a}}\right.}{}$$

$$= -\frac{2(a^{4/3}-b^{4/3})\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d} + \frac{\sin(c+dx)\,(b-a\sin(c+dx}{3abd\,(a+b\sin^3(c}$$

$$= -\frac{2(a^{4/3}-b^{4/3})\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d} + \frac{(a^{4/3}-b^{4/3})\log\left(a^{2/3}-\sqrt[3]{a}\right.}{9}$$

$$= -\frac{2(a^{4/3}+b^{4/3})\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}b^{5/3}d} - \frac{2(a^{4/3}-b^{4/3})\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin\right.}{9a^{5/3}b^{5/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.75, size = 258, normalized size = 1.08

$$\frac{-\frac{4\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{5/3}\sqrt[3]{b}}-\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{a^{5/3}\sqrt[3]{b}}+\frac{9\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)\sin^2(c+dx)}{ab}-\frac{9\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)\sin^2(c+dx)}{ab}+\frac{12}{b(a+b\sin^3(c+dx))}+\frac{6\sin(c+dx)}{a(a+b\sin^3(c+dx))}}{18d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-4*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/
(a^(5/3)*b^(1/3)) + (4*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(5/3)*b^(1/3
)) - (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2
])/(a^(5/3)*b^(1/3)) + (9*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^
3)/a)]*Sin[c + d*x]^2)/(a*b) - (9*Hypergeometric2F1[2/3, 2, 5/3, -((b*Sin[c
 + d*x]^3)/a)]*Sin[c + d*x]^2)/(a*b) + 12/(b*(a + b*Sin[c + d*x]^3)) + (6*S
in[c + d*x])/(a*(a + b*Sin[c + d*x]^3)))/(18*d)
```

Maple [A]

time = 0.88, size = 284, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{-\frac{\sin^2(dx+c)}{3b}+\frac{\sin(dx+c)}{3a}+\frac{2}{3b}}{a+b\left(\sin^3(dx+c)\right)}+\frac{2b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\right.}{}\right)}{3b\left(\frac{a}{b}\right)^{}}\right.}{3}}{}$
default	$\frac{\frac{-\frac{\sin^2(dx+c)}{3b}+\frac{\sin(dx+c)}{3a}+\frac{2}{3b}}{a+b\left(\sin^3(dx+c)\right)}+\frac{2b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\right.}{}\right)}{3b\left(\frac{a}{b}\right)^{}}\right.}{3}}{}$
risch	$-\frac{2i\left(a\,e^{5i(dx+c)}+6a\,e^{3i(dx+c)}-2ib\,e^{4i(dx+c)}+a\,e^{i(dx+c)}+2ib\,e^{2i(dx+c)}\right)}{3abd\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left(729a^5b^5d^3\_Z^3+108a^3b^3d\_\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-1/3*sin(d*x+c)^2/b+1/3/a*sin(d*x+c)+2/3/b)/(a+b*sin(d*x+c)^3)+2/3/a/
b*(b*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*
ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+a*(-1/3/b/(1/b
*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*x+c)^2-
(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))))
```

Maxima [A]

time = 0.51, size = 213, normalized size = 0.89

$$-\frac{\frac{3\left(a\sin(dx+c)^2-b\sin(dx+c)-2a\right)}{ab^2\sin(dx+c)^3+a^2b}-\frac{2\sqrt{3}\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}+b\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}-b\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}-b\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/9*(3*(a*sin(d*x + c)^2 - b*sin(d*x + c) - 2*a)/(a*b^2*sin(d*x + c)^3 + a
^2*b) - 2*sqrt(3)*(a*(a/b)^(1/3) + b)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*
sin(d*x + c))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - (a*(a/b)^(1/3) - b)*log(si
n(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3))
+ 2*(a*(a/b)^(1/3) - b)*log((a/b)^(1/3) + sin(d*x + c))/(a*b^2*(a/b)^(2/3))
)/d
```

Fricas [C] Result contains complex when optimal does not.
time = 63.73, size = 3878, normalized size = 16.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*b^2*cos(d*x + c)^4 - 6*sqrt(1/3)*(a*b^3*d*cos(d*x + c)^6 - 3*a*b^
3*d*cos(d*x + c)^4 + 3*a*b^3*d*cos(d*x + c)^2 - (a^3*b + a*b^3)*d + 2*(a^2*
b^2*d*cos(d*x + c)^2 - a^2*b^2*d)*sin(d*x + c))*sqrt(((4^(1/3)*(I*sqrt(3) +
 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)
*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^
5*b^5*d^3))^(1/3)))^2*a^2*b^2*d^2 + 64)/(a^2*b^2*d^2))*arctan(-1/64*sqrt(1/
3)*((a^9*b^5 + a^5*b^9)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3)
 - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2
*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2*d^3 + 8*
(a^7*b^5 + a^3*b^9)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (
a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a
^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))*d^2*sin(d*x +
c) - 32*(a^8*b^2 + a^4*b^6)*d*sin(d*x + c) + 16*(a^7*b^3 + a^3*b^7)*d + 2*(
(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*
d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d
^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))*a^3*b^5*d^2 - 4*a^4*b^2*d)*sqrt(16
*a^8 + 32*a^6*b^2 + 32*a^4*b^4 + 32*a^2*b^6 + 16*b^8 - ((a^9*b^3 + a^5*b^7)
*d^2*sin(d*x + c) - (a^8*b^4 + a^4*b^8)*d^2)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4
 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt
(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3
))^(1/3)))^2 - 16*(a^8 + 2*a^4*b^4 + b^8)*cos(d*x + c)^2 + 4*((a^6*b^4 + a^
2*b^8)*d*sin(d*x + c) + (a^9*b + a^5*b^5)*d)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4
 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt
(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3
))^(1/3))) - 32*(a^7*b + a^3*b^5)*sin(d*x + c)))*sqrt(((4^(1/3)*(I*sqrt(3)
+ 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3
)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a
^5*b^5*d^3))^(1/3)))^2*a^2*b^2*d^2 + 64)/(a^2*b^2*d^2))/(a^8 + 2*a^4*b^4 +
b^8)) + 6*sqrt(1/3)*(a*b^3*d*cos(d*x + c)^6 - 3*a*b^3*d*cos(d*x + c)^4 + 3*
```

```
a*b^3*d*cos(d*x + c)^2 - (a^3*b + a*b^3)*d + 2*(a^2*b^2*d*cos(d*x + c)^2 -
a^2*b^2*d)*sin(d*x + c))*sqrt(((4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b
^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*
b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2*a
^2*b^2*d^2 + 64)/(a^2*b^2*d^2))*arctan(1/64*sqrt(1/3)*((a^9*b^5 + a^5*b^9)*
(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*
d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d
^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2*d^3 + 8*(a^7*b^5 + a^3*b^9)*(4^(
1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3)
)^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3)
- (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))*d^2*sin(d*x + c) - 32*(a^8*b^2 + a^4*b
^6)*d*sin(d*x + c) + 16*(a^7*b^3 + a^3*b^7)*d - 2*((4^(1/3)*(I*sqrt(3) + 1)
*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-
I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b
^5*d^3))^(1/3)))*a^3*b^5*d^2 - 4*a^4*b^2*d)*sqrt(16*a^8 + 32*a^6*b^2 + 32*a
^4*b^4 + 32*a^2*b^6 + 16*b^8 - ((a^9*b^3 + a^5*b^7)*d^2*sin(d*x + c) - (a^8
*b^4 + a^4*b^8)*d^2)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) -
(a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((
a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2 - 16*(a^8 +
 2*a^4*b^4 + b^8)*cos(d*x + c)^2 + 4*((a^6*b^4 + a^2*b^8)*d*sin(d*x + c) +
(a^9*b + a^5*b^5)*d)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) -
(a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((
a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3))) - 32*(a^7*b +
 a^3*b^5)*sin(d*x + c)))*sqrt(((4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b
^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*
b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2*a
^2*b^2*d^2 + 64)/(a^2*b^2*d^2))/(a^8 + 2*a^4*b^4 + b^8)) + 12*(a^2 - 2*b^2)
*cos(d*x + c)^2 - (a*b^3*d*cos(d*x + c)^6 - 3*a*b^3*d*cos(d*x + c)^4 + 3*a*
b^3*d*cos(d*x + c)^2 - (a^3*b + a*b^3)*d + 2*(a^2*b^2*d*cos(d*x + c)^2 - a^
2*b^2*d)*sin(d*x + c))*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3)
- (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*
((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))*log(64*a^8
+ 128*a^6*b^2 + 128*a^4*b^4 + 128*a^2*b^6 + 64*b^8 - 4*((a^9*b^3 + a^5*b^7)
*d^2*sin(d*x + c) - (a^8*b^4 + a^4*b^8)*d^2)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4
 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt
(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3
))^(1/3)))^2 - 64*(a^8 + 2*a^4*b^4 + b^8)*cos(d*x + c)^2 + 16*((a^6*b^4 + a
^2*b^8)*d*sin(d*x + c) + (a^9*b + a^5*b^5)*d)*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 228, normalized size = 0.96

$$-\frac{\frac{2\left(a\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2b}+\frac{3\left(a\sin(dx+c)^2-b\sin(dx+c)-2a\right)}{\left(b\sin(dx+c)^3+a\right)ab}-\frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b^3}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\left(-ab^2\right)^{\frac{2}{3}}a\right)\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b^3}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(a*(-a/b)^(1/3) + b)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x +
 c)))/(a^2*b) + 3*(a*sin(d*x + c)^2 - b*sin(d*x + c) - 2*a)/((b*sin(d*x + c
)^3 + a)*a*b) - 2*sqrt(3)*((-a*b^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*arctan(1/
3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^2*b^3) - ((-a*b^
2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x
+ c) + (-a/b)^(2/3))/(a^2*b^3))/d
```

Mupad [B]
time = 14.98, size = 203, normalized size = 0.85

$$\frac{\sum_{k=1}^{3}\ln\left(\frac{4b+4a\sin(c+dx)+\mathrm{root}(729a^5b^5d^3+108a^3b^3d-8b^4+8a^4,d,k)^2a^2b^3\,81+\mathrm{root}(729a^5b^5d^3+108a^3b^3d-8b^4+8a^4,d,k)\,b^3\sin(c+dx)\,18}{a\,b\,9}\right)\mathrm{root}(729a^5b^5d^3+108a^3b^3d-8b^4+8a^4,d,k)}{d}+\frac{\frac{\sin(c+dx)}{3a}+\frac{2}{3b}-\frac{\sin(c+dx)^2}{3b}}{d\left(b\sin\left(c+dx\right)^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] symsum(log((4*b + 4*a*sin(c + d*x) + 81*root(729*a^5*b^5*d^3 + 108*a^3*b^3*
d - 8*b^4 + 8*a^4, d, k)^2*a^2*b^3 + 18*root(729*a^5*b^5*d^3 + 108*a^3*b^3*
d - 8*b^4 + 8*a^4, d, k)*b^3*sin(c + d*x))/(9*a*b))*root(729*a^5*b^5*d^3 +
108*a^3*b^3*d - 8*b^4 + 8*a^4, d, k), k, 1, 3)/d + (sin(c + d*x)/(3*a) + 2/
(3*b) - sin(c + d*x)^2/(3*b))/(d*(a + b*sin(c + d*x)^3))
```

3.395 $\int \frac{\cos^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2\right.}{9a^{5/3}\sqrt[3]{b}\,d}$$

```
[Out] 2/9*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^(1/3)/d-1/9*ln(a^(2/3)-a^(1/3)
*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/b^(1/3)/d+1/3*(a+b*sin(d*
x+c))/a/b/d/(a+b*sin(d*x+c)^3)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c)
)/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3302, 1868, 12, 206, 31, 648, 631, 210, 642}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}+\frac{a+b\sin(c+dx)}{3abd\left(a+b\sin^3(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(5/3)*b^(1/3)*d) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b
^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*
x]^2]/(9*a^(5/3)*b^(1/3)*d) + (a + b*Sin[c + d*x])/(3*a*b*d*(a + b*Sin[c +
d*x]^3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{a+b\sin(c+dx)}{3abd\,(a+b\sin^3(c+dx))} - \frac{\text{Subst}\left(\int -\frac{2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ad}$$

$$= \frac{a+b\sin(c+dx)}{3abd\,(a+b\sin^3(c+dx))} + \frac{2\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ad}$$

$$= \frac{a+b\sin(c+dx)}{3abd\,(a+b\sin^3(c+dx))} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(c+dx)\right)}{9a^{5/3}d} + \frac{2\text{Subs}}{}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} + \frac{a+b\sin(c+dx)}{3abd\,(a+b\sin^3(c+dx))} + \frac{\text{Subst}\left(\int \frac{}{a^{2/3}-\sqrt[3]{}}\right.}{}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c\right.}{9a^{5/3}\sqrt[3]{b}\,d}$$

$$= -\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{}\right.}{}$$

Mathematica [A]
time = 0.52, size = 184, normalized size = 1.01

$$\frac{-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}} + \frac{3\sin(c+dx)}{a(a+b\sin^3(c+dx))} + \frac{\frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{5/3}} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{a^{5/3}} + \frac{3}{a+b\sin^3(c+dx)}}{b}}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/
(a^(5/3)*b^(1/3)) + (3*Sin[c + d*x])/(a*(a + b*Sin[c + d*x]^3)) + ((2*b^(2/
3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/a^(5/3) - (b^(2/3)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/a^(5/3) + 3/(a + b*Si
n[c + d*x]^3))/b)/(9*d)
```

Maple [A]
time = 0.96, size = 165, normalized size = 0.90

method	result
risch	$-\frac{4\left(b\,e^{4i(dx+c)}-b\,e^{2i(dx+c)}+2ia\,e^{3i(dx+c)}\right)}{3abd\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}+\left(\sum_{\_R=\mathrm{RootOf}(729a^5b\,d^3\_Z^3-8)}\_R\ln\left(e^{2i(\right.\right.$
derivativedivides	$\frac{\frac{\sin(dx+c)}{3a\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}}{d}$
default	$\frac{\frac{\sin(dx+c)}{3a\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*sin(d*x+c)/a/(a+b*sin(d*x+c)^3)+2/3/a*(1/3/b/(1/b*a)^(2/3)*ln(sin(
d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin
(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1
/b*a)^(1/3)*sin(d*x+c)-1)))+1/3/b/(a+b*sin(d*x+c)^3))
```

Maxima [A]

time = 0.53, size = 163, normalized size = 0.89

$$\frac{\frac{3\,(b\sin(dx+c)+a)}{ab^2\sin(dx+c)^3+a^2b}+\frac{2\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(3*(b*sin(d*x + c) + a)/(a*b^2*sin(d*x + c)^3 + a^2*b) + 2*sqrt(3)*arct
an(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/(a*b*(a/b)^(2/3
)) - log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b*(a/b
)^(2/3)) + 2*log((a/b)^(1/3) + sin(d*x + c))/(a*b*(a/b)^(2/3)))/d
```

Fricas [A]

time = 0.49, size = 665, normalized size = 3.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*sin(d*x + c) + 3*a^3 + 3*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x +
c)^2 - a*b^2)*sin(d*x + c))*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*
sin(d*x + c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2
/3)*sin(d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x
 + c)^2 - a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) + (
a^2*b)^(2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c
)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)
*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(
2/3)))/(a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c)), 1/9
*(3*a^2*b*sin(d*x + c) + 3*a^3 + 6*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2
 - a*b^2)*sin(d*x + c))*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*sin(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(
2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*
b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos
(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(
a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 169, normalized size = 0.92

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b}-\frac{3\left(b\sin(dx+c)+a\right)}{\left(b\sin(dx+c)^3+a\right)ab}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a^2 - 2*sqrt(3)
*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(
1/3))/(a^2*b) - (-a*b^2)^(1/3)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x +
c) + (-a/b)^(2/3))/(a^2*b) - 3*(b*sin(d*x + c) + a)/((b*sin(d*x + c)^3 + a)
*a*b))/d
```

Mupad [B]
time = 0.38, size = 172, normalized size = 0.94

$$\frac{\frac{\sin(c+d\,x)}{3\,a}+\frac{1}{3\,b}}{d\,\left(b\sin\left(c+d\,x\right)^3+a\right)}+\frac{2\,\ln\left(\frac{2\,b^{5/3}}{a^{2/3}}+\frac{2\,b^2\,\sin(c+d\,x)}{a}\right)}{9\,a^{5/3}\,b^{1/3}\,d}+\frac{\ln\left(\frac{2\,b^2\,\sin(c+d\,x)}{a}+\frac{b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}\,d}-\frac{\ln\left(\frac{2\,b^2\,\sin(c+d\,x)}{a}-\frac{b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] (sin(c + d*x)/(3*a) + 1/(3*b))/(d*(a + b*sin(c + d*x)^3)) + (2*log((2*b^(5/
3))/a^(2/3) + (2*b^2*sin(c + d*x))/a))/(9*a^(5/3)*b^(1/3)*d) + (log((2*b^2*
sin(c + d*x))/a + (b^(5/3)*(3^(1/2)*1i - 1))/a^(2/3))*(3^(1/2)*1i - 1))/(9*
a^(5/3)*b^(1/3)*d) - (log((2*b^2*sin(c + d*x))/a - (b^(5/3)*(3^(1/2)*1i + 1
))/a^(2/3))*(3^(1/2)*1i + 1))/(9*a^(5/3)*b^(1/3)*d)
```

3.396 $\int \frac{\cos(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2\right.}{9a^{5/3}\sqrt[3]{b}d}$$

```
[Out] 2/9*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^(1/3)/d-1/9*ln(a^(2/3)-a^(1/3)
*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/b^(1/3)/d+1/3*sin(d*x+c)/
a/d/(a+b*sin(d*x+c)^3)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3
)*3^(1/2))/a^(5/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3302, 205, 206, 31, 648, 631, 210, 642}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}+\frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(5/3)*b^(1/3)*d) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b
^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*
x]^2]/(9*a^(5/3)*b^(1/3)*d) + Sin[c + d*x]/(3*a*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)} + \frac{2\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ad}$$

$$= \frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(c+dx)\right)}{9a^{5/3}d} + \frac{2\text{Subst}}{}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} + \frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{}{a^{2/3}-\sqrt[3]{a}}\right.}{}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c\right.}{9a^{5/3}\sqrt[3]{b}\,d}$$

$$= -\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{}\right.}{}$$

Mathematica [A]
time = 0.34, size = 152, normalized size = 0.86

$$\frac{-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{\sqrt[3]{b}} + \frac{3a^{2/3}\sin(c+dx)}{a+b\sin^3(c+dx)}}{9a^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/
b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - Log[a^(2/3) - a^(1/3)*b^
(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/b^(1/3) + (3*a^(2/3)*Sin[c +
d*x])/(a + b*Sin[c + d*x]^3))/(9*a^(5/3)*d)
```

Maple [A]
time = 0.54, size = 146, normalized size = 0.83

method	result

risch	$-\frac{4\left(e^{4i(dx+c)}-e^{2i(dx+c)}\right)}{3ad\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}+\left(\sum_{\_R=\text{RootOf}(729a^5b\,d^3\_Z^3-8)}\_R\ln\left(e^{2i(\cdots\right.\right.$
derivativedivides	$\frac{\frac{\sin(dx+c)}{3a\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx\cdots}{\left(\frac{a}{b}\right)^{\cdots}}\right.}{3}\cdots\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\cdots}{d}$
default	$\frac{\frac{\sin(dx+c)}{3a\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx\cdots}{\left(\frac{a}{b}\right)^{\cdots}}\right.}{3}\cdots\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\cdots}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*sin(d*x+c)/a/(a+b*sin(d*x+c)^3)+2/3/a*(1/3/b/(1/b*a)^(2/3)*ln(sin(
d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin
(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1
/b*a)^(1/3)*sin(d*x+c)-1))))
```

Maxima [A]
time = 0.54, size = 155, normalized size = 0.88

$$\frac{\frac{3\sin(dx+c)}{ab\sin(dx+c)^3+a^2}+\frac{2\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(3*sin(d*x + c)/(a*b*sin(d*x + c)^3 + a^2) + 2*sqrt(3)*arctan(-1/3*sqrt
(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - log(sin
(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) + 2
*log((a/b)^(1/3) + sin(d*x + c))/(a*b*(a/b)^(2/3)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(137) = 274$.

time = 0.49, size = 655, normalized size = 3.72

$$\left[\frac{3\,a^2b\sin(dx+c)+3\sqrt{\frac{1}{3}}\left(a^2b-\left(ab^2\cos(dx+c)^2-ab^2\right)\sin(dx+c)\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(-\frac{3\,(a^2b)^{\frac{1}{3}}a\sin(dx+c)+a^2+3\sqrt{\frac{1}{3}}\left(2\,ab\cos(dx+c)^2-2\,ab-(a^2b)^{\frac{2}{3}}\sin(dx+c)+(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}+2\left(ab\cos(dx+c)^2-ab\right)\sin(dx+c)}{\left(b\cos(dx+c)^2-b\right)\sin(dx+c)-a}\right)+(a^2b)^{\frac{2}{3}}\left(\left(b\cos(dx+c)^2-b\right)\sin(dx+c)-a\right)\log\left(-ab\cos(dx+c)^2+ab-(a^2b)^{\frac{2}{3}}\sin(dx+c)+(a^2b)^{\frac{1}{3}}a\right)-2\,(a^2b)^{\frac{2}{3}}\left(\left(b\cos(dx+c)^2-b\right)\sin(dx+c)-a\right)\log\left(ab\sin(dx+c)+(a^2b)^{\frac{2}{3}}\right)}{9\left(a^4bd-\left(a^3b^2d\cos(dx+c)^2-a^3b^2d\right)\sin(dx+c)\right)},\ \frac{3\,a^2b\sin(dx+c)+6\sqrt{\frac{1}{3}}\left(a^2b-\left(ab^2\cos(dx+c)^2-ab^2\right)\sin(dx+c)\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}\sin(dx+c)-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)+(a^2b)^{\frac{2}{3}}\left(\left(b\cos(dx+c)^2-b\right)\sin(dx+c)-a\right)\log\left(-ab\cos(dx+c)^2+ab-(a^2b)^{\frac{2}{3}}\sin(dx+c)+(a^2b)^{\frac{1}{3}}a\right)-2\,(a^2b)^{\frac{2}{3}}\left(\left(b\cos(dx+c)^2-b\right)\sin(dx+c)-a\right)\log\left(ab\sin(dx+c)+(a^2b)^{\frac{2}{3}}\right)}{9\left(a^4bd-\left(a^3b^2d\cos(dx+c)^2-a^3b^2d\right)\sin(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*sin(d*x + c) + 3*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2 - a
*b^2)*sin(d*x + c))*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*sin(d*x
+ c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2/3)*sin(
d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x + c)^2
- a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) + (a^2*b)^(
2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*
b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos
(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(
a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c)), 1/9*(3*a^2*
b*sin(d*x + c) + 6*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2 - a*b^2)*sin(d*
x + c))*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*sin(d*x + c
) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*((b*cos(d*x
 + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3
)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos(d*x + c)^2 - b)
*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a^4*b*d - (a^3*b
^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c))]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(162) = 324$.
time = 122.54, size = 617, normalized size = 3.51

$$\begin{cases} \frac{\tilde{\infty} x\cos(c)}{\sin^6(c)} & \text{for } a=0\wedge b=0\wedge d=0 \\ -\frac{1}{5b^2d\sin^5(c+dx)} & \text{for } a=0 \\ \frac{\tilde{\infty}\sin(c+dx)}{d} & \text{for } b=-\frac{a}{\sin^3(c+dx)} \\ \frac{x\cos(c)}{(a+b\sin^3(c))^2} & \text{for } d=0 \\ \frac{\sin(c+dx)}{a^2d} & \text{for } b=0 \\ -\frac{2a\sqrt[3]{-\frac{a}{b}}\log\left(-\sqrt[3]{-\frac{a}{b}}+\sin(c+dx)\right)}{9a^3d+9a^2bd\sin^3(c+dx)}+\frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\sin(c+dx)+4\sin^2(c+dx)\right)}{9a^3d+9a^2bd\sin^3(c+dx)}+\frac{2\sqrt{3}a\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sin(c+dx)}{3\sqrt[3]{-\frac{a}{b}}}\right)}{9a^3d+9a^2bd\sin^3(c+dx)}-\frac{2a\sqrt[3]{-\frac{a}{b}}\log(2)}{9a^3d+9a^2bd\sin^3(c+dx)}+\frac{3a\sin(c+dx)}{9a^3d+9a^2bd\sin^3(c+dx)}-\frac{2b\sqrt[3]{-\frac{a}{b}}\log\left(-\sqrt[3]{-\frac{a}{b}}+\sin(c+dx)\right)\sin^3(c+dx)}{9a^3d+9a^2bd\sin^3(c+dx)}+\frac{b\sqrt[3]{-\frac{a}{b}}\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\sin(c+dx)+4\sin^2(c+dx)\right)\sin^3(c+dx)}{9a^3d+9a^2bd\sin^3(c+dx)}+\frac{2\sqrt{3}b\sqrt[3]{-\frac{a}{b}}\sin^3(c+dx)\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sin(c+dx)}{3\sqrt[3]{-\frac{a}{b}}}\right)}{9a^3d+9a^2bd\sin^3(c+dx)}-\frac{2b\sqrt[3]{-\frac{a}{b}}\log(2)\sin^3(c+dx)}{9a^3d+9a^2bd\sin^3(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-1/(5*
b**2*d*sin(c + d*x)**5), Eq(a, 0)), (zoo*sin(c + d*x)/d, Eq(b, -a/sin(c + d
*x)**3)), (x*cos(c)/(a + b*sin(c)**3)**2, Eq(d, 0)), (sin(c + d*x)/(a**2*d)
, Eq(b, 0)), (-2*a*(-a/b)**(1/3)*log(-(-a/b)**(1/3) + sin(c + d*x))/(9*a**3
*d + 9*a**2*b*d*sin(c + d*x)**3) + a*(-a/b)**(1/3)*log(4*(-a/b)**(2/3) + 4*
(-a/b)**(1/3)*sin(c + d*x) + 4*sin(c + d*x)**2)/(9*a**3*d + 9*a**2*b*d*sin(
c + d*x)**3) + 2*sqrt(3)*a*(-a/b)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*sin(c +
 d*x)/(3*(-a/b)**(1/3)))/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3) - 2*a*(-a/
b)**(1/3)*log(2)/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3) + 3*a*sin(c + d*x)
/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3) - 2*b*(-a/b)**(1/3)*log(-(-a/b)**(
```

```
1/3) + sin(c + d*x))*sin(c + d*x)**3/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3
) + b*(-a/b)**(1/3)*log(4*(-a/b)**(2/3) + 4*(-a/b)**(1/3)*sin(c + d*x) + 4*
sin(c + d*x)**2)*sin(c + d*x)**3/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3) +
2*sqrt(3)*b*(-a/b)**(1/3)*sin(c + d*x)**3*atan(sqrt(3)/3 + 2*sqrt(3)*sin(c
+ d*x)/(3*(-a/b)**(1/3)))/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3) - 2*b*(-a
/b)**(1/3)*log(2)*sin(c + d*x)**3/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3),
True))
```

Giac [A]
time = 0.46, size = 162, normalized size = 0.92

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^{2}}-\frac{3\sin(dx+c)}{\left(b\sin(dx+c)^{3}+a\right)a}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}b}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^{2}b}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a^2 - 3*sin(d*x
 + c)/((b*sin(d*x + c)^3 + a)*a) - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt
(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^2*b) - (-a*b^2)^(1/3)*
log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^2*b))/d
```

Mupad [B]
time = 15.00, size = 165, normalized size = 0.94

$$\frac{\sin\left(c+d\,x\right)}{3\,a\,d\,\left(b\sin\left(c+d\,x\right)^{3}+a\right)}+\frac{2\,\ln\left(\frac{2\,b^{5/3}}{a^{2/3}}+\frac{2\,b^{2}\,\sin(c+d\,x)}{a}\right)}{9\,a^{5/3}\,b^{1/3}\,d}+\frac{\ln\left(\frac{2\,b^{2}\,\sin(c+d\,x)}{a}+\frac{b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}\,d}-\frac{\ln\left(\frac{2\,b^{2}\,\sin(c+d\,x)}{a}-\frac{b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] sin(c + d*x)/(3*a*d*(a + b*sin(c + d*x)^3)) + (2*log((2*b^(5/3))/a^(2/3) +
(2*b^2*sin(c + d*x))/a))/(9*a^(5/3)*b^(1/3)*d) + (log((2*b^2*sin(c + d*x))/
a + (b^(5/3)*(3^(1/2)*1i - 1))/a^(2/3))*(3^(1/2)*1i - 1))/(9*a^(5/3)*b^(1/3
)*d) - (log((2*b^2*sin(c + d*x))/a - (b^(5/3)*(3^(1/2)*1i + 1))/a^(2/3))*(3
^(1/2)*1i + 1))/(9*a^(5/3)*b^(1/3)*d)
```

3.397 $\int \frac{\sec(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=587

$$-\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\left(a^2-b^2\right)d}-\frac{\sqrt[3]{b}\left(a^2-2a^{2/3}b^{4/3}+b^2\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 d}-\log$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^2/d+1/2*ln(1+sin(d*x+c))/(a-b)^2/d-1/9*b^(1/3)*
(a^(4/3)+2*b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/(a^2-b^2)/d-1/3*
b^(1/3)*(a^2+2*a^(2/3)*b^(4/3)+b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(1/3)/
(a^2-b^2)^2/d+1/18*b^(1/3)*(a^(4/3)+2*b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*s
in(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/(a^2-b^2)/d+1/6*b^(1/3)*(a^2+2*a^(2
/3)*b^(4/3)+b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2
)/a^(1/3)/(a^2-b^2)^2/d-2/3*a*b*ln(a+b*sin(d*x+c)^3)/(a^2-b^2)^2/d+1/3*b*(a
-sin(d*x+c)*(b-a*sin(d*x+c)))/a/(a^2-b^2)/d/(a+b*sin(d*x+c)^3)-1/9*b^(1/3)*
(a^(4/3)-2*b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/
2))/a^(5/3)/(a^2-b^2)/d*3^(1/2)-1/3*b^(1/3)*(a^2-2*a^(2/3)*b^(4/3)+b^2)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/(a^2-b^2)^2
/d*3^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3302, 2099, 1868, 1874, 31, 648, 631, 210, 642, 1885, 266}

$$\frac{b(a-\sin(c+dx)(b-a\sin(c+dx)))}{3ad\left(a^2-b^2\right)\left(a+b\sin^3(c+dx)\right)}-\frac{2ab\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)^2}-\frac{\sqrt[3]{b}\left(-2a^{2/3}b^{4/3}+a^2+b^2\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d\left(a^2-b^2\right)^2}-\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}d\left(a^2-b^2\right)}+\frac{\sqrt[3]{b}\left(2a^{2/3}b^{4/3}+a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6\sqrt[3]{a}\,d\left(a^2-b^2\right)^2}+\frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{18a^{5/3}d\left(a^2-b^2\right)}-\frac{\sqrt[3]{b}\left(2a^{2/3}b^{4/3}+a^2+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3\sqrt[3]{a}\,d\left(a^2-b^2\right)^2}-\frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] -1/3*(b^(1/3)*(a^(4/3) - 2*b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x
])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*(a^2 - b^2)*d) - (b^(1/3)*(a^2 - 2*
a^(2/3)*b^(4/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2*d) - Log[1 - Sin[c + d*x]]/(2*(a +
 b)^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*d) - (b^(1/3)*(a^(4/3) + 2*b^
(4/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)*d) - (b^
(1/3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/
(3*a^(1/3)*(a^2 - b^2)^2*d) + (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(2/3) -
a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 -
b^2)*d) + (b^(1/3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2*d) -
 (2*a*b*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + (b*(a - Sin[c + d*
x]*(b - a*Sin[c + d*x])))/(3*a*(a^2 - b^2)*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)^2(-1+x)} + \frac{1}{2(a-b)^2(1+x)} + \frac{b(b-ax+bx^2)}{(-a^2+b^2)(a+bx^3)^2} + \frac{b(-2ab+(a^2+b^2)x-2ab}{(a^2-b^2)^2(a+bx^3)}\right.\right.}{d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{b\text{Subst}\left(\int \frac{-2ab+(a^2+b^2)x-2abx^2}{a+bx^3}\right.}{(a^2-b^2)^2}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{b(a-\sin(c+dx)(b-a\sin(}{3a(a^2-b^2)\,d\,(a+b\sin^3(c}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{2ab\log\left(a+b\sin^3(c+dx)\right)}{3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(\sqrt[3]{a}\right.}{9a^{5/3}\left(a^2-b\right.}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(\sqrt[3]{a}\right.}{9a^{5/3}\left(a^2-b\right.}$$

$$= -\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\left(a^2-b^2\right)d} - \frac{\sqrt[3]{b}\left(a^2-2a^{2/3}b^{4/3}+b^2\right)\tan}{\sqrt{3}\,\sqrt[3]{a}\,\left(a^2-\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.08, size = 503, normalized size = 0.86

$$\frac{-\frac{9\log(1-\sin(c+dx))}{(a+b)^2}+\frac{9\log(1+\sin(c+dx))}{(a-b)^2}-\frac{12\sqrt[3]{a}\,b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{(a^2-b^2)^2}+\frac{6\sqrt[3]{a}\,b^{5/3}\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)\right)}{(a^2-b^2)^2}+\frac{2b^{5/3}\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)\right)}{a^{5/3}(a^2-b^2)}-\frac{12ab\log\left(a+b\sin^3(c+dx)\right)}{(a^2-b^2)^2}+\frac{9b(a^2+b^2)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)\sin^2(c+dx)}{a(a^2-b^2)^2}+\frac{9b\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)\sin^2(c+dx)}{a^3-ab^2}+\frac{6b}{(a^2-b^2)(a+b\sin^3(c+dx))}-\frac{6b^2\sin(c+dx)}{a(a^2-b^2)(a+b\sin^3(c+dx))}}{18d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-9*Log[1 - Sin[c + d*x]])/(a + b)^2 + (9*Log[1 + Sin[c + d*x]])/(a - b)^2
 - (12*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^2 - b^2)^2 +
 (6*a^(1/3)*b^(5/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(S
```

```
qrt[3]*a^(1/3))] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin
[c + d*x]^2]))/(a^2 - b^2)^2 + (2*b^(5/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^
(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] - 2*Log[a^(1/3) + b^(1/3)*Sin[c + d*
x]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])
)/(a^(5/3)*(a^2 - b^2)) - (12*a*b*Log[a + b*Sin[c + d*x]^3])/(a^2 - b^2)^2
+ (9*b*(a^2 + b^2)*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^3)/a)]*
Sin[c + d*x]^2)/(a*(a^2 - b^2)^2) + (9*b*Hypergeometric2F1[2/3, 2, 5/3, -((
b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a^3 - a*b^2) + (6*b)/((a^2 - b^2)*(a
 + b*Sin[c + d*x]^3)) - (6*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(a + b*Sin[c +
d*x]^3)))/(18*d)
```

Maple [A]
time = 1.48, size = 389, normalized size = 0.66 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/(a-b)^2/(a+b)^2*(((1/3*a^2-1/3*b^2)*sin(d*x+c)^2-1/3*b*(a^2-b^2)/a*s
in(d*x+c)+1/3*a^2-1/3*b^2)/(a+b*sin(d*x+c)^3)+2/3/a*((-4*a^2*b+b^3)*(1/3/b/
(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(sin(d*x+c
)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+(2*a^3+a*b^2)*(-1/3/b/(1/b
*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*x+c)^2-
(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))-a^2*ln(a+b*sin(d*x+c)^3)))-1/2
/(a+b)^2*ln(sin(d*x+c)-1)+1/2/(a-b)^2*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.54, size = 483, normalized size = 0.82

$$\frac{\frac{4\sqrt{3}\left(2a^3\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)-2a^2b\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{a}{b}\right)+ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\left(2a^2b\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\right)-2a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+b^3\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{4\left(a^2b\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\right)+2a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}+ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-b^3\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{6\left(ab\sin(dx+c)^2-b^2\sin(dx+c)+ab\right)}{a^4-a^2b^2+(a^3b-ab^3)\sin(dx+c)^3}+\frac{9\log(\sin(dx+c)+1)}{a^2-2ab+b^2}-\frac{9\log(\sin(dx+c)-1)}{a^2+2ab+b^2}}{18d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(4*sqrt(3)*(2*a^3*((a/b)^(2/3) + 1) - 2*a^2*b*(2*(a/b)^(1/3) + a/b) +
a*b^2*(a/b)^(2/3) + b^3*(a/b)^(1/3))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*s
in(d*x + c))/(a/b)^(1/3))/((a^5*(a/b)^(2/3) - 2*a^3*b^2*(a/b)^(2/3) + a*b^4
*(a/b)^(2/3))*(a/b)^(1/3)) - 2*(2*a^2*b*(3*(a/b)^(2/3) - 2) - 2*a^3*(a/b)^(
1/3) - a*b^2*(a/b)^(1/3) + b^3)*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x +
c) + (a/b)^(2/3))/(a^5*(a/b)^(2/3) - 2*a^3*b^2*(a/b)^(2/3) + a*b^4*(a/b)^(2
/3)) - 4*(a^2*b*(3*(a/b)^(2/3) + 4) + 2*a^3*(a/b)^(1/3) + a*b^2*(a/b)^(1/3)
 - b^3)*log((a/b)^(1/3) + sin(d*x + c))/(a^5*(a/b)^(2/3) - 2*a^3*b^2*(a/b)^
(2/3) + a*b^4*(a/b)^(2/3)) + 6*(a*b*sin(d*x + c)^2 - b^2*sin(d*x + c) + a*b
)/(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d*x + c)^3) + 9*log(sin(d*x + c) + 1
)/(a^2 - 2*a*b + b^2) - 9*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 2.70, size = 10855, normalized size = 18.49

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/324*(216*a^3*b - 216*a*b^3 - 108*(a^3*b - a*b^3)*cos(d*x + c)^2 - 2*((a^6
 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)^2 -
 (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))*(4*(9*a^2*b^2/(a^4*d - 2*a^2*
b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3)
 + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2
 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a
^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4
*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b
^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 +
 a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d
^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4
+ b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d -
2*a^2*b^2*d + b^4*d))*log(-56*a^5*b^2 + 20*a^3*b^4 + 1/324*(2*a^11 - 3*a^9*
b^2 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2
 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2
*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)
*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^
2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^
2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d
)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^
2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^
3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3
))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 1
/9*(12*a^8*b + 22*a^6*b^3 - 8*a^4*b^5 + a^2*b^7)*(4*(9*a^2*b^2/(a^4*d - 2*a
^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt
(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*
d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(
8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*
a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^
3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^
2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^
9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b
^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d
 - 2*a^2*b^2*d + b^4*d))*d + 4*(8*a^6*b + 28*a^4*b^3 - 10*a^2*b^5 + b^7)*si
n(d*x + c)) - (324*a^3*b - ((a^6 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3
*b^3 + a*b^5)*d*cos(d*x + c)^2 - (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c
))*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2
```

```
*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d +
 b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*
a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*
b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a
^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a
*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)
) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(
8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*
sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d)) + 3*sqrt(1/3)*((a^6 -
 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)^2 - (
a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))*sqrt((29808*a^4*b^2 + 10368*a^2
*b^4 - 5184*b^6 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(4*(
9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 +
a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)
^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2
*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3
) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3)
)^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((
a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/7
29*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 +
 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3)
 + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b - 2*a^5*b
^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2
- 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*
a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*
(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2
*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2
 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)
^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2
*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 -...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 566, normalized size = 0.96

$$-\frac{\frac{12\,ab\log\left(\left|b\sin(dx+c)^3+a\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{4\left(2\,a^8b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-3\,a^6b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^8\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^7b^3+9\,a^5b^5-6\,a^3b^7+ab^9\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^{11}b-4\,a^9b^3+6\,a^7b^5-4\,a^5b^7+a^3b^9}+\frac{4\left(\left(2\sqrt{3}\,a^3+\sqrt{3}\,ab^2\right)\left(-ab^2\right)^{\frac{2}{3}}+\left(4\sqrt{3}\,a^2b^2-\sqrt{3}\,b^4\right)\left(-ab^2\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^6b-2\,a^4b^3+a^2b^5}-\frac{2\left(\left(2\,a^3+ab^2\right)\left(-ab^2\right)^{\frac{2}{3}}-\left(4\,a^2b^2-b^4\right)\left(-ab^2\right)^{\frac{1}{3}}\right)\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^6b-2\,a^4b^3+a^2b^5}-\frac{9\,\log\left(\left|\sin(dx+c)+1\right|\right)}{a^2-2\,ab+b^2}+\frac{9\,\log\left(\left|\sin(dx+c)-1\right|\right)}{a^2+2\,ab+b^2}-\frac{6\left(2\,a^2b^2\sin(dx+c)^3+a^3b\sin(dx+c)^2-ab^3\sin(dx+c)^2-a^2b^2\sin(dx+c)+b^4\sin(dx+c)+3\,a^3b-ab^3\right)}{\left(a^5-2\,a^3b^2+ab^4\right)\left(b\sin(dx+c)^3+a\right)}}{18\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/18*(12*a*b*log(abs(b*sin(d*x + c)^3 + a))/(a^4 - 2*a^2*b^2 + b^4) + 4*(2
*a^8*b^2*(-a/b)^(1/3) - 3*a^6*b^4*(-a/b)^(1/3) + a^2*b^8*(-a/b)^(1/3) - 4*a
^7*b^3 + 9*a^5*b^5 - 6*a^3*b^7 + a*b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3)
+ sin(d*x + c)))/(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9) + 4
*((2*sqrt(3)*a^3 + sqrt(3)*a*b^2)*(-a*b^2)^(2/3) + (4*sqrt(3)*a^2*b^2 - sqr
t(3)*b^4)*(-a*b^2)^(1/3))*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c)
)/(-a/b)^(1/3))/(a^6*b - 2*a^4*b^3 + a^2*b^5) - 2*((2*a^3 + a*b^2)*(-a*b^2)
^(2/3) - (4*a^2*b^2 - b^4)*(-a*b^2)^(1/3))*log(sin(d*x + c)^2 + (-a/b)^(1/3
)*sin(d*x + c) + (-a/b)^(2/3))/(a^6*b - 2*a^4*b^3 + a^2*b^5) - 9*log(abs(si
n(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + 9*log(abs(sin(d*x + c) - 1))/(a^2 +
2*a*b + b^2) - 6*(2*a^2*b^2*sin(d*x + c)^3 + a^3*b*sin(d*x + c)^2 - a*b^3*s
in(d*x + c)^2 - a^2*b^2*sin(d*x + c) + b^4*sin(d*x + c) + 3*a^3*b - a*b^3)/
((a^5 - 2*a^3*b^2 + a*b^4)*(b*sin(d*x + c)^3 + a)))/d
```

Mupad [B]
time = 15.17, size = 980, normalized size = 1.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x)^3)^2),x)
```

```
[Out] symsum(log(((8*b^6)/27 - (16*a^2*b^4)/27)/(a^7 + a^3*b^4 - 2*a^5*b^2) + roo
t(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108*a
^3*b^2*z - 64*a^2*b + 8*b^3, z, k)*(((32*a*b^7)/27 + (128*a^3*b^5)/27)/(a^7
 + a^3*b^4 - 2*a^5*b^2) - root(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9
*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*z - 64*a^2*b + 8*b^3, z, k)*(root(1458*
a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*
z - 64*a^2*b + 8*b^3, z, k)*((16*a^3*b^9 - 77*a^5*b^7 + 34*a^7*b^5 + 27*a^9
*b^3)/(a^7 + a^3*b^4 - 2*a^5*b^2) + root(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3
 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*z - 64*a^2*b + 8*b^3, z, k)*(
(36*a^4*b^10 + 108*a^6*b^8 - 324*a^8*b^6 + 180*a^10*b^4)/(a^7 + a^3*b^4 - 2
*a^5*b^2) + (sin(c + d*x)*(4374*a^5*b^9 - 7290*a^7*b^7 + 1458*a^9*b^5 + 145
8*a^11*b^3))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2))) + (sin(c + d*x)*(216*a^2*b^1
0 - 864*a^4*b^8 - 1836*a^6*b^6 + 2484*a^8*b^4))/(27*(a^7 + a^3*b^4 - 2*a^5*
b^2))) + ((64*a^2*b^8)/9 - (353*a^4*b^6)/9 + (388*a^6*b^4)/9)/(a^7 + a^3*b^
4 - 2*a^5*b^2) + (sin(c + d*x)*(96*a*b^9 - 408*a^3*b^7 + 447*a^5*b^5))/(27*
(a^7 + a^3*b^4 - 2*a^5*b^2))) + (sin(c + d*x)*(16*b^8 + 134*a^2*b^6 - 236*a
^4*b^4))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2))) + (8*a*b^5*sin(c + d*x))/(9*(a^7
 + a^3*b^4 - 2*a^5*b^2)))*root(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9
*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*z - 64*a^2*b + 8*b^3, z, k), k, 1, 3)/d
 - log(sin(c + d*x) - 1)/(d*(4*a*b + 2*a^2 + 2*b^2)) + log(sin(c + d*x) + 1
```

```
)/(d*(2*a^2 - 4*a*b + 2*b^2)) + (b/(3*(a^2 - b^2)) + (b*sin(c + d*x)^2)/(3*
(a^2 - b^2)) - (b^2*sin(c + d*x))/(3*a*(a^2 - b^2)))/(d*(a + b*sin(c + d*x)
^3))
```

3.398 $\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

Optimal. Leaf size=747

$$-\frac{b^{5/3}\left(4a^2-3a^{4/3}b^{2/3}+2b^2\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\left(a^2-b^2\right)^2 d}-\frac{b^{5/3}\left(4a^{8/3}-9a^2b^{2/3}+8a^{2/3}b^2-3b^{8/3}\right)\tan^{-1}\left(\right.}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^3 d}$$

```
[Out] -1/4*(a+7*b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/4*(a-7*b)*ln(1+sin(d*x+c))/(a-b)^
3/d+1/9*b^(5/3)*(4*a^2+3*a^(4/3)*b^(2/3)+2*b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+
c))/a^(5/3)/(a^2-b^2)^2/d+1/3*b^(5/3)*(3*b^(2/3)*(3*a^2+b^2)+4*a^(2/3)*(a^2
+2*b^2))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(1/3)/(a^2-b^2)^3/d-1/18*b^(5/3)*
(4*a^2+3*a^(4/3)*b^(2/3)+2*b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/
3)*sin(d*x+c)^2)/a^(5/3)/(a^2-b^2)^2/d-1/6*b^(5/3)*(3*b^(2/3)*(3*a^2+b^2)+4
*a^(2/3)*(a^2+2*b^2))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x
+c)^2)/a^(1/3)/(a^2-b^2)^3/d+2/3*a*b*(a^2+5*b^2)*ln(a+b*sin(d*x+c)^3)/(a^2-
b^2)^3/d+1/4/(a+b)^2/d/(1-sin(d*x+c))-1/4/(a-b)^2/d/(1+sin(d*x+c))-1/3*b*(a
*(a^2+2*b^2)-b*sin(d*x+c)*(2*a^2+b^2-3*a*b*sin(d*x+c)))/a/(a^2-b^2)^2/d/(a+
b*sin(d*x+c)^3)-1/9*b^(5/3)*(4*a^2-3*a^(4/3)*b^(2/3)+2*b^2)*arctan(1/3*(a^(
1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/(a^2-b^2)^2/d*3^(1/2)-1
/3*b^(5/3)*(4*a^(8/3)-9*a^2*b^(2/3)+8*a^(2/3)*b^2-3*b^(8/3))*arctan(1/3*(a^
(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/(a^2-b^2)^3/d*3^(1/2)
```

Rubi [A]

time = 0.68, antiderivative size = 747, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3302, 2099, 1868, 1874, 31, 648, 631, 210, 642, 1885, 266}

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] -1/3*(b^(5/3)*(4*a^2 - 3*a^(4/3)*b^(2/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(1/
3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*(a^2 - b^2)^2*d) - (b
^(5/3)*(4*a^(8/3) - 9*a^2*b^(2/3) + 8*a^(2/3)*b^2 - 3*b^(8/3))*ArcTan[(a^(1
/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b
^2)^3*d) - ((a + 7*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a - 7*b)*L
og[1 + Sin[c + d*x]])/(4*(a - b)^3*d) + (b^(5/3)*(4*a^2 + 3*a^(4/3)*b^(2/3)
 + 2*b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)^2*d)
+ (b^(5/3)*(3*b^(2/3)*(3*a^2 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(1/3)
+ b^(1/3)*Sin[c + d*x]])/(3*a^(1/3)*(a^2 - b^2)^3*d) - (b^(5/3)*(4*a^2 + 3*
a^(4/3)*b^(2/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/
3)*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 - b^2)^2*d) - (b^(5/3)*(3*b^(2/3)*(3*a
```

```
^2 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c +
d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^3*d) + (2*a*b*(a^2 +
 5*b^2)*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^3*d) + 1/(4*(a + b)^2*d*(
1 - Sin[c + d*x])) - 1/(4*(a - b)^2*d*(1 + Sin[c + d*x])) - (b*(a*(a^2 + 2*
b^2) - b*Sin[c + d*x]*(2*a^2 + b^2 - 3*a*b*Sin[c + d*x])))/(3*a*(a^2 - b^2)
^2*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
```

```
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)^2(-1+x)^2} + \frac{-a-7b}{4(a+b)^3(-1+x)} + \frac{1}{4(a-b)^2(1+x)^2} + \frac{a-7b}{4(a-b)^3(1+x)} + \frac{b^2(2a^2+b^2}{(a^2-}\right.\right.}{d}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{}{4(a+b)^2d(1}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{}{4(a+b)^2d(1}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{2ab(a^2+5b^2}{3}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{b^{5/3}(4a^2+3}{}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{b^{5/3}(4a^2+3}{}$$

$$= -\frac{b^{5/3}\left(4a^2-3a^{4/3}b^{2/3}+2b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\left(a^2-b^2\right)^2d} - \frac{b^{5/3}\left(4a^{8/3}-9a^2b^{2/3}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.30, size = 657, normalized size = 0.88

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-1/4*((a + 7*b)*Log[1 - Sin[c + d*x]])/(a + b)^3 + ((a - 7*b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^3) + (4*a^(1/3)*b^(5/3)*(a^2 + 2*b^2)*Log[a^(1/3) + b
^(1/3)*Sin[c + d*x]])/(3*(a^2 - b^2)^3) - (2*a^(1/3)*(a^2 + 2*b^2)*(2*Sqrt[
```

```
3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] + b
^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]
))/(3*(a^2 - b^2)^3) + ((2 + b^2/a^2)*(2*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1
/3)*Sin[c + d*x]] - a^(1/3)*(2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*
Sin[c + d*x])/(Sqrt[3]*a^(1/3))] + b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Si
n[c + d*x] + b^(2/3)*Sin[c + d*x]^2])))/(9*(a^2 - b^2)^2) + (2*a*b*(a^2 + 5
*b^2)*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^3) + 1/(4*(a + b)^2*(1 - Si
n[c + d*x])) - (3*b^3*(3*a^2 + b^2)*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin
[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(2*a*(a^2 - b^2)^3) - (3*b^3*Hypergeometri
c2F1[2/3, 2, 5/3, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(2*a*(a^2 - b^2)
^2) - 1/(4*(a - b)^2*(1 + Sin[c + d*x])) - (b*(a^2 + 2*b^2))/(3*(a^2 - b^2)
^2*(a + b*Sin[c + d*x]^3)) + (a*b^2*(2 + b^2/a^2)*Sin[c + d*x])/(3*(a^2 - b
^2)^2*(a + b*Sin[c + d*x]^3)))/d
```

Maple [A]

time = 2.84, size = 483, normalized size = 0.65

method	result
derivativedivides	$\frac{b^2\left(\frac{(-a^2b+b^3)(\sin^2(dx+c))+\frac{(2a^4-a^2b^2-b^4)\sin(dx+c)}{3a}-\frac{a^4+a^2b^2-2b^4}{3b}}{a+b(\sin^3(dx+c))}+\frac{2(8a^4+11a^2b^2-b^4)\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln(s}{}\right.}{}\right.}{}$

default	$b^2\left(\frac{\left(-a^2b+b^3\right)\left(\sin^2(dx+c)\right)+\frac{\left(2a^4-a^2b^2-b^4\right)\sin(dx+c)}{3a}-\frac{a^4+a^2b^2-2b^4}{3b}}{a+b\left(\sin^3(dx+c)\right)}+\frac{2\left(8a^4+11a^2b^2-b^4\right)\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin \ldots\right.}{\ldots}\right.}{\ldots}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2/(a-b)^3/(a+b)^3*(((-a^2*b+b^3)*sin(d*x+c)^2+1/3*(2*a^4-a^2*b^2-b^4
)/a*sin(d*x+c)-1/3*(a^4+a^2*b^2-2*b^4)/b)/(a+b*sin(d*x+c)^3)+2/3/a*((8*a^4+
11*a^2*b^2-b^4)*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/
b*a)^(2/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1
/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+(-1
5*a^3*b-3*a*b^3)*(-1/3/b/(1/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(
1/b*a)^(1/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^
(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+1
/3*(3*a^4+15*a^2*b^2)/b*ln(a+b*sin(d*x+c)^3)))-1/4/(a-b)^2/(1+sin(d*x+c))+1
/4*(a-7*b)/(a-b)^3*ln(1+sin(d*x+c))-1/4/(a+b)^2/(sin(d*x+c)-1)+1/4/(a+b)^3*
(-a-7*b)*ln(sin(d*x+c)-1))
```

Maxima [A]

time = 0.51, size = 788, normalized size = 1.05

$$-\frac{\frac{8\sqrt{3}\left(5a^3b^2\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\right)-a^2b^3\left(11\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{10a}{b}\right)-2a^4b\left(4\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{a}{b}\right)+3ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}+2a^5\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^7\left(\frac{a}{b}\right)^{\frac{2}{3}}-3a^5b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+3a^3b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^6\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{4\left(a^2b^3\left(30\left(\frac{a}{b}\right)^{\frac{2}{3}}-11\right)+2a^4b\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\right)-15a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-3ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}+b^5\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^7\left(\frac{a}{b}\right)^{\frac{2}{3}}-3a^5b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+3a^3b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{8\left(a^2b^3\left(15\left(\frac{a}{b}\right)^{\frac{2}{3}}+11\right)+a^4b\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}+8\right)+15a^3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+3ab^4\left(\frac{a}{b}\right)^{\frac{1}{3}}-b^5\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{a^7\left(\frac{a}{b}\right)^{\frac{2}{3}}-3a^5b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+3a^3b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^6\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{9(a-7b)\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}+\frac{9(a+7b)\log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3}-\frac{6\left(3\left(a^3b+3ab^3\right)\sin(dx+c)^4-8a^3b-4ab^3-2\left(5a^2b^2+b^4\right)\sin(dx+c)^3+2\left(a^3b-ab^3\right)\sin(dx+c)^2+\left(3a^4+7a^2b^2+2b^4\right)\sin(dx+c)\right)}{a^6-2a^4b^2+a^2b^4-\left(a^5b-2a^3b^3+ab^5\right)\sin(dx+c)^5+\left(a^5b-2a^3b^3+ab^5\right)\sin(dx+c)^3-\left(a^6-2a^4b^2+a^2b^4\right)\sin(dx+c)^2}}{36d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/36*(8*sqrt(3)*(5*a^3*b^2*(3*(a/b)^(2/3) + 2) - a^2*b^3*(11*(a/b)^(1/3) +
 10*a/b) - 2*a^4*b*(4*(a/b)^(1/3) + a/b) + 3*a*b^4*(a/b)^(2/3) + b^5*(a/b)^
```

```
(1/3) + 2*a^5)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/
3))/((a^7*(a/b)^(2/3) - 3*a^5*b^2*(a/b)^(2/3) + 3*a^3*b^4*(a/b)^(2/3) - a*b
^6*(a/b)^(2/3))*(a/b)^(1/3)) - 4*(a^2*b^3*(30*(a/b)^(2/3) - 11) + 2*a^4*b*(
3*(a/b)^(2/3) - 4) - 15*a^3*b^2*(a/b)^(1/3) - 3*a*b^4*(a/b)^(1/3) + b^5)*lo
g(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a^7*(a/b)^(2/3)
 - 3*a^5*b^2*(a/b)^(2/3) + 3*a^3*b^4*(a/b)^(2/3) - a*b^6*(a/b)^(2/3)) - 8*(
a^2*b^3*(15*(a/b)^(2/3) + 11) + a^4*b*(3*(a/b)^(2/3) + 8) + 15*a^3*b^2*(a/b
)^(1/3) + 3*a*b^4*(a/b)^(1/3) - b^5)*log((a/b)^(1/3) + sin(d*x + c))/(a^7*(
a/b)^(2/3) - 3*a^5*b^2*(a/b)^(2/3) + 3*a^3*b^4*(a/b)^(2/3) - a*b^6*(a/b)^(2
/3)) - 9*(a - 7*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) +
9*(a + 7*b)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(3*(a
^3*b + 3*a*b^3)*sin(d*x + c)^4 - 8*a^3*b - 4*a*b^3 - 2*(5*a^2*b^2 + b^4)*si
n(d*x + c)^3 + 2*(a^3*b - a*b^3)*sin(d*x + c)^2 + (3*a^4 + 7*a^2*b^2 + 2*b^
4)*sin(d*x + c))/(a^6 - 2*a^4*b^2 + a^2*b^4 - (a^5*b - 2*a^3*b^3 + a*b^5)*s
in(d*x + c)^5 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3 - (a^6 - 2*a^4*b
^2 + a^2*b^4)*sin(d*x + c)^2))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 6.24, size = 15989, normalized size = 21.40

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(18*a^5*b - 36*a^3*b^3 + 18*a*b^5 - 18*(a^5*b + 2*a^3*b^3 - 3*a*b^5)*
cos(d*x + c)^4 + 48*(a^5*b + a^3*b^3 - 2*a*b^5)*cos(d*x + c)^2 + 2*((a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 - ((a^7*b - 3*a^5*b^3 + 3
*a^3*b^5 - a*b^7)*d*cos(d*x + c)^4 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7
)*d*cos(d*x + c)^2)*sin(d*x + c))*(3*4^(2/3)*(-I*sqrt(3) + 1)*(3*(a^3*b + 5
*a*b^3)^2/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)^2 - (3*a^2*b^2 - b^4)
/(a^8*d^2 - 3*a^6*b^2*d^2 + 3*a^4*b^4*d^2 - a^2*b^6*d^2))/((27*a^2*b^3 - b^
5)/(a^11*d^3 - 3*a^9*b^2*d^3 + 3*a^7*b^4*d^3 - a^5*b^6*d^3) + 54*(a^3*b + 5
*a*b^3)^3/(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)^3 - 27*(a^3*b + 5*a*b
^3)*(3*a^2*b^2 - b^4)/((a^8*d^2 - 3*a^6*b^2*d^2 + 3*a^4*b^4*d^2 - a^2*b^6*d
^2)*(a^6*d - 3*a^4*b^2*d + 3*a^2*b^4*d - b^6*d)) + (512*a^6 + 273*a^4*b^2 -
 30*a^2*b^4 + b^6)*b^5/((a^2 - b^2)^6*a^5*d^3))^(1/3) + 4^(1/3)*(I*sqrt(3)
+ 1)*((27*a^2*b^3 - b^5)/(a^11*d^3 - 3*a^9*b^2*d^3 + 3*a^7*b^4*d^3 - a^5*b^
6*d^3) + 54*(a^3*b + 5*a* ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 790, normalized size = 1.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/36*(8*(15*a^10*b^4*(-a/b)^(1/3) - 42*a^8*b^6*(-a/b)^(1/3) + 36*a^6*b^8*(-
a/b)^(1/3) - 6*a^4*b^10*(-a/b)^(1/3) - 3*a^2*b^12*(-a/b)^(1/3) - 8*a^11*b^3
 + 13*a^9*b^5 + 10*a^7*b^7 - 28*a^5*b^9 + 14*a^3*b^11 - a*b^13)*(-a/b)^(1/3
)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^15*b - 6*a^13*b^3 + 15*a^11*b^5
 - 20*a^9*b^7 + 15*a^7*b^9 - 6*a^5*b^11 + a^3*b^13) + 8*(3*(5*sqrt(3)*a^3*b
 + sqrt(3)*a*b^3)*(-a*b^2)^(2/3) + (8*sqrt(3)*a^4*b + 11*sqrt(3)*a^2*b^3 -
sqrt(3)*b^5)*(-a*b^2)^(1/3))*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x +
 c))/(-a/b)^(1/3))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6) - 4*(3*(5*a^3*b
+ a*b^3)*(-a*b^2)^(2/3) - (8*a^4*b + 11*a^2*b^3 - b^5)*(-a*b^2)^(1/3))*log(
sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^8 - 3*a^6*b^2
 + 3*a^4*b^4 - a^2*b^6) + 24*(a^3*b + 5*a*b^3)*log(abs(b*sin(d*x + c)^3 + a
))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 9*(a - 7*b)*log(abs(sin(d*x + c) +
 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 9*(a + 7*b)*log(abs(sin(d*x + c) - 1
))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(3*a^3*b*sin(d*x + c)^4 + 9*a*b^3*si
n(d*x + c)^4 - 10*a^2*b^2*sin(d*x + c)^3 - 2*b^4*sin(d*x + c)^3 + 2*a^3*b*s
in(d*x + c)^2 - 2*a*b^3*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + 7*a^2*b^2*sin
(d*x + c) + 2*b^4*sin(d*x + c) - 8*a^3*b - 4*a*b^3)/((b*sin(d*x + c)^5 - b*
sin(d*x + c)^3 + a*sin(d*x + c)^2 - a)*(a^5 - 2*a^3*b^2 + a*b^4)))/d
```

Mupad [B]
time = 15.75, size = 1605, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x)^3)^2),x)
```

```
[Out] symsum(log(root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b^6*z^3 - 729
*a^11*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z + 324*a^3*b^4
*z + 216*a^2*b^3 - 8*b^5, z, k)*(((32*a*b^11)/27 + (2173*a^3*b^9)/27 + (847
*a^5*b^7)/3 - 20*a^7*b^5)/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b
^2) - root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b^6*z^3 - 729*a^11
*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z + 324*a^3*b^4*z +
216*a^2*b^3 - 8*b^5, z, k)*(((32*a^2*b^12)/3 - (1017*a^4*b^10)/4 + 325*a^6*
```

```
b^8 + (4153*a^8*b^6)/12 - (63*a^10*b^4)/2)/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*
a^7*b^4 - 4*a^9*b^2) + root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b
^6*z^3 - 729*a^11*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z +
 324*a^3*b^4*z + 216*a^2*b^3 - 8*b^5, z, k)*((16*a^3*b^13 - (563*a^5*b^11)/
2 + 303*a^7*b^9 + 188*a^9*b^7 - 239*a^11*b^5 + (27*a^13*b^3)/2)/(a^11 + a^3
*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) + root(2187*a^9*b^2*z^3 - 2187*a^
7*b^4*z^3 + 729*a^5*b^6*z^3 - 729*a^11*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*
z^2 - 972*a^5*b^2*z + 324*a^3*b^4*z + 216*a^2*b^3 - 8*b^5, z, k)*((36*a^4*b
^14 + 36*a^6*b^12 - 504*a^8*b^10 + 936*a^10*b^8 - 684*a^12*b^6 + 180*a^14*b
^4)/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) + (sin(c + d*x)*(1
7496*a^5*b^13 - 64152*a^7*b^11 + 81648*a^9*b^9 - 34992*a^11*b^7 - 5832*a^13
*b^5 + 5832*a^15*b^3))/(108*(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9
*b^2))) - (sin(c + d*x)*(13824*a^4*b^12 - 864*a^2*b^14 + 30780*a^6*b^10 - 9
6660*a^8*b^8 + 50004*a^10*b^6 + 2916*a^12*b^4))/(108*(a^11 + a^3*b^8 - 4*a^
5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))) - (sin(c + d*x)*(7200*a^3*b^11 - 384*a*b^1
3 - 68247*a^5*b^9 + 31542*a^7*b^7 + 10449*a^9*b^5))/(108*(a^11 + a^3*b^8 -
4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))) + (sin(c + d*x)*(64*b^12 + 4758*a^2*b^
10 - 29860*a^4*b^8 - 9234*a^6*b^6))/(108*(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^
7*b^4 - 4*a^9*b^2))) + ((28*b^10)/27 + (122*a^2*b^8)/27 + (10*a^4*b^6)/3)/(
a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) + (sin(c + d*x)*(1080*a
*b^9 + 2568*a^3*b^7))/(108*(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*
b^2)))*root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b^6*z^3 - 729*a^1
1*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z + 324*a^3*b^4*z +
 216*a^2*b^3 - 8*b^5, z, k), k, 1, 3)/d + ((b*sin(c + d*x)^2)/(3*(a^2 - b^2
)) + (sin(c + d*x)^4*((a^2*b)/2 + (3*b^3)/2))/(a^4 + b^4 - 2*a^2*b^2) - (2*
b*(2*a^2 + b^2))/(3*(a^2 - b^2)^2) + (sin(c + d*x)*(3*a^4 + 2*b^4 + 7*a^2*b
^2))/(6*a*(a^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^3*(b^4/3 + (5*a^2*b^2)/3
))/(a*(a^4 + b^4 - 2*a^2*b^2)))/(d*(a - a*sin(c + d*x)^2 + b*sin(c + d*x)^3
 - b*sin(c + d*x)^5)) - (log(sin(c + d*x) - 1)*(a + 7*b))/(d*(12*a*b^2 + 12
*a^2*b + 4*a^3 + 4*b^3)) + (log(sin(c + d*x) + 1)*(a - 7*b))/(d*(12*a*b^2 -
 12*a^2*b + 4*a^3 - 4*b^3))
```

3.399 $\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] Defer[Int][Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] Result contains complex when optimal does not.
time = 0.28, size = 394, normalized size = 15.15

$$\frac{-i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+4ia\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+2a\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1+12b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-6ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-4ia\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^3-2a\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^3+2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right]+\frac{24\cos(c+dx)(a+b\sin(c+dx))}{4a+3b\sin(c+dx)-b\sin(3(c+dx))}}{18abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
, (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2] + (4*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 12*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^2 - (6*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 2*a*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^3 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[
```

```
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^
5) & ] + (24*Cos[c + d*x]*(a + b*Sin[c + d*x]))/(4*a + 3*b*Sin[c + d*x] - b
*Sin[3*(c + d*x)]))/(18*a*b*d)
```

Maple [**A**] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(25) = 50$.
time = 1.64, size = 241, normalized size = 9.27

method	result
derivativedivides	$\frac{\frac{-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{2\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b}+\frac{8\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a}+\frac{2}{3b}}{a\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+ \frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\right.}\right.}{}}{d}$
default	$\frac{\frac{-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{2\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b}+\frac{8\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a}+\frac{2}{3b}}{a\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+ \frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\right.}\right.}{}}{d}$
risch	$-\frac{2\left(2ia\,e^{4i(dx+c)}+b\,e^{5i(dx+c)}+2ia\,e^{2i(dx+c)}-b\,e^{i(dx+c)}\right)}{3abd\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left(531441a^{10}b^{8}d^{6}\_Z^{6}+59049a^{8}b^{6}d\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/3/a*tan(1/2*d*x+1/2*c)^5+1/3/b*tan(1/2*d*x+1/2*c)^4+4/3/a*tan(1/
2*d*x+1/2*c)^3+2/3/b*tan(1/2*d*x+1/2*c)^2+1/3/a*tan(1/2*d*x+1/2*c)+1/3/b)/(
a*tan(1/2*d*x+1/2*c)^6+3*a*tan(1/2*d*x+1/2*c)^4+8*b*tan(1/2*d*x+1/2*c)^3+3*
a*tan(1/2*d*x+1/2*c)^2+a)+2/9/a/b*sum((_R^4*b+_R^3*a+_R*a+b)/(_R^5*a+2*_R^3
*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^
3*b+3*_Z^2*a+a)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c)^3 + a)^2, x)
```

Mupad [A]
time = 15.99, size = 2431, normalized size = 93.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] 2/(3*d*(a*b + 8*b^2*tan(c/2 + (d*x)/2)^3 + 3*a*b*tan(c/2 + (d*x)/2)^2 + 3*a
*b*tan(c/2 + (d*x)/2)^4 + a*b*tan(c/2 + (d*x)/2)^6)) + symsum(log((638976*a
^2*b^4 - 655360*b^6 - 8192*a^6 + 24576*a^4*b^2 - 2949120*root(531441*a^10*b
^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a
^6 - 64*b^6, d, k)*a^3*b^5 + 2138112*root(531441*a^10*b^8*d^6 + 59049*a^8*b
^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*a
^5*b^3 - 9437184*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^
4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*b^8*tan(c/2 + (d*x)/2
) - 786432*a*b^5*tan(c/2 + (d*x)/2) + 98304*a^5*b*tan(c/2 + (d*x)/2) - 2123
3664*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a
^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^2*b^8 + 18579456*root(531441*
a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b
^2 + a^6 - 64*b^6, d, k)^2*a^4*b^6 + 2654208*root(531441*a^10*b^8*d^6 + 590
```

49*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^6*b^4 - 167215104*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^3*a^5*b^7 + 113467392*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^3*a^7*b^5 - 107495424*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^4*a^6*b^8 + 107495424*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^4*a^8*b^6 - 1934917632*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^5*a^7*b^9 + 1451188224*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^5*a^9*b^7 + 688128*a^3*b^3*tan(c/2 + (d*x)/2) - 1179648*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*a*b^7 + 12976128*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*a^2*b^6*tan(c/2 + (d*x)/2) - 6266880*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*a^4*b^4*tan(c/2 + (d*x)/2) + 737280*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*a^6*b^2*tan(c/2 + (d*x)/2) - 53084160*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^3*b^7*tan(c/2 + (d*x)/2) + 50429952*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^5*b^5*tan(c/2 + (d*x)/2) + 2654208*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^7*b^3*tan(c/2 + (d*x)/2) - 59719680*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^3*a^6*b^6*tan(c/2 + (d*x)/2) + 5971968*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^3*a^8*b^4*tan(c/2 + (d*x)/2) - 859963392*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^4*a^5*b^9*tan(c/2 + (d*x)/2) + 859963392*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^4*a^7*b^7*tan(c/2 + (d*x)/2) - 483729408*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^5*a^8*b^8*tan(c/2 + (d*x)/2))/(a^3*b^4))*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k), k, 1, 6)/d + (8*tan(c/2 + (d*x)/2)^3)/(3*d*(3*a^2*tan(c/2 + (d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*b*tan(c/2 + (d*x)/2)^3)) - (2*tan(c/2 + (d*x)/2)^5)/(3*d*(3*a^2*tan(c/2 + (d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*b*tan(c/2 + (d*x)/2)^3)) + (4*tan(c/2 + (d*x)/2)^2)/(3*d*(a*b + 8*b^2*tan(c/2 + (d*x)/2)^3 + 3*a*b*tan(c/2 + (d*x)/2)^2 + 3*a*b*tan(c/2 + (d*x)/2)^4 + a*b*tan(c/2 + (d*x)/2)^6)) + (2*tan(c/2 + (d*x)/2)^4)/(3*d*(a*b + 8*b^2*tan(c/2 +

```
(d*x)/2)^3 + 3*a*b*tan(c/2 + (d*x)/2)^2 + 3*a*b*tan(c/2 + (d*x)/2)^4 + a*b
*tan(c/2 + (d*x)/2)^6)) + (2*tan(c/2 + (d*x)/2))/(3*d*(3*a^2*tan(c/2 + (d*x
)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*
b*tan(c/2 + (d*x)/2)^3))
```

3.400 $\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2} dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}, x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
[Out] Defer[Int][Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2} dx = \int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2} dx$$

Mathematica [A] Result contains complex when optimal does not.
time = 0.18, size = 273, normalized size = 10.50

$$\frac{-i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+12\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2-6i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2+2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right]+\frac{12\sin(2(c+dx))}{4a+3b\sin(c+dx)-b\sin(3(c+dx))}}{18ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
[Out] ((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
, (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2] + 12*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (6*I)*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2
*b*#1^3 + b*#1^5) & ] + (12*Sin[2*(c + d*x)])/(4*a + 3*b*Sin[c + d*x] - b*S
in[3*(c + d*x)]))/(18*a*d)
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(25) = 50$.
time = 2.01, size = 175, normalized size = 6.73

method	result
derivativedivides	$\frac{\frac{-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_\right.}\right.}{9a}}{d}$
default	$\frac{\frac{-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_\right.}\right.}{9a}}{d}$
risch	$-\frac{2\left(e^{5i(dx+c)}-e^{i(dx+c)}\right)}{3ad\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(531441a^{12}b^4d^6-531441a^{10}b^6d^6\right)\_\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/3/a*tan(1/2*d*x+1/2*c)^5+1/3/a*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*
x+1/2*c)^6+3*a*tan(1/2*d*x+1/2*c)^4+8*b*tan(1/2*d*x+1/2*c)^3+3*a*tan(1/2*d*
x+1/2*c)^2+a)+2/9/a*sum((_R^4+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2
*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A] Result contains complex when optimal does not.
time = 3.28, size = 36403, normalized size = 1400.12

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/324*(3*sqrt(2/3)*sqrt(1/6)*(a^2*d - (a*b*d*cos(d*x + c)^2 - a*b*d)*sin(d*
x + c))*sqrt(-((a^4 - a^2*b^2)*((-I*sqrt(3) + 1)*(3/(a^6*b^2*d^4 - a^4*b^4*
d^4) - 1/(a^4*d^2 - a^2*b^2*d^2)^2)/(-1/1062882*(a^4 - 16*a^2*b^2 + 64*b^4)
/(a^12*b^4*d^6 - a^10*b^6*d^6) + 1/118098/((a^6*b^2*d^4 - a^4*b^4*d^4)*(a^4
*d^2 - a^2*b^2*d^2)) - 1/531441/(a^4*d^2 - a^2*b^2*d^2)^3 + 1/1062882*(a^6
+ 28*a^4*b^2 - 80*a^2*b^4 + 64*b^6)/((a^2 - b^2)^2*a^10*b^4*d^6))^(1/3) - 6
561*(I*sqrt(3) + 1)*(-1/1062882*(a^4 - 16*a^2*b^2 + 64*b^4)/(a^12*b^4*d^6 -
 a^10*b^6*d^6) + 1/118098/((a^6*b^2*d^4 - a^4*b^4*d^4)*(a^4*d^2 - a^2*b^2*d
^2)) - 1/531441/(a^4*d^2 - a^2*b^2*d^2)^3 + 1/1062882*(a^6 + 28*a^4*b^2 - 8
0*a^2*b^4 + 64*b^6)/((a^2 - b^2)^2*a^10*b^4*d^6))^(1/3) - 162/(a^4*d^2 - a^
2*b^2*d^2))*d^2 + 3*sqrt(1/3)*(a^4 - a^2*b^2)*d^2*sqrt(-((a^8*b^2 - 2*a^6*b
^4 + a^4*b^6)*((-I*sqrt(3) + 1)*(3/(a^6*b^2*d^4 - a^4*b^4*d^4) - 1/(a^4*d^2
 - a^2*b^2*d^2)^2)/(-1/1062882*(a^4 - 16*a^2*b^2 + 64*b^4)/(a^12*b^4*d^6 -
a^10*b^6*d^6) + 1/118098/ ...
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sin(c + d*x)**3)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a)^2, x)
```

Mupad [A]
time = 15.75, size = 1648, normalized size = 63.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] symsum(log(-((131072*b^2)/243 - (16384*a^2)/243 + (8192*root(531441*a^12*b^
4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*
```

b^2 + a^4 + 64*b^4, d, k)*a^4*tan(c/2 + (d*x)/2))/27 + (1048576*root(531441
*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 -
16*a^2*b^2 + a^4 + 64*b^4, d, k)*b^4*tan(c/2 + (d*x)/2))/27 + (262144*root
(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^
2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^2*b^4)/3 - (131072*root(531441
*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 -
16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^4*b^2)/3 - 98304*root(531441*a^12*b^4
*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b
^2 + a^4 + 64*b^4, d, k)^3*a^5*b^3 + 442368*root(531441*a^12*b^4*d^6 - 5314
41*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 +
64*b^4, d, k)^4*a^6*b^4 + 221184*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6
*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d,
k)^4*a^8*b^2 + 7962624*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 196
83*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^5*a^7*b
^5 - 5971968*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4
*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^5*a^9*b^3 + (1310
72*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729
*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)*a*b^3)/27 - (65536*root(531
441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^
2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)*a^3*b)/27 - (131072*root(531441*a^12*b
^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2
*b^2 + a^4 + 64*b^4, d, k)*a^2*b^2*tan(c/2 + (d*x)/2))/9 - (32768*root(5314
41*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2
- 16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^5*b*tan(c/2 + (d*x)/2))/3 - (131072
*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a
^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^3*b^3*tan(c/2 + (d*x)/2))
/3 + 245760*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*
d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^3*a^6*b^2*tan(c/2
+ (d*x)/2) + 3538944*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683
*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^4*a^5*b^5
*tan(c/2 + (d*x)/2) - 2654208*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^
6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^
4*a^7*b^3*tan(c/2 + (d*x)/2) + 1990656*root(531441*a^12*b^4*d^6 - 531441*a^
10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^
4, d, k)^5*a^8*b^4*tan(c/2 + (d*x)/2))/a^3)*root(531441*a^12*b^4*d^6 - 5314
41*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 +
64*b^4, d, k), k, 1, 6)/d - (2*tan(c/2 + (d*x)/2)^5)/(3*d*(3*a^2*tan(c/2 +
(d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 +
8*a*b*tan(c/2 + (d*x)/2)^3)) + (2*tan(c/2 + (d*x)/2))/(3*d*(3*a^2*tan(c/2 +
(d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 +
8*a*b*tan(c/2 + (d*x)/2)^3))

3.401 $\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

[Out] Unintegrable(1/(a+b*sin(d*x+c)^3)^2,x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*Sin[c + d*x]^3)^(-2),x]

[Out] Defer[Int][(a + b*Sin[c + d*x]^3)^(-2), x]

Rubi steps

$$\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] Result contains complex when optimal does not.
time = 0.37, size = 502, normalized size = 29.53

$$\frac{\frac{i\,\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-ib^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+4iab\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+2ab\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1-24a^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+12b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^2+12ia^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-6ib^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^2-4iab\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^3-2ab\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^3+2b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-ib^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\#1^4}{b\#1-4ia\#1^2-2b\#1^3+b\#1^5}\&\right]}{a^2-b^2}-\frac{12b\cos(c+dx)(-3a+a\cos(2(c+dx))+2b\sin(c+dx))}{(a-b)(a+b)(4a+3b\sin(c+dx)-b\sin(3(c+dx)))}}{18ad}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[c + d*x]^3)^(-2),x]

[Out] ((I*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 24*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 12*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (12*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 2*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2*b^2*ArcTan[S

```
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a^2 - b^2) - (12
*b*Cos[c + d*x]*(-3*a + a*Cos[2*(c + d*x)] + 2*b*Sin[c + d*x]))/((a - b)*(a
 + b)*(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c + d*x)])))/(18*a*d)
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(16) = 32$.
time = 3.13, size = 349, normalized size = 20.53

method	result
derivativedivides	$\frac{\frac{\frac{2b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a\left(a^2-b^2\right)}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-b^2\right)}+\frac{8b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a\left(a^2-b^2\right)}+\frac{8b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-b^2\right)}-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a\left(a^2-b^2\right)}+\frac{2b}{3a^2-3b^2}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\_R=\mathrm{RootOf}(a}{d}$
default	$\frac{\frac{\frac{2b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a\left(a^2-b^2\right)}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-b^2\right)}+\frac{8b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a\left(a^2-b^2\right)}+\frac{8b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-b^2\right)}-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a\left(a^2-b^2\right)}+\frac{2b}{3a^2-3b^2}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}+\_R=\mathrm{RootOf}(a}{d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(1/3*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^5-1/3/(a^2-b^2)*b*tan(1/2*d*
x+1/2*c)^4+4/3*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+4/3/(a^2-b^2)*b*tan(1/2
*d*x+1/2*c)^2-1/3*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)+1/3/(a^2-b^2)*b)/(a*ta
n(1/2*d*x+1/2*c)^6+3*a*tan(1/2*d*x+1/2*c)^4+8*b*tan(1/2*d*x+1/2*c)^3+3*a*ta
n(1/2*d*x+1/2*c)^2+a)+1/9/a/(a^2-b^2)*sum(((3*a^2-2*b^2)*_R^4-2*a*b*_R^3+6*
a^2*_R^2-2*a*_R*b+3*a^2-2*b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d
*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A] Result contains complex when optimal does not.
time = 8.35, size = 70185, normalized size = 4128.53

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(36*a*b*cos(d*x + c)^3 + 36*b^2*cos(d*x + c)*sin(d*x + c) - sqrt(2/3
)*sqrt(1/2)*((a^4 - a^2*b^2)*d - ((a^3*b - a*b^3)*d*cos(d*x + c)^2 - (a^3*b
 - a*b^3)*d)*sin(d*x + c))*sqrt(-(1458*a^4 + 486*a^2*b^2 - 486*b^4 - (a^8 -
 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*((-I*sqrt(3) + 1)*(3*(3*a^4 + a^2*b^2 - b
^4)^2/(a^8*d^2 - 3*a^6*b^2*d^2 + 3*a^4*b^4*d^2 - a^2*b^6*d^2)^2 - (27*a^2 -
 11*b^2)/(a^10*d^4 - 3*a^8*b^2*d^4 + 3*a^6*b^4*d^4 - a^4*b^6*d^4))/(-1/1062
882*(729*a^4 - 432*a^2*b^2 + 64*b^4)/(a^16*d^6 - 3*a^14*b^2*d^6 + 3*a^12*b^
4*d^6 - a^10*b^6*d^6) - 1/19683*(3*a^4 + a^2*b^2 - b^4)^3/(a^8*d^2 - 3*a^6*
b^2*d^2 + 3*a^4*b^4*d^2 - a^2*b^6*d^2)^3 + 1/39366*(3*a^4 + a^2*b^2 - b^4)*
(27*a^2 - 11*b^2)/((a^10*d^4 - 3*a^8*b^2*d^4 + 3*a^6*b^4*d^4 - a^4*b^6*d^4)
*(a^8*d^2 - 3*a^6*b^2*d^2 + 3*a^4*b^4*d^2 - a^2*b^6*d^2)) + 1/1062882*(3375
*a^8 - 4573*a^6*b^2 + 2460*a^4*b^4 - 624*a^2*b^6 + 64*b^8)*b^2/((a^2 - b^2)
^6*a^10*d^6))^(1/3) + 2187*(I*sqrt(3) + 1)*(-1/1062882*(729*a^4 - 432*a^2*b
^2 + 64*b^4)/(a^16*d^6 -  ...
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sin^3\left(c + dx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x)**3)**(-2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^3 + a)^(-2), x)
```

Mupad [A]
time = 17.89, size = 1567, normalized size = 92.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] symsum(log(- (8192*(80*b^6 - 270*a^2*b^4))/(243*(a^7 + a^3*b^4 - 2*a^5*b^2)
) - root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 + 531441*a^10*b^6*d^6
- 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4*d^4 - 177147*a^12*d^
4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 64*b^4 - 729*a^4, d, k
)*((8192*(144*a*b^7 + 648*a^3*b^5 - 2187*a^5*b^3))/(243*(a^7 + a^3*b^4 - 2*
a^5*b^2)) - root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 + 531441*a^10*
b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4*d^4 - 177147
*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 64*b^4 - 729*a
^4, d, k)*(root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 + 531441*a^10*b
^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4*d^4 - 177147*
a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 64*b^4 - 729*a^
4, d, k)*((8192*(26973*a^7*b^5 - 20412*a^5*b^7 + 39366*a^9*b^3))/(243*(a^7
+ a^3*b^4 - 2*a^5*b^2)) - root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6
+ 531441*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^
4*d^4 - 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 -
64*b^4 - 729*a^4, d, k)*(root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 +
 531441*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4
*d^4 - 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 6
4*b^4 - 729*a^4, d, k)*((8192*(236196*a^7*b^9 - 649539*a^9*b^7 + 590490*a^1
1*b^5 - 177147*a^13*b^3))/(243*(a^7 + a^3*b^4 - 2*a^5*b^2)) + (8192*tan(c/2
 + (d*x)/2)*(6561*a^8*b^8 - 13122*a^10*b^6 + 6561*a^12*b^4))/(27*(a^7 + a^3
*b^4 - 2*a^5*b^2))) + (8192*(13122*a^6*b^8 - 85293*a^8*b^6 + 72171*a^10*b^4
))/(243*(a^7 + a^3*b^4 - 2*a^5*b^2)) + (8192*tan(c/2 + (d*x)/2)*(11664*a^5*
b^9 - 40824*a^7*b^7 + 37908*a^9*b^5 - 8748*a^11*b^3))/(27*(a^7 + a^3*b^4 -
2*a^5*b^2))) + (8192*tan(c/2 + (d*x)/2)*(3078*a^6*b^6 - 8181*a^8*b^4))/(27*
(a^7 + a^3*b^4 - 2*a^5*b^2))) - (8192*(2592*a^2*b^8 - 11340*a^4*b^6 + 11664
*a^6*b^4))/(243*(a^7 + a^3*b^4 - 2*a^5*b^2)) + (8192*tan(c/2 + (d*x)/2)*(12
60*a^5*b^5 - 720*a^3*b^7 + 1944*a^7*b^3))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2)))
 + (8192*tan(c/2 + (d*x)/2)*(128*b^8 - 688*a^2*b^6 + 1053*a^4*b^4))/(27*(a^
7 + a^3*b^4 - 2*a^5*b^2))) - (8192*tan(c/2 + (d*x)/2)*(32*a*b^5 - 108*a^3*b
^3))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2)))*root(1594323*a^14*b^2*d^6 - 1594323*
a^12*b^4*d^6 + 531441*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 +
 59049*a^8*b^4*d^4 - 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 4
32*a^2*b^2 - 64*b^4 - 729*a^4, d, k), k, 1, 6)/d + ((2*b)/(3*(a^2 - b^2)) +
 (8*b*tan(c/2 + (d*x)/2)^2)/(3*(a^2 - b^2)) - (2*b*tan(c/2 + (d*x)/2)^4)/(3
*(a^2 - b^2)) - (2*b^2*tan(c/2 + (d*x)/2))/(3*a*(a^2 - b^2)) + (8*b^2*tan(c
/2 + (d*x)/2)^3)/(3*a*(a^2 - b^2)) + (2*b^2*tan(c/2 + (d*x)/2)^5)/(3*a*(a^2
 - b^2)))/(d*(a + 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a*t
an(c/2 + (d*x)/2)^6 + 8*b*tan(c/2 + (d*x)/2)^3))
```

3.402 $\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] Defer[Int][Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] Result contains complex when optimal does not.
time = 1.16, size = 845, normalized size = 32.50

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (((-I)*b*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6
 & , (16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^3*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)] - (8*I)*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2] - I*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (20*I)*a^3*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1 + (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1 + 10*a^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 8*a*b^2*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1 - 120*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
```

```
- #1)]*#1^2 + 12*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (60*I)
*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^3*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^2 - (20*I)*a^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 10*a^3
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 8*a*b^2*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^3 + 16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 2*
b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (8*I)*a^2*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^4 - I*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)
/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a*(a^2 - b^2)^2) + (18*Sin
[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (18*Sin[
(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*b*Cos
[c + d*x]*(-2*a^3 - 7*a*b^2 + 3*a*b^2*Cos[2*(c + d*x)] + 2*b*(2*a^2 + b^2)*
Sin[c + d*x]))/(a*(a - b)^2*(a + b)^2*(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c
+ d*x)])))/(18*d)
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(25) = 50$.
time = 6.41, size = 398, normalized size = 15.31

method	result
derivativedivides	$-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b\left(\frac{-\frac{(2a^2+b^2)b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\left(-\frac{a^2}{3}+\frac{4b^2}{3}\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4b(a^2+}{}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^4\right(}\right.}{}$
default	$-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b\left(\frac{-\frac{(2a^2+b^2)b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\left(-\frac{a^2}{3}+\frac{4b^2}{3}\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4b(a^2+}{}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a\left(\tan^4\right(}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)+2*b
/(a-b)^2/(a+b)^2*((-1/3*(2*a^2+b^2)*b/a*tan(1/2*d*x+1/2*c)^5+(-1/3*a^2+4/3*
b^2)*tan(1/2*d*x+1/2*c)^4-4/3*b*(a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)^3+(-2/3*a^
2-10/3*b^2)*tan(1/2*d*x+1/2*c)^2+1/3*(2*a^2+b^2)*b/a*tan(1/2*d*x+1/2*c)-1/3
*a^2-2/3*b^2)/(a*tan(1/2*d*x+1/2*c)^6+3*a*tan(1/2*d*x+1/2*c)^4+8*b*tan(1/2*
d*x+1/2*c)^3+3*a*tan(1/2*d*x+1/2*c)^2+a)+1/18/a*sum((b*(-11*a^2+2*b^2)*_R^4
+2*a*(5*a^2+4*b^2)*_R^3-54*a^2*b*_R^2+2*a*(5*a^2+4*b^2)*_R-11*a^2*b+2*b^3)/
```

```
(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+
3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**] Result contains complex when optimal does not.
time = 38.67, size = 102913, normalized size = 3958.19

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/108*(108*(a^3*b + 2*a*b^3)*cos(d*x + c)^4 - 108*a^3*b + 108*a*b^3 - sqrt(
2)*sqrt(1/2)*((a^6 - 2*a^4*b^2 + a^2*b^4)*d*cos(d*x + c) - ((a^5*b - 2*a^3*
b^3 + a*b^5)*d*cos(d*x + c)^3 - (a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c))
*sin(d*x + c))*sqrt(-(5670*a^6*b^2 + 31590*a^4*b^4 + 2916*a^2*b^6 - 810*b^8
 - (a^12 - 5*a^10*b^2 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*((-
I*sqrt(3) + 1)*((35*a^6*b^2 + 195*a^4*b^4 + 18*a^2*b^6 - 5*b^8)^2/(a^12*d^2
 - 5*a^10*b^2*d^2 + 10*a^8*b^4*d^2 - 10*a^6*b^6*d^2 + 5*a^4*b^8*d^2 - a^2*b
^10*d^2)^2 - 45*(10*a^2*b^4 - b^6)/(a^14*d^4 - 5*a^12*b^2*d^4 + 10*a^10*b^4
*d^4 - 10*a^8*b^6*d^4 + 5*a^6*b^8*d^4 - a^4*b^10*d^4))/(-1/19683*(35*a^6*b^
2 + 195*a^4*b^4 + 18*a^2*b^6 - 5*b^8)^3/(a^12*d^2 - 5*a^10*b^2*d^2 + 10*a^8
*b^4*d^2 - 10*a^6*b^6*d^2 + 5*a^4*b^8*d^2 - a^2*b^10*d^2)^3 - 1/1062882*(15
625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8)/(a^20*d^6 - 5*a^18*b^2*d^6 + 10*a^16*b
^4*d^6 - 10*a^14*b^6*d^6 + 5*a^12*b^8*d^6 - a^10*b^10*d^6) + 5/1458*(35*a^6
*b^2 + 195*a^4*b^4 + 18*a ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c)^3 + a)^2, x)
```

Mupad [A]
time = 21.21, size = 2500, normalized size = 96.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x)^3)^2),x)
```

```
[Out] symsum(log(5479612416*a^8*b^36 - 180486144*a^6*b^38 - root(5314410*a^16*b^4
*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 -
 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a
^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^
2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(tan(c
/2 + (d*x)/2)*(764411904*a^6*b^40 - 27805483008*a^8*b^38 + 437297356800*a^1
0*b^36 - 3672461721600*a^12*b^34 + 19250011791360*a^14*b^32 - 6915063575347
2*a^16*b^30 + 180165872001024*a^18*b^28 - 352655758540800*a^20*b^26 + 52992
3028377600*a^22*b^24 - 618699706859520*a^24*b^22 + 563713761042432*a^26*b^2
0 - 399760062234624*a^28*b^18 + 218398602240000*a^30*b^16 - 90108039168000*
a^32*b^14 + 27130620764160*a^34*b^12 - 5617221156864*a^36*b^10 + 7135367086
08*a^38*b^8 - 41803776000*a^40*b^6) - root(5314410*a^16*b^4*d^6 - 5314410*a
^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 - 531441*a^10*b^1
0*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a^14*b^2*d^4 + 10
62882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^2 - 98415*a^6*b^
6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(root(5314410*a^16*b^4
*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 -
 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a
^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^
2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(tan(c
/2 + (d*x)/2)*(157695787008*a^12*b^38 - 4039140556800*a^14*b^36 + 391830495
06816*a^16*b^34 - 212750482120704*a^18*b^32 + 750889290203136*a^20*b^30 - 1
854140141887488*a^22*b^28 + 3327952874029056*a^24*b^26 - 4413464400863232*a
^26*b^24 + 4311710468702208*a^28*b^22 - 3009938035433472*a^30*b^20 + 135980
8836452352*a^32*b^18 - 238981192998912*a^34*b^16 - 150898421366784*a^36*b^1
4 + 136937506922496*a^38*b^12 - 52028967665664*a^40*b^10 + 10565134000128*a
^42*b^8 - 976165945344*a^44*b^6 + 12093235200*a^46*b^4) - root(5314410*a^16
```

*b^4*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 - 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(tan(c/2 + (d*x)/2)*(69657034752*a^11*b^41 - 1619526057984*a^13*b^39 + 16404231684096*a^15*b^37 - 99052303417344*a^17*b^35 + 405403942256640*a^19*b^33 - 1203882531618816*a^21*b^31 + 2700324609196032*a^23*b^29 - 4688893637296128*a^25*b^27 + 6394933732442112*a^27*b^25 - 6897962008903680*a^29*b^23 + 5886924977995776*a^31*b^21 - 3949971812646912*a^33*b^19 + 2053768012627968*a^35*b^17 - 806001549115392*a^37*b^15 + 227778503639040*a^39*b^13 - 42212163059712*a^41*b^11 + 3970450980864*a^43*b^9 + 52242776064*a^45*b^7 - 34828517376*a^47*b^5) + 8707129344*a^12*b^40 - 470184984576*a^14*b^38 + 6308315209728*a^16*b^36 - 44092902998016*a^18*b^34 + 197477693521920*a^20*b^32 - 623151832891392*a^22*b^30 + 1459506434899968*a^24*b^28 - 2616109254180864*a^26*b^26 + 3653180601827328*a^28*b^24 - 4009284777738240*a^30*b^22 + 3462677318909952*a^32*b^20 - 2339013569937408*a^34*b^18 + 1217047711186944*a^36*b^16 - 473946464452608*a^38*b^14 + 130868154040320*a^40*b^12 - 22777850363904*a^42*b^10 + 1645647446016*a^44*b^8 + 156728328192*a^46*b^6 - 30474952704*a^48*b^4 + root(5314410*a^16*b^4*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 - 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(tan(c/2 + (d*x)/2)*(39182082048*a^14*b^40 - 705277476864*a^16*b^38 + 5994858553344*a^18*b^36 - 31972578951168*a^20*b^34 + 119897171066880*a^22*b^32 - 335712078987264*a^24*b^30 + 727376171139072*a^26*b^28 - 1246930579095552*a^28*b^26 + 1714529546256384*a^30*b^24 - 1905032829173760*a^32*b^22 + 1714529546256384*a^34*b^20 - 1246930579095552*a^36*b^18 + 727376171139072*a^38*b^16 - 335712078987264*a^40*b^14 + 119897171066880*a^42*b^12 - 31972578951168*a^44*b^10 + 5994858553344*a^46*b^8 - 705277476864*a^48*b^6 + 39182082048*a^50*b^4) + 156728328192*a^13*b^41 - 2938656153600*a^15*b^39 + 26095266643968*a^17*b^37 - 145874891464704*a^19*b^35 + 575506421121024*a^21*b^33 - 1702539829149696*a^23*b^31 + 3916640921518080*a^25*b^29 - 7169850829799424*a^27*b^27 + 10598909922312192*a^29*b^25 - 12763719955464192*a^31*b^23 + 12573216672546816*a^33*b^21 - 10131310955151360*a^35*b^19 + 6650296421842944*a^37*b^17 - 3524976829366272*a^39*b^15 + 1486724921229312*a^41*b^13 - 487581829005312*a^43*b^11 + 119897171066880*a^45*b^9 - 20805685567488*a^47*b^7 + 2272560758784*a^49*b^5 - 117546246144*a^51*b^3)) - 59982446592*a^11*b^39 + 1080651497472*a^13*b^37 - 6860250464256*a^15*b^35 + 16482112118784*a^17*b^33 + 27170113388544*a^19*b^31 - 327284061511680*a^21*b^29 + 1194949984370688*a^23*b^27 - 2698934854606848*a^25...

3.403 $\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] Defer[Int][Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] Result contains complex when optimal does not.
time = 1.26, size = 1158, normalized size = 44.54

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((4*I)*b^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1
^6 & , (14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 74*a^2*b^2*ArcTan
[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x
] - #1)] - (7*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (37*I)*a^2*b^2*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2] - I*b^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] +
(144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + (36*I)*a*b^3*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 72*a^3*b*Log[1 - 2*Cos[c + d*x]
```

```
*#1 + #1^2]*#1 + 18*a*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 180*a^4*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 372*a^2*b^2*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 + 12*b^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^2 + (90*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (186*I)*a^2*
b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^4*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^2 - (144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - (36*I)*a*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 72*a^3
*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 18*a*b^3*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^3 + 14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 7
4*a^2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 2*b^4*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (7*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1^4 - (37*I)*a^2*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - I*b^4*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b
*#1^5) & ] + (3*Sec[c + d*x]^3*(48*a^5*b + 568*a^3*b^3 + 14*a*b^5 + (78*a^5
*b + 606*a^3*b^3 + 81*a*b^5)*Cos[2*(c + d*x)] + 18*a*b^3*(4*a^2 + b^2)*Cos[
4*(c + d*x)] + 2*a^5*b*Cos[6*(c + d*x)] - 30*a^3*b^3*Cos[6*(c + d*x)] - 17*
a*b^5*Cos[6*(c + d*x)] + 48*a^6*Sin[c + d*x] - 244*a^4*b^2*Sin[c + d*x] + 2
0*a^2*b^4*Sin[c + d*x] - 4*b^6*Sin[c + d*x] + 16*a^6*Sin[3*(c + d*x)] - 194
*a^4*b^2*Sin[3*(c + d*x)] - 86*a^2*b^4*Sin[3*(c + d*x)] - 6*b^6*Sin[3*(c +
d*x)] - 14*a^4*b^2*Sin[5*(c + d*x)] - 74*a^2*b^4*Sin[5*(c + d*x)] - 2*b^6*S
in[5*(c + d*x)]))/(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c + d*x)]))/(72*a*(a^2
 - b^2)^3*d)
```

Maple [A] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(25) = 50$.

time = 8.93, size = 525, normalized size = 20.19

method	result
derivativedivides	$-\frac{1}{3(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{a-4b}{(a-b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{3(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{}{2(a+b)^2(\mathrm{ta}}$
default	$-\frac{1}{3(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{a-4b}{(a-b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{3(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{}{2(a+b)^2(\mathrm{ta}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(-1/3/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2-(a-4*b)/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)-1/3/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^2-(4*b+a)/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+2*b^2/(a-b)^3/(a+b)^3*((1/3*(a^4+7*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)^5-3*b^3*tan(1/2*d*x+1/2*c)^4+4*b^2*(2*a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3+(6*a^2*b+6*b^3)*tan(1/2*d*x+1/2*c)^2-1/3*(a^4+7*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+2*a^2*b+b^3)/(a*tan(1/2*d*x+1/2*c)^6+3*a*tan(1/2*d*x+1/2*c)^4+8*b*tan(1/2*d*x+1/2*c)^3+3*a*tan(1/2*d*x+1/2*c)^2+a)+1/18/a*sum(((19*a^4+28*a^2*b^2-2*b^4)*_R^4+18*a*b*(-4*a^2-b^2)*_R^3+6*a^2*(11*a^2+34*b^2)*_R^2+18*a*b*(-4*a^2-b^2)*_R+19*a^4+28*a^2*b^2-2*b^4)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [**A**] Result contains complex when optimal does not.
time = 104.28, size = 133123, normalized size = 5120.12

too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")

[Out] 1/108*(36*(2*a^5*b - 30*a^3*b^3 - 17*a*b^5)*cos(d*x + c)^6 - 36*a^5*b + 72*a^3*b^3 - 36*a*b^5 - 108*(a^5*b - 21*a^3*b^3 - 10*a*b^5)*cos(d*x + c)^4 + sqrt(2/3)*sqrt(1/6)*((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^3 - ((a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c)^5 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c)^3)*sin(d*x + c))*sqrt(-(573480*a^8*b^4 + 4293324*a^6*b^6 + 3847662*a^4*b^8 + 159894*a^2*b^10 - 17010*b^12 - (a^16 - 7*a^14*b^2 + 21*a^12*b^4 - 35*a^10*b^6 + 35*a^8*b^8 - 21*a^6*b^10 + 7*a^4*b^12 - a^2*b^14)*((-I*sqrt(3) + 1)*((1180*a^8*b^4 + 8834*a^6*b^6 + 7917*a^4*b^8 + 329*a^2*b^10 - 35*b^12)^2/(a^16*d^2 - 7*a^14*b^2*d^2 + 21*a^12*b^4*d^2 - 35*a^10*b^6*d^2 + 35*a^8*b^8*d^2 - 21*a^6*b^10*d^2 + 7*a^4*b^12*d^2 - a^2*b^14*d^2)^2 + 15*(1029*a^4*b^6 - 3173*a^2*b^8 + 119*b^10)/(a^18*d^4 - 7*a^16*b^2*d^4 + 21*a^14*b^4*d^4 - 35*a^12*b^6*d^4 + 35*a^10*b^8*d^4 - 21*a^8*b^10*d^4 + 7*a^6*b^12*d^4 - a^4*b^14*d^4))/(-1/531441*(1180*a^8*b^4 + 8834*a^6*b^6 + 7917 ...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c)^3 + a)^2, x)
```

Mupad [**A**]
time = 25.44, size = 2500, normalized size = 96.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x)^3)^2),x)
```

```
[Out] symsum(log(26838024192*a^8*b^54 - tan(c/2 + (d*x)/2)*(7962624000*a^7*b^55 -
 508612608000*a^9*b^53 + 8841498624000*a^11*b^51 - 82283765760000*a^13*b^49
 + 501714984960000*a^15*b^47 - 2205295497216000*a^17*b^45 + 737918163763200
0*a^19*b^43 - 19451488075776000*a^21*b^41 + 41318016122880000*a^23*b^39 - 7
1811432161280000*a^25*b^37 + 103155513237504000*a^27*b^35 - 123224906907648
000*a^29*b^33 + 122756816093184000*a^31*b^31 - 101967282708480000*a^33*b^29
 + 70396872007680000*a^35*b^27 - 40129785593856000*a^37*b^25 + 186876255928
32000*a^39*b^23 - 6994754113536000*a^41*b^21 + 2053854351360000*a^43*b^19 -
 455730831360000*a^45*b^17 + 71860690944000*a^47*b^15 - 7177310208000*a^49*
b^13 + 341397504000*a^51*b^11) - 392822784*a^6*b^56 - root(18600435*a^18*b^
6*d^6 - 18600435*a^16*b^8*d^6 - 11160261*a^20*b^4*d^6 + 11160261*a^14*b^10*
d^6 + 3720087*a^22*b^2*d^6 - 3720087*a^12*b^12*d^6 + 531441*a^10*b^14*d^6 -
 531441*a^24*d^6 - 173879622*a^14*b^6*d^4 - 155830311*a^12*b^8*d^4 - 232259
40*a^16*b^4*d^4 - 6475707*a^10*b^10*d^4 + 688905*a^8*b^12*d^4 - 11565585*a^
8*b^8*d^2 + 3750705*a^10*b^6*d^2 + 433755*a^6*b^10*d^2 - 117649*a^4*b^8 + 5
488*a^2*b^10 - 64*b^12, d, k)*(tan(c/2 + (d*x)/2)*(764411904*a^6*b^58 - 614
39606784*a^8*b^56 + 2110475575296*a^10*b^54 - 33643637121024*a^12*b^52 + 31
```

9697763065856*a^14*b^50 - 2067381036048384*a^16*b^48 + 9810082122817536*a^1
8*b^46 - 35797302942326784*a^20*b^44 + 103613766013034496*a^22*b^42 - 24300
4699498881024*a^24*b^40 + 468678655511248896*a^26*b^38 - 750973819695611904
*a^28*b^36 + 1006348379003928576*a^30*b^34 - 1132028278205497344*a^32*b^32
+ 1070100496146087936*a^34*b^30 - 848821864657895424*a^36*b^28 + 5626355927
01198336*a^38*b^26 - 309384400894377984*a^40*b^24 + 139566181489975296*a^42
*b^22 - 50807786761396224*a^44*b^20 + 14569217952178176*a^46*b^18 - 3172130
021597184*a^48*b^16 + 494158536400896*a^50*b^14 - 49418889191424*a^52*b^12
+ 2463538323456*a^54*b^10 - 14338695168*a^56*b^8) + 95551488*a^7*b^57 + 358
79583744*a^9*b^55 - 1812522147840*a^11*b^53 + 29896430247936*a^13*b^51 - 27
3690491977728*a^15*b^49 + 1665068560662528*a^17*b^47 - 7358934856605696*a^1
9*b^45 + 24887080515133440*a^21*b^43 - 66575487905316864*a^23*b^41 + 144045
035942510592*a^25*b^39 - 255939373888192512*a^27*b^37 + 377317716543258624*
a^29*b^35 - 464589495171809280*a^31*b^33 + 479470084160126976*a^33*b^31 - 4
15092174607761408*a^35*b^29 + 300910589340991488*a^37*b^27 - 18182304326703
5136*a^39*b^25 + 90863416678809600*a^41*b^23 - 37111903240495104*a^43*b^21
+ 12175612162301952*a^45*b^19 - 3127996467412992*a^47*b^17 + 60541899359846
4*a^49*b^15 - 82897275985920*a^51*b^13 + 7145262637056*a^53*b^11 - 29087067
3408*a^55*b^9 + root(18600435*a^18*b^6*d^6 - 18600435*a^16*b^8*d^6 - 111602
61*a^20*b^4*d^6 + 11160261*a^14*b^10*d^6 + 3720087*a^22*b^2*d^6 - 3720087*a
^12*b^12*d^6 + 531441*a^10*b^14*d^6 - 531441*a^24*d^6 - 173879622*a^14*b^6*
d^4 - 155830311*a^12*b^8*d^4 - 23225940*a^16*b^4*d^4 - 6475707*a^10*b^10*d^
4 + 688905*a^8*b^12*d^4 - 11565585*a^8*b^8*d^2 + 3750705*a^10*b^6*d^2 + 433
755*a^6*b^10*d^2 - 117649*a^4*b^8 + 5488*a^2*b^10 - 64*b^12, d, k)*(tan(c/2
 + (d*x)/2)*(45578059776*a^9*b^57 - 1988020371456*a^11*b^55 + 2172525517209
6*a^13*b^53 - 78629462802432*a^15*b^51 - 330769869373440*a^17*b^49 + 533728
8405614592*a^19*b^47 - 32144913894998016*a^21*b^45 + 126404118900965376*a^2
3*b^43 - 367050326151462912*a^25*b^41 + 829818883454238720*a^27*b^39 - 1502
808604998893568*a^29*b^37 + 2216700870917750784*a^31*b^35 - 268852344938260
0704*a^33*b^33 + 2692902186903011328*a^35*b^31 - 2227622993351147520*a^37*b
^29 + 1515332894269243392*a^39*b^27 - 839694861496221696*a^41*b^25 + 372789
943915216896*a^43*b^23 - 128854679612424192*a^45*b^21 + 32863270985072640*a
^47*b^19 - 5445156193763328*a^49*b^17 + 316457498640384*a^51*b^15 + 9146398
6446336*a^53*b^13 - 25165538721792*a^55*b^11 + 2461645209600*a^57*b^9 - 737
41860864*a^59*b^7) + root(18600435*a^18*b^6*d^6 - 18600435*a^16*b^8*d^6 - 1
1160261*a^20*b^4*d^6 + 11160261*a^14*b^10*d^6 + 3720087*a^22*b^2*d^6 - 3720
087*a^12*b^12*d^6 + 531441*a^10*b^14*d^6 - 531441*a^24*d^6 - 173879622*a^14
*b^6*d^4 - 155830311*a^12*b^8*d^4 - 23225940*a^16*b^4*d^4 - 6475707*a^10*b^
10*d^4 + 688905*a^8*b^12*d^4 - 11565585*a^8*b^8*d^2 + 3750705*a^10*b^6*d^2
+ 433755*a^6*b^10*d^2 - 117649*a^4*b^8 + 5488*a^2*b^10 - 64*b^12, d, k)*(ro
ot(18600435*a^18*b^6*d^6 - 18600435*a^16*b^8*d^6 - 11160261*a^20*b^4*d^6 +
11160261*a^14*b^10*d^6 + 3720087*a^22*b^2*d^6 - 3720087*a^12*b^12*d^6 + 531
441*a^10*b^14*d^6 - 531441*a^24*d^6 - 173879622*a^14*b^6*d^4 - 155830311*a^
12*b^8*d^4 - 23225940*a^16*b^4*d^4 - 6475707*a^10*b^10*d^4 + 688905*a^8*b^1
2*d^4 - 11565585*a^8*b^8*d^2 + 3750705*a^10*b^6*d^2 + 433755*a^6*b^10*d^2 -

117649*a^4*b^8 + 5488*a^2*b^10 - 64*b^12, d, k)*(tan(c/2 + (d*x)/2)*(69657034752*a^11*b^59 - 2855938424832*a^13*b^57 + 46200028299264*a^15*b^55 - 432918470983680*a^17*b^53 + 2732993758494720*a^19*b^51 - 12560556506480640*a^21*b^49 + 43925900257198080*a^23*b^47 - 119837962587340800*a^25*b^45 + 257651619562782720*a^27*b^43 - 433619569038458880*a^...

3.404 $\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^3\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{3\sin(c+dx)}{bd}+\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] -3*sin(d*x+c)/b/d+1/3*sin(d*x+c)^3/b/d-1/2*arctanh(b^(1/4)*sin(d*x+c)/a^(1/
4))*(a^(1/2)-b^(1/2))^3/a^(3/4)/b^(7/4)/d+1/2*arctan(b^(1/4)*sin(d*x+c)/a^(
1/4))*(a^(1/2)+b^(1/2))^3/a^(3/4)/b^(7/4)/d
```

Rubi [A]
time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3302, 1185, 1181, 211, 214}

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^3\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}+\frac{\sin^3(c+dx)}{3bd}-\frac{3\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b])^3*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b
^(7/4)*d) - ((Sqrt[a] - Sqrt[b])^3*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])
/(2*a^(3/4)*b^(7/4)*d) - (3*Sin[c + d*x])/(b*d) + Sin[c + d*x]^3/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{3}{b}+\frac{x^2}{b}+\frac{3a+b-(a+3b)x^2}{b(a-bx^4)}\right)dx, x, \sin(c+dx)\right)}{d} \\
&= -\frac{3\sin(c+dx)}{bd}+\frac{\sin^3(c+dx)}{3bd}+\frac{\text{Subst}\left(\int \frac{3a+b+(-a-3b)x^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{bd} \\
&= -\frac{3\sin(c+dx)}{bd}+\frac{\sin^3(c+dx)}{3bd}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin\right.}{2\sqrt{a}\,bd} \\
&= \frac{\left(\sqrt{a}+\sqrt{b}\right)^3\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 207, normalized size = 1.58

$$\frac{3\left(\sqrt{a}-\sqrt{b}\right)^3\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)+3i\left(\sqrt{a}+\sqrt{b}\right)^3\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)-3i\left(\sqrt{a}+\sqrt{b}\right)^3\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right)-3\left(\sqrt{a}-\sqrt{b}\right)^3\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)-36a^{3/4}b^{3/4}\sin(c+dx)+4a^{3/4}b^{3/4}\sin^3(c+dx)}{12a^{3/4}b^{7/4}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (3*(Sqrt[a] - Sqrt[b])^3*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]] + (3*I)*(Sqrt[
a] + Sqrt[b])^3*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - (3*I)*(Sqrt[a] + Sq
rt[b])^3*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] - 3*(Sqrt[a] - Sqrt[b])^3*Lo
g[a^(1/4) + b^(1/4)*Sin[c + d*x]] - 36*a^(3/4)*b^(3/4)*Sin[c + d*x] + 4*a^(
3/4)*b^(3/4)*Sin[c + d*x]^3)/(12*a^(3/4)*b^(7/4)*d)
```

Maple [A]
time = 1.16, size = 176, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{\frac{\left(\sin^{3}(dx+c)\right)}{3}-3\sin(dx+c)}{b}+\frac{\frac{(3a+b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{(-a-3b)\left(2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\right.}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}}{d}$
default	$\frac{\frac{\frac{\left(\sin^{3}(dx+c)\right)}{3}-3\sin(dx+c)}{b}+\frac{\frac{(3a+b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{(-a-3b)\left(2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\right.}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}}{d}$
risch	$\frac{11i e^{i(dx+c)}}{8bd}-\frac{11i e^{-i(dx+c)}}{8bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(256a^{3}b^{7}d^{4}\_Z^{4}+\left(192a^{4}b^{4}d^{2}+640a^{3}b^{5}d^{2}+192a^{2}b^{6}d^{2}\right)\_Z^{2}-a^{6}+6a^{5}b-\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b*(1/3*sin(d*x+c)^3-3*sin(d*x+c))+1/b*(1/4*(3*a+b)*(1/b*a)^(1/4)/a*(
ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))+2*arctan(sin(d*x+
c)/(1/b*a)^(1/4)))-1/4*(-a-3*b)/b/(1/b*a)^(1/4)*(2*arctan(sin(d*x+c)/(1/b*a
)^(1/4))-ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4))))))
```

Maxima [A]
time = 0.53, size = 177, normalized size = 1.35

$$\frac{\frac{4\left(\sin(dx+c)^{3}-9\sin(dx+c)\right)}{b}+\frac{3\left(\frac{2\left(b\left(3\sqrt{a}+\sqrt{b}\right)+a^{\frac{3}{2}}+3a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(3\sqrt{a}-\sqrt{b}\right)+a^{\frac{3}{2}}-3a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(sin(d*x + c)^3 - 9*sin(d*x + c))/b + 3*(2*(b*(3*sqrt(a) + sqrt(b))
 + a^(3/2) + 3*a*sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b*(3*sqrt(a) - sqrt(b)) + a^(3
/2) - 3*a*sqrt(b))*log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt
(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)))/b)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1429 vs. $2(101) = 202$.

time = 0.61, size = 1429, normalized size = 10.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*d*sqrt(-(a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b
^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)
/(a*b^3*d^2))*log(1/2*(a^6 + 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5
- b^6)*sin(d*x + c) + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b +
 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) -
 (3*a^5*b^2 + 46*a^4*b^3 + 60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt(-(a*b^3
*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*
b^5 + b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))) - 3*b*d*s
qrt((a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b
^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))*
log(1/2*(a^6 + 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*sin(d*x
 + c) + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 +
 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + (3*a^5*b^2 +
46*a^4*b^3 + 60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt((a*b^3*d^2*sqrt((a^6
+ 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3
*b^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))) - 3*b*d*sqrt(-(a*b^3*d^2
*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5
+ b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))*log(-1/2*(a^6
+ 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*sin(d*x + c) + 1/2*(
(a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3
+ 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - (3*a^5*b^2 + 46*a^4*b^3 +
60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt(-(a*b^3*d^2*sqrt((a^6 + 30*a^5*b +
 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) +
 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))) + 3*b*d*sqrt((a*b^3*d^2*sqrt((a^6 +
30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b
^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))*log(-1/2*(a^6 + 12*a^5*b -
27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*sin(d*x + c) + 1/2*((a^4*b^5 + 3*
a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4
 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + (3*a^5*b^2 + 46*a^4*b^3 + 60*a^3*b^4 +
18*a^2*b^5 + a*b^6)*d)*sqrt((a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 +
 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - 6*a^2 - 20*a*
b - 6*b^2)/(a*b^3*d^2))) - 4*(cos(d*x + c)^2 + 8)*sin(d*x + c))/(b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a-b*sin(d*x+c)**4),x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(101) = 202.
time = 0.81, size = 360, normalized size = 2.75

$$\frac{\frac{8\left(b^2\sin(dx+c)^3-9b^2\sin(dx+c)\right)}{b^3}-\frac{6\sqrt{2}\left(\left(-ab^3\right)^{\frac34}(a+3b)-\left(-ab^3\right)^{\frac14}(3ab^2+b^3)\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{ab^4}-\frac{6\sqrt{2}\left(\left(-ab^3\right)^{\frac34}(a+3b)-\left(-ab^3\right)^{\frac14}(3ab^2+b^3)\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac14}}\right)}{ab^4}+\frac{3\sqrt{2}\left(\left(-ab^3\right)^{\frac34}(a+3b)+\left(-ab^3\right)^{\frac14}(3ab^2+b^3)\right)\log\left(\sin(dx+c)^2+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{ab^4}-\frac{3\sqrt{2}\left(\left(-ab^3\right)^{\frac34}(a+3b)+\left(-ab^3\right)^{\frac14}(3ab^2+b^3)\right)\log\left(\sin(dx+c)^2-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac14}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{ab^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="giac")
[Out] 1/24*(8*(b^2*sin(d*x + c)^3 - 9*b^2*sin(d*x + c))/b^3 - 6*sqrt(2)*((-a*b^3)
^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(1/2*sqrt(2)*(sqrt
(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^4) - 6*sqrt(2)*((-a*b
^3)^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(-1/2*sqrt(2)*(
sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^4) + 3*sqrt(2)*((
-a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^
2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^4) - 3*sqrt(2)*((-
a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^2
 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^4))/d
```

Mupad [B]
time = 0.77, size = 1931, normalized size = 14.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a - b*sin(c + d*x)^4),x)
[Out] (atan((a^3*sin(c + d*x)*(- (a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3) - 5/(4*
b^2) - 3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(16*a*b^6) - (15*(a^3*b^7)^(1/2))/(
16*a^2*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*8i)/(92*a*b + (120*(a^3*b
^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 + (36*(a^3*b^7)
^(1/2))/(a*b^2) + (2*(a^3*b^7)^(1/2))/(a^2*b) + (6*a^2*(a^3*b^7)^(1/2))/b^5
 + (92*a*(a^3*b^7)^(1/2))/b^4) + (b^3*sin(c + d*x)*(- (a^3*b^7)^(1/2)/(16*b
^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(16*a*b^
6) - (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2
)*8i)/(92*a*b + (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b +
(2*a^4)/b^2 + (36*(a^3*b^7)^(1/2))/(a*b^2) + (2*(a^3*b^7)^(1/2))/(a^2*b) +
(6*a^2*(a^3*b^7)^(1/2))/b^5 + (92*a*(a^3*b^7)^(1/2))/b^4) + (a*b^2*sin(c +
d*x)*(- (a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*b) -
(15*(a^3*b^7)^(1/2))/(16*a*b^6) - (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) - (a^3*
b^7)^(1/2)/(16*a^3*b^4))^(1/2)*120i)/(92*a*b + (120*(a^3*b^7)^(1/2))/b^3 +
120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 + (36*(a^3*b^7)^(1/2))/(a*b^2) +
```

(2*(a^3*b^7)^(1/2))/(a^2*b) + (6*a^2*(a^3*b^7)^(1/2))/b^5 + (92*a*(a^3*b^7
)^(1/2))/b^4) + (a^2*b*sin(c + d*x)*(- (a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*
b^3) - 5/(4*b^2) - 3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(16*a*b^6) - (15*(a^3*b
^7)^(1/2))/(16*a^2*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*120i)/(92*a*b
 + (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 +
 (36*(a^3*b^7)^(1/2))/(a*b^2) + (2*(a^3*b^7)^(1/2))/(a^2*b) + (6*a^2*(a^3*b
^7)^(1/2))/b^5 + (92*a*(a^3*b^7)^(1/2))/b^4))*(-(a^3*(a^3*b^7)^(1/2) + b^3*
(a^3*b^7)^(1/2) + 6*a^2*b^6 + 20*a^3*b^5 + 6*a^4*b^4 + 15*a*b^2*(a^3*b^7)^(
1/2) + 15*a^2*b*(a^3*b^7)^(1/2))/(16*a^3*b^7))^(1/2)*2i)/d - (3*sin(c + d*x
))/(b*d) + (atan((a^3*sin(c + d*x)*((a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3
) - 5/(4*b^2) - 3/(8*a*b) + (15*(a^3*b^7)^(1/2))/(16*a*b^6) + (15*(a^3*b^7)
^(1/2))/(16*a^2*b^5) + (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*8i)/(92*a*b - (1
20*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 - (36*
(a^3*b^7)^(1/2))/(a*b^2) - (2*(a^3*b^7)^(1/2))/(a^2*b) - (6*a^2*(a^3*b^7)^(
1/2))/b^5 - (92*a*(a^3*b^7)^(1/2))/b^4) + (b^3*sin(c + d*x)*((a^3*b^7)^(1/2
)/(16*b^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*b) + (15*(a^3*b^7)^(1/2))/(
16*a*b^6) + (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) + (a^3*b^7)^(1/2)/(16*a^3*b^4
))^(1/2)*8i)/(92*a*b - (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^
3)/b + (2*a^4)/b^2 - (36*(a^3*b^7)^(1/2))/(a*b^2) - (2*(a^3*b^7)^(1/2))/(a^
2*b) - (6*a^2*(a^3*b^7)^(1/2))/b^5 - (92*a*(a^3*b^7)^(1/2))/b^4) + (a*b^2*s
in(c + d*x)*((a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*
b) + (15*(a^3*b^7)^(1/2))/(16*a*b^6) + (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) +
(a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*120i)/(92*a*b - (120*(a^3*b^7)^(1/2))/b
^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 - (36*(a^3*b^7)^(1/2))/(a*b
^2) - (2*(a^3*b^7)^(1/2))/(a^2*b) - (6*a^2*(a^3*b^7)^(1/2))/b^5 - (92*a*(a^
3*b^7)^(1/2))/b^4) + (a^2*b*sin(c + d*x)*((a^3*b^7)^(1/2)/(16*b^7) - (3*a)/
(8*b^3) - 5/(4*b^2) - 3/(8*a*b) + (15*(a^3*b^7)^(1/2))/(16*a*b^6) + (15*(a^
3*b^7)^(1/2))/(16*a^2*b^5) + (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*120i)/(92*
a*b - (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^
2 - (36*(a^3*b^7)^(1/2))/(a*b^2) - (2*(a^3*b^7)^(1/2))/(a^2*b) - (6*a^2*(a^
3*b^7)^(1/2))/b^5 - (92*a*(a^3*b^7)^(1/2))/b^4))*((a^3*(a^3*b^7)^(1/2) + b^
3*(a^3*b^7)^(1/2) - 6*a^2*b^6 - 20*a^3*b^5 - 6*a^4*b^4 + 15*a*b^2*(a^3*b^7)
^(1/2) + 15*a^2*b*(a^3*b^7)^(1/2))/(16*a^3*b^7))^(1/2)*2i)/d + sin(c + d*x)
^3/(3*b*d)

3.405 $\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=113

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(a-2\sqrt{a}\sqrt{b}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}-\frac{\sin(c+dx)}{bd}$$

```
[Out] -sin(d*x+c)/b/d+1/2*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))^2/
a^(3/4)/b^(5/4)/d+1/2*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))*(a+b-2*a^(1/2)*b^
(1/2))/a^(3/4)/b^(5/4)/d
```

Rubi [A]
time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3302, 1185, 1181, 211, 214}

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\text{ArcTan}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(-2\sqrt{a}\sqrt{b}+a+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b])^2*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b
^(5/4)*d) + ((a - 2*Sqrt[a]*Sqrt[b] + b)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(
1/4)])/(2*a^(3/4)*b^(5/4)*d) - Sin[c + d*x]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{a+b-2bx^2}{b\left(a-bx^4\right)}\right)\,dx, x, \sin(c+dx)\right)}{d}\\
&= -\frac{\sin(c+dx)}{bd}+\frac{\text{Subst}\left(\int \frac{a+b-2bx^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{bd}\\
&= -\frac{\sin(c+dx)}{bd}-\frac{\left(2\sqrt{b}-\frac{a+b}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{b}\,d}-\\
&= \frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 189, normalized size = 1.67

$$
\frac{-\left(\sqrt{a}-\sqrt{b}\right)^2\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)+i\left(\left(\sqrt{a}+\sqrt{b}\right)^2\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)-\left(\sqrt{a}+\sqrt{b}\right)^2\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right)-i\left(\sqrt{a}-\sqrt{b}\right)^2\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)\right)-4a^{3/4}\sqrt[4]{b}\sin(c+dx)}{4a^{3/4}b^{5/4}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-((Sqrt[a] - Sqrt[b])^2*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]]) + I*((Sqrt[a]
 + Sqrt[b])^2*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - (Sqrt[a] + Sqrt[b])^2
*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] - I*(Sqrt[a] - Sqrt[b])^2*Log[a^(1/4
) + b^(1/4)*Sin[c + d*x]]) - 4*a^(3/4)*b^(1/4)*Sin[c + d*x])/(4*a^(3/4)*b^(
5/4)*d)
```

Maple [A]
time = 0.83, size = 152, normalized size = 1.35

method	result
derivativedivides	$\frac{-\frac{\sin(dx+c)}{b}+\frac{\frac{(a+b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}}{d}$
default	$\frac{-\frac{\sin(dx+c)}{b}+\frac{\frac{(a+b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}}{d}$
risch	$\frac{ie^{i(dx+c)}}{2bd}-\frac{ie^{-i(dx+c)}}{2bd}+\left(\sum_{\_R=\mathrm{RootOf}(256a^3b^5d^4\_Z^4+(128a^3b^3d^2+128a^2b^4d^2)\_Z^2-a^4+4a^3b-6a^2b^2+4a\,b^3-b^4)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*sin(d*x+c)+1/b*(1/4*(a+b)*(1/b*a)^(1/4)/a*(ln((sin(d*x+c)+(1/b*a)
^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))+2*arctan(sin(d*x+c)/(1/b*a)^(1/4)))+1/2
/(1/b*a)^(1/4)*(2*arctan(sin(d*x+c)/(1/b*a)^(1/4))-ln((sin(d*x+c)+(1/b*a)^(
1/4))/(sin(d*x+c)-(1/b*a)^(1/4))))))
```

Maxima [A]
time = 0.50, size = 158, normalized size = 1.40

$$\frac{\frac{\frac{2\left(b\left(2\sqrt{a}+\sqrt{b}\right)+a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(2\sqrt{a}-\sqrt{b}\right)-a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{b}-\frac{4\sin(dx+c)}{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*((2*(b*(2*sqrt(a) + sqrt(b)) + a*sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b*(2*sqrt(
a) - sqrt(b)) - a*sqrt(b))*log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)
))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))*sqrt(b)))/b - 4*sin(d*x + c)/b)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(85) = 170.

time = 0.56, size = 1041, normalized size = 9.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b
^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))*log(1/2*(a^4 + 4*a^3*b - 10*a^
2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a^3
*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a
^2*b^3 + a*b^4)*d)*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*
a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))) - b*d*sqrt((a*b^2*d^
2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) - 4*a
- 4*b)/(a*b^2*d^2))*log(1/2*(a^4 + 4*a^3*b - 10*a^2*b^2 + 4*a*b^3 + b^4)*si
n(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^
3 + b^4)/(a^3*b^5*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt((
a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)
) - 4*a - 4*b)/(a*b^2*d^2))) - b*d*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b +
38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))*log(-
1/2*(a^4 + 4*a^3*b - 10*a^2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*
b^4*d^3*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4))
- (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-(a*b^2*d^2*sqrt((a^4 + 1
2*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d
^2))) + b*d*sqrt((a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 +
b^4)/(a^3*b^5*d^4)) - 4*a - 4*b)/(a*b^2*d^2))*log(-1/2*(a^4 + 4*a^3*b - 10*
a^2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a
^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + (a^4*b + 7*a^3*b^2 + 7
*a^2*b^3 + a*b^4)*d)*sqrt((a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12
*a*b^3 + b^4)/(a^3*b^5*d^4)) - 4*a - 4*b)/(a*b^2*d^2))) + 4*sin(d*x + c))/(
b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(85) = 170.
time = 0.78, size = 311, normalized size = 2.75

$$-\frac{\frac{8\sin(dx+c)}{b}-\frac{2\sqrt{2}\left((-ab^3)^{\frac{1}{4}}(ab+b^2)-2(-ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}-\frac{2\sqrt{2}\left((-ab^3)^{\frac{1}{4}}(ab+b^2)-2(-ab^3)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}-\frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}(ab+b^2)+2(-ab^3)^{\frac{3}{4}}\right)\log\left(\sin(dx+c)^2+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{ab^3}+\frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}(ab+b^2)+2(-ab^3)^{\frac{3}{4}}\right)\log\left(\sin(dx+c)^2-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{ab^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/8*(8*sin(d*x + c)/b - 2*sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) - 2*(-a*b^3)
^(3/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(
1/4))/(a*b^3) - 2*sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) - 2*(-a*b^3)^(3/4))*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a
*b^3) - sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*log(sin(d*x
 + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3) + sqrt(2)
*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*log(sin(d*x + c)^2 - sqrt(
2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3))/d
```

Mupad [B]
time = 15.78, size = 1097, normalized size = 9.71

$$\frac{2\,\mathrm{atanh}\left(\frac{8 b^3 \sin(c+d x)\sqrt{\frac{\sqrt{a^3 b^5}}{16 a b^5}-\frac{1}{4 a b}-\frac{1}{4 b^2}+\frac{3 \sqrt{a^3 b^5}}{8 a^2 b^4}+\frac{\sqrt{a^3 b^5}}{16 a^3 b^3}}}{\frac{2 \sqrt{a^3 b^5}}{a^2}-24 a b+\frac{14 \sqrt{a^3 b^5}}{b^2}-4 a^2-4 b^2+\frac{14 \sqrt{a^3 b^5}}{a b}+\frac{2 a \sqrt{a^3 b^5}}{b^3}}+\frac{48 a b^2 \sin(c+d x)\sqrt{\frac{\sqrt{a^3 b^5}}{16 a b^5}-\frac{1}{4 a b}-\frac{1}{4 b^2}+\frac{3 \sqrt{a^3 b^5}}{8 a^2 b^4}+\frac{\sqrt{a^3 b^5}}{16 a^3 b^3}}}{\frac{2 \sqrt{a^3 b^5}}{a^2}-24 a b+\frac{14 \sqrt{a^3 b^5}}{b^2}-4 a^2-4 b^2+\frac{14 \sqrt{a^3 b^5}}{a b}+\frac{2 a \sqrt{a^3 b^5}}{b^3}}+\frac{8 a^2 b \sin(c+d x)\sqrt{\frac{\sqrt{a^3 b^5}}{16 a b^5}-\frac{1}{4 a b}-\frac{1}{4 b^2}+\frac{3 \sqrt{a^3 b^5}}{8 a^2 b^4}+\frac{\sqrt{a^3 b^5}}{16 a^3 b^3}}}{\frac{2 \sqrt{a^3 b^5}}{a^2}-24 a b+\frac{14 \sqrt{a^3 b^5}}{b^2}-4 a^2-4 b^2+\frac{14 \sqrt{a^3 b^5}}{a b}+\frac{2 a \sqrt{a^3 b^5}}{b^3}}\right)\sqrt{\frac{a^2 \sqrt{a^3 b^5}+b^2 \sqrt{a^3 b^5}-4 a^2 b^4-4 a^3 b^3+6 a b \sqrt{a^3 b^5}}{16 a^3 b^5}}}{d}-\frac{2\,\mathrm{atanh}\left(\frac{8 b^3 \sin(c+d x)\sqrt{-\frac{1}{4 b^2}-\frac{1}{4 a b}-\frac{\sqrt{a^3 b^5}}{16 a b^5}-\frac{3 \sqrt{a^3 b^5}}{8 a^2 b^4}-\frac{\sqrt{a^3 b^5}}{16 a^3 b^3}}}{24 a b+\frac{2 \sqrt{a^3 b^5}}{a^2}+\frac{14 \sqrt{a^3 b^5}}{b^2}+4 a^2+4 b^2+\frac{14 \sqrt{a^3 b^5}}{a b}+\frac{2 a \sqrt{a^3 b^5}}{b^3}}+\frac{48 a b^2 \sin(c+d x)\sqrt{-\frac{1}{4 b^2}-\frac{1}{4 a b}-\frac{\sqrt{a^3 b^5}}{16 a b^5}-\frac{3 \sqrt{a^3 b^5}}{8 a^2 b^4}-\frac{\sqrt{a^3 b^5}}{16 a^3 b^3}}}{24 a b+\frac{2 \sqrt{a^3 b^5}}{a^2}+\frac{14 \sqrt{a^3 b^5}}{b^2}+4 a^2+4 b^2+\frac{14 \sqrt{a^3 b^5}}{a b}+\frac{2 a \sqrt{a^3 b^5}}{b^3}}+\frac{8 a^2 b \sin(c+d x)\sqrt{-\frac{1}{4 b^2}-\frac{1}{4 a b}-\frac{\sqrt{a^3 b^5}}{16 a b^5}-\frac{3 \sqrt{a^3 b^5}}{8 a^2 b^4}-\frac{\sqrt{a^3 b^5}}{16 a^3 b^3}}}{24 a b+\frac{2 \sqrt{a^3 b^5}}{a^2}+\frac{14 \sqrt{a^3 b^5}}{b^2}+4 a^2+4 b^2+\frac{14 \sqrt{a^3 b^5}}{a b}+\frac{2 a \sqrt{a^3 b^5}}{b^3}}\right)\sqrt{-\frac{a^2 \sqrt{a^3 b^5}+b^2 \sqrt{a^3 b^5}+4 a^2 b^4+4 a^3 b^3+6 a b \sqrt{a^3 b^5}}{16 a^3 b^5}}}{d}-\frac{\sin(c+d x)}{b d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (2*atanh((8*b^3*sin(c + d*x)*((a^3*b^5)^(1/2)/(16*a*b^5) - 1/(4*a*b) - 1/(4
*b^2) + (3*(a^3*b^5)^(1/2))/(8*a^2*b^4) + (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/
2))/((2*(a^3*b^5)^(1/2))/a^2 - 24*a*b + (14*(a^3*b^5)^(1/2))/b^2 - 4*a^2 -
4*b^2 + (14*(a^3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (48*a*b^2
*sin(c + d*x)*((a^3*b^5)^(1/2)/(16*a*b^5) - 1/(4*a*b) - 1/(4*b^2) + (3*(a^3
*b^5)^(1/2))/(8*a^2*b^4) + (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/((2*(a^3*b^
5)^(1/2))/a^2 - 24*a*b + (14*(a^3*b^5)^(1/2))/b^2 - 4*a^2 - 4*b^2 + (14*(a^
3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (8*a^2*b*sin(c + d*x)*((
a^3*b^5)^(1/2)/(16*a*b^5) - 1/(4*a*b) - 1/(4*b^2) + (3*(a^3*b^5)^(1/2))/(8*
a^2*b^4) + (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/((2*(a^3*b^5)^(1/2))/a^2 -
24*a*b + (14*(a^3*b^5)^(1/2))/b^2 - 4*a^2 - 4*b^2 + (14*(a^3*b^5)^(1/2))/(a
*b) + (2*a*(a^3*b^5)^(1/2))/b^3))*((a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/
2) - 4*a^2*b^4 - 4*a^3*b^3 + 6*a*b*(a^3*b^5)^(1/2))/(16*a^3*b^5))^(1/2))/d
- (2*atanh((8*b^3*sin(c + d*x)*(- 1/(4*b^2) - 1/(4*a*b) - (a^3*b^5)^(1/2)/(
16*a*b^5) - (3*(a^3*b^5)^(1/2))/(8*a^2*b^4) - (a^3*b^5)^(1/2)/(16*a^3*b^3))
^(1/2))/(24*a*b + (2*(a^3*b^5)^(1/2))/a^2 + (14*(a^3*b^5)^(1/2))/b^2 + 4*a^
2 + 4*b^2 + (14*(a^3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (48*a
*b^2*sin(c + d*x)*(- 1/(4*b^2) - 1/(4*a*b) - (a^3*b^5)^(1/2)/(16*a*b^5) - (
3*(a^3*b^5)^(1/2))/(8*a^2*b^4) - (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/(24*a
*b + (2*(a^3*b^5)^(1/2))/a^2 + (14*(a^3*b^5)^(1/2))/b^2 + 4*a^2 + 4*b^2 + (
14*(a^3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (8*a^2*b*sin(c + d
*x)*(- 1/(4*b^2) - 1/(4*a*b) - (a^3*b^5)^(1/2)/(16*a*b^5) - (3*(a^3*b^5)^(1
/2))/(8*a^2*b^4) - (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/(24*a*b + (2*(a^3*b
^5)^(1/2))/a^2 + (14*(a^3*b^5)^(1/2))/b^2 + 4*a^2 + 4*b^2 + (14*(a^3*b^5)^(
1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3))*(-(a^2*(a^3*b^5)^(1/2) + b^2*(a^3
```

```
*b^5)^(1/2) + 4*a^2*b^4 + 4*a^3*b^3 + 6*a*b*(a^3*b^5)^(1/2))/(16*a^3*b^5))^
(1/2))/d - sin(c + d*x)/(b*d)
```

3.406 $\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

```
[Out] -1/2*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4)/b^(3/4)/
d+1/2*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)/
d
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3302, 1181, 211, 214}

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b])*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(
3/4)*d) - ((Sqrt[a] - Sqrt[b])*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*
a^(3/4)*b^(3/4)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2d} - \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\right.}{}$$

$$= \frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 160, normalized size = 1.68

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)+i\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)-i\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right)-\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)}{4a^{3/4}b^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] - Sqrt[b])*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]] + I*(Sqrt[a] + Sqr
t[b])*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - I*(Sqrt[a] + Sqrt[b])*Log[a^(
1/4) + I*b^(1/4)*Sin[c + d*x]] - (Sqrt[a] - Sqrt[b])*Log[a^(1/4) + b^(1/4)*
Sin[c + d*x]])/(4*a^(3/4)*b^(3/4)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(67) = 134$.
time = 0.71, size = 136, normalized size = 1.43

method	result
derivativedivides	$\frac{\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{d}$

default	$\frac{\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(256a^3b^3d^4\_Z^4+64a^2b^2d^2\_Z^2-a^2+2ab-b^2\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(-\frac{128ia^3b^2d^3\_R^3}{a^2-b^2}+\left(-\frac{24ib}{a^2-}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*(1/b*a)^(1/4)/a*(ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)
^(1/4)))+2*arctan(sin(d*x+c)/(1/b*a)^(1/4)))+1/4/b/(1/b*a)^(1/4)*(2*arctan(
sin(d*x+c)/(1/b*a)^(1/4))-ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)
^(1/4)))))
```

Maxima [A]
time = 0.53, size = 121, normalized size = 1.27

$$\frac{\frac{2\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(sqrt(a) + sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (sqrt(a) - sqrt(b))*log((sqrt(
b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(
a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(67) = 134.
time = 0.46, size = 631, normalized size = 6.64

$$\frac{1}{4}\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\log\left(\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}-\left(a^2b+ab^2\right)d\right)\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\right)-\frac{1}{4}\sqrt{\frac{abd^2\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}-2}{abd^2}}\log\left(\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}+\left(a^2b+ab^2\right)d\right)\sqrt{\frac{abd^2\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}-2}{abd^2}}\right)-\frac{1}{4}\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\log\left(-\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}-\left(a^2b+ab^2\right)d\right)\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\right)+\frac{1}{4}\sqrt{\frac{abd^2\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}-2}{abd^2}}\log\left(-\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}+\left(a^2b+ab^2\right)d\right)\sqrt{\frac{abd^2\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}-2}{abd^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))*
log(1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2
)/(a^3*b^3*d^4)) - (a^2*b + a*b^2)*d)*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^
```

```
2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))) - 1/4*sqrt((a*b*d^2*sqrt((a^2 + 2*a*b +
b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))*log(1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*
(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + (a^2*b + a*b^2)*d)*s
qrt((a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))) - 1/4
*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))*log
(-1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/
(a^3*b^3*d^4)) - (a^2*b + a*b^2)*d)*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)
/(a^3*b^3*d^4)) + 2)/(a*b*d^2))) + 1/4*sqrt((a*b*d^2*sqrt((a^2 + 2*a*b + b^
2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))*log(-1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(
a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + (a^2*b + a*b^2)*d)*sq
rt((a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(67) = 134.
time = 0.71, size = 280, normalized size = 2.95

$$\frac{\frac{2\sqrt{2}\left(\left(-ab^{3}\right)^{\frac{1}{4}}b^{2}-\left(-ab^{3}\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin\left(dx+c\right)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^{3}}+\frac{2\sqrt{2}\left(\left(-ab^{3}\right)^{\frac{1}{4}}b^{2}-\left(-ab^{3}\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin\left(dx+c\right)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^{3}}+\frac{\sqrt{2}\left(\left(-ab^{3}\right)^{\frac{1}{4}}b^{2}+\left(-ab^{3}\right)^{\frac{3}{4}}\right)\log\left(\sin\left(dx+c\right)^{2}+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin\left(dx+c\right)+\sqrt{-\frac{a}{b}}\right)}{ab^{3}}-\frac{\sqrt{2}\left(\left(-ab^{3}\right)^{\frac{1}{4}}b^{2}+\left(-ab^{3}\right)^{\frac{3}{4}}\right)\log\left(\sin\left(dx+c\right)^{2}-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin\left(dx+c\right)+\sqrt{-\frac{a}{b}}\right)}{ab^{3}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(sq
rt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^3) + 2*sqrt(2)*((-a
*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4)
 - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^3) + sqrt(2)*((-a*b^3)^(1/4)*b^2 + (-
a*b^3)^(3/4))*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt
(-a/b))/(a*b^3) - sqrt(2)*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*log(sin(d*x
 + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3))/d
```

Mupad [**B**]
time = 15.81, size = 489, normalized size = 5.15

$$-\frac{2\,\mathrm{atanh}\left(\frac{8\,b^3\,\sin\left(c+d\,x\right)\,\sqrt{-\frac{1}{8\,a\,b}-\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b+\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2+\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}+\frac{8\,a\,b^2\,\sin\left(c+d\,x\right)\,\sqrt{-\frac{1}{8\,a\,b}-\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b+\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2+\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}\right)\,\sqrt{-\frac{a\,\sqrt{a^3\,b^3}+b\,\sqrt{a^3\,b^3}+2\,a^2\,b^2}{16\,a^3\,b^3}}}{d}-\frac{2\,\mathrm{atanh}\left(\frac{8\,b^3\,\sin\left(c+d\,x\right)\,\sqrt{\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{1}{8\,a\,b}+\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b-\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2-\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}+\frac{8\,a\,b^2\,\sin\left(c+d\,x\right)\,\sqrt{\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{1}{8\,a\,b}+\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b-\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2-\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}\right)\,\sqrt{\frac{a\,\sqrt{a^3\,b^3}+b\,\sqrt{a^3\,b^3}-2\,a^2\,b^2}{16\,a^3\,b^3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a - b*sin(c + d*x)^4),x)

[Out] - (2*atanh((8*b^3*sin(c + d*x)*(- 1/(8*a*b) - (a^3*b^3)^(1/2)/(16*a^2*b^3)
- (a^3*b^3)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b + (2*(a^3*b^3)^(1/2))/a + 2*b
^2 + (2*b*(a^3*b^3)^(1/2))/a^2) + (8*a*b^2*sin(c + d*x)*(- 1/(8*a*b) - (a^3
*b^3)^(1/2)/(16*a^2*b^3) - (a^3*b^3)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b + (2
*(a^3*b^3)^(1/2))/a + 2*b^2 + (2*b*(a^3*b^3)^(1/2))/a^2))*(-(a*(a^3*b^3)^(1
/2) + b*(a^3*b^3)^(1/2) + 2*a^2*b^2)/(16*a^3*b^3))^(1/2))/d - (2*atanh((8*b
^3*sin(c + d*x)*((a^3*b^3)^(1/2)/(16*a^2*b^3) - 1/(8*a*b) + (a^3*b^3)^(1/2)
/(16*a^3*b^2))^(1/2))/(2*a*b - (2*(a^3*b^3)^(1/2))/a + 2*b^2 - (2*b*(a^3*b^
3)^(1/2))/a^2) + (8*a*b^2*sin(c + d*x)*((a^3*b^3)^(1/2)/(16*a^2*b^3) - 1/(8
*a*b) + (a^3*b^3)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b - (2*(a^3*b^3)^(1/2))/a
 + 2*b^2 - (2*b*(a^3*b^3)^(1/2))/a^2))*((a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1
/2) - 2*a^2*b^2)/(16*a^3*b^3))^(1/2))/d
```

3.407 $\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}$$

```
[Out] 1/2*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))/a^(3/4)/b^(1/4)/d+1/2*arctanh(b^(1/4
)*sin(d*x+c)/a^(1/4))/a^(3/4)/b^(1/4)/d
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3302, 218, 214, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)*d) + ArcTanh[(b^(
1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
```

```
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}\,d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}+\sqrt{b}\,x^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}\,d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.76

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a - b*Sin[c + d*x]^4),x]
[Out] (ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)] + ArcTanh[(b^(1/4)*Sin[c + d*x])/a^
(1/4)])/(2*a^(3/4)*b^(1/4)*d)
```

Maple [A]
time = 0.30, size = 68, normalized size = 0.96

method	result	size
risch	$\sum_{\_R=\text{RootOf}\left(256a^3b\,d^4\_Z^4-1\right)} \_R\ln\left(e^{2i(dx+c)}+8iad\_R\,e^{i(dx+c)}-1\right)$	48
derivativedivides	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4da}$	68
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4da}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(1/b*a)^(1/4)/a*(ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1
/4)))+2*arctan(sin(d*x+c)/(1/b*a)^(1/4)))
```

Maxima [A]
time = 0.50, size = 100, normalized size = 1.41

$$\frac{\frac{2\,\arctan\left(\frac{\sqrt{b}\,\sin\left(dx+c\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}-\frac{\log\left(\frac{\sqrt{b}\,\sin\left(dx+c\right)-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,\sin\left(dx+c\right)+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(2*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))) - log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b
)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(51) = 102.
time = 58.43, size = 330, normalized size = 4.65

$$\frac{1}{2}\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(a^2bd^3\left(\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\sin\left(dx+c\right)+\sqrt{a^2d^2\sqrt{\frac{1}{a^3bd^4}}-\cos\left(dx+c\right)^2+1}\,a^2bd^3\left(\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\right)-\frac{1}{2}\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(-a^2bd^3\left(\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\sin\left(dx+c\right)+\sqrt{a^2d^2\sqrt{\frac{1}{a^3bd^4}}-\cos\left(dx+c\right)^2+1}\,a^2bd^3\left(\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\right)+\frac{1}{8}\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}a^2d^2\sqrt{\frac{1}{a^3bd^4}}+\frac{1}{2}ad\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\sin\left(dx+c\right)-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)-\frac{1}{8}\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}a^2d^2\sqrt{\frac{1}{a^3bd^4}}-\frac{1}{2}ad\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\sin\left(dx+c\right)-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(1/(a^3*b*d^4))^(1/4)*arctan(a^2*b*d^3*(1/(a^3*b*d^4))^(3/4)*sin(d*x +
c) + sqrt(a^2*d^2*sqrt(1/(a^3*b*d^4)) - cos(d*x + c)^2 + 1)*a^2*b*d^3*(1/(a
^3*b*d^4))^(3/4)) - 1/2*(1/(a^3*b*d^4))^(1/4)*arctan(-a^2*b*d^3*(1/(a^3*b*d
^4))^(3/4)*sin(d*x + c) + sqrt(a^2*d^2*sqrt(1/(a^3*b*d^4)) - cos(d*x + c)^2
 + 1)*a^2*b*d^3*(1/(a^3*b*d^4))^(3/4)) + 1/8*(1/(a^3*b*d^4))^(1/4)*log(1/4*
a^2*d^2*sqrt(1/(a^3*b*d^4)) + 1/2*a*d*(1/(a^3*b*d^4))^(1/4)*sin(d*x + c) -
1/4*cos(d*x + c)^2 + 1/4) - 1/8*(1/(a^3*b*d^4))^(1/4)*log(1/4*a^2*d^2*sqrt(
1/(a^3*b*d^4)) - 1/2*a*d*(1/(a^3*b*d^4))^(1/4)*sin(d*x + c) - 1/4*cos(d*x +
 c)^2 + 1/4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.

time = 2.59, size = 129, normalized size = 1.82

$$\begin{cases} \frac{\tilde{\infty} x \cos{(c)}}{\sin^4{(c)}} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\sin{(c+dx)}}{ad} & \text{for } b = 0 \\ \frac{1}{3bd\sin^3{(c+dx)}} & \text{for } a = 0 \\ \frac{x\cos{(c)}}{a - b\sin^4{(c)}} & \text{for } d = 0 \\ -\frac{\sqrt[4]{\frac{a}{b}}\log{\left(-\sqrt[4]{\frac{a}{b}} + \sin{(c+dx)}\right)}}{4ad} + \frac{\sqrt[4]{\frac{a}{b}}\log{\left(\sqrt[4]{\frac{a}{b}} + \sin{(c+dx)}\right)}}{4ad} + \frac{\sqrt[4]{\frac{a}{b}}\operatorname{atan}{\left(\frac{\sin{(c+dx)}}{\sqrt[4]{\frac{a}{b}}}\right)}}{2ad} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sin(c
+ d*x)/(a*d), Eq(b, 0)), (1/(3*b*d*sin(c + d*x)**3), Eq(a, 0)), (x*cos(c)/(
a - b*sin(c)**4), Eq(d, 0)), (-(a/b)**(1/4)*log(-(a/b)**(1/4) + sin(c + d*x
))/(4*a*d) + (a/b)**(1/4)*log((a/b)**(1/4) + sin(c + d*x))/(4*a*d) + (a/b)*
*(1/4)*atan(sin(c + d*x)/(a/b)**(1/4))/(2*a*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(51) = 102$.

time = 0.70, size = 224, normalized size = 3.15

$$\frac{\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab} + \frac{2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab} + \frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\log\left(\sin(dx+c)^2+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{ab} - \frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\log\left(\sin(dx+c)^2-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{ab}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2)*(-a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*
sin(d*x + c))/(-a/b)^(1/4))/(a*b) + 2*sqrt(2)*(-a*b^3)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b) + sqrt(2)
*(-a*b^3)^(1/4)*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sq
rt(-a/b))/(a*b) - sqrt(2)*(-a*b^3)^(1/4)*log(sin(d*x + c)^2 - sqrt(2)*(-a/b
)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b))/d
```

Mupad [B]

time = 0.11, size = 40, normalized size = 0.56

$$\frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sin(c+d\,x)}{a^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\,\sin(c+d\,x)}{a^{1/4}}\right)}{2\,a^{3/4}\,b^{1/4}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((b^(1/4)*sin(c + d*x))/a^(1/4)) + atanh((b^(1/4)*sin(c + d*x))/a^(1/4
)))/(2*a^(3/4)*b^(1/4)*d)
```

3.408 $\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)d}+\frac{\tanh^{-1}(\sin(c+dx))}{(a-b)d}-\frac{\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)d}$$

```
[Out] arctanh(sin(d*x+c))/(a-b)/d-1/2*b^(1/4)*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))
/a^(3/4)/d/(a^(1/2)-b^(1/2))+1/2*b^(1/4)*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))
/a^(3/4)/d/(a^(1/2)+b^(1/2))
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3302, 1185, 213, 1181, 211, 214}

$$\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(1/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])*d) + ArcTanh[Sin[c + d*x]]/((a - b)*d) - (b^(1/4)*ArcTanh[(b^(1/4)*Sin
[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-bx^4)}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)(-1+x^2)} - \frac{b(1+x^2)}{(a-b)(a-bx^4)}\right)\,dx, x, \sin(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(c+dx)\right)}{(a-b)d} - \frac{b\text{Subst}\left(\int \frac{1+x^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{(a-b)d}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{(a-b)d} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)d} - \frac{b\text{Subst}\left(\int -\right.}{2}\\
&= \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)d} + \frac{\tanh^{-1}(\sin(c+dx))}{(a-b)d} - \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.14, size = 184, normalized size = 1.57

$$
\frac{4a^{3/4}\tanh^{-1}(\sin(c+dx)) + \sqrt[4]{b}\left(\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right) + i\left(\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right) + \left(-\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right) + i\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)\right)\right)}{4a^{3/4}(a-b)d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (4*a^(3/4)*ArcTanh[Sin[c + d*x]] + b^(1/4)*((Sqrt[a] + Sqrt[b])*Log[a^(1/4)
 - b^(1/4)*Sin[c + d*x]] + I*((Sqrt[a] - Sqrt[b])*Log[a^(1/4) - I*b^(1/4)*S
in[c + d*x]] + (-Sqrt[a] + Sqrt[b])*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] +
 I*(Sqrt[a] + Sqrt[b])*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])))/(4*a^(3/4)*(a
 - b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(89) = 178$.

time = 0.68, size = 185, normalized size = 1.58

method	result
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2a-2b}+\frac{b\left(-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)}{\sin(dx+c)-\left(\frac{a}{b}\right)}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a-b}}{d}$
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2a-2b}+\frac{b\left(-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)}{\sin(dx+c)-\left(\frac{a}{b}\right)}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a-b}}{d}$
risch	$\frac{\ln(e^{i(dx+c)}+i)}{d(a-b)}-\frac{\ln(e^{i(dx+c)}-i)}{d(a-b)}+2\left(\sum_{\_R=\text{RootOf}((4096a^5d^4-8192a^4b\,d^4+4096a^3b^2d^4)\_Z^4-256a^2b\,d^2\_Z^2-b)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(2*a-2*b)*ln(sin(d*x+c)-1)+b/(a-b)*(-1/4*(1/b*a)^(1/4)/a*(ln((sin(d
*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))+2*arctan(sin(d*x+c)/(1/b*a
)^(1/4)))+1/4/b/(1/b*a)^(1/4)*(2*arctan(sin(d*x+c)/(1/b*a)^(1/4))-ln((sin(d
*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))))+1/(2*a-2*b)*ln(1+sin(d*x
+c)))
```

Maxima [A]

time = 0.54, size = 167, normalized size = 1.43

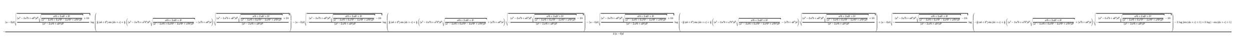
$$\frac{\frac{b\left(\frac{2\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{a-b}+\frac{2\log(\sin(dx+c)+1)}{a-b}-\frac{2\log(\sin(dx+c)-1)}{a-b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(b*(2*(sqrt(a) - sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (sqrt(a) + sqrt(b))*log((sq
rt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sq
rt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a - b) + 2*log(s
in(d*x + c) + 1)/(a - b) - 2*log(sin(d*x + c) - 1)/(a - b))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. $2(89) = 178$.
time = 0.54, size = 1329, normalized size = 11.36



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a - b)*d*sqrt(((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b
^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 -
 2*a^2*b + a*b^2)*d^2))*log(1/2*(a*b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^
4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2
 - 4*a^4*b^3 + a^3*b^4)*d^4)) - (a^2*b + a*b^2)*d)*sqrt(((a^3 - 2*a^2*b + a
*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*
b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))) - (a - b)*d*sqr
t(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*
b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*
d^2))*log(1/2*(a*b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3
*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3
*b^4)*d^4)) + (a^2*b + a*b^2)*d)*sqrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a
^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^
4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))) - (a - b)*d*sqrt(((a^3 - 2*a^2*b
 + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*
a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))*log(-1/2*(a*
b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*
a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - (a^
2*b + a*b^2)*d)*sqrt(((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b
^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 -
 2*a^2*b + a*b^2)*d^2))) + (a - b)*d*sqrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqr
t((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4
)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))*log(-1/2*(a*b + b^2)*sin(d*x
+ c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^
7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + (a^2*b + a*b^2)*d)*s
qrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^
6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2
)*d^2))) - 2*log(sin(d*x + c) + 1) + 2*log(-sin(d*x + c) + 1))/((a - b)*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec{(c + dx)}}{a - b\sin^4{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(sec(c + d*x)/(a - b*sin(c + d*x)**4), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(89) = 178.
time = 0.74, size = 370, normalized size = 3.16

$$-\frac{\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2-\sqrt{2}ab^3}+\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2-\sqrt{2}ab^3}+\frac{\left(\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}b^2-\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(\sin(dx+c)^2+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{a^2b^2-ab^3}-\frac{\left(\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}b^2-\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(\sin(dx+c)^2-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{a^2b^2-ab^3}-\frac{4\log(|\sin(dx+c)+1|)}{a-b}+\frac{4\log(|\sin(dx+c)-1|)}{a-b}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/8*(4*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(
-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^2*b^2 - sqrt(2)*a*b^
3) + 4*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(
-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^2*b^2 - sqrt(2)*a*b^
3) + (sqrt(2)*(-a*b^3)^(1/4)*b^2 - sqrt(2)*(-a*b^3)^(3/4))*log(sin(d*x + c)
^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^2*b^2 - a*b^3) - (s
qrt(2)*(-a*b^3)^(1/4)*b^2 - sqrt(2)*(-a*b^3)^(3/4))*log(sin(d*x + c)^2 - sq
rt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^2*b^2 - a*b^3) - 4*log(abs
(sin(d*x + c) + 1))/(a - b) + 4*log(abs(sin(d*x + c) - 1))/(a - b))/d
```

Mupad [**B**]
time = 17.91, size = 2500, normalized size = 21.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(((b^5*sin(c + d*x)*3i + ((((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128
*a^4*b^4 - (sin(c + d*x)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5
*b^4))/(4*(a - b)))/(2*(a - b)) + (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^
2*b^5))/2)/(2*(a - b)) - 10*a*b^5 + 2*b^6)*1i)/(2*(a - b)))/(a - b) + (b^5*
sin(c + d*x)*3i - ((((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128*a^4*b^4 + (
sin(c + d*x)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4))/(4*(a
 - b)))/(2*(a - b)) - (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))/2)/(
```

2*(a - b)) - 10*a*b^5 + 2*b^6)*1i)/(2*(a - b)))/(a - b))/((3*b^5*sin(c + d*x) + (((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128*a^4*b^4 - (sin(c + d*x)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4))/(4*(a - b)))/(2*(a - b)) + (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))/2)/(2*(a - b)) - 10*a*b^5 + 2*b^6)/(2*(a - b)))/(a - b) - (3*b^5*sin(c + d*x) - (((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128*a^4*b^4 + (sin(c + d*x)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4))/(4*(a - b)))/(2*(a - b)) - (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))/2)/(2*(a - b)) - 10*a*b^5 + 2*b^6)/(2*(a - b)))/(a - b)))*1i)/(d*(a - b)) - (atan((((((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 256*a^4*b^4 + sin(c + d*x)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) - sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 6*b^5*sin(c + d*x))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*1i - (((((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 256*a^4*b^4 - sin(c + d*x)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) + sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) + 6*b^5*sin(c + d*x))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*1i)/((((((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 256*a^4*b^4 + sin(c + d*x)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) - sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 6*b^5*sin(c + d*x))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) + (((((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 256*a^4*b^4 - sin(c + d*x)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) + sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) + 6*b^5*sin(c + d*x))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*2i)/d - (atan((((((-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a

```
^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b^6
- 448*a^3*b^5 + 256*a^4*b^4 + sin(c + d*x)*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b
*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*a^
3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) - sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240
*a^2*b^5))*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4
*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*
(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 6*b^5*sin(c + d*x))*
(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^
2)))^(1/2)*1i - ((((-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5
 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 256*a
^4*b^4 - sin(c + d*x)*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(
a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 +
 512*a^5*b^4)) + sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))*(-(a*(a^3*
b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)
 - 20*a*b^5 + 4*b^6)*(-(a*(a^3*b)^(1/2) - 2*a^2...
```

3.409 $\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2d}+\frac{(a-5b)\tanh^{-1}(\sin(c+dx))}{2(a-b)^2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2d}+\frac{1}{4(a-b)d(1-\sin(c$$

```
[Out] 1/2*(a-5*b)*arctanh(sin(d*x+c))/(a-b)^2/d+1/4/(a-b)/d/(1-sin(d*x+c))-1/4/(a
-b)/d/(1+sin(d*x+c))+1/2*b^(3/4)*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))/a^(3/4
)/d/(a^(1/2)-b^(1/2))^2+1/2*b^(3/4)*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))/a^(3
/4)/d/(a^(1/2)+b^(1/2))^2
```

Rubi [A]
time = 0.14, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3302, 1185, 213, 1181, 211, 214}

$$\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^2}+\frac{1}{4d(a-b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}+\frac{(a-5b)\tanh^{-1}(\sin(c+dx))}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])^2*d) + ((a - 5*b)*ArcTanh[Sin[c + d*x]])/(2*(a - b)^2*d) + (b^(3/4)*Ar
cTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])^2*d)
+ 1/(4*(a - b)*d*(1 - Sin[c + d*x])) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-bx^4)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a-b)(-1+x)^2} + \frac{1}{4(a-b)(1+x)^2} + \frac{-a+5b}{2(a-b)^2(-1+x^2)} + \frac{b(a+b+2bx^2)}{(a-b)^2(a-bx^4)}\right) dx, x, \sin(\ldots}{d}$$

$$= \frac{1}{4(a-b)d(1-\sin(c+dx))} - \frac{1}{4(a-b)d(1+\sin(c+dx))} - \frac{(a-5b)\text{Subst}\left(\int \ldots\right.}{2(a\ldots}$$

$$= \frac{(a-5b)\tanh^{-1}(\sin(c+dx))}{2(a-b)^2d} + \frac{1}{4(a-b)d(1-\sin(c+dx))} - \frac{1}{4(a-b)d(1+\sin(\ldots}$$

$$= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2d} + \frac{(a-5b)\tanh^{-1}(\sin(c+dx))}{2(a-b)^2d} + \frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\text{si}\ldots}{\sqrt[4]{\ldots}}\right.}{2a^{3/4}\left(\sqrt{a}-\sqrt{\ldots}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 255, normalized size = 1.46

$$-\frac{-\frac{2(a-5b)\tanh^{-1}(\sin(c+dx))}{(a-b)^2} + \frac{b^{3/4}\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2} - \frac{ib^{3/4}\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2} + \frac{ib^{3/4}\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2} - \frac{b^{3/4}\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2} + \frac{1}{(a-b)(-1+\sin(c+dx))} + \frac{1}{(a-b)(1+\sin(c+dx))}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/4*((-2*(a - 5*b)*ArcTanh[Sin[c + d*x]])/(a - b)^2 + (b^(3/4)*Log[a^(1/4)
 - b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^2) - (I*b^(3/4)*Log[
a^(1/4) - I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[b])^2) + (I*b^(
3/4)*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[b])^2)
 - (b^(3/4)*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b
])^2) + 1/((a - b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/d
```

Maple [A]
time = 1.26, size = 241, normalized size = 1.38

method	result
derivativedivides	$\frac{-\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(a-5b)\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a-4b)(\sin(dx+c)-1)}+\frac{(-a+5b)\ln(\sin(dx+c)-1)}{4(a-b)^2}-\frac{b\left(\frac{(-a-b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(\cdots}{\sin(\cdots}\right.\right.}{\cdots}\right.}{\cdots}}{d}$
default	$\frac{-\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(a-5b)\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a-4b)(\sin(dx+c)-1)}+\frac{(-a+5b)\ln(\sin(dx+c)-1)}{4(a-b)^2}-\frac{b\left(\frac{(-a-b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(\cdots}{\sin(\cdots}\right.\right.}{\cdots}\right.}{\cdots}}{d}$
risch	$-\frac{i\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d(a-b)\left(e^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{i(dx+c)}+i\right)a}{2(a^2-2ab+b^2)d}-\frac{5\ln\left(e^{i(dx+c)}+i\right)b}{2(a^2-2ab+b^2)d}-\frac{\ln\left(e^{i(dx+c)}-i\right)a}{2d(a^2-2ab+b^2)}+\frac{5\ln\left(e^{i(dx+c)}-i\right)b}{2d(a^2-2ab+b^2)}+8\left(\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(4*a-4*b)/(1+sin(d*x+c))+1/4*(a-5*b)/(a-b)^2*ln(1+sin(d*x+c))-1/(4*
a-4*b)/(sin(d*x+c)-1)+1/4/(a-b)^2*(-a+5*b)*ln(sin(d*x+c)-1)-b/(a-b)^2*(1/4*
(-a-b)*(1/b*a)^(1/4)/a*(ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(
1/4)))+2*arctan(sin(d*x+c)/(1/b*a)^(1/4)))+1/2/(1/b*a)^(1/4)*(2*arctan(sin(
d*x+c)/(1/b*a)^(1/4))-ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/
4))))))
```

Maxima [A]
time = 0.52, size = 244, normalized size = 1.39

$$-\frac{\frac{b\left(\frac{2\left(b\left(2\sqrt{a}-\sqrt{b}\right)-a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(2\sqrt{a}+\sqrt{b}\right)+a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{a^2-2ab+b^2}-\frac{(a-5b)\log(\sin(dx+c)+1)}{a^2-2ab+b^2}+\frac{(a-5b)\log(\sin(dx+c)-1)}{a^2-2ab+b^2}+\frac{2\sin(dx+c)}{(a-b)\sin(dx+c)^2-a+b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(2*(b*(2*sqrt(a) - sqrt(b)) - a*sqrt(b))*arctan(sqrt(b)*sin(d*x + c
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b*(2*sq
rt(a) + sqrt(b)) + a*sqrt(b))*log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt
(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))*sqrt(b)))/(a^2 - 2*a*b + b^2) - (a - 5*b)*log(sin(d*x + c) + 1)/(
a^2 - 2*a*b + b^2) + (a - 5*b)*log(sin(d*x + c) - 1)/(a^2 - 2*a*b + b^2) +
2*sin(d*x + c)/((a - b)*sin(d*x + c)^2 - a + b))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2529 vs. $2(139) = 278$.
time = 0.89, size = 2529, normalized size = 14.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a^2 - 2*a*b + b^2)*d*sqrt(((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 +
a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11
 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^
6 - 8*a^4*b^7 + a^3*b^8)*d^4)) + 4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b
^2 - 4*a^2*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(1/2*(a^2*b^2 + 6*a*b^3 + b
^4)*sin(d*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)
*d^3*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a
^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*
a^4*b^7 + a^3*b^8)*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(
((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3
*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8
*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) +
4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) -
(a^2 - 2*a*b + b^2)*d*sqrt(-((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4
)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*
a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8
*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*b^2 - 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 -
4*a^2*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(1/2*(a^2*b^2 + 6*a*b^3 + b^4)*s
in(d*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^3*
sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b
 + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b
^7 + a^3*b^8)*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-((a^
5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4
 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3
 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*
```

```
b^2 - 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) - (a^2
 - 2*a*b + b^2)*d*sqrt(((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2
*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*
b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*
b^7 + a^3*b^8)*d^4)) + 4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2
*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(-1/2*(a^2*b^2 + 6*a*b^3 + b^4)*sin(d
*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^3*sqrt
((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 2
8*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 +
 a^3*b^8)*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(((a^5 - 4
*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38
*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70
*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) + 4*a*b^2 +
 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) + (a^2 - 2*
a*b + b^2)*d*sqrt(-((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqr
t((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b +
28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7
+ a^3*b^8)*d^4)) - 4*a*b^2 - 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
 + a*b^4)*d^2))*cos(d*x + c)^2*log(-1/2*(a^2*b^2 + 6*a*b^3 + b^4)*sin(d*x +
 c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^3*sqrt((a^
4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^
9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3
*b^8)*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-((a^5 - 4*a^
4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^
2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^
7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*b^2 - 4*
b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) - (a - 5*b)*co
s(d*x + c)^2*log(sin(d*x + c) + 1) + (a - 5*b)*cos(d*x + c)^2*log(-sin(d*x
+ c) + 1) - 2*(a - b)*sin(d*x + c))/((a^2 - 2*a*b + b^2)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a - b*sin(c + d*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(139) = 278.
time = 0.78, size = 475, normalized size = 2.71

$$\frac{\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)+2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3b-2\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)+2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3b-2\sqrt{2}a^2b^2+\sqrt{2}ab^3}-\frac{\left(2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}-\left(-ab^3\right)^{\frac{1}{4}}\left(\sqrt{2}ab+\sqrt{2}b^2\right)\right)\log\left(\sin(dx+c)^2+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{a^3b-2a^2b^2+ab^3}+\frac{\left(2\sqrt{2}\left(-ab^3\right)^{\frac{3}{4}}-\left(-ab^3\right)^{\frac{1}{4}}\left(\sqrt{2}ab+\sqrt{2}b^2\right)\right)\log\left(\sin(dx+c)^2-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{a^3b-2a^2b^2+ab^3}+\frac{2(a-5b)\log(|\sin(dx+c)+1|)}{a^2-2ab+b^2}-\frac{2(a-5b)\log(|\sin(dx+c)-1|)}{a^2-2ab+b^2}-\frac{4\sin(dx+c)}{\left(\sin(dx+c)^2-1\right)(a-b)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(4*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(
sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^3*b - 2*sqr
t(2)*a^2*b^2 + sqrt(2)*a*b^3) + 4*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^
(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(
1/4))/(sqrt(2)*a^3*b - 2*sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) - (2*sqrt(2)*(-a*
b^3)^(3/4) - (-a*b^3)^(1/4)*(sqrt(2)*a*b + sqrt(2)*b^2))*log(sin(d*x + c)^2
 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^3*b - 2*a^2*b^2 + a*b
^3) + (2*sqrt(2)*(-a*b^3)^(3/4) - (-a*b^3)^(1/4)*(sqrt(2)*a*b + sqrt(2)*b^2
))*log(sin(d*x + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^
3*b - 2*a^2*b^2 + a*b^3) + 2*(a - 5*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*
a*b + b^2) - 2*(a - 5*b)*log(abs(sin(d*x + c) - 1))/(a^2 - 2*a*b + b^2) - 4
*sin(d*x + c)/((sin(d*x + c)^2 - 1)*(a - b)))/d
```

Mupad [B]
time = 19.19, size = 2500, normalized size = 14.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan((((((((128*a*b^11 + 256*a^2*b^10 - 3456*a^3*b^9 + 8960*a^4*b^8 - 10880
*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*b^4)/(2*(a^4 - 4*a^3*b - 4
*a*b^3 + b^4 + 6*a^2*b^2)) - (sin(c + d*x)*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^
3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*
a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*(512*a^2*b^11 - 2560*a^3*b
^10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7 + 4608*a^7*b^6 - 2560*a^8*
b^5 + 512*a^9*b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^
3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3
)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (s
in(c + d*x)*(48*a*b^10 - 16*b^11 + 1024*a^2*b^9 - 2208*a^3*b^8 + 1264*a^4*b
^7 - 144*a^5*b^6 + 32*a^6*b^5))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)
)*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*
a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2))
)^(1/2) - (200*a*b^9 + 480*a^2*b^8 - 784*a^3*b^7 + 96*a^4*b^6 + 8*a^5*b^5)/
(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(a^2*(a^3*b^3)^(1/2) + b
^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^
7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (sin(c + d*x)*(11*
a*b^8 + 27*b^9 - 7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a
^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b
^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^
5*b^2)))^(1/2)*1i - (((((128*a*b^11 + 256*a^2*b^10 - 3456*a^3*b^9 + 8960*a^
```

4*b^8 - 10880*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*b^4)/(2*(a^4
- 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (sin(c + d*x)*(-(a^2*(a^3*b^3)^(1
/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/
(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*(512*a^2*b^11
 - 2560*a^3*b^10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7 + 4608*a^7*b^
6 - 2560*a^8*b^5 + 512*a^9*b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2
))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6
*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)
))^(1/2) - (sin(c + d*x)*(48*a*b^10 - 16*b^11 + 1024*a^2*b^9 - 2208*a^3*b^8
 + 1264*a^4*b^7 - 144*a^5*b^6 + 32*a^6*b^5))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4
 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4
*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3
+ 6*a^5*b^2)))^(1/2) - (200*a*b^9 + 480*a^2*b^8 - 784*a^3*b^7 + 96*a^4*b^6
+ 8*a^5*b^5)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(a^2*(a^3*b
^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(
1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (sin(
c + d*x)*(11*a*b^8 + 27*b^9 - 7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a^3*b - 4*a*b^
3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*
b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a
^4*b^3 + 6*a^5*b^2)))^(1/2)*1i)/((((((128*a*b^11 + 256*a^2*b^10 - 3456*a^3*
b^9 + 8960*a^4*b^8 - 10880*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*
b^4)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (sin(c + d*x)*(-(a^2
*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3
b^3)^(1/2))/(16(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*
(512*a^2*b^11 - 2560*a^3*b^10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7
+ 4608*a^7*b^6 - 2560*a^8*b^5 + 512*a^9*b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^
4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 -
4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3
 + 6*a^5*b^2)))^(1/2) + (sin(c + d*x)*(48*a*b^10 - 16*b^11 + 1024*a^2*b^9 -
 2208*a^3*b^8 + 1264*a^4*b^7 - 144*a^5*b^6 + 32*a^6*b^5))/(a^4 - 4*a^3*b -
4*a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) -
4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4
 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (200*a*b^9 + 480*a^2*b^8 - 784*a^3*b^7
+ 96*a^4*b^6 + 8*a^5*b^5)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*
(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*
b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^
(1/2) + (sin(c + d*x)*(11*a*b^8 + 27*b^9 - 7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(
1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b +
a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (((((128*a*b^11 + 256*a^2*b^10 -
 3456*a^3*b^9 + 8960*a^4*b^8 - 10880*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5
+ 256*a^8*b^4)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (sin(c + d
x)(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 +
6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2
)))^(1/2)*(512*a^2*b^11 - 2560*a^3*b^10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 256

```
0*a^6*b^7 + 4608*a^7*b^6 - 2560*a^8*b^5 + 512*a...
```

3.410 $\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=249

$$\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3 d}+\frac{(3a^2-6ab+35b^2)\tanh^{-1}(\sin(c+dx))}{8(a-b)^3d}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3 d}+\frac{}{16(a-b)d}$$

```
[Out] 1/8*(3*a^2-6*a*b+35*b^2)*arctanh(sin(d*x+c))/(a-b)^3/d+1/16/(a-b)/d/(1-sin(
d*x+c))^2+1/16*(3*a-11*b)/(a-b)^2/d/(1-sin(d*x+c))-1/16/(a-b)/d/(1+sin(d*x+
c))^2+1/16*(-3*a+11*b)/(a-b)^2/d/(1+sin(d*x+c))-1/2*b^(5/4)*arctanh(b^(1/4)
*sin(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-b^(1/2))^3+1/2*b^(5/4)*arctan(b^(1/
4)*sin(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)+b^(1/2))^3
```

Rubi [A]
time = 0.21, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3302, 1185, 213, 1181, 211, 214}

$$\frac{b^{5/4}\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{(3a^2-6ab+35b^2)\tanh^{-1}(\sin(c+dx))}{8d(a-b)^3}+\frac{3a-11b}{16d(a-b)^2(1-\sin(c+dx))}-\frac{3a-11b}{16d(a-b)^2(\sin(c+dx)+1)}+\frac{1}{16d(a-b)(1-\sin(c+dx))^2}-\frac{1}{16d(a-b)(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(5/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])^3*d) + ((3*a^2 - 6*a*b + 35*b^2)*ArcTanh[Sin[c + d*x]])/(8*(a - b)^3*d
) - (b^(5/4)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] -
 Sqrt[b])^3*d) + 1/(16*(a - b)*d*(1 - Sin[c + d*x])^2) + (3*a - 11*b)/(16*(
a - b)^2*d*(1 - Sin[c + d*x])) - 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) - (3
*a - 11*b)/(16*(a - b)^2*d*(1 + Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a-bx^4)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a-b)(-1+x)^3}+\frac{3a-11b}{16(a-b)^2(-1+x)^2}+\frac{1}{8(a-b)(1+x)^3}+\frac{3a-11b}{16(a-b)^2(1+x)^2}+\frac{-3a^2+6}{8(a-b)^3}\right.\right.}{d}$$

$$= \frac{1}{16(a-b)d(1-\sin(c+dx))^2}+\frac{3a-11b}{16(a-b)^2d(1-\sin(c+dx))}-\frac{1}{16(a-b)d(1+}$$

$$= \frac{(3a^2-6ab+35b^2)\tanh^{-1}(\sin(c+dx))}{8(a-b)^3d}+\frac{1}{16(a-b)d(1-\sin(c+dx))^2}+\frac{}{16(a}$$

$$= \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3d}+\frac{(3a^2-6ab+35b^2)\tanh^{-1}(\sin(c+dx))}{8(a-b)^3d}-\frac{b^{5/4}\tanh}{2a^{3/4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.75, size = 317, normalized size = 1.27

$$\frac{\frac{2(3a^2-6ab+35b^2)\tanh^{-1}(\sin(c+dx))}{(a-b)^3}+\frac{4b^{5/4}\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{4ib^{5/4}\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{4ib^{5/4}\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{4b^{5/4}\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{1}{(a-b)(-1+\sin(c+dx))^2}+\frac{-3a+11b}{(a-b)^2(-1+\sin(c+dx))}-\frac{1}{(a-b)(1+\sin(c+dx))^2}+\frac{-3a+11b}{(a-b)^2(1+\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((2*(3*a^2 - 6*a*b + 35*b^2)*ArcTanh[Sin[c + d*x]])/(a - b)^3 + (4*b^(5/4)*
Log[a^(1/4) - b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^3) + ((4*
I)*b^(5/4)*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[
b])^3) - ((4*I)*b^(5/4)*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sq
rt[a] + Sqrt[b])^3) - (4*b^(5/4)*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])/(a^(3
/4)*(Sqrt[a] - Sqrt[b])^3) + 1/((a - b)*(-1 + Sin[c + d*x])^2) + (-3*a + 11
*b)/((a - b)^2*(-1 + Sin[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (-
3*a + 11*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

Maple [A]
time = 1.92, size = 322, normalized size = 1.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(8*a-8*b)/(1+sin(d*x+c))^2-1/16*(3*a-11*b)/(a-b)^2/(1+sin(d*x+c))
+1/16*(3*a^2-6*a*b+35*b^2)/(a-b)^3*ln(1+sin(d*x+c))+1/2/(8*a-8*b)/(sin(d*x+
c)-1)^2-1/16*(3*a-11*b)/(a-b)^2/(sin(d*x+c)-1)+1/16/(a-b)^3*(-3*a^2+6*a*b-3
5*b^2)*ln(sin(d*x+c)-1)+b^2/(a-b)^3*(1/4*(-3*a-b)*(1/b*a)^(1/4)/a*(ln((sin(
d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))+2*arctan(sin(d*x+c)/(1/b*
a)^(1/4)))-1/4*(-a-3*b)/b/(1/b*a)^(1/4)*(2*arctan(sin(d*x+c)/(1/b*a)^(1/4))
-ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4))))))
```

Maxima [A]
time = 0.51, size = 363, normalized size = 1.46

$$\frac{\frac{4b^2\left(\frac{2\left(b\left(3\sqrt{a}-\sqrt{b}\right)+a^{\frac{3}{2}}-3a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(3\sqrt{a}+\sqrt{b}\right)+a^{\frac{3}{2}}+3a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{a^3-3a^2b+3ab^2-b^3}+\frac{(3a^2-6ab+35b^2)\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}-\frac{(3a^2-6ab+35b^2)\log(\sin(dx+c)-1)}{a^3-3a^2b+3ab^2-b^3}-\frac{2\left((3a-11b)\sin(dx+c)^3-(5a-13b)\sin(dx+c)\right)}{(a^2-2ab+b^2)\sin(dx+c)^4-2(a^2-2ab+b^2)\sin(dx+c)^2+a^2-2ab+b^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/16*(4*b^2*(2*(b*(3*sqrt(a) - sqrt(b)) + a^(3/2) - 3*a*sqrt(b))*arctan(sqr
t(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + (b*(3*sqrt(a) + sqrt(b)) + a^(3/2) + 3*a*sqrt(b))*log((sqrt(b)*sin(
d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt
```

```
(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3 - 3*a^2*b + 3*a*b^2 -
b^3) + (3*a^2 - 6*a*b + 35*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) - (3*a^2 - 6*a*b + 35*b^2)*log(sin(d*x + c) - 1)/(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3) - 2*((3*a - 11*b)*sin(d*x + c)^3 - (5*a - 13*b)*sin(d*x +
c))/((a^2 - 2*a*b + b^2)*sin(d*x + c)^4 - 2*(a^2 - 2*a*b + b^2)*sin(d*x + c
)^2 + a^2 - 2*a*b + b^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3703 vs. $2(207) = 414$.

time = 1.59, size = 3703, normalized size = 14.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/16*(4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b
^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*
b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b
^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 49
5*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a
^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*
a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4
*log(1/2*(a^3*b^4 + 15*a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10
- 3*a^9*b - 3*a^8*b^2 + 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 +
 3*a^3*b^7)*d^3*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 25
5*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b
^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8
- 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) - (3*a^5*b^3 +
46*a^4*b^4 + 60*a^3*b^5 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4
 + 6*b^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^
5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255
*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^
3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 -
 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b
 + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) - 4*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (a^7 -
6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqr
t((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^
10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 -
 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*
a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20
*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(1/2*(a^
3*b^4 + 15*a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10 - 3*a^9*b -
3*a^8*b^2 + 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 + 3*a^3*b^7)*
```

```
d^3*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 +
30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^1
1*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^
9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) + (3*a^5*b^3 + 46*a^4*b^4 +
 60*a^3*b^5 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (
a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d
^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 3
0*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11
*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9
 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^
2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) - 4*(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 + (a^7 - 6*a^6*b + 15
*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 +
 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/(
(a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^
5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 1
2*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 1
5*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(-1/2*(a^3*b^4 + 15*
a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10 - 3*a^9*b - 3*a^8*b^2 +
 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 + 3*a^3*b^7)*d^3*sqrt((a
^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 +
 b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792
*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*
b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) - (3*a^5*b^3 + 46*a^4*b^4 + 60*a^3*b^5
 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 + (a^7 - 6*a^6
*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^
6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 +
b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*
a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b
^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*
b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) + 4*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (a^7 - 6*a^6*b + 15*a^5*b^2 -
20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6
 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*
a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9
*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11
+ a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 -
 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(-1...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a - b*sin(c + d*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(207) = 414$.
time = 0.76, size = 630, normalized size = 2.53

$$-\frac{\frac{8\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3b)+\left(-ab^3\right)^{\frac{1}{4}}\left(3ab^2+b^3\right)\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^4b-3\sqrt{2}a^3b^2+3\sqrt{2}a^2b^3-\sqrt{2}ab^4}+\frac{8\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3b)+\left(-ab^3\right)^{\frac{1}{4}}\left(3ab^2+b^3\right)\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^4b-3\sqrt{2}a^3b^2+3\sqrt{2}a^2b^3-\sqrt{2}ab^4}-\frac{4\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3b)-\left(-ab^3\right)^{\frac{1}{4}}\left(3ab^2+b^3\right)\right)\log\left(\sin(dx+c)^2+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{\sqrt{2}a^4b-3\sqrt{2}a^3b^2+3\sqrt{2}a^2b^3-\sqrt{2}ab^4}+\frac{4\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3b)-\left(-ab^3\right)^{\frac{1}{4}}\left(3ab^2+b^3\right)\right)\log\left(\sin(dx+c)^2-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{-\frac{a}{b}}\right)}{\sqrt{2}a^4b-3\sqrt{2}a^3b^2+3\sqrt{2}a^2b^3-\sqrt{2}ab^4}-\frac{\left(3a^2-6ab+35b^2\right)\log\left(\left|\sin(dx+c)+1\right|\right)}{a^3-3a^2b+3ab^2-b^3}+\frac{\left(3a^2-6ab+35b^2\right)\log\left(\left|\sin(dx+c)-1\right|\right)}{a^3-3a^2b+3ab^2-b^3}+\frac{2\left(3a\sin(dx+c)^3-11b\sin(dx+c)^3-5a\sin(dx+c)+13b\sin(dx+c)\right)}{\left(a^2-2ab+b^2\right)\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/16*(8*((-a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan
(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)
*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2)*a*b^4) + 8*((-a*b^
3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(-1/2*sqrt(2)*(s
qrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^4*b - 3*sqrt
(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2)*a*b^4) - 4*((-a*b^3)^(3/4)*(a + 3
*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(
1/4)*sin(d*x + c) + sqrt(-a/b))/(sqrt(2)*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt
(2)*a^2*b^3 - sqrt(2)*a*b^4) + 4*((-a*b^3)^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)
*(3*a*b^2 + b^3))*log(sin(d*x + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) +
sqrt(-a/b))/(sqrt(2)*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2
)*a*b^4) - (3*a^2 - 6*a*b + 35*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) + (3*a^2 - 6*a*b + 35*b^2)*log(abs(sin(d*x + c) - 1))/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 2*(3*a*sin(d*x + c)^3 - 11*b*sin(d*x + c)^
3 - 5*a*sin(d*x + c) + 13*b*sin(d*x + c))/((a^2 - 2*a*b + b^2)*(sin(d*x + c
)^2 - 1)^2))/d
```

Mupad [B]
time = 20.57, size = 2500, normalized size = 10.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(((((18064*a*b^13 + 256*b^14 + 119760*a^2*b^12 - 275888*a^3*b^11 + 116
624*a^4*b^10 + 28848*a^5*b^9 - 13712*a^6*b^8 + 6768*a^7*b^7 - 720*a^8*b^6)/
(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 -
 56*a^5*b^3 + 28*a^6*b^2)) - (((4096*a*b^15 + 12288*a^2*b^14 - 251904*a^3*b
^13 + 1087488*a^4*b^12 - 2457600*a^5*b^11 + 3440640*a^6*b^10 - 3182592*a^7*
b^9 + 2002944*a^8*b^8 - 872448*a^9*b^7 + 266240*a^10*b^6 - 55296*a^11*b^5 +
 6144*a^12*b^4)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^
5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (sin(c + d*x)*(-(a^3*(a^3*b^5)
```

^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^
2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6
- 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*(8192*a^2*b^15
- 73728*a^3*b^14 + 286720*a^4*b^13 - 614400*a^5*b^12 + 737280*a^6*b^11 - 34
4064*a^7*b^10 - 344064*a^8*b^9 + 737280*a^9*b^8 - 614400*a^10*b^7 + 286720*
a^11*b^6 - 73728*a^12*b^5 + 8192*a^13*b^4))/(16*(a^8 - 8*a^7*b - 8*a*b^7 +
b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(
a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*
b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8
*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) -
(sin(c + d*x)*(256*b^15 - 50464*a^2*b^13 + 190720*a^3*b^12 - 280960*a^4*b^1
1 + 212736*a^5*b^10 - 111296*a^6*b^9 + 57088*a^7*b^8 - 20096*a^8*b^7 + 2304
*a^9*b^6 - 288*a^10*b^5))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 -
56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2
) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^
3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a
^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2))*(-(a^3*(a^3*b^5)^(1
/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(
a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6
*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) - (sin(c + d*x)*(6
802*a*b^12 + 1257*b^13 - 857*a^2*b^11 + 892*a^3*b^10 + 71*a^4*b^9 + 18*a^5*
b^8 + 9*a^6*b^7))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*
b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*
(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(
1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 +
15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*1i - (((18064*a*b^13 + 256*b
^14 + 119760*a^2*b^12 - 275888*a^3*b^11 + 116624*a^4*b^10 + 28848*a^5*b^9 -
13712*a^6*b^8 + 6768*a^7*b^7 - 720*a^8*b^6)/(64*(a^8 - 8*a^7*b - 8*a*b^7 +
b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (
((4096*a*b^15 + 12288*a^2*b^14 - 251904*a^3*b^13 + 1087488*a^4*b^12 - 24576
00*a^5*b^11 + 3440640*a^6*b^10 - 3182592*a^7*b^9 + 2002944*a^8*b^8 - 872448
*a^9*b^7 + 266240*a^10*b^6 - 55296*a^11*b^5 + 6144*a^12*b^4)/(64*(a^8 - 8*a
^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 +
28*a^6*b^2)) + (sin(c + d*x)*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) -
6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(
a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*
a^6*b^3 + 15*a^7*b^2)))^(1/2)*(8192*a^2*b^15 - 73728*a^3*b^14 + 286720*a^4*
b^13 - 614400*a^5*b^12 + 737280*a^6*b^11 - 344064*a^7*b^10 - 344064*a^8*b^9
+ 737280*a^9*b^8 - 614400*a^10*b^7 + 286720*a^11*b^6 - 73728*a^12*b^5 + 81
92*a^13*b^4))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5
+ 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3
*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2)
+ 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*
a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) + (sin(c + d*x)*(256*b^15 - 5046
4*a^2*b^13 + 190720*a^3*b^12 - 280960*a^4*b^11 + 212736*a^5*b^10 - 111296*a

```
^6*b^9 + 57088*a^7*b^8 - 20096*a^8*b^7 + 2304*a^9*b^6 - 288*a^10*b^5))/(16*
(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*
a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^
2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b
^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b
^3 + 15*a^7*b^2)))^(1/2))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*
a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3
*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6
*b^3 + 15*a^7*b^2)))^(1/2) + (sin(c + d*x)*(6802*a*b^12 + 1257*b^13 - 857*a
^2*b^11 + 892*a^3*b^10 + 71*a^4*b^9 + 18*a^5*b^8 + 9*a^6*b^7))/(16*(a^8 - 8
*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3
+ 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 -
20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2
))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 1...
```

3.411 $\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=252

$$-\frac{17x}{16b}-\frac{4(a+b)x}{b^2}-\frac{(a+3b)x}{2b^2}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\tan^{-1}\left(\right.}{2a^{3/}}$$

[Out] -17/16*x/b-4*(a+b)*x/b^2-1/2*(a+3*b)*x/b^2-17/16*cos(d*x+c)*sin(d*x+c)/b/d-1/2*(a+3*b)*cos(d*x+c)*sin(d*x+c)/b^2/d-17/24*cos(d*x+c)^3*sin(d*x+c)/b/d-1/6*cos(d*x+c)^5*sin(d*x+c)/b/d-1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))^(9/2)/a^(3/4)/b^(5/2)/d+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))^(9/2)/a^(3/4)/b^(5/2)/d

Rubi [A]
time = 0.31, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {3303, 1184, 205, 209, 1180, 211}

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}-\frac{(a+3b)\sin(c+dx)\cos(c+dx)}{2b^2d}-\frac{4x(a+b)}{b^2}-\frac{x(a+3b)}{2b^2}-\frac{\sin(c+dx)\cos^5(c+dx)}{6bd}-\frac{17\sin(c+dx)\cos^3(c+dx)}{24bd}-\frac{17\sin(c+dx)\cos(c+dx)}{16bd}-\frac{17x}{16b}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^10/(a - b*Sin[c + d*x]^4),x]

[Out] (-17*x)/(16*b) - (4*(a + b)*x)/b^2 - ((a + 3*b)*x)/(2*b^2) - ((Sqrt[a] - Sqrt[b])^(9/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(5/2)*d) + ((Sqrt[a] + Sqrt[b])^(9/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(5/2)*d) - (17*Cos[c + d*x]*Sin[c + d*x])/(16*b*d) - ((a + 3*b)*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (17*Cos[c + d*x]^3*Sin[c + d*x])/(24*b*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(6*b*d)

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3303

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)^4} - \frac{2}{b(1+x^2)^3} + \frac{-a-3b}{b^2(1+x^2)^2} - \frac{4(a+b)}{b^2(1+x^2)} + \frac{5a^2+10ab+b^2+4(a^2-b^2)x^2}{b^2(a+2ax^2+(a-b)x^4)}\right) dx, \right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{5a^2+10ab+b^2+4(a^2-b^2)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{b^2 d} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4}\,dx, x, \tan(c\right.}{bd}$$

$$= -\frac{4(a+b)x}{b^2} - \frac{(a+3b)\cos(c+dx)\sin(c+dx)}{2b^2 d} - \frac{\cos^3(c+dx)\sin(c+dx)}{2bd} - \frac{\cos^5}{}$$

$$= -\frac{4(a+b)x}{b^2} - \frac{(a+3b)x}{2b^2} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d} +$$

$$= -\frac{3x}{4b} - \frac{4(a+b)x}{b^2} - \frac{(a+3b)x}{2b^2} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right.}{2a^{3/4}b^{5/2}d}$$

$$= -\frac{17x}{16b} - \frac{4(a+b)x}{b^2} - \frac{(a+3b)x}{2b^2} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+d}{\sqrt[4]{a}}\right.}{2a^{3/4}b^{5/2}d}$$

Mathematica [A]
time = 0.63, size = 233, normalized size = 0.92

$$-\frac{36b(24a+35b)(c+dx) - \frac{96\left(\sqrt{a}+\sqrt{b}\right)^5\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}} - \frac{96\left(\sqrt{a}-\sqrt{b}\right)^5\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}} + 3b(16a+95b)\sin(2(c+dx)) + 21b^2\sin(4(c+dx)) + b^2\sin(6(c+dx))}{192b^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/192*(36*b*(24*a + 35*b)*(c + d*x) - (96*(Sqrt[a] + Sqrt[b])^5*Sqrt[b]*Ar
cTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a
]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - (96*(Sqrt[a] - Sqrt[b])^5*Sqrt[b]*ArcTanh[((
Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt
[-a + Sqrt[a]*Sqrt[b]]) + 3*b*(16*a + 95*b)*Sin[2*(c + d*x)] + 21*b^2*Sin[4
*(c + d*x)] + b^2*Sin[6*(c + d*x)])/(b^3*d)
```

Maple [A]
time = 1.44, size = 258, normalized size = 1.02

method	result
derivativedivides	$\frac{(a-b)\left(\frac{\left(4a\sqrt{ab}+4\sqrt{ab}\,b+a^2+6ab+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(4a\sqrt{ab}+4\sqrt{ab}\,b-a^2-6ab-\cdots\right.}{2\sqrt{ab}\sqrt{(\cdots}}\right)}{b^2}$
default	$\frac{(a-b)\left(\frac{\left(4a\sqrt{ab}+4\sqrt{ab}\,b+a^2+6ab+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(4a\sqrt{ab}+4\sqrt{ab}\,b-a^2-6ab-\cdots\right.}{2\sqrt{ab}\sqrt{(\cdots}}\right)}{b^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*(a-b)*(1/2*(4*a*(a*b)^(1/2)+4*(a*b)^(1/2)*b+a^2+6*a*b+b^2)/(a*b)
^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2
)-a)*(a-b))^(1/2))+1/2*(4*a*(a*b)^(1/2)+4*(a*b)^(1/2)*b-a^2-6*a*b-b^2)/(a*b
)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)
+a)*(a-b))^(1/2)))-1/b^2*(((1/2*a+41/16*b)*tan(d*x+c)^5+(a+35/6*b)*tan(d*x+
c)^3+(1/2*a+55/16*b)*tan(d*x+c))/(tan(d*x+c)^2+1)^3+3/16*(24*a+35*b)*arctan
(tan(d*x+c))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/192*(192*b^2*d*integrate(-4*(4*(a^2*b + 10*a*b^2 + 5*b^3)*cos(6*d*x + 6*
c)^2 + 4*(72*a^3 + 53*a^2*b - 54*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(a^2
*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c)^2 + 4*(a^2*b + 10*a*b^2 + 5*b^3)*si
```

```
n(6*d*x + 6*c)^2 + 4*(72*a^3 + 53*a^2*b - 54*a*b^2 + 9*b^3)*sin(4*d*x + 4*c
)^2 + 2*(8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*(a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c)^2 - ((a^2*b + 10*a*b
^2 + 5*b^3)*cos(6*d*x + 6*c) + 2*(9*a^2*b + 10*a*b^2 - 3*b^3)*cos(4*d*x + 4
*c) + (a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a^2*
b + 10*a*b^2 + 5*b^3 - 2*(8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*cos(4*d*x
+ 4*c) - 8*(a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) -
2*(9*a^2*b + 10*a*b^2 - 3*b^3 - (8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*cos
(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*
c) - ((a^2*b + 10*a*b^2 + 5*b^3)*sin(6*d*x + 6*c) + 2*(9*a^2*b + 10*a*b^2 -
 3*b^3)*sin(4*d*x + 4*c) + (a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 2*((8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*sin(4*d*x + 4*c)
 + 4*(a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(b^4*co
s(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 + 16*b^4*cos(2*d*x + 2*c)^2 +
b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*c)^2 + 16*b^4*sin(2*d*x + 2*c
)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*cos(
4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^2 + 16*
(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^4*cos(6*d*x +
6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))
*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*c
os(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^3 - 3*b^4 - 4*(8*a*b^3 - 3*b^4
)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^4*sin(6*d*x + 6*c) + 2*b^4*si
n(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*
(2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(6*d*x + 6
*c)), x) + 36*(24*a + 35*b)*d*x + b*sin(6*d*x + 6*c) + 21*b*sin(4*d*x + 4*c
) + 3*(16*a + 95*b)*sin(2*d*x + 2*c))/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2948 vs. $2(200) = 400$.
time = 1.54, size = 2948, normalized size = 11.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^2*d*sqrt((a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21
816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)
/(a^3*b^9*d^4)) - a^4 - 36*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d
^2))*log(9/4*a^8 + 12*a^7*b - 39*a^6*b^2 + 143/2*a^4*b^4 - 52*a^3*b^5 - 3*a
^2*b^6 + 8*a*b^7 + 1/4*b^8 - 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286*a^4*
b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 + 1/2*(4*(a
^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*
b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^
9*d^4))*cos(d*x + c)*sin(d*x + c) + (9*a^7*b^2 + 138*a^6*b^3 + 639*a^5*b^4
```

+ 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x +
c))*sqrt((a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^
3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*
d^4)) - a^4 - 36*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2)) + 1/4
(2(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2*cos(d*x + c
)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2)*sqrt((81
*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3
*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))) - 6*b^2*d*sqrt((a*b^5
*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b
^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) - a^4 - 3
6*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2))*log(9/4*a^8 + 12*a^7
*b - 39*a^6*b^2 + 143/2*a^4*b^4 - 52*a^3*b^5 - 3*a^2*b^6 + 8*a*b^7 + 1/4*b^
8 - 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286*a^4*b^4 - 208*a^3*b^5 - 12*a^
2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 - 1/2*(4*(a^4*b^7 + a^3*b^8)*d^3*sqr
t((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 924
0*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))*cos(d*x + c)*sin(
d*x + c) + (9*a^7*b^2 + 138*a^6*b^3 + 639*a^5*b^4 + 876*a^4*b^5 + 343*a^3*b
^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^5*d^2*sqrt(
(81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*
a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) - a^4 - 36*a^3*b -
126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2)) + 1/4*(2*(a^6*b^4 - 4*a^5*b^5
+ 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2*cos(d*x + c)^2 - (a^6*b^4 - 4*a^5*b^
5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2)*sqrt((81*a^8 + 1512*a^7*b + 9324*
a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*
a*b^7 + b^8)/(a^3*b^9*d^4))) + 6*b^2*d*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512
*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548
*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 +
84*a*b^3 + 9*b^4)/(a*b^5*d^2))*log(-9/4*a^8 - 12*a^7*b + 39*a^6*b^2 - 143/2
*a^4*b^4 + 52*a^3*b^5 + 3*a^2*b^6 - 8*a*b^7 - 1/4*b^8 + 1/4*(9*a^8 + 48*a^7
*b - 156*a^6*b^2 + 286*a^4*b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)
*cos(d*x + c)^2 + 1/2*(4*(a^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b
+ 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^
6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))*cos(d*x + c)*sin(d*x + c) - (9*a^7*b^2 +
138*a^6*b^3 + 639*a^5*b^4 + 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8
)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b +
9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6
+ 72*a*b^7 + b^8)/(a^3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 + 84*a*b^3
+ 9*b^4)/(a*b^5*d^2)) + 1/4*(2*(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^
7 + a^2*b^8)*d^2*cos(d*x + c)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*
b^7 + a^2*b^8)*d^2)*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^
3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*
d^4))) - 6*b^2*d*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2
+ 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 +
b^8)/(a^3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 + 84*a*b^3 + 9*b^4)/(a*b
^5*d^2))*log(-9/4*a^8 - 12*a^7*b + 39*a^6*b^2 - 143/2*a^4*b^4 + 52*a^3*b^5

```
+ 3*a^2*b^6 - 8*a*b^7 - 1/4*b^8 + 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286
*a^4*b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 - 1/2*
(4*(a^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816
*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a
^3*b^9*d^4))*cos(d*x + c)*sin(d*x + c) - (9*a^7*b^2 + 138*a^6*b^3 + 639*a^5
*b^4 + 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d
*x + c))*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*
a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^
3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 + 84*a*b^3 + 9*b^4)/(a*b^5*d^2))
 + 1/4*(2*(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2*cos(d
*x + c)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2)*sq
rt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 92
40*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(200) = 400.
time = 0.87, size = 896, normalized size = 3.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/48*(24*(15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 62*sqrt(a^2 - a*b
+ sqrt(a*b)*(a - b))*a^2*b^3 - 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4
 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^5 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*a^4 - 24*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
3*b + 46*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 40*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b + sqrt(a*b)
*(a - b))*sqrt(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c
)/sqrt((a*b^2 + sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-
a + b)/(3*a^5*b^3 - 12*a^4*b^4 + 14*a^3*b^5 - 4*a^2*b^6 - a*b^7) + 24*(15*s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 62*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*a^2*b^3 - 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 -
a*b - sqrt(a*b)*(a - b))*b^5 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a^4 + 24*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 46*sqrt(
```

```
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 40*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt
(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2
- sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^5*
b^3 - 12*a^4*b^4 + 14*a^3*b^5 - 4*a^2*b^6 - a*b^7) - 9*(d*x + c)*(24*a + 35
*b)/b^2 - (24*a*tan(d*x + c)^5 + 123*b*tan(d*x + c)^5 + 48*a*tan(d*x + c)^3
 + 280*b*tan(d*x + c)^3 + 24*a*tan(d*x + c) + 165*b*tan(d*x + c))/((tan(d*x
 + c)^2 + 1)^3*b^2))/d
```

Mupad [B]
time = 18.80, size = 2500, normalized size = 9.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^10/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan(((((tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11
 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534
*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6) + (3*
(((9873*a*b^13)/16 + 8*b^14 + (198963*a^2*b^12)/16 - (13467*a^3*b^11)/8 - (
240165*a^4*b^10)/8 + (68805*a^5*b^9)/16 + (307047*a^6*b^8)/16 - 1929*a^7*b^
7 - 2766*a^8*b^6 - 152*a^9*b^5)/b^8 + (3*((3*((64*a*b^15 + 1216*a^2*b^14 -
2064*a^3*b^13 - 480*a^4*b^12 + 1968*a^5*b^11 - 704*a^6*b^10)/b^8 - (3*tan(c
 + d*x)*(a*24i + b*35i)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 +
 49152*a^5*b^10))/(2048*b^8))*(a*24i + b*35i))/(32*b^2) - (tan(c + d*x)*(61
7264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9
- 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6
))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i)*3i
)/(32*b^2) + (((tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 1115
3*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 -
387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6)
 - (3*(((9873*a*b^13)/16 + 8*b^14 + (198963*a^2*b^12)/16 - (13467*a^3*b^11)
/8 - (240165*a^4*b^10)/8 + (68805*a^5*b^9)/16 + (307047*a^6*b^8)/16 - 1929*
a^7*b^7 - 2766*a^8*b^6 - 152*a^9*b^5)/b^8 + (3*((3*((64*a*b^15 + 1216*a^2*b
^14 - 2064*a^3*b^13 - 480*a^4*b^12 + 1968*a^5*b^11 - 704*a^6*b^10)/b^8 + (3
*tan(c + d*x)*(a*24i + b*35i)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*
b^11 + 49152*a^5*b^10))/(2048*b^8))*(a*24i + b*35i))/(32*b^2) + (tan(c + d*
x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^
4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(
64*b^6))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*3
5i)*3i)/(32*b^2))/(((92769*a*b^11)/64 - (39*a^11*b)/8 + 9*a^12 - (11865*b^1
2)/64 - (76467*a^2*b^10)/16 + (133839*a^3*b^9)/16 - (243927*a^4*b^8)/32 + (
58743*a^5*b^7)/32 + (50967*a^6*b^6)/16 - (52227*a^7*b^5)/16 + (61119*a^8*b^
4)/64 + (12729*a^9*b^3)/64 - (1137*a^10*b^2)/8)/b^8 - (3*((tan(c + d*x)*(12
```

3962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6) + (3*(((9873*a*b^13)/16 + 8*b^14 + (198963*a^2*b^12)/16 - (13467*a^3*b^11)/8 - (240165*a^4*b^10)/8 + (68805*a^5*b^9)/16 + (307047*a^6*b^8)/16 - 1929*a^7*b^7 - 2766*a^8*b^6 - 152*a^9*b^5)/b^8 + (3*((3*((64*a*b^15 + 1216*a^2*b^14 - 2064*a^3*b^13 - 480*a^4*b^12 + 1968*a^5*b^11 - 704*a^6*b^10)/b^8 - (3*tan(c + d*x)*(a*24i + b*35i)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(2048*b^8))*(a*24i + b*35i))/(32*b^2) - (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2) + (3*((tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6) - (3*(((9873*a*b^13)/16 + 8*b^14 + (198963*a^2*b^12)/16 - (13467*a^3*b^11)/8 - (240165*a^4*b^10)/8 + (68805*a^5*b^9)/16 + (307047*a^6*b^8)/16 - 1929*a^7*b^7 - 2766*a^8*b^6 - 152*a^9*b^5)/b^8 + (3*((3*((64*a*b^15 + 1216*a^2*b^14 - 2064*a^3*b^13 - 480*a^4*b^12 + 1968*a^5*b^11 - 704*a^6*b^10)/b^8 + (3*tan(c + d*x)*(a*24i + b*35i)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(2048*b^8))*(a*24i + b*35i))/(32*b^2) + (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2)))*(a*24i + b*35i)*3i)/(16*b^2*d) - (atan(((((78984*a*b^13 + 1024*b^14 + 1591704*a^2*b^12 - 215472*a^3*b^11 - 3842640*a^4*b^10 + 550440*a^5*b^9 + 2456376*a^6*b^8 - 246912*a^7*b^7 - 354048*a^8*b^6 - 19456*a^9*b^5)/(128*b^8) + (((8192*a*b^15 + 155648*a^2*b^14 - 264192*a^3*b^13 - 61440*a^4*b^12 + 251904*a^5*b^11 - 90112*a^6*b^10)/(128*b^8) - (tan(c + d*x)*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) - (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^...

3.412 $\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=186

$$-\frac{11x}{8b}-\frac{(a+3b)x}{b^2}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+
$$

```
[Out] -11/8*x/b-(a+3*b)*x/b^2-11/8*cos(d*x+c)*sin(d*x+c)/b/d-1/4*cos(d*x+c)^3*sin
(d*x+c)/b/d+1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)
-b^(1/2))^(7/2)/a^(3/4)/b^2/d+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)
/a^(1/4))*(a^(1/2)+b^(1/2))^(7/2)/a^(3/4)/b^2/d
```

Rubi [A]
time = 0.21, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3303, 1184, 205, 209, 1180, 211}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}-\frac{x(a+3b)}{b^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}-\frac{11\sin(c+dx)\cos(c+dx)}{8bd}-\frac{11x}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-11*x)/(8*b) - ((a + 3*b)*x)/b^2 + ((Sqrt[a] - Sqrt[b])^(7/2)*ArcTan[(Sqrt
[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^2*d) + ((Sqrt[a] +
 Sqrt[b])^(7/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*
a^(3/4)*b^2*d) - (11*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]^3*S
in[c + d*x])/(4*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3303

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)^3} - \frac{2}{b(1+x^2)^2} + \frac{-a-3b}{b^2(1+x^2)} + \frac{a^2+6ab+b^2+(a-b)(a+3b)x^2}{b^2(a+2ax^2+(a-b)x^4)}\right)\,dx, x, \tan(c+\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+6ab+b^2+(a-b)(a+3b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{b^2d} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, \tan(\right.}{bd}$$

$$= -\frac{(a+3b)x}{b^2} - \frac{\cos(c+dx)\sin(c+dx)}{bd} - \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\left(\left(\sqrt{a}-\right.\right.}{}$$

$$= -\frac{x}{b} - \frac{(a+3b)x}{b^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d} + \frac{\left(\sqrt{a}\right.}{}$$

$$= -\frac{11x}{8b} - \frac{(a+3b)x}{b^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d} + \frac{\left(\sqrt{a}\right.}{}$$

Mathematica [A]
time = 0.48, size = 200, normalized size = 1.08

$$-\frac{4(8a+35b)(c+dx) - \frac{16\left(\sqrt{a}+\sqrt{b}\right)^4\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}} + \frac{16\left(\sqrt{a}-\sqrt{b}\right)^4\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}} + 24b\sin(2(c+dx)) + b\sin(4(c+dx))}{32b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/32*(4*(8*a + 35*b)*(c + d*x) - (16*(Sqrt[a] + Sqrt[b])^4*ArcTan[((Sqrt[a
] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sq
rt[a]*Sqrt[b]]) + (16*(Sqrt[a] - Sqrt[b])^4*ArcTanh[((Sqrt[a] - Sqrt[b])*Ta
n[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]
]) + 24*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(b^2*d)
```

Maple [A]
time = 1.08, size = 224, normalized size = 1.20

method	result

derivativedivides	$\frac{\frac{(a-b)\left(\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b+3ab+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b-3ab-b^2\right)\arctan\left(\right.}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}\right.}}\right.}{b^2}}{d}$
default	$\frac{\frac{(a-b)\left(\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b+3ab+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b-3ab-b^2\right)\arctan\left(\right.}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}\right.}}\right.}{b^2}}{d}$
risch	$-\frac{35x}{8b}-\frac{ax}{b^2}+\frac{3i\,e^{2i(dx+c)}}{8bd}-\frac{3i\,e^{-2i(dx+c)}}{8bd}+\Bigg(\_R=\text{RootOf}\left(256a^3b^8d^4\_Z^4+(32a^5b^4d^2+672a^4b^5d^2+1120a^3b^6d^2\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*(a-b)*(1/2*(a*(a*b)^(1/2)+3*(a*b)^(1/2)*b+3*a*b+b^2)/(a*b)^(1/2)
/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(
a-b))^(1/2))+1/2*(a*(a*b)^(1/2)+3*(a*b)^(1/2)*b-3*a*b-b^2)/(a*b)^(1/2)/(((a
*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(
1/2)))-1/b^2*((11/8*b*tan(d*x+c)^3+13/8*b*tan(d*x+c))/(tan(d*x+c)^2+1)^2+1/
8*(35*b+8*a)*arctan(tan(d*x+c))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/32*(32*b^2*d*integrate(-16*(4*(a*b^2 + b^3)*cos(6*d*x + 6*c)^2 + 2*(8*a^
3 + 29*a^2*b - 20*a*b^2 + 3*b^3)*cos(4*d*x + 4*c)^2 + 4*(a*b^2 + b^3)*cos(2
*d*x + 2*c)^2 + 4*(a*b^2 + b^3)*sin(6*d*x + 6*c)^2 + 2*(8*a^3 + 29*a^2*b -
```

```
20*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)^2 + 2*(10*a^2*b + 13*a*b^2 - 5*b^3)*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(a*b^2 + b^3)*sin(2*d*x + 2*c)^2 - ((a*b^
2 + b^3)*cos(6*d*x + 6*c) + (a^2*b + 4*a*b^2 - b^3)*cos(4*d*x + 4*c) + (a*b
^2 + b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a*b^2 + b^3 - 2*(10*a^2*b +
 13*a*b^2 - 5*b^3)*cos(4*d*x + 4*c) - 8*(a*b^2 + b^3)*cos(2*d*x + 2*c))*cos
(6*d*x + 6*c) - (a^2*b + 4*a*b^2 - b^3 - 2*(10*a^2*b + 13*a*b^2 - 5*b^3)*co
s(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b^2 + b^3)*cos(2*d*x + 2*c) - ((a*b^2
 + b^3)*sin(6*d*x + 6*c) + (a^2*b + 4*a*b^2 - b^3)*sin(4*d*x + 4*c) + (a*b^
2 + b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((10*a^2*b + 13*a*b^2 - 5*b
^3)*sin(4*d*x + 4*c) + 4*(a*b^2 + b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/
(b^4*cos(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 + 16*b^4*cos(2*d*x + 2*
c)^2 + b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*c)^2 + 16*b^4*sin(2*d*
x + 2*c)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^
4)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^
2 + 16*(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^4*cos(6
*d*x + 6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x
+ 4*c))*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3
*b^4)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^3 - 3*b^4 - 4*(8*a*b^3
- 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^4*sin(6*d*x + 6*c) + 2
*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c
) + 16*(2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(6*
d*x + 6*c)), x) + 4*(8*a + 35*b)*d*x + b*sin(4*d*x + 4*c) + 24*b*sin(2*d*x
+ 2*c))/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2433 vs. $2(144) = 288$.
time = 1.03, size = 2433, normalized size = 13.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*d*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*
a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2*b + 3
5*a*b^2 + 7*b^3)/(a*b^4*d^2))*log(7/4*a^6 + 7/2*a^5*b - 63/4*a^4*b^2 + 9*a^
3*b^3 + 25/4*a^2*b^4 - 9/2*a*b^5 - 1/4*b^6 - 1/4*(7*a^6 + 14*a^5*b - 63*a^4
*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 + 1/2*((a^4
*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^
3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*cos(d*x + c)*sin(d*x + c)
- (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2*b^5 + a*b^6)*d*cos(d*x +
c)*sin(d*x + c))*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 +
 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2
*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b
^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b
```

^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) - b^2*d*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2))*log(7/4*a^6 + 7/2*a^5*b - 63/4*a^4*b^2 + 9*a^3*b^3 + 25/4*a^2*b^4 - 9/2*a*b^5 - 1/4*b^6 - 1/4*(7*a^6 + 14*a^5*b - 63*a^4*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 - 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*cos(d*x + c)*sin(d*x + c) - (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) + b^2*d*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) - a^3 - 21*a^2*b - 35*a*b^2 - 7*b^3)/(a*b^4*d^2))*log(-7/4*a^6 - 7/2*a^5*b + 63/4*a^4*b^2 - 9*a^3*b^3 - 25/4*a^2*b^4 + 9/2*a*b^5 + 1/4*b^6 + 1/4*(7*a^6 + 14*a^5*b - 63*a^4*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*cos(d*x + c)*sin(d*x + c) + (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) - a^3 - 21*a^2*b - 35*a*b^2 - 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) - b^2*d*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) - a^3 - 21*a^2*b - 35*a*b^2 - 7*b^3)/(a*b^4*d^2))*log(-7/4*a^6 - 7/2*a^5*b + 63/4*a^4*b^2 - 9*a^3*b^3 - 25/4*a^2*b^4 + 9/2*a*b^5 + 1/4*b^6 + 1/4*(7*a^6 + 14*a^5*b - 63*a^4*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 - 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*cos(d*x + c)*sin(d*x + c) + (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) - a^3 - 21*a^2*b - 35*a*b^2 - 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) - (8*a + 35*b)*d*x - (2*b*cos(d*x + c)^3 + 11*b*cos(d*x + c))*sin(d*x + c))/(b^2*d)

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 836 vs. 2(144) = 288.
time = 0.86, size = 836, normalized size = 4.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(4*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4 + 12*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*a^3*b - 34*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 - 12
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*b^4 - 12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 + 12*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 28*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^2 + 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2 +
sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^5*b^
2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6) + 4*(3*sqrt(a^2 - a*b - sq
rt(a*b)*(a - b))*a^4 + 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b - 34*sq
rt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 12*sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4 + 12*sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^2*b - 28*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2
- 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(pi*floor((d*x + c)/
pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2 - sqrt(a^2*b^4 - (a*b^2 - b^3)*
a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 -
 4*a^2*b^5 - a*b^6) - (d*x + c)*(8*a + 35*b)/b^2 - (11*tan(d*x + c)^3 + 13*
tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*b))/d
```

Mupad [B]
time = 17.46, size = 2500, normalized size = 13.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a - b*sin(c + d*x)^4),x)
```

[Out] - (atan(((((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 352
96*a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/
(64*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4*b^
9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(-(7*a^3*(a^3*b
^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5
*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(
1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16
b^4))(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*
b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^
(1/2))/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a
^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 23
04*a^7*b^4))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a
^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*
a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*
(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*
(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (tan(c +
d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^
3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^
4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6
 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/
2))/(16*a^3*b^8))^(1/2)*1i - (((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9
- 17552*a*b^10 + 35296*a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7
*b^4 + 768*a^8*b^3)/(64*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3
*b^10 + 69632*a^4*b^9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) + (tan(c +
d*x)*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^
6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1
/2))/(16*a^3*b^8))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 +
 12288*a^5*b^8))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) +
 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) +
 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) - (tan(c + d*x)*(256*a*b^10
+ 256*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6
- 29696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(
a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(
a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2))*(-(7*a^3*(a
^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 +
 a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8
))^(1/2) - (tan(c + d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 2149
9*a^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1
721*a^7*b^2))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*
a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35
*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*1i)/((11696*a*b^8 + 1247*a^8*b
- 344*a^9 - 1505*b^9 - 39388*a^2*b^7 + 74648*a^3*b^6 - 86086*a^4*b^5 + 6020
0*a^5*b^4 - 22876*a^6*b^3 + 2408*a^7*b^2)/(32*b^5) + (((373728*a^3*b^8 - 25
6*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 35296*a^4*b^7 - 240464*a^5*b^6 + 2
9040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/(64*b^5) - (((4096*a*b^12 + 532

48*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4*b^9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 35296*a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/(64*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4*b^9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*(...

3.413 $\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{5x}{2b}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}$$

```
[Out] -5/2*x/b-1/2*cos(d*x+c)*sin(d*x+c)/b/d-1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*t
an(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))^(5/2)/a^(3/4)/b^(3/2)/d+1/2*arctan((a^
(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))^(5/2)/a^(3/4)/b^
(3/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3303, 1184, 205, 209, 1180, 211}

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}-\frac{5x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-5*x)/(2*b) - ((Sqrt[a] - Sqrt[b])^(5/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(3/2)*d) + ((Sqrt[a] + Sqrt[b])^(5/2)*A
rcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(3/2)*d
) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3303

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)^2}-\frac{2}{b(1+x^2)}+\frac{3a+b+2(a-b)x^2}{b(a+2ax^2+(a-b)x^4)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{bd} + \frac{\text{Subst}\left(\int \frac{3a+b+2(a-b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+d\right.}{bd}$$

$$= -\frac{2x}{b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2bd} + \frac{\left(2a-2b\right.}{}$$

$$= -\frac{5x}{2b} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\text{t}}{}$$

Mathematica [A]
time = 0.33, size = 194, normalized size = 1.25

$$\frac{-10b(c+dx)+\frac{2\left(\sqrt{a}+\sqrt{b}\right)^3\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{2\left(\sqrt{a}-\sqrt{b}\right)^3\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}-b\sin(2(c+dx))}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-10*b*(c + d*x) + (2*(Sqrt[a] + Sqrt[b])^3*Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt
[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sq
rt[b]]) + (2*(Sqrt[a] - Sqrt[b])^3*Sqrt[b]*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan
[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]
) - b*Sin[2*(c + d*x)])/(4*b^2*d)
```

Maple [A]
time = 0.78, size = 179, normalized size = 1.15

method	result
derivativedivides	$\frac{\frac{(a-b)\left(\frac{\left(-a+2\sqrt{ab}-b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(a+2\sqrt{ab}+b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-}}\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{b}}{d}$
default	$\frac{\frac{(a-b)\left(\frac{\left(-a+2\sqrt{ab}-b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(a+2\sqrt{ab}+b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-}}\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{b}}{d}$
risch	$-\frac{5x}{2b}+\frac{i e^{2i(dx+c)}}{8bd}-\frac{i e^{-2i(dx+c)}}{8bd}+\left(\sum_{\_R=\text{RootOf}\left(256a^3b^6d^4\_Z^4+(32a^4b^3d^2+320a^3b^4d^2+160a^2b^5d^2)\_Z^2+a^5-5\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b*(a-b)*(1/2*(-a+2*(a*b)^(1/2)-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b)
)^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(a+2*(a*
b)^(1/2)+b)/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*
x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2)))-1/b*(1/2*tan(d*x+c)/(tan(d*x+c)^2+1)+5
/2*arctan(tan(d*x+c))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b*d*integrate(-4*(4*(a*b + 3*b^2)*cos(6*d*x + 6*c)^2 + 4*(40*a^2 -
23*a*b + 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(a*b + 3*b^2)*cos(2*d*x + 2*c)^2 + 4
*(a*b + 3*b^2)*sin(6*d*x + 6*c)^2 + 4*(40*a^2 - 23*a*b + 3*b^2)*sin(4*d*x +
 4*c)^2 + 2*(8*a^2 + 41*a*b - 13*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*(a*b + 3*b^2)*sin(2*d*x + 2*c)^2 - ((a*b + 3*b^2)*cos(6*d*x + 6*c) + 2*(5*
a*b - b^2)*cos(4*d*x + 4*c) + (a*b + 3*b^2)*cos(2*d*x + 2*c))*cos(8*d*x + 8
*c) - (a*b + 3*b^2 - 2*(8*a^2 + 41*a*b - 13*b^2)*cos(4*d*x + 4*c) - 8*(a*b
+ 3*b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*(5*a*b - b^2 - (8*a^2 + 41*
a*b - 13*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b + 3*b^2)*cos(2*d*x
+ 2*c) - ((a*b + 3*b^2)*sin(6*d*x + 6*c) + 2*(5*a*b - b^2)*sin(4*d*x + 4*c)
 + (a*b + 3*b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^2 + 41*a*b -
13*b^2)*sin(4*d*x + 4*c) + 4*(a*b + 3*b^2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x
+ 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(
2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*
b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^
2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6
*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x
+ 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3
*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2
- 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2
*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c
) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*
d*x + 6*c)), x) + 10*d*x + sin(2*d*x + 2*c))/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(111) = 222$.
time = 0.77, size = 1751, normalized size = 11.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")

[Out] 1/8*(b*d*sqrt((a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2))*log(5/4*a^4 - 7/2*a^2*b^2 + 2*a*b^3 + 1/4*b^4 - 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(d*x + c)^2 + 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) + (5*a^4*b + 15*a^3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) - b*d*sqrt((a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2))*log(5/4*a^4 - 7/2*a^2*b^2 + 2*a*b^3 + 1/4*b^4 - 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(d*x + c)^2 - 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) + (5*a^4*b + 15*a^3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) + b*d*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2))*log(-5/4*a^4 + 7/2*a^2*b^2 - 2*a*b^3 - 1/4*b^4 + 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(d*x + c)^2 + 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) - (5*a^4*b + 15*a^3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) - b*d*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2))*log(-5/4*a^4 + 7/2*a^2*b^2 - 2*a*b^3 - 1/4*b^4 + 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(d*x + c)^2 - 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) - (5*a^4*b + 15*a^3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) - 20*d*x - 4*cos(d*x + c)*sin(d*x + c))/(b*d)

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 995 vs. 2(111) = 222.
time = 0.80, size = 995, normalized size = 6.42

$$-\frac{\frac{5(dx+c)}{b}+\frac{\left(2\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}\,a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}\,ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}\,b^2\right)b^2|-a+b|-\left(9\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,a^3b-15\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,a^2b^2-9\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,ab^3-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,b^4\right)|-a+b||b|+\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}\,a^3b-3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}\,a^2b^2-7\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}\,ab^3-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}\,b^4\right)|-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{ab+\sqrt{a^2b^2-(ab-b^2)ab}}{ab-b^2}}}\right)\right)}{(3a^5b^2-12a^4b^3+14a^3b^4-4a^2b^5-ab^6)|b|}-\frac{\left(2\left(3\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}\,a^2-6\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}\,ab-\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}\,b^2\right)b^2|-a+b|+\left(9\sqrt{a^2-ab-\sqrt{ab}(a-b)}\,a^3b-15\sqrt{a^2-ab-\sqrt{ab}(a-b)}\,a^2b^2-9\sqrt{a^2-ab-\sqrt{ab}(a-b)}\,ab^3-\sqrt{a^2-ab-\sqrt{ab}(a-b)}\,b^4\right)|-a+b||b|+\left(3\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}\,a^3b-3\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}\,a^2b^2-7\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}\,ab^3-\sqrt{a^2-ab-\sqrt{ab}(a-b)}\sqrt{ab}\,b^4\right)|-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{ab-\sqrt{a^2b^2-(ab-b^2)ab}}{ab-b^2}}}\right)\right)}{(3a^5b^2-12a^4b^3+14a^3b^4-4a^2b^5-ab^6)|b|}+\frac{\tan(dx+c)}{(\tan(dx+c)^2+1)b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)/b + (2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a
^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (9*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*a^3*b - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 - 9
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*b^4)*abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt
(a*b)*a^3*b - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 7*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + a
rctan(tan(d*x + c)/sqrt((a*b + sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^2)
)))/((3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6)*abs(b)) - (2
*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt
(a*b)*b^2)*b^2*abs(-a + b) + (9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b -
 15*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 9*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4)*abs(-a + b)*a
bs(b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 3*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 7*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt(
(a*b - sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^2))))/((3*a^5*b^2 - 12*a^4
*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6)*abs(b)) + tan(d*x + c)/((tan(d*x + c
)^2 + 1)*b))/d
```

Mupad [B]
time = 18.04, size = 2500, normalized size = 16.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(a - b*sin(c + d*x)^4),x)

[Out] (atan((a^4*b^8*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*240i - a^3*b^9*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*108i - a^5*b^7*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*80i - a^6*b^6*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*120i + a^7*b^5*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*60i + a^8*b^4*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*8i - a^3*b^11*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*64i + a^4*b^10*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*128i + a^5*b^9*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*6080i + a^6*b^8*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*4032i + a^7*b^7*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*320i + a^5*b^11*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(5/2)*3072i + a^6*b^10*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(5/2)*3072i)/(55*a^2*b^9*cos(c + d*x) + 540*a^3*b^8*cos(c + d*x) + 1035*a^4*b^7*cos(c + d*x) + 45*a^5*b^6*cos(c + d*x) + a^6*b^5*cos(c + d*x) + 110*a^7*b^4*cos(c + d*x) + 5*a^8*b^3*cos(c + d*x) + 50*a^6*cos(c + d*x)*(a^3*b^7)^(1/2) + 10*b^6*cos(c + d*x)*(a^3*b^7)^(1/2) + a*b^10*cos(c + d*x) + 195*a*b^5*cos(c + d*x)*(a^3*b^7)^(1/2) + 75*a^5*b*cos(c + d*x)*(a^3*b^7)^(1/2) + 1002*a^2*b^4*cos(c + d*x)*(a^3*b^7)^(1/2) + 490*a^3*b^3*cos(c + d*x)*(a^3*b^7)^(1/2) - 30*a^4*b^2*cos(c + d*x)*(a^3*b^7)^(1/2)))*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*2i)/d + (atan((a^4*b^8*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*240i - a^3*b^9*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*108i - a^5*b^7*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 -

```
a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*80i - a^6*b^6*sin(c +
 d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^
4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*120i + a^7*b^5*si
n(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a
^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*60i + a^8*b^
4*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 -
10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*8i - a^3
*b^11*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^
5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*64i
+ a^4*b^10*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a
^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)
*128i + a^5*b^9*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2)
- 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^
(3/2)*6080i + a^6*b^8*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^
(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*
b^6))^(3/2)*4032i + a^7*b^7*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3
*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(1
6*a^3*b^6))^(3/2)*320i + a^5*b^11*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^
2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/
2))/(16*a^3*b^6))^(5/2)*3072i + a^6*b^10*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/
2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b
^7)^(1/2))/(16*a^3*b^6))^(5/2)*3072i)/(55*a^2*b^9*cos(c + d*x) + 540*a^3*b^
8*cos(c + d*x) + 1035*a^4*b^7*cos(c + d*x) + 45*a^5*b^6*cos(c + d*x) + a^6*
b^5*cos(c + d*x) + 110*a^7*b^4*cos(c + d*x) + 5*a^8*b^3*cos(c + d*x) - 50*a
^6*cos(c + d*x)*(a^3*b^7)^(1/2) - 10*b^6*cos(c + d*x)*(a^3*b^7)^(1/2) + a*b
^10*cos(c + d*x) - 195*a*b^5*cos(c + d*x)*(a^3*...
```

3.414 $\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=127

$$-\frac{x}{b}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}$$

```
[Out] -x/b+1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2
))^(3/2)/a^(3/4)/b/d+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))
*(a^(1/2)+b^(1/2))^(3/2)/a^(3/4)/b/d
```

Rubi [A]
time = 0.15, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3303, 1184, 209, 1180, 211}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(x/b) + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(2*a^(3/4)*b*d) + ((Sqrt[a] + Sqrt[b])^(3/2)*ArcTan[(Sqrt[
Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3303

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)}+\frac{a+b+(a-b)x^2}{b(a+2ax^2+(a-b)x^4)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd}+\frac{\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{bd}$$

$$= -\frac{x}{b}+\frac{\left(\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)(a-b)\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2bd}+$$

$$= -\frac{x}{b}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan}{}$$

Mathematica [A]
time = 0.18, size = 171, normalized size = 1.35

$$\frac{-2(c+dx)+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-2*(c + d*x) + ((Sqrt[a] + Sqrt[b])^2*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - ((S
qrt[a] - Sqrt[b])^2*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sq
rt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]))/(2*b*d)
```

Maple [A]
time = 0.86, size = 148, normalized size = 1.17

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(dx+c))}{b}+\frac{(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\arctan\left(\frac{(a-b)\,\mathrm{t}}{\sqrt{\left(\sqrt{ab}\,-\right.}}\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)}}\right.}{b}}{d}$
default	$\frac{-\frac{\arctan(\tan(dx+c))}{b}+\frac{(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\arctan\left(\frac{(a-b)\,\mathrm{t}}{\sqrt{\left(\sqrt{ab}\,-\right.}}\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)}}\right.}{b}}{d}$
risch	$-\frac{x}{b}+\left(\sum_{\_R=\mathrm{RootOf}\left(256a^3b^4d^4\_Z^4+(32a^3b^2d^2+96d^2b^3a^2)\_Z^2+a^3-3a^2b+3a\,b^2-b^3\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{128}{3a}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*arctan(tan(d*x+c))+1/b*(a-b)*(1/2*((a*b)^(1/2)+b)/(a*b)^(1/2)/(((
a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2))+1/2*((a*b)^(1/2)-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arct
an((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -(b*integrate(-8*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + 4*b
^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2 + 4*(8*a^2 - 3*a*b)*cos(4*
d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) + 4*(8*a^2 - 3*a*b)*sin(4*d*x + 4*c)^2
+ 6*(4*a*b - b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^2*cos(6*d*x + 6*c)
 + 2*a*b*cos(4*d*x + 4*c) + b^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (8*b^2
*cos(2*d*x + 2*c) - b^2 + 6*(4*a*b - b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c
) - 2*(a*b - 3*(4*a*b - b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^2*sin(
6*d*x + 6*c) + 2*a*b*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + 2*(4*b^2*sin(2*d*x + 2*c) + 3*(4*a*b - b^2)*sin(4*d*x + 4*c))*sin(6*d
*x + 6*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos
(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b
^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b
^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x
+ 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^
3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos
(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*
b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8
*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6
*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*
x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))
*sin(6*d*x + 6*c)), x) + x)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1197 vs. $2(91) = 182$.
time = 0.61, size = 1197, normalized size = 9.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b*sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)
/(a*b^2*d^2))*log(1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 - 3/4*a^2 + 1/2*
a*b + 1/4*b^2 + 1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*
cos(d*x + c)*sin(d*x + c) + (3*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*
sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2
*d^2)) - 1/4*(2*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^
2)*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))) - b*sqrt((a*b^2*d^2*sqrt((9*a
^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2*d^2))*log(1/4*(3*a^2 - 2
*a*b - b^2)*cos(d*x + c)^2 - 3/4*a^2 + 1/2*a*b + 1/4*b^2 - 1/2*(a^3*b^2*d^3
*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a
^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*
```

```
a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2*d^2)) - 1/4*(2*(a^3*b - a^2*b^2
)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/(a
^3*b^3*d^4))) + b*sqrt(-(a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)
) + a + 3*b)/(a*b^2*d^2))*log(-1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 + 3
/4*a^2 - 1/2*a*b - 1/4*b^2 + 1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a
^3*b^3*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^2*b + a*b^2)*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-(a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) + a
 + 3*b)/(a*b^2*d^2)) - 1/4*(2*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b
 - a^2*b^2)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))) - b*sqrt(-(a*b^
2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) + a + 3*b)/(a*b^2*d^2))*log
(-1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 + 3/4*a^2 - 1/2*a*b - 1/4*b^2 -
1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*cos(d*x + c)*sin
(d*x + c) - (3*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^2*d^2
*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) + a + 3*b)/(a*b^2*d^2)) - 1/4*(2
*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^2)*sqrt((9*a^2
+ 6*a*b + b^2)/(a^3*b^3*d^4))) - 8*x)/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 906 vs. 2(91) = 182.
time = 0.79, size = 906, normalized size = 7.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/b + ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2
 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a^3*b - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 - 7*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*b^4)*abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^2*b^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x +
c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b + sqrt(a^2*b^2 - (a*b - b^2)*a
*b))/(a*b - b^2))))/((3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b
```

```
^6)*abs(b)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqr
t(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*a^3*b - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 7*sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4)*
abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b
^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi +
1/2) + arctan(tan(d*x + c)/sqrt((a*b - sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*
b - b^2)))))/((3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6)*abs(
b)))/d
```

Mupad [B]
time = 16.40, size = 2500, normalized size = 19.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a - b*sin(c + d*x)^4),x)
```

```
[Out] atan((90*a^4*tan(c + d*x))/(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 - 68*a^2*
b^2 + (18*a^5)/b) - (18*a^5*tan(c + d*x))/(10*a*b^4 - 90*a^4*b + 18*a^5 - 2
*b^5 - 68*a^2*b^3 + 132*a^3*b^2) + (2*b^4*tan(c + d*x))/(10*a*b^3 + 132*a^3
*b - 90*a^4 - 2*b^4 - 68*a^2*b^2 + (18*a^5)/b) + (68*a^2*b^2*tan(c + d*x))/
(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 - 68*a^2*b^2 + (18*a^5)/b) - (10*a*b
^3*tan(c + d*x))/(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 - 68*a^2*b^2 + (18*
a^5)/b) - (132*a^3*b*tan(c + d*x))/(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 -
 68*a^2*b^2 + (18*a^5)/b))/(b*d) + (atan(((tan(c + d*x)*(30*a*b^4 - 30*a^4*
b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) + (-(3*a*(a^3*b^5)^(1/2) + b*(
a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^
5*b - 4*b^6 + ((-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3
*b^2)/(16*a^3*b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b
^4 - 192*a^5*b^3 + tan(c + d*x)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2)
+ 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768
*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 -
480*a^3*b^4 + 48*a^4*b^3 + 144*a^5*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^
5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b
^3 + 12*a^4*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 +
a^3*b^2)/(16*a^3*b^4))^(1/2)*1i + (tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^
5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) - (-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)
^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*
b^6 + ((-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(1
6*a^3*b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192
*a^5*b^3 - tan(c + d*x)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*
b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5
```

+ 768*a^5*b^4)) - tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*b^4 + 48*a^4*b^3 + 144*a^5*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*1i)/((tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) + (-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*b^6 + ((-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*b^4 + 48*a^4*b^3 + 144*a^5*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2) - (tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) - (-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*b^6 + ((-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192*a^5*b^3 - tan(c + d*x)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) - tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*b^4 + 48*a^4*b^3 + 144*a^5*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*2i)/d + (atan(((tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) + ((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*b^6 + (((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*b^4 + 48*a^4*b^3 + 144*a^5*b^2))*((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2))*((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*1i + (tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) - ((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*b^6 + (((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^...

3.415 $\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d}+\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d}$$

```
[Out] -1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))^(
1/2)/a^(3/4)/d/b^(1/2)+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4
))*(a^(1/2)+b^(1/2))^(1/2)/a^(3/4)/d/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3303, 1107, 211}

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])
/a^(1/4)])/(a^(3/4)*Sqrt[b]*d) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(Sqrt[Sqrt
[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3303

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
```

```
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2\sqrt{a}\sqrt{b}\,d} - \frac{(a-b)\text{Subst}\left(\int \frac{}{a+\sqrt{a}}\right.}{2}$$
$$= -\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d} + \frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{}}}{}\right.}{2a^{3/4}\sqrt{b}}$$

Mathematica [A]
time = 0.20, size = 158, normalized size = 1.26

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}+b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{a}\sqrt{b}-b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\,bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (((Sqrt[a]*Sqrt[b] + b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + ((Sqrt[a]*Sqrt[b] - b)*ArcTa
nh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a
+ Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*b*d)
```

Maple [A]
time = 0.56, size = 115, normalized size = 0.92

method	result
risch	$\sum_{\_R=\text{RootOf}\left(256a^3b^2d^4\_Z^4+32a^2b\,d^2\_Z^2+a-b\right)} \_R\ln\left(e^{2i(dx+c)}+32a^2d^2\_R^2+8iad\_R+\frac{2a}{b}-1\right)$

derivativedivides	$\frac{(a-b)\left(-\frac{\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{d}$
default	$\frac{(a-b)\left(-\frac{\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a-b)*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(
d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(cos(d*x + c)^2/(b*sin(d*x + c)^4 - a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(85) = 170$.
time = 0.50, size = 541, normalized size = 4.33

$$-\frac{1}{8}\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\log\left(\frac{1}{2}ad\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\cos(dx+c)\sin(dx+c)+\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\left(2a^2d^2\cos(dx+c)^2-a^2d^2\right)\sqrt{\frac{1}{a^3bd^4}}-\frac{1}{4}\right)+\frac{1}{8}\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\log\left(-\frac{1}{2}ad\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\cos(dx+c)\sin(dx+c)+\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\left(2a^2d^2\cos(dx+c)^2-a^2d^2\right)\sqrt{\frac{1}{a^3bd^4}}-\frac{1}{4}\right)+\frac{1}{8}\sqrt{\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}-1}{abd^2}}\log\left(\frac{1}{2}ad\sqrt{\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}-1}{abd^2}}\cos(dx+c)\sin(dx+c)-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\left(2a^2d^2\cos(dx+c)^2-a^2d^2\right)\sqrt{\frac{1}{a^3bd^4}}+\frac{1}{4}\right)-\frac{1}{8}\sqrt{\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}-1}{abd^2}}\log\left(-\frac{1}{2}ad\sqrt{\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}-1}{abd^2}}\cos(dx+c)\sin(dx+c)-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\left(2a^2d^2\cos(dx+c)^2-a^2d^2\right)\sqrt{\frac{1}{a^3bd^4}}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*log(1/2*a*d*sqrt(-(
a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*cos(d*x + c)*sin(d*x + c) + 1/4
*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^2*d^2)*sqrt(1/(a^3*b*d^
4)) - 1/4) + 1/8*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*log(-1/
2*a*d*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*cos(d*x + c)*sin(d
*x + c) + 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^2*d^2)*sqr
t(1/(a^3*b*d^4)) - 1/4) + 1/8*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d
^2))*log(1/2*a*d*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d^2))*cos(d*x
+ c)*sin(d*x + c) - 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^
2*d^2)*sqrt(1/(a^3*b*d^4)) + 1/4) - 1/8*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) -
 1)/(a*b*d^2))*log(-1/2*a*d*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d^2
))*cos(d*x + c)*sin(d*x + c) - 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x
+ c)^2 - a^2*d^2)*sqrt(1/(a^3*b*d^4)) + 1/4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(85) = 170.
time = 0.83, size = 558, normalized size = 4.46

$$\frac{\frac{\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^2b-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,ab^2-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,b^3-3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^2+6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,ab+\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a+\sqrt{-16(a-b)a+16a^2}}{a-b}}}\right)\right)|a-b|}{3a^5b-12a^4b^2+14a^3b^3-4a^2b^4-ab^5}+\frac{\left(3\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,a^2b-6\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,ab^2-\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,b^3-3\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^2+6\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,ab+\sqrt{a^2-ab-\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a-\sqrt{-16(a-b)a+16a^2}}{a-b}}}\right)\right)|a-b|}{3a^5b-12a^4b^2+14a^3b^3-4a^2b^4-ab^5}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b - 6*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^3 - 3*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 + 6*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*sqrt(a*b)*a*b + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*(
pi*floor((d*x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a + sqrt(-16*(
a - b)*a + 16*a^2))/(a - b))))*abs(a - b)/(3*a^5*b - 12*a^4*b^2 + 14*a^3*b^
3 - 4*a^2*b^4 - a*b^5) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b - 6*s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*b^3 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 + 6*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b + sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*b^2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sq
rt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/(a - b))))*abs(a - b)/(3*a^5*b - 12
*a^4*b^2 + 14*a^3*b^3 - 4*a^2*b^4 - a*b^5))/d
```

Mupad [B]

time = 15.66, size = 1409, normalized size = 11.27

$$\frac{\mathrm{atan}\left(\frac{\left(\tan(c+d\,x)\,(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3)+\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\left(16\,a\,b^3+16\,a^3\,b-32\,a^2\,b^2+\tan(c+d\,x)\,(64\,a^4\,b-128\,a^3\,b^2+64\,a^2\,b^3)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\right)\right)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\,1\mathrm{i}+\left(\tan(c+d\,x)\,(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3)-\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\left(16\,a\,b^3+16\,a^3\,b-32\,a^2\,b^2-\tan(c+d\,x)\,(64\,a^4\,b-128\,a^3\,b^2+64\,a^2\,b^3)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\right)\right)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\,1\mathrm{i}}{\left(\tan(c+d\,x)\,(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3)+\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\left(16\,a\,b^3+16\,a^3\,b-32\,a^2\,b^2+\tan(c+d\,x)\,(64\,a^4\,b-128\,a^3\,b^2+64\,a^2\,b^3)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\right)\right)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}-\left(\tan(c+d\,x)\,(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3)-\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\left(16\,a\,b^3+16\,a^3\,b-32\,a^2\,b^2-\tan(c+d\,x)\,(64\,a^4\,b-128\,a^3\,b^2+64\,a^2\,b^3)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\right)\right)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}}\right)\sqrt{-\frac{\sqrt{a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}\,2\mathrm{i}}{d}+\frac{\mathrm{atan}\left(\frac{\left(\tan(c+d\,x)\,(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3)+\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\left(16\,a\,b^3+16\,a^3\,b-32\,a^2\,b^2+\tan(c+d\,x)\,(64\,a^4\,b-128\,a^3\,b^2+64\,a^2\,b^3)\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\right)\right)\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\,1\mathrm{i}+\left(\tan(c+d\,x)\,(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3)-\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\left(16\,a\,b^3+16\,a^3\,b-32\,a^2\,b^2-\tan(c+d\,x)\,(64\,a^4\,b-128\,a^3\,b^2+64\,a^2\,b^3)\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\right)\right)\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\,1\mathrm{i}}{\left(\tan(c+d\,x)\,(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3)+\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\left(16\,a\,b^3+16\,a^3\,b-32\,a^2\,b^2+\tan(c+d\,x)\,(64\,a^4\,b-128\,a^3\,b^2+64\,a^2\,b^3)\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\right)\right)\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}-\left(\tan(c+d\,x)\,(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3)-\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\left(16\,a\,b^3+16\,a^3\,b-32\,a^2\,b^2-\tan(c+d\,x)\,(64\,a^4\,b-128\,a^3\,b^2+64\,a^2\,b^3)\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\right)\right)\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}}\right)\sqrt{\frac{\sqrt{a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan(((tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) + (-((a^3*b^3)^(
1/2) + a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 + tan(c
 + d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(
16*a^3*b^2))^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)*1i + (
tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) - (-((a^3*b^3)^(1/2) + a
^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 - tan(c + d*x)*
(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b
^2))^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)*1i)/((tan(c +
d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) + (-((a^3*b^3)^(1/2) + a^2*b)/(1
6*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 + tan(c + d*x)*(64*a^4*
b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/
2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - (tan(c + d*x)*(12*a*
b^2 - 12*a^2*b + 4*a^3 - 4*b^3) - (-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))
^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 - tan(c + d*x)*(64*a^4*b + 64*a^2*
b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)))*(-((a^
3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16
*a^3*b^2))^(1/2)*2i)/d + (atan(((tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3
- 4*b^3) + (((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^
3*b - 32*a^2*b^2 + tan(c + d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(((a^
3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)))*(((a^3*b^3)^(1/2) - a^2*b)/(16*
a^3*b^2))^(1/2)*1i + (tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) -
(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^
2*b^2 - tan(c + d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(((a^3*b^3)^(1/2
) - a^2*b)/(16*a^3*b^2))^(1/2)))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(
1/2)*1i)/((tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) + (((a^3*b^3)
^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 + tan
(c + d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(((a^3*b^3)^(1/2) - a^2*b)/
(16*a^3*b^2))^(1/2)))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2) - (tan
(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) - (((a^3*b^3)^(1/2) - a^2*b
)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 - tan(c + d*x)*(64*
a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^
(1/2)))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)))*(((a^3*b^3)^(1/2)
- a^2*b)/(16*a^3*b^2))^(1/2)*2i)/d
```

3.416 $\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}+\frac{\tan(c+dx)}{(a-b)d}$$

```
[Out] -1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1/2)/a^(3/4)/d/(
a^(1/2)-b^(1/2))^(3/2)+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4
))*b^(1/2)/a^(3/4)/d/(a^(1/2)+b^(1/2))^(3/2)+tan(d*x+c)/(a-b)/d
```

Rubi [A]
time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3303, 1184, 1180, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan(c+dx)}{d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*(Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(a^(3
/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]
*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + Tan[c +
d*x]/((a - b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3303

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a-b}-\frac{b(1+2x^2)}{(a-b)(a+2ax^2+(a-b)x^4)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan(c+dx)}{(a-b)d}-\frac{b\text{Subst}\left(\int \frac{1+2x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{(a-b)d}$$

$$= \frac{\tan(c+dx)}{(a-b)d}-\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+\right.}{2\sqrt{a}(a-b)d}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}$$

Mathematica [A]
time = 0.34, size = 175, normalized size = 1.23

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}-b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{a}\sqrt{b}+b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}+2\tan(c+dx)}{2(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (((Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + ((Sqrt[a]*Sqrt[b]
+ b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]
)/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 2*Tan[c + d*x])/(2*(a - b)*d)
```

Maple [A]
time = 0.77, size = 166, normalized size = 1.17

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{a-b} - b\left(\frac{\left(a+2\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}} + \frac{\left(-a+2\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(}{\sqrt{\left(\sqrt{ab}-a\right.}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a}}\right.}{d}$
default	$\frac{\frac{\tan(dx+c)}{a-b} - b\left(\frac{\left(a+2\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}} + \frac{\left(-a+2\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(}{\sqrt{\left(\sqrt{ab}-a\right.}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a}}\right.}{d}$
risch	$\frac{2i}{d(a-b)\left(e^{2i(dx+c)}+1\right)} + 4\left(\sum_{\_R=\operatorname{RootOf}\left(\left(65536a^6d^4-196608a^5b\,d^4+196608a^4b^2d^4-65536a^3b^3d^4\right)\_Z^4+\left(512a^3b\,d^2+\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a-b)*tan(d*x+c)-b*(1/2*(a+2*(a*b)^(1/2)+b)/(a*b)^(1/2)/(a-b)/(((a*b
)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/
2))+1/2*(-a+2*(a*b)^(1/2)-b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2
)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] (((a - b)*d*cos(2*d*x + 2*c)^2 + (a - b)*d*sin(2*d*x + 2*c)^2 + 2*(a - b)*d
*cos(2*d*x + 2*c) + (a - b)*d)*integrate(4*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^
2*cos(2*d*x + 2*c)^2 + 4*b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2
- 12*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) - 12*(8*a*b
- 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(8*a*b - 15*b^2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) - (b^2*cos(6*d*x + 6*c) - 6*b^2*cos(4*d*x + 4*c) + b^2*cos(2*d*x +
2*c))*cos(8*d*x + 8*c) + (8*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 15*b^2)
*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(3*b^2 + (8*a*b - 15*b^2)*cos(2*d*x
 + 2*c))*cos(4*d*x + 4*c) - (b^2*sin(6*d*x + 6*c) - 6*b^2*sin(4*d*x + 4*c)
+ b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*(4*b^2*sin(2*d*x + 2*c) + (8*a
*b - 15*b^2)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c))/(a*b^2 - b^3 + (a*b^2 - b^
3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 1
12*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d*x + 4*c)^2 + 16*(a*b^2 - b^3)*cos(2*d*
x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*sin(6*d*x
+ 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*sin(4*d*x + 4*c)^2 + 1
6*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^
2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b^2 - b^3 - 4*(a*b^2 - b^3)*cos(6*d*x +
6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*co
s(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^2 - b^3 - 2*(8*a^2*b - 11*a*b^2 +
 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*
c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3 - 4*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(2*d
*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^2 - b^3)*cos(2*d*x + 2*c) - 4*(2*(a*b^
2 - b^3)*sin(6*d*x + 6*c) + (8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) +
 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b - 11*a*b
^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c)), x) + 2*sin(2*d*x + 2*c))/((a - b)*d*cos(2*d*x + 2*c)^2 + (a - b)*d
*sin(2*d*x + 2*c)^2 + 2*(a - b)*d*cos(2*d*x + 2*c) + (a - b)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2589 vs. $2(102) = 204$.
time = 0.73, size = 2589, normalized size = 18.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((a - b)*d*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^
3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 -
 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*
b^3)*d^2))*cos(d*x + c)*log(3/4*a*b^2 + 1/4*b^3 - 1/4*(3*a*b^2 + b^3)*cos(d
*x + c)^2 + 1/2*(2*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^
```

3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a^3*b + 4*a^2*b^2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))) - (a - b)*d*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2))*cos(d*x + c)*log(3/4*a*b^2 + 1/4*b^3 - 1/4*(3*a*b^2 + b^3)*cos(d*x + c)^2 - 1/2*(2*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a^3*b + 4*a^2*b^2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))) + (a - b)*d*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2))*cos(d*x + c)*log(-3/4*a*b^2 - 1/4*b^3 + 1/4*(3*a*b^2 + b^3)*cos(d*x + c)^2 + 1/2*(2*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^3*b + 4*a^2*b^2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))) - (a - b)*d*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2))*cos(d*x + c)*log(-3/4*a*b^2 - 1/4*b^3 + 1/4*(3*a*b^2 + b^3)*cos(d*x + c)^2 - 1/2*(2*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^3*b + 4*a^2*b^2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b

```
^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2
)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5
 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a
^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*
d^4))) + 8*sin(d*x + c))/((a - b)*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

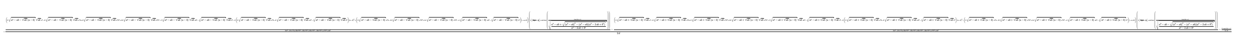
$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a - b*sin(c + d*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1211 vs. $2(102) = 204$.
time = 0.81, size = 1211, normalized size = 8.53



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5 - 9*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(a^2 - a*b - sqrt(a*b)*(a -
 b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a
^2*b^3 - 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a
*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a - b)^2 + (3*sq
rt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*a^3*b^2 + 14*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 - 4*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^5
)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^
2 - a*b + sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - 2*a
*b + b^2))))/(3*a^8 - 21*a^7*b + 59*a^6*b^2 - 85*a^5*b^3 + 65*a^4*b^4 - 23*
a^3*b^5 + a^2*b^6 + a*b^7) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^5 - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a^2*b^3 - 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*a*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)
```

```
*b^3)*(a - b)^2 - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 12*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*a^3*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*a^2*b^3 - 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arcta
n(tan(d*x + c)/sqrt((a^2 - a*b - sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*
a*b + b^2)))/(a^2 - 2*a*b + b^2))))/(3*a^8 - 21*a^7*b + 59*a^6*b^2 - 85*a^5
*b^3 + 65*a^4*b^4 - 23*a^3*b^5 + a^2*b^6 + a*b^7) - 2*tan(d*x + c)/(a - b))
/d
```

Mupad [B]
time = 16.94, size = 2832, normalized size = 19.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] tan(c + d*x)/(d*(a - b)) + (atan(((((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/
(a - b) - (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b
 + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b -
 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*b^3)^(1/2) + b*
(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b
^2)))^(1/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*((3*a*(a^
3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 +
a^3*b^3 - 3*a^4*b^2)))^(1/2)*1i - (((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/
(a - b) + (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b
 + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b -
 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*b^3)^(1/2) + b*
(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b
^2)))^(1/2) + (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*((3*a*(a^
3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 +
a^3*b^3 - 3*a^4*b^2)))^(1/2)*1i)/((((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/
(a - b) - (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b
 + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b -
 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*b^3)^(1/2) + b*
(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b
^2)))^(1/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*((3*a*(a^
3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 +
a^3*b^3 - 3*a^4*b^2)))^(1/2) - (4*b^3)/(a - b) + (((2*(8*a*b^4 - 16*a^2*b^3
 + 8*a^3*b^2))/(a - b) + (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3
)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1
/2)*(16*a^5*b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*
b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^
3*b^3 - 3*a^4*b^2)))^(1/2) + (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a
- b))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3
```

*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*2i)/d + (atan(((((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/(a - b) - (4*tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*1i - (((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/(a - b) + (4*tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2) + (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*1i)/((((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/(a - b) - (4*tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2) - (4*b^3)/(a - b) + (((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/(a - b) + (4*tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2) + (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*2i)/d

3.417 $\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d}+\frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan^3(c+dx)}{3(a-b)d}$$

```
[Out] 1/2*b*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)
-b^(1/2))^(5/2)+1/2*b*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^
(3/4)/d/(a^(1/2)+b^(1/2))^(5/2)+(a-3*b)*tan(d*x+c)/(a-b)^2/d+1/3*tan(d*x+c)
^3/(a-b)/d
```

Rubi [A]

time = 0.22, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3303, 1184, 1180, 211}

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{b\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a-b)}+\frac{(a-3b)\tan(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt
[a] - Sqrt[b])^(5/2)*d) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/
a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) + ((a - 3*b)*Tan[c + d*x]
)/((a - b)^2*d) + Tan[c + d*x]^3/(3*(a - b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3303

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{(a-b)^2}+\frac{x^2}{a-b}+\frac{b(a+b)+b(a+3b)x^2}{(a-b)^2(a+2ax^2+(a-b)x^4)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan^3(c+dx)}{3(a-b)d}+\frac{\text{Subst}\left(\int \frac{b(a+b)+b(a+3b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+d\right.}{(a-b)^2d}$$

$$= \frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan^3(c+dx)}{3(a-b)d}+\frac{\left(\left(\sqrt{a}-\sqrt{b}\right)^3b\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}}\right.}{2\sqrt{a}\,(a-b)^2}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d}+\frac{(a-}{}$$

Mathematica [A]
time = 0.71, size = 205, normalized size = 1.27

$$\frac{\frac{3\left(\sqrt{a}-\sqrt{b}\right)^2b\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{3\left(\sqrt{a}+\sqrt{b}\right)^2b\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}+4(a-4b)\tan(c+dx)+2(a-b)\sec^2(c+dx)\tan(c+dx)}{6(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((3*(Sqrt[a] - Sqrt[b])^2*b*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[
a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - (3*(Sqrt[a] +
Sqrt[b])^2*b*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*S
qrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 4*(a - 4*b)*Tan[c + d*x] +
 2*(a - b)*Sec[c + d*x]^2*Tan[c + d*x])/(6*(a - b)^2*d)
```

Maple [A]
time = 1.38, size = 230, normalized size = 1.43

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^3(dx+c)\right)}{3}-\frac{b\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)a-3b\tan(dx+c)}{(a-b)^2}+b\left(\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b+3ab+b^2\right)\arctan\left(\frac{(a-b)\tan(dx+}{\sqrt{\left(\sqrt{ab}+a\right)(}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right.}{d}$
default	$\frac{\frac{\frac{a\left(\tan^3(dx+c)\right)}{3}-\frac{b\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)a-3b\tan(dx+c)}{(a-b)^2}+b\left(\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b+3ab+b^2\right)\arctan\left(\frac{(a-b)\tan(dx+}{\sqrt{\left(\sqrt{ab}+a\right)(}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right.}{d}$
risch	$-\frac{4i\left(3b\,e^{4i(dx+c)}-3a\,e^{2i(dx+c)}+9b\,e^{2i(dx+c)}-a+4b\right)}{3d(a-b)^2\left(e^{2i(dx+c)}+1\right)^3}+16\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16777216a^8d^4-83886080a^7b\,d^4+1677721\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a-b)^2*(1/3*a*tan(d*x+c)^3-1/3*b*tan(d*x+c)^3+tan(d*x+c)*a-3*b*tan(
d*x+c))+b/(a-b)*(1/2*(a*(a*b)^(1/2)+3*(a*b)^(1/2)*b+3*a*b+b^2)/(a*b)^(1/2)/
(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a
)*(a-b))^(1/2))+1/2*(a*(a*b)^(1/2)+3*(a*b)^(1/2)*b-3*a*b-b^2)/(a*b)^(1/2)/(
a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-
a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/3*(36*(a - 2*b)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 12*(b*sin(4*d*x + 4*
c) - (a - 3*b)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 3*((a^2 - 2*a*b + b^2)*
d*cos(6*d*x + 6*c)^2 + 9*(a^2 - 2*a*b + b^2)*d*cos(4*d*x + 4*c)^2 + 9*(a^2
- 2*a*b + b^2)*d*cos(2*d*x + 2*c)^2 + (a^2 - 2*a*b + b^2)*d*sin(6*d*x + 6*c
)^2 + 9*(a^2 - 2*a*b + b^2)*d*sin(4*d*x + 4*c)^2 + 18*(a^2 - 2*a*b + b^2)*d
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*(a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*
c)^2 + 6*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d + 2
*(3*(a^2 - 2*a*b + b^2)*d*cos(4*d*x + 4*c) + 3*(a^2 - 2*a*b + b^2)*d*cos(2*
d*x + 2*c) + (a^2 - 2*a*b + b^2)*d)*cos(6*d*x + 6*c) + 6*(3*(a^2 - 2*a*b +
b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d)*cos(4*d*x + 4*c) + 6*((a^2
 - 2*a*b + b^2)*d*sin(4*d*x + 4*c) + (a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c))*integrate(-8*(4*b^3*cos(6*d*x + 6*c)^2 + 4*b^3*cos(2*d*
x + 2*c)^2 + 4*b^3*sin(6*d*x + 6*c)^2 + 4*b^3*sin(2*d*x + 2*c)^2 - b^3*cos(
2*d*x + 2*c) - 4*(8*a^2*b + 13*a*b^2 - 6*b^3)*cos(4*d*x + 4*c)^2 - 4*(8*a^2
*b + 13*a*b^2 - 6*b^3)*sin(4*d*x + 4*c)^2 + 2*(4*a*b^2 - 11*b^3)*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) - (b^3*cos(6*d*x + 6*c) + b^3*cos(2*d*x + 2*c) - 2*
(a*b^2 + 2*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + (8*b^3*cos(2*d*x + 2*c
) - b^3 + 2*(4*a*b^2 - 11*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(a*b^
2 + 2*b^3 + (4*a*b^2 - 11*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^3*si
n(6*d*x + 6*c) + b^3*sin(2*d*x + 2*c) - 2*(a*b^2 + 2*b^3)*sin(4*d*x + 4*c))
*sin(8*d*x + 8*c) + 2*(4*b^3*sin(2*d*x + 2*c) + (4*a*b^2 - 11*b^3)*sin(4*d*
x + 4*c))*sin(6*d*x + 6*c))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2 - 2*a*b^3 +
 b^4)*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - 2*a*b^3 + b^4)*cos(6*d*x + 6*c)^2
+ 4*(64*a^4 - 176*a^3*b + 169*a^2*b^2 - 66*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^
2 + 16*(a^2*b^2 - 2*a*b^3 + b^4)*cos(2*d*x + 2*c)^2 + (a^2*b^2 - 2*a*b^3 +
b^4)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - 2*a*b^3 + b^4)*sin(6*d*x + 6*c)^2 +
 4*(64*a^4 - 176*a^3*b + 169*a^2*b^2 - 66*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^2
 + 16*(8*a^3*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 16*(a^2*b^2 - 2*a*b^3 + b^4)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - 2*a
*b^3 + b^4 - 4*(a^2*b^2 - 2*a*b^3 + b^4)*cos(6*d*x + 6*c) - 2*(8*a^3*b - 19
*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - 2*a*b^3 + b^4)
*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^2 - 2*a*b^3 + b^4 - 2*(8*a^3
*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - 2*a*b^3
 + b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 19*a^2*b^2 + 14*a
*b^3 - 3*b^4 - 4*(8*a^3*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos(2*d*x + 2*c)
)*cos(4*d*x + 4*c) - 8*(a^2*b^2 - 2*a*b^3 + b^4)*cos(2*d*x + 2*c) - 4*(2*(a
^2*b^2 - 2*a*b^3 + b^4)*sin(6*d*x + 6*c) + (8*a^3*b - 19*a^2*b^2 + 14*a*b^3
```

```
 - 3*b^4)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - 2*a*b^3 + b^4)*sin(2*d*x + 2*c))*
sin(8*d*x + 8*c) + 16*((8*a^3*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*sin(4*d*x
+ 4*c) + 2*(a^2*b^2 - 2*a*b^3 + b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x
) + 4*(3*b*cos(4*d*x + 4*c) - 3*(a - 3*b)*cos(2*d*x + 2*c) - a + 4*b)*sin(6
*d*x + 6*c) - 12*(3*(a - 2*b)*cos(2*d*x + 2*c) + a - 3*b)*sin(4*d*x + 4*c)
+ 12*b*sin(2*d*x + 2*c))/((a^2 - 2*a*b + b^2)*d*cos(6*d*x + 6*c)^2 + 9*(a^2
 - 2*a*b + b^2)*d*cos(4*d*x + 4*c)^2 + 9*(a^2 - 2*a*b + b^2)*d*cos(2*d*x +
2*c)^2 + (a^2 - 2*a*b + b^2)*d*sin(6*d*x + 6*c)^2 + 9*(a^2 - 2*a*b + b^2)*d
*sin(4*d*x + 4*c)^2 + 18*(a^2 - 2*a*b + b^2)*d*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 9*(a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*c)^2 + 6*(a^2 - 2*a*b + b^2)*
d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d + 2*(3*(a^2 - 2*a*b + b^2)*d*cos
(4*d*x + 4*c) + 3*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b
^2)*d)*cos(6*d*x + 6*c) + 6*(3*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^
2 - 2*a*b + b^2)*d)*cos(4*d*x + 4*c) + 6*((a^2 - 2*a*b + b^2)*d*sin(4*d*x +
 4*c) + (a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4113 vs. $2(123) = 246$.
time = 0.92, size = 4113, normalized size = 25.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5*
a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 +
 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b
^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 +
 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 -
10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(d*x + c)^3*log(5/4*a^2*b^4 + 5/2*
a*b^5 + 1/4*b^6 - 1/4*(5*a^2*b^4 + 10*a*b^5 + b^6)*cos(d*x + c)^2 + 1/2*((a
^9 - 2*a^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25*a^5*b^4 + 14*a^4*b^5 - 3*a^3*b^6
)*d^3*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13
 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210
*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))*cos(d*x
+ c)*sin(d*x + c) + (15*a^4*b^3 + 35*a^3*b^4 + 13*a^2*b^5 + a*b^6)*d*cos(d*
x + c)*sin(d*x + c))*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5*a^5*b + 1
0*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*
b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*
a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b
^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^
3 + 5*a^2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*
a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^7*b - 5*a^6*b^2 + 10
*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2)*sqrt((25*a^4*b^5 + 100*a^
```

3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 12
0*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5
*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))) - 3*(a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b
^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^
4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^
9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*
b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))
)/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(
d*x + c)^3*log(5/4*a^2*b^4 + 5/2*a*b^5 + 1/4*b^6 - 1/4*(5*a^2*b^4 + 10*a*b^
5 + b^6)*cos(d*x + c)^2 - 1/2*((a^9 - 2*a^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25
*a^5*b^4 + 14*a^4*b^5 - 3*a^3*b^6)*d^3*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110
*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3
+ 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a
^4*b^9 + a^3*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (15*a^4*b^3 + 35*a^3*b
^4 + 13*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^2*b^2 + 10*a
*b^3 + 5*b^4 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5
)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13
 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210
*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 -
 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7
*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x
 + c)^2 - (a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^
6)*d^2)*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^
13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 2
10*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))) + 3*(
a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 + (a^6 - 5*a^5*b + 1
0*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*
b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*
a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b
^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^
3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(d*x + c)^3*log(-5/4*a^2*b^4 - 5/2*a*b^5 -
1/4*b^6 + 1/4*(5*a^2*b^4 + 10*a*b^5 + b^6)*cos(d*x + c)^2 + 1/2*((a^9 - 2*a
^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25*a^5*b^4 + 14*a^4*b^5 - 3*a^3*b^6)*d^3*sq
rt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^
12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6
 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))*cos(d*x + c)*sin
(d*x + c) - (15*a^4*b^3 + 35*a^3*b^4 + 13*a^2*b^5 + a*b^6)*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 + (a^6 - 5*a^5*b + 10*a^4*b^
2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 11
0*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3
 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*
a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^
2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4
+ 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^7*b - 5*a^6*b^2 + 10*a^5*b^3
 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2)*sqrt((25*a^4*b^5 + 100*a^3*b^6 +

```
110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b
^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a - b*sin(c + d*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2183 vs. $2(123) = 246$.
time = 0.90, size = 2183, normalized size = 13.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/6*(2*(a^2*tan(d*x + c)^3 - 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c)^3 + 3*
a^2*tan(d*x + c) - 12*a*b*tan(d*x + c) + 9*b^2*tan(d*x + c))/(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3) - 3*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3
*b + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 19*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*b^4)*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)^2*abs(-a + b) - (3*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*a^6*b^2 + 23*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^3 - 3*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*a^4*b^4 - 23*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*a^3*b^5 + 19*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^6 - 3*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*a*b^7 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^8)*a
bs(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*abs(-a + b) - (9*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a^9*b - 69*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*a^8*b^2 + 216*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^3 - 3
52*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 + 306*sqrt(a^2 - a
*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 114*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*a^4*b^6 - 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^3*b^7 + 24*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^8 - 3*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^9 - sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*sqrt(a*b)*b^10)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) +
arctan(tan(d*x + c)/sqrt((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3 + sqrt((a^4 - 3
*a^3*b + 3*a^2*b^2 - a*b^3)^2 - (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
```

```
 + b^4))))/((3*a^11 - 30*a^10*b + 131*a^9*b^2 - 328*a^8*b^3 + 518*a^7*b^4 -
 532*a^6*b^5 + 350*a^5*b^6 - 136*a^4*b^7 + 23*a^3*b^8 + 2*a^2*b^9 - a*b^10)
*abs(a^3 - 3*a^2*b + 3*a*b^2 - b^3)) + 3*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*sqrt(a*b)*a^3*b + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2
*b^2 - 19*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 3*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)^2
*abs(-a + b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^7*b - 15*sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*a^6*b^2 + 23*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
a^5*b^3 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^4 - 23*sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*a^3*b^5 + 19*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*
b^6 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^7 - sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*b^8)*abs(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*abs(-a + b) - (9*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b - 69*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a^8*b^2 + 216*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a^7*b^3 - 352*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4
 + 306*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 114*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^6 - 16*sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*sqrt(a*b)*a^3*b^7 + 24*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt
(a*b)*a^2*b^8 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^9 - sqr
t(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^10)*abs(-a + b))*(pi*floor((d*
x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4 - 3*a^3*b + 3*a^2*b^2 - a*
b^3 - sqrt((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)^2 - (a^4 - 3*a^3*b + 3*a^2*b
^2 - a*b^3)*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)))/(a^4 - 4*a^3*b +
6*a^2*b^2 - 4*a*b^3 + b^4))))/((3*a^11 - 30*a^10*b + 131*a^9*b^2 - 328*a^8*
b^3 + 518*a^7*b^4 - 532*a^6*b^5 + 350*a^5*b^6 - 136*a^4*b^7 + 23*a^3*b^8 +
2*a^2*b^9 - a*b^10)*abs(a^3 - 3*a^2*b + 3*a*b^2 - b^3)))/d
```

Mupad [B]
time = 17.74, size = 2500, normalized size = 15.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] tan(c + d*x)^3/(3*d*(a - b)) - (tan(c + d*x)*((2*a)/(a - b)^2 - 3/(a - b)))
/d + (atan(((((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 -
3*a^2*b + a^3 - b^3) - (4*tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b
^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*
(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16
*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))
/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(
1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^
7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) - (4*tan
(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3
```

- b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*1i - (((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (4*tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) + (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*1i)/((((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (4*tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) - (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) - (2*(a*b^4 + 3*b^5))/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (4*tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) + (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*2i)/d + (atan(((((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (4*tan(c + d*x)*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 -

```
80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(-(5*a^2*(a^3*b^5)^(1/2) + b^
2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/
2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(
1/2) - (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 -
3*a^2*b + a^3 - b^3))*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^
2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 +
 a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*1i - (((16*a*b^6 -
32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (4*
tan(c + d*x)*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 1
0*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5
- 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*
a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^...
```

3.418 $\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=204

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{7/2}d}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{7/2}d}+\frac{(a^2-3ab+6b^2)\tan(c+dx)}{(a-b)^3d}$$

```
[Out] -1/2*b^(3/2)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(
a^(1/2)-b^(1/2))^(7/2)+1/2*b^(3/2)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c
)/a^(1/4))/a^(3/4)/d/(a^(1/2)+b^(1/2))^(7/2)+(a^2-3*a*b+6*b^2)*tan(d*x+c)/(
a-b)^3/d+2/3*(a-2*b)*tan(d*x+c)^3/(a-b)^2/d+1/5*tan(d*x+c)^5/(a-b)/d
```

Rubi [A]

time = 0.26, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3303, 1184, 1180, 211}

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{7/2}}+\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{7/2}}+\frac{(a^2-3ab+6b^2)\tan(c+dx)}{d(a-b)^3}+\frac{\tan^5(c+dx)}{5d(a-b)}+\frac{2(a-2b)\tan^3(c+dx)}{3d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*(b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(a^(3
/4)*(Sqrt[a] - Sqrt[b])^(7/2)*d) + (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]
*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(7/2)*d) + ((a^2 -
3*a*b + 6*b^2)*Tan[c + d*x])/((a - b)^3*d) + (2*(a - 2*b)*Tan[c + d*x]^3)/(
3*(a - b)^2*d) + Tan[c + d*x]^5/(5*(a - b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3303

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^4}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2-3ab+6b^2}{(a-b)^3} + \frac{2(a-2b)x^2}{(a-b)^2} + \frac{x^4}{a-b} - \frac{b^2(3a+b)+4b^2(a+b)x^2}{(a-b)^3(a+2ax^2+(a-b)x^4)}\right)\,dx, x, \tan(c+dx\right.}{d}$$

$$= \frac{(a^2-3ab+6b^2)\tan(c+dx)}{(a-b)^3d} + \frac{2(a-2b)\tan^3(c+dx)}{3(a-b)^2d} + \frac{\tan^5(c+dx)}{5(a-b)d} - \frac{\operatorname{Subst}}{}$$

$$= \frac{(a^2-3ab+6b^2)\tan(c+dx)}{(a-b)^3d} + \frac{2(a-2b)\tan^3(c+dx)}{3(a-b)^2d} + \frac{\tan^5(c+dx)}{5(a-b)d} + \frac{\left(\left(\sqrt{a}\right.\right.}{}$$

$$= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{7/2}d} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{7/2}d}$$

Mathematica [A]

time = 0.91, size = 253, normalized size = 1.24

$$\frac{\frac{15\left(\sqrt{a}-\sqrt{b}\right)^3 b^{3/2}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}} + \frac{15\left(\sqrt{a}+\sqrt{b}\right)^3 b^{3/2}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}} + 2(8a^2-21ab+73b^2)\tan(c+dx) + 4(2a-7b)(a-b)\sec^2(c+dx)\tan(c+dx) + 6(a-b)^2\sec^4(c+dx)\tan(c+dx)}{30(a-b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((15*(Sqrt[a] - Sqrt[b])^3*b^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x]
)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + (15*(Sq
rt[a] + Sqrt[b])^3*b^(3/2)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[
-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 2*(8*a^2 - 2
1*a*b + 73*b^2)*Tan[c + d*x] + 4*(2*a - 7*b)*(a - b)*Sec[c + d*x]^2*Tan[c +
 d*x] + 6*(a - b)^2*Sec[c + d*x]^4*Tan[c + d*x])/(30*(a - b)^3*d)
```

Maple [A]
time = 1.46, size = 311, normalized size = 1.52 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a-b)^3*(1/5*a^2*tan(d*x+c)^5-2/5*a*b*tan(d*x+c)^5+1/5*b^2*tan(d*x+c
)^5+2/3*a^2*tan(d*x+c)^3-2*a*b*tan(d*x+c)^3+4/3*b^2*tan(d*x+c)^3+a^2*tan(d*
x+c)-3*a*b*tan(d*x+c)+6*b^2*tan(d*x+c))-b^2/(a-b)^2*(1/2*(4*a*(a*b)^(1/2)+4
*(a*b)^(1/2)*b+a^2+6*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/
2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(4*a*(a*b)^(1
/2)+4*(a*b)^(1/2)*b-a^2-6*a*b-b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/15*(300*(a*b - 5*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 10*(48*b^2*sin(
6*d*x + 6*c) + 3*(a*b + 3*b^2)*sin(8*d*x + 8*c) + 2*(8*a^2 - 21*a*b + 49*b^
2)*sin(4*d*x + 4*c) + 8*(a^2 - 3*a*b + 8*b^2)*sin(2*d*x + 2*c))*cos(10*d*x
+ 10*c) + 50*(6*(a*b - 5*b^2)*sin(6*d*x + 6*c) - 16*(a^2 - 3*a*b + 5*b^2)*s
in(4*d*x + 4*c) - (8*a^2 - 27*a*b + 55*b^2)*sin(2*d*x + 2*c))*cos(8*d*x + 8
*c) - 200*((8*a^2 - 21*a*b + 25*b^2)*sin(4*d*x + 4*c) + 4*(a^2 - 3*a*b + 5*
b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 15*((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*d*cos(10*d*x + 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(8*d*x
+ 8*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c)^2 + 100*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*cos(2*d*x + 2*c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(
10*d*x + 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2
+ 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c)^2 + 100*(a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*d*sin(2*d*x + 2*c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x +
```

2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 2*(5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(8*d*x + 8*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(10*d*x + 10*c) + 10*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(8*d*x + 8*c) + 20*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(6*d*x + 6*c) + 20*(5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(4*d*x + 4*c) + 10*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 50*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(4*(4*(a*b^3 + 3*b^4)*cos(6*d*x + 6*c)^2 - 4*(56*a^2*b^2 + 19*a*b^3 - 15*b^4)*cos(4*d*x + 4*c)^2 + 4*(a*b^3 + 3*b^4)*cos(2*d*x + 2*c)^2 + 4*(a*b^3 + 3*b^4)*sin(6*d*x + 6*c)^2 - 4*(56*a^2*b^2 + 19*a*b^3 - 15*b^4)*sin(4*d*x + 4*c)^2 + 2*(8*a^2*b^2 - 7*a*b^3 - 29*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(a*b^3 + 3*b^4)*sin(2*d*x + 2*c)^2 - ((a*b^3 + 3*b^4)*cos(6*d*x + 6*c) - 2*(7*a*b^3 + 5*b^4)*cos(4*d*x + 4*c) + (a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a*b^3 + 3*b^4 - 2*(8*a^2*b^2 - 7*a*b^3 - 29*b^4)*cos(4*d*x + 4*c) - 8*(a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(7*a*b^3 + 5*b^4 + (8*a^2*b^2 - 7*a*b^3 - 29*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b^3 + 3*b^4)*cos(2*d*x + 2*c) - ((a*b^3 + 3*b^4)*sin(6*d*x + 6*c) - 2*(7*a*b^3 + 5*b^4)*sin(4*d*x + 4*c) + (a*b^3 + 3*b^4)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^2*b^2 - 7*a*b^3 - 29*b^4)*sin(4*d*x + 4*c) + 4*(a*b^3 + 3*b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 240*a^4*b + 345*a^3*b^2 - 235*a^2*b^3 + 75*a*b^4 - 9*b^5)*cos(4*d*x + 4*c)^2 + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(2*d*x + 2*c)^2 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 240*a^4*b + 345*a^3*b^2 - 235*a^2*b^3 + 75*a*b^4 - 9*b^5)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(2*d*x + 2*c)^2 + 2*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 - 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(6*d*x + 6*c) - 2*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 - 2*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*

```
b^5)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(2*d*x +
 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 +
 3*b^5 - 4*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*cos(2*d*x
 + 2*c))*cos(4*d*x + 4*c) - 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(2*d
*x + 2*c) - 4*(2*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(6*d*x + 6*c) + (
8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + ...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5587 vs. $2(160) = 320$.
time = 1.45, size = 5587, normalized size = 27.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/120*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 3
5*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 2
1*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^
4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b
 + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b
^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*
b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^
6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2))*cos
(d*x + c)^5*log(7/4*a^3*b^5 + 35/4*a^2*b^6 + 21/4*a*b^7 + 1/4*b^8 - 1/4*(7*
a^3*b^5 + 35*a^2*b^6 + 21*a*b^7 + b^8)*cos(d*x + c)^2 + 1/2*(4*(a^11 - 6*a^
10*b + 14*a^9*b^2 - 14*a^8*b^3 + 14*a^6*b^5 - 14*a^5*b^6 + 6*a^4*b^7 - a^3*
b^8)*d^3*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 51
1*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*
b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*
a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^
4*b^13 + a^3*b^14)*d^4))*cos(d*x + c)*sin(d*x + c) + (7*a^6*b^3 + 77*a^5*b^
4 + 238*a^4*b^5 + 162*a^3*b^6 + 27*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x
+ c))*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*
a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqr
t((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 +
 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a
^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 20
02*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3
*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a
^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2)) - 1/4*(2*(a^9*b - 7*a^8*b^2 + 21*a^7*b^3
- 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2*cos(d*x +
 c)^2 - (a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*
b^6 + 7*a^3*b^7 - a^2*b^8)*d^2)*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b
^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b +
```

```
91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6
- 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^1
1 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))) - 15*(a^3 - 3*a^2*b + 3*a*
b^2 - b^3)*d*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 - (a^8 - 7*a^7*
b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*
d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2
*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 +
 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b
^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^1
3 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4
 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2))*cos(d*x + c)^5*log(7/4*a^3*b^5 + 3
5/4*a^2*b^6 + 21/4*a*b^7 + 1/4*b^8 - 1/4*(7*a^3*b^5 + 35*a^2*b^6 + 21*a*b^7
 + b^8)*cos(d*x + c)^2 - 1/2*(4*(a^11 - 6*a^10*b + 14*a^9*b^2 - 14*a^8*b^3
+ 14*a^6*b^5 - 14*a^5*b^6 + 6*a^4*b^7 - a^3*b^8)*d^3*sqrt((49*a^6*b^7 + 490
*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/
((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12
*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a
^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))*cos(d*
x + c)*sin(d*x + c) + (7*a^6*b^3 + 77*a^5*b^4 + 238*a^4*b^5 + 162*a^3*b^6 +
 27*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^3*b^3 + 21*a^2*b
^4 + 35*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b
^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1
519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*
a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*
a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 36
4*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b +
 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2
)) - 1/4*(2*(a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*
a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2*cos(d*x + c)^2 - (a^9*b - 7*a^8*b^2 + 21
*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2)
*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^
11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 10
01*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8
- 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 +
 a^3*b^14)*d^4))) + 15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt(-(a^3*b^3 + 2
1*a^2*b^4 + 35*a*b^5 + 7*b^6 + (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 3
5*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*
b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3106 vs. $2(160) = 320$.
time = 0.94, size = 3106, normalized size = 15.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/30*(2*(3*a^4*tan(d*x + c)^5 - 12*a^3*b*tan(d*x + c)^5 + 18*a^2*b^2*tan(d*
x + c)^5 - 12*a*b^3*tan(d*x + c)^5 + 3*b^4*tan(d*x + c)^5 + 10*a^4*tan(d*x
+ c)^3 - 50*a^3*b*tan(d*x + c)^3 + 90*a^2*b^2*tan(d*x + c)^3 - 70*a*b^3*tan
(d*x + c)^3 + 20*b^4*tan(d*x + c)^3 + 15*a^4*tan(d*x + c) - 75*a^3*b*tan(d*
x + c) + 195*a^2*b^2*tan(d*x + c) - 225*a*b^3*tan(d*x + c) + 90*b^4*tan(d*x
 + c))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) + 15*(4*(3
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 - 3*sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 7*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*a*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^5)*(a^
5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)^2*abs(-a + b) - (9*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^9*b^2 - 69*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*a^8*b^3 + 216*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b^4 - 352*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b^5 + 306*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*a^5*b^6 - 114*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^7 - 16*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^8 + 24*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*a^2*b^9 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^10 - sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*b^11)*abs(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3
+ 5*a*b^4 - b^5)*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*a^14*b - 18*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^13*b^2 - 1
9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^12*b^3 + 508*sqrt(a^2 - a
*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^11*b^4 - 2221*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*sqrt(a*b)*a^10*b^5 + 5314*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sq
rt(a*b)*a^9*b^6 - 8139*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^
7 + 8328*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^8 - 5631*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^9 + 2322*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*sqrt(a*b)*a^5*b^10 - 417*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*a^4*b^11 - 68*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
3*b^12 + 41*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^13 - 2*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^14 - sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*b^15)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + ar
ctan(tan(d*x + c)/sqrt((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4
 - a*b^5 + sqrt((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^
5)^2 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*(a^6 -
```

```
 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)))/(a^6 - 6
*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6))))/((3*a^14
- 39*a^13*b + 230*a^12*b^2 - 814*a^11*b^3 + 1925*a^10*b^4 - 3201*a^9*b^5 +
3828*a^8*b^6 - 3300*a^7*b^7 + 2013*a^6*b^8 - 825*a^5*b^9 + 198*a^4*b^10 - 1
4*a^3*b^11 - 5*a^2*b^12 + a*b^13)*abs(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b
^3 + 5*a*b^4 - b^5)) - 15*(4*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*
b)*a^3*b^2 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 7*sq
rt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*b^5)*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*
b^4 - b^5)^2*abs(-a + b) + (9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^9*b^2 -
 69*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^8*b^3 + 216*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*a^7*b^4 - 352*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6*b^5 +
306*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b^6 - 114*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*a^4*b^7 - 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^8 + 2
4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^9 - 3*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*a*b^10 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^11)*abs(a^5 - 5*a
^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*abs(-a + b) - (3*sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^14*b - 18*sqrt(a^2 - a*b - sqrt(a*b)*
(a - b))*sqrt(a*b)*a^13*b^2 - 19*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a^12*b^3 + 508*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^11*b^4 -
 2221*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^10*b^5 + 5314*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b^6 - 8139*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a^8*b^7 + 8328*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*sqrt(a*b)*a^7*b^8 - 5631*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6
*b^9 + 2322*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^10 - 417*sq
rt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^11 - 68*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^12 + 41*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^2*b^13 - 2*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*
b^14 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^15)*abs(-a + b))*(pi
*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^6 - 5*a^5*b + 10*a
^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5 - sqrt((a^6 - 5*a^5*b + 10*a^4*b^2
- 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)^2 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*
b^3 + 5*a^2*b^4 - a*b^5)*(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*
b^4 - 6*a*b^5 + b^6)))/(a^6 - 6*a^5*b + 15*a^4*...
```

Mupad [B]
time = 18.23, size = 2500, normalized size = 12.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(((((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*b^4 + 1
2*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2) - (4*
```

tan(c + d*x)*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 112*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) - (4*tan(c + d*x)*(28*a*b^7 + b^8 + 70*a^2*b^6 + 28*a^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*1i - (((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2) + (4*tan(c + d*x)*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 112*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) + (4*tan(c + d*x)*(28*a*b^7 + b^8 + 70*a^2*b^6 + 28*a^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*1i)/((((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2) - (4*tan(c + d*x)*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 112*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) - (4*tan(c + d*x)*(28*a*b^7 + b^8 + 70*a^2*b^6 + 28*a^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a

^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) + (((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2) + (4*tan(c + d*x)*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 112*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) + (4*tan(c + d*x)*(28*a*b^7 + b^8 + 70*a^2*b^6 + 28*a^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) - (8*(a*b^6 + b^7))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2)))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*...

3.419 $\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Defer[Int][Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A]
time = 8.28, size = 0, normalized size = 0.00

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int \left(\cos^m(fx+e)\right)\left(a+b\left(\sin^4(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^m, x)
```

Fricas [A]
time = 0.72, size = 37, normalized size = 1.42

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^m,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^{m}\,{\left(b\,{\sin\left(e+f\,x\right)}^{4}+a\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^m*(a + b*sin(e + f*x)^4)^p,x)
[Out] int(cos(e + f*x)^m*(a + b*sin(e + f*x)^4)^p, x)
```

3.420 $\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=197

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)}-\frac{(a-b(5+4p))\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+\right.}{bf(5+4p)}$$

```
[Out] sin(f*x+e)*(a+b*sin(f*x+e)^4)^(1+p)/b/f/(5+4*p)-(a-b*(5+4*p))*hypergeom([1/
4, -p],[5/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f*x+e)^4)^p/b/f/(5+4*p)
/((1+b*sin(f*x+e)^4/a)^p)-2/3*hypergeom([3/4, -p],[7/4],-b*sin(f*x+e)^4/a)*
sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)
```

Rubi [A]
time = 0.15, antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3302, 1221, 1218, 252, 251, 372, 371}

$$\frac{\left(1-\frac{a}{4bp+5b}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}-\frac{2\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{3f}+\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{p+1}}{bf(4p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^(1 + p))/(b*f*(5 + 4*p)) + ((1 - a/(5*
b + 4*b*p))*Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e
+ f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) - (2*Hype
rgeometric2F1[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*(a + b*
Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+bx^4\right)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} + \frac{\text{Subst}\left(\int (-a+b(5+4p)\right.}{}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} + \frac{\text{Subst}\left(\int\left(-a\left(1-\frac{b(5+4p)}{a}\right.\right.\right.}{}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} - \frac{2\text{Subst}\left(\int x^2\left(a+bx^4\right)^p\,dx\right.}{f}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} - \frac{\left(2\left(a+b\sin^4(e+fx)\right)^p\left(\right.\right.}{}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} - \frac{(a-b(5+4p))\,{}_2F_1\left(\frac{1}{4},-\right.}{}$$

Mathematica [A]
time = 0.11, size = 141, normalized size = 0.72

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\left(15\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)-10\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin^2(e+fx)+3\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin^4(e+fx)\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^p*(15*Hypergeometric2F1[1/4, -p, 5/4,
-((b*Sin[e + f*x]^4)/a)] - 10*Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e +
f*x]^4)/a)]*Sin[e + f*x]^2 + 3*Hypergeometric2F1[5/4, -p, 9/4, -((b*Sin[e +
 f*x]^4)/a)]*Sin[e + f*x]^4))/(15*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Maple [F]
time = 0.88, size = 0, normalized size = 0.00

$$\int \left(\cos^5(fx+e)\right)\left(a+b\left(\sin^4(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^5, x)
```

Fricas [F]
time = 0.42, size = 37, normalized size = 0.19

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^5,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^{5}\,\left(b\sin\left(e+f\,x\right)^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^4)^p,x)
[Out] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^4)^p, x)
```

3.421 $\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=140

$$\frac{{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f}-\frac{{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)\ldots}{}$$

```
[Out] hypergeom([1/4, -p],[5/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f*x+e)^4)^
p/f/((1+b*sin(f*x+e)^4/a)^p)-1/3*hypergeom([3/4, -p],[7/4],-b*sin(f*x+e)^4/
a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3302, 1218, 252, 251, 372, 371}

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}-\frac{\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) - (Hypergeometric2F1
[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^
4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int (1-x^2)\left(a+bx^4\right)^p\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left((a+bx^4)^p-x^2(a+bx^4)^p\right)\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int (a+bx^4)^p\,dx,x,\sin(e+fx)\right)}{f}-\frac{\text{Subst}\left(\int x^2(a+bx^4)^p\ \ldots\right.}{f}\\
&= \frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^4}{a}\right)^p \ldots\right.}{f}\\
&= \frac{{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\ldots\right.}{f}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 106, normalized size = 0.76

$$
-\frac{\sin(e+fx)\left(-3\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)+{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin^2(e+fx)\right)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{3f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] -1/3*(Sin[e + f*x]*(-3*Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)
/a)] + Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x
]^2)*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Maple [**F**]
time = 1.82, size = 0, normalized size = 0.00

$$\int \left(\cos^{3}\left(fx+e\right)\right) \left(a+b\big(\sin^{4}\left(fx+e\right)\big)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.42, size = 37, normalized size = 0.26

$$\text{integral}\Big(\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{3},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^3,
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,\left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^4)^p,x)
```

```
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^4)^p, x)
```

3.422 $\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] hypergeom([1/4, -p],[5/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f*x+e)^4)^
p/f/((1+b*sin(f*x+e)^4/a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3302, 252, 251}

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int (a+bx^4)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^4}{a}\right)^p\,d\right.}{f}$$

$$= \frac{{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\right.}{f}$$

Mathematica [**A**]
time = 0.01, size = 67, normalized size = 1.00

$$\frac{{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Maple [**F**]
time = 0.42, size = 0, normalized size = 0.00

$$\int\cos(fx+e)\left(a+b\left(\sin^4(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e), x)

Fricas [**F**]
time = 0.46, size = 35, normalized size = 0.52

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e), x)
```

Mupad [**B**]
time = 15.62, size = 64, normalized size = 0.96

$$\frac{\sin\left(e+f\,x\right)\,\left(b\,\sin\left(e+f\,x\right)^4+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\,\sin\left(e+f\,x\right)^4}{a}\right)}{f\,\left(\frac{b\,\sin\left(e+f\,x\right)^4}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^4)^p,x)
```

```
[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^4)^p*hypergeom([1/4, -p], 5/4, -(b*sin(e
+ f*x)^4)/a))/(f*((b*sin(e + f*x)^4)/a + 1)^p)
```

3.423 $\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=158

$$\frac{F_1\left(\frac{1}{4};1,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f}+\frac{F_1\left(\frac{3}{4};1,-p;\right.}{}$$

```
[Out] AppellF1(1/4,1,-p,5/4,sin(f*x+e)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f
*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)+1/3*AppellF1(3/4,1,-p,7/4,sin(f*x+e)^
4,-b*sin(f*x+e)^4/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4
/a)^p)
```

Rubi [A]

time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3302, 1254, 441, 440, 525, 524}

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{4};1,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{f}+\frac{\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{4};1,-p;\frac{7}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (AppellF1[1/4, 1, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (AppellF
1[3/4, 1, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*
(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}
\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^4)^p}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+bx^4)^p}{1-x^4}-\frac{x^2(a+bx^4)^p}{-1+x^4}\right)dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx^4)^p}{1-x^4}\,dx,x,\sin(e+fx)\right)}{f}-\frac{\text{Subst}\left(\int \frac{x^2(a+bx^4)^p}{-1+x^4}\,dx,x,\right.}{f}\\
&= \frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{1-x^4}\,dx\right.}{f}\\
&= \frac{F_1\left(\frac{1}{4};1,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4\right.}{f}
\end{aligned}$$

Mathematica [F]
time = 8.36, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [F]
time = 0.56, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e), x)
```

Fricas [F]
time = 0.41, size = 35, normalized size = 0.22

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^4+a\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x), x)
```

3.424 $\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=239

$$\frac{F_1\left(\frac{1}{4};2,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f}+\frac{2F_1\left(\frac{3}{4};2,-p\right.}{}$$

```
[Out] AppellF1(1/4,2,-p,5/4,sin(f*x+e)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f
*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)+2/3*AppellF1(3/4,2,-p,7/4,sin(f*x+e)^
4,-b*sin(f*x+e)^4/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4
/a)^p)+1/5*AppellF1(5/4,2,-p,9/4,sin(f*x+e)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)
^5*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)
```

Rubi [A]

time = 0.15, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3302, 1254, 441, 440, 525, 524}

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{4};2,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{f}+\frac{\sin^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{4};2,-p;\frac{9}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{5f}+\frac{2\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{4};2,-p;\frac{7}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (AppellF1[1/4, 2, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (2*Appel
lF1[3/4, 2, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^
3*(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (AppellF1[
5/4, 2, -p, 9/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^5*(a
 + b*Sin[e + f*x]^4)^p)/(5*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^4)^p}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+bx^4)^p}{(-1+x^4)^2} + \frac{2x^2(a+bx^4)^p}{(-1+x^4)^2} + \frac{x^4(a+bx^4)^p}{(-1+x^4)^2}\right) dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx^4)^p}{(-1+x^4)^2}\,dx, x, \sin(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int \frac{x^4(a+bx^4)^p}{(-1+x^4)^2}\,dx, x,\right.}{f}$$
$$= \frac{\left((a+b\sin^4(e+fx))^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{(-1+x^4)^2}\,dx\right.}{f}$$
$$= \frac{F_1\left(\frac{1}{4}; 2, -p; \frac{5}{4}; \sin^4(e+fx), -\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(\right.}{f}$$

Mathematica [F]
time = 13.74, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int \left(\sec^3(fx+e)\right)\left(a+b\left(\sin^4(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^3, x)
```

Fricas [F]
time = 0.51, size = 37, normalized size = 0.15

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^3,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,{\sin\left(e+f\,x\right)}^4+a\right)}^p}{{\cos\left(e+f\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^3, x)
```

3.425 $\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Defer[Int][Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A]
time = 5.02, size = 0, normalized size = 0.00

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A]
time = 2.29, size = 0, normalized size = 0.00

$$\int \left(\cos^4(fx+e)\right)\left(a+b\left(\sin^4(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^4, x)
```

Fricas [A]
time = 0.44, size = 37, normalized size = 1.42

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^4,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^4\,{\left(b\,{\sin\left(e+f\,x\right)}^4+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^4)^p,x)
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^4)^p, x)
```

3.426 $\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A]
time = 16.65, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A]
time = 2.87, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^2, x)
```

Fricas [A]
time = 0.46, size = 37, normalized size = 1.42

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^2,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^2\,{\left(b\,{\sin\left(e+f\,x\right)}^4+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^4)^p,x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^4)^p, x)
```

3.427 $\int (a+b\sin^4(e+fx))^p\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left((a+b\sin^4(e+fx))^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^4(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int (a+b\sin^4(e+fx))^p\,dx = \int (a+b\sin^4(e+fx))^p\,dx$$

Mathematica [A]
time = 2.27, size = 0, normalized size = 0.00

$$\int (a+b\sin^4(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int((a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p, x)
```

Fricas [A]
time = 0.41, size = 28, normalized size = 1.65

$$\mathrm{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p,x)
[Out] int((a + b*sin(e + f*x)^4)^p, x)
```

3.428 $\int \sec^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sec^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p ,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \sec^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \sec^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$$

Mathematica [A]
time = 9.38, size = 0, normalized size = 0.00

$$\int \sec^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \left(\sec^2\left(fx+e\right)\right) \left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^2, x)
```

Fricas [A]
time = 0.44, size = 37, normalized size = 1.42

$$\mathrm{integral}\Big(\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\sec\left(fx+e\right)^{2},x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^2,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\sin\left(e+f\,x\right)^4+a\right)^p}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^2,x)
[Out] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^2, x)
```

3.429 $\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A]
time = 12.99, size = 0, normalized size = 0.00

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \left(\sec^4(fx+e)\right)\left(a+b\left(\sin^4(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^4, x)
```

Fricas [A]
time = 0.48, size = 37, normalized size = 1.42

$$\text{integral}\Big(\big(b\cos{(fx+e)}^4-2\,b\cos{(fx+e)}^2+a+b\big)^p\sec{(fx+e)}^4\,,x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^4,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**4)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\sin\left(e+f\,x\right)^4+a\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^4,x)
[Out] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^4, x)
```

3.430 $\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A]
time = 8.28, size = 0, normalized size = 0.00

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(\cos^m(fx+e)\right)\left(a+b\left(\sin^n(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

Fricas [A]
time = 0.42, size = 25, normalized size = 0.96

$$\text{integral}((b\sin(fx+e)^n+a)^p\cos(fx+e)^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos(e+f\,x)^m\,(a+b\sin(e+f\,x)^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^m*(a + b*sin(e + f*x)^n)^p,x)
[Out] int(cos(e + f*x)^m*(a + b*sin(e + f*x)^n)^p, x)
```

3.431 $\int \cos^5(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

Optimal. Leaf size=226

$$\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{f}-\frac{2\,{}_2F_1\left(\frac{3}{n},-p;\frac{3+n}{n};-\frac{b s}{}\right.}{}$$

```
[Out] hypergeom([-p, 1/n],[1+1/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)*(a+b*sin(f*x+e)^n
)^p/f/((1+b*sin(f*x+e)^n/a)^p)-2/3*hypergeom([-p, 3/n],[(3+n)/n],-b*sin(f*x
+e)^n/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^n)^p/f/((1+b*sin(f*x+e)^n/a)^p)+1/5*h
ypergeom([-p, 5/n],[(5+n)/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)^5*(a+b*sin(f*x+e
)^n)^p/f/((1+b*sin(f*x+e)^n/a)^p)
```

Rubi [A]
time = 0.12, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3302, 1907, 252, 251, 372, 371}

$$\frac{\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}+\frac{\sin^5(e+fx)\,(a+b\sin^n(e+fx))^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{5f}-\frac{2\sin^3(e+fx)\,(a+b\sin^n(e+fx))^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p) - (2*Hyper
geometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a
 + b*Sin[e + f*x]^n)^p)/(3*f*(1 + (b*Sin[e + f*x]^n)/a)^p) + (Hypergeometri
c2F1[5/n, -p, (5 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^5*(a + b*Sin
[e + f*x]^n)^p)/(5*f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos^5(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \frac{\text{Subst}\left(\int (1-x^2)^2\,(a+bx^n)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int ((a+bx^n)^p - 2x^2(a+bx^n)^p + x^4(a+bx^n)^p)\,dx, x, \sin(\right.}{f}$$

$$= \frac{\text{Subst}\left(\int (a+bx^n)^p\,dx, x, \sin(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x^4(a+bx^n)^p\right.}{f}$$

$$= \frac{\left((a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^n}{a}\right)^p\right.}{f}$$

$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\,(a+b\sin^n(e+fx))}{f}$$

Mathematica [A]
time = 0.15, size = 155, normalized size = 0.69

$$\frac{\sin(e+fx)\left(15\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)-10\,{}_2F_1\left(\frac{3}{n},-p;\frac{3+n}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin^2(e+fx)+3\,{}_2F_1\left(\frac{5}{n},-p;\frac{5+n}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin^4(e+fx)\right)(a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Sin[e + f*x]*(15*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*
x]^n)/a)] - 10*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a
)]*Sin[e + f*x]^2 + 3*Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*Sin[e + f*
x]^n)/a)]*Sin[e + f*x]^4)*(a + b*Sin[e + f*x]^n)^p)/(15*f*(1 + (b*Sin[e + f
*x]^n)/a)^p)
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(\cos^5(fx+e)\right)\left(a+b(\sin^n(fx+e))\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

Fricas [F]
time = 0.45, size = 25, normalized size = 0.11

$$\mathrm{integral}\left((b\sin(fx+e)^n+a)^p\cos(fx+e)^5\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(e + f\,x)^5\,\left(a + b\sin(e + f\,x)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^n)^p,x)
```

```
[Out] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^n)^p, x)
```

3.432 $\int \cos^3(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

Optimal. Leaf size=148

$$\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{f} - \frac{{}_2F_1\left(\frac{3}{n},-p;\frac{3+n}{n};-\frac{b\sin}{}\right.}{}$$

```
[Out] hypergeom([-p, 1/n],[1+1/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)*(a+b*sin(f*x+e)^n
)^p/f/((1+b*sin(f*x+e)^n/a)^p)-1/3*hypergeom([-p, 3/n],[(3+n)/n],-b*sin(f*x
+e)^n/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^n)^p/f/((1+b*sin(f*x+e)^n/a)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3302, 1907, 252, 251, 372, 371}

$$\frac{\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f} - \frac{\sin^3(e+fx)\,(a+b\sin^n(e+fx))^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p) - (Hyperge
ometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a +
 b*Sin[e + f*x]^n)^p)/(3*f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+bx^n\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\left(a+bx^n\right)^p-x^2\left(a+bx^n\right)^p\right)\,dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(a+bx^n\right)^p\,dx,x,\sin(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int x^2\left(a+bx^n\right)^p$$
$$= \frac{\left(\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^n}{a}\right)^p$$
$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)$$

Mathematica [A]
time = 0.07, size = 114, normalized size = 0.77

$$-\frac{\sin(e+fx)\left(-3\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)+{}_2F_1\left(\frac{3}{n},-p;\frac{3+n}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin^2(e+fx)\right)\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] -1/3*(Sin[e + f*x]*(-3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e
 + f*x]^n)/a)] + Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)
/a)]*Sin[e + f*x]^2)*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a
)^p)
```

Maple [**F**]
time = 0.41, size = 0, normalized size = 0.00

$$\int \left(\cos^{3}\left(fx+e\right)\right)\left(a+b(\sin^{n}\left(fx+e\right))\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.39, size = 25, normalized size = 0.17

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,{\left(a+b\,{\sin\left(e+f\,x\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^n)^p,x)
```

```
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^n)^p, x)
```

3.433 $\int \cos(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

Optimal. Leaf size=69

$$\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] hypergeom([-p, 1/n],[1+1/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)*(a+b*sin(f*x+e)^n
)^p/f/((1+b*sin(f*x+e)^n/a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3302, 252, 251}

$$\frac{\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \frac{\text{Subst}\left(\int (a+bx^n)^p\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\left((a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^n}{a}\right)^p$$
$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\,(a+b\sin^n(e+fx))^p}{f}$$

Mathematica [A]
time = 0.02, size = 69, normalized size = 1.00

$$\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \cos(fx+e)\,(a+b(\sin^n(fx+e)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)

Fricas [F]
time = 0.42, size = 23, normalized size = 0.33

$$\text{integral}\left((b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right),x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")

[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^{n}\left(e+fx\right)\right)^{p}\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**n)**p,x)

[Out] Integral((a + b*sin(e + f*x)**n)**p*cos(e + f*x), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")

[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)

Mupad [B]
time = 15.91, size = 70, normalized size = 1.01

$$\frac{\sin\left(e+f\,x\right)\,{\left(a+b\,{\sin\left(e+f\,x\right)}^n\right)}^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,{\sin\left(e+f\,x\right)}^n}{a}\right)}{f\,{\left(\frac{b\,{\sin\left(e+f\,x\right)}^n}{a}+1\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^n)^p,x)

[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*si
n(e + f*x)^n)/a))/(f*((b*sin(e + f*x)^n)/a + 1)^p)

3.434 $\int \sec(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}(\sec(e+fx)\,(a+b\sin^n(e+fx))^p\,,x)$$

```
[Out] Unintegrable(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \int \sec(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Mathematica [A]
time = 6.82, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\,(a+b(\sin^n(fx+e)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

Fricas [A]
time = 0.40, size = 23, normalized size = 0.96

$$\mathrm{integral}((b\sin(fx+e)^n+a)^p\sec(fx+e),x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\sin(e+f\,x)^n)^p}{\cos(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x),x)
[Out] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x), x)
```

3.435 $\int \sec^3(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sec^3(e+fx)\,(a+b\sin^n(e+fx))^p\,,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^3(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec^3(e+fx)\,(a+b\sin^n(e+fx))^p\,dx=\int \sec^3(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Mathematica [A]
time = 13.77, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\sec^3(fx+e)\right)\left(a+b(\sin^n(fx+e))\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

Fricas [A]
time = 0.43, size = 25, normalized size = 0.96

$$\text{integral}\left((b\sin\left(fx+e\right)^{n}+a)^{p}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**n)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\sin\left(e+f\,x\right)^{n}\right)^{p}}{\cos\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^3,x)
[Out] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^3, x)
```

3.436 $\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A]
time = 35.37, size = 0, normalized size = 0.00

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \left(\cos^4\left(fx+e\right)\right)\left(a+b(\sin^n\left(fx+e\right))\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

Fricas [A]
time = 0.41, size = 25, normalized size = 0.96

$$\text{integral}\left((b\sin\left(fx+e\right)^{n}+a)^{p}\cos\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^4\,{\left(a+b\,{\sin\left(e+f\,x\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^n)^p,x)
```

```
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^n)^p, x)
```

3.437 $\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A]
time = 21.54, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b(\sin^n\left(fx+e\right))\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

Fricas [A]
time = 0.41, size = 25, normalized size = 0.96

$$\text{integral}\left((b\sin\left(fx+e\right)^n+a)^p\cos\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin^n\left(e+fx\right))^p\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**n)**p,x)
[Out] Integral((a + b*sin(e + f*x)**n)**p*cos(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+b\,{\sin\left(e+f\,x\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^n)^p,x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^n)^p, x)
```

3.438 $\int (a+b\sin^n(e+fx))^p \, dx$

Optimal. Leaf size=17

$$\text{Int}\left((a+b\sin^n(e+fx))^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^n(e+fx))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int (a+b\sin^n(e+fx))^p \, dx = \int (a+b\sin^n(e+fx))^p \, dx$$

Mathematica [A]

time = 2.64, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(e+fx))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A]

time = 0.07, size = 0, normalized size = 0.00

$$\int (a+b(\sin^n(fx+e)))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int((a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p, x)
```

Fricas [A]
time = 0.37, size = 16, normalized size = 0.94

$$\text{integral}((b\sin(fx+e)^n+a)^p,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(e+fx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**n)**p,x)
[Out] Integral((a + b*sin(e + f*x)**n)**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int (a+b\sin(e+f\,x)^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p,x)
[Out] int((a + b*sin(e + f*x)^n)^p, x)
```

3.439 $\int \sec^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sec^2(e+fx)\,(a+b\sin^n(e+fx))^p\,,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \int \sec^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Mathematica [A]
time = 8.76, size = 0, normalized size = 0.00

$$\int \sec^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(\sec^2(fx+e)\right)\left(a+b(\sin^n(fx+e))\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

Fricas [A]
time = 0.42, size = 25, normalized size = 0.96

$$\text{integral}\left((b\sin\left(fx+e\right)^{n}+a)^{p}\sec\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(a+b\sin\left(e+f\,x\right)^{n}\right)}^{p}}{{\cos\left(e+f\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^2,x)
[Out] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^2, x)
```

3.440 $\int \sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \int \sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Mathematica [A]
time = 17.58, size = 0, normalized size = 0.00

$$\int \sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(\sec^4(fx+e)\right)\left(a+b(\sin^n(fx+e))\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

Fricas [A]
time = 0.39, size = 25, normalized size = 0.96

$$\text{integral}\left((b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**n)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\sin\left(e+f\,x\right)^n\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^4,x)
[Out] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^4, x)
```

3.441 $\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=128

$$\frac{a^3\log(\cos(c+dx))}{(a+b)^4d}-\frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^4d}+\frac{(3a^2+3ab+b^2)\sec^2(c+dx)}{2(a+b)^3d}-\frac{(3a+2b)\sec^4(c+dx)}{4(a+b)^2d}+\frac{\sec^6}{6(}$$

```
[Out] a^3*ln(cos(d*x+c))/(a+b)^4/d-1/2*a^3*ln(a+b*sin(d*x+c)^2)/(a+b)^4/d+1/2*(3*
a^2+3*a*b+b^2)*sec(d*x+c)^2/(a+b)^3/d-1/4*(3*a+2*b)*sec(d*x+c)^4/(a+b)^2/d+
1/6*sec(d*x+c)^6/(a+b)/d
```

Rubi [A]
time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3273, 90}

$$-\frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^4}+\frac{a^3\log(\cos(c+dx))}{d(a+b)^4}+\frac{(3a^2+3ab+b^2)\sec^2(c+dx)}{2d(a+b)^3}+\frac{\sec^6(c+dx)}{6d(a+b)}-\frac{(3a+2b)\sec^4(c+dx)}{4d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/((a + b)^4*d) - (a^3*Log[a + b*Sin[c + d*x]^2])/(2*
(a + b)^4*d) + ((3*a^2 + 3*a*b + b^2)*Sec[c + d*x]^2)/(2*(a + b)^3*d) - ((3
*a + 2*b)*Sec[c + d*x]^4)/(4*(a + b)^2*d) + Sec[c + d*x]^6/(6*(a + b)*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^4(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^4} + \frac{3a+2b}{(a+b)^2(-1+x)^3} + \frac{3a^2+3ab+b^2}{(a+b)^3(-1+x)^2} + \frac{a^3}{(a+b)^4(-1+x)} - \frac{a^3b}{(a+b)^4(a+bx)}\right.\right.}{2d}$$
$$= \frac{a^3\log(\cos(c+dx))}{(a+b)^4d} - \frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^4d} + \frac{(3a^2+3ab+b^2)\sec^2(c+dx)}{2(a+b)^3d}$$

Mathematica [A]
time = 0.22, size = 113, normalized size = 0.88

$$\frac{\frac{12a^3\log(\cos(c+dx))}{(a+b)^4} - \frac{6a^3\log\left(a+b\sin^2(c+dx)\right)}{(a+b)^4} + \frac{6(3a^2+3ab+b^2)\sec^2(c+dx)}{(a+b)^3} - \frac{3(3a+2b)\sec^4(c+dx)}{(a+b)^2} + \frac{2\sec^6(c+dx)}{a+b}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((12*a^3*Log[Cos[c + d*x]])/(a + b)^4 - (6*a^3*Log[a + b*Sin[c + d*x]^2])/(
a + b)^4 + (6*(3*a^2 + 3*a*b + b^2)*Sec[c + d*x]^2)/(a + b)^3 - (3*(3*a + 2
*b)*Sec[c + d*x]^4)/(a + b)^2 + (2*Sec[c + d*x]^6)/(a + b))/(12*d)
```

Maple [A]
time = 0.49, size = 114, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{a^3\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^4}+\frac{a^3\ln(\cos(dx+c))}{(a+b)^4}-\frac{3a+2b}{4(a+b)^2\cos(dx+c)^4}-\frac{-3a^2-3ab-b^2}{2(a+b)^3\cos(dx+c)^2}+\frac{1}{6(a+b)\cos(dx+c)^6}}{d}$
default	$\frac{-\frac{a^3\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^4}+\frac{a^3\ln(\cos(dx+c))}{(a+b)^4}-\frac{3a+2b}{4(a+b)^2\cos(dx+c)^4}-\frac{-3a^2-3ab-b^2}{2(a+b)^3\cos(dx+c)^2}+\frac{1}{6(a+b)\cos(dx+c)^6}}{d}$
risch	$\frac{6a^2e^{10i(dx+c)}+6ab\,e^{10i(dx+c)}+2b^2e^{10i(dx+c)}+12a^2e^{8i(dx+c)}+4b\,e^{8i(dx+c)}a+\frac{68a^2e^{6i(dx+c)}}{3}+\frac{52ab\,e^{6i(dx+c)}}{3}+\frac{20b^2e^{6i(dx+c)}}{3}}{d(a+b)^3\left(e^{2i(dx+c)}+1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^3/(a+b)^4*ln(a+b-b*cos(d*x+c)^2)+a^3/(a+b)^4*ln(cos(d*x+c))-1/4
*(3*a+2*b)/(a+b)^2/cos(d*x+c)^4-1/2*(-3*a^2-3*a*b-b^2)/(a+b)^3/cos(d*x+c)^2
+1/6/(a+b)/cos(d*x+c)^6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(120) = 240.
time = 0.34, size = 273, normalized size = 2.13

$$-\frac{\frac{6\,a^3\log\left(b\sin(dx+c)^2+a\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{6\,a^3\log\left(\sin(dx+c)^2-1\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{6\,(3\,a^2+3\,ab+b^2)\sin(dx+c)^4-3\,(9\,a^2+7\,ab+2\,b^2)\sin(dx+c)^2+11\,a^2+7\,ab+2\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\sin(dx+c)^6-3\,(a^3+3\,a^2b+3\,ab^2+b^3)\sin(dx+c)^4-a^3-3\,a^2b-3\,ab^2-b^3+3\,(a^3+3\,a^2b+3\,ab^2+b^3)\sin(dx+c)^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*a^3*log(b*sin(d*x + c)^2 + a)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4) - 6*a^3*log(sin(d*x + c)^2 - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4) + (6*(3*a^2 + 3*a*b + b^2)*sin(d*x + c)^4 - 3*(9*a^2 + 7*a*b + 2*b
^2)*sin(d*x + c)^2 + 11*a^2 + 7*a*b + 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*sin(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(d*x + c)^4 - a^3
- 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(d*x + c)^
2))/d
```

Fricas [A]
time = 0.62, size = 179, normalized size = 1.40

$$-\frac{6\,a^3\cos\left(dx+c\right)^6\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-12\,a^3\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)-6\,(3\,a^3+6\,a^2b+4\,ab^2+b^3)\cos\left(dx+c\right)^4-2\,a^3-6\,a^2b-6\,ab^2-2\,b^3+3\,(3\,a^3+8\,a^2b+7\,ab^2+2\,b^3)\cos\left(dx+c\right)^2}{12\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a^3*cos(d*x + c)^6*log(-b*cos(d*x + c)^2 + a + b) - 12*a^3*cos(d*x
 + c)^6*log(-cos(d*x + c)) - 6*(3*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*cos(d*x +
c)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 + 3*(3*a^3 + 8*a^2*b + 7*a*b^2 + 2
*b^3)*cos(d*x + c)^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cos(d*
x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^7\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**7/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(120) = 240.
time = 3.29, size = 603, normalized size = 4.71

$$-\frac{\frac{30\,a^3\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{147\,a^3+\frac{1002\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{120\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2925\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{960\,a^2b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{240\,ab^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4780\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{3600\,a^2b(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2400\,ab^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{640\,b^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2925\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{960\,a^2b(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{240\,ab^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{1002\,a^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{120\,a^2b(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{147\,a^3(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/60*(30*a^3*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 60*a^3*log(abs(-(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4) + (147*a^3 + 1002*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120*a^2*
b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2925*a^3*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 960*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2
40*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4780*a^3*(cos(d*x + c)
 - 1)^3/(cos(d*x + c) + 1)^3 + 3600*a^2*b*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 2400*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 640*b^3*(
cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*a^3*(cos(d*x + c) - 1)^4/(c
os(d*x + c) + 1)^4 + 960*a^2*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
240*a*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^3*(cos(d*x + c
) - 1)^5/(cos(d*x + c) + 1)^5 + 120*a^2*b*(cos(d*x + c) - 1)^5/(cos(d*x + c
) + 1)^5 + 147*a^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)
^6))/d
```

Mupad [B]
time = 14.34, size = 115, normalized size = 0.90

$$\frac{\tan(c+d\,x)^6}{6\,d\,(a+b)}+\frac{a^2\tan(c+d\,x)^2}{2\,d\,(a+b)^3}-\frac{a^3\,\ln\left((a+b)\,\tan(c+d\,x)^2+a\right)}{d\,\left(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4\right)}-\frac{a\tan(c+d\,x)^4}{4\,d\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + b*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)^6/(6*d*(a + b)) + (a^2*tan(c + d*x)^2)/(2*d*(a + b)^3) - (a^3*
log(a + tan(c + d*x)^2*(a + b)))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12
*a^2*b^2)) - (a*tan(c + d*x)^4)/(4*d*(a + b)^2)
```

3.442 $\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{a^2\log(\cos(c+dx))}{(a+b)^3d}+\frac{a^2\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^3d}-\frac{(2a+b)\sec^2(c+dx)}{2(a+b)^2d}+\frac{\sec^4(c+dx)}{4(a+b)d}$$

```
[Out] -a^2*ln(cos(d*x+c))/(a+b)^3/d+1/2*a^2*ln(a+b*sin(d*x+c)^2)/(a+b)^3/d-1/2*(2
*a+b)*sec(d*x+c)^2/(a+b)^2/d+1/4*sec(d*x+c)^4/(a+b)/d
```

Rubi [A]
time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3273, 90}

$$\frac{a^2\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^3}-\frac{a^2\log(\cos(c+dx))}{d(a+b)^3}+\frac{\sec^4(c+dx)}{4d(a+b)}-\frac{(2a+b)\sec^2(c+dx)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/((a + b)^3*d)) + (a^2*Log[a + b*Sin[c + d*x]^2])/
(2*(a + b)^3*d) - ((2*a + b)*Sec[c + d*x]^2)/(2*(a + b)^2*d) + Sec[c + d*x]
^4/(4*(a + b)*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)^3} + \frac{-2a-b}{(a+b)^2(-1+x)^2} - \frac{a^2}{(a+b)^3(-1+x)} + \frac{a^2b}{(a+b)^3(a+bx)}\right) dx, x, \sin^2(\right.}{2d}$$
$$= -\frac{a^2\log(\cos(c+dx))}{(a+b)^3d} + \frac{a^2\log(a+b\sin^2(c+dx))}{2(a+b)^3d} - \frac{(2a+b)\sec^2(c+dx)}{2(a+b)^2d} + \frac{\sec}{4}$$

Mathematica [A]
time = 0.19, size = 78, normalized size = 0.83

$$\frac{2a^2(-2\log(\cos(c+dx)) + \log(a+b\sin^2(c+dx))) - 2(2a^2+3ab+b^2)\sec^2(c+dx) + (a+b)^2\sec^4(c+dx)}{4(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (2*a^2*(-2*Log[Cos[c + d*x]] + Log[a + b*Sin[c + d*x]^2]) - 2*(2*a^2 + 3*a*
b + b^2)*Sec[c + d*x]^2 + (a + b)^2*Sec[c + d*x]^4)/(4*(a + b)^3*d)
```

Maple [A]
time = 0.47, size = 83, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{2a+b}{2(a+b)^2\cos(dx+c)^2}+\frac{1}{4(a+b)\cos(dx+c)^4}-\frac{a^2\ln(\cos(dx+c))}{(a+b)^3}+\frac{a^2\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^3}}{d}$
default	$\frac{-\frac{2a+b}{2(a+b)^2\cos(dx+c)^2}+\frac{1}{4(a+b)\cos(dx+c)^4}-\frac{a^2\ln(\cos(dx+c))}{(a+b)^3}+\frac{a^2\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^3}}{d}$
risch	$-\frac{2\left(2a\,e^{6i(dx+c)}+b\,e^{6i(dx+c)}+2a\,e^{4i(dx+c)}+2a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}\right)}{d(a+b)^2\left(e^{2i(dx+c)}+1\right)^4}-\frac{a^2\ln\left(e^{2i(dx+c)}+1\right)}{d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{a^2\ln\left(e^{4i(dx+c)}\right.}{2d(a^3+3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*(2*a+b)/(a+b)^2/cos(d*x+c)^2+1/4/(a+b)/cos(d*x+c)^4-a^2/(a+b)^3*l
n(cos(d*x+c))+1/2*a^2/(a+b)^3*ln(a+b-b*cos(d*x+c)^2))
```

Maxima [A]
time = 0.34, size = 159, normalized size = 1.69

$$\frac{\frac{2\,a^2\log\left(b\sin(dx+c)^2+a\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\,a^2\log\left(\sin(dx+c)^2-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\,(2\,a+b)\sin(dx+c)^2-3\,a-b}{(a^2+2\,ab+b^2)\sin(dx+c)^4-2\,(a^2+2\,ab+b^2)\sin(dx+c)^2+a^2+2\,ab+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*log(b*sin(d*x + c)^2 + a)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*a^
2*log(sin(d*x + c)^2 - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (2*(2*a + b)*si
n(d*x + c)^2 - 3*a - b)/((a^2 + 2*a*b + b^2)*sin(d*x + c)^4 - 2*(a^2 + 2*a*
b + b^2)*sin(d*x + c)^2 + a^2 + 2*a*b + b^2))/d
```

Fricas [A]
time = 0.49, size = 118, normalized size = 1.26

$$\frac{2\,a^2\cos\left(dx+c\right)^4\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-4\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)-2\left(2\,a^2+3\,ab+b^2\right)\cos\left(dx+c\right)^2+a^2+2\,ab+b^2}{4\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c)^4*log(-b*cos(d*x + c)^2 + a + b) - 4*a^2*cos(d*x +
c)^4*log(-cos(d*x + c)) - 2*(2*a^2 + 3*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*
a*b + b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^5\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(88) = 176.
time = 1.63, size = 393, normalized size = 4.18

$$\frac{\frac{6\,a^2\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{25\,a^2+\frac{124\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{24\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{246\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{144\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{48\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{124\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{24\,ab(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{25\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/12*(6*a^2*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 12*a^2*log(abs(-(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (25*a^2 + 124*a^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24*a*b*(cos(d*x + c) - 1)/(cos(d*x +
```

c) + 1) + 246*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 144*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 48*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 24*a*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 25*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^4))/d

Mupad [B]
time = 14.35, size = 90, normalized size = 0.96

$$\frac{a^2\left(\frac{\ln\left((a+b)\tan(c+d\,x)^2+a\right)}{2}-\frac{\tan(c+d\,x)^2}{2}+\frac{\tan(c+d\,x)^4}{4}\right)+\frac{b^2\tan(c+d\,x)^4}{4}-a\,b\left(\frac{\tan(c+d\,x)^2}{2}-\frac{\tan(c+d\,x)^4}{2}\right)}{d\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (a^2*(log(a + tan(c + d*x)^2*(a + b))/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4/4) + (b^2*tan(c + d*x)^4)/4 - a*b*(tan(c + d*x)^2/2 - tan(c + d*x)^4/2))/(d*(a + b)^3)
```

3.443 $\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{a\log(\cos(c+dx))}{(a+b)^2d}-\frac{a\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^2d}+\frac{\sec^2(c+dx)}{2(a+b)d}$$

```
[Out] a*ln(cos(d*x+c))/(a+b)^2/d-1/2*a*ln(a+b*sin(d*x+c)^2)/(a+b)^2/d+1/2*sec(d*x
+c)^2/(a+b)/d
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3273, 78}

$$\frac{\sec^2(c+dx)}{2d(a+b)}-\frac{a\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^2}+\frac{a\log(\cos(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a*Log[Cos[c + d*x]])/((a + b)^2*d) - (a*Log[a + b*Sin[c + d*x]^2])/(2*(a +
 b)^2*d) + Sec[c + d*x]^2/(2*(a + b)*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2} + \frac{a}{(a+b)^2(-1+x)} - \frac{ab}{(a+b)^2(a+bx)}\right)\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{a\log(\cos(c+dx))}{(a+b)^2 d} - \frac{a\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^2 d} + \frac{\sec^2(c+dx)}{2(a+b)d}$$

Mathematica [A]
time = 0.07, size = 52, normalized size = 0.81

$$\frac{a\left(2\log(\cos(c+dx)) - \log\left(a+b\sin^2(c+dx)\right)\right) + (a+b)\sec^2(c+dx)}{2(a+b)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a*(2*Log[Cos[c + d*x]] - Log[a + b*Sin[c + d*x]^2]) + (a + b)*Sec[c + d*x]
^2)/(2*(a + b)^2*d)
```

Maple [A]
time = 0.36, size = 58, normalized size = 0.91

method	result	size
derivativedivides	$\frac{-\frac{a\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^2}+\frac{a\ln(\cos(dx+c))}{(a+b)^2}+\frac{1}{2(a+b)\cos(dx+c)^2}}{d}$	58
default	$\frac{-\frac{a\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^2}+\frac{a\ln(\cos(dx+c))}{(a+b)^2}+\frac{1}{2(a+b)\cos(dx+c)^2}}{d}$	58
risch	$\frac{2\,\mathrm{e}^{2i(dx+c)}}{d(a+b)\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d(a^2+2ab+b^2)}-\frac{a\ln\left(\mathrm{e}^{4i(dx+c)}-\frac{2(2a+b)\mathrm{e}^{2i(dx+c)}}{b}+1\right)}{2d(a^2+2ab+b^2)}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a/(a+b)^2*ln(a+b-b*cos(d*x+c)^2)+a/(a+b)^2*ln(cos(d*x+c))+1/2/(a+
b)/cos(d*x+c)^2)
```

Maxima [A]
time = 0.34, size = 82, normalized size = 1.28

$$-\frac{\frac{a\log\left(b\sin(dx+c)^2+a\right)}{a^2+2\,ab+b^2} - \frac{a\log\left(\sin(dx+c)^2-1\right)}{a^2+2\,ab+b^2} + \frac{1}{(a+b)\sin(dx+c)^2-a-b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(b*sin(d*x + c)^2 + a)/(a^2 + 2*a*b + b^2) - a*log(sin(d*x + c)^
2 - 1)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(d*x + c)^2 - a - b))/d
```

Fricas [A]
time = 0.43, size = 78, normalized size = 1.22

$$-\frac{a\cos\left(dx+c\right)^{2}\log\left(-b\cos\left(dx+c\right)^{2}+a+b\right)-2\,a\cos\left(dx+c\right)^{2}\log\left(-\cos\left(dx+c\right)\right)-a-b}{2\,(a^{2}+2\,ab+b^{2})d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)^2*log(-b*cos(d*x + c)^2 + a + b) - 2*a*cos(d*x + c)^2*
log(-cos(d*x + c)) - a - b)/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{3}\left(c+dx\right)}{a+b\sin^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(60) = 120.
time = 0.76, size = 234, normalized size = 3.66

$$-\frac{\frac{a\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{a^{2}+2\,ab+b^{2}}-\frac{2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^{2}+2\,ab+b^{2}}+\frac{3\,a+\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{(a^{2}+2\,ab+b^{2})\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(
a^2 + 2*a*b + b^2) - 2*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1
))/(a^2 + 2*a*b + b^2) + (3*a + 10*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/((a^2 + 2*a*b + b^2)*((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1)^2))/d
```

Mupad [B]
time = 14.28, size = 52, normalized size = 0.81

$$-\frac{a\left(\frac{\ln\left((a+b)\tan(c+dx)^2+a\right)}{2}-\frac{\tan(c+dx)^2}{2}\right)-\frac{b\tan(c+dx)^2}{2}}{d\left(a+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x)^2),x)
```

```
[Out] -(a*(log(a + tan(c + d*x)^2*(a + b))/2 - tan(c + d*x)^2/2) - (b*tan(c + d*x
)^2)/2)/(d*(a + b)^2)
```

3.444 $\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=43

$$-\frac{\log(\cos(c+dx))}{(a+b)d}+\frac{\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)d}$$

```
[Out] -ln(cos(d*x+c))/(a+b)/d+1/2*ln(a+b*sin(d*x+c)^2)/(a+b)/d
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3273, 36, 31}

$$\frac{\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)}-\frac{\log(\cos(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(Log[Cos[c + d*x]]/((a + b)*d)) + Log[a + b*Sin[c + d*x]^2]/(2*(a + b)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sin^2(c+dx)\right)}{2(a+b)d} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \sin^2(c+dx)\right)}{2(a+b)d}$$
$$= -\frac{\log(\cos(c+dx))}{(a+b)d} + \frac{\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)d}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 0.86

$$\frac{-2\log(\cos(c+dx)) + \log\left(a+b-b\cos^2(c+dx)\right)}{2ad+2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]^2),x]
[Out] (-2*Log[Cos[c + d*x]] + Log[a + b - b*Cos[c + d*x]^2])/(2*a*d + 2*b*d)
```

Maple [A]
time = 0.38, size = 42, normalized size = 0.98

method	result	size
derivativedivides	$\frac{\frac{\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a+2b}-\frac{\ln(\cos(dx+c))}{a+b}}{d}$	42
default	$\frac{\frac{\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a+2b}-\frac{\ln(\cos(dx+c))}{a+b}}{d}$	42
risch	$-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d(a+b)}+\frac{\ln\left(e^{4i(dx+c)}-\frac{2(2a+b)e^{2i(dx+c)}}{b}+1\right)}{2d(a+b)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2/(a+b)*ln(a+b-b*cos(d*x+c)^2)-1/(a+b)*ln(cos(d*x+c)))
```

Maxima [A]
time = 0.32, size = 43, normalized size = 1.00

$$\frac{\frac{\log\left(b\sin(dx+c)^2+a\right)}{a+b} - \frac{\log\left(\sin(dx+c)^2-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*(log(b*sin(d*x + c)^2 + a)/(a + b) - log(sin(d*x + c)^2 - 1)/(a + b))/d
```

Fricas [A]
time = 0.41, size = 37, normalized size = 0.86

$$\frac{\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-2\,\log\left(-\cos\left(dx+c\right)\right)}{2\left(a+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*(log(-b*cos(d*x + c)^2 + a + b) - 2*log(-cos(d*x + c)))/((a + b)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)**2),x)
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(41) = 82.
time = 0.48, size = 110, normalized size = 2.56

$$\frac{\frac{\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a+b}-\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*(log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a +
 b) - 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/(a + b))/d
```

Mupad [B]
time = 14.51, size = 28, normalized size = 0.65

$$\frac{\ln\left(\left(a+b\right)\,\tan(c+d\,x)^2+a\right)}{d\,\left(2\,a+2\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*sin(c + d*x)^2),x)
[Out] log(a + tan(c + d*x)^2*(a + b))/(d*(2*a + 2*b))
```

3.445 $\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

```
[Out] ln(sin(d*x+c))/a/d-1/2*ln(a+b*sin(d*x+c)^2)/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3273, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]^2]/(2*a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(c+dx)\right)}{2ad} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \sin^2(c+dx)\right)}{2ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]^2]/(2*a*d)
```

Maple [A]
time = 0.26, size = 35, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\frac{\ln(\sin(dx+c))}{a}-\frac{\ln\left(a+\left(\sin^2(dx+c)\right)b\right)}{2a}}{d}$	35
default	$\frac{\frac{\ln(\sin(dx+c))}{a}-\frac{\ln\left(a+\left(\sin^2(dx+c)\right)b\right)}{2a}}{d}$	35
risch	$\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}-\frac{\ln\left(e^{4i(dx+c)}-\frac{2(2a+b)e^{2i(dx+c)}}{b}+1\right)}{2ad}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(sin(d*x+c))-1/2/a*ln(a+sin(d*x+c)^2*b))
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.97

$$-\frac{\frac{\log\left(b\sin(dx+c)^2+a\right)}{a}-\frac{\log\left(\sin(dx+c)^2\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b*sin(d*x + c)^2 + a)/a - log(sin(d*x + c)^2)/a)/d
```

Fricas [A]
time = 0.42, size = 35, normalized size = 0.92

$$-\frac{\log\left(-b\cos\left(dx+c\right)^{2}+a+b\right)-2\,\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(log(-b*cos(d*x + c)^2 + a + b) - 2*log(1/2*sin(d*x + c)))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(c+dx\right)}{a+b\sin^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

Giac [A]
time = 0.47, size = 38, normalized size = 1.00

$$\frac{\frac{\log\left(\sin(dx+c)^{2}\right)}{a}-\frac{\log\left(\left|b\sin(dx+c)^{2}+a\right|\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(log(sin(d*x + c)^2)/a - log(abs(b*sin(d*x + c)^2 + a))/a)/d
```

Mupad [B]
time = 14.43, size = 41, normalized size = 1.08

$$-\frac{\ln\left(a+a\tan(c+d\,x)^{2}+b\tan(c+d\,x)^{2}\right)-2\,\ln\left(\tan(c+d\,x)\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x)^2),x)
```

```
[Out] -(log(a + a*tan(c + d*x)^2 + b*tan(c + d*x)^2) - 2*log(tan(c + d*x)))/(2*a*
d)
```

3.446 $\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=63

$$-\frac{\csc^2(c+dx)}{2ad}-\frac{(a+b)\log(\sin(c+dx))}{a^2d}+\frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2/a/d-(a+b)*ln(sin(d*x+c))/a^2/d+1/2*(a+b)*ln(a+b*sin(d*x+c
)^2)/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3273, 78}

$$\frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}-\frac{(a+b)\log(\sin(c+dx))}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/2*Csc[c + d*x]^2/(a*d) - ((a + b)*Log[Sin[c + d*x]])/(a^2*d) + ((a + b)*
Log[a + b*Sin[c + d*x]^2])/(2*a^2*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2} + \frac{-a-b}{a^2x} + \frac{b(a+b)}{a^2(a+bx)}\right)\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= -\frac{\csc^2(c+dx)}{2ad} - \frac{(a+b)\log(\sin(c+dx))}{a^2d} + \frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}$$

Mathematica [A]
time = 0.11, size = 50, normalized size = 0.79

$$-\frac{a\csc^2(c+dx) + (a+b)\left(2\log(\sin(c+dx)) - \log\left(a+b\sin^2(c+dx)\right)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/2*(a*Csc[c + d*x]^2 + (a + b)*(2*Log[Sin[c + d*x]] - Log[a + b*Sin[c + d
*x]^2]))/(a^2*d)
```

Maple [A]
time = 0.53, size = 101, normalized size = 1.60

method	result
derivativedivides	$\frac{\frac{1}{4a(\cos(dx+c)-1)}+\frac{(-a-b)\ln(\cos(dx+c)-1)}{2a^2}-\frac{1}{4a(1+\cos(dx+c))}+\frac{(-a-b)\ln(1+\cos(dx+c))}{2a^2}+\frac{(a+b)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^2}}{d}$
default	$\frac{\frac{1}{4a(\cos(dx+c)-1)}+\frac{(-a-b)\ln(\cos(dx+c)-1)}{2a^2}-\frac{1}{4a(1+\cos(dx+c))}+\frac{(-a-b)\ln(1+\cos(dx+c))}{2a^2}+\frac{(a+b)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^2}}{d}$
risch	$\frac{2\,e^{2i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}-\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}+\frac{\ln\left(e^{4i(dx+c)}-\frac{2(2a+b)e^{2i(dx+c)}}{b}+1\right)}{2ad}+\ln\left(e^{4i(dx+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a/(cos(d*x+c)-1)+1/2*(-a-b)/a^2*ln(cos(d*x+c)-1)-1/4/a/(1+cos(d*x+
c))+1/2*(-a-b)/a^2*ln(1+cos(d*x+c))+1/2*(a+b)/a^2*ln(a+b-b*cos(d*x+c)^2))
```

Maxima [A]
time = 0.31, size = 56, normalized size = 0.89

$$\frac{\frac{(a+b)\log\left(b\sin(dx+c)^2+a\right)}{a^2} - \frac{(a+b)\log\left(\sin(dx+c)^2\right)}{a^2} - \frac{1}{a\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + b)*log(b*sin(d*x + c)^2 + a)/a^2 - (a + b)*log(sin(d*x + c)^2)/a^
2 - 1/(a*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.44, size = 91, normalized size = 1.44

$$\frac{\left((a+b)\cos\left(dx+c\right)^{2}-a-b\right)\log\left(-b\cos\left(dx+c\right)^{2}+a+b\right)-2\left((a+b)\cos\left(dx+c\right)^{2}-a-b\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+a}{2\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(((a + b)*cos(d*x + c)^2 - a - b)*log(-b*cos(d*x + c)^2 + a + b) - 2*((
a + b)*cos(d*x + c)^2 - a - b)*log(1/2*sin(d*x + c)) + a)/(a^2*d*cos(d*x +
c)^2 - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{3}\left(c+dx\right)}{a+b\sin^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

Giac [A]
time = 0.55, size = 108, normalized size = 1.71

$$\frac{\frac{\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}}{a}+\frac{4\,(a+b)\log\left(\left|-a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+2\,a+4\,b\right|\right)}{a^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x +
 c) + 1))/a + 4*(a + b)*log(abs(-a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) +
 (cos(d*x + c) - 1)/(cos(d*x + c) + 1)) + 2*a + 4*b))/a^2)/d
```

Mupad [B]
time = 14.46, size = 69, normalized size = 1.10

$$\frac{\ln\left(a+a\tan(c+d\,x)^{2}+b\tan(c+d\,x)^{2}\right)\,(a+b)}{2\,a^{2}\,d}-\frac{\cot(c+d\,x)^{2}}{2\,a\,d}-\frac{\ln\left(\tan(c+d\,x)\right)\,(a+b)}{a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (log(a + a*tan(c + d*x)^2 + b*tan(c + d*x)^2)*(a + b))/(2*a^2*d) - cot(c +
d*x)^2/(2*a*d) - (log(tan(c + d*x))*(a + b))/(a^2*d)
```

3.447 $\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{(2a+b)\csc^2(c+dx)}{2a^2d}-\frac{\csc^4(c+dx)}{4ad}+\frac{(a+b)^2\log(\sin(c+dx))}{a^3d}-\frac{(a+b)^2\log\left(a+b\sin^2(c+dx)\right)}{2a^3d}$$

```
[Out] 1/2*(2*a+b)*csc(d*x+c)^2/a^2/d-1/4*csc(d*x+c)^4/a/d+(a+b)^2*ln(sin(d*x+c))/
a^3/d-1/2*(a+b)^2*ln(a+b*sin(d*x+c)^2)/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3273, 90}

$$-\frac{(a+b)^2\log\left(a+b\sin^2(c+dx)\right)}{2a^3d}+\frac{(a+b)^2\log(\sin(c+dx))}{a^3d}+\frac{(2a+b)\csc^2(c+dx)}{2a^2d}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((2*a + b)*Csc[c + d*x]^2)/(2*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a + b)^2*
Log[Sin[c + d*x]])/(a^3*d) - ((a + b)^2*Log[a + b*Sin[c + d*x]^2])/(2*a^3*d
)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} + \frac{-2a-b}{a^2x^2} + \frac{(a+b)^2}{a^3x} - \frac{b(a+b)^2}{a^3(a+bx)}\right)dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{(2a+b)\csc^2(c+dx)}{2a^2d} - \frac{\csc^4(c+dx)}{4ad} + \frac{(a+b)^2\log(\sin(c+dx))}{a^3d} - \frac{(a+b)^2\log(}{}$$

Mathematica [A]
time = 0.36, size = 72, normalized size = 0.81

$$\frac{2a(2a+b)\csc^2(c+dx) - a^2\csc^4(c+dx) + 2(a+b)^2\left(2\log(\sin(c+dx)) - \log\left(a+b\sin^2(c+dx)\right)\right)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
[Out] (2*a*(2*a + b)*Csc[c + d*x]^2 - a^2*Csc[c + d*x]^4 + 2*(a + b)^2*(2*Log[Sin
[c + d*x]] - Log[a + b*Sin[c + d*x]^2]))/(4*a^3*d)
```

Maple [A]
time = 0.54, size = 161, normalized size = 1.81

method	result
derivativedivides	$\frac{-\frac{1}{16a(1+\cos(dx+c))^2} - \frac{-7a-4b}{16a^2(1+\cos(dx+c))} + \frac{(a^2+2ab+b^2)\ln(1+\cos(dx+c))}{2a^3} - \frac{1}{16a(\cos(dx+c)-1)^2} - \frac{7a+4b}{16a^2(\cos(dx+c)-1)} + \frac{(a^2+2}{}}{d}$
default	$\frac{-\frac{1}{16a(1+\cos(dx+c))^2} - \frac{-7a-4b}{16a^2(1+\cos(dx+c))} + \frac{(a^2+2ab+b^2)\ln(1+\cos(dx+c))}{2a^3} - \frac{1}{16a(\cos(dx+c)-1)^2} - \frac{7a+4b}{16a^2(\cos(dx+c)-1)} + \frac{(a^2+2}{}}{d}$
risch	$-\frac{2\left(2a\,e^{6i(dx+c)} + b\,e^{6i(dx+c)} - 2a\,e^{4i(dx+c)} - 2b\,e^{4i(dx+c)} + 2a\,e^{2i(dx+c)} + b\,e^{2i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^4} + \frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad} + \frac{2b\ln(e^{2i}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/16/a/(1+cos(d*x+c))^2-1/16*(-7*a-4*b)/a^2/(1+cos(d*x+c))+1/2*(a^2+2
*a*b+b^2)/a^3*ln(1+cos(d*x+c))-1/16/a/(cos(d*x+c)-1)^2-1/16*(7*a+4*b)/a^2/(
cos(d*x+c)-1)+1/2*(a^2+2*a*b+b^2)/a^3*ln(cos(d*x+c)-1)-1/2*(a^2+2*a*b+b^2)/
a^3*ln(a+b-b*cos(d*x+c)^2))
```

Maxima [A]
time = 0.32, size = 92, normalized size = 1.03

$$-\frac{\frac{2\,(a^2+2\,ab+b^2)\log\left(b\sin(dx+c)^2+a\right)}{a^3} - \frac{2\,(a^2+2\,ab+b^2)\log\left(\sin(dx+c)^2\right)}{a^3} - \frac{2\,(2\,a+b)\sin(dx+c)^2-a}{a^2\sin(dx+c)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(a^2 + 2*a*b + b^2)*log(b*sin(d*x + c)^2 + a)/a^3 - 2*(a^2 + 2*a*b
+ b^2)*log(sin(d*x + c)^2)/a^3 - (2*(2*a + b)*sin(d*x + c)^2 - a)/(a^2*sin(
d*x + c)^4))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(83) = 166$.
time = 0.45, size = 198, normalized size = 2.22

$$-\frac{2\left(2a^2+ab\right)\cos\left(dx+c\right)^2-3a^2-2ab+2\left(\left(a^2+2ab+b^2\right)\cos\left(dx+c\right)^4-2\left(a^2+2ab+b^2\right)\cos\left(dx+c\right)^2+a^2+2ab+b^2\right)\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-4\left(\left(a^2+2ab+b^2\right)\cos\left(dx+c\right)^4-2\left(a^2+2ab+b^2\right)\cos\left(dx+c\right)^2+a^2+2ab+b^2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)}{4\left(a^3d\cos\left(dx+c\right)^4-2a^3d\cos\left(dx+c\right)^2+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*a^2 + a*b)*cos(d*x + c)^2 - 3*a^2 - 2*a*b + 2*((a^2 + 2*a*b + b^
2)*cos(d*x + c)^4 - 2*(a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^
2)*log(-b*cos(d*x + c)^2 + a + b) - 4*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 -
 2*(a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*log(1/2*sin(d*x
+ c)))/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(83) = 166$.
time = 0.52, size = 205, normalized size = 2.30

$$-\frac{\frac{a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+12\,a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+8\,b\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)}{a^2}+\frac{32\left(a^2+2\,ab+b^2\right)\log\left(\left|-a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+2\,a+4\,b\right|\right)}{a^3}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*((a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(
d*x + c) + 1))^2 + 12*a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x +
 c) - 1)/(cos(d*x + c) + 1)) + 8*b*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) +
```

```
 (cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/a^2 + 32*(a^2 + 2*a*b + b^2)*log(a
bs(-a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x
+ c) + 1)) + 2*a + 4*b))/a^3)/d
```

Mupad [B]
time = 14.50, size = 103, normalized size = 1.16

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(a^2+2\,a\,b+b^2\right)}{a^3\,d}-\frac{\ln\left(a+a\tan(c+d\,x)^2+b\tan(c+d\,x)^2\right)\,\left(a^2+2\,a\,b+b^2\right)}{2\,a^3\,d}-\frac{\frac{1}{4\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{2\,a^2}}{d\tan\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (log(tan(c + d*x))*(2*a*b + a^2 + b^2))/(a^3*d) - (log(a + a*tan(c + d*x)^2
 + b*tan(c + d*x)^2)*(2*a*b + a^2 + b^2))/(2*a^3*d) - (1/(4*a) - (tan(c + d
*x)^2*(a + b))/(2*a^2))/(d*tan(c + d*x)^4)
```

3.448 $\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{(3a^2+3ab+b^2)\csc^2(c+dx)}{2a^3d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}-\frac{\csc^6(c+dx)}{6ad}-\frac{(a+b)^3\log(\sin(c+dx))}{a^4d}+\frac{(a+b)^3\log}{}$$

```
[Out] -1/2*(3*a^2+3*a*b+b^2)*csc(d*x+c)^2/a^3/d+1/4*(3*a+b)*csc(d*x+c)^4/a^2/d-1/
6*csc(d*x+c)^6/a/d-(a+b)^3*ln(sin(d*x+c))/a^4/d+1/2*(a+b)^3*ln(a+b*sin(d*x+
c)^2)/a^4/d
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$,
Rules used = {3273, 90}

$$\frac{(a+b)^3\log\left(a+b\sin^2(c+dx)\right)}{2a^4d}-\frac{(a+b)^3\log(\sin(c+dx))}{a^4d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}-\frac{(3a^2+3ab+b^2)\csc^2(c+dx)}{2a^3d}-\frac{\csc^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/2*((3*a^2 + 3*a*b + b^2)*Csc[c + d*x]^2)/(a^3*d) + ((3*a + b)*Csc[c + d*
x]^4)/(4*a^2*d) - Csc[c + d*x]^6/(6*a*d) - ((a + b)^3*Log[Sin[c + d*x]])/(a
^4*d) + ((a + b)^3*Log[a + b*Sin[c + d*x]^2])/(2*a^4*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^4(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{-3a-b}{a^2x^3}+\frac{3a^2+3ab+b^2}{a^3x^2}-\frac{(a+b)^3}{a^4x}+\frac{b(a+b)^3}{a^4(a+bx)}\right)dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= -\frac{(3a^2+3ab+b^2)\csc^2(c+dx)}{2a^3d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}-\frac{\csc^6(c+dx)}{6ad}-\frac{(a+b}{}$$

Mathematica [A]
time = 0.18, size = 100, normalized size = 0.83

$$-\frac{6a(3a^2+3ab+b^2)\csc^2(c+dx)-3a^2(3a+b)\csc^4(c+dx)+2a^3\csc^6(c+dx)+12(a+b)^3\log(\sin(c+dx))-6(a+b)^3\log\left(a+b\sin^2(c+dx)\right)}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/12*(6*a*(3*a^2 + 3*a*b + b^2)*Csc[c + d*x]^2 - 3*a^2*(3*a + b)*Csc[c + d
*x]^4 + 2*a^3*Csc[c + d*x]^6 + 12*(a + b)^3*Log[Sin[c + d*x]] - 6*(a + b)^3
*Log[a + b*Sin[c + d*x]^2])/(a^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(113) = 226$.
time = 0.57, size = 253, normalized size = 2.09

method	result
derivativedivides	$\frac{\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^4}-\frac{1}{48a(1+\cos(dx+c))^3}-\frac{-5a-2b}{32a^2(1+\cos(dx+c))^2}-\frac{19a^2+22ab+8b^2}{32a^3(1+\cos(dx+c))}+\frac{\left(-a^3-3a^2b-\right.}{}}{}$
default	$\frac{\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^4}-\frac{1}{48a(1+\cos(dx+c))^3}-\frac{-5a-2b}{32a^2(1+\cos(dx+c))^2}-\frac{19a^2+22ab+8b^2}{32a^3(1+\cos(dx+c))}+\frac{\left(-a^3-3a^2b-\right.}{}}{}$
risch	$\frac{6a^2\mathrm{e}^{10i(dx+c)}+6ab\,\mathrm{e}^{10i(dx+c)}+2b^2\mathrm{e}^{10i(dx+c)}-12a^2\mathrm{e}^{8i(dx+c)}-20b\,\mathrm{e}^{8i(dx+c)}a-8b^2\mathrm{e}^{8i(dx+c)}+\frac{68a^2\mathrm{e}^{6i(dx+c)}}{3}+28ab\,\mathrm{e}^{6i(d}}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(a^3+3*a^2*b+3*a*b^2+b^3)/a^4*ln(a+b-b*cos(d*x+c)^2)-1/48/a/(1+cos
(d*x+c))^3-1/32*(-5*a-2*b)/a^2/(1+cos(d*x+c))^2-1/32*(19*a^2+22*a*b+8*b^2)/
a^3/(1+cos(d*x+c))+1/2*(-a^3-3*a^2*b-3*a*b^2-b^3)/a^4*ln(1+cos(d*x+c))+1/48
/a/(cos(d*x+c)-1)^3-1/32*(-5*a-2*b)/a^2/(cos(d*x+c)-1)^2-1/32*(-19*a^2-22*a
*b-8*b^2)/a^3/(cos(d*x+c)-1)+1/2*(-a^3-3*a^2*b-3*a*b^2-b^3)/a^4*ln(cos(d*x+
c)-1))
```

Maxima [A]
time = 0.31, size = 137, normalized size = 1.13

$$\frac{\frac{6\,(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(b\sin(dx+c)^2+a\right)}{a^4}-\frac{6\,(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(\sin(dx+c)^2\right)}{a^4}-\frac{6\,(3\,a^2+3\,ab+b^2)\sin(dx+c)^4-3\,(3\,a^2+ab)\sin(dx+c)^2+2\,a^2}{a^3\sin(dx+c)^6}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(b*sin(d*x + c)^2 + a)/a^4 - 6*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(sin(d*x + c)^2)/a^4 - (6*(3*a^2 + 3*a*b
+ b^2)*sin(d*x + c)^4 - 3*(3*a^2 + a*b)*sin(d*x + c)^2 + 2*a^2)/(a^3*sin(d*
x + c)^6))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(113) = 226.
time = 0.50, size = 371, normalized size = 3.07

$$\frac{6\,(3\,a^3+3\,a^2b+ab^2)\cos(dx+c)^4+11\,a^3+15\,a^2b+6\,ab^2-3\,(9\,a^3+11\,a^2b+4\,ab^2)\cos(dx+c)^2+6\,\left((a^3+3\,a^2b+3\,ab^2+b^3)\cos(dx+c)^6-3\,(a^3+3\,a^2b+3\,ab^2+b^3)\cos(dx+c)^4-a^3-3\,a^2b-3\,ab^2-b^3+3\,(a^3+3\,a^2b+3\,ab^2+b^3)\cos(dx+c)^2\right)\log\left(-b\cos(dx+c)^2+a+b\right)-12\,\left((a^3+3\,a^2b+3\,ab^2+b^3)\cos(dx+c)^6-3\,(a^3+3\,a^2b+3\,ab^2+b^3)\cos(dx+c)^4-a^3-3\,a^2b-3\,ab^2-b^3+3\,(a^3+3\,a^2b+3\,ab^2+b^3)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)}{12\,(a^4d\cos(dx+c)^6-3\,a^4d\cos(dx+c)^4+3\,a^4d\cos(dx+c)^2-a^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(3*a^3 + 3*a^2*b + a*b^2)*cos(d*x + c)^4 + 11*a^3 + 15*a^2*b + 6*a*
b^2 - 3*(9*a^3 + 11*a^2*b + 4*a*b^2)*cos(d*x + c)^2 + 6*((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x +
c)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
s(d*x + c)^2)*log(-b*cos(d*x + c)^2 + a + b) - 12*((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4 -
 a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x
+ c)^2)*log(1/2*sin(d*x + c)))/(a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)
^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**7/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(113) = 226.
time = 0.54, size = 353, normalized size = 2.92

$$\frac{\frac{a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^3+12\,a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+6\,ab\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+84\,a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+120\,ab\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+48\,b^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)}{a^3}+\frac{192\,(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(\left|-a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+2\,a+4\,b\right|\right)}{a^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/384*((a^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(co
s(d*x + c) + 1))^3 + 12*a^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d
*x + c) - 1)/(cos(d*x + c) + 1))^2 + 6*a*b*((cos(d*x + c) + 1)/(cos(d*x + c
) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2 + 84*a^2*((cos(d*x + c) +
 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1)) + 120*a*b*(
(cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) +
1)) + 48*b^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/a^3 + 192*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-a*((c
os(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1)
) + 2*a + 4*b))/a^4)/d
```

Mupad [B]
time = 15.35, size = 138, normalized size = 1.14

$$\frac{\ln\left(a+a\tan(c+d\,x)^2+b\tan(c+d\,x)^2\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{2\,a^4\,d}-\frac{\frac{1}{6\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{4\,a^2}+\frac{\tan(c+d\,x)^4\,(a+b)^2}{2\,a^3}}{d\tan\left(c+d\,x\right)^6}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (log(a + a*tan(c + d*x)^2 + b*tan(c + d*x)^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^
3))/(2*a^4*d) - (1/(6*a) - (tan(c + d*x)^2*(a + b))/(4*a^2) + (tan(c + d*x)
^4*(a + b)^2)/(2*a^3))/(d*tan(c + d*x)^6) - (log(tan(c + d*x))*(3*a*b^2 + 3
*a^2*b + a^3 + b^3))/(a^4*d)
```

3.449 $\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}d}-\frac{a^3\tan(c+dx)}{(a+b)^4d}+\frac{a^2\tan^3(c+dx)}{3(a+b)^3d}-\frac{a\tan^5(c+dx)}{5(a+b)^2d}+\frac{\tan^7(c+dx)}{7(a+b)d}$$

```
[Out] a^(7/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(9/2)/d-a^3*tan(d*x+c)
/(a+b)^4/d+1/3*a^2*tan(d*x+c)^3/(a+b)^3/d-1/5*a*tan(d*x+c)^5/(a+b)^2/d+1/7*
tan(d*x+c)^7/(a+b)/d
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3274, 308, 211}

$$\frac{a^{7/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{9/2}}-\frac{a^3\tan(c+dx)}{d(a+b)^4}+\frac{a^2\tan^3(c+dx)}{3d(a+b)^3}+\frac{\tan^7(c+dx)}{7d(a+b)}-\frac{a\tan^5(c+dx)}{5d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(9/2)*d) - (a
^3*Tan[c + d*x])/((a + b)^4*d) + (a^2*Tan[c + d*x]^3)/(3*(a + b)^3*d) - (a*
Tan[c + d*x]^5)/(5*(a + b)^2*d) + Tan[c + d*x]^7/(7*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3274

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a^3}{(a+b)^4}+\frac{a^2x^2}{(a+b)^3}-\frac{ax^4}{(a+b)^2}+\frac{x^6}{a+b}+\frac{a^4}{(a+b)^4(a+(a+b)x^2)}\right)\,dx, x, \tan(c+dx)\right.}{d}$$

$$= -\frac{a^3\tan(c+dx)}{(a+b)^4d}+\frac{a^2\tan^3(c+dx)}{3(a+b)^3d}-\frac{a\tan^5(c+dx)}{5(a+b)^2d}+\frac{\tan^7(c+dx)}{7(a+b)d}+\frac{a^4\text{Subst}}{}$$

$$= \frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}d}-\frac{a^3\tan(c+dx)}{(a+b)^4d}+\frac{a^2\tan^3(c+dx)}{3(a+b)^3d}-\frac{a\tan^5(c+}{5(a+b)}$$

Mathematica [A]
time = 1.59, size = 147, normalized size = 1.22

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}d}+\frac{(-176a^3-122a^2b-66ab^2-15b^3+(122a^3+254a^2b+177ab^2+45b^3)\sec^2(c+dx)-3(a+b)^2(22a+15b)\sec^4(c+dx)+15(a+b)^3\sec^6(c+dx))\tan(c+dx)}{105(a+b)^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(9/2)*d) + ((
-176*a^3 - 122*a^2*b - 66*a*b^2 - 15*b^3 + (122*a^3 + 254*a^2*b + 177*a*b^2
 + 45*b^3)*Sec[c + d*x]^2 - 3*(a + b)^2*(22*a + 15*b)*Sec[c + d*x]^4 + 15*(
a + b)^3*Sec[c + d*x]^6)*Tan[c + d*x])/(105*(a + b)^4*d)
```

Maple [A]
time = 0.66, size = 120, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{\frac{(a+b)\left(a^2+2ab+b^2\right)\left(\tan^7(dx+c)\right)}{7}-\frac{a\left(a^2+2ab+b^2\right)\left(\tan^5(dx+c)\right)}{5}+\frac{a^2\left(\tan^3(dx+c)\right)(a+b)}{3}-a^3\tan(dx+c)}{(a+b)^4}+\frac{a^4\arctan\left(\frac{\tan(dx+c}{\sqrt{a(a}}\right.}{(a+b)^4\sqrt{a(a-}}}{d}$
default	$\frac{\frac{\frac{(a+b)\left(a^2+2ab+b^2\right)\left(\tan^7(dx+c)\right)}{7}-\frac{a\left(a^2+2ab+b^2\right)\left(\tan^5(dx+c)\right)}{5}+\frac{a^2\left(\tan^3(dx+c)\right)(a+b)}{3}-a^3\tan(dx+c)}{(a+b)^4}+\frac{a^4\arctan\left(\frac{\tan(dx+c}{\sqrt{a(a}}\right.}{(a+b)^4\sqrt{a(a-}}}{d}$

risch	$-\frac{2i(224b\,e^{2i(dx+c)}a^2+66a\,b^2+15b^3+176a^3+122a^2b+315b^3e^{4i(dx+c)}+420a\,b^2e^{6i(dx+c)}+1176a\,b^2e^{4i(dx+c)}+1722b\,e^{4i(d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a+b)^4*(1/7*(a+b)*(a^2+2*a*b+b^2)*tan(d*x+c)^7-1/5*a*(a^2+2*a*b+b^2
)*tan(d*x+c)^5+1/3*a^2*tan(d*x+c)^3*(a+b)-a^3*tan(d*x+c))+a^4/(a+b)^4/(a*(a
+b))^(1/2)*arctan(tan(d*x+c)*(a+b)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.54, size = 180, normalized size = 1.50

$$\frac{\frac{105\,a^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)a}}+\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\tan(dx+c)^7-21\,(a^3+2\,a^2b+ab^2)\tan(dx+c)^5-105\,a^3\tan(dx+c)+35\,(a^3+a^2b)\tan(dx+c)^3}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*a^4*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*a)) + (15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*tan(d*x + c)^7 - 21*(a^3 + 2*a^2*b + a*b^2)*tan(d*x + c)^5 - 105*
a^3*tan(d*x + c) + 35*(a^3 + a^2*b)*tan(d*x + c)^3)/(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(106) = 212$.
time = 0.45, size = 602, normalized size = 5.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/420*(105*a^3*sqrt(-a/(a + b))*cos(d*x + c)^7*log(((8*a^2 + 8*a*b + b^2)*
cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b
 + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))
*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(
d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*((176*a^3 + 122*a^2*b + 66*a*b^2 + 15*
b^3)*cos(d*x + c)^6 - (122*a^3 + 254*a^2*b + 177*a*b^2 + 45*b^3)*cos(d*x +
c)^4 - 15*a^3 - 45*a^2*b - 45*a*b^2 - 15*b^3 + 3*(22*a^3 + 59*a^2*b + 52*a*
b^2 + 15*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4
```

```
*a*b^3 + b^4)*d*cos(d*x + c)^7), -1/210*(105*a^3*sqrt(a/(a + b))*arctan(1/2
*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x
 + c)))*cos(d*x + c)^7 + 2*((176*a^3 + 122*a^2*b + 66*a*b^2 + 15*b^3)*cos(d
*x + c)^6 - (122*a^3 + 254*a^2*b + 177*a*b^2 + 45*b^3)*cos(d*x + c)^4 - 15*
a^3 - 45*a^2*b - 45*a*b^2 - 15*b^3 + 3*(22*a^3 + 59*a^2*b + 52*a*b^2 + 15*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*d*cos(d*x + c)^7)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**8/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(106) = 212.
time = 4.93, size = 472, normalized size = 3.93

$$\frac{\frac{105\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^4}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\sqrt{a^2+ab}}+\frac{15a^6\tan(dx+c)^7+90a^5b\tan(dx+c)^7+225a^4b^2\tan(dx+c)^7+300a^3b^3\tan(dx+c)^7+225a^2b^4\tan(dx+c)^7+90ab^5\tan(dx+c)^7+15b^6\tan(dx+c)^7-21a^6\tan(dx+c)^5-105a^5b\tan(dx+c)^5-210a^4b^2\tan(dx+c)^5-210a^3b^3\tan(dx+c)^5-105a^2b^4\tan(dx+c)^5-21ab^5\tan(dx+c)^5+35a^6\tan(dx+c)^3+140a^5b\tan(dx+c)^3+210a^4b^2\tan(dx+c)^3+140a^3b^3\tan(dx+c)^3+35a^2b^4\tan(dx+c)^3-105a^6\tan(dx+c)-315a^5b\tan(dx+c)-315a^4b^2\tan(dx+c)-105a^3b^3\tan(dx+c)}{a^7+7a^6b+21a^5b^2+35a^4b^3+35a^3b^4+21a^2b^5+7ab^6+b^7}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(105*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
 + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^4/((a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*sqrt(a^2 + a*b)) + (15*a^6*tan(d*x + c)^7 + 90*a^5*b*tan(d*
x + c)^7 + 225*a^4*b^2*tan(d*x + c)^7 + 300*a^3*b^3*tan(d*x + c)^7 + 225*a^
2*b^4*tan(d*x + c)^7 + 90*a*b^5*tan(d*x + c)^7 + 15*b^6*tan(d*x + c)^7 - 21
*a^6*tan(d*x + c)^5 - 105*a^5*b*tan(d*x + c)^5 - 210*a^4*b^2*tan(d*x + c)^5
 - 210*a^3*b^3*tan(d*x + c)^5 - 105*a^2*b^4*tan(d*x + c)^5 - 21*a*b^5*tan(d
*x + c)^5 + 35*a^6*tan(d*x + c)^3 + 140*a^5*b*tan(d*x + c)^3 + 210*a^4*b^2*
tan(d*x + c)^3 + 140*a^3*b^3*tan(d*x + c)^3 + 35*a^2*b^4*tan(d*x + c)^3 - 1
05*a^6*tan(d*x + c) - 315*a^5*b*tan(d*x + c) - 315*a^4*b^2*tan(d*x + c) - 1
05*a^3*b^3*tan(d*x + c))/(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*
b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7))/d
```

Mupad [B]
time = 14.78, size = 141, normalized size = 1.18

$$\frac{\tan(c+d\,x)^7}{7\,d\,(a+b)}+\frac{a^2\tan(c+d\,x)^3}{3\,d\,(a+b)^3}+\frac{a^{7/2}\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}{2\,\sqrt{a}\,(a+b)^{9/2}}\right)}{d\,(a+b)^{9/2}}-\frac{a\tan(c+d\,x)^5}{5\,d\,(a+b)^2}-\frac{a^3\tan(c+d\,x)}{d\,(a+b)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8/(a + b*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)^7/(7*d*(a + b)) + (a^2*tan(c + d*x)^3)/(3*d*(a + b)^3) + (a^(7
/2)*atan((tan(c + d*x)*(2*a + 2*b)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b
^2))/(2*a^(1/2)*(a + b)^(9/2))))/(d*(a + b)^(9/2)) - (a*tan(c + d*x)^5)/(5*
d*(a + b)^2) - (a^3*tan(c + d*x))/(d*(a + b)^4)
```

3.450 $\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{7/2}d}+\frac{a^2\tan(c+dx)}{(a+b)^3d}-\frac{a\tan^3(c+dx)}{3(a+b)^2d}+\frac{\tan^5(c+dx)}{5(a+b)d}$$

```
[Out] -a^(5/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(7/2)/d+a^2*tan(d*x+c
)/(a+b)^3/d-1/3*a*tan(d*x+c)^3/(a+b)^2/d+1/5*tan(d*x+c)^5/(a+b)/d
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3274, 308, 211}

$$-\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{7/2}}+\frac{a^2\tan(c+dx)}{d(a+b)^3}+\frac{\tan^5(c+dx)}{5d(a+b)}-\frac{a\tan^3(c+dx)}{3d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(7/2)*d)) +
 (a^2*Tan[c + d*x])/((a + b)^3*d) - (a*Tan[c + d*x]^3)/(3*(a + b)^2*d) + Ta
n[c + d*x]^5/(5*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3274

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{(a+b)^3} - \frac{ax^2}{(a+b)^2} + \frac{x^4}{a+b} - \frac{a^3}{(a+b)^3(a+(a+b)x^2)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{a^2\tan(c+dx)}{(a+b)^3d} - \frac{a\tan^3(c+dx)}{3(a+b)^2d} + \frac{\tan^5(c+dx)}{5(a+b)d} - \frac{a^3\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan\right.}{(a+b)^3d}$$
$$= -\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{7/2}d} + \frac{a^2\tan(c+dx)}{(a+b)^3d} - \frac{a\tan^3(c+dx)}{3(a+b)^2d} + \frac{\tan^5(c+d}{5(a+b)d}$$

Mathematica [A]
time = 0.54, size = 111, normalized size = 1.14

$$\frac{-15a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right) + \sqrt{a+b}\,(23a^2+11ab+3b^2-(11a^2+17ab+6b^2)\sec^2(c+dx)+3(a+b)^2\sec^4(c+dx))\tan(c+dx)}{15(a+b)^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-15*a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a + b]*(23*a
^2 + 11*a*b + 3*b^2 - (11*a^2 + 17*a*b + 6*b^2)*Sec[c + d*x]^2 + 3*(a + b)^
2*Sec[c + d*x]^4)*Tan[c + d*x])/(15*(a + b)^(7/2)*d)
```

Maple [A]
time = 0.55, size = 121, normalized size = 1.25

method	result
derivativedivides	$\frac{\frac{\frac{a^2(\tan^5(dx+c))}{5}+\frac{2ab(\tan^5(dx+c))}{5}+\frac{b^2(\tan^5(dx+c))}{5}-\frac{a^2(\tan^3(dx+c))}{3}-\frac{ab(\tan^3(dx+c))}{3}+a^2\tan(dx+c)}{(a+b)^3}-\frac{a^3\arctan\left(\frac{\tan(dx+c}{\sqrt{a(a}}\right.}{(a+b)^3\sqrt{a(a-}}}{d}$
default	$\frac{\frac{\frac{a^2(\tan^5(dx+c))}{5}+\frac{2ab(\tan^5(dx+c))}{5}+\frac{b^2(\tan^5(dx+c))}{5}-\frac{a^2(\tan^3(dx+c))}{3}-\frac{ab(\tan^3(dx+c))}{3}+a^2\tan(dx+c)}{(a+b)^3}-\frac{a^3\arctan\left(\frac{\tan(dx+c}{\sqrt{a(a}}\right.}{(a+b)^3\sqrt{a(a-}}}{d}$

risch	$\frac{2i(45a^2e^{8i(dx+c)}+45b\,e^{8i(dx+c)}a+15b^2e^{8i(dx+c)}+90a^2e^{6i(dx+c)}+30ab\,e^{6i(dx+c)}+140a^2e^{4i(dx+c)}+80ab\,e^{4i(dx+c)}+30b}{15d(a+b)^3\left(e^{2i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a+b)^3*(1/5*a^2*tan(d*x+c)^5+2/5*a*tan(d*x+c)^5*b+1/5*b^2*tan(d*x+c
)^5-1/3*a^2*tan(d*x+c)^3-1/3*a*tan(d*x+c)^3*b+a^2*tan(d*x+c))-a^3/(a+b)^3/(
a*(a+b))^(1/2)*arctan(tan(d*x+c)*(a+b)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.52, size = 130, normalized size = 1.34

$$-\frac{\frac{15\,a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)a}}-\frac{3\,(a^2+2\,ab+b^2)\tan(dx+c)^5-5\,(a^2+ab)\tan(dx+c)^3+15\,a^2\tan(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*a^3*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*sqrt((a + b)*a)) - (3*(a^2 + 2*a*b + b^2)*tan(d*x + c)^5 -
 5*(a^2 + a*b)*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(85) = 170$.
time = 0.43, size = 472, normalized size = 4.87

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{a+b}}\cos(dx+c)^5\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+2\,ab+b^2)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)+4\left((23\,a^2+11\,ab+3\,b^2)\cos(dx+c)^4-(11\,a^2+17\,ab+6\,b^2)\cos(dx+c)^2+3\,a^2+6\,ab+3\,b^2\right)\sin(dx+c)}{60\,(a^3+3\,a^2b+3\,ab^2+b^3)d\cos(dx+c)^5},\ \frac{15\,a^2\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)\cos(dx+c)^5+2\left((23\,a^2+11\,ab+3\,b^2)\cos(dx+c)^4-(11\,a^2+17\,ab+6\,b^2)\cos(dx+c)^2+3\,a^2+6\,ab+3\,b^2\right)\sin(dx+c)}{30\,(a^3+3\,a^2b+3\,ab^2+b^3)d\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*a^2*sqrt(-a/(a + b))*cos(d*x + c)^5*log(((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b +
 b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*s
in(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*
x + c)^2 + a^2 + 2*a*b + b^2)) + 4*((23*a^2 + 11*a*b + 3*b^2)*cos(d*x + c)^
4 - (11*a^2 + 17*a*b + 6*b^2)*cos(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sin(d
*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^5), 1/30*(15*a^2*s
qrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b)
```

```
)/(a*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^5 + 2*((23*a^2 + 11*a*b + 3*b
^2)*cos(d*x + c)^4 - (11*a^2 + 17*a*b + 6*b^2)*cos(d*x + c)^2 + 3*a^2 + 6*a
*b + 3*b^2)*sin(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^5
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**6/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(85) = 170.
time = 2.19, size = 296, normalized size = 3.05

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^3}{(a^3+3a^2b+3ab^2+b^3)\sqrt{a^2+ab}}-\frac{3a^4\tan(dx+c)^5+12a^3b\tan(dx+c)^5+18a^2b^2\tan(dx+c)^5+12ab^3\tan(dx+c)^5+3b^4\tan(dx+c)^5-5a^4\tan(dx+c)^3-15a^3b\tan(dx+c)^3-15a^2b^2\tan(dx+c)^3-5ab^3\tan(dx+c)^3+15a^4\tan(dx+c)+30a^3b\tan(dx+c)+15a^2b^2\tan(dx+c)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
+ c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^3/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*sqrt(a^2 + a*b)) - (3*a^4*tan(d*x + c)^5 + 12*a^3*b*tan(d*x + c)^5 + 18*
a^2*b^2*tan(d*x + c)^5 + 12*a*b^3*tan(d*x + c)^5 + 3*b^4*tan(d*x + c)^5 - 5
*a^4*tan(d*x + c)^3 - 15*a^3*b*tan(d*x + c)^3 - 15*a^2*b^2*tan(d*x + c)^3 -
 5*a*b^3*tan(d*x + c)^3 + 15*a^4*tan(d*x + c) + 30*a^3*b*tan(d*x + c) + 15*
a^2*b^2*tan(d*x + c))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
b^5))/d
```

Mupad [B]
time = 15.11, size = 112, normalized size = 1.15

$$\frac{\tan(c+d\,x)^5}{5\,d\,(a+b)}-\frac{a^{5/2}\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{2\,\sqrt{a}\,(a+b)^{7/2}}\right)}{d\,(a+b)^{7/2}}-\frac{a\tan(c+d\,x)^3}{3\,d\,(a+b)^2}+\frac{a^2\tan(c+d\,x)}{d\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + b*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)^5/(5*d*(a + b)) - (a^(5/2)*atan((tan(c + d*x)*(2*a + 2*b)*(3*a
*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2))))/(d*(a + b)^(7/2))
- (a*tan(c + d*x)^3)/(3*d*(a + b)^2) + (a^2*tan(c + d*x))/(d*(a + b)^3)
```

3.451 $\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}d}-\frac{a\tan(c+dx)}{(a+b)^2d}+\frac{\tan^3(c+dx)}{3(a+b)d}$$

```
[Out] a^(3/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(5/2)/d-a*tan(d*x+c)/(
a+b)^2/d+1/3*tan(d*x+c)^3/(a+b)/d
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3274, 308, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a+b)}-\frac{a\tan(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(5/2)*d) - (a
*Tan[c + d*x])/((a + b)^2*d) + Tan[c + d*x]^3/(3*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3274

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(a+b)^2}+\frac{x^2}{a+b}+\frac{a^2}{(a+b)^2(a+(a+b)x^2)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{a\tan(c+dx)}{(a+b)^2 d}+\frac{\tan^3(c+dx)}{3(a+b)d}+\frac{a^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)^2 d}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}d}-\frac{a\tan(c+dx)}{(a+b)^2 d}+\frac{\tan^3(c+dx)}{3(a+b)d}$$

Mathematica [A]
time = 0.23, size = 75, normalized size = 1.01

$$\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a+b}\,(-4a-b+(a+b)\sec^2(c+dx))\tan(c+dx)}{3(a+b)^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
[Out] (3*a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a + b]*(-4*a -
 b + (a + b)*Sec[c + d*x]^2)*Tan[c + d*x])/(3*(a + b)^(5/2)*d)
```

Maple [A]
time = 0.52, size = 78, normalized size = 1.05

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^3(dx+c)\right)}{3}+\frac{b\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)a}{(a+b)^2}+\frac{a^2\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{(a+b)^2\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^3(dx+c)\right)}{3}+\frac{b\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)a}{(a+b)^2}+\frac{a^2\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{(a+b)^2\sqrt{a(a+b)}}}{d}$
risch	$-\frac{2i\left(6a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}+6a\,e^{2i(dx+c)}+4a+b\right)}{3d(a+b)^2\left(e^{2i(dx+c)}+1\right)^3}+\frac{\sqrt{-a(a+b)}\,a\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+b)}-2a-}{b}\right.}{2(a+b)^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a+b)^2*(1/3*a*tan(d*x+c)^3+1/3*b*tan(d*x+c)^3-tan(d*x+c)*a)+a^2/(a+
b)^2/(a*(a+b))^(1/2)*arctan(tan(d*x+c)*(a+b)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.54, size = 85, normalized size = 1.15

$$\frac{\frac{3\,a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+2\,ab+b^2)}+\frac{(a+b)\tan(dx+c)^3-3\,a\tan(dx+c)}{a^2+2\,ab+b^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a
^2 + 2*a*b + b^2)) + ((a + b)*tan(d*x + c)^3 - 3*a*tan(d*x + c))/(a^2 + 2*a
*b + b^2))/d
```

Fricas [A]
time = 0.45, size = 366, normalized size = 4.95

$$\left[\frac{3\,a\sqrt{-\frac{a}{a+b}}\cos\left(dx+c\right)^3\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2-4\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+2\,ab+b^2)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)-4\left((4\,a+b)\cos\left(dx+c\right)^2-a-b\right)\sin\left(dx+c\right)}{12\,(a^2+2\,ab+b^2)d\cos\left(dx+c\right)^3},\ -\frac{3\,a\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)\cos\left(dx+c\right)^3+2\left((4\,a+b)\cos\left(dx+c\right)^2-a-b\right)\sin\left(dx+c\right)}{6\,(a^2+2\,ab+b^2)d\cos\left(dx+c\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(-a/(a + b))*cos(d*x + c)^3*log(((8*a^2 + 8*a*b + b^2)*cos(d
*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^
2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(
d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x +
 c)^2 + a^2 + 2*a*b + b^2)) - 4*((4*a + b)*cos(d*x + c)^2 - a - b)*sin(d*x
+ c))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^3), -1/6*(3*a*sqrt(a/(a + b))*arc
tan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*
sin(d*x + c)))*cos(d*x + c)^3 + 2*((4*a + b)*cos(d*x + c)^2 - a - b)*sin(d*
x + c))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^4\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(64) = 128.
time = 1.01, size = 164, normalized size = 2.22

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^2}{(a^2+2ab+b^2)\sqrt{a^2+ab}}+\frac{a^2\tan(dx+c)^3+2ab\tan(dx+c)^3+b^2\tan(dx+c)^3-3a^2\tan(dx+c)-3ab\tan(dx+c)}{a^3+3a^2b+3ab^2+b^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^2/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a
*b)) + (a^2*tan(d*x + c)^3 + 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c)^3 - 3*
a^2*tan(d*x + c) - 3*a*b*tan(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

Mupad [B]
time = 15.18, size = 83, normalized size = 1.12

$$\frac{\tan(c+dx)^3}{3d\,(a+b)}-\frac{a\tan(c+dx)}{d\,(a+b)^2}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\tan(c+dx)\,(2a+2b)\,(a^2+2ab+b^2)}{2\sqrt{a}\,(a+b)^{5/2}}\right)}{d\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)^3/(3*d*(a + b)) - (a*tan(c + d*x))/(d*(a + b)^2) + (a^(3/2)*at
an((tan(c + d*x)*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2))
))/(d*(a + b)^(5/2))
```

3.452 $\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}d}+\frac{\tan(c+dx)}{(a+b)d}$$

```
[Out] -arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1/2)/(a+b)^(3/2)/d+tan(d*x+c)/(a
+b)/d
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3274, 327, 211}

$$\frac{\tan(c+dx)}{d(a+b)}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(3/2)*d)) +
 Tan[c + d*x]/((a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3274

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
```

p]

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{\tan(c+dx)}{(a+b)d} - \frac{a\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)d} \\
&= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}d} + \frac{\tan(c+dx)}{(a+b)d}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 53, normalized size = 1.00

$$
-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}d} + \frac{\tan(c+dx)}{(a+b)d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(3/2)*d)) +
 Tan[c + d*x]/((a + b)*d)
```

Maple [A]
time = 0.50, size = 51, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{a+b}-\frac{a\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{(a+b)\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{\tan(dx+c)}{a+b}-\frac{a\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{(a+b)\sqrt{a(a+b)}}}{d}$

risch	$\frac{2i}{d(a+b)\left(e^{2i(dx+c)}+1\right)}-\frac{\sqrt{-a\left(a+b\right)}\,\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a\left(a+b\right)}-2a-b}{b}\right)}{2(a+b)^2d}+\frac{\sqrt{-a\left(a+b\right)}\,\ln\Big(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a+b)*tan(d*x+c)-a/(a+b)/(a*(a+b))^(1/2)*arctan(tan(d*x+c)*(a+b)/(a*
(a+b))^(1/2)))
```

Maxima [A]
time = 0.53, size = 51, normalized size = 0.96

$$-\frac{\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a+b)}-\frac{\tan(dx+c)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(a*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b))
- tan(d*x + c)/(a + b))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(45) = 90.
time = 0.41, size = 300, normalized size = 5.66

$$\left[\frac{\sqrt{-\frac{a}{a+b}}\cos\left(dx+c\right)\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(dx+c)^2+4\left(\left(2a^2+3ab+b^2\right)\cos(dx+c)^3-\left(a^2+2ab+b^2\right)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2+2ab+b^2}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2ab+b^2}\right)+4\sin\left(dx+c\right)}{4\left(a+b\right)d\cos\left(dx+c\right)},\ \frac{\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left(\left(2a+b\right)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2a\cos(dx+c)\sin(dx+c)}\right)\cos\left(dx+c\right)+2\sin\left(dx+c\right)}{2\left(a+b\right)d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/(a + b))*cos(d*x + c)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)
^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(
d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c
) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 +
 a^2 + 2*a*b + b^2)) + 4*sin(d*x + c))/((a + b)*d*cos(d*x + c)), 1/2*(sqrt(
a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a
*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c) + 2*sin(d*x + c))/((a + b)*d*cos(
d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

Giac [A]
time = 0.61, size = 86, normalized size = 1.62

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}\,(a+b)}-\frac{\tan(dx+c)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*a/(sqrt(a^2 + a*b)*(a + b)) - tan(d*x + c)
/(a + b))/d
```

Mupad [B]
time = 14.92, size = 53, normalized size = 1.00

$$\frac{\tan(c+d\,x)}{d\,(a+b)}-\frac{\sqrt{a}\,\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)}{d\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)/(d*(a + b)) - (a^(1/2)*atan((tan(c + d*x)*(2*a + 2*b))/(2*a^(1
/2)*(a + b)^(1/2))))/(d*(a + b)^(3/2))
```

3.453 $\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d-arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*(a+b)^(1/2)/a^(3/2)/
d
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3274, 331, 211}

$$-\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - C
ot[c + d*x]/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3274

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{\cot(c+dx)}{ad} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{ad}\\
&= -\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)}{ad}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 52, normalized size = 1.00

$$
\frac{-\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right) - \sqrt{a}\,\cot(c+dx)}{a^{3/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
[Out] (-(Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]) - Sqrt[a]*Cot[c
+ d*x])/(a^(3/2)*d)
```

Maple [A]
time = 0.52, size = 55, normalized size = 1.06

method	result
derivativedivides	$\frac{\frac{(-a-b)\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}} - \frac{1}{a\tan(dx+c)}}{d}$
default	$\frac{\frac{(-a-b)\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}} - \frac{1}{a\tan(dx+c)}}{d}$
risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)} - \frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)} + \frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2a^2d} + \frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*(-a-b)/(a*(a+b))^(1/2)*arctan(tan(d*x+c)*(a+b)/(a*(a+b))^(1/2))-1/
a/tan(d*x+c))
```

Maxima [A]
time = 0.51, size = 50, normalized size = 0.96

$$-\frac{\frac{(a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -((a + b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a)
+ 1/(a*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(44) = 88.
time = 0.41, size = 290, normalized size = 5.58

$$\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2(4a^2+5ab+b^2)\cos(dx+c)^2+4\left((2a^2+ab)\cos(dx+c)^3-(a^2+ab)\cos(dx+c)\right)\sqrt{-\frac{a+b}{a}}\sin(dx+c)+a^2+2ab+b^2}{b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right)\sin(dx+c)-4\cos(dx+c)}{4ad\sin(dx+c)}, \frac{\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2(a+b)\cos(dx+c)\sin(dx+c)}\right)\sin(dx+c)-2\cos(dx+c)}{2ad\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2
 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a
*b)*cos(d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*c
os(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x
+ c) - 4*cos(d*x + c))/(a*d*sin(d*x + c)), 1/2*(sqrt((a + b)/a)*arctan(1/2*
((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(d*x + c)*si
n(d*x + c)))*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

Giac [A]
time = 0.50, size = 85, normalized size = 1.63

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}\,a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*(a + b)/(sqrt(a^2 + a*b)*a) + 1/(a*tan(d*x
 + c)))/d
```

Mupad [B]
time = 14.71, size = 44, normalized size = 0.85

$$-\frac{\cot(c+d\,x)}{a\,d}-\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)\,\sqrt{a+b}}{a^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x)^2),x)
```

```
[Out] - cot(c + d*x)/(a*d) - (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a + b)^
(1/2))/(a^(3/2)*d)
```

3.454 $\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

```
[Out] (a+b)^(3/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(5/2)/d+(a+b)*cot(d*x+
c)/a^2/d-1/3*cot(d*x+c)^3/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3274, 331, 211}

$$\frac{(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*d) + ((
a + b)*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3274

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^3(c+dx)}{3ad} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{x^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{(a+b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx$$

$$= \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad}$$

Mathematica [A]
time = 0.22, size = 72, normalized size = 1.01

$$\frac{3(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right) + \sqrt{a}\cot(c+dx)\left(4a+3b-a\csc^2(c+dx)\right)}{3a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (3*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Cot[c
 + d*x]*(4*a + 3*b - a*Csc[c + d*x]^2))/(3*a^(5/2)*d)
```

Maple [A]
time = 0.57, size = 79, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{\left(a^2+2ab+b^2\right)\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}} - \frac{1}{3a\tan(dx+c)^3} - \frac{-a-b}{a^2\tan(dx+c)}}{d}$
default	$\frac{\frac{\left(a^2+2ab+b^2\right)\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}} - \frac{1}{3a\tan(dx+c)^3} - \frac{-a-b}{a^2\tan(dx+c)}}{d}$
risch	$\frac{2i\left(6a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-6a\,e^{2i(dx+c)}-6b\,e^{2i(dx+c)}+4a+3b\right)}{3d\,a^2\left(e^{2i(dx+c)}-1\right)^3} + \frac{\sqrt{-a(a+b)}\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+}}{b}\right.}{2a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((a^2+2*a*b+b^2)/a^2/(a*(a+b))^(1/2)*arctan(tan(d*x+c)*(a+b)/(a*(a+b))^
(1/2))-1/3/a/tan(d*x+c)^3-1/a^2*(-a-b)/tan(d*x+c))
```

Maxima [A]
time = 0.56, size = 76, normalized size = 1.07

$$\frac{\frac{3\left(a^2+2\,ab+b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^2}+\frac{3\,(a+b)\tan(dx+c)^2-a}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2 + 2*a*b + b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sq
rt((a + b)*a)*a^2) + (3*(a + b)*tan(d*x + c)^2 - a)/(a^2*tan(d*x + c)^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(61) = 122.
time = 0.43, size = 402, normalized size = 5.66

$$\left[\frac{4\,(4\,a+3\,b)\cos(dx+c)^3+3\,\left((a+b)\cos(dx+c)^2-a-b\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2-4\,\left((2\,a^2+ab)\cos(dx+c)^3-(a^2+ab)\cos(dx+c)\right)\sqrt{-\frac{a+b}{a}}\sin(dx+c)+a^2+2\,ab+b^2}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)\sin(dx+c)-12\,(a+b)\cos(dx+c)}{12\,(a^2d\cos(dx+c)^2-a^2d)\sin(dx+c)},\ \frac{2\,(4\,a+3\,b)\cos(dx+c)^3-3\,\left((a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2\,(a+b)\cos(dx+c)\sin(dx+c)}\right)\sin(dx+c)-6\,(a+b)\cos(dx+c)}{6\,(a^2d\cos(dx+c)^2-a^2d)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(4*a + 3*b)*cos(d*x + c)^3 + 3*((a + b)*cos(d*x + c)^2 - a - b)*sq
rt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b
 + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*b)*cos(
d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x +
 c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 1
2*(a + b)*cos(d*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c)), 1/6*
(2*(4*a + 3*b)*cos(d*x + c)^3 - 3*((a + b)*cos(d*x + c)^2 - a - b)*sqrt((a
+ b)/a)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a +
 b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 6*(a + b)*cos(d*x + c))/((a^
2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

Giac [A]
time = 0.51, size = 120, normalized size = 1.69

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(a^2+2ab+b^2)}{\sqrt{a^2+ab}\,a^2}+\frac{3a\tan(dx+c)^2+3b\tan(dx+c)^2-a}{a^2\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*
a^2) + (3*a*tan(d*x + c)^2 + 3*b*tan(d*x + c)^2 - a)/(a^2*tan(d*x + c)^3))/
d
```

Mupad [B]
time = 15.09, size = 64, normalized size = 0.90

$$\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)\,(a+b)^{3/2}}{a^{5/2}\,d}-\frac{\frac{1}{3\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{a^2}}{d\tan\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a + b)^(3/2))/(a^(5/2)*d) - (1
/(3*a) - (tan(c + d*x)^2*(a + b))/a^2)/(d*tan(c + d*x)^3)
```

3.455 $\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d}-\frac{(a+b)^2\cot(c+dx)}{a^3d}+\frac{(a+b)\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5ad}$$

```
[Out] -(a+b)^(5/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/d-(a+b)^2*cot(d
*x+c)/a^3/d+1/3*(a+b)*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+c)^5/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3274, 331, 211}

$$-\frac{(a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d}-\frac{(a+b)^2\cot(c+dx)}{a^3d}+\frac{(a+b)\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(7/2)*d)) -
 ((a + b)^2*Cot[c + d*x])/(a^3*d) + ((a + b)*Cot[c + d*x]^3)/(3*a^2*d) - Co
t[c + d*x]^5/(5*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3274

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
```

p]

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cot^5(c+dx)}{5ad} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{x^4(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{(a+b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{x^2(a+(a+b)x^2)}\,dx, x, \tan(c+\right.}{a^2d}$$
$$= -\frac{(a+b)^2\cot(c+dx)}{a^3d} + \frac{(a+b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad} - \frac{(a+b)^3\text{Subst}\left(\int\right.}{}$$
$$= -\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d} - \frac{(a+b)^2\cot(c+dx)}{a^3d} + \frac{(a+b)\cot^3(c+}{3a^2d}$$

Mathematica [A]
time = 0.62, size = 101, normalized size = 1.05

$$\frac{-15(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right) - \sqrt{a}\,\cot(c+dx)\left(23a^2+35ab+15b^2-a(11a+5b)\csc^2(c+dx)+3a^2\csc^4(c+dx)\right)}{15a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-15*(a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] - Sqrt[a]*Cot
[c + d*x]*(23*a^2 + 35*a*b + 15*b^2 - a*(11*a + 5*b)*Csc[c + d*x]^2 + 3*a^2
*Csc[c + d*x]^4))/(15*a^(7/2)*d)
```

Maple [A]
time = 0.57, size = 115, normalized size = 1.20

method	result
derivativedivides	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{a^2+2ab+b^2}{a^3\tan(dx+c)}-\frac{-a-b}{3a^2\tan(dx+c)^3}+\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{d}$

default	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{a^2+2ab+b^2}{a^3\tan(dx+c)}-\frac{-a-b}{3a^2\tan(dx+c)^3}+\frac{(-a^3-3a^2b-3a\,b^2-b^3)\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{d}$
risch	$-\frac{2i(45a^2e^{8i(dx+c)}+45b\,e^{8i(dx+c)}a+15b^2e^{8i(dx+c)}-90a^2e^{6i(dx+c)}-150ab\,e^{6i(dx+c)}-60b^2e^{6i(dx+c)}+140a^2e^{4i(dx+c)}+}{15d\,a^3(e^{2i(dx+c)}-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5/a/tan(d*x+c)^5-1/a^3*(a^2+2*a*b+b^2)/tan(d*x+c)-1/3*(-a-b)/a^2/ta
n(d*x+c)^3+1/a^3*(-a^3-3*a^2*b-3*a*b^2-b^3)/(a*(a+b))^(1/2)*arctan(tan(d*x+
c)*(a+b)/(a*(a+b))^(1/2)))
```

Maxima [A]
time = 0.51, size = 111, normalized size = 1.16

$$-\frac{\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^3}+\frac{15\,(a^2+2\,ab+b^2)\tan(dx+c)^4-5\,(a^2+ab)\tan(dx+c)^2+3\,a^2}{a^3\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan((a + b)*tan(d*x + c)/sqrt(
(a + b)*a))/(sqrt((a + b)*a)*a^3) + (15*(a^2 + 2*a*b + b^2)*tan(d*x + c)^4
- 5*(a^2 + a*b)*tan(d*x + c)^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(84) = 168$.
time = 0.43, size = 576, normalized size = 6.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(23*a^2 + 35*a*b + 15*b^2)*cos(d*x + c)^5 - 20*(7*a^2 + 13*a*b +
6*b^2)*cos(d*x + c)^3 - 15*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 - 2*(a^2 + 2
*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^
2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 +
4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*b)*cos(d*x + c))*sqrt(-(a + b)/a
```

```
)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos
(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) + 60*(a^2 + 2*a*b + b^2)*cos
(d*x + c))/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x
 + c)), -1/30*(2*(23*a^2 + 35*a*b + 15*b^2)*cos(d*x + c)^5 - 10*(7*a^2 + 13
*a*b + 6*b^2)*cos(d*x + c)^3 - 15*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 - 2*(
a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt((a + b)/a)*arct
an(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(d*x
+ c)*sin(d*x + c)))*sin(d*x + c) + 30*(a^2 + 2*a*b + b^2)*cos(d*x + c))/((a
^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(84) = 168.
time = 0.53, size = 171, normalized size = 1.78

$$-\frac{\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}\,a^3}+\frac{15\,a^2\tan(dx+c)^4+30\,ab\tan(dx+c)^4+15\,b^2\tan(dx+c)^4-5\,a^2\tan(dx+c)^2-5\,ab\tan(dx+c)^2+3\,a^2}{a^3\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn
(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/(s
qrt(a^2 + a*b)*a^3) + (15*a^2*tan(d*x + c)^4 + 30*a*b*tan(d*x + c)^4 + 15*b
^2*tan(d*x + c)^4 - 5*a^2*tan(d*x + c)^2 - 5*a*b*tan(d*x + c)^2 + 3*a^2)/(a
^3*tan(d*x + c)^5))/d
```

Mupad [B]
time = 16.15, size = 82, normalized size = 0.85

$$-\frac{\frac{1}{5\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{3\,a^2}+\frac{\tan(c+d\,x)^4\,(a+b)^2}{a^3}}{d\tan\left(c+d\,x\right)^5}-\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)\,{(a+b)}^{5/2}}{a^{7/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + b*sin(c + d*x)^2),x)
```

```
[Out] - (1/(5*a) - (tan(c + d*x)^2*(a + b))/(3*a^2) + (tan(c + d*x)^4*(a + b)^2)/
a^3)/(d*tan(c + d*x)^5) - (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a +
b)^(5/2))/(a^(7/2)*d)
```

3.456 $\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d}+\frac{(a+b)^3\cot(c+dx)}{a^4d}-\frac{(a+b)^2\cot^3(c+dx)}{3a^3d}+\frac{(a+b)\cot^5(c+dx)}{5a^2d}-\frac{\cot^7(c+dx)}{7ad}$$

```
[Out] (a+b)^(7/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(9/2)/d+(a+b)^3*cot(d*
x+c)/a^4/d-1/3*(a+b)^2*cot(d*x+c)^3/a^3/d+1/5*(a+b)*cot(d*x+c)^5/a^2/d-1/7*
cot(d*x+c)^7/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3274, 331, 211}

$$\frac{(a+b)^{7/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d}+\frac{(a+b)^3\cot(c+dx)}{a^4d}-\frac{(a+b)^2\cot^3(c+dx)}{3a^3d}+\frac{(a+b)\cot^5(c+dx)}{5a^2d}-\frac{\cot^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((a + b)^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*d) + ((
a + b)^3*Cot[c + d*x])/(a^4*d) - ((a + b)^2*Cot[c + d*x]^3)/(3*a^3*d) + ((a
 + b)*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*x]^7/(7*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3274

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
```

p]

Rubi steps

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^8(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{x^6(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{(a+b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7ad} + \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{x^4(a+(a+b)x^2)}\,dx, x, \tan(c+\right.}{a^2d}$$

$$= -\frac{(a+b)^2\cot^3(c+dx)}{3a^3d} + \frac{(a+b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7ad} - \frac{(a+b)^3\text{Subst}\left(\int\right.}{}$$

$$= \frac{(a+b)^3\cot(c+dx)}{a^4d} - \frac{(a+b)^2\cot^3(c+dx)}{3a^3d} + \frac{(a+b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx}{7ad}$$

$$= \frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d} + \frac{(a+b)^3\cot(c+dx)}{a^4d} - \frac{(a+b)^2\cot^3(c+d}{3a^3d}$$

Mathematica [A]
time = 0.76, size = 135, normalized size = 1.15

$$\frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d} + \frac{\cot(c+dx)\left(176a^3+406a^2b+350ab^2+105b^3-a(122a^2+112ab+35b^2)\csc^2(c+dx)+3a^2(22a+7b)\csc^4(c+dx)-15a^3\csc^6(c+dx)\right)}{105a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((a + b)^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*d) + (C
ot[c + d*x]*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3 - a*(122*a^2 + 112*a
*b + 35*b^2)*Csc[c + d*x]^2 + 3*a^2*(22*a + 7*b)*Csc[c + d*x]^4 - 15*a^3*Cs
c[c + d*x]^6))/(105*a^4*d)
```

Maple [A]
time = 0.65, size = 155, normalized size = 1.32

method	result

derivativedivides	$\frac{\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{a^4\sqrt{a(a+b)}}-\frac{-a^3-3a^2b-3a\,b^2-b^3}{a^4\tan(dx+c)}-\frac{-a-b}{5a^2\tan(dx+c)^5}-\frac{a^2+2ab+b^2}{3a^3\tan(dx+c)^3}-\frac{1}{7a\tan(d}}{d}$
default	$\frac{\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{a(a+b)}}\right)}{a^4\sqrt{a(a+b)}}-\frac{-a^3-3a^2b-3a\,b^2-b^3}{a^4\tan(dx+c)}-\frac{-a-b}{5a^2\tan(dx+c)^5}-\frac{a^2+2ab+b^2}{3a^3\tan(dx+c)^3}-\frac{1}{7a\tan(d}}{d}$
risch	$2i\left(-2212b\,e^{2i(dx+c)}a^2+350a\,b^2+105b^3+176a^3+406a^2b+1575b^3e^{4i(dx+c)}-630b^3e^{2i(dx+c)}-6860a\,b^2e^{6i(dx+c)}+5040a\,b\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8/(a+sin(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)/a^4/(a*(a+b))^(1/2)*arctan(tan(d*x
+c)*(a+b)/(a*(a+b))^(1/2))-1/a^4*(-a^3-3*a^2*b-3*a*b^2-b^3)/tan(d*x+c)-1/5*
(-a-b)/a^2/tan(d*x+c)^5-1/3/a^3*(a^2+2*a*b+b^2)/tan(d*x+c)^3-1/7/a/tan(d*x+
c)^7)
```

Maxima [A]
time = 0.52, size = 154, normalized size = 1.32

$$\frac{\frac{105\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^4}+\frac{105\left(a^3+3a^2b+3ab^2+b^3\right)\tan(dx+c)^6-35\left(a^3+2a^2b+ab^2\right)\tan(dx+c)^4-15a^3+21\left(a^3+a^2b\right)\tan(dx+c)^2}{a^4\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*arctan((a + b)*tan(d
*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^4) + (105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*tan(d*x + c)^6 - 35*(a^3 + 2*a^2*b + a*b^2)*tan(d*x + c)^4 - 15*
a^3 + 21*(a^3 + a^2*b)*tan(d*x + c)^2)/(a^4*tan(d*x + c)^7))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(103) = 206.
time = 0.50, size = 834, normalized size = 7.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/420*(4*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3)*cos(d*x + c)^7 - 28*(
58*a^3 + 158*a^2*b + 145*a*b^2 + 45*b^3)*cos(d*x + c)^5 + 140*(10*a^3 + 29*
a^2*b + 28*a*b^2 + 9*b^3)*cos(d*x + c)^3 + 105*((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4 - a^
3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c
)^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2
+ 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*
b)*cos(d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*co
s(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x +
 c) - 420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c))/((a^4*d*cos(d*x + c
)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c)
), 1/210*(2*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3)*cos(d*x + c)^7 - 14
*(58*a^3 + 158*a^2*b + 145*a*b^2 + 45*b^3)*cos(d*x + c)^5 + 70*(10*a^3 + 29
*a^2*b + 28*a*b^2 + 9*b^3)*cos(d*x + c)^3 - 105*((a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4 - a
^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x +
c)^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a
 + b)/a)/((a + b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 210*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cos(d*x + c))/((a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*
x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(103) = 206.
time = 0.52, size = 238, normalized size = 2.03

$$\frac{\frac{105\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}\,a^4}+\frac{105\,a^3\tan(dx+c)^6+315\,a^2b\tan(dx+c)^6+315\,ab^2\tan(dx+c)^6+105\,b^3\tan(dx+c)^6-35\,a^3\tan(dx+c)^4-70\,a^2b\tan(dx+c)^4-35\,ab^2\tan(dx+c)^4+21\,a^3\tan(dx+c)^2+21\,a^2b\tan(dx+c)^2-15\,a^3}{a^4\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(pi*floor((d*x + c)/
pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^
2 + a*b)))/(sqrt(a^2 + a*b)*a^4) + (105*a^3*tan(d*x + c)^6 + 315*a^2*b*tan(
d*x + c)^6 + 315*a*b^2*tan(d*x + c)^6 + 105*b^3*tan(d*x + c)^6 - 35*a^3*tan
(d*x + c)^4 - 70*a^2*b*tan(d*x + c)^4 - 35*a*b^2*tan(d*x + c)^4 + 21*a^3*ta
n(d*x + c)^2 + 21*a^2*b*tan(d*x + c)^2 - 15*a^3)/(a^4*tan(d*x + c)^7))/d
```

Mupad [B]
time = 18.91, size = 100, normalized size = 0.85

$$\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)\,(a+b)^{7/2}}{a^{9/2}\,d}-\frac{\frac{1}{7\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{5\,a^2}+\frac{\tan(c+d\,x)^4\,(a+b)^2}{3\,a^3}-\frac{\tan(c+d\,x)^6\,(a+b)^3}{a^4}}{d\tan\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a + b)^(7/2))/(a^(9/2)*d) - (1
/(7*a) - (tan(c + d*x)^2*(a + b))/(5*a^2) + (tan(c + d*x)^4*(a + b)^2)/(3*a
^3) - (tan(c + d*x)^6*(a + b)^3)/a^4)/(d*tan(c + d*x)^7)
```

3.457 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^5(e+fx)\,dx$

Optimal. Leaf size=64

$$\frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{2a}{f\sqrt{a\cos^2(e+fx)}}-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] 1/3*a^2/f/(a*cos(f*x+e)^2)^(3/2)-2*a/f/(a*cos(f*x+e)^2)^(1/2)-(a*cos(f*x+e)
^2)^(1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {3255, 3284, 16, 45}

$$\frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{2a}{f\sqrt{a\cos^2(e+fx)}}-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] a^2/(3*f*(a*Cos[e + f*x]^2)^(3/2)) - (2*a)/(f*Sqrt[a*Cos[e + f*x]^2]) - Sqr
t[a*Cos[e + f*x]^2]/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
```

```
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx &= \int \sqrt{a\cos^2(e+fx)}\tan^5(e+fx)\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)^2\sqrt{ax}}{x^3}\,dx,x,\cos^2(e+fx)\right)}{2f}\\ &= -\frac{a^3\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{5/2}}\,dx,x,\cos^2(e+fx)\right)}{2f}\\ &= -\frac{a^3\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}}-\frac{2}{a(ax)^{3/2}}+\frac{1}{a^2\sqrt{ax}}\right)dx,x,\cos^2(e+fx)\right.}{2f}\\ &= \frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{2a}{f\sqrt{a\cos^2(e+fx)}}-\frac{\sqrt{a\cos^2(e+fx}}{f}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.80

$$-\frac{\sqrt{a\cos^2(e+fx)}\left(-1+6\cos^2(e+fx)+3\cos^4(e+fx)\right)\sec^4(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
[Out] -1/3*(Sqrt[a*Cos[e + f*x]^2]*(-1 + 6*Cos[e + f*x]^2 + 3*Cos[e + f*x]^4)*Sec
[e + f*x]^4)/f
```

Maple [A]
time = 14.03, size = 48, normalized size = 0.75

method	result
default	$-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\left(3\left(\cos^4(fx+e)\right)+6\left(\cos^2(fx+e)\right)-1\right)}{3\cos(fx+e)^4 f}$
risch	$-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}{2f\left(e^{2i(fx+e)}+1\right)}-\frac{4\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/cos(f*x+e)^4*(a*cos(f*x+e)^2)^(1/2)*(3*cos(f*x+e)^4+6*cos(f*x+e)^2-1)/
f
```

Maxima [A]
time = 0.31, size = 72, normalized size = 1.12

$$-\frac{3\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,a^{3}-\frac{6\left(a\sin(fx+e)^{2}-a\right)a^{4}+a^{5}}{\left(-a\sin(fx+e)^{2}+a\right)^{\frac{3}{2}}}}{3\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(-a*sin(f*x + e)^2 + a)*a^3 - (6*(a*sin(f*x + e)^2 - a)*a^4 + a
^5)/(-a*sin(f*x + e)^2 + a)^(3/2))/(a^3*f)
```

Fricas [A]
time = 0.39, size = 47, normalized size = 0.73

$$-\frac{\left(3\,\cos\left(fx+e\right)^{4}+6\,\cos\left(fx+e\right)^{2}-1\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{3\,f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(f*x + e)^4 + 6*cos(f*x + e)^2 - 1)*sqrt(a*cos(f*x + e)^2)/(f*co
s(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\,\tan^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**5, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(59) = 118.

time = 1.55, size = 136, normalized size = 2.12

$$\frac{2\sqrt{a}\left(\frac{3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1}-\frac{3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-12\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+5\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{3}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 2/3*sqrt(a)*(3*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/(tan(1/2*f*x + 1/2*e)^2 + 1)
 - (3*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^4 - 12*sgn(tan(1
/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^2 + 5*sgn(tan(1/2*f*x + 1/2*e)^
4 - 1))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [**B**]

time = 19.71, size = 326, normalized size = 5.09

$$-\frac{\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{f}-\frac{8\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{16\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{3\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{16\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{3\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (16*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2))/(3*f*(exp(e*2i + f*x*2i) + 1)^2*(exp(e*1i + f*x*1i)
+ exp(e*3i + f*x*3i))) - (8*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i
)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(f*(exp(e*2i + f*x*2i) + 1)*
(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (a - a*((exp(- e*1i - f*x*1i)*
1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)/f - (16*exp(e*3i + f*x*3i)*(a -
 a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(3*f
*(exp(e*2i + f*x*2i) + 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.458 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^3(e+fx)\,dx$

Optimal. Leaf size=38

$$\frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] a/f/(a*cos(f*x+e)^2)^(1/2)+(a*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3255, 3284, 16, 45}

$$\frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] a/(f*Sqrt[a*Cos[e + f*x]^2]) + Sqrt[a*Cos[e + f*x]^2]/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
```

```
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\,\tan^3(e+fx)\,dx=\int \sqrt{a\cos^2(e+fx)}\,\tan^3(e+fx)\,dx$$
$$=-\frac{\mathrm{Subst}\left(\int \frac{(1-x)\sqrt{ax}}{x^2}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{a^2\mathrm{Subst}\left(\int \frac{1-x}{(ax)^{3/2}}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{a^2\mathrm{Subst}\left(\int \left(\frac{1}{(ax)^{3/2}}-\frac{1}{a\sqrt{ax}}\right)dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=\frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Mathematica [A]
time = 0.06, size = 29, normalized size = 0.76

$$\frac{a(1+\cos^2(e+fx))}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
[Out] (a*(1 + Cos[e + f*x]^2))/(f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 13.21, size = 35, normalized size = 0.92

method	result
default	$\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(\cos^2(fx+e)+1\right)}{\cos(fx+e)^2 f}$
risch	$\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\;e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}+\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}{2f\left(e^{2i(fx+e)}+1\right)}+\frac{2\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-}}}{f\left(e^{2i(fx+e)}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

[Out] 1/cos(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2+1)/f

Maxima [A]
time = 0.30, size = 48, normalized size = 1.26

$$\frac{\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,a^{2}+\frac{a^{3}}{\sqrt{-a\sin\left(fx+e\right)^{2}+a}}}{a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")

[Out] (sqrt(-a*sin(f*x + e)^2 + a)*a^2 + a^3/sqrt(-a*sin(f*x + e)^2 + a))/(a^2*f)

Fricas [A]
time = 0.38, size = 34, normalized size = 0.89

$$\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\,\left(\cos\left(fx+e\right)^{2}+1\right)}{f\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")

[Out] sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2 + 1)/(f*cos(f*x + e)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\,\tan^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)

[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**3, x)

Giac [A]
time = 0.71, size = 39, normalized size = 1.03

$$\frac{4\sqrt{a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")

```
[Out] 4*sqrt(a)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/((tan(1/2*f*x + 1/2*e)^4 - 1)*f)
```

Mupad [B]
time = 0.72, size = 69, normalized size = 1.82

$$\frac{\sqrt{2}\ \sqrt{a\ (\cos(2\,e+2\,f\,x)+1)}\ (8\ \cos(2\,e+2\,f\,x)+\cos(4\,e+4\,f\,x)+7)}{2\,f\ (4\ \cos(2\,e+2\,f\,x)+\cos(4\,e+4\,f\,x)+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*(a*(cos(2*e + 2*f*x) + 1))^(1/2)*(8*cos(2*e + 2*f*x) + cos(4*e + 4
*f*x) + 7))/(2*f*(4*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3))
```

3.459 $\int \sqrt{a - a\sin^2(e + fx)} \tan(e + fx)\, dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

[Out] -(a*cos(f*x+e)^2)^(1/2)/f

Rubi [A]
time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3255, 3284, 16, 32}

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x],x]

[Out] -(Sqrt[a*Cos[e + f*x]^2]/f)

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\tan(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{x}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a\text{Subst}\left(\int \frac{1}{\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] -(Sqrt[a*Cos[e + f*x]^2]/f)
```

Maple [A]
time = 0.14, size = 21, normalized size = 1.11

method	result	size
derivativedivides	$-\frac{\sqrt{a-a\left(\sin^2\left(fx+e\right)\right)}}{f}$	21
default	$-\frac{\sqrt{a-a\left(\sin^2\left(fx+e\right)\right)}}{f}$	21
risch	$-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}\; e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}}{2f\left(e^{2i(fx+e)}+1\right)}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -(a-a*sin(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.54, size = 21, normalized size = 1.11

$$-\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -sqrt(-a*sin(f*x + e)^2 + a)/f
```

Fricas [A]
time = 0.37, size = 17, normalized size = 0.89

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -sqrt(a*cos(f*x + e)^2)/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.
time = 0.49, size = 39, normalized size = 2.05

$$\frac{2\sqrt{a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/((tan(1/2*f*x + 1/2*e)^2 + 1)*f)
```

Mupad [B]
time = 15.25, size = 20, normalized size = 1.05

$$-\frac{\sqrt{a-a\sin\left(e+f\,x\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] -(a - a*sin(e + f*x)^2)^(1/2)/f
```

3.460 $\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] -arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3255, 3284, 52, 65, 212}

$$\frac{\sqrt{a\cos^2(e+fx)}}{f}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a*Cos[e + f*x
]^2]/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx &= \int \sqrt{a\cos^2(e+fx)}\cot(e+fx)\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a\cos^2(e+fx)}}{f} - \frac{a\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a\cos^2(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{f}\\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a\cos^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 1.10

$$\frac{\sqrt{a\cos^2(e+fx)}\left(\cos(e+fx)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a - a*Sin[e + f*x]^2],x]
```

[Out] (Sqrt[a*Cos[e + f*x]^2]*(Cos[e + f*x] - Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]])*Sec[e + f*x])/f

Maple [A]
time = 11.22, size = 52, normalized size = 1.04

method	result
default	$\frac{\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}-\sqrt{a}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}+2a}{\sin\left(fx+e\right)}\right)}{f}$
risch	$\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^{2}a\,e^{-2i(fx+e)}}\;e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}+\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^{2}a\,e^{-2i(fx+e)}}}{2f\left(e^{2i(fx+e)}+1\right)}-\frac{\ln\left(e^{ifx}+e^{-ie}\right)\sqrt{\left(e^{2i(fx+e)}\right.}}{f\left(e^{2i(f\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((a*cos(f*x+e)^2)^(1/2)-a^(1/2)*ln(2/sin(f*x+e)*(a^(1/2)*(a*cos(f*x+e)^2)^(1/2)+a)))/f

Maxima [A]
time = 0.52, size = 74, normalized size = 1.48

$$-\frac{\sqrt{a}\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,\sqrt{a}}{|\sin\left(fx+e\right)|}+\frac{2a}{|\sin\left(fx+e\right)|}\right)-\sqrt{-a\sin\left(fx+e\right)^{2}+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] -(sqrt(a)*log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs(sin(f*x + e))) - sqrt(-a*sin(f*x + e)^2 + a))/f

Fricas [A]
time = 0.40, size = 57, normalized size = 1.14

$$\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(2\cos\left(fx+e\right)-\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}\right)\right)}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(a*cos(f*x + e)^2)*(2*cos(f*x + e) - log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1)))/(f*cos(f*x + e))

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x), x)
```

Giac [**A**]
time = 0.45, size = 55, normalized size = 1.10

$$\frac{a\left(\frac{\arctan\left(\frac{\sqrt{-a\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{-a\sin\left(fx+e\right)^{2}+a}}{a}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*(arctan(sqrt(-a*sin(f*x + e)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(-a*sin(f*x
+ e)^2 + a)/a)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{cot}\left(e+f\,x\right)\,\sqrt{a-a\,{\sin\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)*(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.461 $\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=87

$$\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{3\sqrt{a\cos^2(e+fx)}}{2f}-\frac{(a\cos^2(e+fx))^{3/2}\csc^2(e+fx)}{2af}$$

```
[Out] -1/2*(a*cos(f*x+e)^2)^(3/2)*csc(f*x+e)^2/a/f+3/2*arctanh((a*cos(f*x+e)^2)^(
1/2)/a^(1/2))*a^(1/2)/f-3/2*(a*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3255, 3284, 16, 43, 52, 65, 212}

$$-\frac{3\sqrt{a\cos^2(e+fx)}}{2f}-\frac{\csc^2(e+fx)\,(a\cos^2(e+fx))^{3/2}}{2af}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]])/(2*f) - (3*Sqrt[a*Cos[e
 + f*x]^2])/(2*f) - ((a*Cos[e + f*x]^2)^(3/2)*Csc[e + f*x]^2)/(2*a*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{a\cos^2(e+fx)}\cot^3(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x\sqrt{ax}}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(ax)^{3/2}}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2af}$$
$$= -\frac{(a\cos^2(e+fx))^{3/2}\csc^2(e+fx)}{2af} + \frac{3\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cos^2\right.}{4f}$$
$$= -\frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{(a\cos^2(e+fx))^{3/2}\csc^2(e+fx)}{2af} + \frac{(3a)}{}$$
$$= -\frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{(a\cos^2(e+fx))^{3/2}\csc^2(e+fx)}{2af} + \frac{3\text{Sub}}{}$$
$$= \frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{(a\,c}{}$$

Mathematica [A]
time = 0.30, size = 88, normalized size = 1.01

$$-\frac{\sqrt{a\cos^2(e+fx)}\left(8\cos(e+fx)+\csc^2\left(\frac{1}{2}(e+fx)\right)-12\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+12\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\sec^2\left(\frac{1}{2}(e+fx)\right)\right)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] -1/8*(Sqrt[a*Cos[e + f*x]^2]*(8*Cos[e + f*x] + Csc[(e + f*x)/2]^2 - 12*Log[
Cos[(e + f*x)/2]] + 12*Log[Sin[(e + f*x)/2]] - Sec[(e + f*x)/2]^2)*Sec[e +
f*x])/f
```

Maple [A]
time = 11.07, size = 78, normalized size = 0.90

method	result
default	$\frac{-\sqrt{a\left(\cos^2\left(fx+e\right)\right)}-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}}{2\sin\left(fx+e\right)^2}+\frac{3\sqrt{a}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2\left(fx+e\right)\right)}+2a}{\sin\left(fx+e\right)}\right)}{2}}{f}$

risch	$-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}\, e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)} - \frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}}{2f\left(e^{2i(fx+e)}+1\right)} + \frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2}}}{\left(e^{2i(fx+e)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(a*cos(f*x+e)^2)^(1/2)-1/2/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)+3/2*a^(1/2
)*ln((2*a+2*a^(1/2)*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.55, size = 105, normalized size = 1.21

$$\frac{3\sqrt{a}\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^2+a}\,\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)-3\sqrt{-a\sin\left(fx+e\right)^2+a}-\frac{\left(-a\sin(fx+e)^2+a\right)^{\frac{3}{2}}}{a\sin(fx+e)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*sqrt(a)*log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e))
+ 2*a/abs(sin(f*x + e))) - 3*sqrt(-a*sin(f*x + e)^2 + a) - (-a*sin(f*x + e)
^2 + a)^(3/2)/(a*sin(f*x + e)^2))/f
```

Fricas [A]
time = 0.39, size = 88, normalized size = 1.01

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(4\cos\left(fx+e\right)^3+3\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-6\cos\left(fx+e\right)\right)}{4\left(f\cos\left(fx+e\right)^3-f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*(4*cos(f*x + e)^3 + 3*(cos(f*x + e)^2 - 1)*log(
-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 6*cos(f*x + e))/(f*cos(f*x + e)^3
 - f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\,\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a-a*sin(f*x+e)**2)**(1/2),x)
```

[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(75) = 150.
time = 0.57, size = 171, normalized size = 1.97

$$-\frac{\left(\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-6\,\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)+\frac{3\,\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-14\,\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}\right)\sqrt{a}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -1/8*(sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^2 - 6*log(tan(1/2*f*x + 1/2*e)^2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + (3*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^4 - 14*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^2 - sgn(tan(1/2*f*x + 1/2*e)^4 - 1))/(tan(1/2*f*x + 1/2*e)^4 + tan(1/2*f*x + 1/2*e)^2))*sqrt(a)/f

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^3\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(e + f*x)^3*(a - a*sin(e + f*x)^2)^(1/2),x)

[Out] int(cot(e + f*x)^3*(a - a*sin(e + f*x)^2)^(1/2), x)

3.462 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^6(e+fx)\,dx$

Optimal. Leaf size=120

$$\frac{15\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)}{8f}-\frac{15\sqrt{a\cos^2(e+fx)}\,\tan(e+fx)}{8f}-\frac{5\sqrt{a\cos^2(e+fx)}}{8f}$$

```
[Out] 15/8*arctanh(sin(f*x+e))*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-15/8*(a*cos(f*
x+e)^2)^(1/2)*tan(f*x+e)/f-5/8*(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f+1/4*(a
*cos(f*x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3255, 3286, 2672, 294, 327, 212}

$$\frac{\tan^5(e+fx)\sqrt{a\cos^2(e+fx)}}{4f}-\frac{5\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}-\frac{15\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}+\frac{15\sec(e+fx)\sqrt{a\cos^2(e+fx)}\,\tanh^{-1}(\sin(e+fx))}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] (15*ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/(8*f) - (15*
Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/(8*f) - (5*Sqrt[a*Cos[e + f*x]^2]*Tan[
e + f*x]^3)/(8*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x]^5)/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\,\tan^6(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\,\tan^6(e+fx)\,dx$$

$$= \left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\int \sin(e+fx)\tan^5(e+fx)\,dx$$

$$= \frac{\left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \sin(e+\right.}{f}$$

$$= \frac{\sqrt{a\cos^2(e+fx)}\,\tan^5(e+fx)}{4f} - \frac{\left(5\sqrt{a\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= -\frac{5\sqrt{a\cos^2(e+fx)}\,\tan^3(e+fx)}{8f} + \frac{\sqrt{a\cos^2(e+fx)}\,\tan^5(e\cdots}{4f}$$

$$= -\frac{15\sqrt{a\cos^2(e+fx)}\,\tan(e+fx)}{8f} - \frac{5\sqrt{a\cos^2(e+fx)}\,\tan^3(e\cdots}{8f}$$

$$= \frac{15\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)}{8f} - \frac{15\sqrt{a\cdots}}{}$$

Mathematica [A]

time = 0.23, size = 75, normalized size = 0.62

$$\frac{\sqrt{a\cos^2(e+fx)}\,\sec^5(e+fx)\left(60\tanh^{-1}(\sin(e+fx))\cos^4(e+fx)-5\sin(e+fx)-15\sin(3(e+fx))-2\sin(5(e+fx))\right)}{32f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^6,x]

[Out] (Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]^5*(60*ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^4 - 5*Sin[e + f*x] - 15*Sin[3*(e + f*x)] - 2*Sin[5*(e + f*x)]))/(32*f)

Maple [A]
time = 7.11, size = 120, normalized size = 1.00

method	result
default	$\frac{a(16(\cos^4(fx+e))\sin(fx+e)+18(\cos^2(fx+e))\sin(fx+e)-4\sin(fx+e)+(15\ln(\sin(fx+e)-1)-15\ln(1+\sin(fx+e)))(\cos^4(fx+e)))}{16(1+\sin(fx+e))(\sin(fx+e)-1)\cos(fx+e)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$
risch	$\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}-\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}{2\left(e^{2i(fx+e)}+1\right)f}+\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)

[Out] 1/16*a*(16*cos(f*x+e)^4*sin(f*x+e)+18*cos(f*x+e)^2*sin(f*x+e)-4*sin(f*x+e)+(15*ln(sin(f*x+e)-1)-15*ln(1+sin(f*x+e)))*cos(f*x+e)^4)/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2144 vs. 2(113) = 226.
time = 2.12, size = 2144, normalized size = 17.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")

[Out] 1/16*(8*(sin(9*f*x + 9*e) + 4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*cos(10*f*x + 10*e) - 20*(3*sin(8*f*x + 8*e) + sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - 3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) + 60*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*cos(8*f*x + 8*e) - 80*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - 3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 20*(6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*cos(6*f*x + 6*e) + 120*(sin(4*f*x + 4*e) + 3*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 20*(4*sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) + 15*(2*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e

```
) + cos(f*x + e))*cos(9*f*x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*cos(5*f*x +
5*e) + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(7*f*x + 7*e) + 16*cos(7*f*x +
 7*e)^2 + 12*(4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(5*f*x + 5*e) + 36*cos(
5*f*x + 5*e)^2 + 16*cos(3*f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*cos(f*x + e) +
cos(f*x + e)^2 + 2*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x +
 3*e) + sin(f*x + e))*sin(9*f*x + 9*e) + sin(9*f*x + 9*e)^2 + 8*(6*sin(5*f*
x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(7*f*x + 7*e) + 16*sin(7*f
*x + 7*e)^2 + 12*(4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + 36*
sin(5*f*x + 5*e)^2 + 16*sin(3*f*x + 3*e)^2 + 8*sin(3*f*x + 3*e)*sin(f*x + e
) + sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) +
1) - 15*(2*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) +
cos(f*x + e))*cos(9*f*x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*cos(5*f*x + 5*e)
 + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(7*f*x + 7*e) + 16*cos(7*f*x + 7*e
)^2 + 12*(4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(5*f*x + 5*e) + 36*cos(5*f*
x + 5*e)^2 + 16*cos(3*f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*cos(f*x + e) + cos(
f*x + e)^2 + 2*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e
) + sin(f*x + e))*sin(9*f*x + 9*e) + sin(9*f*x + 9*e)^2 + 8*(6*sin(5*f*x +
5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(7*f*x + 7*e) + 16*sin(7*f*x +
 7*e)^2 + 12*(4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + 36*sin(
5*f*x + 5*e)^2 + 16*sin(3*f*x + 3*e)^2 + 8*sin(3*f*x + 3*e)*sin(f*x + e) +
sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) -
 8*(cos(9*f*x + 9*e) + 4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*
x + 3*e) + cos(f*x + e))*sin(10*f*x + 10*e) + 4*(15*cos(8*f*x + 8*e) + 5*co
s(6*f*x + 6*e) - 5*cos(4*f*x + 4*e) - 15*cos(2*f*x + 2*e) - 2)*sin(9*f*x +
9*e) - 60*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) + c
os(f*x + e))*sin(8*f*x + 8*e) + 16*(5*cos(6*f*x + 6*e) - 5*cos(4*f*x + 4*e)
 - 15*cos(2*f*x + 2*e) - 2)*sin(7*f*x + 7*e) - 20*(6*cos(5*f*x + 5*e) + 4*c
os(3*f*x + 3*e) + cos(f*x + e))*sin(6*f*x + 6*e) - 24*(5*cos(4*f*x + 4*e) +
 15*cos(2*f*x + 2*e) + 2)*sin(5*f*x + 5*e) + 20*(4*cos(3*f*x + 3*e) + cos(f
*x + e))*sin(4*f*x + 4*e) - 16*(15*cos(2*f*x + 2*e) + 2)*sin(3*f*x + 3*e) +
 240*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 60*cos(f*x + e)*sin(2*f*x + 2*e) -
 60*cos(2*f*x + 2*e)*sin(f*x + e) - 8*sin(f*x + e))*sqrt(a)/((2*(4*cos(7*f*
x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(9*f*
x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e)
+ cos(f*x + e))*cos(7*f*x + 7*e) + 16*cos(7*f*x + 7*e)^2 + 12*(4*cos(3*f*x
+ 3*e) + cos(f*x + e))*cos(5*f*x + 5*e) + 36*cos(5*f*x + 5*e)^2 + 16*cos(3*
f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(4*sin(
7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(
9*f*x + 9*e) + sin(9*f*x + 9*e)^2 + 8*(6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3
*e) + sin(f*x + e))*sin(7*f*x + 7*e) + 16*sin(7*f*x + 7*e)^2 + 12*(4*sin(3*
f*x + 3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + 36*sin(5*f*x + 5*e)^2 + 16*si
n(3*f*x + 3*e)^2 + 8*sin(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*f)
```

Fricas [**A**]

time = 0.41, size = 87, normalized size = 0.72

$$-\frac{\left(15 \cos \left(fx+e\right)^{4} \log \left(-\frac{\sin(fx+e)-1}{\sin(fx+e)+1}\right)+2 \left(8 \cos \left(fx+e\right)^{4}+9 \cos \left(fx+e\right)^{2}-2\right) \sin \left(fx+e\right)\right) \sqrt{a \cos \left(fx+e\right)^{2}}}{16 f \cos \left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/16*(15*cos(f*x + e)^4*log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) + 2*(8
*cos(f*x + e)^4 + 9*cos(f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2
)/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \left(\sin \left(e+fx\right)-1\right) \left(\sin \left(e+fx\right)+1\right)} \tan^{6}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(113) = 226.
time = 1.97, size = 251, normalized size = 2.09

$$-\frac{\left(15 \log \left(\left|\frac{1}{\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)}+\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)+2\right|\right) \mathrm{sgn}\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}-1\right)-15 \log \left(\left|\frac{1}{\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)}+\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)-2\right|\right) \mathrm{sgn}\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}-1\right)-\frac{32 \mathrm{sgn}\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}-1\right)}{\frac{1}{\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)}+\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)}-\frac{4 \left(7 \left(\frac{1}{\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)}+\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)\right)^{3} \mathrm{sgn}\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}-1\right)-36 \left(\frac{1}{\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)}+\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)\right) \mathrm{sgn}\left(\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}-1\right)\right)}{\left(\left(\frac{1}{\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)}+\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)\right)^{2}-4\right)^{2}}\right) \sqrt{a}}{16 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] -1/16*(15*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))*sgn(t
an(1/2*f*x + 1/2*e)^4 - 1) - 15*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*
x + 1/2*e) - 2))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 32*sgn(tan(1/2*f*x + 1/2
*e)^4 - 1)/(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e)) - 4*(7*(1/tan(1/
2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) -
36*(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))*sgn(tan(1/2*f*x + 1/2*e)
^4 - 1))/((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^2 - 4)^2)*sqrt(a)
/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{tan}(e+f\,x)^6\,\sqrt{a-a\,{\sin\left(e+f\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a - a*sin(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^6*(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.463 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=91

$$-\frac{3\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)}{2f}+\frac{3\sqrt{a\cos^2(e+fx)}\,\tan(e+fx)}{2f}+\frac{\sqrt{a\cos^2(e+fx)}\,\mathrm{t}}{2f}$$

```
[Out] -3/2*arctanh(sin(f*x+e))*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f+3/2*(a*cos(f*x
+e)^2)^(1/2)*tan(f*x+e)/f+1/2*(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3255, 3286, 2672, 294, 327, 212}

$$\frac{\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}+\frac{3\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}-\frac{3\sec(e+fx)\sqrt{a\cos^2(e+fx)}\,\tanh^{-1}(\sin(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (-3*ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/(2*f) + (3*S
qrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/(2*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e +
 f*x]^3)/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\sin^2(e+fx)}\,\tan^4(e+fx)\,dx &= \int \sqrt{a\cos^2(e+fx)}\,\tan^4(e+fx)\,dx\\ &= \left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\int \sin(e+fx)\tan^3(e+fx)\,dx\\ &= \frac{\left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\sin(e+\right.}{f}\\ &= \frac{\sqrt{a\cos^2(e+fx)}\,\tan^3(e+fx)}{2f}-\frac{\left(3\sqrt{a\cos^2(e+fx)}\,\sec(e+\right.}{}\\ &= \frac{3\sqrt{a\cos^2(e+fx)}\,\tan(e+fx)}{2f}+\frac{\sqrt{a\cos^2(e+fx)}\,\tan^3(e+f}{2f}\\ &= -\frac{3\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)}{2f}+\frac{3\sqrt{a\,c}}{}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 55, normalized size = 0.60

$$\frac{a\left(-3\tanh^{-1}(\sin(e+fx))\cos(e+fx)+(2+\cos(2(e+fx)))\tan(e+fx)\right)}{2f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (a*(-3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x] + (2 + Cos[2*(e + f*x)])*Tan[e +
f*x]))/(2*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 7.00, size = 84, normalized size = 0.92

method	result
default	$-\frac{a\left(-4\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-2\sin\left(fx+e\right)+\left(-3\ln\left(\sin\left(fx+e\right)-1\right)+3\ln\left(1+\sin\left(fx+e\right)\right)\right)\left(\cos^{2}\left(fx+e\right)\right)\right)}{4\cos\left(fx+e\right)\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}\, f}$
risch	$-\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^{2} a\, e^{-2i(fx+e)}}\; e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}+\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^{2} a\, e^{-2i(fx+e)}}}{2\left(e^{2i(fx+e)}+1\right)f}-\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^{2} a\, e}}{f\left(e^{2i\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a*(-4*cos(f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+(-3*ln(sin(f*x+e)-1)+3*ln(1
+sin(f*x+e)))*cos(f*x+e)^2)/cos(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(86) = 172$.
time = 0.67, size = 910, normalized size = 10.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(sin(5*f*x + 5*e) + 2*sin(3*f*x + 3*e) + sin(f*x + e))*cos(6*f*x +
6*e) - 6*(sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 6*(2*sin(
3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) + 3*(2*(2*cos(3*f*x + 3*e) +
cos(f*x + e))*cos(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2
+ 4*cos(3*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e)
+ sin(f*x + e))*sin(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e)^
2 + 4*sin(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*log(cos(f*x + e)^2 +
sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 3*(2*(2*cos(3*f*x + 3*e) + cos(f*x +
 e))*cos(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2 + 4*cos(3
*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e) + sin(f*x
 + e))*sin(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e)^2 + 4*sin
(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x +
 e)^2 - 2*sin(f*x + e) + 1) - 2*(cos(5*f*x + 5*e) + 2*cos(3*f*x + 3*e) + co
s(f*x + e))*sin(6*f*x + 6*e) + 2*(3*cos(4*f*x + 4*e) - 3*cos(2*f*x + 2*e) -
```

```
1)*sin(5*f*x + 5*e) - 6*(2*cos(3*f*x + 3*e) + cos(f*x + e))*sin(4*f*x + 4*
e) - 4*(3*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) + 12*cos(3*f*x + 3*e)*sin(
2*f*x + 2*e) + 6*cos(f*x + e)*sin(2*f*x + 2*e) - 6*cos(2*f*x + 2*e)*sin(f*x
 + e) - 2*sin(f*x + e))*sqrt(a)/((2*(2*cos(3*f*x + 3*e) + cos(f*x + e))*cos
(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2 + 4*cos(3*f*x + 3
*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e) + sin(f*x + e))*s
in(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e)^2 + 4*sin(3*f*x +
 3*e)*sin(f*x + e) + sin(f*x + e)^2)*f)
```

Fricas [**A**]
time = 0.40, size = 77, normalized size = 0.85

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(3\,\cos\left(fx+e\right)^{2}\log\left(-\frac{\sin(fx+e)+1}{\sin(fx+e)-1}\right)-2\left(2\,\cos\left(fx+e\right)^{2}+1\right)\sin\left(fx+e\right)\right)}{4\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2*log(-(sin(f*x + e) + 1)/(sin(
f*x + e) - 1)) - 2*(2*cos(f*x + e)^2 + 1)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\,\tan^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(86) = 172.
time = 0.96, size = 211, normalized size = 2.32

$$\frac{\left(3\,\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\right|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)-3\,\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\right|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)-\frac{4\left(3\left(\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)-8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\right)}{\left(\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)^{3}-\frac{4}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}-4\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}\right)\sqrt{a}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/4*(3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))*sgn(tan(
1/2*f*x + 1/2*e)^4 - 1) - 3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x +
1/2*e) - 2))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*(3*(1/tan(1/2*f*x + 1/2*e)
 + tan(1/2*f*x + 1/2*e))^2*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 8*sgn(tan(1/2*
```

```
f*x + 1/2*e)^4 - 1))/((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3 - 4
/tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^4\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a - a*sin(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^4*(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.464 $\int \sqrt{a-a\sin^2(e+fx)}\tan^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{f}$$

```
[Out] arctanh(sin(f*x+e))*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-(a*cos(f*x+e)^2)^(1
/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.192$, Rules used = {3255, 3286, 2672, 327, 212}

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\tanh^{-1}(\sin(e+fx))}{f}-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/f - (Sqrt[a*Cos
[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\sin^2(e+fx)}\,\tan^2(e+fx)\,dx &= \int \sqrt{a\cos^2(e+fx)}\,\tan^2(e+fx)\,dx\\ &= \left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\int \sin(e+fx)\tan(e+fx)\,dx\\ &= \frac{\left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}\\ &= -\frac{\sqrt{a\cos^2(e+fx)}\,\tan(e+fx)}{f}+\frac{\left(\sqrt{a\cos^2(e+fx)}\,\sec(e+f\right.}{}\\ &= \frac{\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e}}{}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 40, normalized size = 0.70

$$\frac{\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\left(\tanh^{-1}(\sin(e+fx))-\sin(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*(ArcTanh[Sin[e + f*x]] - Sin[e + f*x])
)/f
```

Maple [**A**]
time = 7.43, size = 54, normalized size = 0.95

method	result
default	$-\frac{a\cos(fx+e)(2\sin(fx+e)+\ln(\sin(fx+e)-1)-\ln(1+\sin(fx+e)))}{2\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$
risch	$\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}\; e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)} - \frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}}{2\left(e^{2i(fx+e)}+1\right)f} + \frac{\ln\left(e^{ifx}+ie^{-ie}\right)\sqrt{\left(e^{2i(fx+e}\right.}}{f\left(e^{2i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*cos(f*x+e)*(2*sin(f*x+e)+ln(sin(f*x+e)-1)-ln(1+sin(f*x+e)))/(a*cos(f
*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.56, size = 80, normalized size = 1.40

$$\frac{\sqrt{a}\left(\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2+2\sin\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)^2+\sin\left(fx+e\right)^2-2\sin\left(fx+e\right)+1\right)-2\sin\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - lo
g(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 2*sin(f*x + e))/f
```

Fricas [A]
time = 0.41, size = 55, normalized size = 0.96

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\log\left(-\frac{\sin(fx+e)-1}{\sin(fx+e)+1}\right)+2\sin\left(fx+e\right)\right)}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(a*cos(f*x + e)^2)*(log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) +
2*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(58) = 116.
time = 0.60, size = 136, normalized size = 2.39

$$-\frac{\left(\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)-\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)-\frac{4\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}{\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}\right)\sqrt{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))*sgn(tan(1
/2*f*x + 1/2*e)^4 - 1) - log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2
*e) - 2))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*sgn(tan(1/2*f*x + 1/2*e)^4 -
1)/(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.465 $\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{a\cos^2(e+fx)}\,\csc(e+fx)\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e+fx)}\,\tan(e+fx)}{f}$$

```
[Out] -csc(f*x+e)*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-(a*cos(f*x+e)^2)^(1/2)*tan(
f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3286, 2670, 14}

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f) - (Sqrt[a*Cos[e + f
*x]^2]*Tan[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
```

```
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx &= \int \sqrt{a\cos^2(e+fx)}\cot^2(e+fx)\,dx\\ &= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \cos(e+fx)\cot^2(e+fx)\,dx\\ &= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,-\sin(e+\right.}{f}\\ &= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,-\text{si}\right.}{f}\\ &= -\frac{\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e+fx)}}{f}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 35, normalized size = 0.61

$$-\frac{\sqrt{a\cos^2(e+fx)}\,(1+\csc^2(e+fx))\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] -((Sqrt[a*Cos[e + f*x]^2]*(1 + Csc[e + f*x]^2)*Tan[e + f*x])/f)
```

Maple [A]
time = 4.58, size = 43, normalized size = 0.75

method	result
default	$-\frac{\cos(fx+e)a\left(\sin^2(fx+e)+1\right)}{\sin(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$
risch	$\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\;e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}-\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}{2\left(e^{2i(fx+e)}+1\right)f}-\frac{2i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-}}}{\left(e^{2i(fx+e)}-1\right)f\left(e^{2i(}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(f*x+e)*a*(sin(f*x+e)^2+1)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.52, size = 45, normalized size = 0.79

$$-\frac{2\sqrt{a}\tan\left(fx+e\right)^{2}+\sqrt{a}}{\sqrt{\tan\left(fx+e\right)^{2}+1}\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*sqrt(a)*tan(f*x + e)^2 + sqrt(a))/(sqrt(tan(f*x + e)^2 + 1)*f*tan(f*x +
 e))
```

Fricas [A]
time = 0.40, size = 42, normalized size = 0.74

$$\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\cos\left(fx+e\right)^{2}-2\right)}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2 - 2)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**2, x)
```

Giac [A]
time = 0.51, size = 90, normalized size = 1.58

$$\frac{\left(\left(\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)+\frac{4\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}{\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}\right)\sqrt{a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))*sgn(tan(1/2*f*x + 1/2*
e)^4 - 1) + 4*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/(1/tan(1/2*f*x + 1/2*e) + tan
(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

Mupad [B]
time = 18.53, size = 88, normalized size = 1.54

$$\frac{\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,\left(-\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,6\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{f\,\left(\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] ((a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*
(exp(e*4i + f*x*4i)*1i - exp(e*2i + f*x*2i)*6i + 1i))/(f*(exp(e*4i + f*x*4i
) - 1))
```

3.466 $\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=91

$$\frac{2\sqrt{a\cos^2(e+fx)}\,\csc(e+fx)\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e+fx)}\,\csc^3(e+fx)\sec(e+fx)}{3f}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] 2*csc(f*x+e)*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-1/3*csc(f*x+e)^3*sec(f*x+e
)*(a*cos(f*x+e)^2)^(1/2)/f+(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3286, 2670, 276}

$$\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{3f}+\frac{2\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (2*Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f - (Sqrt[a*Cos[e + f*
x]^2]*Csc[e + f*x]^3*Sec[e + f*x])/(3*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e +
f*x])/f
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
```

```
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx &= \int \sqrt{a\cos^2(e+fx)}\cot^4(e+fx)\,dx\\ &= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \cos(e+fx)\cot^4(e+fx)\,dx\\ &= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,-\sin(e\right.}{f}\\ &= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx,x,-\right.}{f}\\ &= \frac{2\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e+fx)}\,\mathrm{c}}{}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 47, normalized size = 0.52

$$-\frac{\sqrt{a\cos^2(e+fx)}\left(-3-6\csc^2(e+fx)+\csc^4(e+fx)\right)\tan(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] -1/3*(Sqrt[a*Cos[e + f*x]^2]*(-3 - 6*Csc[e + f*x]^2 + Csc[e + f*x]^4)*Tan[e
 + f*x])/f
```

Maple [A]
time = 3.92, size = 55, normalized size = 0.60

method	result
default	$\frac{\cos(fx+e)a\left(3\left(\sin^4(fx+e)\right)+6\left(\sin^2(fx+e)\right)-1\right)}{3\sin(fx+e)^3\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$
risch	$-\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}+\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}{2\left(e^{2i(fx+e)}+1\right)f}+\frac{4i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}}}{3(e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*cos(f*x+e)*a*(3*sin(f*x+e)^4+6*sin(f*x+e)^2-1)/sin(f*x+e)^3/(a*cos(f*x+e)^2)^(1/2)/f

Maxima [A]
time = 0.50, size = 61, normalized size = 0.67

$$\frac{8\sqrt{a}\tan\left(fx+e\right)^{4}+4\sqrt{a}\tan\left(fx+e\right)^{2}-\sqrt{a}}{3\sqrt{\tan\left(fx+e\right)^{2}+1}\,f\tan\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] 1/3*(8*sqrt(a)*tan(f*x + e)^4 + 4*sqrt(a)*tan(f*x + e)^2 - sqrt(a))/(sqrt(tan(f*x + e)^2 + 1)*f*tan(f*x + e)^3)

Fricas [A]
time = 0.41, size = 66, normalized size = 0.73

$$-\frac{\left(3\cos\left(fx+e\right)^{4}-12\cos\left(fx+e\right)^{2}+8\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{3\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] -1/3*(3*cos(f*x + e)^4 - 12*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((f*cos(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**4*(a-a*sin(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**4, x)

Giac [A]
time = 0.53, size = 132, normalized size = 1.45

$$\frac{\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)-24\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)-\frac{48\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}{\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}\right)\sqrt{a}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3*sgn(tan(1/2*f*x + 1
/2*e)^4 - 1) - 24*(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))*sgn(tan(1
/2*f*x + 1/2*e)^4 - 1) - 48*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/(1/tan(1/2*f*x
+ 1/2*e) + tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

Mupad [B]
time = 18.42, size = 364, normalized size = 4.00

$$\frac{\left(\frac{1i}{f}-\frac{e^{e\,2i+f\,x\,2i}\,1i}{f}\right)\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}}{e^{e\,2i+f\,x\,2i}+1}+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,8i}{f\,\left(e^{e\,2i+f\,x\,2i}-1\right)\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,16i}{3\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^2\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,16i}{3\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^3\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] ((1i/f - (exp(e*2i + f*x*2i)*1i)/f)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(exp(e*2i + f*x*2i) + 1) + (exp(e*3i +
f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)
^(1/2)*8i)/(f*(exp(e*2i + f*x*2i) - 1)*(exp(e*1i + f*x*1i) + exp(e*3i + f*x
*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2)^2)^(1/2)*16i)/(3*f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i
+ f*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i
- f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*16i)/(3*f*(exp(e*2i +
 f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.467 $\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=124

$$-\frac{3\sqrt{a\cos^2(e+fx)}\,\csc(e+fx)\sec(e+fx)}{f}+\frac{\sqrt{a\cos^2(e+fx)}\,\csc^3(e+fx)\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e+f}}{}$$

```
[Out] -3*csc(f*x+e)*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f+csc(f*x+e)^3*sec(f*x+e)*(
a*cos(f*x+e)^2)^(1/2)/f-1/5*csc(f*x+e)^5*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/
f-(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3286, 2670, 276}

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^5(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{5f}+\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{3\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (-3*Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f + (Sqrt[a*Cos[e + f
*x]^2]*Csc[e + f*x]^3*Sec[e + f*x])/f - (Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x
]^5*Sec[e + f*x])/(5*f) - (Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
```

```
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx &= \int \sqrt{a\cos^2(e+fx)}\cot^6(e+fx)\,dx\\
&= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \cos(e+fx)\cot^6(e+fx)\,dx\\
&= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx,x,-\sin(e\right.}{f}\\
&= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)\right.}{f}\\
&= -\frac{3\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f}+\frac{\sqrt{a\cos^2(e+fx)}}{}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 67, normalized size = 0.54

$$\frac{\sqrt{a\cos^2(e+fx)}\,(-182+235\cos(2(e+fx))-90\cos(4(e+fx))+5\cos(6(e+fx)))\csc^5(e+fx)\sec(e+fx)}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] (Sqrt[a*Cos[e + f*x]^2]*(-182 + 235*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)]
 + 5*Cos[6*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/(160*f)
```

Maple [A]
time = 4.88, size = 65, normalized size = 0.52

method	result
default	$-\frac{\cos(fx+e)a\left(5(\sin^6(fx+e))+15(\sin^4(fx+e))-5(\sin^2(fx+e))+1\right)}{5\sin(fx+e)^5\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$
risch	$\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}-\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}{2\left(e^{2i(fx+e)}+1\right)f}-\frac{2i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*cos(f*x+e)*a*(5*sin(f*x+e)^6+15*sin(f*x+e)^4-5*sin(f*x+e)^2+1)/sin(f*x
+e)^5/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [A]
time = 0.53, size = 73, normalized size = 0.59

$$-\frac{16\sqrt{a}\tan\left(fx+e\right)^{6}+8\sqrt{a}\tan\left(fx+e\right)^{4}-2\sqrt{a}\tan\left(fx+e\right)^{2}+\sqrt{a}}{5\sqrt{\tan\left(fx+e\right)^{2}+1}\,f\tan\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*(16*sqrt(a)*tan(f*x + e)^6 + 8*sqrt(a)*tan(f*x + e)^4 - 2*sqrt(a)*tan(
f*x + e)^2 + sqrt(a))/(sqrt(tan(f*x + e)^2 + 1)*f*tan(f*x + e)^5)
```

Fricas [A]
time = 0.39, size = 86, normalized size = 0.69

$$\frac{\left(5\cos\left(fx+e\right)^{6}-30\cos\left(fx+e\right)^{4}+40\cos\left(fx+e\right)^{2}-16\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{5\left(f\cos\left(fx+e\right)^{5}-2f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(5*cos(f*x + e)^6 - 30*cos(f*x + e)^4 + 40*cos(f*x + e)^2 - 16)*sqrt(a*
cos(f*x + e)^2)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))*s
in(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**6, x)
```

Giac [A]
time = 0.55, size = 174, normalized size = 1.40

$$\frac{\left(\left(\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)^5\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1\right)-20\left(\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1\right)+240\left(\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1\right)+\frac{320\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1\right)}{\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}\right)\sqrt{a}}{160\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/160*((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^5*sgn(tan(1/2*f*x +
1/2*e)^4 - 1) - 20*(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3*sgn(ta
n(1/2*f*x + 1/2*e)^4 - 1) + 240*(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2
*e))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + 320*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/
(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

Mupad [B]
time = 26.64, size = 555, normalized size = 4.48

$$-\frac{\left(\frac{1\mathrm{i}}{f}-\frac{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}}{f}\right)\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,12\mathrm{i}}{f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,16\mathrm{i}}{f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)^2\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,144\mathrm{i}}{5\,f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)^3\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,128\mathrm{i}}{5\,f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)^4\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,64\mathrm{i}}{5\,f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)^5\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] - ((1i/f - (exp(e*2i + f*x*2i)*1i)/f)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 -
 (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(exp(e*2i + f*x*2i) + 1) - (exp(e*3i
+ f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^
2)^(1/2)*12i)/(f*(exp(e*2i + f*x*2i) - 1)*(exp(e*1i + f*x*1i) + exp(e*3i +
f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2)^2)^(1/2)*16i)/(f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i
 + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*144i)/(5*f*(exp(e*2i
 + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i +
f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)
^(1/2)*128i)/(5*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1i + f*x*1i) + exp(e*3i
 + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*64i)/(5*f*(exp(e*2i + f*x*2i) - 1)^5*(exp
(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.468 $\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=65

$$\frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}+\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 1/5*a^2/f/(a*cos(f*x+e)^2)^(5/2)-2/3*a/f/(a*cos(f*x+e)^2)^(3/2)+1/f/(a*cos(
f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3255, 3284, 16, 45}

$$\frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}+\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] a^2/(5*f*(a*Cos[e + f*x]^2)^(5/2)) - (2*a)/(3*f*(a*Cos[e + f*x]^2)^(3/2)) +
 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
```

```
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx=\int \frac{\tan^5(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$=-\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{ax}}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{a^3\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{7/2}}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{a^3\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{2}{a(ax)^{5/2}}+\frac{1}{a^2(ax)^{3/2}}\right)\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=\frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}+\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 0.66

$$\frac{15-10\sec^2(e+fx)+3\sec^4(e+fx)}{15f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] (15 - 10*Sec[e + f*x]^2 + 3*Sec[e + f*x]^4)/(15*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 14.43, size = 51, normalized size = 0.78

method	result	size
default	$\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(15(\cos^4(fx+e))-10(\cos^2(fx+e))+3\right)}{15a\cos(fx+e)^6 f}$	51
risch	$\frac{2\,e^{8i(fx+e)}+\frac{8\,e^{6i(fx+e)}}{3}+\frac{116\,e^{4i(fx+e)}}{15}+\frac{8\,e^{2i(fx+e)}}{3}+2}{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,\left(e^{2i(fx+e)}+1\right)^4 f}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/a/cos(f*x+e)^6*(a*cos(f*x+e)^2)^(1/2)*(15*cos(f*x+e)^4-10*cos(f*x+e)^2
+3)/f
```

Maxima [A]
time = 0.30, size = 72, normalized size = 1.11

$$\frac{15\left(a\sin\left(fx+e\right)^{2}-a\right)^{2}a^{3}+10\left(a\sin\left(fx+e\right)^{2}-a\right)a^{4}+3\,a^{5}}{15\left(-a\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*(a*sin(f*x + e)^2 - a)^2*a^3 + 10*(a*sin(f*x + e)^2 - a)*a^4 + 3*a
^5)/((-a*sin(f*x + e)^2 + a)^(5/2)*a^3*f)
```

Fricas [A]
time = 0.39, size = 50, normalized size = 0.77

$$\frac{\left(15\,\cos\left(fx+e\right)^{4}-10\,\cos\left(fx+e\right)^{2}+3\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{15\,af\cos\left(fx+e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*cos(f*x + e)^4 - 10*cos(f*x + e)^2 + 3)*sqrt(a*cos(f*x + e)^2)/(a*
f*cos(f*x + e)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{5}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [A]
time = 1.61, size = 71, normalized size = 1.09

$$\frac{16\left(10\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-5\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{15\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{5}\sqrt{a}\,f\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/15*(10*tan(1/2*f*x + 1/2*e)^4 - 5*tan(1/2*f*x + 1/2*e)^2 + 1)/((tan(1/2*
f*x + 1/2*e)^2 - 1)^5*sqrt(a)*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

Mupad [B]
time = 23.61, size = 486, normalized size = 7.48

$$\frac{4\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{a\,f\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{32\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{3\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{352\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{15\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{128\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{5\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{64\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{5\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^5\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (4*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2)^2)^(1/2))/(a*f*(exp(e*2i + f*x*2i) + 1)*(exp(e*1i + f*x*1i) + e
xp(e*3i + f*x*3i))) - (32*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*
1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(3*a*f*(exp(e*2i + f*x*2i) + 1
)^2*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (352*exp(e*3i + f*x*3i)*(a
 - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(1
5*a*f*(exp(e*2i + f*x*2i) + 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
 - (128*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
+ f*x*1i)*1i)/2)^2)^(1/2))/(5*a*f*(exp(e*2i + f*x*2i) + 1)^4*(exp(e*1i + f*
x*1i) + exp(e*3i + f*x*3i))) + (64*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i -
 f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(5*a*f*(exp(e*2i + f*
x*2i) + 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.469 $\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$\frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 1/3*a/f/(a*cos(f*x+e)^2)^(3/2)-1/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3255, 3284, 16, 45}

$$\frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] a/(3*f*(a*Cos[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
```

```
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx &= \int \frac{\tan^3(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{a^2\text{Subst}\left(\int \frac{1-x}{(ax)^{5/2}}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{a^2\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}}-\frac{1}{a(ax)^{3/2}}\right)\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cos^2(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 0.74

$$\frac{-3+\sec^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] (-3 + Sec[e + f*x]^2)/(3*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 11.01, size = 41, normalized size = 0.98

method	result	size
default	$-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(3\left(\cos^2\left(fx+e\right)\right)-1\right)}{3a\cos\left(fx+e\right)^4 f}$	41
risch	$-\frac{2\left(3\,\mathrm{e}^{4i(fx+e)}+2\,\mathrm{e}^{2i(fx+e)}+3\right)}{3\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 f}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/cos(f*x+e)^4*(a*cos(f*x+e)^2)^(1/2)*(3*cos(f*x+e)^2-1)/f
```

Maxima [A]
time = 0.30, size = 48, normalized size = 1.14

$$\frac{3\left(a\sin\left(fx+e\right)^{2}-a\right)a^{2}+a^{3}}{3\left(-a\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a*sin(f*x + e)^2 - a)*a^2 + a^3)/((-a*sin(f*x + e)^2 + a)^(3/2)*a^2
*f)
```

Fricas [A]
time = 0.41, size = 40, normalized size = 0.95

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(3\cos\left(fx+e\right)^{2}-1\right)}{3\,af\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2 - 1)/(a*f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{3}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [A]
time = 0.85, size = 57, normalized size = 1.36

$$\frac{4\left(3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)}{3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{3}\sqrt{a}\,f\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*tan(1/2*f*x + 1/2*e)^2 - 1)/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*sqrt(a)*
f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

Mupad [B]
time = 19.50, size = 100, normalized size = 2.38

$$-\frac{4\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+3\,{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+3\right)}{3\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] -(4*exp(e*2i + f*x*2i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + 3*exp(e*4i + f*x*4i) + 3))/(3
*a*f*(exp(e*2i + f*x*2i) + 1)^4)
```

3.470 $\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

[Out] 1/f/(a*cos(f*x+e)^2)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3255, 3284, 16, 32}

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]

[Out] 1/(f*Sqrt[a*Cos[e + f*x]^2])

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx &= \int \frac{\tan(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{ax}}\,dx,x,\cos^2(e+fx)\right)}{2f} \\
&= -\frac{a\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx,x,\cos^2(e+fx)\right)}{2f} \\
&= \frac{1}{f\sqrt{a\cos^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 0.13, size = 20, normalized size = 1.11

method	result	size
derivativedivides	$\frac{1}{\sqrt{a-a\left(\sin^2\left(fx+e\right)\right)}\,f}$	20
default	$\frac{1}{\sqrt{a-a\left(\sin^2\left(fx+e\right)\right)}\,f}$	20
risch	$\frac{2}{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,f}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(a-a*sin(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(17) = 34$.
time = 0.52, size = 69, normalized size = 3.83

$$\frac{\frac{\sqrt{-a\sin(fx+e)^2+a}}{a\sin(fx+e)+a} - \frac{\sqrt{-a\sin(fx+e)^2+a}}{a\sin(fx+e)-a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(-a*sin(f*x + e)^2 + a)/(a*sin(f*x + e) + a) - sqrt(-a*sin(f*x + e
)^2 + a)/(a*sin(f*x + e) - a))/f
```

Fricas [A]
time = 0.40, size = 27, normalized size = 1.50

$$\frac{\sqrt{a\cos(fx+e)^2}}{af\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(e+fx)}{\sqrt{-a(\sin(e+fx)-1)(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.
time = 0.58, size = 41, normalized size = 2.28

$$\frac{2}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\sqrt{a}\,f\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/((tan(1/2*f*x + 1/2*e)^2 - 1)*sqrt(a)*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

Mupad [B]
time = 0.39, size = 61, normalized size = 3.39

$$\frac{2\sqrt{2}\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\sqrt{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}}{a\,f\,\left(4\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (2*2^(1/2)*(cos(2*e + 2*f*x) + 1)*(a*(cos(2*e + 2*f*x) + 1))^(1/2))/(a*f*(4
*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3))
```

3.471 $\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

[Out] -arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3255, 3284, 65, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]

[Out] -(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx &= \int \frac{\cot(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{af}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 1.58

$$\frac{\cos(e+fx)\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] (Cos[e + f*x]*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]))/(f*Sqrt[a*C
os[e + f*x]^2])
```

Maple [A]
time = 5.46, size = 40, normalized size = 1.29

method	result	size
default	$-\frac{\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2\left(fx+e\right)\right)}+2a}{\sin(fx+e)}\right)}{\sqrt{a}\,f}$	40

risch	$\frac{2\ln(e^{ifx}-e^{-ie})\cos(fx+e)}{f\sqrt{(e^{2i(fx+e)}+1)^2 a\,e^{-2i(fx+e)}}}-\frac{2\ln(e^{ifx}+e^{-ie})\cos(fx+e)}{f\sqrt{(e^{2i(fx+e)}+1)^2 a\,e^{-2i(fx+e)}}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e))/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(26) = 52$.
time = 0.29, size = 54, normalized size = 1.74

$$-\frac{\log\left(\frac{2\sqrt{-a\sin(fx+e)^2+a}\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs(sin(
f*x + e)))/(sqrt(a)*f)
```

Fricas [A]
time = 0.40, size = 84, normalized size = 2.71

$$\left[-\frac{\sqrt{a\cos(fx+e)^2}\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}\right)}{2\,af\cos(fx+e)},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(fx+e)^2}\sqrt{-a}}{a}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(f*x + e)^2)*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1))/(a
*f*cos(f*x + e)), sqrt(-a)*arctan(sqrt(a*cos(f*x + e)^2)*sqrt(-a)/a)/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot(e+fx)}{\sqrt{-a(\sin(e+fx)-1)(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [A]
time = 0.44, size = 32, normalized size = 1.03

$$\frac{\arctan\left(\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(e+f\,x)}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.472 $\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2af}$$

```
[Out] 1/2*arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*csc(f*x+e)^2*(a*c
os(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3255, 3284, 16, 43, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(2*Sqrt[a]*f) - (Sqrt[a*Cos[e + f*x
]^2]*Csc[e + f*x]^2)/(2*a*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx &= \int \frac{\cot^3(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{ax}}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{(1-x)^2}\,dx,x,\cos^2(e+fx)\right)}{2af}\\
&= -\frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2af}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx,x,\cos^2(e+fx)\right)}{4f}\\
&= -\frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2af}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a\cos^2(e+fx)}\right)}{2af}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2af}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 80, normalized size = 1.21

$$\frac{\cos(e+fx)\left(-\csc^2\left(\frac{1}{2}(e+fx)\right)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sec^2\left(\frac{1}{2}(e+fx)\right)\right)}{8f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-Csc[(e + f*x)/2]^2 + 4*Log[Cos[(e + f*x)/2]] - 4*Log[Sin[(e
 + f*x)/2]] + Sec[(e + f*x)/2]^2))/(8*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 9.31, size = 67, normalized size = 1.02

method	result
default	$\frac{-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}}{2a\sin(fx+e)^2}+\frac{\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2\left(fx+e\right)\right)}+2a}{\sin(fx+e)}\right)}{2\sqrt{a}}}{f}$
risch	$\frac{\left(e^{2i(fx+e)}+1\right)^2}{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,f\left(e^{2i(fx+e)}-1\right)^2}+\frac{\ln\left(e^{ifx}+e^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}-\frac{\ln\left(e^{ifx}-e^{-ie}\right.}{f\sqrt{\left(e^{2i(fx+e)}+1\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)+1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(
a*cos(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.58, size = 86, normalized size = 1.30

$$\frac{\frac{\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^2+a}\,\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)}{\sqrt{a}}-\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}}{a\sin(fx+e)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs(
sin(f*x + e)))/sqrt(a) - sqrt(-a*sin(f*x + e)^2 + a)/(a*sin(f*x + e)^2))/f
```

Fricas [A]
time = 0.40, size = 79, normalized size = 1.20

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\left(\cos\left(fx+e\right)^{2}-1\right)\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2\cos\left(fx+e\right)\right)}{4\left(af\cos\left(fx+e\right)^{3}-af\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*((cos(f*x + e)^2 - 1)*log(-(cos(f*x + e) - 1)/(
cos(f*x + e) + 1)) - 2*cos(f*x + e))/(a*f*cos(f*x + e)^3 - a*f*cos(f*x + e)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{3}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(57) = 114.
time = 0.63, size = 117, normalized size = 1.77

$$-\frac{\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}-\frac{2\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}+\frac{2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}}{8\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(tan(1/2*f*x + 1/2*e)^2/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 2*log(tan(1/
2*f*x + 1/2*e)^2)/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + (2*tan(1/2*f*x + 1/2*e)
^2 - 1)/(sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^2))/(sqrt(a)*
f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\cot(e+f\,x)^{3}}{\sqrt{a-a\sin\left(e+f\,x\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.473 $\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{3\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{8f\sqrt{a\cos^2(e+fx)}}-\frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}}+\frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 3/8*arctanh(sin(f*x+e))*cos(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)-3/8*tan(f*x+e)/
f/(a*cos(f*x+e)^2)^(1/2)+1/4*tan(f*x+e)^3/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3286, 2691, 3855}

$$\frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}}-\frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*f*Sqrt[a*Cos[e + f*x]^2]) - (3*Ta
n[e + f*x])/(8*f*Sqrt[a*Cos[e + f*x]^2]) + Tan[e + f*x]^3/(4*f*Sqrt[a*Cos[e
 + f*x]^2])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \sec(e+fx)\tan^4(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}} - \frac{(3\cos(e+fx))\int \sec(e+fx)\tan^2(e+fx)\,dx}{4\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} + \frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}} + \frac{(3\cos(e+fx))\int \sec(e+f}{8\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{3\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} - \frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} + \frac{\tan^3(e+f}{4f\sqrt{a\cos^2(e+}$$

Mathematica [A]
time = 0.09, size = 66, normalized size = 0.73

$$\frac{3\tanh^{-1}(\sin(e+fx))\cos(e+fx)+\tan(e+fx)\left(3-6\sec^2(e+fx)+8\tan^2(e+fx)\right)}{8f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x] + Tan[e + f*x]*(3 - 6*Sec[e + f*x]^2
+ 8*Tan[e + f*x]^2))/(8*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 11.08, size = 103, normalized size = 1.13

method	result
default	$\frac{10\left(\cos^2(fx+e)\right)\sin(fx+e)-4\sin(fx+e)+\left(3\ln(\sin(fx+e)-1)-3\ln(1+\sin(fx+e))\right)\left(\cos^4(fx+e)\right)}{16(1+\sin(fx+e))(\sin(fx+e)-1)\cos(fx+e)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$

risch	$\frac{i(5\,e^{6i(fx+e)}-3\,e^{4i(fx+e)}+3\,e^{2i(fx+e)}-5)}{4\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,\left(e^{2i(fx+e)}+1\right)^3 f} - \frac{3\ln\left(e^{ifx}-ie^{-ie}\right)\cos(fx+e)}{4f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}} + \frac{3\ln\left(e^{ifx}+ie^{-\right.}}{4f\sqrt{\left(e^{2i(fx+e)}+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(10*cos(f*x+e)^2*sin(f*x+e)-4*sin(f*x+e)+(3*ln(sin(f*x+e)-1)-3*ln(1+si
n(f*x+e)))*cos(f*x+e)^4)/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(a*cos(f*
x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1652 vs. $2(86) = 172$.
time = 0.92, size = 1652, normalized size = 18.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*(5*sin(7*f*x + 7*e) - 3*sin(5*f*x + 5*e) + 3*sin(3*f*x + 3*e) - 5*
sin(f*x + e))*cos(8*f*x + 8*e) - 40*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e
) + 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(3*sin(5*f*x + 5*e) - 3*sin(3
*f*x + 3*e) + 5*sin(f*x + e))*cos(6*f*x + 6*e) + 24*(3*sin(4*f*x + 4*e) + 2
*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 24*(3*sin(3*f*x + 3*e) - 5*sin(f*x +
e))*cos(4*f*x + 4*e) - 3*(2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*co
s(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x
+ 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 +
 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*
cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f
*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) +
 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*
x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 +
 8*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x +
e) + 1) + 3*(2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e
) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*co
s(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2
*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*si
n(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x
+ 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 +
48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x
+ 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(
5*cos(7*f*x + 7*e) - 3*cos(5*f*x + 5*e) + 3*cos(3*f*x + 3*e) - 5*cos(f*x +
e))*sin(8*f*x + 8*e) + 20*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(
```

```
2*f*x + 2*e) + 1)*sin(7*f*x + 7*e) + 16*(3*cos(5*f*x + 5*e) - 3*cos(3*f*x +
 3*e) + 5*cos(f*x + e))*sin(6*f*x + 6*e) - 12*(6*cos(4*f*x + 4*e) + 4*cos(2
*f*x + 2*e) + 1)*sin(5*f*x + 5*e) - 24*(3*cos(3*f*x + 3*e) - 5*cos(f*x + e)
)*sin(4*f*x + 4*e) + 12*(4*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) - 48*cos(
3*f*x + 3*e)*sin(2*f*x + 2*e) + 80*cos(f*x + e)*sin(2*f*x + 2*e) - 80*cos(2
*f*x + 2*e)*sin(f*x + e) - 20*sin(f*x + e))/((2*(4*cos(6*f*x + 6*e) + 6*cos
(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)
^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*
cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(
4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*
x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(
3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x +
6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*
sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*sqrt(a)*f)
```

Fricas [A]
time = 0.43, size = 80, normalized size = 0.88

$$-\frac{\left(3\cos\left(fx+e\right)^{4}\log\left(-\frac{\sin(fx+e)-1}{\sin(fx+e)+1}\right)+2\left(5\cos\left(fx+e\right)^{2}-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{16\,af\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] -1/16*(3*cos(f*x + e)^4*log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) + 2*(5*
cos(f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)^
5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{4}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a-a*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**4/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(86) = 172.
time = 1.14, size = 201, normalized size = 2.21

$$-\frac{\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}-\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\right|\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}-\frac{4\left(3\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^{3}-\frac{20}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}-20\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^{2}-4\right)^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}}{16\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*(3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))/sgn(ta
n(1/2*f*x + 1/2*e)^4 - 1) - 3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x
+ 1/2*e) - 2))/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*(3*(1/tan(1/2*f*x + 1/2*
e) + tan(1/2*f*x + 1/2*e))^3 - 20/tan(1/2*f*x + 1/2*e) - 20*tan(1/2*f*x + 1
/2*e))/(((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^2 - 4)^2*sgn(tan(1
/2*f*x + 1/2*e)^4 - 1)))/(sqrt(a)*f)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^4}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.474 $\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=62

$$-\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}+\frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -1/2*arctanh(sin(f*x+e))*cos(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)+1/2*tan(f*x+e)
/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3255, 3286, 2691, 3855}

$$\frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -1/2*(ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(f*Sqrt[a*Cos[e + f*x]^2]) + Tan[
e + f*x]/(2*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx &= \int \frac{\tan^2(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx \\
&= \frac{\cos(e+fx)\int \sec(e+fx)\tan^2(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}} \\
&= \frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}} - \frac{\cos(e+fx)\int \sec(e+fx)\,dx}{2\sqrt{a\cos^2(e+fx)}} \\
&= -\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2f\sqrt{a\cos^2(e+fx)}} + \frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.69

$$
\frac{-\tanh^{-1}(\sin(e+fx))\cos(e+fx)+\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (-(ArcTanh[Sin[e + f*x]]*Cos[e + f*x]) + Tan[e + f*x])/(2*f*Sqrt[a*Cos[e +
f*x]^2])
```

Maple [A]
time = 9.58, size = 65, normalized size = 1.05

method	result
default	$\frac{\frac{\sin(fx+e)}{2}+\frac{(\ln(\sin(fx+e)-1)-\ln(1+\sin(fx+e)))\left(\cos^2(fx+e)\right)}{4}}{\cos(fx+e)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$
risch	$-\frac{i\left(\mathrm{e}^{2i(fx+e)}-1\right)}{\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)f}-\frac{\ln\left(\mathrm{e}^{ifx}+i\mathrm{e}^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}}+\frac{\ln\left(\mathrm{e}^{ifx}-i\mathrm{e}^{-ie}\right.}{f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*sin(f*x+e)+1/4*(ln(sin(f*x+e)-1)-ln(1+sin(f*x+e)))*cos(f*x+e)^2)/cos(f
*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(59) = 118$.
time = 0.62, size = 575, normalized size = 9.27

$$\frac{4\left(\sin\left(3fx+3e\right)-\sin\left(fx+e\right)\right)\cos\left(4fx+4e\right)-\left(2\left(2\cos\left(2fx+2e\right)+1\right)\cos\left(4fx+4e\right)+\cos\left(4fx+4e\right)^{2}+4\cos\left(2fx+2e\right)^{2}+\sin\left(4fx+4e\right)^{2}+4\sin\left(4fx+4e\right)\sin\left(2fx+2e\right)+4\sin\left(2fx+2e\right)^{2}+4\cos\left(2fx+2e\right)+1\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}+2\sin\left(fx+e\right)+1\right)+\left(2\left(2\cos\left(2fx+2e\right)+1\right)\cos\left(4fx+4e\right)+\cos\left(4fx+4e\right)^{2}+4\cos\left(2fx+2e\right)^{2}+\sin\left(4fx+4e\right)^{2}+4\sin\left(4fx+4e\right)\sin\left(2fx+2e\right)+4\sin\left(2fx+2e\right)^{2}+4\cos\left(2fx+2e\right)+1\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}-2\sin\left(fx+e\right)+1\right)-4\left(\cos\left(3fx+3e\right)-\cos\left(fx+e\right)\right)\sin\left(4fx+4e\right)+4\left(2\cos\left(2fx+2e\right)+1\right)\sin\left(3fx+3e\right)-8\cos\left(3fx+3e\right)\sin\left(2fx+2e\right)+8\cos\left(fx+e\right)\sin\left(2fx+2e\right)-8\cos\left(2fx+2e\right)\sin\left(fx+e\right)-4\sin\left(fx+e\right)}{4\left(2\left(2\cos\left(2fx+2e\right)+1\right)\cos\left(4fx+4e\right)+\cos\left(4fx+4e\right)^{2}+4\cos\left(2fx+2e\right)^{2}+\sin\left(4fx+4e\right)^{2}+4\sin\left(4fx+4e\right)\sin\left(2fx+2e\right)+4\sin\left(2fx+2e\right)^{2}+4\cos\left(2fx+2e\right)+1\right)\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*f*x + 3*e) - sin(f*x + e))*cos(4*f*x + 4*e) - (2*(2*cos(2*f*x
 + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 +
 sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2
*e)^2 + 4*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin
(f*x + e) + 1) + (2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x +
 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*si
n(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*log(cos(f*x
 + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(cos(3*f*x + 3*e) - cos(
f*x + e))*sin(4*f*x + 4*e) + 4*(2*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) -
8*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 8*cos(f*x + e)*sin(2*f*x + 2*e) - 8*c
os(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e))/((2*(2*cos(2*f*x + 2*e) + 1)
*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x +
 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*co
s(2*f*x + 2*e) + 1)*sqrt(a)*f)
```

Fricas [A]
time = 0.40, size = 67, normalized size = 1.08

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\cos\left(fx+e\right)^{2}\log\left(-\frac{\sin\left(fx+e\right)+1}{\sin\left(fx+e\right)-1}\right)-2\,\sin\left(fx+e\right)\right)}{4\,af\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2*log(-(sin(f*x + e) + 1)/(sin(f*
x + e) - 1)) - 2*sin(f*x + e))/(a*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt{-a(\sin(e+fx)-1)(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(59) = 118.
time = 0.73, size = 169, normalized size = 2.73

$$\frac{\frac{\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}-\frac{\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}-\frac{4\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^2-4\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}}{4\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))/sgn(tan(1/
2*f*x + 1/2*e)^4 - 1) - log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*
e) - 2))/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*(1/tan(1/2*f*x + 1/2*e) + tan(
1/2*f*x + 1/2*e))/(((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^2 - 4)*
sgn(tan(1/2*f*x + 1/2*e)^4 - 1)))/(sqrt(a)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^2}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.475 $\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=25

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

[Out] -cot(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)

Rubi [A]

time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3286, 2686, 8}

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]

[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]))

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx &= \int \frac{\cot^2(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx\\ &= \frac{\cos(e+fx)\int \cot(e+fx)\csc(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cos(e+fx)\text{Subst}\left(\int 1\,dx,x,\csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]))
```

Maple [A]
time = 0.30, size = 32, normalized size = 1.28

method	result	size
default	$-\frac{\cos(fx+e)}{\sin(fx+e)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$	32
risch	$-\frac{2i\left(e^{2i(fx+e)}+1\right)}{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,f\left(e^{2i(fx+e)}-1\right)}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -cos(f*x+e)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(25) = 50$.
time = 0.57, size = 98, normalized size = 3.92

$$-\frac{2\left(\cos\left(fx+e\right)\sin\left(2\,fx+2\,e\right)-\cos\left(2\,fx+2\,e\right)\sin\left(fx+e\right)+\sin\left(fx+e\right)\right)\sqrt{a}}{\left(a\cos\left(2\,fx+2\,e\right)^{2}+a\sin\left(2\,fx+2\,e\right)^{2}-2\,a\cos\left(2\,fx+2\,e\right)+a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(cos(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e)*sin(f*x + e) + sin(f*x
 + e))*sqrt(a)/((a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 - 2*a*cos(2*f*
x + 2*e) + a)*f)
```

Fricas [A]
time = 0.39, size = 36, normalized size = 1.44

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}}{af\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{2}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(25) = 50$.
time = 0.57, size = 67, normalized size = 2.68

$$\frac{\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}+\frac{1}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*f*x + 1/2*e)/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + 1/(sgn(tan(1/2*
f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)))/(sqrt(a)*f)
```

Mupad [B]
time = 15.06, size = 37, normalized size = 1.48

$$-\frac{\sqrt{2\,a\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}}{a\,f\,\sin\left(2\,e+2\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a - a*sin(e + f*x)^2)^(1/2),x)
[Out] -(2*a*(cos(2*e + 2*f*x) + 1))^(1/2)/(a*f*sin(2*e + 2*f*x))
```

3.476 $\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] cot(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2/f/(a*cos(f*
x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3255, 3286, 2686}

$$\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*
f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx &= \int \frac{\cot^4(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx \\
&= \frac{\cos(e+fx)\int \cot^3(e+fx)\csc(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}} \\
&= -\frac{\cos(e+fx)\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}} \\
&= \frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 0.62

$$-\frac{\cot(e+fx)\left(-3+\csc^2(e+fx)\right)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -1/3*(Cot[e + f*x]*(-3 + Csc[e + f*x]^2))/(f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 3.82, size = 44, normalized size = 0.73

method	result	size
default	$\frac{\cos(fx+e)\left(3\left(\sin^2(fx+e)\right)-1\right)}{3\sin(fx+e)^3\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$	44
risch	$\frac{2i\left(e^{2i(fx+e)}+1\right)\left(3\,e^{4i(fx+e)}-2\,e^{2i(fx+e)}+3\right)}{3\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,f\left(e^{2i(fx+e)}-1\right)^3}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*cos(f*x+e)*(3*sin(f*x+e)^2-1)/sin(f*x+e)^3/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(59) = 118.
time = 0.63, size = 571, normalized size = 9.52

$$-\frac{2\left(\left(3\sin(5fx+5e)-2\sin(3fx+3e)+3\sin(fx+e)\right)\cos(6fx+6e)+9\left(\sin(4fx+4e)-\sin(2fx+2e)\right)\cos(5fx+5e)+3\left(2\sin(3fx+3e)-3\sin(fx+e)\right)\cos(4fx+4e)-\left(3\cos(5fx+5e)-2\cos(3fx+3e)+3\cos(fx+e)\right)\sin(6fx+6e)-3\left(3\cos(4fx+4e)-3\cos(2fx+2e)+1\right)\sin(5fx+5e)-3\left(2\cos(3fx+3e)-3\cos(fx+e)\right)\sin(4fx+4e)-2\left(3\cos(2fx+2e)-1\right)\sin(3fx+3e)+6\cos(3fx+3e)\sin(2fx+2e)-9\cos(fx+e)\sin(2fx+2e)+9\cos(2fx+2e)\sin(fx+e)-3\sin(fx+e)\right)\sqrt{a}}{3\left(a\cos(6fx+6e)^2+9a\cos(4fx+4e)^2+9a\cos(2fx+2e)^2+a\sin(6fx+6e)^2+9a\sin(4fx+4e)^2-18a\sin(4fx+4e)\sin(2fx+2e)+9a\sin(2fx+2e)^2-2\left(3a\cos(4fx+4e)-3a\cos(2fx+2e)+a\right)\cos(6fx+6e)-6\left(3a\cos(2fx+2e)-a\right)\cos(4fx+4e)-6a\cos(2fx+2e)-6\left(a\sin(4fx+4e)-a\sin(2fx+2e)\right)\sin(6fx+6e)+a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*((3*sin(5*f*x + 5*e) - 2*sin(3*f*x + 3*e) + 3*sin(f*x + e))*cos(6*f*x
+ 6*e) + 9*(sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 3*(2*si
n(3*f*x + 3*e) - 3*sin(f*x + e))*cos(4*f*x + 4*e) - (3*cos(5*f*x + 5*e) - 2
*cos(3*f*x + 3*e) + 3*cos(f*x + e))*sin(6*f*x + 6*e) - 3*(3*cos(4*f*x + 4*e
) - 3*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e) - 3*(2*cos(3*f*x + 3*e) - 3*co
s(f*x + e))*sin(4*f*x + 4*e) - 2*(3*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e)
+ 6*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 9*cos(f*x + e)*sin(2*f*x + 2*e) + 9
*cos(2*f*x + 2*e)*sin(f*x + e) - 3*sin(f*x + e))*sqrt(a)/((a*cos(6*f*x + 6*
e)^2 + 9*a*cos(4*f*x + 4*e)^2 + 9*a*cos(2*f*x + 2*e)^2 + a*sin(6*f*x + 6*e)
^2 + 9*a*sin(4*f*x + 4*e)^2 - 18*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a*
sin(2*f*x + 2*e)^2 - 2*(3*a*cos(4*f*x + 4*e) - 3*a*cos(2*f*x + 2*e) + a)*co
s(6*f*x + 6*e) - 6*(3*a*cos(2*f*x + 2*e) - a)*cos(4*f*x + 4*e) - 6*a*cos(2*
f*x + 2*e) - 6*(a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
 a)*f)
```

Fricas [A]
time = 0.38, size = 58, normalized size = 0.97

$$\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\,\left(3\,\cos\left(fx+e\right)^{2}-2\right)}{3\,\left(af\cos\left(fx+e\right)^{3}-af\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2 - 2)/((a*f*cos(f*x + e)^3 - a*
f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{4}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [A]
time = 0.67, size = 99, normalized size = 1.65

$$\frac{\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-9\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}-\frac{9\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}}{24\,\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((tan(1/2*f*x + 1/2*e)^3 - 9*tan(1/2*f*x + 1/2*e))/sgn(tan(1/2*f*x + 1
/2*e)^4 - 1) - (9*tan(1/2*f*x + 1/2*e)^2 - 1)/(sgn(tan(1/2*f*x + 1/2*e)^4 -
 1)*tan(1/2*f*x + 1/2*e)^3))/(sqrt(a)*f)
```

Mupad [B]
time = 19.19, size = 118, normalized size = 1.97

$$\frac{4\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,\left(-{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,2\mathrm{i}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)}{3\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (4*exp(e*2i + f*x*2i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2)^2)^(1/2)*(exp(e*4i + f*x*4i)*3i - exp(e*2i + f*x*2i)*2i + 3i))/
(3*a*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*2i + f*x*2i) + 1))
```

3.477 $\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -cot(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)+2/3*cot(f*x+e)*csc(f*x+e)^2/f/(a*cos(f
*x+e)^2)^(1/2)-1/5*cot(f*x+e)*csc(f*x+e)^4/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3286, 2686, 200}

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2])) + (2*Cot[e + f*x]*Csc[e + f*x]^2
)/(3*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x]^4)/(5*f*Sqrt[a*
Cos[e + f*x]^2])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx &= \int \frac{\cot^6(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx \\
&= \frac{\cos(e+fx)\int \cot^5(e+fx)\csc(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}} \\
&= -\frac{\cos(e+fx)\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}} \\
&= -\frac{\cos(e+fx)\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}} \\
&= -\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}} + \frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.51

$$-\frac{\cot(e+fx)\left(15-10\csc^2(e+fx)+3\csc^4(e+fx)\right)}{15f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -1/15*(Cot[e + f*x]*(15 - 10*Csc[e + f*x]^2 + 3*Csc[e + f*x]^4))/(f*Sqrt[a*
Cos[e + f*x]^2])
```

Maple [A]
time = 5.45, size = 54, normalized size = 0.56

method	result	size
default	$-\frac{\cos(fx+e)\left(15\left(\sin^4(fx+e)\right)-10\left(\sin^2(fx+e)\right)+3\right)}{15\sin(fx+e)^5\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$	54

risch	$-\frac{2i(e^{2i(fx+e)}+1)(15e^{8i(fx+e)}-20e^{6i(fx+e)}+58e^{4i(fx+e)}-20e^{2i(fx+e)}+15)}{15\sqrt{(e^{2i(fx+e)}+1)^2 a e^{-2i(fx+e)}}\, f(e^{2i(fx+e)}-1)^5}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*cos(f*x+e)*(15*sin(f*x+e)^4-10*sin(f*x+e)^2+3)/sin(f*x+e)^5/(a*cos(f*
x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(94) = 188$.
time = 0.67, size = 1344, normalized size = 14.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*((15*sin(9*f*x + 9*e) - 20*sin(7*f*x + 7*e) + 58*sin(5*f*x + 5*e) - 20
*sin(3*f*x + 3*e) + 15*sin(f*x + e))*cos(10*f*x + 10*e) + 75*(sin(8*f*x + 8
*e) - 2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(9*f*x
 + 9*e) + 5*(20*sin(7*f*x + 7*e) - 58*sin(5*f*x + 5*e) + 20*sin(3*f*x + 3*e
) - 15*sin(f*x + e))*cos(8*f*x + 8*e) + 100*(2*sin(6*f*x + 6*e) - 2*sin(4*f
*x + 4*e) + sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 10*(58*sin(5*f*x + 5*e) -
20*sin(3*f*x + 3*e) + 15*sin(f*x + e))*cos(6*f*x + 6*e) + 290*(2*sin(4*f*x
+ 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 50*(4*sin(3*f*x + 3*e) - 3*si
n(f*x + e))*cos(4*f*x + 4*e) - (15*cos(9*f*x + 9*e) - 20*cos(7*f*x + 7*e) +
 58*cos(5*f*x + 5*e) - 20*cos(3*f*x + 3*e) + 15*cos(f*x + e))*sin(10*f*x +
10*e) - 15*(5*cos(8*f*x + 8*e) - 10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e)
- 5*cos(2*f*x + 2*e) + 1)*sin(9*f*x + 9*e) - 5*(20*cos(7*f*x + 7*e) - 58*co
s(5*f*x + 5*e) + 20*cos(3*f*x + 3*e) - 15*cos(f*x + e))*sin(8*f*x + 8*e) -
20*(10*cos(6*f*x + 6*e) - 10*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) - 1)*sin
(7*f*x + 7*e) - 10*(58*cos(5*f*x + 5*e) - 20*cos(3*f*x + 3*e) + 15*cos(f*x
+ e))*sin(6*f*x + 6*e) - 58*(10*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) + 1)*
sin(5*f*x + 5*e) - 50*(4*cos(3*f*x + 3*e) - 3*cos(f*x + e))*sin(4*f*x + 4*e
) - 20*(5*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) + 100*cos(3*f*x + 3*e)*sin
(2*f*x + 2*e) - 75*cos(f*x + e)*sin(2*f*x + 2*e) + 75*cos(2*f*x + 2*e)*sin(
f*x + e) - 15*sin(f*x + e))*sqrt(a)/((a*cos(10*f*x + 10*e)^2 + 25*a*cos(8*f
*x + 8*e)^2 + 100*a*cos(6*f*x + 6*e)^2 + 100*a*cos(4*f*x + 4*e)^2 + 25*a*co
s(2*f*x + 2*e)^2 + a*sin(10*f*x + 10*e)^2 + 25*a*sin(8*f*x + 8*e)^2 + 100*a
*sin(6*f*x + 6*e)^2 + 100*a*sin(4*f*x + 4*e)^2 - 100*a*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 25*a*sin(2*f*x + 2*e)^2 - 2*(5*a*cos(8*f*x + 8*e) - 10*a*co
s(6*f*x + 6*e) + 10*a*cos(4*f*x + 4*e) - 5*a*cos(2*f*x + 2*e) + a)*cos(10*f
*x + 10*e) - 10*(10*a*cos(6*f*x + 6*e) - 10*a*cos(4*f*x + 4*e) + 5*a*cos(2*
```

```
f*x + 2*e) - a)*cos(8*f*x + 8*e) - 20*(10*a*cos(4*f*x + 4*e) - 5*a*cos(2*f*
x + 2*e) + a)*cos(6*f*x + 6*e) - 20*(5*a*cos(2*f*x + 2*e) - a)*cos(4*f*x +
4*e) - 10*a*cos(2*f*x + 2*e) - 10*(a*sin(8*f*x + 8*e) - 2*a*sin(6*f*x + 6*e
) + 2*a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) - 50*(2*a
*sin(6*f*x + 6*e) - 2*a*sin(4*f*x + 4*e) + a*sin(2*f*x + 2*e))*sin(8*f*x +
8*e) - 100*(2*a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + a
)*f)
```

Fricas [A]
time = 0.44, size = 79, normalized size = 0.82

$$-\frac{\left(15\cos\left(fx+e\right)^{4}-20\cos\left(fx+e\right)^{2}+8\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{15\left(af\cos\left(fx+e\right)^{5}-2\,af\cos\left(fx+e\right)^{3}+af\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] -1/15*(15*cos(f*x + e)^4 - 20*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((
a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{6}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a-a*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**6/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [A]
time = 0.76, size = 128, normalized size = 1.33

$$\frac{\frac{3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-25\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+150\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}+\frac{150\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-25\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+3}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}}}{480\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] 1/480*((3*tan(1/2*f*x + 1/2*e)^5 - 25*tan(1/2*f*x + 1/2*e)^3 + 150*tan(1/2*
f*x + 1/2*e))/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + (150*tan(1/2*f*x + 1/2*e)^4
 - 25*tan(1/2*f*x + 1/2*e)^2 + 3)/(sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*
f*x + 1/2*e)^5))/(sqrt(a)*f)
```

Mupad [B]
time = 22.47, size = 491, normalized size = 5.11

$$-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,4\mathrm{i}}{a\,f\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,32\mathrm{i}}{3\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^2\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,352\mathrm{i}}{15\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,128\mathrm{i}}{5\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^4\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,64\mathrm{i}}{5\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^5\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2)^2)^(1/2)*4i)/(a*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*1i + f*x*1i)
+ exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*
1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*32i)/(3*a*f*(exp(e*2i + f*x*2i)
 - 1)^2*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a
 - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*352
i)/(15*a*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*
3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
 + f*x*1i)*1i)/2)^2)^(1/2)*128i)/(5*a*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1
i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1
i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*64i)/(5*a*f*(exp(e*
2i + f*x*2i) - 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.478 $\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{a^2}{7f\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}+\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/7*a^2/f/(a*cos(f*x+e)^2)^(7/2)-2/5*a/f/(a*cos(f*x+e)^2)^(5/2)+1/3/f/(a*co
s(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3284, 16, 45}

$$\frac{a^2}{7f\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}+\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] a^2/(7*f*(a*Cos[e + f*x]^2)^(7/2)) - (2*a)/(5*f*(a*Cos[e + f*x]^2)^(5/2)) +
 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
```

```
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tan^5(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{a^3\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{9/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{a^3\text{Subst}\left(\int \left(\frac{1}{(ax)^{9/2}}-\frac{2}{a(ax)^{7/2}}+\frac{1}{a^2(ax)^{5/2}}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{a^2}{7f\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}+\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.75

$$\frac{\left(15-42\cos^2(e+fx)+35\cos^4(e+fx)\right)\sec^4(e+fx)}{105f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a - a*Sin[e + f*x]^2)^(3/2),x]
[Out] ((15 - 42*Cos[e + f*x]^2 + 35*Cos[e + f*x]^4)*Sec[e + f*x]^4)/(105*f*(a*Cos
[e + f*x]^2)^(3/2))
```

Maple [A]
time = 13.60, size = 51, normalized size = 0.75

method	result	size
default	$\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(35\left(\cos^4(fx+e)\right)-42\left(\cos^2(fx+e)\right)+15\right)}{105a^2\cos(fx+e)^8 f}$	51
risch	$\frac{\frac{8\,e^{10i(fx+e)}}{3}-\frac{32\,e^{8i(fx+e)}}{15}+\frac{304\,e^{6i(fx+e)}}{35}-\frac{32\,e^{4i(fx+e)}}{15}+\frac{8\,e^{2i(fx+e)}}{3}}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,\left(e^{2i(fx+e)}+1\right)^6 a}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105/a^2/cos(f*x+e)^8*(a*cos(f*x+e)^2)^(1/2)*(35*cos(f*x+e)^4-42*cos(f*x+e
)^2+15)/f
```

Maxima [A]
time = 0.33, size = 72, normalized size = 1.06

$$\frac{35 \left(a \sin \left(fx+e\right)^{2}-a\right)^{2} a^{3}+42 \left(a \sin \left(fx+e\right)^{2}-a\right) a^{4}+15 \, a^{5}}{105 \left(-a \sin \left(fx+e\right)^{2}+a\right)^{\frac{7}{2}} a^{3} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/105*(35*(a*sin(f*x + e)^2 - a)^2*a^3 + 42*(a*sin(f*x + e)^2 - a)*a^4 + 15
*a^5)/((-a*sin(f*x + e)^2 + a)^(7/2)*a^3*f)
```

Fricas [A]
time = 0.39, size = 50, normalized size = 0.74

$$\frac{\left(35 \, \cos \left(fx+e\right)^{4}-42 \, \cos \left(fx+e\right)^{2}+15\right) \sqrt{a \cos \left(fx+e\right)^{2}}}{105 \, a^{2} f \cos \left(fx+e\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*cos(f*x + e)^4 - 42*cos(f*x + e)^2 + 15)*sqrt(a*cos(f*x + e)^2)/(
a^2*f*cos(f*x + e)^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}{\left(e+fx\right)}}{\left(-a\left(\sin{\left(e+fx\right)}-1\right)\left(\sin{\left(e+fx\right)}+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [A]
time = 0.82, size = 64, normalized size = 0.94

$$\frac{15\left(a\tan\left(fx+e\right)^{2}+a\right)^{\frac{7}{2}}-42\left(a\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}a+35\left(a\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{2}}{105\,a^{5}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(a*tan(f*x + e)^2 + a)^(7/2) - 42*(a*tan(f*x + e)^2 + a)^(5/2)*a
+ 35*(a*tan(f*x + e)^2 + a)^(3/2)*a^2)/(a^5*f)
```

Mupad [B]
time = 33.91, size = 583, normalized size = 8.57

$$\frac{16\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{3\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{464\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{15\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{3072\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{35\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{4736\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{35\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^5\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{768\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{7\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^6\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{256\,{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{7\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^7\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (16*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2))/(3*a^2*f*(exp(e*2i + f*x*2i) + 1)^2*(exp(e*1i + f*x*
1i) + exp(e*3i + f*x*3i))) - (464*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i -
f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(15*a^2*f*(exp(e*2i +
f*x*2i) + 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (3072*exp(e*3i
+ f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^
2)^(1/2))/(35*a^2*f*(exp(e*2i + f*x*2i) + 1)^4*(exp(e*1i + f*x*1i) + exp(e*
3i + f*x*3i))) - (4736*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)
/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(35*a^2*f*(exp(e*2i + f*x*2i) + 1
)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (768*exp(e*3i + f*x*3i)*(a
 - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(7
*a^2*f*(exp(e*2i + f*x*2i) + 1)^6*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))
) - (256*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
 + f*x*1i)*1i)/2)^2)^(1/2))/(7*a^2*f*(exp(e*2i + f*x*2i) + 1)^7*(exp(e*1i +
 f*x*1i) + exp(e*3i + f*x*3i)))
```

3.479 $\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/5*a/f/(a*cos(f*x+e)^2)^(5/2)-1/3/f/(a*cos(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3284, 16, 45}

$$\frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] a/(5*f*(a*Cos[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
```

```
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tan^3(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{a^2\text{Subst}\left(\int \frac{1-x}{(ax)^{7/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{a^2\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{1}{a(ax)^{5/2}}\right)\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 34, normalized size = 0.77

$$\frac{a(3-5\cos^2(e+fx))}{15f\left(a\cos^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
[Out] (a*(3 - 5*Cos[e + f*x]^2))/(15*f*(a*Cos[e + f*x]^2)^(5/2))
```

Maple [A]
time = 13.66, size = 41, normalized size = 0.93

method	result	size
default	$-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(5\left(\cos^2(fx+e)\right)-3\right)}{15a^2\cos(fx+e)^6 f}$	41
risch	$-\frac{8\left(5\,e^{6i(fx+e)}-2\,e^{4i(fx+e)}+5\,e^{2i(fx+e)}\right)}{15f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,\left(e^{2i(fx+e)}+1\right)^4 a}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/a^2/cos(f*x+e)^6*(a*cos(f*x+e)^2)^(1/2)*(5*cos(f*x+e)^2-3)/f
```

Maxima [A]
time = 0.33, size = 50, normalized size = 1.14

$$\frac{5\left(a\sin\left(fx+e\right)^{2}-a\right)a^{2}+3\,a^{3}}{15\left(-a\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(5*(a*sin(f*x + e)^2 - a)*a^2 + 3*a^3)/((-a*sin(f*x + e)^2 + a)^(5/2)*
a^2*f)
```

Fricas [A]
time = 0.38, size = 40, normalized size = 0.91

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(5\,\cos\left(fx+e\right)^{2}-3\right)}{15\,a^{2}f\cos\left(fx+e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(a*cos(f*x + e)^2)*(5*cos(f*x + e)^2 - 3)/(a^2*f*cos(f*x + e)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{3}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [A]
time = 0.70, size = 44, normalized size = 1.00

$$\frac{3\left(a\tan\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}-5\left(a\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a}{15\,a^{4}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(a*tan(f*x + e)^2 + a)^(5/2) - 5*(a*tan(f*x + e)^2 + a)^(3/2)*a)/(a
^4*f)
```

Mupad [B]
time = 20.21, size = 389, normalized size = 8.84

$$-\frac{16\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{3\,a^2\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{272\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{15\,a^2\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{128\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{5\,a^2\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{64\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{5\,a^2\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^5\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (272*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2)^2)^(1/2))/(15*a^2*f*(exp(e*2i + f*x*2i) + 1)^3*(exp(e*1i + f*
x*1i) + exp(e*3i + f*x*3i))) - (16*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i -
 f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(3*a^2*f*(exp(e*2i +
f*x*2i) + 1)^2*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (128*exp(e*3i +
 f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2
)^(1/2))/(5*a^2*f*(exp(e*2i + f*x*2i) + 1)^4*(exp(e*1i + f*x*1i) + exp(e*3i
 + f*x*3i))) + (64*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 -
 (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(5*a^2*f*(exp(e*2i + f*x*2i) + 1)^5*(
exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.480 $\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=21

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/3/f/(a*cos(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3255, 3284, 16, 32}

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a\text{Subst}\left(\int \frac{1}{(ax)^{5/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 21, normalized size = 1.00

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Maple [A]
time = 0.11, size = 21, normalized size = 1.00

method	result	size
derivativedivides	$\frac{1}{3\left(a-a\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}} f}$	21
default	$\frac{1}{3\left(a-a\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}} f}$	21
risch	$\frac{8\,e^{2i(fx+e)}}{3f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,\left(e^{2i(fx+e)}+1\right)^2 a}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(a-a*sin(f*x+e)^2)^(3/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(18) = 36.
time = 0.56, size = 101, normalized size = 4.81

$$\frac{\frac{1}{\sqrt{-a\sin\left(fx+e\right)^2+a}\,a\sin\left(fx+e\right)+\sqrt{-a\sin\left(fx+e\right)^2+a}\,a} - \frac{1}{\sqrt{-a\sin\left(fx+e\right)^2+a}\,a\sin\left(fx+e\right)-\sqrt{-a\sin\left(fx+e\right)^2+a}\,a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(1/(sqrt(-a*sin(f*x + e)^2 + a)*a*sin(f*x + e) + sqrt(-a*sin(f*x + e)^2
 + a)*a) - 1/(sqrt(-a*sin(f*x + e)^2 + a)*a*sin(f*x + e) - sqrt(-a*sin(f*x
+ e)^2 + a)*a))/f
```

Fricas [A]
time = 0.43, size = 28, normalized size = 1.33

$$\frac{\sqrt{a\cos\left(fx+e\right)^{2}}}{3\,a^{2}f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*cos(f*x + e)^2)/(a^2*f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [A]
time = 0.66, size = 23, normalized size = 1.10

$$\frac{\left(a\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}{3\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(a*tan(f*x + e)^2 + a)^(3/2)/(a^3*f)
```

Mupad [B]
time = 18.31, size = 72, normalized size = 3.43

$$\frac{16\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^{2}}}{3\,a^{2}\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (16*exp(e*4i + f*x*4i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2))/(3*a^2*f*(exp(e*2i + f*x*2i) + 1)^4)
```

3.481 $\int \frac{\cot(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}} \, dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a*cos(f*x+e)^2)^(
1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3255, 3284, 53, 65, 212}

$$\frac{1}{af\sqrt{a\cos^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cos[
e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\cot(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{1}{af\sqrt{a\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2af} \\
&= \frac{1}{af\sqrt{a\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{a^2f} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a\cos^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 1.04

$$
\frac{1+\cos(e+fx)\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{af\sqrt{a\cos^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (1 + Cos[e + f*x]*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]))/(a*f*Sq
rt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 16.43, size = 75, normalized size = 1.42

method	result
default	$\frac{-\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}+2a}{\sin(fx+e)}\right)a^{2}\left(\cos^{2}(fx+e)\right)+\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}\,a^{\frac{3}{2}}}{a^{\frac{7}{2}}\cos(fx+e)^{2}f}$
risch	$\frac{2}{a\sqrt{\left(e^{2i(fx+e)}+1\right)^{2}a\,e^{-2i(fx+e)}}\,f}+\frac{2\ln\left(e^{ifx}-e^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^{2}a\,e^{-2i(fx+e)}}\,a}-\frac{2\ln\left(e^{ifx}+e^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^{2}a\,e^{-2i(fx}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(7/2)/cos(f*x+e)^2*(-ln(2/sin(f*x+e)*(a^(1/2)*(a*cos(f*x+e)^2)^(1/2)+a)
)*a^2*cos(f*x+e)^2+(a*cos(f*x+e)^2)^(1/2)*a^(3/2))/f
```

Maxima [A]
time = 0.31, size = 77, normalized size = 1.45

$$-\frac{\frac{\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}}-\frac{1}{\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs(sin
(f*x + e)))/a^(3/2) - 1/(sqrt(-a*sin(f*x + e)^2 + a)*a))/f
```

Fricas [A]
time = 0.41, size = 58, normalized size = 1.09

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\cos\left(fx+e\right)\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}\right)-2\right)}{2\,a^{2}f\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] -1/2*sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1)) - 2)/(a^2*f*cos(f*x + e)^2)

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\left(-a\left(\sin(e+fx)-1\right)\left(\sin(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)**2)**(3/2),x)

[Out] Integral(cot(e + f*x)/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2), x)

Giac [**A**]

time = 0.43, size = 59, normalized size = 1.11

$$\frac{\arctan\left(\frac{\sqrt{-a\sin(fx+e)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,af}+\frac{1}{\sqrt{-a\sin(fx+e)^2+a}\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] arctan(sqrt(-a*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*a*f) + 1/(sqrt(-a*sin(f*x + e)^2 + a)*a*f)

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(e+f\,x)}{\left(a-a\,\sin\left(e+f\,x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(3/2),x)

[Out] int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(3/2), x)

3.482 $\int \frac{\cot^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2a^2f}$$

```
[Out] -1/2*arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csc(f*x+e)^2*(a*
cos(f*x+e)^2)^(1/2)/a^2/f
```

Rubi [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3255, 3284, 16, 44, 65, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - (Sqrt[a*Cos[e +
f*x]^2]*Csc[e + f*x]^2)/(2*a^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\cot^3(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)^2\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2af} \\
&= -\frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2a^2f} - \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{4af} \\
&= -\frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2a^2f} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{2a^2f} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2a^2f}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 1.24

$$-\frac{\cos^3(e+fx)\left(\csc^2\left(\frac{1}{2}(e+fx)\right)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\sec^2\left(\frac{1}{2}(e+fx)\right)\right)}{8f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*(Cos[e + f*x]^3*(Csc[(e + f*x)/2]^2 + 4*Log[Cos[(e + f*x)/2]] - 4*Log[
Sin[(e + f*x)/2]] - Sec[(e + f*x)/2]^2))/(f*(a*Cos[e + f*x]^2)^(3/2))
```

Maple [A]
time = 8.81, size = 67, normalized size = 1.02

method	result
default	$\frac{-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}}{2a^2\sin\left(fx+e\right)^2}-\frac{\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2\left(fx+e\right)\right)}+2a}{\sin\left(fx+e\right)}\right)}{2a^{\frac{3}{2}}}}{f}$
risch	$\frac{\left(e^{2i(fx+e)}+1\right)^2}{a\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}\; f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{ifx}+e^{-ie}\right)\cos\left(fx+e\right)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}\; a}+\frac{\ln\left(e^{ifx}-e^{-i}\right.}{f\sqrt{\left(e^{2i(fx+e)}+\right.}}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a^2/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)-1/2/a^(3/2)*ln((2*a+2*a^(1/2)
*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.28, size = 106, normalized size = 1.61

$$-\frac{\frac{\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^2+a}\,\sqrt{a}}{|\sin\left(fx+e\right)|}+\frac{2\,a}{|\sin\left(fx+e\right)|}\right)}{a^{\frac{3}{2}}}-\frac{1}{\sqrt{-a\sin\left(fx+e\right)^2+a}\;a}+\frac{1}{\sqrt{-a\sin\left(fx+e\right)^2+a}\;a\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs
(sin(f*x + e)))/a^(3/2) - 1/(sqrt(-a*sin(f*x + e)^2 + a)*a) + 1/(sqrt(-a*si
n(f*x + e)^2 + a)*a*sin(f*x + e)^2))/f
```

Fricas [A]
time = 0.43, size = 83, normalized size = 1.26

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\left(\cos\left(fx+e\right)^{2}-1\right)\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}\right)-2\cos\left(fx+e\right)\right)}{4\left(a^{2}f\cos\left(fx+e\right)^{3}-a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*((cos(f*x + e)^2 - 1)*log(-(cos(f*x + e) + 1)/(
cos(f*x + e) - 1)) - 2*cos(f*x + e))/(a^2*f*cos(f*x + e)^3 - a^2*f*cos(f*x
+ e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{3}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(57) = 114.
time = 0.73, size = 129, normalized size = 1.95

$$-\frac{\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}+\frac{2\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}-\frac{2\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+\sqrt{a}}{a^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(tan(1/2*f*x + 1/2*e)^2/(a^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)) + 2*
log(tan(1/2*f*x + 1/2*e)^2)/(a^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)) - (2*
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a))/(a^2*sgn(tan(1/2*f*x + 1/2*e)^4 -
 1)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\mathrm{cot}\left(e+f\,x\right)}^{3}}{{\left(a-a\,{\sin\left(e+f\,x\right)}^{2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(3/2), x)
```

3.483 $\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}-\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -1/8*arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a*cos(f*x+e)^2)^(1/2)-1/8*tan(f*x+
e)/a/f/(a*cos(f*x+e)^2)^(1/2)+1/4*sec(f*x+e)^2*tan(f*x+e)/a/f/(a*cos(f*x+e)
^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3255, 3286, 2691, 3853, 3855}

$$-\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*(ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(a*f*Sqrt[a*Cos[e + f*x]^2]) - Ta
n[e + f*x]/(8*a*f*Sqrt[a*Cos[e + f*x]^2]) + (Sec[e + f*x]^2*Tan[e + f*x])/(
4*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^2(e+fx)}{(a\cos^2(e+fx))^{3/2}}\,dx$$
$$= \frac{\cos(e+fx)\int \sec^3(e+fx)\tan^2(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}} - \frac{\cos(e+fx)\int \sec^3(e+fx)\,dx}{4a\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}} + \frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}} - \frac{\cos(e+fx)\int \sec(e+}{8a\sqrt{a\cos^2(e+f}$$
$$= -\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{8af\sqrt{a\cos^2(e+fx)}} - \frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}} + \frac{\sec^2(e+fx)\,\mathrm{t}}{4af\sqrt{a\cos^2}}$$

Mathematica [A]
time = 0.07, size = 59, normalized size = 0.56

$$\frac{-\tanh^{-1}(\sin(e+fx))\cos(e+fx)+(-1+2\sec^2(e+fx))\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(ArcTanh[Sin[e + f*x]]*Cos[e + f*x]) + (-1 + 2*Sec[e + f*x]^2)*Tan[e + f*
x])/(8*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A]
time = 8.65, size = 104, normalized size = 0.98

method	result
default	$-\frac{-2\left(\cos^2(fx+e)\right)\sin(fx+e)+4\sin(fx+e)+\left(\ln(\sin(fx+e)-1)-\ln(1+\sin(fx+e))\right)\left(\cos^4(fx+e)\right)}{16a(1+\sin(fx+e))(\sin(fx+e)-1)\cos(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$
risch	$\frac{i\left(e^{6i(fx+e)}-7\,e^{4i(fx+e)}+7\,e^{2i(fx+e)}-1\right)}{4a\left(e^{2i(fx+e)}+1\right)^3\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,f}-\frac{\ln\left(e^{ifx}+ie^{-ie}\right)\cos(fx+e)}{4f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,a}+\frac{\ln(e^{ifx}}{4f\sqrt{(e^{2i(fx+e}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/a*(-2*cos(f*x+e)^2*sin(f*x+e)+4*sin(f*x+e)+(ln(sin(f*x+e)-1)-ln(1+sin
(f*x+e)))*cos(f*x+e)^4)/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(a*cos(f*x
+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1666 vs. 2(102) = 204.
time = 0.87, size = 1666, normalized size = 15.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*(sin(7*f*x + 7*e) - 7*sin(5*f*x + 5*e) + 7*sin(3*f*x + 3*e) - sin(
f*x + e))*cos(8*f*x + 8*e) - 8*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2
*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(7*sin(5*f*x + 5*e) - 7*sin(3*f*x
+ 3*e) + sin(f*x + e))*cos(6*f*x + 6*e) + 56*(3*sin(4*f*x + 4*e) + 2*sin(2*
f*x + 2*e))*cos(5*f*x + 5*e) + 24*(7*sin(3*f*x + 3*e) - sin(f*x + e))*cos(4
*f*x + 4*e) + (2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2
*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(
2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x +
 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*
sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2
+ 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*
x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - (
2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*
f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e
) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) +
1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*
sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e
```

```
) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6
*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*l
og(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(cos(7*f*x + 7
*e) - 7*cos(5*f*x + 5*e) + 7*cos(3*f*x + 3*e) - cos(f*x + e))*sin(8*f*x + 8
*e) + 4*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*
sin(7*f*x + 7*e) + 16*(7*cos(5*f*x + 5*e) - 7*cos(3*f*x + 3*e) + cos(f*x +
e))*sin(6*f*x + 6*e) - 28*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*sin
(5*f*x + 5*e) - 24*(7*cos(3*f*x + 3*e) - cos(f*x + e))*sin(4*f*x + 4*e) + 2
8*(4*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) - 112*cos(3*f*x + 3*e)*sin(2*f*
x + 2*e) + 16*cos(f*x + e)*sin(2*f*x + 2*e) - 16*cos(2*f*x + 2*e)*sin(f*x +
 e) - 4*sin(f*x + e))/((a*cos(8*f*x + 8*e)^2 + 16*a*cos(6*f*x + 6*e)^2 + 36
*a*cos(4*f*x + 4*e)^2 + 16*a*cos(2*f*x + 2*e)^2 + a*sin(8*f*x + 8*e)^2 + 16
*a*sin(6*f*x + 6*e)^2 + 36*a*sin(4*f*x + 4*e)^2 + 48*a*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 16*a*sin(2*f*x + 2*e)^2 + 2*(4*a*cos(6*f*x + 6*e) + 6*a*cos
(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*cos(8*f*x + 8*e) + 8*(6*a*cos(4*f
*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*cos(6*f*x + 6*e) + 12*(4*a*cos(2*f*x
+ 2*e) + a)*cos(4*f*x + 4*e) + 8*a*cos(2*f*x + 2*e) + 4*(2*a*sin(6*f*x + 6*
e) + 3*a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*(3*
a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + a)*sqrt(a)*f)
```

Fricas [A]
time = 0.40, size = 77, normalized size = 0.73

$$-\frac{\left(\cos\left(fx+e\right)^{4}\log\left(-\frac{\sin(fx+e)+1}{\sin(fx+e)-1}\right)+2\left(\cos\left(fx+e\right)^{2}-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{16\,a^{2}f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(cos(f*x + e)^4*log(-(sin(f*x + e) + 1)/(sin(f*x + e) - 1)) + 2*(cos(
f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2)/(a^2*f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{2}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [A]
time = 0.73, size = 84, normalized size = 0.79

$$\frac{\sqrt{a\tan\left(fx+e\right)^{2}+a}\left(\frac{2\tan(fx+e)^{2}}{a}+\frac{1}{a}\right)\tan\left(fx+e\right)+\frac{\log\left(\left|-\sqrt{a}\tan(fx+e)+\sqrt{a\tan\left(fx+e\right)^{2}+a}\right|\right)}{\sqrt{a}}}{8\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(a*tan(f*x + e)^2 + a)*(2*tan(f*x + e)^2/a + 1/a)*tan(f*x + e) + l
og(abs(-sqrt(a)*tan(f*x + e) + sqrt(a*tan(f*x + e)^2 + a)))/sqrt(a))/(a*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{\left(a-a\sin\left(e+f\,x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2), x)
```

3.484 $\int \frac{\cot^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a*cos(f*x+e)^2)^(1/2)-cot(f*x+e)/a/f/(a
*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3255, 3286, 2701, 327, 213}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(a*f*Sqrt[a*Cos[e + f*x]^2]) - Cot[e +
 f*x]/(a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx&=\int \frac{\cot^2(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\&=\frac{\cos(e+fx)\int \csc^2(e+fx)\sec(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}\\&=-\frac{\cos(e+fx)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\&=-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\&=\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}\end{aligned}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 44, normalized size = 0.70

$$-\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[e + f*x]^2])/(a*f*Sqrt[
a*Cos[e + f*x]^2]))
```

Maple [A]
time = 9.73, size = 65, normalized size = 1.03

method	result
default	$-\frac{\cos(fx+e)(2+\sin(fx+e)(\ln(\sin(fx+e)-1)-\ln(1+\sin(fx+e))))}{2a\sin(fx+e)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\, f}$
risch	$-\frac{2i(\mathrm{e}^{2i(fx+e)}+1)}{a\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\, f(\mathrm{e}^{2i(fx+e)}-1)}+\frac{2\ln(\mathrm{e}^{ifx}+i\mathrm{e}^{-ie})\cos(fx+e)}{f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\, a}-\frac{2\ln(\mathrm{e}^{ifx}-i\mathrm{e}}{f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+\right.}}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*cos(f*x+e)*(2+sin(f*x+e)*(ln(sin(f*x+e)-1)-ln(1+sin(f*x+e))))/sin(f*
x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(64) = 128.
time = 0.54, size = 240, normalized size = 3.81

$$\frac{(\cos(2fx+2e)^2+\sin(2fx+2e)^2-2\cos(2fx+2e)+1)\log(\cos(fx+e)^2+\sin(fx+e)^2+2\sin(fx+e)+1)-(\cos(2fx+2e)^2+\sin(2fx+2e)^2-2\cos(2fx+2e)+1)\log(\cos(fx+e)^2+\sin(fx+e)^2-2\sin(fx+e)+1)-4\cos(fx+e)\sin(2fx+2e)+4\cos(2fx+2e)\sin(fx+e)-4\sin(fx+e)}{2\left(a\cos(2fx+2e)^2+a\sin(2fx+2e)^2-2a\cos(2fx+2e)+a\right)\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1)*log
(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - (cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f
*x + e)^2 - 2*sin(f*x + e) + 1) - 4*cos(f*x + e)*sin(2*f*x + 2*e) + 4*cos(2
*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e))/((a*cos(2*f*x + 2*e)^2 + a*sin(2
*f*x + 2*e)^2 - 2*a*cos(2*f*x + 2*e) + a)*sqrt(a)*f)
```

Fricas [A]
time = 0.42, size = 66, normalized size = 1.05

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\log\left(-\frac{\sin(fx+e)-1}{\sin(fx+e)+1}\right)\sin\left(fx+e\right)+2\right)}{2\,a^2 f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(a*cos(f*x + e)^2)*(log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1))*si
n(f*x + e) + 2)/(a^2*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{(-a(\sin(e+fx)-1)(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [A]
time = 0.69, size = 70, normalized size = 1.11

$$\frac{\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}+\frac{1}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*f*x + 1/2*e)/(a^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)) + 1/(a^
(3/2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(e+f\,x)^2}{\left(a-a\sin\left(e+f\,x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2), x)
```

3.485 $\int \frac{\cot^4(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -1/3*cot(f*x+e)*csc(f*x+e)^2/a/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3286, 2686, 30}

$$-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*(Cot[e + f*x]*Csc[e + f*x]^2)/(a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx &= \int \frac{\cot^4(e+fx)}{(a\cos^2(e+fx))^{3/2}}\,dx\\ &= \frac{\cos(e+fx)\int \cot(e+fx)\csc^3(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cos(e+fx)\mathrm{Subst}\left(\int x^2\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.76

$$-\frac{\cot^3(e+fx)}{3f\,(a\cos^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a - a*Sin[e + f*x]^2)^(3/2),x]
[Out] -1/3*Cot[e + f*x]^3/(f*(a*Cos[e + f*x]^2)^(3/2))
```

Maple [A]
time = 1.95, size = 35, normalized size = 0.92

method	result	size
default	$-\frac{\cos(fx+e)}{3a\sin(fx+e)^3\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$	35
risch	$\frac{8i\left(e^{4i(fx+e)}+e^{2i(fx+e)}\right)}{3\left(e^{2i(fx+e)}-1\right)^3 f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,a}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3/a*cos(f*x+e)/sin(f*x+e)^3/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(37) = 74.
time = 0.54, size = 411, normalized size = 10.82

$$\frac{8\left(\cos(3fx+3e)\sin(6fx+6e)-3\cos(3fx+3e)\sin(4fx+4e)-(3\cos(2fx+2e)-1)\sin(3fx+3e)-\cos(6fx+6e)\sin(3fx+3e)+3\cos(4fx+4e)\sin(3fx+3e)+3\cos(3fx+3e)\sin(2fx+2e)\right)\sqrt{a}}{3\left(a^2\cos(6fx+6e)^2+9a^2\cos(4fx+4e)^2+9a^2\cos(2fx+2e)^2+a^2\sin(6fx+6e)^2+9a^2\sin(4fx+4e)^2-18a^2\sin(4fx+4e)\sin(2fx+2e)+9a^2\sin(2fx+2e)^2-6a^2\cos(2fx+2e)+a^2-2\left(3a^2\cos(4fx+4e)-3a^2\cos(2fx+2e)+a^2\right)\cos(6fx+6e)-6\left(3a^2\cos(2fx+2e)-a^2\right)\cos(4fx+4e)-6\left(a^2\sin(4fx+4e)-a^2\sin(2fx+2e)\right)\sin(6fx+6e)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*(cos(3*f*x + 3*e)*sin(6*f*x + 6*e) - 3*cos(3*f*x + 3*e)*sin(4*f*x + 4*e
) - (3*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - cos(6*f*x + 6*e)*sin(3*f*x
+ 3*e) + 3*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) + 3*cos(3*f*x + 3*e)*sin(2*f*x
 + 2*e))*sqrt(a)/((a^2*cos(6*f*x + 6*e)^2 + 9*a^2*cos(4*f*x + 4*e)^2 + 9*a^
2*cos(2*f*x + 2*e)^2 + a^2*sin(6*f*x + 6*e)^2 + 9*a^2*sin(4*f*x + 4*e)^2 -
18*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a^2*sin(2*f*x + 2*e)^2 - 6*a^2
*cos(2*f*x + 2*e) + a^2 - 2*(3*a^2*cos(4*f*x + 4*e) - 3*a^2*cos(2*f*x + 2*e
) + a^2)*cos(6*f*x + 6*e) - 6*(3*a^2*cos(2*f*x + 2*e) - a^2)*cos(4*f*x + 4*
e) - 6*(a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Fricas [A]
time = 0.39, size = 50, normalized size = 1.32

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}}{3\left(a^2f\cos\left(fx+e\right)^3-a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*cos(f*x + e)^2)/((a^2*f*cos(f*x + e)^3 - a^2*f*cos(f*x + e))*sin
(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(37) = 74.
time = 0.78, size = 113, normalized size = 2.97

$$\frac{\frac{3\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}+\frac{a^{\frac{9}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3a^{\frac{9}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*sqrt(a)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a))/(a^2*sgn(tan(1/2*f*x + 1
/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^3) + (a^(9/2)*tan(1/2*f*x + 1/2*e)^3 + 3*
a^(9/2)*tan(1/2*f*x + 1/2*e))/(a^6*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)))/f
```

Mupad [B]
time = 18.69, size = 88, normalized size = 2.32

$$\frac{e^{e\,4i+f\,x\,4i}\,\sqrt{a-a\,{\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}^2}\,16i}{3\,a^2\,f\,{\left(e^{e\,2i+f\,x\,2i}-1\right)}^3\,\left(e^{e\,2i+f\,x\,2i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (exp(e*4i + f*x*4i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2)^2)^(1/2)*16i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*2i + f*x
*2i) + 1))
```

3.486 $\int \frac{\cot^6(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 1/3*cot(f*x+e)*csc(f*x+e)^2/a/f/(a*cos(f*x+e)^2)^(1/2)-1/5*cot(f*x+e)*csc(f
*x+e)^4/a/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3286, 2686, 14}

$$\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x
]*Csc[e + f*x]^4)/(5*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^6(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\cot^6(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\
&= \frac{\cos(e+fx)\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}\\
&= -\frac{\cos(e+fx)\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\
&= -\frac{\cos(e+fx)\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\
&= \frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.53

$$-\frac{\cot^3(e+fx)\left(-5+3\csc^2(e+fx)\right)}{15f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a - a*Sin[e + f*x]^2)^(3/2),x]
[Out] -1/15*(Cot[e + f*x]^3*(-5 + 3*Csc[e + f*x]^2))/(f*(a*Cos[e + f*x]^2)^(3/2))
```

Maple [A]
time = 6.49, size = 67, normalized size = 0.87

method	result	size
default	$-\frac{\cos(fx+e)\left(5\left(\cos^2(fx+e)\right)-2\right)}{15(1+\cos(fx+e))^2(-1+\cos(fx+e))^2 a\sin(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$	67
risch	$-\frac{8i\left(e^{2i(fx+e)}+1\right)\left(5\,e^{6i(fx+e)}+2\,e^{4i(fx+e)}+5\,e^{2i(fx+e)}\right)}{15\left(e^{2i(fx+e)}-1\right)^5 f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,a}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*cos(f*x+e)*(5*cos(f*x+e)^2-2)/(1+cos(f*x+e))^2/(-1+cos(f*x+e))^2/a/si
n(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(75) = 150$.
time = 0.69, size = 1146, normalized size = 14.88

$$\frac{8\left(\left(5\sin\left(7fx+7e\right)+2\sin\left(5fx+5e\right)+5\sin\left(3fx+3e\right)\right)\cos\left(10fx+10e\right)-5\left(5\sin\left(7fx+7e\right)+2\sin\left(5fx+5e\right)+5\sin\left(3fx+3e\right)\right)\cos\left(8fx+8e\right)-25\left(2\sin\left(6fx+6e\right)-2\sin\left(4fx+4e\right)+\sin\left(2fx+2e\right)\right)\cos\left(7fx+7e\right)+10\left(2\sin\left(5fx+5e\right)+5\sin\left(3fx+3e\right)\right)\cos\left(6fx+6e\right)+10\left(2\sin\left(4fx+4e\right)-\sin\left(2fx+2e\right)\right)\cos\left(5fx+5e\right)-\left(5\cos\left(7fx+7e\right)+2\cos\left(5fx+5e\right)+5\cos\left(3fx+3e\right)\right)\sin\left(10fx+10e\right)+5\left(5\cos\left(7fx+7e\right)+2\cos\left(5fx+5e\right)+5\cos\left(3fx+3e\right)\right)\sin\left(8fx+8e\right)+5\left(10\cos\left(6fx+6e\right)-10\cos\left(4fx+4e\right)+5\cos\left(2fx+2e\right)-1\right)\sin\left(7fx+7e\right)-10\left(2\cos\left(5fx+5e\right)+5\cos\left(3fx+3e\right)\right)\sin\left(6fx+6e\right)-2\left(10\cos\left(4fx+4e\right)-5\cos\left(2fx+2e\right)+1\right)\sin\left(5fx+5e\right)+50\cos\left(3fx+3e\right)\sin\left(4fx+4e\right)+5\left(5\cos\left(2fx+2e\right)-1\right)\sin\left(3fx+3e\right)-50\cos\left(4fx+4e\right)\sin\left(3fx+3e\right)-25\cos\left(3fx+3e\right)\sin\left(2fx+2e\right)\right)\sqrt{a}}{15\left(a^{2}\cos\left(10fx+10e\right)^{2}+25a^{2}\cos\left(8fx+8e\right)^{2}+100a^{2}\cos\left(6fx+6e\right)^{2}+100a^{2}\cos\left(4fx+4e\right)^{2}+25a^{2}\cos\left(2fx+2e\right)^{2}+a^{2}\sin\left(10fx+10e\right)^{2}+25a^{2}\sin\left(8fx+8e\right)^{2}+100a^{2}\sin\left(6fx+6e\right)^{2}+100a^{2}\sin\left(4fx+4e\right)^{2}-100a^{2}\sin\left(4fx+4e\right)\sin\left(2fx+2e\right)+25a^{2}\sin\left(2fx+2e\right)^{2}-10a^{2}\cos\left(2fx+2e\right)+a^{2}-2\left(5a^{2}\cos\left(8fx+8e\right)-10a^{2}\cos\left(6fx+6e\right)+10a^{2}\cos\left(4fx+4e\right)-5a^{2}\cos\left(2fx+2e\right)+a^{2}\right)\cos\left(10fx+10e\right)-10\left(10a^{2}\cos\left(6fx+6e\right)-10a^{2}\cos\left(4fx+4e\right)+5a^{2}\cos\left(2fx+2e\right)-a^{2}\right)\cos\left(8fx+8e\right)-20\left(10a^{2}\cos\left(4fx+4e\right)-5a^{2}\cos\left(2fx+2e\right)+a^{2}\right)\cos\left(6fx+6e\right)-20\left(5a^{2}\cos\left(2fx+2e\right)-a^{2}\right)\cos\left(4fx+4e\right)-10\left(a^{2}\sin\left(8fx+8e\right)-2a^{2}\sin\left(6fx+6e\right)+2a^{2}\sin\left(4fx+4e\right)-a^{2}\sin\left(2fx+2e\right)\right)\sin\left(10fx+10e\right)-50\left(2a^{2}\sin\left(6fx+6e\right)-2a^{2}\sin\left(4fx+4e\right)+a^{2}\sin\left(2fx+2e\right)\right)\sin\left(8fx+8e\right)-100\left(2a^{2}\sin\left(4fx+4e\right)-a^{2}\sin\left(2fx+2e\right)\right)\sin\left(6fx+6e\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/15*((5*sin(7*f*x + 7*e) + 2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3*e))*cos(10
*f*x + 10*e) - 5*(5*sin(7*f*x + 7*e) + 2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3
*e))*cos(8*f*x + 8*e) - 25*(2*sin(6*f*x + 6*e) - 2*sin(4*f*x + 4*e) + sin(2
*f*x + 2*e))*cos(7*f*x + 7*e) + 10*(2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3*e)
)*cos(6*f*x + 6*e) + 10*(2*sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x +
 5*e) - (5*cos(7*f*x + 7*e) + 2*cos(5*f*x + 5*e) + 5*cos(3*f*x + 3*e))*sin(
10*f*x + 10*e) + 5*(5*cos(7*f*x + 7*e) + 2*cos(5*f*x + 5*e) + 5*cos(3*f*x +
 3*e))*sin(8*f*x + 8*e) + 5*(10*cos(6*f*x + 6*e) - 10*cos(4*f*x + 4*e) + 5*
cos(2*f*x + 2*e) - 1)*sin(7*f*x + 7*e) - 10*(2*cos(5*f*x + 5*e) + 5*cos(3*f
*x + 3*e))*sin(6*f*x + 6*e) - 2*(10*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) +
 1)*sin(5*f*x + 5*e) + 50*cos(3*f*x + 3*e)*sin(4*f*x + 4*e) + 5*(5*cos(2*f*
x + 2*e) - 1)*sin(3*f*x + 3*e) - 50*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) - 25*
cos(3*f*x + 3*e)*sin(2*f*x + 2*e))*sqrt(a)/((a^2*cos(10*f*x + 10*e)^2 + 25*
a^2*cos(8*f*x + 8*e)^2 + 100*a^2*cos(6*f*x + 6*e)^2 + 100*a^2*cos(4*f*x + 4
*e)^2 + 25*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(10*f*x + 10*e)^2 + 25*a^2*sin(8
*f*x + 8*e)^2 + 100*a^2*sin(6*f*x + 6*e)^2 + 100*a^2*sin(4*f*x + 4*e)^2 - 1
00*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 25*a^2*sin(2*f*x + 2*e)^2 - 10*a
^2*cos(2*f*x + 2*e) + a^2 - 2*(5*a^2*cos(8*f*x + 8*e) - 10*a^2*cos(6*f*x +
6*e) + 10*a^2*cos(4*f*x + 4*e) - 5*a^2*cos(2*f*x + 2*e) + a^2)*cos(10*f*x +
 10*e) - 10*(10*a^2*cos(6*f*x + 6*e) - 10*a^2*cos(4*f*x + 4*e) + 5*a^2*cos(
2*f*x + 2*e) - a^2)*cos(8*f*x + 8*e) - 20*(10*a^2*cos(4*f*x + 4*e) - 5*a^2*
cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) - 20*(5*a^2*cos(2*f*x + 2*e) - a^2
)*cos(4*f*x + 4*e) - 10*(a^2*sin(8*f*x + 8*e) - 2*a^2*sin(6*f*x + 6*e) + 2*
a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) - 50*(2*a^2
*sin(6*f*x + 6*e) - 2*a^2*sin(4*f*x + 4*e) + a^2*sin(2*f*x + 2*e))*sin(8*f*
x + 8*e) - 100*(2*a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(6*f*x +
6*e))*f)
```

Fricas [A]
time = 0.42, size = 75, normalized size = 0.97

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(5\,\cos\left(fx+e\right)^{2}-2\right)}{15\,\left(a^{2}f\,\cos\left(fx+e\right)^{5}-2\,a^{2}f\,\cos\left(fx+e\right)^{3}+a^{2}f\,\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(a*cos(f*x + e)^2)*(5*cos(f*x + e)^2 - 2)/((a^2*f*cos(f*x + e)^5
- 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{6}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(75) = 150.
time = 0.93, size = 151, normalized size = 1.96

$$-\frac{\frac{30\,\sqrt{a}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+5\,\sqrt{a}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-3\,\sqrt{a}}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}}-\frac{3\,a^{\frac{17}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-5\,a^{\frac{17}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-30\,a^{\frac{17}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{10}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*((30*sqrt(a)*tan(1/2*f*x + 1/2*e)^4 + 5*sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - 3*sqrt(a))/(a^2*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^5
) - (3*a^(17/2)*tan(1/2*f*x + 1/2*e)^5 - 5*a^(17/2)*tan(1/2*f*x + 1/2*e)^3
- 30*a^(17/2)*tan(1/2*f*x + 1/2*e))/(a^10*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)))
/f
```

Mupad [B]
time = 20.61, size = 393, normalized size = 5.10

$$-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,16\mathrm{i}}{3\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^2\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,272\mathrm{i}}{15\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,128\mathrm{i}}{5\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^4\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,64\mathrm{i}}{5\,a^2\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^5\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2)^2)^(1/2)*16i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i + f
*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f
*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*272i)/(15*a^2*f*(exp(e*2
i + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i +
 f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2
)^(1/2)*128i)/(5*a^2*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1i + f*x*1i) + exp
(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2
 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*64i)/(5*a^2*f*(exp(e*2i + f*x*2i) -
1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.487 $\int \frac{\cot^8(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^6(e+fx)}{7af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -1/3*cot(f*x+e)*csc(f*x+e)^2/a/f/(a*cos(f*x+e)^2)^(1/2)+2/5*cot(f*x+e)*csc(
f*x+e)^4/a/f/(a*cos(f*x+e)^2)^(1/2)-1/7*cot(f*x+e)*csc(f*x+e)^6/a/f/(a*cos(
f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3255, 3286, 2686, 276}

$$-\frac{\cot(e+fx)\csc^6(e+fx)}{7af\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^8/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*(Cot[e + f*x]*Csc[e + f*x]^2)/(a*f*Sqrt[a*Cos[e + f*x]^2]) + (2*Cot[e
 + f*x]*Csc[e + f*x]^4)/(5*a*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e
 + f*x]^6)/(7*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^8(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx &= \int \frac{\cot^8(e+fx)}{(a\cos^2(e+fx))^{3/2}}\,dx\\ &= \frac{\cos(e+fx)\int \cot^5(e+fx)\csc^3(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cos(e+fx)\text{Subst}\left(\int x^2(-1+x^2)^2\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cos(e+fx)\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^6(}{7af\sqrt{a\cos^2(e+}}\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 51, normalized size = 0.44

$$-\frac{\cot^3(e+fx)\left(35-42\csc^2(e+fx)+15\csc^4(e+fx)\right)}{105f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^8/(a - a*Sin[e + f*x]^2)^(3/2),x]
[Out] -1/105*(Cot[e + f*x]^3*(35 - 42*Csc[e + f*x]^2 + 15*Csc[e + f*x]^4))/(f*(a*
Cos[e + f*x]^2)^(3/2))
```

Maple [**A**]
time = 4.52, size = 57, normalized size = 0.50

method	result	size
default	$-\frac{\cos(fx+e)\left(35\left(\cos^4(fx+e)\right)-28\left(\cos^2(fx+e)\right)+8\right)}{105a\sin(fx+e)^7\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$	57

risch	$\frac{8i(e^{2i(fx+e)}+1)(35e^{10i(fx+e)}+28e^{8i(fx+e)}+114e^{6i(fx+e)}+28e^{4i(fx+e)}+35e^{2i(fx+e)})}{105(e^{2i(fx+e)}-1)^7 f\sqrt{(e^{2i(fx+e)}+1)^2 a\,e^{-2i(fx+e)}}\,a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105/a*cos(f*x+e)*(35*cos(f*x+e)^4-28*cos(f*x+e)^2+8)/sin(f*x+e)^7/(a*cos
(f*x+e)^2)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2187 vs. $2(112) = 224$.
time = 0.59, size = 2187, normalized size = 19.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/105*((35*sin(11*f*x + 11*e) + 28*sin(9*f*x + 9*e) + 114*sin(7*f*x + 7*e)
 + 28*sin(5*f*x + 5*e) + 35*sin(3*f*x + 3*e))*cos(14*f*x + 14*e) - 7*(35*si
n(11*f*x + 11*e) + 28*sin(9*f*x + 9*e) + 114*sin(7*f*x + 7*e) + 28*sin(5*f*
x + 5*e) + 35*sin(3*f*x + 3*e))*cos(12*f*x + 12*e) - 245*(3*sin(10*f*x + 10
*e) - 5*sin(8*f*x + 8*e) + 5*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + sin(2*
f*x + 2*e))*cos(11*f*x + 11*e) + 21*(28*sin(9*f*x + 9*e) + 114*sin(7*f*x +
7*e) + 28*sin(5*f*x + 5*e) + 35*sin(3*f*x + 3*e))*cos(10*f*x + 10*e) + 196*
(5*sin(8*f*x + 8*e) - 5*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) - sin(2*f*x +
 2*e))*cos(9*f*x + 9*e) - 35*(114*sin(7*f*x + 7*e) + 28*sin(5*f*x + 5*e) +
35*sin(3*f*x + 3*e))*cos(8*f*x + 8*e) - 798*(5*sin(6*f*x + 6*e) - 3*sin(4*f
*x + 4*e) + sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 245*(4*sin(5*f*x + 5*e) +
5*sin(3*f*x + 3*e))*cos(6*f*x + 6*e) + 196*(3*sin(4*f*x + 4*e) - sin(2*f*x
+ 2*e))*cos(5*f*x + 5*e) - (35*cos(11*f*x + 11*e) + 28*cos(9*f*x + 9*e) + 1
14*cos(7*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(14*f*x
 + 14*e) + 7*(35*cos(11*f*x + 11*e) + 28*cos(9*f*x + 9*e) + 114*cos(7*f*x +
 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(12*f*x + 12*e) + 35*
(21*cos(10*f*x + 10*e) - 35*cos(8*f*x + 8*e) + 35*cos(6*f*x + 6*e) - 21*cos
(4*f*x + 4*e) + 7*cos(2*f*x + 2*e) - 1)*sin(11*f*x + 11*e) - 21*(28*cos(9*f
*x + 9*e) + 114*cos(7*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e
))*sin(10*f*x + 10*e) - 28*(35*cos(8*f*x + 8*e) - 35*cos(6*f*x + 6*e) + 21*
cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) + 1)*sin(9*f*x + 9*e) + 35*(114*cos(7
*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(8*f*x + 8*e) +
 114*(35*cos(6*f*x + 6*e) - 21*cos(4*f*x + 4*e) + 7*cos(2*f*x + 2*e) - 1)*s
in(7*f*x + 7*e) - 245*(4*cos(5*f*x + 5*e) + 5*cos(3*f*x + 3*e))*sin(6*f*x +
 6*e) - 28*(21*cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e)
+ 735*cos(3*f*x + 3*e)*sin(4*f*x + 4*e) + 35*(7*cos(2*f*x + 2*e) - 1)*sin(3
```

```
*f*x + 3*e) - 735*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) - 245*cos(3*f*x + 3*e)*
sin(2*f*x + 2*e))*sqrt(a)/((a^2*cos(14*f*x + 14*e)^2 + 49*a^2*cos(12*f*x +
12*e)^2 + 441*a^2*cos(10*f*x + 10*e)^2 + 1225*a^2*cos(8*f*x + 8*e)^2 + 1225
*a^2*cos(6*f*x + 6*e)^2 + 441*a^2*cos(4*f*x + 4*e)^2 + 49*a^2*cos(2*f*x + 2
*e)^2 + a^2*sin(14*f*x + 14*e)^2 + 49*a^2*sin(12*f*x + 12*e)^2 + 441*a^2*si
n(10*f*x + 10*e)^2 + 1225*a^2*sin(8*f*x + 8*e)^2 + 1225*a^2*sin(6*f*x + 6*e
)^2 + 441*a^2*sin(4*f*x + 4*e)^2 - 294*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 49*a^2*sin(2*f*x + 2*e)^2 - 14*a^2*cos(2*f*x + 2*e) + a^2 - 2*(7*a^2*co
s(12*f*x + 12*e) - 21*a^2*cos(10*f*x + 10*e) + 35*a^2*cos(8*f*x + 8*e) - 35
*a^2*cos(6*f*x + 6*e) + 21*a^2*cos(4*f*x + 4*e) - 7*a^2*cos(2*f*x + 2*e) +
a^2)*cos(14*f*x + 14*e) - 14*(21*a^2*cos(10*f*x + 10*e) - 35*a^2*cos(8*f*x
+ 8*e) + 35*a^2*cos(6*f*x + 6*e) - 21*a^2*cos(4*f*x + 4*e) + 7*a^2*cos(2*f*
x + 2*e) - a^2)*cos(12*f*x + 12*e) - 42*(35*a^2*cos(8*f*x + 8*e) - 35*a^2*c
os(6*f*x + 6*e) + 21*a^2*cos(4*f*x + 4*e) - 7*a^2*cos(2*f*x + 2*e) + a^2)*c
os(10*f*x + 10*e) - 70*(35*a^2*cos(6*f*x + 6*e) - 21*a^2*cos(4*f*x + 4*e) +
 7*a^2*cos(2*f*x + 2*e) - a^2)*cos(8*f*x + 8*e) - 70*(21*a^2*cos(4*f*x + 4*
e) - 7*a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) - 42*(7*a^2*cos(2*f*x +
 2*e) - a^2)*cos(4*f*x + 4*e) - 14*(a^2*sin(12*f*x + 12*e) - 3*a^2*sin(10*f
*x + 10*e) + 5*a^2*sin(8*f*x + 8*e) - 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*
f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(14*f*x + 14*e) - 98*(3*a^2*sin(10*f*
x + 10*e) - 5*a^2*sin(8*f*x + 8*e) + 5*a^2*sin(6*f*x + 6*e) - 3*a^2*sin(4*f
*x + 4*e) + a^2*sin(2*f*x + 2*e))*sin(12*f*x + 12*e) - 294*(5*a^2*sin(8*f*x
 + 8*e) - 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x +
 2*e))*sin(10*f*x + 10*e) - 490*(5*a^2*sin(6*f*x + 6*e) - 3*a^2*sin(4*f*x +
 4*e) + a^2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 490*(3*a^2*sin(4*f*x + 4*e
) - a^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Fricas [A]
time = 0.42, size = 100, normalized size = 0.87

$$\frac{\left(35\,\cos\left(fx+e\right)^{4}-28\,\cos\left(fx+e\right)^{2}+8\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{105\left(a^{2}f\cos\left(fx+e\right)^{7}-3\,a^{2}f\cos\left(fx+e\right)^{5}+3\,a^{2}f\cos\left(fx+e\right)^{3}-a^{2}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*cos(f*x + e)^4 - 28*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((
a^2*f*cos(f*x + e)^7 - 3*a^2*f*cos(f*x + e)^5 + 3*a^2*f*cos(f*x + e)^3 - a^
2*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**8/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.96, size = 184, normalized size = 1.60

$$\frac{\frac{525\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+35\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-63\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+15\sqrt{a}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}+\frac{15a^{\frac{25}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-63a^{\frac{25}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+35a^{\frac{25}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+525a^{\frac{25}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{14}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}}{13440f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13440*((525*sqrt(a)*tan(1/2*f*x + 1/2*e)^6 + 35*sqrt(a)*tan(1/2*f*x + 1/2
*e)^4 - 63*sqrt(a)*tan(1/2*f*x + 1/2*e)^2 + 15*sqrt(a))/(a^2*sgn(tan(1/2*f*
x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^7) + (15*a^(25/2)*tan(1/2*f*x + 1/2*
e)^7 - 63*a^(25/2)*tan(1/2*f*x + 1/2*e)^5 + 35*a^(25/2)*tan(1/2*f*x + 1/2*e
)^3 + 525*a^(25/2)*tan(1/2*f*x + 1/2*e))/(a^14*sgn(tan(1/2*f*x + 1/2*e)^4 -
 1)))/f
```

Mupad [B]
time = 33.01, size = 589, normalized size = 5.12

$$\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,16i}{3a^2f\left(e^{e\,2i+f\,x\,2i}-1\right)^2\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,464i}{15a^2f\left(e^{e\,2i+f\,x\,2i}-1\right)^3\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,3072i}{35a^2f\left(e^{e\,2i+f\,x\,2i}-1\right)^4\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,4736i}{35a^2f\left(e^{e\,2i+f\,x\,2i}-1\right)^5\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,768i}{7a^2f\left(e^{e\,2i+f\,x\,2i}-1\right)^6\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,256i}{7a^2f\left(e^{e\,2i+f\,x\,2i}-1\right)^7\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^8/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2)^2)^(1/2)*16i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i + f*x
*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x
*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*464i)/(15*a^2*f*(exp(e*2i
+ f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f
*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^
(1/2)*3072i)/(35*a^2*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1i + f*x*1i) + exp
(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2
 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*4736i)/(35*a^2*f*(exp(e*2i + f*x*2i)
 - 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a
 - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*768
i)/(7*a^2*f*(exp(e*2i + f*x*2i) - 1)^6*(exp(e*1i + f*x*1i) + exp(e*3i + f*x
*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2)^2)^(1/2)*256i)/(7*a^2*f*(exp(e*2i + f*x*2i) - 1)^7*(exp(
e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.488 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^5(e+fx)\,dx$

Optimal. Leaf size=177

$$\frac{(8a^2+24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f}-\frac{(8a^2+24ab+15b^2)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f}-\frac{(8a+7b}{}$$

[Out] 1/8*(8*a^2+24*a*b+15*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-1/8*(8*a+7*b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/f+1/4*sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)/f-1/8*(8*a^2+24*a*b+15*b^2)*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f

Rubi [A]
time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 52, 65, 214}

$$-\frac{(8a^2+24ab+15b^2)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)^2}+\frac{(8a^2+24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}+\frac{\sec^4(e+fx)\,(a+b\sin^2(e+fx))^{3/2}}{4f(a+b)}-\frac{(8a+7b)\sec^2(e+fx)\,(a+b\sin^2(e+fx))^{3/2}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]

[Out] ((8*a^2 + 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)*f) - ((8*a^2 + 24*a*b + 15*b^2)*Sqrt[a + b*Sin[e + f*x]^2])/(8*(a + b)^2*f) - ((8*a + 7*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(8*(a + b)^2*f) + (Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2))/(4*(a + b)*f)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{(1-x)^3}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\,\left(\frac{1}{2}(4a+3\right.}{(1-x)^2}\right.}{4(}$$

$$= -\frac{(8a+7b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\,(}{4}$$

$$= -\frac{(8a^2+24ab+15b^2)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} - \frac{(8a+7b)\sec^2(e+}{8}$$

$$= -\frac{(8a^2+24ab+15b^2)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} - \frac{(8a+7b)\sec^2(e+}{8}$$

$$= \frac{(8a^2+24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f} - \frac{(8a^2+2}{}$$

Mathematica [A]
time = 0.39, size = 143, normalized size = 0.81

$$\frac{-\left((8a+7b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\right)+2(a+b)\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}+\left(8a^2+24ab+15b^2\right)\left(\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)-\sqrt{a+b\sin^2(e+fx)}\right)}{8(a+b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] (-((8*a + 7*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2)) + 2*(a + b)*Sec
[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2) + (8*a^2 + 24*a*b + 15*b^2)*(Sqrt[
a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] - Sqrt[a + b*Sin[e +
 f*x]^2]))/(8*(a + b)^2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. 2(157) = 314.
time = 35.18, size = 721, normalized size = 4.07

method	result

default	$\frac{\left(-16(a+b)^{\frac{3}{2}}\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,a^2-48(a+b)^{\frac{3}{2}}\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,ab-30b^2\sqrt{a+b-b\left(\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((-16*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2-48*(a+b)^(3/2)*(a+b-b
*cos(f*x+e)^2)^(1/2)*a*b-30*b^2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)+8*ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
a^4+40*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*
x+e)+a))*a^3*b+71*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))*a^2*b^2+54*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+15*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4+8*ln(2/(sin(f*x+e)-1)*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+40*ln(2/(sin(f*x+e)
-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b+71*ln(2/
(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2
*b^2+54*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f
*x+e)+a))*a*b^3+15*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-2*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(
3/2)*(8*a+7*b)*cos(f*x+e)^2+4*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a+4*b*
(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2))/(a+b)^(3/2)/cos(f*x+e)^4/(a^2+2*a*b
+b^2)/f
```

Maxima [A]

time = 0.51, size = 237, normalized size = 1.34

$$-\frac{16\sqrt{b\sin\left(fx+e\right)^2+a}\,b^3+\frac{\left(8\,a^2b^3+24\,ab^4+15\,b^5\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}}-\frac{2\left(\left(8\,ab^4+9\,b^5\right)\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}-\left(8\,a^2b^4+15\,ab^5+7\,b^6\right)\sqrt{b\sin\left(fx+e\right)^2+a}\right)}{\left(b\sin(fx+e)^2+a\right)^2(a+b)+a^3+3\,a^2b+3\,ab^2+b^3-2\left(b\sin(fx+e)^2+a\right)(a^2+2\,ab+b^2)}}{16\,b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(16*sqrt(b*sin(f*x + e)^2 + a)*b^3 + (8*a^2*b^3 + 24*a*b^4 + 15*b^5)*
log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a)
+ sqrt(a + b)))/(a + b)^(3/2) - 2*((8*a*b^4 + 9*b^5)*(b*sin(f*x + e)^2 + a)
^(3/2) - (8*a^2*b^4 + 15*a*b^5 + 7*b^6)*sqrt(b*sin(f*x + e)^2 + a))/((b*sin
(f*x + e)^2 + a)^2*(a + b) + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(b*sin(f*x +
 e)^2 + a)*(a^2 + 2*a*b + b^2)))/(b^3*f)
```

Fricas [A]

time = 0.93, size = 354, normalized size = 2.00

$$\left[\frac{\left(8\,a^2+24\,ab+15\,b^2\right)\sqrt{a+b}\,\cos\left(fx+e\right)^4\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)-2\left(8\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^4+\left(8\,a^2+17\,ab+9\,b^2\right)\cos\left(fx+e\right)^2-2\,a^2-4\,ab-2\,b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{16\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^4},\ -\frac{\left(8\,a^2+24\,ab+15\,b^2\right)\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)\cos\left(fx+e\right)^4+\left(8\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^4+\left(8\,a^2+17\,ab+9\,b^2\right)\cos\left(fx+e\right)^2-2\,a^2-4\,ab-2\,b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{8\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/16*((8*a^2 + 24*a*b + 15*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*x
+ e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x
 + e)^2) - 2*(8*(a^2 + 2*a*b + b^2)*cos(f*x + e)^4 + (8*a^2 + 17*a*b + 9*b^
2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))
/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4), -1/8*((8*a^2 + 24*a*b + 15*b^2)*sq
rt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos
(f*x + e)^4 + (8*(a^2 + 2*a*b + b^2)*cos(f*x + e)^4 + (8*a^2 + 17*a*b + 9*b
^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)
)/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2(e + fx)}\,\tan^5(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2790 vs. $2(163) = 326$.
time = 2.05, size = 2790, normalized size = 15.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] -1/4*((8*a^2 + 24*a*b + 15*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) - 16*(
(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b - sqrt(a)*b)/((sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) + a + 4*b) - 2*(8*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 + 16*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))^7*a*b + 7*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
```

tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(5/2) - 80*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(3/2)*b - 17*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*b^2 - 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 - 464*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b - 425*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 60*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^3 + 136*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) + 144*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b - 425*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 - 468*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b^3 + 344*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^4 + 1520*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3*b + 2093*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b^2 + 712*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^3 - 240*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^4 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(9/2) + 592*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2)*b + 2165*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b^2 + 2808*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^3 + 1232*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^4 - 232*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^5 - 1072*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*

```
f*x + 1/2*e)^2 + a))*a^4*b - 1675*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))*a^3*b^2 - 652*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))*a^2*b^3 + 624*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^
4 + 448*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^5 - 104*a^(1
1/2) - 656*a^(9/2)*b - 1723*a^(7/2)*b^2 - 2340*...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^5\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.489 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^3(e+fx)\,dx$

Optimal. Leaf size=118

$$-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}\,f}+\frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f}+\frac{\sec^2(e+fx)\,(a+b\sin^2(e+}{2(a+b)f}$$

```
[Out] 1/2*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)/f-1/2*(2*a+3*b)*arctanh((a+
b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)+1/2*(2*a+3*b)*(a+b*sin(f*x
+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 52, 65, 214}

$$\frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}+\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] -1/2*((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(Sqrt[a
+ b]*f) + ((2*a + 3*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*(a + b)*f) + (Sec[e +
 f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(2*(a + b)*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{(1-x)^2}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)f} - \frac{(2a+3b)\text{Subst}\left(\int \frac{\sqrt{a+b}}{1-x}\right.}{4(a+}$$

$$= \frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} + \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx\right.}{2(a+b)f}$$

$$= \frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} + \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx\right.}{2(a+b)f}$$

$$= -\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}\,f} + \frac{(2a+3b)\sqrt{a+}}{2(a+}$$

Mathematica [A]
time = 0.28, size = 84, normalized size = 0.71

$$\frac{-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+(2+\cos(2(e+fx)))\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] (-(((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b
]) + (2 + Cos[2*(e + f*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*f
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(102) = 204$.
time = 31.09, size = 403, normalized size = 3.42

method	result
default	$-\left(-4\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,a-6b\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,\sqrt{a+b}+2\ln\left(\frac{2\sqrt{a+b}}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-(-4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a-6*b*(a+b-b*cos(f*x+e)^2)
^(1/2)*(a+b)^(1/2)+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)+b*sin(f*x+e)+a))*a^2+5*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/
2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+5*ln(2/(1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b+3*ln(2/(1+sin(f*
x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2)*cos(f*x
+e)^2+2*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2))/(a+b)^(3/2)/cos(f*x+e)^2/f
```

Maxima [A]
time = 0.55, size = 132, normalized size = 1.12

$$\frac{4\sqrt{b\sin\left(fx+e\right)^2+a}\,b^2-\frac{2\sqrt{b\sin\left(fx+e\right)^2+a}\,b^3}{b\sin\left(fx+e\right)^2-b}+\frac{\left(2\,ab^2+3\,b^3\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{\sqrt{a+b}}}{4\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*sqrt(b*sin(f*x + e)^2 + a)*b^2 - 2*sqrt(b*sin(f*x + e)^2 + a)*b^3/(b
*sin(f*x + e)^2 - b) + (2*a*b^2 + 3*b^3)*log((sqrt(b*sin(f*x + e)^2 + a) -
sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/sqrt(a + b))/(b^2*
f)
```

Fricas [**A**]
time = 0.63, size = 234, normalized size = 1.98

$$\left[\frac{(2a+3b)\sqrt{a+b}\cos\left(fx+e\right)^2\log\left(\frac{b\cos(fx+e)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)+2\left(2\left(a+b\right)\cos\left(fx+e\right)^2+a+b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{4\left(a+b\right)f\cos\left(fx+e\right)^2}, \frac{(2a+3b)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)\cos\left(fx+e\right)^2+\left(2\left(a+b\right)\cos\left(fx+e\right)^2+a+b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{2\left(a+b\right)f\cos\left(fx+e\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a + 3*b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(2
*(a + b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*
f*cos(f*x + e)^2), 1/2*((2*a + 3*b)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^2 + (2*(a + b)*cos(f*x + e)
^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*f*cos(f*x + e)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\tan^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(106) = 212$.
time = 0.83, size = 1011, normalized size = 8.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] ((2*a + 3*b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
```

```
) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 4*((sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))*b - sqrt(a)*b)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))
*sqrt(a) + a + 4*b) - 2*(2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a))^3*a + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b + 2*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) + 5*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b - 2*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
+ a))*b^2 - 2*a^(5/2) - 5*a^(3/2)*b - 4*sqrt(a)*b^2)/((sqrt(a)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4
*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^2)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+fx)^3\sqrt{b\sin(e+fx)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.490 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}-\frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

```
[Out] arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f-(a+b*sin(f*x+e)
^2)^(1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3273, 52, 65, 214}

$$\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}-\frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] (Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/f - Sqrt[a +
b*Sin[e + f*x]^2]/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, s\right.}{2f}$$

$$= -\frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{bf}$$

$$= \frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f} - \frac{\sqrt{a+b\sin^2(e+fx}}{f}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 1.03

$$\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b-b\cos^2(e+fx)}}{\sqrt{a+b}}\right) - \sqrt{a+b-b\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] (Sqrt[a + b]*ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]] - Sqrt[a +
 b - b*Cos[e + f*x]^2])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(50) = 100$.
time = 28.33, size = 129, normalized size = 2.22

method	result
default	$\frac{\frac{\sqrt{a+b}\,\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)}{2}+\frac{\sqrt{a+b}\,\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b(c}{1+s}\right.}{2}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(a+b)^(1/2)*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)+b*sin(f*x+e)+a))+1/2*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*
cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))-(a+b-b*cos(f*x+e)^2)^(1/2))/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(52) = 104$.
time = 0.52, size = 129, normalized size = 2.22

$$-\frac{\sqrt{a+b}\,\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)+1)}\right)-\sqrt{a+b}\,\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)-1)}\right)+2\sqrt{b\sin(fx+e)^2+a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(a + b)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a
/(sqrt(a*b)*(sin(f*x + e) + 1))) - sqrt(a + b)*arcsinh(-b*sin(f*x + e)/(sqr
t(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1))) + 2*sqrt(b*s
in(f*x + e)^2 + a))/f
```

Fricas [A]
time = 0.50, size = 145, normalized size = 2.50

$$\left[\frac{\sqrt{a+b}\,\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)-2\sqrt{-b\cos(fx+e)^2+a+b}}{2f},\ -\frac{\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)+\sqrt{-b\cos(fx+e)^2+a+b}}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2*sqrt(-b*cos(f*x + e)^2 + a +
b))/f, -(sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(
a + b)) + sqrt(-b*cos(f*x + e)^2 + a + b))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(52) = 104.
time = 0.58, size = 329, normalized size = 5.67

$$-\frac{2\left(\frac{(a+b)\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}-\sqrt{a}}{2\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{2\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)b-\sqrt{a}\,b\right)}{\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{2}+2\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)\sqrt{a}+a+4b}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] -2*((a + b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
 - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 2*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))*b - sqrt(a)*b)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*
sqrt(a) + a + 4*b))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.491 $\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

```
[Out] -arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sin(f*x+e)^2)^(1/
2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3273, 52, 65, 214}

$$\frac{\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sin
[e + f*x]^2]/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{bf} \\
&= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sin^2(e+fx)}}{f}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.98

$$
-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a+b\sin^2(e+fx)}}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] - Sqrt[a + b*Sin[e +
 f*x]^2])/f)
```

Maple [A]
time = 8.72, size = 58, normalized size = 1.07

method	result	size
default	$\frac{\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a+b*sin(f*x+e)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2
))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.30, size = 45, normalized size = 0.83

$$-\frac{\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)-\sqrt{b\sin\left(fx+e\right)^{2}+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -(sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - sqrt(b*sin(f*x + e)^2
+ a))/f
```

Fricas [A]
time = 0.78, size = 135, normalized size = 2.50

$$\left[\frac{\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^{2}+2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a}-2a-b\right)}{\cos(fx+e)^{2}-1}\right)+2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{2f},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-a}}{a}\right)+\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*(sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)
)/f, (sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) + sqrt(-b
*cos(f*x + e)^2 + a + b))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^{2}\left(e+fx\right)}\,\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x), x)
```

Giac [A]
time = 0.51, size = 49, normalized size = 0.91

$$\frac{\frac{a\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b\sin\left(fx+e\right)^{2}+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a*arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*sin(f*x +
e)^2 + a))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)\,\sqrt{b\sin\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.492 $\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=110

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af}$$

[Out] -1/2*csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/a/f+1/2*(2*a-b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*(2*a-b)*(a+b*sin(f*x+e)^2)^(1/2)/a/f

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 52, 65, 214}

$$-\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af}+\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]

[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) - ((2*a - b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*a*f) - (Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(2*a*f)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)\sqrt{a+bx}}{x^2}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af} - \frac{(2a-b)\text{Subst}\left(\int \frac{\sqrt{a+}}{x}\right.}{4}$$

$$= -\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{\csc^2(e+fx)\left(a+b\sin^2(e+\right.}{2af}$$

$$= -\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{\csc^2(e+fx)\left(a+b\sin^2(e+\right.}{2af}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f} - \frac{(2a-b)\sqrt{a+b\sin}}{2af}$$

Mathematica [A]
time = 0.14, size = 77, normalized size = 0.70

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a}\,(2+\csc^2(e+fx))\sqrt{a+b\sin^2(e+fx)}}{2\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] - Sqrt[a]*(2 + Csc[e
 + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2])/(2*Sqrt[a]*f)
```

Maple [A]
time = 9.70, size = 122, normalized size = 1.11

method	result
default	$\frac{-\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}-\frac{\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{2\sin\left(fx+e\right)^2}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{2\sqrt{a}}+\sqrt{a}\,\ln\left(\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(a+b*sin(f*x+e)^2)^(1/2)-1/2/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-1/2/a^
(1/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+a^(1/2)*ln(
(2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.33, size = 119, normalized size = 1.08

$$\frac{2\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)-\frac{b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{\sqrt{a}}-2\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{\sqrt{b\sin\left(fx+e\right)^2+a}\,b}{a}-\frac{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}{a\sin(fx+e)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - b*arcsinh(a/(sqrt
(a*b)*abs(sin(f*x + e))))/sqrt(a) - 2*sqrt(b*sin(f*x + e)^2 + a) + sqrt(b*s
in(f*x + e)^2 + a)*b/a - (b*sin(f*x + e)^2 + a)^(3/2)/(a*sin(f*x + e)^2))/f
```

Fricas [A]

time = 0.96, size = 239, normalized size = 2.17

$$\left[-\frac{\left((2a-b)\cos(fx+e)^2-2a+b\right)\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right)+2\left(2a\cos(fx+e)^2-3a\right)\sqrt{-b\cos(fx+e)^2+a+b}}{4\left(af\cos(fx+e)^2-af\right)}, -\frac{\left((2a-b)\cos(fx+e)^2-2a+b\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-a}}{a}\right)+\left(2a\cos(fx+e)^2-3a\right)\sqrt{-b\cos(fx+e)^2+a+b}}{2\left(af\cos(fx+e)^2-af\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a)*log(2*(b*cos(f*x + e)^2
 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1
)) + 2*(2*a*cos(f*x + e)^2 - 3*a)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a*f*cos
(f*x + e)^2 - a*f), -1/2*(((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a)*arc
tan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) + (2*a*cos(f*x + e)^2 - 3*a
)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a*f*cos(f*x + e)^2 - a*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2(e+fx)}\,\cot^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**3, x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{4096,[8,8]%%%}+%%%{%%%{16384,[1]%%%},[8,7]%%%}+%%%{%%%{24576,
[2]%%%
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^3\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.493 $\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=165

$$-\frac{(8a^2-8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{(8a^2-8ab-b^2)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}+\frac{(8a+b)\csc^2(}{}$$

```
[Out] -1/8*(8*a^2-8*a*b-b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+
1/8*(8*a+b)*csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/a^2/f-1/4*csc(f*x+e)^4*(a
+b*sin(f*x+e)^2)^(3/2)/a/f+1/8*(8*a^2-8*a*b-b^2)*(a+b*sin(f*x+e)^2)^(1/2)/a
^2/f
```

Rubi [A]
time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 52, 65, 214}

$$\frac{(8a^2-8ab-b^2)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}+\frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8a^2f}-\frac{(8a^2-8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/8*((8*a^2 - 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(a
^(3/2)*f) + ((8*a^2 - 8*a*b - b^2)*Sqrt[a + b*Sin[e + f*x]^2])/(8*a^2*f) +
((8*a + b)*Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(8*a^2*f) - (Csc[e
+ f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2))/(4*a*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2\sqrt{a+bx}}{x^3}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4af} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-8a-b)+2ax\right)\sqrt{}}{x^2}\right.}{}$$

$$= \frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8a^2f} - \frac{\csc^4(e+fx)\,(a+}{4a}$$

$$= \frac{(8a^2-8ab-b^2)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} + \frac{(8a+b)\csc^2(e+fx)\,(a}{8a^2 f}$$

$$= \frac{(8a^2-8ab-b^2)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} + \frac{(8a+b)\csc^2(e+fx)\,(a}{8a^2 f}$$

$$= -\frac{(8a^2-8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{(8a^2-8a}{}$$

Mathematica [A]
time = 0.39, size = 103, normalized size = 0.62

$$\frac{(-8a^2+8ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\,\left(8a+(8a-b)\csc^2(e+fx)-2a\csc^4(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}{8a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
[Out] ((-8*a^2 + 8*a*b + b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + Sqrt[
a]*(8*a + (8*a - b)*Csc[e + f*x]^2 - 2*a*Csc[e + f*x]^4)*Sqrt[a + b*Sin[e +
 f*x]^2])/(8*a^(3/2)*f)
```

Maple [A]
time = 10.74, size = 212, normalized size = 1.28

method	result

default	$\frac{\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}-\sqrt{a}\ \ln\left(\frac{2a+2\sqrt{a}\ \sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin(fx+e)}\right)+\frac{b\ln\left(\frac{2a+2\sqrt{a}\ \sqrt{a+b\left(\sin^2\left(fx\right.\right.}}{\sin(fx+e)}\right.}{\sqrt{a}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*sin(f*x+e)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2
))/sin(f*x+e))+1/a^(1/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(
f*x+e))-1/8*b/a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+1/8*b^2/a^(3/2)*ln((2
*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+1/sin(f*x+e)^2*(a+b*sin(
f*x+e)^2)^(1/2)-1/4/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2))/f
```

Maxima [A]
time = 0.28, size = 227, normalized size = 1.38

$$-\frac{8\sqrt{a}\ \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\ |\sin(fx+e)|}\right)-\frac{8\,b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\ |\sin(fx+e)|}\right)}{\sqrt{a}}-\frac{b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\ |\sin(fx+e)|}\right)}{a^{\frac{3}{2}}}-8\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{8\sqrt{b\sin\left(fx+e\right)^2+a}\ b}{a}+\frac{\sqrt{b\sin\left(fx+e\right)^2+a}\ b^2}{a^2}-\frac{8\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}{a\sin(fx+e)^2}-\frac{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b}{a^2\sin(fx+e)^2}+\frac{2\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}{a\sin(fx+e)^4}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 8*b*arcsinh(a/(s
qrt(a*b)*abs(sin(f*x + e))))/sqrt(a) - b^2*arcsinh(a/(sqrt(a*b)*abs(sin(f*x
 + e))))/a^(3/2) - 8*sqrt(b*sin(f*x + e)^2 + a) + 8*sqrt(b*sin(f*x + e)^2 +
 a)*b/a + sqrt(b*sin(f*x + e)^2 + a)*b^2/a^2 - 8*(b*sin(f*x + e)^2 + a)^(3/
2)/(a*sin(f*x + e)^2) - (b*sin(f*x + e)^2 + a)^(3/2)*b/(a^2*sin(f*x + e)^2)
 + 2*(b*sin(f*x + e)^2 + a)^(3/2)/(a*sin(f*x + e)^4))/f
```

Fricas [A]
time = 1.24, size = 415, normalized size = 2.52

$$\left[-\frac{\left(\left(8a^2-8ab-b^2\right)\cos\left(fx+e\right)^4-2\left(8a^2-8ab-b^2\right)\cos\left(fx+e\right)^2+8a^2-8ab-b^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\ \sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right)-2\left(8a^2\cos\left(fx+e\right)^4-\left(24a^2-ab\right)\cos\left(fx+e\right)^2+14a^2-ab\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{16\left(a^2f\cos\left(fx+e\right)^4-2a^2f\cos\left(fx+e\right)^2+a^2f\right)},\ \frac{\left(\left(8a^2-8ab-b^2\right)\cos\left(fx+e\right)^4-2\left(8a^2-8ab-b^2\right)\cos\left(fx+e\right)^2+8a^2-8ab-b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\ \sqrt{-a}}{a}\right)+\left(8a^2\cos\left(fx+e\right)^4-\left(24a^2-ab\right)\cos\left(fx+e\right)^2+14a^2-ab\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{8\left(a^2f\cos\left(fx+e\right)^4-2a^2f\cos\left(fx+e\right)^2+a^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 - 8*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 8*a*b - b^2)*cos
(f*x + e)^2 + 8*a^2 - 8*a*b - b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(8
*a^2*cos(f*x + e)^4 - (24*a^2 - a*b)*cos(f*x + e)^2 + 14*a^2 - a*b)*sqrt(-b
*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 +
a^2*f), 1/8*(((8*a^2 - 8*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 8*a*b - b^2
```

```
)*cos(f*x + e)^2 + 8*a^2 - 8*a*b - b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(-a)/a) + (8*a^2*cos(f*x + e)^4 - (24*a^2 - a*b)*cos(f*x +
 e)^2 + 14*a^2 - a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^
4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2{(e + fx)}}\,\cot^5{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.45Unable to divide
, perhaps due to rounding error%%%{1,[4,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1
]%%%}]
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}(e+f\,x)^5\,\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.494 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=234

$$\frac{(7a+8b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{4a\sqrt{\cos^2(e+fx)}\,F$$

```
[Out] 1/3*(7*a+8*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^
(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-4/3*a*Ell
ipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*
x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(3*a+4*b)*(a+b*sin(f*x+e)^2)
^(1/2)*tan(f*x+e)/(a+b)/f+1/3*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.19, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 478, 592, 538, 437, 435, 432, 430}

$$-\frac{4a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{(7a+8b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\tan^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] ((7*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*
x]^2)/a]) - (4*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x
]^2]) - ((3*a + 4*b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f)
 + (Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3)/(3*f)
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan^4(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{(1-x^2)^{5/2}}\,dx, x, \sin(\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan^3(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sqrt{a+b\sin^2(e\ }}{}$$

$$= -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sqrt{a+b\sin^2(e\ }}{}$$

$$= -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sqrt{a+b\sin^2(e\ }}{}$$

$$= \frac{(7a+8b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]

time = 1.40, size = 198, normalized size = 0.85

$$\frac{2a(7a+8b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right) - 8a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right) - \frac{(8a^2+12ab+b^2+4(4a^2+6ab+b^2)\cos(2(e+fx))-b(4a+5b)\cos(4(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2\sqrt{2}}}{6(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (2*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
```

```
f*x, -(b/a)] - ((8*a^2 + 12*a*b + b^2 + 4*(4*a^2 + 6*a*b + b^2)*Cos[2*(e +
f*x)] - b*(4*a + 5*b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqr
t[2]))/(6*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 17.99, size = 380, normalized size = 1.62

method	result
default	$-\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b(4a+5b)\sin(fx+e)(\cos^4(fx+e))-2\sqrt{-b\left(\cos^4\left(fx+e\right)\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(4*a+5*b)*sin(f*x+e)*cos
(f*x+e)^4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2+5*a*b+3*b^2)*
cos(f*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a
*b+b^2)*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b
)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*x+e),
(-1/a*b)^(1/2))*a-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(
a+b)/(sin(f*x+e)-1)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/
2)/(1+sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.14, size = 27, normalized size = 0.12

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e+f\,x)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.495 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=171

$$-\frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{a\sqrt{\cos^2(e+fx)}\,F\left(\sin^{-1}(s\right.}{}$$

```
[Out] -2*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*
sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+a*EllipticF(sin(f*x+e),(-b
/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+
b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.10, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3275, 478, 538, 437, 435, 432, 430}

$$\frac{a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a + b*Sin
[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan^2(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{(1-x^2)^{3/2}}\,dx, x, \sin(e\ldots}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+f\ldots}{\ldots}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+\ldots}{\ldots}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+\ldots}{\ldots}$$

$$= -\frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\ldots}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.34, size = 140, normalized size = 0.82

$$\frac{-2\sqrt{2}\,a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+(2a+b-b\cos(2(e+fx)))\tan(e+fx)}{\sqrt{2}\,f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
[Out] (-2*Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)] + Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x,
-(b/a)] + (2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[2*a
+ b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 16.10, size = 294, normalized size = 1.72

method	result

default	$-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b\sin(fx+e)(\cos^2(fx+e))+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)+a*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-2*a*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2)))/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin
(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.10, size = 27, normalized size = 0.16

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^2\left(e+fx\right)}\,\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^2\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.496 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=51

$$\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3257, 3256}

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx=\frac{\sqrt{a+b\sin^2(e+fx)}\int\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$
$$=\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 1.20

$$\frac{a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
[Out] (a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 5.29, size = 71, normalized size = 1.39

method	result	size
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)}{\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F]
time = 0.10, size = 18, normalized size = 0.35

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases}\frac{\sqrt{a}\ \mathrm{E}\left(e+f\,x\middle|-\frac{b}{a}\right)}{f} & \text{if}\ \ 0<a\\ \int\sqrt{b\sin\left(e+f\,x\right)^{2}+a}\ dx & \text{if}\ \ \neg 0<a\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] piecewise(0 < a, (a^(1/2)*ellipticE(e + f*x, -b/a))/f, ~0 < a, int((a + b*s
in(e + f*x)^2)^(1/2), x))
```

3.497 $\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=174

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e-}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-2*EllipticE(sin(f*x+e),(-b/a)^(1/2))
*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)
^2/a)^(1/2)+(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)
^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3275, 484, 538, 437, 435, 432, 430}

$$\frac{(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - (2*Sqrt[Cos[e + f*x]^2]*El
lipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2
)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\sqrt{a+bx^2}}{x^2}dx,\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\right.}{}$$

Mathematica [A]
time = 0.39, size = 143, normalized size = 0.82

$$\frac{-((2a+b-b\cos(2(e+fx)))\cot(e+fx))-2\sqrt{2}\,a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{\sqrt{2}\,f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-((2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*Sqrt[2]*a*Sqrt[(2*a + b
 - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sqrt[2]*(a + b)*Sqrt
[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(Sqrt[2]*f*S
qrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 9.20, size = 156, normalized size = 0.90

method	result

default	$\frac{\sin(fx+e)\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\left(\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a+\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\right.\right.}{\sin(fx+e)\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a)+b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/s
in(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Fricas [F]
time = 0.13, size = 27, normalized size = 0.16

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^2\left(e+fx\right)}\,\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^2\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.498 $\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=232

$$\frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(7a-b)\sqrt{\cos^2(e+fx)}\,E(}{}$$

```
[Out] 1/3*(3*a-b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-1/3*cot(f*x+e)^3*(a+b*s
in(f*x+e)^2)^(1/2)/f+1/3*(7*a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x
+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(f*x+e)^2/a)^
(1/2)-4/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2
)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 484, 594, 538, 437, 435, 432, 430}

$$-\frac{4(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{(7a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,-\frac{b}{a}\right)}{3af\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((3*a - b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (Cot[e + f*x]
^3*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + ((7*a - b)*Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]
)/(3*a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (4*(a + b)*Sqrt[Cos[e + f*x]^2]*
EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*
x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
```

```
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1-x^2)^{3/2}\sqrt{a+bx^2}}{x^4}\,dx,\right.}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e\right.}{}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+}}{3f}$$

Mathematica [A]
time = 2.20, size = 197, normalized size = 0.85

$$\frac{-\frac{(-8a^2-4ab+3b^2+4(4a^2+2ab-b^2)\cos(2(e+fx))+b(-4a+b)\cos(4(e+fx)))\cot(e+fx)\csc^2(e+fx)}{2\sqrt{2}} + 2a(7a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right) - 8a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{6af\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-1/2*((-8*a^2 - 4*a*b + 3*b^2 + 4*(4*a^2 + 2*a*b - b^2)*Cos[2*(e + f*x)] +
 b*(-4*a + b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] + 2*a*
(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)]
```

```
 - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(
b/a)])/(6*a*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 10.21, size = 351, normalized size = 1.51

method	result
default	$-\frac{4\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2(\sin^3(fx+e))+4b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+4*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-7*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*sin(f*x+e)^3+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+4*a*b*sin(f*x+e)^6-b^2
*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-6*a*b*sin(f*x+e)^4+b^2*sin(f*x+e)^4-5*a^2*
sin(f*x+e)^2+2*a*b*sin(f*x+e)^2+a^2)/a/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x
+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Fricas [F]
time = 0.15, size = 27, normalized size = 0.12

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2{(e + fx)}}\cot^4{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{cot}\left(e+f\,x\right)}^4\,\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.499 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$

Optimal. Leaf size=220

$$\frac{(8a^2+40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}\,f}-\frac{(8a^2+40ab+35b^2)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)f}-\frac{(8a^2+4$$

```
[Out] -1/24*(8*a^2+40*a*b+35*b^2)*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/f-1/8*(8*a+9*b
)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(5/2)/(a+b)^2/f+1/4*sec(f*x+e)^4*(a+b*sin
(f*x+e)^2)^(5/2)/(a+b)/f+1/8*(8*a^2+40*a*b+35*b^2)*arctanh((a+b*sin(f*x+e)^
2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)-1/8*(8*a^2+40*a*b+35*b^2)*(a+b*sin(f*x+
e)^2)^(1/2)/(a+b)/f
```

Rubi [A]
time = 0.17, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 52, 65, 214}

$$-\frac{(8a^2+40ab+35b^2)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24f(a+b)^2}-\frac{(8a^2+40ab+35b^2)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)}+\frac{(8a^2+40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f\sqrt{a+b}}+\frac{\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{4f(a+b)}-\frac{(8a+9b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] ((8*a^2 + 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])
/(8*Sqrt[a + b]*f) - ((8*a^2 + 40*a*b + 35*b^2)*Sqrt[a + b*Sin[e + f*x]^2])
/(8*(a + b)*f) - ((8*a^2 + 40*a*b + 35*b^2)*(a + b*Sin[e + f*x]^2)^(3/2))/(
24*(a + b)^2*f) - ((8*a + 9*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))
/(8*(a + b)^2*f) + (Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2))/(4*(a + b)
*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{(1-x)^3}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(\frac{1}{2}(4a+5b\right.}{(1-x)^2}\right.}{4(}$$

$$= -\frac{(8a+9b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{8(a+b)^2f} + \frac{\sec^4(e+fx)}{}$$

$$= -\frac{(8a^2+40ab+35b^2)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24(a+b)^2f} - \frac{(8a+9b)\sec^2(}{}$$

$$= -\frac{(8a^2+40ab+35b^2)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)f} - \frac{(8a^2+40ab+35}{2}$$

$$= -\frac{(8a^2+40ab+35b^2)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)f} - \frac{(8a^2+40ab+35}{2}$$

$$= \frac{(8a^2+40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}\,f} - \frac{(8a^2+}{}$$

Mathematica [A]
time = 1.34, size = 160, normalized size = 0.73

$$-\frac{3(8a+9b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2} - 6(a+b)\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2} + (8a^2+40ab+35b^2)\left(-3(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right) + \sqrt{a+b\sin^2(e+fx)}\,(4a+3b+b\sin^2(e+fx))\right)}{24(a+b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] -1/24*(3*(8*a + 9*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2) - 6*(a + b
)*Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2) + (8*a^2 + 40*a*b + 35*b^2)*(
-3*(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] + Sqrt[a +
 b*Sin[e + f*x]^2]*(4*a + 3*b + b*Sin[e + f*x]^2)))/((a + b)^2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(196) = 392$.
time = 25.31, size = 711, normalized size = 3.23

method	result
default	$\frac{16\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(a+b)^{\frac{5}{2}} b\left(\cos^6(fx+e)\right)+\left(-64\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(a+b)^{\frac{5}{2}} a-160\sqrt{a-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(16*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*b*cos(f*x+e)^6+(-64*(a+b-b*
cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*a-160*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2
)*b+24*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*
x+e)+a))*a^4+168*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)+b*sin(f*x+e)+a))*a^3*b+369*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^2+330*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+105*ln(2/(sin(f*x+e)-1)*
((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4+24*ln(2/(1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4+168*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
*a^3*b+369*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*si
n(f*x+e)+a))*a^2*b^2+330*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+105*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-6*(a+b-b*cos(f*x+e
)^2)^(1/2)*(a+b)^(5/2)*(8*a+13*b)*cos(f*x+e)^2+12*(a+b-b*cos(f*x+e)^2)^(1/2
)*(a+b)^(5/2)*a+12*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*b)/(a+b)^(5/2)/co
s(f*x+e)^4/f
```

Maxima [A]

time = 0.51, size = 246, normalized size = 1.12

$$-\frac{16\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^3+48\left(ab^3+3b^4\right)\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{3\left(8a^2b^3+40ab^4+35b^5\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{6\left(\left(8ab^4+13b^5\right)\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}-\left(8a^2b^4+19ab^5+11b^6\right)\sqrt{b\sin\left(fx+e\right)^2+a}\right)}{\left(b\sin(fx+e)^2+a\right)^2-2\left(b\sin(fx+e)^2+a\right)(a+b)+a^2+2ab+b^2}}{48b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/48*(16*(b*sin(f*x + e)^2 + a)^(3/2)*b^3 + 48*(a*b^3 + 3*b^4)*sqrt(b*sin(
f*x + e)^2 + a) + 3*(8*a^2*b^3 + 40*a*b^4 + 35*b^5)*log((sqrt(b*sin(f*x + e
)^2 + a) - sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/sqrt(a
+ b) - 6*((8*a*b^4 + 13*b^5)*(b*sin(f*x + e)^2 + a)^(3/2) - (8*a^2*b^4 + 19
*a*b^5 + 11*b^6)*sqrt(b*sin(f*x + e)^2 + a))/((b*sin(f*x + e)^2 + a)^2 - 2*
(b*sin(f*x + e)^2 + a)*(a + b) + a^2 + 2*a*b + b^2))/(b^3*f)
```

Fricas [A]

time = 0.79, size = 385, normalized size = 1.75

$$\left[\frac{3\,{\left(8\,a^{2} + 40\,a b + 35\,b^{2}\right)} \sqrt{a + b} \cos\left(f x + e\right)^{4} \log\left(\frac{b \cos\left(f x + e\right)^{2} - 2\,\sqrt{-b \cos\left(f x + e\right)^{2} + a + b} \sqrt{a + b} - 2\,a - 2\,b}{\cos\left(f x + e\right)^{2}}\right) + 2\,{\left(8\,{\left(a b + b^{2}\right)} \cos\left(f x + e\right)^{6} - 16\,{\left(2\,a^{2} + 7\,a b + 5\,b^{2}\right)} \cos\left(f x + e\right)^{4} - 3\,{\left(8\,a^{2} + 21\,a b + 13\,b^{2}\right)} \cos\left(f x + e\right)^{2} + 6\,a^{2} + 12\,a b + 6\,b^{2}\right)} \sqrt{-b \cos\left(f x + e\right)^{2} + a + b}}{48\,{\left(a + b\right)} f \cos\left(f x + e\right)^{4}}, -\frac{3\,{\left(8\,a^{2} + 40\,a b + 35\,b^{2}\right)} \sqrt{-a - b} \arctan\left(\frac{\sqrt{-b \cos\left(f x + e\right)^{2} + a + b} \sqrt{-a - b}}{a + b}\right) \cos\left(f x + e\right)^{4} - {\left(8\,{\left(a b + b^{2}\right)} \cos\left(f x + e\right)^{6} - 16\,{\left(2\,a^{2} + 7\,a b + 5\,b^{2}\right)} \cos\left(f x + e\right)^{4} - 3\,{\left(8\,a^{2} + 21\,a b + 13\,b^{2}\right)} \cos\left(f x + e\right)^{2} + 6\,a^{2} + 12\,a b + 6\,b^{2}\right)} \sqrt{-b \cos\left(f x + e\right)^{2} + a + b}}{24\,{\left(a + b\right)} f \cos\left(f x + e\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(8*a^2 + 40*a*b + 35*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*
x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f
*x + e)^2) + 2*(8*(a*b + b^2)*cos(f*x + e)^6 - 16*(2*a^2 + 7*a*b + 5*b^2)*c
os(f*x + e)^4 - 3*(8*a^2 + 21*a*b + 13*b^2)*cos(f*x + e)^2 + 6*a^2 + 12*a*b
 + 6*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*f*cos(f*x + e)^4), -1/2
4*(3*(8*a^2 + 40*a*b + 35*b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 +
 a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^4 - (8*(a*b + b^2)*cos(f*x + e)^
6 - 16*(2*a^2 + 7*a*b + 5*b^2)*cos(f*x + e)^4 - 3*(8*a^2 + 21*a*b + 13*b^2)
*cos(f*x + e)^2 + 6*a^2 + 12*a*b + 6*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/
((a + b)*f*cos(f*x + e)^4)]
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.54Error: Bad Argum
ent Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e+f\,x)^5\,\left(b\,\sin\left(e+f\,x\right)^2+a\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.500 $\int (a+b\sin^2(e+fx))^{3/2}\tan^3(e+fx)\,dx$

Optimal. Leaf size=148

$$-\frac{\sqrt{a+b}\,(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}+\frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(2a+5b)\,(a+b\sin}{6(a+b}$$

[Out] 1/6*(2*a+5*b)*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)/f+1/2*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(5/2)/(a+b)/f-1/2*(2*a+5*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f+1/2*(2*a+5*b)*(a+b*sin(f*x+e)^2)^(1/2)/f

Rubi [A]
time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 52, 65, 214}

$$\frac{(2a+5b)\,(a+b\sin^2(e+fx))^{3/2}}{6f(a+b)}+\frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{\sqrt{a+b}\,(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}+\frac{\sec^2(e+fx)\,(a+b\sin^2(e+fx))^{5/2}}{2f(a+b)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]

[Out] -1/2*(Sqrt[a + b]*(2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/f + ((2*a + 5*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*f) + ((2*a + 5*b)*(a + b*Sin[e + f*x]^2)^(3/2))/(6*(a + b)*f) + (Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/(2*(a + b)*f)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{(1-x)^2}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)f} - \frac{(2a+5b)\text{Subst}\left(\int \frac{(a+bx}{1-}\right.}{4(a}$$

$$= \frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6(a+b)f} + \frac{\sec^2(e+fx)\left(a+b\sin^2(e\right.}{2(a+b)f}$$

$$= \frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{(2a+5b)\left(a+b\sin^2(e+fx\right.}{6(a+b)f}$$

$$= \frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{(2a+5b)\left(a+b\sin^2(e+fx\right.}{6(a+b)f}$$

$$= -\frac{\sqrt{a+b}\,(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f} + \frac{(2a+}{}$$

Mathematica [A]
time = 0.34, size = 116, normalized size = 0.78

$$\frac{3\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}+(2a+5b)\left(-3(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)+\sqrt{a+b\sin^2(e+fx)}\,(4a+3b+b\sin^2(e+fx))\right)}{6(a+b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] (3*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2) + (2*a + 5*b)*(-3*(a + b)^(3
/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] + Sqrt[a + b*Sin[e + f*
x]^2]*(4*a + 3*b + b*Sin[e + f*x]^2)))/(6*(a + b)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(128) = 256.
time = 24.25, size = 567, normalized size = 3.83

method	result

default	$\frac{-4\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(a+b)^{\frac{3}{2}}b\left(\cos^4(fx+e)\right)-\left(-16\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(a+b)^{\frac{3}{2}}a-28\sqrt{a+b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(-4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*b*cos(f*x+e)^4-(-16*(a+b-b*
cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a-28*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)
*b+6*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+
e)+a))*a^3+27*a^2*b*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)+b*sin(f*x+e)+a))+36*a*b^2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+15*b^3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a
+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+6*ln(2/(1+sin(f*x+e))*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3+27*a^2*b*ln(2/(1+sin(f*x+
e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+36*a*b^2*ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+15*
b^3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e
)+a)))*cos(f*x+e)^2+6*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a+6*(a+b-b*cos
(f*x+e)^2)^(1/2)*(a+b)^(3/2)*b)/(a+b)^(3/2)/cos(f*x+e)^2/f
```

Maxima [A]

time = 0.58, size = 175, normalized size = 1.18

$$\frac{4\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^2+12\left(ab^2+2\,b^3\right)\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{3\left(2\,a^2b^2+7\,ab^3+5\,b^4\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{6\left(ab^3+b^4\right)\sqrt{b\sin\left(fx+e\right)^2+a}}{b\sin(fx+e)^2-b}}{12\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(b*sin(f*x + e)^2 + a)^(3/2)*b^2 + 12*(a*b^2 + 2*b^3)*sqrt(b*sin(f*
x + e)^2 + a) + 3*(2*a^2*b^2 + 7*a*b^3 + 5*b^4)*log((sqrt(b*sin(f*x + e)^2
+ a) - sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/sqrt(a + b)
 - 6*(a*b^3 + b^4)*sqrt(b*sin(f*x + e)^2 + a)/(b*sin(f*x + e)^2 - b))/(b^2*
f)
```

Fricas [A]

time = 0.62, size = 265, normalized size = 1.79

$$\left[\frac{3\left(2\,a+5\,b\right)\sqrt{a+b}\,\cos\left(fx+e\right)^2\log\left(\frac{b\cos(fx+e)^2+2\,\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos(fx+e)^2}\right)-2\left(2\,b\cos\left(fx+e\right)^4-2\left(4\,a+7\,b\right)\cos\left(fx+e\right)^2-3\,a-3\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{12\,f\cos\left(fx+e\right)^2},\ \frac{3\left(2\,a+5\,b\right)\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)\cos\left(fx+e\right)^2-\left(2\,b\cos\left(fx+e\right)^4-2\left(4\,a+7\,b\right)\cos\left(fx+e\right)^2-3\,a-3\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{6\,f\cos\left(fx+e\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*a + 5*b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*s
qrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2
*(2*b*cos(f*x + e)^4 - 2*(4*a + 7*b)*cos(f*x + e)^2 - 3*a - 3*b)*sqrt(-b*co
s(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2), 1/6*(3*(2*a + 5*b)*sqrt(-a - b)*
arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^2
 - (2*b*cos(f*x + e)^4 - 2*(4*a + 7*b)*cos(f*x + e)^2 - 3*a - 3*b)*sqrt(-b*
cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2185 vs. 2(133) = 266.
time = 1.45, size = 2185, normalized size = 14.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^2 + 7*a*b + 5*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*
f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 6*(2*(sqrt(a)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1
/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2 + 3*(sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^3*b^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a))^2*a^(5/2) + 7*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a
^(3/2)*b + 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)
*b^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 - 3*(sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*
```

```
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b + 3*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^2 + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))*b^3 - 2*a^(7/2) - 7*a^(5/2)*b - 9*a^(3/2)*b^2 - 4*s
qrt(a)*b^3)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^2 -
8*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*
tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b + 3*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^2 + 9*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b + 15*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b^2 + 6*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^3*a^2*b + 30*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^3*a*b^2 + 32*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))^3*b^3 - 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(
5/2)*b + 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b
^2 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^3
- 9*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b - 33*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b^2 + 48*(sqrt(a)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^4 - 3*a^(7/2)*b - 21*a^(5/2)*b^2 - 5
6*a^(3/2)*b^3 - 48*sqrt(a)*b^4)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(
a) + a + 4*b)^3)/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^3\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.501 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx$

Optimal. Leaf size=84

$$\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}-\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] (a+b)^(3/2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f-1/3*(a+b*sin(f*
x+e)^2)^(3/2)/f-(a+b)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {3273, 52, 65, 214}

$$-\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] ((a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/f - ((a + b
)*Sqrt[a + b*Sin[e + f*x]^2])/f - (a + b*Sin[e + f*x]^2)^(3/2)/(3*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx)^{3/2}}{1-x}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$=-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b)\text{Subst}\left(\int\frac{\sqrt{a+bx}}{1-x}\,dx,x,\sin\right.}{2f}$$

$$=-\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a}{}$$

$$=-\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a}{}$$

$$=\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}-\frac{(a+b)\sqrt{a+b\sin}}{f}$$

Mathematica [A]
time = 0.10, size = 79, normalized size = 0.94

$$\frac{3(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b-b\cos^2(e+fx)}}{\sqrt{a+b}}\right)+\sqrt{a+b-b\cos^2(e+fx)}\,\left(-4(a+b)+b\cos^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
[Out] (3*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]] + Sqrt
[a + b - b*Cos[e + f*x]^2]*(-4*(a + b) + b*Cos[e + f*x]^2))/(3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(72) = 144$.
time = 20.98, size = 393, normalized size = 4.68

method	result
default	$-\frac{-2\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,\sqrt{a+b}\,b\left(\cos^2(fx+e)\right)+8\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,a+8b\sqrt{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(a+b)^(1/2)*(-2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)*b*cos(f*x+e)^2+
8*a*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)+8*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(
a+b)^(1/2)-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*
sin(f*x+e)+a))*a^2-6*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)-b*sin(f*x+e)+a))*a*b-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2-3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2-6*ln(2/(sin(f*x+e)-1)*((a+b)^(1/
2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b-3*ln(2/(sin(f*x+e)-1)*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(75) = 150$.
time = 0.54, size = 166, normalized size = 1.98

$$-\frac{3\,(a+b)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)+1)}\right)-3\,(a+b)^{\frac{3}{2}}\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)-1)}\right)+2\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}+6\sqrt{b\sin\left(fx+e\right)^2+a}\,a+6\sqrt{b\sin\left(fx+e\right)^2+a}\,b}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a + b)^(3/2)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1))
 - a/(sqrt(a*b)*(sin(f*x + e) + 1))) - 3*(a + b)^(3/2)*arcsinh(-b*sin(f*x +
 e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1))) + 2*
(b*sin(f*x + e)^2 + a)^(3/2) + 6*sqrt(b*sin(f*x + e)^2 + a)*a + 6*sqrt(b*si
n(f*x + e)^2 + a)*b)/f
```

Fricas [A]
time = 0.63, size = 186, normalized size = 2.21

$$\left[\frac{3\,(a+b)^{\frac{3}{2}}\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)+2\left(b\cos\left(fx+e\right)^2-4\,a-4\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{6\,f},\ -\frac{3\,(a+b)\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)-\left(b\cos\left(fx+e\right)^2-4\,a-4\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{3\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a + b)^(3/2)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(b*cos(f*x + e)^2 - 4*a -
 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/f, -1/3*(3*(a + b)*sqrt(-a - b)*arct
an(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - (b*cos(f*x + e)^
2 - 4*a - 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}} \tan\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1338 vs. $2(75) = 150$.
time = 0.80, size = 1338, normalized size = 15.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] -2/3*(3*(a^2 + 2*a*b + b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 2*(6*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b + 3*(sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*t
an(1/2*f*x + 1/2*e)^2 + a))^5*b^2 + 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^4*a^(3/2)*b + 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))^4*sqrt(a)*b^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a))^3*a^2*b + 54*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
3*a*b^2 + 40*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^3 - 1
2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan
(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b + 18*(sq
```

```
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^2 + 24*(sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^3 - 18*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b - 57*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b^2 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))*a*b^3 + 48*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a))*b^4 - 6*a^(7/2)*b - 39*a^(5/2)*b^2 - 88*a^(3/2)*b^3 - 48*sqrt(
a)*b^4)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) + a + 4*b)^3)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)\ \left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.502 $\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sin(f*x+e)^2)
^(3/2)/f+a*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 52, 65, 214}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sin[e + f*x]^2])/f + (a + b*Sin[e + f*x]^2)^(3/2)/(3*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^2(e+f\right.}{2f}$$

$$= \frac{a\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\text{Subst}\left(\int\right.}{}$$

$$= \frac{a\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sin^2(e+fx}}{f}$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 0.88

$$\frac{-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a+b\sin^2(e+fx)}\left(4a+b\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sin[e
+ f*x]^2]*(4*a + b*Sin[e + f*x]^2))/(3*f)
```

Maple [A]
time = 9.76, size = 86, normalized size = 1.10

method	result
default	$\frac{\frac{b\left(\sin^{2}\left(fx+e\right)\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{3}+\frac{4a\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{3}-a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}\left(fx+e\right.\right.}}{\sin\left(fx+e\right)}\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+4/3*a*(a+b*sin(f*x+e)^2)^(1/2)
-a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.31, size = 64, normalized size = 0.82

$$-\frac{3\,a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin\left(fx+e\right)|}\right)-\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}-3\,\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - (b*sin(f*x + e)^
2 + a)^(3/2) - 3*sqrt(b*sin(f*x + e)^2 + a)*a)/f
```

Fricas [A]
time = 0.87, size = 175, normalized size = 2.24

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{2\left(b\cos\left(fx+e\right)^{2}+2\,\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a}-2\,a-b\right)}{\cos\left(fx+e\right)^{2}-1}\right)-2\left(b\cos\left(fx+e\right)^{2}-4\,a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{6\,f},\ \frac{3\,\sqrt{-a}\,a\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-a}}{a}\right)-\left(b\cos\left(fx+e\right)^{2}-4\,a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{3\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^(3/2)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(b*cos(f*x + e)^2 - 4*a - b)*
sqrt(-b*cos(f*x + e)^2 + a + b))/f, 1/3*(3*sqrt(-a)*a*arctan(sqrt(-b*cos(f*
x + e)^2 + a + b)*sqrt(-a)/a) - (b*cos(f*x + e)^2 - 4*a - b)*sqrt(-b*cos(f*
x + e)^2 + a + b))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}} \cot\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

Giac [A]
time = 0.43, size = 71, normalized size = 0.91

$$\frac{\frac{3\,a^2 \arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} + 3\sqrt{b\sin\left(fx+e\right)^2+a}\;a}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/sqrt(-a) + (b*sin(f*
x + e)^2 + a)^(3/2) + 3*sqrt(b*sin(f*x + e)^2 + a)*a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e + f\,x)\,\left(b\sin\left(e + f\,x\right)^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.503 $\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{(2a-3b)\left(a+b\sin^2(e+\right.}{6af}$$

```
[Out] -1/6*(2*a-3*b)*(a+b*sin(f*x+e)^2)^(3/2)/a/f-1/2*csc(f*x+e)^2*(a+b*sin(f*x+e
)^2)^(5/2)/a/f+1/2*(2*a-3*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))*a^(1
/2)/f-1/2*(2*a-3*b)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 52, 65, 214}

$$-\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af}-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*f) - (
(2*a - 3*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*f) - ((2*a - 3*b)*(a + b*Sin[e +
 f*x]^2)^(3/2))/(6*a*f) - (Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/(2*
a*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)(a+bx)^{3/2}}{x^2}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2af} - \frac{(2a-3b)\text{Subst}\left(\int \frac{(a+b}{}\right.}{4}$$

$$= -\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af} - \frac{\csc^2(e+fx)\left(a+b\sin^2(e\right.}{2af}$$

$$= -\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f} - \frac{(2a-3b)\left(a+b\sin^2(e+fx\right.}{6af}$$

$$= -\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f} - \frac{(2a-3b)\left(a+b\sin^2(e+fx\right.}{6af}$$

$$= \frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{(2a-3b)\sqrt{a}}{}$$

Mathematica [A]
time = 0.31, size = 90, normalized size = 0.64

$$\frac{3\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+(-8a+5b+b\cos(2(e+fx))-3a\csc^2(e+fx))\sqrt{a+b\sin^2(e+fx)}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + (-8*a
+ 5*b + b*Cos[2*(e + f*x)] - 3*a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2]
)/(6*f)
```

Maple [A]
time = 8.41, size = 165, normalized size = 1.18

method	result
default	$-\frac{b\left(\sin^2(fx+e)\right)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{3}-\frac{4a\sqrt{a+b\left(\sin^2(fx+e)\right)}}{3}+b\sqrt{a+b\left(\sin^2(fx+e)\right)}-\frac{a\sqrt{a+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-4/3*a*(a+b*sin(f*x+e)^2)^(1/2
)+b*(a+b*sin(f*x+e)^2)^(1/2)-1/2*a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-3/
2*a^(1/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+a^(3/2)
*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.30, size = 156, normalized size = 1.11

$$\frac{6a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)-9\sqrt{a}\,b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)-2\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}-6\sqrt{b\sin(fx+e)^2+a}\,a+9\sqrt{b\sin(fx+e)^2+a}\,b+\frac{3\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b}{a}-\frac{3\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}}{a\sin(fx+e)^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 9*sqrt(a)*b*arcsi
nh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 2*(b*sin(f*x + e)^2 + a)^(3/2) - 6*sq
rt(b*sin(f*x + e)^2 + a)*a + 9*sqrt(b*sin(f*x + e)^2 + a)*b + 3*(b*sin(f*x
+ e)^2 + a)^(3/2)*b/a - 3*(b*sin(f*x + e)^2 + a)^(5/2)/(a*sin(f*x + e)^2))/
f
```

Fricas [A]
time = 1.06, size = 282, normalized size = 2.01

$$\left[-\frac{3\,\left(\left(2\,a-3\,b\right)\cos\left(fx+e\right)^{2}-2\,a+3\,b\right)\sqrt{a}\,\log\left(\frac{2\,\left(b\cos(fx+e)^{2}+2\,\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a}-2\,a-b\right)}{\cos(fx+e)^{2}-1}\right)-2\,\left(2\,b\cos\left(fx+e\right)^{4}-2\,\left(4\,a-b\right)\cos\left(fx+e\right)^{2}+11\,a-4\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{12\,\left(f\cos\left(fx+e\right)^{2}-f\right)}, -\frac{3\,\left(\left(2\,a-3\,b\right)\cos\left(fx+e\right)^{2}-2\,a+3\,b\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-a}}{a}\right)-\left(2\,b\cos\left(fx+e\right)^{4}-2\,\left(4\,a-b\right)\cos\left(fx+e\right)^{2}+11\,a-4\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{6\,\left(f\cos\left(fx+e\right)^{2}-f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [-1/12*(3*((2*a - 3*b)*cos(f*x + e)^2 - 2*a + 3*b)*sqrt(a)*log(2*(b*cos(f*x
 + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e
)^2 - 1)) - 2*(2*b*cos(f*x + e)^4 - 2*(4*a - b)*cos(f*x + e)^2 + 11*a - 4*b
)*sqrt(-b*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2 - f), -1/6*(3*((2*a -
3*b)*cos(f*x + e)^2 - 2*a + 3*b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(-a)/a) - (2*b*cos(f*x + e)^4 - 2*(4*a - b)*cos(f*x + e)^2 + 11*a
 - 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2 - f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.71Unable to divide
, perhaps due to rounding error%%%{65536,[8,11]%%%}+%%%{%%%{393216,[1]%%%},
[8,10]
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^3\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.504 $\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=208

$$-\frac{(8a^2-24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f}+\frac{(8a^2-24ab+3b^2)\sqrt{a+b\sin^2(e+fx)}}{8af}+\frac{(8a^2-}{}$$

```
[Out] 1/24*(8*a^2-24*a*b+3*b^2)*(a+b*sin(f*x+e)^2)^(3/2)/a^2/f+1/8*(8*a-b)*csc(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(5/2)/a^2/f-1/4*csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(
5/2)/a/f-1/8*(8*a^2-24*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))
/f/a^(1/2)+1/8*(8*a^2-24*a*b+3*b^2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.13, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 52, 65, 214}

$$\frac{(8a^2-24ab+3b^2)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24a^2f}+\frac{(8a^2-24ab+3b^2)\sqrt{a+b\sin^2(e+fx)}}{8af}-\frac{(8a^2-24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f}+\frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{8a^2f}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])
/(Sqrt[a]*f) + ((8*a^2 - 24*a*b + 3*b^2)*Sqrt[a + b*Sin[e + f*x]^2])/(8*a*f
) + ((8*a^2 - 24*a*b + 3*b^2)*(a + b*Sin[e + f*x]^2)^(3/2))/(24*a^2*f) + ((
8*a - b)*Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csc[e +
f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2))/(4*a*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2(a+bx)^{3/2}}{x^3}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{4af} + \frac{\text{Subst}\left(\int \frac{(\frac{1}{2}(-8a+b)+2a}{x^2}\right.}{}$$

$$= \frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{8a^2f} - \frac{\csc^4(e+fx)\,(a}{}$$

$$= \frac{(8a^2-3(8a-b)b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24a^2f} + \frac{(8a-b)\csc^2(e+}{}$$

$$= \frac{(8a^2-3(8a-b)b)\sqrt{a+b\sin^2(e+fx)}}{8af} + \frac{(8a^2-3(8a-b)b)}{2}$$

$$= \frac{(8a^2-3(8a-b)b)\sqrt{a+b\sin^2(e+fx)}}{8af} + \frac{(8a^2-3(8a-b)b)}{2}$$

$$= -\frac{(8a^2-3(8a-b)b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f} + \frac{(8a^2}{}$$

Mathematica [A]

time = 0.56, size = 123, normalized size = 0.59

$$\frac{-3(8a^2-24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a+b\sin^2(e+fx)}\,(3(8a-5b)\csc^2(e+fx)-6a\csc^4(e+fx)+8(4a-6b+b\sin^2(e+fx)))}{24\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] +
Sqrt[a]*Sqrt[a + b*Sin[e + f*x]^2]*(3*(8*a - 5*b)*Csc[e + f*x]^2 - 6*a*Csc[
e + f*x]^4 + 8*(4*a - 6*b + b*Sin[e + f*x]^2)))/(24*Sqrt[a]*f)
```

Maple [A]

time = 12.80, size = 257, normalized size = 1.24

method	result

default	$\frac{\frac{b\left(\sin^2(fx+e)\right)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{3}+\frac{4a\sqrt{a+b\left(\sin^2(fx+e)\right)}}{3}-2b\sqrt{a+b\left(\sin^2(fx+e)\right)}+\frac{a\sqrt{a+b}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+4/3*a*(a+b*sin(f*x+e)^2)^(1/2)
-2*b*(a+b*sin(f*x+e)^2)^(1/2)+a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+3*a^(
1/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-5/8*b/sin(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-3/8/a^(1/2)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(
f*x+e)^2)^(1/2))/sin(f*x+e))-a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(
1/2))/sin(f*x+e))-1/4*a/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2))/f
```

Maxima [A]
time = 0.29, size = 287, normalized size = 1.38

$$-\frac{24\,a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)-72\,\sqrt{a}\,b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)+\frac{9\,b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{\sqrt{a}}-8\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}-24\sqrt{b\sin(fx+e)^2+a}\,a+72\sqrt{b\sin(fx+e)^2+a}\,b+\frac{24\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b}{a}-\frac{3\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b^2}{a^2}-\frac{9\sqrt{b\sin(fx+e)^2+a}\,b^2}{a}-\frac{24\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}}{a\sin(fx+e)^2}+\frac{3\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}b}{a^2\sin(fx+e)^2}+\frac{6\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}}{a\sin(fx+e)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*(24*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 72*sqrt(a)*b*a
rcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) + 9*b^2*arcsinh(a/(sqrt(a*b)*abs(si
n(f*x + e))))/sqrt(a) - 8*(b*sin(f*x + e)^2 + a)^(3/2) - 24*sqrt(b*sin(f*x
+ e)^2 + a)*a + 72*sqrt(b*sin(f*x + e)^2 + a)*b + 24*(b*sin(f*x + e)^2 + a)
^(3/2)*b/a - 3*(b*sin(f*x + e)^2 + a)^(3/2)*b^2/a^2 - 9*sqrt(b*sin(f*x + e)
^2 + a)*b^2/a - 24*(b*sin(f*x + e)^2 + a)^(5/2)/(a*sin(f*x + e)^2) + 3*(b*s
in(f*x + e)^2 + a)^(5/2)*b/(a^2*sin(f*x + e)^2) + 6*(b*sin(f*x + e)^2 + a)^
(5/2)/(a*sin(f*x + e)^4))/f
```

Fricas [A]
time = 1.36, size = 442, normalized size = 2.12

$$\left[\frac{3\left((8a^2-24ab+3b^2)\cos(fx+e)^4-2(8a^2-24ab+3b^2)\cos(fx+e)^2+8a^2-24ab+3b^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right)-2\left(8ab\cos(fx+e)^6-8(4a^2-3ab)\cos(fx+e)^4+(88a^2-87ab)\cos(fx+e)^2-50a^2+55ab\right)\sqrt{-b\cos(fx+e)^2+a+b}}{48\left(af\cos(fx+e)^4-2af\cos(fx+e)^2+af\right)},\ \frac{3\left((8a^2-24ab+3b^2)\cos(fx+e)^4-2(8a^2-24ab+3b^2)\cos(fx+e)^2+8a^2-24ab+3b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-a}}{a}\right)-\left(8ab\cos(fx+e)^6-8(4a^2-3ab)\cos(fx+e)^4+(88a^2-87ab)\cos(fx+e)^2-50a^2+55ab\right)\sqrt{-b\cos(fx+e)^2+a+b}}{24\left(af\cos(fx+e)^4-2af\cos(fx+e)^2+af\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2 - 24*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 24*a*b + 3*b
^2)*cos(f*x + e)^2 + 8*a^2 - 24*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^
2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 -
1)) - 2*(8*a*b*cos(f*x + e)^6 - 8*(4*a^2 - 3*a*b)*cos(f*x + e)^4 + (88*a^2
```

```
- 87*a*b)*cos(f*x + e)^2 - 50*a^2 + 55*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b)
)/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f), 1/24*(3*((8*a^2 - 24*a
*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 24*a*b + 3*b^2)*cos(f*x + e)^2 + 8*
a^2 - 24*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
-a)/a) - (8*a*b*cos(f*x + e)^6 - 8*(4*a^2 - 3*a*b)*cos(f*x + e)^4 + (88*a^2
 - 87*a*b)*cos(f*x + e)^2 - 50*a^2 + 55*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b
))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.95Unable to divide
, perhaps due to rounding error%%%{524288,[8,12]%%%}+%%%{%%%{3670016,[1]%%%
},[8,1
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(e+f\,x)^5\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.505 $\int (a+b\sin^2(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=275

$$-\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{8(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\right|-}{3f\sqrt{1+\frac{b\sin^2(e+}{a}}}$$

```
[Out] -1/3*(3*a+8*b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+8/3*(a+2*b)
*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*si
n(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(5*a+8*b)*EllipticF(si
n(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)
^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)-(a+2*b)*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(
1/2)*tan(f*x+e)/f+1/3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.23, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3275, 478, 591, 596, 538, 437, 435, 432, 430}

$$-\frac{a(5a+8b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{8(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\tan^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{(3a+8b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] -1/3*((3*a + 8*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f +
 (8*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a
]) - (a*(5*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e +
f*x]^2]) - ((a + 2*b)*Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x
])/f + ((a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3)/(3*f)
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^{5/2}}\,dx, x, \sin(\right.}{f}$$

$$= \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e\right.}{}$$

$$= -\frac{(a+2b)\sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} + \frac{(a}{}$$

$$= -\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a}{}$$

$$= -\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a}{}$$

$$= -\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a}{}$$

$$= -\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{8(}{}$$

Mathematica [A]
time = 1.91, size = 211, normalized size = 0.77

$$\frac{32a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-4a(5a+8b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-\frac{\left(32a^2+108ab+18b^2+\left(64a^2+160ab+17b^2\right)\cos(2(e+fx))-2b(6a+17b)\cos(4(e+fx))-b^2\cos(6(e+fx))\right)\sec^2(e+fx)\tan(e+fx)}{4\sqrt{2}}}{12f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (32*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 4*a*(5*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
 + f*x, -(b/a)] - ((32*a^2 + 108*a*b + 18*b^2 + (64*a^2 + 160*a*b + 17*b^2)
*Cos[2*(e + f*x)] - 2*b*(6*a + 17*b)*Cos[4*(e + f*x)] - b^2*Cos[6*(e + f*x)
])*Sec[e + f*x]^2*Tan[e + f*x])/(4*Sqrt[2]))/(12*f*Sqrt[2*a + b - b*Cos[2*(
e + f*x)]])
```

Maple [A]
time = 16.84, size = 419, normalized size = 1.52

method	result
default	$-\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b^2 \sin(fx+e)(\cos^6(fx+e))+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*sin(f*x+e)*cos(f*x+e)^
6+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(3*a+7*b)*cos(f*x+e)^4*sin(f
*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(4*a^2+13*a*b+9*b^2)*cos(f
*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^
2)*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/
2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(5*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a+8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-8*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a-16*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(sin(
f*x+e)-1)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(1+sin(
f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.14, size = 45, normalized size = 0.16

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*
x + e)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e+f\,x)^4\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.506 $\int (a+b\sin^2(e+fx))^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=222

$$\frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(7a+8b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] 4/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*(7*a+8*b)*Ellipt
icE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+4/3*a*(a+b)*EllipticF(sin(f*x+e),(-
b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a
+b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.15, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 478, 542, 538, 437, 435, 432, 430}

$$\frac{4a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a+8b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}+\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (4*b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((7*a +
8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f
*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*a
*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/f
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
```

```
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^{3/2}}\,dx,x,\sin(e\right.}{f}$$

$$= \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\ \sec(e+\right.}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{(a+b\sin^2(e}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{(a+b\sin^2(e}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{(a+b\sin^2(e}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(7a+8b)\sqrt{c}}{}$$

Mathematica [A]
time = 1.97, size = 174, normalized size = 0.78

$$\frac{-8a(7a+8b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+32a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\left(24a^2+40ab+13b^2-4b(2a+3b)\cos(2(e+fx))-b^2\cos(4(e+fx))\right)\tan(e+fx)}{24f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (-8*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
 -(b/a)] + 32*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
+ f*x, -(b/a)] + Sqrt[2]*(24*a^2 + 40*a*b + 13*b^2 - 4*b*(2*a + 3*b)*Cos[2*
```

```
(e + f*x)] - b^2*Cos[4*(e + f*x)])*Tan[e + f*x])/(24*f*Sqrt[2*a + b - b*Cos
[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(202) = 404$.
time = 18.77, size = 515, normalized size = 2.32

method	result
default	$\frac{-\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^{2}\left(fx+e\right)\right)}\, b^{2}\sin(fx+e)(\cos^{4}(fx+e))-2\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)+\left(a\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*sin(f*x+e)*cos(f*x+e)^
4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+b)*cos(f*x+e)^2*sin(f*x
+e)+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e)
+4*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+4*a*(cos
(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1
/a*b)^(1/2))*b*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-7*(cos(f*x+e)^2)^
(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2-8*(cos(f*x+e)^2)^(1/2)*(-b
/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos
(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*b)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1
)*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.13, size = 45, normalized size = 0.20

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*
x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}} \tan^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{tan}\left(e+f\,x\right)}^2\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.507 $\int (a+b\sin^2(e+fx))^{3/2}\,dx$

Optimal. Leaf size=154

$$-\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{a(a+b)}{}$$

```
[Out] -1/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+2/3*(2*a+b)*(cos(f*
x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)
^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+
b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int\frac{a(3a+b)+2b(2a+}{\sqrt{a+b\sin^2(}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{1}{3}(a(a+b))\int\frac{}{\sqrt{a+b\,\mathrm{si}}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left(2(2a+b)\sqrt{a+b\sin^2(e}\right.}{3\sqrt{1}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{1+\frac{b\sin}{}}}$$

Mathematica [A]
time = 0.53, size = 156, normalized size = 1.01

$$\frac{4\sqrt{2}\,a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}\,a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+b(-2a-b+b\cos(2(e+fx)))\sin(2(e+fx))}{6\sqrt{2}\,f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 11.44, size = 266, normalized size = 1.73

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3-1/3*s
in(f*x+e)*a*b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.10, size = 18, normalized size = 0.12

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^{2}\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2), x)
```

3.508 $\int \cot^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx$

Optimal. Leaf size=223

$$\frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}-\frac{(7a-b)\sqrt{\cos^2(e+\ldots}}{}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+4/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin
(f*x+e)^2)^(1/2)/f-1/3*(7*a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e
)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2
)+4/3*a*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^
(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 484, 542, 538, 437, 435, 432, 430}

$$\frac{4a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (Cot[e +
 f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/f - ((7*a - b)*Sqrt[Cos[e + f*x]^2]*Ell
ipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2
])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*a*(a + b)*Sqrt[Cos[e + f*x]^2]
*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f
*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
```

```
e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\,(a+bx^2)^{3/2}}{x^2}\,dx\right.}{f}$$

$$= -\frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec\right.}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx}{}$$

Mathematica [A]
time = 1.50, size = 173, normalized size = 0.78

$$\frac{\sqrt{2}\left(-24a^2-8ab+3b^2+4(2a-b)b\cos(2(e+fx))+b^2\cos(4(e+fx))\right)\cot(e+fx)-8a(7a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+32a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{24f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*(-24*a^2 - 8*a*b + 3*b^2 + 4*(2*a - b)*b*Cos[2*(e + f*x)] + b^2*Co
s[4*(e + f*x)])*Cot[e + f*x] - 8*a*(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f
*x)])/a]*EllipticE[e + f*x, -(b/a)] + 32*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(24*f*Sqrt[2*a + b - b*Cos[2*(e
+ f*x)]])
```

Maple [A]
time = 9.17, size = 204, normalized size = 0.91

method	result
default	$\frac{\sin(fx+e)\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\, a\left(4\,\mathrm{EllipticF}\left(\sin(fx+e), \sqrt{-\frac{b}{a}}\right)a+4\,\mathrm{EllipticF}\left(\sin(fx+e), \sqrt{\right.\right.}{3\sin(fx+e)\cos}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(4
*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*b-7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b
)^(1/2))*b)+b^2*cos(f*x+e)^6+(2*a*b-2*b^2)*cos(f*x+e)^4+(-3*a^2-2*a*b+b^2)*
cos(f*x+e)^2)/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Fricas [F]
time = 0.14, size = 45, normalized size = 0.20

$$\mathrm{integral}\left(-\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cot\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*
x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(e+f\,x)^2\,{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.509 $\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=276

$$\frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(3a-5b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

```
[Out] -1/3*cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)/f+(a-b)*cos(f*x+e)^2*cot(f*x+e)*
(a+b*sin(f*x+e)^2)^(1/2)/f+1/3*(3*a-5*b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x
+e)^2)^(1/2)/f+8/3*(a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos
(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*
(5*a-3*b)*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2
)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3275, 484, 594, 542, 538, 437, 435, 432, 430}

$$-\frac{(5a-3b)(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{8(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(3a-5b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)*Cos[e + f*x]^2*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + ((3*a
- 5*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (Cot[e
 + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2))/(3*f) + (8*(a - b)*Sqrt[Cos[e + f*x
]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((5*a - 3*b)*(a + b)*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 484

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^p*((c + d*x^n)^q/(e*(m
+ 1))), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1-x^2)^{3/2}(a+bx^2)^{3/2}}{x^4}\,dx,\right.}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\,\mathrm{se}\right.}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\cot^3(}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-}{}$$

Mathematica [A]
time = 3.28, size = 218, normalized size = 0.79

$$\frac{-\frac{(-32a^2+44ab+58b^2+(64a^2-32ab-79b^2)\cos(2(e+fx))-2(6a-11b)b\cos(4(e+fx))-b^2\cos(6(e+fx)))\cot(e+fx)\csc^2(e+fx)}{4\sqrt{2}}+32a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-4(5a^2+2ab-3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{12f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-1/4*((-32*a^2 + 44*a*b + 58*b^2 + (64*a^2 - 32*a*b - 79*b^2)*Cos[2*(e + f
*x)] - 2*(6*a - 11*b)*b*Cos[4*(e + f*x)] - b^2*Cos[6*(e + f*x)])*Cot[e + f*
x]*Csc[e + f*x]^2)/Sqrt[2] + 32*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)
])/a]*EllipticE[e + f*x, -(b/a)] - 4*(5*a^2 + 2*a*b - 3*b^2)*Sqrt[(2*a + b
```

```
- b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(12*f*Sqrt[2*a + b - b
*Cos[2*(e + f*x)]])
```

Maple [A]
time = 10.51, size = 419, normalized size = 1.52

method	result
default	$-\frac{-b^2\left(\sin^8(fx+e)\right)+5\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\text{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3(fx+e)\right)+2b\sqrt{\frac{\cos(2\ldots}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-b^2*sin(f*x+e)^8+5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)
*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+2*b*(cos(f*x+e)^2)^(
1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*si
n(f*x+e)^3-3*b^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*sin(f*x+e)^3-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+8*
(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/
a*b)^(1/2))*a*b*sin(f*x+e)^3+3*a*b*sin(f*x+e)^6-3*b^2*sin(f*x+e)^6+4*a^2*si
n(f*x+e)^4-8*a*b*sin(f*x+e)^4+4*b^2*sin(f*x+e)^4-5*a^2*sin(f*x+e)^2+5*a*b*s
in(f*x+e)^2+a^2)/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Fricas [F]
time = 0.19, size = 45, normalized size = 0.16

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*
x + e)^4, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(e+f\,x)^4\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.510 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=134

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f}-\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f}+\frac{\sec^4(e+$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)
^(5/2)/f-1/8*(8*a+5*b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f+1/4*
sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]
time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 91, 79, 65, 214}

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}+\frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4f(a+b)}-\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(
8*(a + b)^(5/2)*f) - ((8*a + 5*b)*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]
)/(8*(a + b)^2*f) + (Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2])/(4*(a + b)*
f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+b)+2(a+b)x}{(1-x)^2\sqrt{a+bx}}\,dx, x, \sin^2(e+\ldots\right.}{4(a+b)f}$$

$$= -\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+\ldots}}{4(a+b)f}$$

$$= -\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+\ldots}}{4(a+b)f}$$

$$= \frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f} - \frac{(8a+5b)\sec^2(e+fx)\ldots}{8(a+\ldots}$$

Mathematica [A]
time = 0.33, size = 108, normalized size = 0.81

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)+\sqrt{a+b}\,\sec^2(e+fx)\left(-8a-5b+2(a+b)\sec^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] +
Sqrt[a + b]*Sec[e + f*x]^2*(-8*a - 5*b + 2*(a + b)*Sec[e + f*x]^2)*Sqrt[a +
 b*Sin[e + f*x]^2])/(8*(a + b)^(5/2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(118) = 236$.
time = 27.45, size = 644, normalized size = 4.81

method	result
default	$\left(8\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)a^4+24\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+\ldots\right.\right.}}{\sin(fx+e)-1}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((8*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*a^4+24*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))*a^3*b+27*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^2+14*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+3*ln(2/(sin(f*x+e)-1)*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4+8*ln(2/(1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4+24*ln(2/(
1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*
b+27*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+
e)+a))*a^2*b^2+14*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))*a*b^3+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-2*(a+b-b*cos(f*x+e)^2)^(1/2
)*(a+b)^(5/2)*(8*a+5*b)*cos(f*x+e)^2+4*(a+b)^(5/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)*a+4*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2))/(a+b)^(5/2)/cos(f*x+e)^4/(
a^2+2*a*b+b^2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(123) = 246$.
time = 0.57, size = 254, normalized size = 1.90

$$-\frac{\frac{(8\,a^2b^3+8\,ab^4+3\,b^5)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{(a^2+2\,ab+b^2)\sqrt{a+b}}-\frac{2\left((8\,ab^4+5\,b^5)\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}-(8\,a^2b^4+11\,ab^5+3\,b^6)\sqrt{b\sin\left(fx+e\right)^2+a}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4+\left(b\sin(fx+e)^2+a\right)^2(a^2+2\,ab+b^2)-2\,(a^3+3\,a^2b+3\,ab^2+b^3)\left(b\sin(fx+e)^2+a\right)}}{16\,b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a^2*b^3 + 8*a*b^4 + 3*b^5)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt
(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*s
qrt(a + b)) - 2*((8*a*b^4 + 5*b^5)*(b*sin(f*x + e)^2 + a)^(3/2) - (8*a^2*b^
4 + 11*a*b^5 + 3*b^6)*sqrt(b*sin(f*x + e)^2 + a))/(a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4 + (b*sin(f*x + e)^2 + a)^2*(a^2 + 2*a*b + b^2) - 2*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*(b*sin(f*x + e)^2 + a)))/(b^3*f)
```

Fricas [A]
time = 0.51, size = 328, normalized size = 2.45

$$\left[\frac{(8\,a^2+8\,ab+3\,b^2)\sqrt{a+b}\,\cos\left(fx+e\right)^4\log\left(\frac{b\cos(fx+e)^2-2\,\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos(fx+e)^2}\right)-2\left((8\,a^2+13\,ab+5\,b^2)\cos\left(fx+e\right)^2-2\,a^2-4\,ab-2\,b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)f\cos\left(fx+e\right)^4},\ -\frac{(8\,a^2+8\,ab+3\,b^2)\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)\cos\left(fx+e\right)^4+\left((8\,a^2+13\,ab+5\,b^2)\cos\left(fx+e\right)^2-2\,a^2-4\,ab-2\,b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)f\cos\left(fx+e\right)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*a^2 + 8*a*b + 3*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*x +
e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x +
 e)^2) - 2*((8*a^2 + 13*a*b + 5*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2
)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f
*x + e)^4), -1/8*((8*a^2 + 8*a*b + 3*b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^4 + ((8*a^2 + 13*a*b
+ 5*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a
 + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2580 vs. 2(123) = 246.
time = 1.91, size = 2580, normalized size = 19.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((8*a^2 + 8*a*b + 3*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a -
b)) - 2*(8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 + 8*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a*b + 3*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(5/2) - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^6*a^(3/2)*b - 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^6*sqrt(a)*b^2 - 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt
```

(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 - 408*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b - 269*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 44*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^3 + 136*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) + 40*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b - 493*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 - 292*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b^3 + 344*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^4 + 1304*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3*b + 1345*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b^2 + 104*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^3 - 176*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^4 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(9/2) + 600*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2)*b + 1865*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b^2 + 1880*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^3 + 528*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^4 - 232*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^5 - 904*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^4*b - 1079*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b^2 - 60*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b^3 + 560*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^4 + 192*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e

```
)^2 + a))*b^5 - 104*a^(11/2) - 584*a^(9/2)*b - 1351*a^(7/2)*b^2 - 1588*a^(5
/2)*b^3 - 912*a^(3/2)*b^4 - 192*sqrt(a)*b^5)/(((sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a))*sqrt(a) - 3*a - 4*b)^4*(a^2 + 2*a*b + b^2)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^5}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.511 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=81

$$-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f}+\frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f}$$

```
[Out] -1/2*(2*a+b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f+1/
2*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3273, 79, 65, 214}

$$\frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/((a + b)^(
3/2)*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*(a + b)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{a+bx}}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} - \frac{(2a+b)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx,x,\sin\right.}{4(a+b)f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} - \frac{(2a+b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\,\mathrm{s}}\right.}{2b(a+b)f}$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+f}}{2(a+b)f}$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 0.95

$$-\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/2*(((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b)^(
3/2) - (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(a + b))/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(69) = 138$.
time = 22.77, size = 353, normalized size = 4.36

method	result
default	$-\frac{\left(2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}-2b\sin\left(fx+e\right)+2a}{1+\sin\left(fx+e\right)}\right)a^2+3\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx\right.\right.}}{1+\sin\left(fx+e\right)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-(2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(
f*x+e)+a))*a^2+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)-b*sin(f*x+e)+a))*a*b+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)-b*sin(f*x+e)+a))*b^2+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b+ln(2/(sin(f*x+e)-1)*((a+b)^(1/
2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^2+2*(a+b-b*c
os(f*x+e)^2)^(1/2)*(a+b)^(3/2))/(a+b)^(5/2)/cos(f*x+e)^2/f
```

Maxima [A]
time = 0.55, size = 128, normalized size = 1.58

$$-\frac{\frac{2\sqrt{b\sin\left(fx+e\right)^2+a}\,b^3}{\left(b\sin\left(fx+e\right)^2+a\right)(a+b)-a^2-2\,ab-b^2}-\frac{\left(2\,ab^2+b^3\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}}}{4\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(b*sin(f*x + e)^2 + a)*b^3/((b*sin(f*x + e)^2 + a)*(a + b) - a^
2 - 2*a*b - b^2) - (2*a*b^2 + b^3)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a
 + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/(a + b)^(3/2))/(b^2*f)
```

Fricas [A]
time = 0.48, size = 220, normalized size = 2.72

$$\left[\frac{(2\,a+b)\sqrt{a+b}\,\cos\left(fx+e\right)^2\log\left(\frac{b\cos(fx+e)^2+2\,\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos(fx+e)^2}\right)+2\,\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,(a+b)}{4\,(a^2+2\,ab+b^2)f\cos\left(fx+e\right)^2},\ \frac{(2\,a+b)\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)\cos\left(fx+e\right)^2+\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,(a+b)}{2\,(a^2+2\,ab+b^2)f\cos\left(fx+e\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*a + b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)
, 1/2*((2*a + b)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-
a - b)/(a + b))*cos(f*x + e)^2 + sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b))/(
(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. 2(72) = 144.
time = 0.91, size = 793, normalized size = 9.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((2*a + b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
- sqrt(a))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) - 2*(2*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^3*b + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))^2*a^(3/2) + 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqr
t(a)*b - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - (sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
```

```
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^2 - 2*a^(5/2) - 5*a^(3/2)*b - 4*sqrt(a)
*b^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^2*(a + b)
))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^3}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.512 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,f}$$

```
[Out] arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3273, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{bf}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,f}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b-b\cos^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(30) = 60$.
time = 19.82, size = 103, normalized size = 2.86

method	result
default	$\frac{\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)+\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}-}{1+\sin(fx+e)}\right.}{2\sqrt{a+b}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+
e)+a))+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*
x+e)+a)))/(a+b)^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(31) = 62$.
time = 0.55, size = 112, normalized size = 3.11

$$-\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)+1)}\right)}{\sqrt{a+b}}-\frac{\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)-1)}\right)}{\sqrt{a+b}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*
(sin(f*x + e) + 1)))/sqrt(a + b) - arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(
f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/sqrt(a + b))/f
```

Fricas [A]
time = 0.45, size = 112, normalized size = 3.11

$$\left[\frac{\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos(fx+e)^2}\right)}{2\,\sqrt{a+b}\,f},\,-\frac{\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)}{(a+b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)
- 2*a - 2*b)/cos(f*x + e)^2)/(sqrt(a + b)*f), -sqrt(-a - b)*arctan(sqrt(-b*
cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))/((a + b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(31) = 62$.
time = 0.61, size = 98, normalized size = 2.72

$$-\frac{2 \arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2-\sqrt{a\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^4+2 a\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2+4 b\tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2+a}-\sqrt{a}}{2 \sqrt{-a-b}}\right)}{\sqrt{-a-b} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sqrt(a
))/sqrt(-a - b))/(sqrt(-a - b)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(e+f\,x)}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.513 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3273, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
```

```
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Maple [A]
time = 5.68, size = 42, normalized size = 1.27

method	result	size
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin(fx+e)}\right)}{\sqrt{a}\,f}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))/f
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.79

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/(sqrt(a)*f)
```

Fricas [A]
time = 0.44, size = 100, normalized size = 3.03

$$\left[\frac{\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right)}{2\sqrt{a}\,f}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a}}{a}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) -
2*a - b)/(cos(f*x + e)^2 - 1))/(sqrt(a)*f), sqrt(-a)*arctan(sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/a)/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [A]
time = 0.45, size = 31, normalized size = 0.94

$$\frac{\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(e+f\,x)}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.514 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=75

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af}$$

```
[Out] 1/2*(2*a+b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csc(f*x
+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3273, 79, 65, 214}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - (Cs
c[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{(2a+b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin\right.}{4af}$$

$$= -\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{(2a+b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{2abf}$$

$$= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+f}{2af}$$

Mathematica [A]
time = 0.12, size = 71, normalized size = 0.95

$$\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/a^(3/2) - (Csc[e +
 f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/a)/(2*f)
```

Maple [A]
time = 9.04, size = 109, normalized size = 1.45

method	result
default	$\frac{-\frac{\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{2a\sin(fx+e)^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin(fx+e)}\right)}{2a^{\frac{3}{2}}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin(fx+e)}\right)}{\sqrt{a}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+1/2*b/a^(3/2)*ln((2*a+2*a^(1/
2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*s
in(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.28, size = 81, normalized size = 1.08

$$\frac{\frac{2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,\left|\sin\left(fx+e\right)\right|}\right)}{\sqrt{a}}+\frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{b\sin\left(fx+e\right)^2+a}}{a\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/sqrt(a) + b*arcsinh(a/(sqrt
(a*b)*abs(sin(f*x + e))))/a^(3/2) - sqrt(b*sin(f*x + e)^2 + a)/(a*sin(f*x +
 e)^2))/f
```

Fricas [A]
time = 0.47, size = 220, normalized size = 2.93

$$\left[\frac{\left((2\,a+b)\cos\left(fx+e\right)^2-2\,a-b\right)\sqrt{a}\,\log\left(\frac{2\left(b\cos(fx+e)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}-2\,a-b\right)}{\cos(fx+e)^2-1}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a}{4\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)},\ -\frac{\left((2\,a+b)\cos\left(fx+e\right)^2-2\,a-b\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a}}{a}\right)-\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a}{2\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a)*log(2*(b*cos(f*x + e)^2
- 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)
) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*f*cos(f*x + e)^2 - a^2*f), -1
/2*(((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x +
e)^2 + a + b)*sqrt(-a)/a) - sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*f*cos(f
*x + e)^2 - a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e + fx)}{\sqrt{a + b\sin^2(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e + f\,x)^3}{\sqrt{b\sin\left(e + f\,x\right)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.515 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}-\frac{\csc^4(e+}{}$$

```
[Out] -1/8*(8*a^2+8*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/
f+1/8*(8*a+3*b)*csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f-1/4*csc(f*x+e)^
4*(a+b*sin(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 91, 79, 65, 214}

$$\frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}-\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/8*((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/
(a^(5/2)*f) + ((8*a + 3*b)*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(8*a^
2*f) - (Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2])/(4*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-8a-3b)+2ax}{x^2\sqrt{a+bx}}\,dx, x, \sin^2(e+\right.}{4af}$$

$$= \frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} - \frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+f}{4af}$$

$$= \frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} - \frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+f}{4af}$$

$$= -\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} + \frac{(8a+3b)\csc^2(e+fx}{8a}$$

Mathematica [A]
time = 0.26, size = 101, normalized size = 0.80

$$\frac{-\left((8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)\right)+\sqrt{a}\,\csc^2(e+fx)\,(8a+3b-2a\csc^2(e+fx))\,\sqrt{a+b\sin^2(e+fx)}}{8a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]) + S
qrt[a]*Csc[e + f*x]^2*(8*a + 3*b - 2*a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f
*x]^2])/(8*a^(5/2)*f)
```

Maple [A]
time = 11.56, size = 205, normalized size = 1.63

method	result
default	$\frac{-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b(\sin^2(fx+e))}}{\sin(fx+e)}\right)}{\sqrt{a}}+\frac{\sqrt{a+b(\sin^2(fx+e))}}{a\sin(fx+e)^2}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b(\sin^2(fx+e)}}{\sin(fx+e)}\right.}{a^{\frac{3}{2}}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+1/a/sin (f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-b/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x +e)^2)^(1/2))/sin(f*x+e))-1/4/a/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)+3/8/a ^2*b/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-3/8/a^(5/2)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f

Maxima [A]
time = 0.32, size = 167, normalized size = 1.33

$$-\frac{\frac{8\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{\sqrt{a}}+\frac{8\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}}+\frac{3\,b^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}}-\frac{8\,\sqrt{b\sin(fx+e)^{2}+a}}{a\sin(fx+e)^{2}}-\frac{3\,\sqrt{b\sin(fx+e)^{2}+a}\,b}{a^{2}\sin(fx+e)^{2}}+\frac{2\,\sqrt{b\sin(fx+e)^{2}+a}}{a\sin(fx+e)^{4}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] -1/8*(8*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/sqrt(a) + 8*b*arcsinh(a/(s qrt(a*b)*abs(sin(f*x + e))))/a^(3/2) + 3*b^2*arcsinh(a/(sqrt(a*b)*abs(sin(f *x + e))))/a^(5/2) - 8*sqrt(b*sin(f*x + e)^2 + a)/(a*sin(f*x + e)^2) - 3*sq rt(b*sin(f*x + e)^2 + a)*b/(a^2*sin(f*x + e)^2) + 2*sqrt(b*sin(f*x + e)^2 + a)/(a*sin(f*x + e)^4))/f

Fricas [A]
time = 0.53, size = 388, normalized size = 3.08

$$\left[\frac{\left((8a^{2}+8ab+3b^{2})\cos(fx+e)^{4}-2\,(8a^{2}+8ab+3b^{2})\cos(fx+e)^{2}+8a^{2}+8ab+3b^{2}\right)\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^{2}+2\sqrt{-b\cos(fx+e)^{2}+a+b}\,\sqrt{a}-2a-b\right)}{\cos(fx+e)^{2}-1}\right)-2\left((8a^{2}+3ab)\cos(fx+e)^{2}-6a^{2}-3ab\right)\sqrt{-b\cos(fx+e)^{2}+a+b}}{16\left(a^{3}f\cos(fx+e)^{4}-2a^{3}f\cos(fx+e)^{2}+a^{3}f\right)},\ \frac{\left((8a^{2}+8ab+3b^{2})\cos(fx+e)^{4}-2\,(8a^{2}+8ab+3b^{2})\cos(fx+e)^{2}+8a^{2}+8ab+3b^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^{2}+a+b}\,\sqrt{-a}}{a}\right)-\left((8a^{2}+3ab)\cos(fx+e)^{2}-6a^{2}-3ab\right)\sqrt{-b\cos(fx+e)^{2}+a+b}}{8\left(a^{3}f\cos(fx+e)^{4}-2a^{3}f\cos(fx+e)^{2}+a^{3}f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [1/16*(((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 8*a*b + 3*b^2)* cos(f*x + e)^2 + 8*a^2 + 8*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2 *sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*((8*a^2 + 3*a*b)*cos(f*x + e)^2 - 6*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/8*(((8 *a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 8*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - ((8*a^2 + 3*a*b)*cos(f*x + e)^2 - 6*a^2 - 3*a*b)*sqrt(-b*c os(f*x + e)^2 + a + b))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^ 3*f)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(115) = 230.
time = 0.83, size = 898, normalized size = 7.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a)*(tan(1/2*f*x + 1/2*e)^2/a - (13*a + 6*b)/a^2) - 8*
(8*a^2 + 8*a*b + 3*b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 4*(8*a^(5/2) + 8*a^(3/2)*b + 3*sqrt(a)
*b^2)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(
3/2) - 2*sqrt(a)*b))/a^3 + 4*(6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))^3*a^2 + 16*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3
*a*b + 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) - 8*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 - 20*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b - 10*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*a*b^2 - 7*a^(7/2) - 4*a^(5/2)*b)/(((sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^2*a^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^5}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.516 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=246

$$\frac{2(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{a\sqrt{\cos^2(e+fx)}\,F(\mathrm{s}$$

[Out] 2/3*(2*a+b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(2*a+b)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)^2/f+1/3*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f

Rubi [A]
time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 481, 541, 538, 437, 435, 432, 430}

$$-\frac{a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\mathrm{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{3f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\mathrm{ArcSin}(\sin(e+fx))|-\frac{b}{a}\right)}{3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]

[Out] (2*(2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - (2*(2*a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)^2*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f)

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^{5/2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right.}{f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+f\right.}{}$$

$$= -\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+}}{3(a+b)f}$$

$$= -\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+}}{3(a+b)f}$$

$$= -\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+}}{3(a+b)f}$$

$$= \frac{2(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 1.50, size = 188, normalized size = 0.76

$$\frac{4a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)-2a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-\frac{(2(4a^2+3ab+b^2)\cos(2(e+fx))+(2a+b)(2a-b-b\cos(4(e+fx))))\sec^2(e+fx)\tan(e+fx)}{\sqrt{2}}}{6(a+b)^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

[Out] (4*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] - 2*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*
x, -(b/a)] - ((2*(4*a^2 + 3*a*b + b^2)*Cos[2*(e + f*x)] + (2*a + b)*(2*a -
b - b*Cos[4*(e + f*x)]))*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[2])/(6*(a + b)^2
*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])

Maple [A]
time = 18.12, size = 377, normalized size = 1.53

method	result
default	$-\frac{2\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b(2a+b)\sin(fx+e)(\cos^4(fx+e))-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(2*a+b)*sin(f*x+e)*cos
(f*x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(4*a^2+7*a*b+3*b^2)*co
s(f*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b
+b^2)*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*c
os(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*a-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(sin(f*
x+e)-1)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(1+sin(f*
x+e))/(a+b)^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tan(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)

Fricas [C] Result contains complex when optimal does not.
time = 0.20, size = 845, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

```
[Out] 1/3*((2*(2*I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 -
 (-4*I*a^2*b - 4*I*a*b^2 - I*b^3)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((
a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-2*I*a*b^2 - I*b^3)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (4*I*a^2*b + 4*I*a*b^2 + I*b^3)
*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ell
iptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e)
- I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)
/b^2))/b^2) + (2*(-3*I*a^2*b - 5*I*a*b^2 - 2*I*b^3)*sqrt(-b)*sqrt((a^2 + a*
b)/b^2)*cos(f*x + e)^3 - (-6*I*a^3 - 5*I*a^2*b - I*a*b^2)*sqrt(-b)*cos(f*x
+ e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqr
t((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e)))
, (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(
3*I*a^2*b + 5*I*a*b^2 + 2*I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e
)^3 - (6*I*a^3 + 5*I*a^2*b + I*a*b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sq
rt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a
*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b
^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (a*b^2 + b^3 - 2*(2*a*b^
2 + b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^
2*b^2 + 2*a*b^3 + b^4)*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(e+f\,x)^4}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.517 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f}$$

```
[Out] -EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*si
n(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(a+b*sin(f*x+e)^2)^(1/
2)*tan(f*x+e)/(a+b)/f
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$,
Rules used = {3275, 482, 437, 435}

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}-\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) +
 (Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/((a + b)*f)
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{(1-x^2)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\right.}{(a+b)}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin\right.}{(a+b)}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A]
time = 0.27, size = 100, normalized size = 0.92

$$\frac{-2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,(2a+b-b\cos(2(e+fx)))\tan(e+fx)}{2(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + S
qrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*(a + b)*f*Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(102) = 204$.
time = 17.38, size = 222, normalized size = 2.04

method	result
default	$\frac{-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b\sin(fx+e)(\cos^2(fx+e))+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+}{(a+b)\sqrt{-\left(a+b\left(\sin^2\left(fx+\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)-a*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^
(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))/(a+b)/(-(a+b*sin(f*x+e)^2)*(sin
(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.16, size = 730, normalized size = 6.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^2*sin(f*x + e) - (2*I*sqrt(-b)*b^2
*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (2*I*a*b + I*b^2)*sqrt(-b)*cos(f*x +
e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8
```

```
*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (-2*I*sq
rt(-b)*b^2*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (-2*I*a*b - I*b^2)*sqrt(-b)
*cos(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arc
sin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x
 + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)
 - 2*(2*(-I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (2*I
*a^2 + I*a*b)*sqrt(-b)*cos(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
+ b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(co
s(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt(
(a^2 + a*b)/b^2))/b^2) - 2*(2*(I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2
)*cos(f*x + e) + (-2*I*a^2 - I*a*b)*sqrt(-b)*cos(f*x + e))*sqrt((2*b*sqrt((
a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/((a*b^2 + b^3)*f*cos(f*x + e)
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.518 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin
(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3262, 3261}

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sin^2(e+fx)}}$$
$$= \frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.06, size = 60, normalized size = 1.18

$$\frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 60, normalized size = 1.18

method	result	size
default	$\frac{\sqrt{-\frac{b(\cos^2(fx+e))-a-b}{a}}\,\mathrm{am}^{-1}\left(fx+e\middle|\frac{i\sqrt{b}}{\sqrt{a}}\right)}{f\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/(a+b-b*cos(f*x+e)^2)^(1/2)*(-(b*cos(f*x+e)^2-a-b)/a)^(1/2)*InverseJacob
iAM(f*x+e,I/a^(1/2)*b^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.11, size = 305, normalized size = 5.98

$$-\frac{\left(2i\sqrt{-b}\,b\sqrt{\frac{a^2+ab}{b^2}}+(-2i\,a-i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,(\cos(fx+e)+i\,\sin(fx+e))\right)\,|\,\frac{8\,a^2+8\,ab+b^2-4\,(2\,ab+b^2)\sqrt{\frac{a^2+ab}{b^2}}}{b^2})+\left(-2i\sqrt{-b}\,b\sqrt{\frac{a^2+ab}{b^2}}+(2i\,a+i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\,(\cos(fx+e)-i\,\sin(fx+e))\right)\,|\,\frac{8\,a^2+8\,ab+b^2-4\,(2\,ab+b^2)\sqrt{\frac{a^2+ab}{b^2}}}{b^2})}{b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -((2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2) + (-2*I*a - I*b)*sqrt(-b))*sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2
 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b
 + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b*sqr
t((a^2 + a*b)/b^2) + (2*I*a + I*b)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2
) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^
2)*sqrt((a^2 + a*b)/b^2))/b^2))/(b^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.519 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}-\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-EllipticE(sin(f*x+e),(-b/a)^(1/2))
*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(f*x+
e)^2/a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3275, 486, 21, 437, 435}

$$-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{af\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*f)) - (Sqrt[Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x
]^2])/(a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\Big(\ldots}{\ldots}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\Big(\ldots}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\Big(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin\ldots}}{af\sqrt{\ldots}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|\ldots\right.}{af\sqrt{1+\frac{b\sin\ldots}{\ldots}}}$$

Mathematica [A]
time = 0.26, size = 101, normalized size = 0.95

$$-\frac{\sqrt{2a+b-b\cos(2(e+fx))}\,\cot(e+fx)}{\sqrt{2}\,af} - \frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*x])/(Sqrt[2]*a*f)) - (Sqrt[
(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 7.06, size = 120, normalized size = 1.13

method	result

default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,a\sin(fx+e)\,\text{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-b(\cos^4(fx+e))+(a+b)(\cos}{a\sin(fx+e)\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*sin(f*x+e)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)/a/sin(f*
x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.15, size = 723, normalized size = 6.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^2*cos(f*x + e) + (2*I*sqrt(-b)*b^
2*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (2*I*a*b + I*b^2)*sqrt(-b)*sin(f*x +
 e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((
2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (
8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*s
qrt(-b)*b^2*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (-2*I*a*b - I*b^2)*sqrt(-b
)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(ar
csin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*
x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2
) + 2*(2*(-I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (2*
I*a^2 + I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(c
os(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt
((a^2 + a*b)/b^2))/b^2) + 2*(2*(I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^
2)*sin(f*x + e) + (-2*I*a^2 - I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt(
```

```
(a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2
- 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(a*b^2*f*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^2}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.520 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=240

$$\frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}+\frac{2(2a+b)\sqrt{\mathrm{co}}}{}$$

```
[Out] 2/3*(2*a+b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)*csc(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a/f+2/3*(2*a+b)*EllipticE(sin(f*x+e),(-b/a)
^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f/(1+b
*sin(f*x+e)^2/a)^(1/2)-1/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x
+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a/f/(a+b*sin(f*x+e)^2)^
(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 485, 597, 538, 437, 435, 432, 430}

$$\frac{2(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}-\frac{(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3af\sqrt{a+b\sin^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (2*(2*a + b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*f) - (Cot[e +
f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) + (2*(2*a + b)*Sqrt
[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
a + b*Sin[e + f*x]^2])/(3*a^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((a + b)*
Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*S
qrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{x^4\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+\right.}{}$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+}}{3af}$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+}}{3af}$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+}}{3af}$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+}}{3af}$$

Mathematica [A]

time = 2.70, size = 186, normalized size = 0.78

$$\frac{\frac{(-2(4a^2+5ab+2b^2)\cos(2(e+fx))+(2a+b)(2a+3b+b\cos(4(e+fx))))\cot(e+fx)\csc^2(e+fx)}{\sqrt{2}} + 4a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right) - 2a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{6a^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

[Out] (((-2*(4*a^2 + 5*a*b + 2*b^2)*Cos[2*(e + f*x)] + (2*a + b)*(2*a + 3*b + b*C
os[4*(e + f*x)]))*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] + 4*a*(2*a + b)*Sqrt
[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] - 2*a*(a + b)
*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6*a^2*
f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])

Maple [A]
time = 10.26, size = 351, normalized size = 1.46

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2(\sin^3(fx+e))+b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{a+}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-4*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ell
ipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+4*a*b*sin(f*x+e)^6+2*b^2
*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4-2*b^2*sin(f*x+e)^4-5*a^
2*sin(f*x+e)^2-a*b*sin(f*x+e)^2+a^2)/a^2/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f
*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(cot(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)

Fricas [C] Result contains complex when optimal does not.
time = 0.17, size = 1045, normalized size = 4.35

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

```
[Out] 1/3*((2*(-2*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b)*
sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (4*I*a^2*b + 4*I*a*b^2 + I*b^3 + (-4*I
*a^2*b - 4*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2
 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b
 + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(2*I*a*b^2 + I*b^
3 + (-2*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin
(f*x + e) - (-4*I*a^2*b - 4*I*a*b^2 - I*b^3 + (4*I*a^2*b + 4*I*a*b^2 + I*b^
3)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b
)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*
sqrt((a^2 + a*b)/b^2))/b^2) + (2*(3*I*a^2*b + 5*I*a*b^2 + 2*I*b^3 + (-3*I*a
^2*b - 5*I*a*b^2 - 2*I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*
sin(f*x + e) - (6*I*a^3 + 5*I*a^2*b + I*a*b^2 + (-6*I*a^3 - 5*I*a^2*b - I*a
*b^2)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2
) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b
)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^
2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^2*b - 5*I*a*b^2 - 2*I*b^3 + (3*
I*a^2*b + 5*I*a*b^2 + 2*I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^
2)*sin(f*x + e) - (-6*I*a^3 - 5*I*a^2*b - I*a*b^2 + (6*I*a^3 + 5*I*a^2*b +
I*a*b^2)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
+ b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
 b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(2*a*b^2 + b^3)*cos(f*x + e)^3 - (3*
a*b^2 + 2*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^2*b^2*f*c
os(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(e+fx)^4}{\sqrt{b\sin(e+fx)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.521 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{(8a^2-8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f}-\frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(8a+3b)\sec^2(e}{8(a+b)^2f\sqrt{a+b\sin}}$$

```
[Out] 1/8*(8*a^2-8*a*b-b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(
7/2)/f+1/8*(-8*a^2+8*a*b+b^2)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)-1/8*(8*a+3
*b)*sec(f*x+e)^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/4*sec(f*x+e)^4/(a+b)/
f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 53, 65, 214}

$$-\frac{8a^2-8ab-b^2}{8f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{(8a^2-8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}+\frac{\sec^4(e+fx)}{4f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(8a+3b)\sec^2(e+fx)}{8f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((8*a^2 - 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(8*
(a + b)^(7/2)*f) - (8*a^2 - 8*a*b - b^2)/(8*(a + b)^3*f*Sqrt[a + b*Sin[e +
f*x]^2]) - ((8*a + 3*b)*Sec[e + f*x]^2)/(8*(a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2]) + Sec[e + f*x]^4/(4*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)}{4(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a-b)+2(a+b)x}{(1-x)^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx\right.}{4(a+b)f}$$

$$= -\frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^4(e+fx)}{4(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a^2}{}$$

$$= -\frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{}{4(}$$

$$= -\frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{}{4(}$$

$$= \frac{(8a^2-8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{8a^2-8ab-}{8(a+b)^3f\sqrt{a+b\sin}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.33, size = 107, normalized size = 0.60

$$\frac{(-8a^2+8ab+b^2)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin^2(e+fx)}{a+b}\right) - \frac{1}{2}(a+b)(4a-b+(8a+3b)\cos(2(e+fx)))\sec^4(e+fx)}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-8*a^2 + 8*a*b + b^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[e + f*x]
^2)/(a + b)] - ((a + b)*(4*a - b + (8*a + 3*b)*Cos[2*(e + f*x)])*Sec[e + f*
x]^4)/2)/(8*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3762 vs. $2(155) = 310$.
time = 90.77, size = 3763, normalized size = 21.26

method	result	size
default	Expression too large to display	3763

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(a^4*b^2*cos(f*x+e)^4+4*a^3*b^3*cos(f*x+e)^4+6*a^2*b^4*cos(f*x+e)^4+4
*a*b^5*cos(f*x+e)^4+b^6*cos(f*x+e)^4-2*a^5*b*cos(f*x+e)^2-10*a^4*b^2*cos(f*
x+e)^2-20*a^3*b^3*cos(f*x+e)^2-20*a^2*b^4*cos(f*x+e)^2-10*a*b^5*cos(f*x+e)^
2-2*b^6*cos(f*x+e)^2+a^6+6*a^5*b+15*a^4*b^2+20*a^3*b^3+15*a^2*b^4+6*a*b^5+b
^6)/cos(f*x+e)^4/(a+b)^(3/2)*(-4*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*b^3
-8*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)
+a))*a^6*cos(f*x+e)^4+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)+b*sin(f*x+e)+a))*b^6*cos(f*x+e)^4-8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^6*cos(f*x+e)^4+ln(2/(1+sin(f
*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^6*cos(f*x
+e)^4+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x
+e)+a))*b^6*cos(f*x+e)^8+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)-b*sin(f*x+e)+a))*b^6*cos(f*x+e)^8-2*ln(2/(sin(f*x+e)-1)*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^6*cos(f*x+e)^6-2*ln(2/(1+
sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^6*co
s(f*x+e)^6-4*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*a^3-2*(a+b-b*cos(f*x+e)
^2)^(1/2)*(a+b)^(3/2)*b^4*cos(f*x+e)^4-8*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^
(1/2)*(a+b)^(3/2)*a^4*cos(f*x+e)^4+16*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2
)*a^3*cos(f*x+e)^2+6*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*b^3*cos(f*x+e)^
2-12*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*a^2*b-12*(a+b-b*cos(f*x+e)^2)^(
3/2)*(a+b)^(3/2)*a*b^2-8*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^8-8*ln(2/(sin(f*x+e)-1)*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^8+9*
ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a)
)*a^2*b^4*cos(f*x+e)^8+10*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^8-8*ln(2/(1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^8-8*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
*a^3*b^3*cos(f*x+e)^8+9*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^4*cos(f*x+e)^8+10*ln(2/(1+sin(f*x+e))*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^8+16*l
n(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*a^5*b*cos(f*x+e)^6+32*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)+b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^6-2*ln(2/(sin(f*x+e)-1)*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^6-38*l
n(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*a^2*b^4*cos(f*x+e)^6-22*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
```

```
^2)^(1/2)+b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^6+16*ln(2/(1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^5*b*cos(f*x+e)^6+32*ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
a^4*b^2*cos(f*x+e)^6-2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)-b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^6-38*ln(2/(1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^4*cos(f*x+e)^6-22*
ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)
)*a*b^5*cos(f*x+e)^6-24*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)+b*sin(f*x+e)+a))*a^5*b*cos(f*x+e)^4-15*ln(2/(sin(f*x+e)-1)*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^4+20*l
n(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*a^3*b^3*cos(f*x+e)^4+30*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^4*cos(f*x+e)^4+12*ln(2/(sin(f*x+e)-1)*((a+
b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^4-24*
ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)
)*a^5*b*cos(f*x+e)^4-15*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^4+20*ln(2/(1+sin(f*x+e))*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^4+30
*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a
))*a^2*b^4*cos(f*x+e)^4+12*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^4-2*(a+b-b*cos(f*x+e)^2)^(1/2
)*(a+b)^(3/2)*b^4*cos(f*x+e)^8-2*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*b^3
*cos(f*x+e)^6+4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*b^4*cos(f*x+e)^6+8*(
-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*(a+b)^(3/2)*a^3*cos(f*x+e)^4+16*(a+b
-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a^4*cos(f*x+e)^4-16*(a+b-b*cos(f*x+e)^2)
^(1/2)*(a+b)^(3/2)*a*b^3*cos(f*x+e)^8-8*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(
1/2)*(a+b)^(3/2)*a^2*b^2*cos(f*x+e)^8+40*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^
(1/2)*(a+b)^(3/2)*a*b^3*cos(f*x+e)^8+16*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3
/2)*a*b^2*cos(f*x+e)^6+8*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*(a+b)^(3/2
)*a^2*b*cos(f*x+e)^6+8*(-b*cos(f*x+e)^2+(a*b^2+...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(163) = 326$.
time = 0.52, size = 341, normalized size = 1.93

$$-\frac{\frac{\left(8\,a^2b^3-8\,ab^4-b^5\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{a+b}}+\frac{2\left(8\,a^4b^3+16\,a^3b^4+8\,a^2b^5+\left(8\,a^2b^3-8\,ab^4-b^5\right)\left(b\sin(fx+e)^2+a\right)^2-\left(16\,a^3b^3+8\,a^2b^4-7\,ab^5+b^6\right)\left(b\sin(fx+e)^2+a\right)\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}-2\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}+\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5\right)\sqrt{b\sin\left(fx+e\right)^2+a}}}{16\,b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a^2*b^3 - 8*a*b^4 - b^5)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a
 + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*sqrt(a + b)) + 2*(8*a^4*b^3 + 16*a^3*b^4 + 8*a^2*b^5 + (8*a^2*b^3
```

```
- 8*a*b^4 - b^5)*(b*sin(f*x + e)^2 + a)^2 - (16*a^3*b^3 + 8*a^2*b^4 - 7*a*b
^5 + b^6)*(b*sin(f*x + e)^2 + a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*sin(f
*x + e)^2 + a)^(5/2) - 2*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*sin
(f*x + e)^2 + a)^(3/2) + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5)*sqrt(b*sin(f*x + e)^2 + a)))/(b^3*f)
```

Fricas [A]
time = 0.56, size = 593, normalized size = 3.35

$$\left[-\frac{\left(\left(8\,a^{2}b-8\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{6}-\left(8\,a^{3}-9\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{4}\right)\sqrt{a+b}\,\log\left(\frac{b\cos(fx+e)^{2}+2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a+b}-2a-2b}{\cos(fx+e)^{2}}\right)-2\left(\left(8\,a^{3}-9\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{4}-2\,a^{3}-6\,a^{2}b-6\,ab^{2}-2\,b^{3}+\left(8\,a^{3}+19\,a^{2}b+14\,ab^{2}+3\,b^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{16\left(\left(a^{4}b+4\,a^{3}b^{2}+6\,a^{2}b^{3}+4\,ab^{4}+b^{5}\right)f\cos\left(fx+e\right)^{6}-\left(a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5}\right)f\cos\left(fx+e\right)^{4}\right)},\,-\frac{\left(\left(8\,a^{2}b-8\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{6}-\left(8\,a^{3}-9\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{4}\right)\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-a-b}}{a+b}\right)-\left(\left(8\,a^{3}-9\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{4}-2\,a^{3}-6\,a^{2}b-6\,ab^{2}-2\,b^{3}+\left(8\,a^{3}+19\,a^{2}b+14\,ab^{2}+3\,b^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{8\left(\left(a^{4}b+4\,a^{3}b^{2}+6\,a^{2}b^{3}+4\,ab^{4}+b^{5}\right)f\cos\left(fx+e\right)^{6}-\left(a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5}\right)f\cos\left(fx+e\right)^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2*b - 8*a*b^2 - b^3)*cos(f*x + e)^6 - (8*a^3 - 9*a*b^2 - b^3)
*cos(f*x + e)^4)*sqrt(a + b)*log((b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)
^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2*((8*a^3 - 9*a*b^2
- b^3)*cos(f*x + e)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 + (8*a^3 + 19*a^2
*b + 14*a*b^2 + 3*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a
^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*cos(f*x + e)^6 - (a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*f*cos(f*x + e)^4), -1/8*(((
8*a^2*b - 8*a*b^2 - b^3)*cos(f*x + e)^6 - (8*a^3 - 9*a*b^2 - b^3)*cos(f*x +
 e)^4)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a
+ b)) - ((8*a^3 - 9*a*b^2 - b^3)*cos(f*x + e)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2
 - 2*b^3 + (8*a^3 + 19*a^2*b + 14*a*b^2 + 3*b^3)*cos(f*x + e)^2)*sqrt(-b*co
s(f*x + e)^2 + a + b))/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*c
os(f*x + e)^6 - (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*f
*cos(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2776 vs. $2(163) = 326$.
time = 2.13, size = 2776, normalized size = 15.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] -1/4*(4*((a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*tan(1/2*f*x + 1/2*e)^2/(a^7*b + 7*a^6*b^2 + 21*a^5*b^3 + 35*a^4*b^4 + 35*a^3*b^5 + 21*a^2*b^6 + 7*a*b^7 + b^8) + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)/(a^7*b + 7*a^6*b^2 + 21*a^5*b^3 + 35*a^4*b^4 + 35*a^3*b^5 + 21*a^2*b^6 + 7*a*b^7 + b^8))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + (8*a^2 - 8*a*b - b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a - b)) - 2*(8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(5/2) - 32*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(3/2)*b - 25*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*b^2 - 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 - 352*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b - 113*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 28*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^3 + 136*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) - 64*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b - 561*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 - 116*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b^3 + 344*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^4 + 1088*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3*b + 597*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b^2 - 504*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^3

```
 - 112*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*
a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^4 + 24*(sqr
t(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f
*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(9/2) + 608*(sqrt(a)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1
/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2)*b + 1565*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b^2 + 952*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^3 - 176*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^4 - 232*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^5 - 736*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))*a^4*b - 483*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))*a^3*b^2 + 532*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))*a^2*b^3 + 496*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^
4 - 64*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*
a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^5 - 104*a^(11
/2) - 512*a^(9/2)*b - 979*a^(7/2)*b^2 - 836*a^(5/2)*b^3 - 208*a^(3/2)*b^4 +
 64*sqrt(a)*b^5)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*((sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^5}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.522 $\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f}+\frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -1/2*(2*a-b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f+1/
2*(2*a-b)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/2*sec(f*x+e)^2/(a+b)/f/(a+b*
sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 53, 65, 214}

$$\frac{2a-b}{2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}+\frac{\sec^2(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/((a + b)^(
5/2)*f) + (2*a - b)/(2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + Sec[e + f*
x]^2/(2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a-b)\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \sin^2(\right.}{4(a+b)f}$$

$$= \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a-}{}$$

$$= \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a-}{}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} + \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 75, normalized size = 0.64

$$\frac{(2a-b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin^2(e+fx)}{a+b}\right)+(a+b)\sec^2(e+fx)}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a - b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[e + f*x]^2)/(a + b)]
+ (a + b)*Sec[e + f*x]^2)/(2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2198 vs. 2(102) = 204.
time = 71.23, size = 2199, normalized size = 18.64

method	result	size
default	Expression too large to display	2199

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/4/(a^3*b^2*cos(f*x+e)^4+3*a^2*b^3*cos(f*x+e)^4+3*a*b^4*cos(f*x+e)^4+b^5*cos(f*x+e)^4-2*a^4*b*cos(f*x+e)^2-8*a^3*b^2*cos(f*x+e)^2-12*a^2*b^3*cos(f*x+e)^2-8*a*b^4*cos(f*x+e)^2-2*b^5*cos(f*x+e)^2+a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3+5*a*b^4+b^5)/cos(f*x+e)^2/(a+b)^(1/2)*(2*b^3/(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)+2/(a+b)^(1/2)*a^3*(a+b-b*cos(f*x+e)^2)^(3/2)+6*b/(a+b)^(1/2)*a^2*(a+b-b*cos(f*x+e)^2)^(3/2)+6*b^2/(a+b)^(1/2)*a*(a+b-b*cos(f*x+e)^2)^(3/2)-cos(f*x+e)^6*(2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2+2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*a-6*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*b-4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a+2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b)*b^2+2*cos(f*x+e)^4*((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*(a+b)^(1/2)*a+(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*(a+b)^(1/2)*b+(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*b+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^3+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^3+2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*a^2-4*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*a*b-6*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*b^2-4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2-2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a*b+2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b^2)*b-cos(f*x+e)^2*(-2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*(a+b)^(1/2)*a^2+2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*(a+b)^(1/2)*b^2+4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a*b+4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*b^2+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4+5*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+5*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1

```
/2)+b*sin(f*x+e)+a))*a^2*b^2-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4+2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b
^2)^(1/2)*(a+b)^(1/2)*a^3-2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(
1/2)*a^2*b-10*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*a*b^2-6*(
-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*b^3-4*(a+b)^(1/2)*(a+b-b
*cos(f*x+e)^2)^(1/2)*a^3-6*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2*b+2*(
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b^3))/f
```

Maxima [A]
time = 0.52, size = 198, normalized size = 1.68

$$\frac{\frac{(2\,ab^2-b^3)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{(a^2+2\,ab+b^2)\sqrt{a+b}}-\frac{2\left(2\,a^2b^2+2\,ab^3-(2\,ab^2-b^3)\left(b\sin(fx+e)^2+a\right)\right)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}(a^2+2\,ab+b^2)-(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{b\sin\left(fx+e\right)^2+a}}}{4\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*a*b^2 - b^3)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a + b))/(sqrt(b
*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 2*
(2*a^2*b^2 + 2*a*b^3 - (2*a*b^2 - b^3)*(b*sin(f*x + e)^2 + a))/((b*sin(f*x
+ e)^2 + a)^(3/2)*(a^2 + 2*a*b + b^2) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqr
t(b*sin(f*x + e)^2 + a)))/(b^2*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(102) = 204.
time = 0.51, size = 445, normalized size = 3.77

$$\left[-\frac{\left((2\,ab-b^2)\cos\left(fx+e\right)^4-(2\,a^2+ab-b^2)\cos\left(fx+e\right)^2\right)\sqrt{a+b}\,\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos(fx+e)^2}\right)+2\left((2\,a^2+ab-b^2)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{4\left((a^3b+3\,a^2b^2+3\,ab^3+b^4)f\cos\left(fx+e\right)^4-(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)f\cos\left(fx+e\right)^2\right)},\ \frac{\left((2\,ab-b^2)\cos\left(fx+e\right)^4-(2\,a^2+ab-b^2)\cos\left(fx+e\right)^2\right)\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)-\left((2\,a^2+ab-b^2)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{2\left((a^3b+3\,a^2b^2+3\,ab^3+b^4)f\cos\left(fx+e\right)^4-(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)f\cos\left(fx+e\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a*b - b^2)*cos(f*x + e)^4 - (2*a^2 + a*b - b^2)*cos(f*x + e)^2)*
sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*((2*a^2 + a*b - b^2)*cos(f*x + e)^2
 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b + 3*a^2*b^2
+ 3*a*b^3 + b^4)*f*cos(f*x + e)^4 - (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*f*cos(f*x + e)^2), 1/2*(((2*a*b - b^2)*cos(f*x + e)^4 - (2*a^2 + a*b -
 b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(-a - b)/(a + b)) - ((2*a^2 + a*b - b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*
```

```
f*cos(f*x + e)^4 - (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*f*cos(f*x +
e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. 2(106) = 212.
time = 1.09, size = 1011, normalized size = 8.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(1/2*f*x + 1/2*e)^2/(a^5*b + 5
*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6) + (a^4*b + 3*a^3*b^2 +
3*a^2*b^3 + a*b^4)/(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 +
 b^6))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a) + (2*a - b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqr
t(-a - b)) - 2*(2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a
+ (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan
(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b + 2*(sqrt(a)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2
*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) + 5*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b - 2*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))*a^2 - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^2
 - 2*a^(5/2) - 5*a^(3/2)*b - 4*sqrt(a)*b^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
```

```
/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*sqrt(a) - 3*a - 4*b)^2*(a^2 + 2*a*b + b^2)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^3}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.523 $\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f}-\frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-1/(a+b)/f/(a+b*
sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 53, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{1}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(3/2)*f) - 1/((a +
 b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \sin^2(e+f\right.}{2(a+b)f}$$

$$= -\frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+}\right.}{b(a+b)f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 54, normalized size = 0.86

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{b\cos^2(e+fx)}{a+b}\right)}{(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*Cos[e + f*x]^2)/(a + b)]/((a + b)*
f*Sqrt[a + b - b*Cos[e + f*x]^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(55) = 110$.
time = 59.16, size = 1317, normalized size = 20.90

method	result	size
default	Expression too large to display	1317

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/(a^2*b^2*cos(f*x+e)^4+2*a*b^3*cos(f*x+e)^4+b^4*cos(f*x+e)^4-2*a^3*b*c
os(f*x+e)^2-6*a^2*b^2*cos(f*x+e)^2-6*a*b^3*cos(f*x+e)^2-2*b^4*cos(f*x+e)^2+
a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2
)*b^3+(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*b^2-4*a^2*b*(a+b-b*cos(f*x+e)
^2)^(1/2)+a^2*b*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)-2*(a+b-b*cos(f*x+e)
^2)^(1/2)*a^3+(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^3-2*(a+b-b*cos(f*x+
e)^2)^(1/2)*a*b^2-a*b^2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)-(-b*cos(f*x
+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a^2+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*co
s(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)*a^3+ln(2/(sin(f*x+e)-1)*((a+
b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2)*a^3+ln(2/(
1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+b
)^(1/2)*a*b^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b
*sin(f*x+e)+a))*(a+b)^(1/2)*a*b^2+2*a^2*b*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)+2*a^2*b*ln(2/(sin(f
*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/
2)+b^2*(ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f
*x+e)+a))*(a+b)^(1/2)*a+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2)*a-2*(a+b-b*cos(f*x+e)^2)^(1/2)*a+(-b*
cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/
2)*b)*cos(f*x+e)^4-b*(2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)*a^2+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)*a*b+2*ln(2/(sin(f
*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/
2)*a^2+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*(a+b)^(1/2)*a*b+(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a+(-b*co
s(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*b-4*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2-4*(a+b
-b*cos(f*x+e)^2)^(1/2)*a*b+2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2-2*
(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^2)*cos(f*x+e)^2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(57) = 114$.

time = 0.52, size = 151, normalized size = 2.40

$$-\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)+1)}\right)}{(a+b)^{\frac{3}{2}}}-\frac{\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)-1)}\right)}{(a+b)^{\frac{3}{2}}}+\frac{2}{\sqrt{b\sin(fx+e)^2+a}\,a+\sqrt{b\sin(fx+e)^2+a}\,b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*
(sin(f*x + e) + 1)))/(a + b)^(3/2) - arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(si
n(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/(a + b)^(3/2) + 2/(sqr
t(b*sin(f*x + e)^2 + a)*a + sqrt(b*sin(f*x + e)^2 + a)*b))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(55) = 110.
time = 0.45, size = 281, normalized size = 4.46

$$\left[\frac{(b\cos(fx+e)^2-a-b)\sqrt{a+b}\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)+2\sqrt{-b\cos(fx+e)^2+a+b}\,(a+b)}{2\left((a^2b+2ab^2+b^3)f\cos(fx+e)^2-(a^3+3a^2b+3ab^2+b^3)f\right)},\ -\frac{(b\cos(fx+e)^2-a-b)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{a+b}\right)-\sqrt{-b\cos(fx+e)^2+a+b}\,(a+b)}{(a^2b+2ab^2+b^3)f\cos(fx+e)^2-(a^3+3a^2b+3ab^2+b^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cos(f*x + e)^2 - a - b)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2*b + 2*a*b^2 + b^3)*f*cos(f*x +
 e)^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f), -((b*cos(f*x + e)^2 - a - b)*sq
rt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - s
qrt(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2*b + 2*a*b^2 + b^3)*f*cos(f*x
+ e)^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(57) = 114.

time = 0.74, size = 250, normalized size = 3.97

$$-\frac{\frac{\frac{\left(a^{2} b+2 a b^{2}+b^{3}\right) \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}}{a^{3} b+3 a^{2} b^{2}+3 a b^{3}+b^{4}}+\frac{a^{2} b+2 a b^{2}+b^{3}}{a^{3} b+3 a^{2} b^{2}+3 a b^{3}+b^{4}}}{\sqrt{a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+2 a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+4 b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+a}}+\frac{2 \arctan\left(-\frac{\sqrt{a} \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-\sqrt{a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+2 a \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+4 b \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+a}-\sqrt{a}}{2 \sqrt{-a-b}}\right)}{(a+b) \sqrt{-a-b}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(((a^2*b + 2*a*b^2 + b^3)*tan(1/2*f*x + 1/2*e)^2/(a^3*b + 3*a^2*b^2 + 3*a*
b^3 + b^4) + (a^2*b + 2*a*b^2 + b^3)/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))/s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a) + 2*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a + b)*sqrt(-a - b)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.524 $\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sin(f*x+e)^
2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3273, 53, 65, 214}

$$\frac{1}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
+ b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}\\
&= \frac{1}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2af}\\
&= \frac{1}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{abf}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sin^2(e+fx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 46, normalized size = 0.81

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\sin^2(e+fx)}{a}\right)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sin[e + f*x]^2)/a]/(a*f*Sqrt[a + b*S
in[e + f*x]^2])
```

Maple [A]
time = 8.88, size = 62, normalized size = 1.09

method	result	size
default	$\frac{\frac{1}{a\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{a^{\frac{3}{2}}}}{f}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/a/(a+b*sin(f*x+e)^2)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2
)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.84

$$-\frac{\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin\left(fx+e\right)|}\right)}{a^{\frac{3}{2}}}-\frac{1}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\;a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) - 1/(sqrt(b*sin(f*x + e)
^2 + a)*a))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.
time = 0.47, size = 225, normalized size = 3.95

$$\left[\frac{\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{a}\,\log\left(\frac{2\left(b\cos(fx+e)^{2}+2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a}-2a-b\right)}{\cos(fx+e)^{2}-1}\right)-2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\;a}{2\left(a^{2}bf\cos\left(fx+e\right)^{2}-\left(a^{3}+a^{2}b\right)f\right)},\ \frac{\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{-a}}{a}\right)-\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\;a}{a^{2}bf\cos\left(fx+e\right)^{2}-\left(a^{3}+a^{2}b\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cos(f*x + e)^2 - a - b)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*b*f*cos(f*x + e)^2 - (a^3 + a^2*b)*f),
((b*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(-a)/a) - sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*b*f*cos(f*x + e)^2 -
 (a^3 + a^2*b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot (e+fx)}{(a+b\sin^2 (e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [A]
time = 0.51, size = 57, normalized size = 1.00

$$\frac{\arctan\left(\frac{\sqrt{b\sin{(fx+e)}^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,af}+\frac{1}{\sqrt{b\sin{(fx+e)}^2+a}\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*a*f) + 1/(sqrt(b*sin(
f*x + e)^2 + a)*a*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(e+f\,x)}{\left(b\sin{(e+f\,x)}^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.525 $\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}} \, dx$

Optimal. Leaf size=110

$$\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] 1/2*(2*a+3*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(-2*a
-3*b)/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/2*csc(f*x+e)^2/a/f/(a+b*sin(f*x+e)^2
)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 53, 65, 214}

$$\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) - (
2*a + 3*b)/(2*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - Csc[e + f*x]^2/(2*a*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sin^2(e+f\right.}{4af}$$

$$= -\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a+3b)\text{Subst}}{}$$

$$= -\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a+3b)\text{Subst}}{}$$

$$= \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 70, normalized size = 0.64

$$\frac{-a\csc^2(e+fx) - (2a+3b)\,{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; 1 + \frac{b\sin^2(e+fx)}{a}\right)}{2a^2f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a*Csc[e + f*x]^2) - (2*a + 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*S
in[e + f*x]^2)/a])/(2*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [A]
time = 10.36, size = 148, normalized size = 1.35

method	result

default	$\dfrac{-\frac{1}{2a\sin(fx+e)^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{3b}{2a^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{3b\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\left(\sin^2(fx+e\right.}}{\sin(fx+e)}\right.}{2a^{\frac{5}{2}}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)-3/2/a^2*b/(a+b*sin(f*x+e)^2)^
(1/2)+3/2/a^(5/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))
-1/a/(a+b*sin(f*x+e)^2)^(1/2)+1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2
)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.32, size = 123, normalized size = 1.12

$$\frac{\frac{2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}}+\frac{3\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}}-\frac{2}{\sqrt{b\sin(fx+e)^2+a}\,a}-\frac{3\,b}{\sqrt{b\sin(fx+e)^2+a}\,a^2}-\frac{1}{\sqrt{b\sin(fx+e)^2+a}\,a\sin(fx+e)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) + 3*b*arcsinh(a/(sq
rt(a*b)*abs(sin(f*x + e))))/a^(5/2) - 2/(sqrt(b*sin(f*x + e)^2 + a)*a) - 3*
b/(sqrt(b*sin(f*x + e)^2 + a)*a^2) - 1/(sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*
x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(94) = 188.
time = 0.47, size = 406, normalized size = 3.69

$$\left[\frac{\left((2\,ab+3\,b^2)\cos(fx+e)^4-(2\,a^2+7\,ab+6\,b^2)\cos(fx+e)^2+2\,a^2+5\,ab+3\,b^2\right)\sqrt{a}\,\log\left(\frac{2\left(b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right)+2\left((2\,a^2+3\,ab)\cos(fx+e)^2-3\,a^2-3\,ab\right)\sqrt{-b\cos(fx+e)^2+a+b}}{4\left(a^3bf\cos(fx+e)^4-(a^4+2\,a^3b)f\cos(fx+e)^2+(a^4+a^3b)f\right)},\ -\frac{\left((2\,ab+3\,b^2)\cos(fx+e)^4-(2\,a^2+7\,ab+6\,b^2)\cos(fx+e)^2+2\,a^2+5\,ab+3\,b^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\,\sqrt{-a}}{a}\right)-\left((2\,a^2+3\,ab)\cos(fx+e)^2-3\,a^2-3\,ab\right)\sqrt{-b\cos(fx+e)^2+a+b}}{2\left(a^3bf\cos(fx+e)^4-(a^4+2\,a^3b)f\cos(fx+e)^2+(a^4+a^3b)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a*b + 3*b^2)*cos(f*x + e)^4 - (2*a^2 + 7*a*b + 6*b^2)*cos(f*x + e
)^2 + 2*a^2 + 5*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*((2*a^2
+ 3*a*b)*cos(f*x + e)^2 - 3*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(
a^3*b*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b)*f*cos(f*x + e)^2 + (a^4 + a^3*b)*f
), -1/2*(((2*a*b + 3*b^2)*cos(f*x + e)^4 - (2*a^2 + 7*a*b + 6*b^2)*cos(f*x
```

```
+ e)^2 + 2*a^2 + 5*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(-a)/a) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 3*a^2 - 3*a*b)*sqrt(-b
*cos(f*x + e)^2 + a + b))/(a^3*b*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b)*f*cos(f
*x + e)^2 + (a^4 + a^3*b)*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(98) = 196.

time = 0.81, size = 525, normalized size = 4.77

$$-\frac{\frac{\left(\frac{(a^5b+a^4b^2)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^6b+a^5b^2}+\frac{2(5a^5b+11a^4b^2+6a^3b^3)}{a^6b+a^5b^2}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\frac{9a^5b+17a^4b^2+8a^3b^3}{a^6b+a^5b^2}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}+\frac{2\left(2a^{\frac{3}{2}}+3\sqrt{a}\,b\right)\log\left(\left|-\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)a-a^{\frac{3}{2}}-2\sqrt{a}\,b\right|\right)}{a^3}-\frac{2\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)a^{\frac{3}{2}}+2\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)\sqrt{a}\,b+a^2\right)}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\sqrt{a}-a^{\frac{3}{2}}\right)a^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*((((a^5*b + a^4*b^2)*tan(1/2*f*x + 1/2*e)^2/(a^6*b + a^5*b^2) + 2*(5*a
^5*b + 11*a^4*b^2 + 6*a^3*b^3)/(a^6*b + a^5*b^2))*tan(1/2*f*x + 1/2*e)^2 +
(9*a^5*b + 17*a^4*b^2 + 8*a^3*b^3)/(a^6*b + a^5*b^2))/sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + 2
*(2*a^(3/2) + 3*sqrt(a)*b)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a^3 - 2*((sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))*a^(3/2) + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a))*sqrt(a)*b + a^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^2*sqrt(a) - a^(3/2))*a^2))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cot}\left(e+f\,x\right)}^3}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.526 $\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{(8a^2+24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}}+\frac{(8a+5b)\csc^2(e+f\ldots}{8a^2f\sqrt{a+b\sin^2(e+\ldots}}$$

```
[Out] -1/8*(8*a^2+24*a*b+15*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2
)/f+1/8*(8*a^2+24*a*b+15*b^2)/a^3/f/(a+b*sin(f*x+e)^2)^(1/2)+1/8*(8*a+5*b)*
csc(f*x+e)^2/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/4*csc(f*x+e)^4/a/f/(a+b*sin(f
*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 53, 65, 214}

$$\frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{(8a^2+24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((8*a^2 + 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]
)/(a^(7/2)*f) + (8*a^2 + 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sin[e + f*x]^
2]) + ((8*a + 5*b)*Csc[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - C
sc[e + f*x]^4/(4*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-8a-5b)+2ax}{x^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{4af}$$

$$= \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a^2+24ab+15b^2}{}$$

$$= \frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx}{4af\sqrt{a+b\sin^2(e}}$$

$$= \frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx}{4af\sqrt{a+b\sin^2(e}}$$

$$= -\frac{(8a^2+24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{8a^2+24ab+}{8a^3f\sqrt{a+b\sin^2(}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.22, size = 94, normalized size = 0.56

$$\frac{a\csc^2(e+fx)\left(8a+5b-2a\csc^2(e+fx)\right)+\left(8a^2+24ab+15b^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\sin^2(e+fx)}{a}\right)}{8a^3f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Csc[e + f*x]^2*(8*a + 5*b - 2*a*Csc[e + f*x]^2) + (8*a^2 + 24*a*b + 15*b
^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sin[e + f*x]^2)/a])/(8*a^3*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Maple [A]
time = 10.39, size = 265, normalized size = 1.59

method	result
default	$\frac{\frac{1}{a\sin(fx+e)^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{3b}{a^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{3b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right.}{a^{\frac{5}{2}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/a/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)+3/a^2*b/(a+b*sin(f*x+e)^2)^(1/2)
-3/a^(5/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+1/a/(a
+b*sin(f*x+e)^2)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)
)/sin(f*x+e))-1/4/a/sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2)+5/8/a^2*b/sin(f*x
+e)^2/(a+b*sin(f*x+e)^2)^(1/2)+15/8/a^3*b^2/(a+b*sin(f*x+e)^2)^(1/2)-15/8/a
^(7/2)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

Maxima [A]
time = 0.29, size = 231, normalized size = 1.38

$$-\frac{\frac{8\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}}+\frac{24\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}}+\frac{15\,b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{7}{2}}}-\frac{8}{\sqrt{b\sin(fx+e)^2+a}\,a}-\frac{24\,b}{\sqrt{b\sin(fx+e)^2+a}\,a^2}-\frac{15\,b^2}{\sqrt{b\sin(fx+e)^2+a}\,a^3}-\frac{8}{\sqrt{b\sin(fx+e)^2+a}\,a\sin(fx+e)^2}-\frac{5\,b}{\sqrt{b\sin(fx+e)^2+a}\,a^2\sin(fx+e)^2}+\frac{2}{\sqrt{b\sin(fx+e)^2+a}\,a\sin(fx+e)^4}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) + 24*b*arcsinh(a/(
sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) + 15*b^2*arcsinh(a/(sqrt(a*b)*abs(sin
(f*x + e))))/a^(7/2) - 8/(sqrt(b*sin(f*x + e)^2 + a)*a) - 24*b/(sqrt(b*sin(
f*x + e)^2 + a)*a^2) - 15*b^2/(sqrt(b*sin(f*x + e)^2 + a)*a^3) - 8/(sqrt(b*
sin(f*x + e)^2 + a)*a*sin(f*x + e)^2) - 5*b/(sqrt(b*sin(f*x + e)^2 + a)*a^2
*sin(f*x + e)^2) + 2/(sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*x + e)^4))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(147) = 294.
time = 0.53, size = 652, normalized size = 3.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2*b + 24*a*b^2 + 15*b^3)*cos(f*x + e)^6 - (8*a^3 + 48*a^2*b +
87*a*b^2 + 45*b^3)*cos(f*x + e)^4 - 8*a^3 - 32*a^2*b - 39*a*b^2 - 15*b^3 +
(16*a^3 + 72*a^2*b + 102*a*b^2 + 45*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*c
```

```
os(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f
*x + e)^2 - 1)) - 2*((8*a^3 + 24*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 14*a^3
+ 29*a^2*b + 15*a*b^2 - (24*a^3 + 53*a^2*b + 30*a*b^2)*cos(f*x + e)^2)*sqrt
(-b*cos(f*x + e)^2 + a + b))/(a^4*b*f*cos(f*x + e)^6 - (a^5 + 3*a^4*b)*f*co
s(f*x + e)^4 + (2*a^5 + 3*a^4*b)*f*cos(f*x + e)^2 - (a^5 + a^4*b)*f), 1/8*(
((8*a^2*b + 24*a*b^2 + 15*b^3)*cos(f*x + e)^6 - (8*a^3 + 48*a^2*b + 87*a*b^
2 + 45*b^3)*cos(f*x + e)^4 - 8*a^3 - 32*a^2*b - 39*a*b^2 - 15*b^3 + (16*a^3
 + 72*a^2*b + 102*a*b^2 + 45*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-a)/a) - ((8*a^3 + 24*a^2*b + 15*a*b^2)*cos(f*x
 + e)^4 + 14*a^3 + 29*a^2*b + 15*a*b^2 - (24*a^3 + 53*a^2*b + 30*a*b^2)*cos
(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^4*b*f*cos(f*x + e)^6 - (a^
5 + 3*a^4*b)*f*cos(f*x + e)^4 + (2*a^5 + 3*a^4*b)*f*cos(f*x + e)^2 - (a^5 +
 a^4*b)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1150 vs. 2(153) = 306.
time = 1.05, size = 1150, normalized size = 6.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*(((((a^8*b + a^7*b^2)*tan(1/2*f*x + 1/2*e)^2/(a^9*b + a^8*b^2) - (11*
a^8*b + 21*a^7*b^2 + 10*a^6*b^3)/(a^9*b + a^8*b^2))*tan(1/2*f*x + 1/2*e)^2
- (89*a^8*b + 297*a^7*b^2 + 328*a^6*b^3 + 120*a^5*b^4)/(a^9*b + a^8*b^2))*t
an(1/2*f*x + 1/2*e)^2 - (77*a^8*b + 219*a^7*b^2 + 206*a^6*b^3 + 64*a^5*b^4)
/(a^9*b + a^8*b^2))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - 8*(8*a^2 + 24*a*b + 15*b^2)*arctan(
-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a
^3) - 4*(8*a^(5/2) + 24*a^(3/2)*b + 15*sqrt(a)*b^2)*log(abs(-(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a^4 + 4*
```

```
(6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2 + 20*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 14*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))^2*a^(3/2)*b - 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*a^3 - 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b
 - 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a
*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^2 - 7*a^(7/2
) - 8*a^(5/2)*b)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 -
 a)^2*a^3))/f
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.527 $\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=292

$$\frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}+\frac{(7a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin}}{3(a+b)^3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] 1/3*(7*a-b)*b*cos(f*x+e)*sin(f*x+e)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*
(7*a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*
(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)-4/3*a*Ellipti
cF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)
^2/a)^(1/2)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-4/3*a*tan(f*x+e)/(a+b)^2/f/(
a+b*sin(f*x+e)^2)^(1/2)+1/3*sec(f*x+e)^2*tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)
^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 481, 541, 538, 437, 435, 432, 430}

$$-\frac{4a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{(7a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f(a+b)^3\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((7*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^3*f*Sqrt[a + b*Sin[e + f
*x]^2]) + ((7*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -
(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^3*f*Sqrt[1 + (b*
Sin[e + f*x]^2)/a]) - (4*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)^2*f*Sq
rt[a + b*Sin[e + f*x]^2]) - (4*a*Tan[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b*Si
n[e + f*x]^2]) + (Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*f*Sqrt[a + b*Sin[
e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^{5/2}(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{}{(1} \right.}{3(a+b)}$$

$$= -\frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{c\right.}}{}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{se}}{3(a}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{se}}{3(a}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{se}}{3(a}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+\right.}{3(a+b)^3f\sqrt{1}}$$

Mathematica [A]

time = 1.60, size = 197, normalized size = 0.67

$$\frac{2a(7a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-8a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-\frac{(8a^2-21ab-5b^2+4(4a^2-3ab+b^2)\cos(2(e+fx))+b(-7a+b)\cos(4(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2\sqrt{2}}}{6(a+b)^3f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*a*(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*
x, -(b/a)] - ((8*a^2 - 21*a*b - 5*b^2 + 4*(4*a^2 - 3*a*b + b^2)*Cos[2*(e +
f*x)] + b*(-7*a + b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqrt
[2]))/(6*(a + b)^3*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 18.77, size = 368, normalized size = 1.26

method	result
default	$-\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\,b(7a-b)\sin(fx+e)(\cos^4(fx+e))-4\sqrt{-b\left(\cos^4\left(fx+e\right)\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(7*a-b)*sin(f*x+e)*cos(f
*x+e)^4-4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*cos(f*x+e)^2*s
in(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*
x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^
4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(1+sin(f*x+e))/(-
(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(a+b
)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains complex when optimal does not.
time = 0.23, size = 1228, normalized size = 4.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*((7*I*a*b^3 - I*b^4)*cos(f*x + e)^5 + (-7*I*a^2*b^2 - 6*I*a*b^3 + I
*b^4)*cos(f*x + e)^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-14*I*a^2*b^2 - 5*
I*a*b^3 + I*b^4)*cos(f*x + e)^5 + (14*I*a^3*b + 19*I*a^2*b^2 + 4*I*a*b^3 -
I*b^4)*cos(f*x + e)^3)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*
x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2
 + a*b)/b^2))/b^2) + (2*((-7*I*a*b^3 + I*b^4)*cos(f*x + e)^5 + (7*I*a^2*b^2
 + 6*I*a*b^3 - I*b^4)*cos(f*x + e)^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((14
*I*a^2*b^2 + 5*I*a*b^3 - I*b^4)*cos(f*x + e)^5 + (-14*I*a^3*b - 19*I*a^2*b^
2 - 4*I*a*b^3 + I*b^4)*cos(f*x + e)^3)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b
+ b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*((3*I*a^2*b^2 + 2*I*a*b^3 - I*b^4
)*cos(f*x + e)^5 + (-3*I*a^3*b - 5*I*a^2*b^2 - I*a*b^3 + I*b^4)*cos(f*x + e
)^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((-6*I*a^3*b + 7*I*a^2*b^2 + 5*I*a*b^
3)*cos(f*x + e)^5 + (6*I*a^4 - I*a^3*b - 12*I*a^2*b^2 - 5*I*a*b^3)*cos(f*x
+ e)^3)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(
arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(
f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b
^2) - 2*(2*((-3*I*a^2*b^2 - 2*I*a*b^3 + I*b^4)*cos(f*x + e)^5 + (3*I*a^3*b
+ 5*I*a^2*b^2 + I*a*b^3 - I*b^4)*cos(f*x + e)^3)*sqrt(-b)*sqrt((a^2 + a*b)/
b^2) + ((6*I*a^3*b - 7*I*a^2*b^2 - 5*I*a*b^3)*cos(f*x + e)^5 + (-6*I*a^4 +
I*a^3*b + 12*I*a^2*b^2 + 5*I*a*b^3)*cos(f*x + e)^3)*sqrt(-b))*sqrt((2*b*sqr
t((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^
2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*((7*a*b^3 - b^4)*cos(f*
x + e)^4 + a^2*b^2 + 2*a*b^3 + b^4 - 4*(a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^3*b^3 + 3*a^2*b^4 + 3*a*b^5
 + b^6)*f*cos(f*x + e)^5 - (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6
)*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral(tan(e + f*x)**4/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(e+f\,x)^4}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.528 $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+f}}{(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] -2*b*cos(f*x+e)*sin(f*x+e)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-2*EllipticE(s
in(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^
(1/2)/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+EllipticF(sin(f*x+e),(-b/a)^(1/2
))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/(a+b)/f/(a+b*
sin(f*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 482, 541, 538, 437, 435, 432, 430}

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2b\sin(e+fx)\cos(e+fx)}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*b*Cos[e + f*x]*Sin[e + f*x])/((a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) -
 (2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
 + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) +
 Tan[e + f*x]/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
```

```
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^2}{(1-x^2)^{3/2}(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{}}\right.}{(a+b)f}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2\sqrt{\cos\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2\sqrt{\cos\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\right|-}{(a+b)^2f\sqrt{1+\frac{b}{}}}$$

Mathematica [A]
time = 0.62, size = 145, normalized size = 0.65

$$\frac{-2\sqrt{2}\,a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+2(a-b\cos(2(e+fx)))\tan(e+fx)}{\sqrt{2}\,(a+b)^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)] + Sqrt[2]*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
```

```
f*x, -(b/a)] + 2*(a - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(Sqrt[2]*(a + b)^2
*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 18.81, size = 278, normalized size = 1.24

method	result
default	$\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\,\left(a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\ \text{Elliptic}\right.}{(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))+b*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))-2*a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE
(sin(f*x+e),(-1/a*b)^(1/2))-2*sin(f*x+e)*cos(f*x+e)^2*b+a*sin(f*x+e)+b*sin(
f*x+e))/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/c
os(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.20, size = 1048, normalized size = 4.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((2*(-I*b^3*cos(f*x + e)^3 + (I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b)*sqrt(
(a^2 + a*b)/b^2) - ((2*I*a*b^2 + I*b^3)*cos(f*x + e)^3 + (-2*I*a^2*b - 3*I*
a*b^2 - I*b^3)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*
a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(
cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqr
```

```
t((a^2 + a*b)/b^2))/b^2) + (2*(I*b^3*cos(f*x + e)^3 + (-I*a*b^2 - I*b^3)*co
s(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a*b^2 - I*b^3)*cos(f*x
+ e)^3 + (2*I*a^2*b + 3*I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 +
 a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b +
 b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^2 + I*b^3)*
cos(f*x + e)^3 + (-I*a^2*b - 2*I*a*b^2 - I*b^3)*cos(f*x + e))*sqrt(-b)*sqrt
((a^2 + a*b)/b^2) - ((2*I*a^2*b - I*a*b^2 - I*b^3)*cos(f*x + e)^3 + (-2*I*a
^3 - I*a^2*b + 2*I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((-I*a*b^2 - I*b^3)*cos(f*
x + e)^3 + (I*a^2*b + 2*I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 +
 a*b)/b^2) - ((-2*I*a^2*b + I*a*b^2 + I*b^3)*cos(f*x + e)^3 + (2*I*a^3 + I*
a^2*b - 2*I*a*b^2 - I*b^3)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2
*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*
b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*b^3*cos(f*x + e)^2 - a*b^2 - b^3)
*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^2*b^3 + 2*a*b^4 + b^5)*f
*cos(f*x + e)^3 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral(tan(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(e+f\,x)^2}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.529 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(cos(f*x+e)^2)^(
1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)
/a/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3263, 21, 3257, 3256}

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
```

```
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a-b\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)} \\
&= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{a(a+b)} \\
&= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{a(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\
&= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 0.89

$$\frac{2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b\sin(2(e+fx))}{2a(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]

[Out] (2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sq
rt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)
```

Maple [A]
time = 10.73, size = 103, normalized size = 1.02

method	result	size
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+\sin(fx+e)(\cos^2(fx+e))b}{a(a+b)\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))+sin(f*x+e)*cos(f*x+e)^2*b)/a/(a+b)/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.17, size = 938, normalized size = 9.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^3*cos(f*x + e)*sin(f*x + e) - (2*
(I*b^3*cos(f*x + e)^2 - I*a*b^2 - I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (
2*I*a^2*b + 3*I*a*b^2 + I*b^3 + (-2*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqrt(-
b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8
*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(-I*b
^3*cos(f*x + e)^2 + I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-2*I
*a^2*b - 3*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b))*
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2
 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(2*(-I*a^2
*b - 2*I*a*b^2 - I*b^3 + (I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a
```

```
^2 + a*b)/b^2) + (2*I*a^3 + 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b - I*a*b^2)*co
s(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellip
tic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) +
I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b
^2))/b^2) + 2*(2*(I*a^2*b + 2*I*a*b^2 + I*b^3 + (-I*a*b^2 - I*b^3)*cos(f*x
+ e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + (-2*I*a^3 - 3*I*a^2*b - I*a*b^2 +
(2*I*a^2*b + I*a*b^2)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
+ b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
 b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/((a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 - (a
^3*b^2 + 2*a^2*b^3 + a*b^4)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*sin(e + f*x)**2)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(1/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.530 $\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f}-\frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\,\mathrm{s}}{a^2f\sqrt{1+\frac{b\sin^2(e-}{a}}}$$

```
[Out] cot(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)-2*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/
2)/a^2/f-2*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/
2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+EllipticF(sin(
f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(
1/2)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 480, 597, 538, 437, 435, 432, 430}

$$-\frac{2\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\mathrm{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f}+\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\mathrm{ArcSin}(\sin(e+fx))\,|-\frac{b}{a}\right)}{af\sqrt{a+b\sin^2(e+fx)}}+\frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] Cot[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2]) - (2*Cot[e + f*x]*Sqrt[a + b*
Sin[e + f*x]^2])/(a^2*f) - (2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e +
 f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a^2*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]]
, -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(a*f*Sqrt[a + b*Sin[
e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{}{x^2\sqrt{1-}}\right.}{af}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} + \frac{\left(\sqrt{\cos^2(e\right.}{}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} - \frac{\left(2\sqrt{\cos^2(e\right.}{}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} - \frac{\left(2\sqrt{\cos^2(e\right.}{}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} - \frac{2\sqrt{\cos^2(e}}{}$$

Mathematica [A]
time = 0.53, size = 142, normalized size = 0.68

$$\frac{-2(a+b-b\cos(2(e+fx)))\cot(e+fx)-2\sqrt{2}\,a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{\sqrt{2}\,a^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

[Out] (-2*(a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x] - 2*Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(Sqrt[2]*a^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])

Maple [A]
time = 9.34, size = 141, normalized size = 0.67

method	result
default	$\frac{\sin(fx+e)\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,a\left(\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-2\,\mathrm{EllipticE}\left(\sin(fx+e),\sqrt{-\right.\right.}{\sin(fx+e)a^2\cos(fx+e)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))+2*b*cos(f*x+e)^4+(-a-2*b)*cos(f*x+e)^2)/sin(f*x+e)/a^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)

Fricas [C] Result contains complex when optimal does not.
time = 0.16, size = 996, normalized size = 4.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] ((2*(-I*b^3*cos(f*x + e)^2 + I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (-2*I*a^2*b - 3*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*b^3*cos(f*x + e)^2 - I*a*b^2 - I*b^3)*sqrt(-b)

```
*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (2*I*a^2*b + 3*I*a*b^2 + I*b^3 + (-2*
I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2
 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2
) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*
(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-I*a^2*b - 3*I*a*b^2 - 2*I*
b^3 + (I*a*b^2 + 2*I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*si
n(f*x + e) - (-2*I*a^3 - 3*I*a^2*b - I*a*b^2 + (2*I*a^2*b + I*a*b^2)*cos(f*
x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*
x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2
 + a*b)/b^2))/b^2) + (2*(I*a^2*b + 3*I*a*b^2 + 2*I*b^3 + (-I*a*b^2 - 2*I*b^
3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (2*I*a^3 +
 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b - I*a*b^2)*cos(f*x + e)^2)*sqrt(-b)*sin(
f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(s
qrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e)
)), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2
*b^3*cos(f*x + e)^3 - (a*b^2 + 2*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2
+ a + b))/((a^2*b^3*f*cos(f*x + e)^2 - (a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(e+f\,x)^2}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.531 $\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=297

$$\frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}+\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f}-\frac{(3a+4b)\cot(e+fx)\csc^2(e+\cdots}{3a^2b\cdots}$$

```
[Out] (a+b)*cot(f*x+e)*csc(f*x+e)^2/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(7*a+8*b)*
cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/f-1/3*(3*a+4*b)*cot(f*x+e)*csc(f*x+
e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a^2/b/f+1/3*(7*a+8*b)*EllipticE(sin(f*x+e),(-
b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/f/
(1+b*sin(f*x+e)^2/a)^(1/2)-4/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec
(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a^2/f/(a+b*sin(f*x+
e)^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 479, 597, 538, 437, 435, 432, 430}

$$\frac{(7a+8b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f}-\frac{4(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{(3a+4b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2bf}+\frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*Cot[e + f*x]*Csc[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((7*a + 8*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*f) - ((3*a + 4
*b)*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b*f) + (
(7*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)
/a]) - (4*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/
a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a^2*f*Sqrt[a + b*Sin[e
+ f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subst}\left(\int\right.}{ab}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{(3a+4b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+}}{3a^2bf}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx}}{3a^3f}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx}}{3a^3f}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx}}{3a^3f}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx}}{3a^3f}$$

Mathematica [A]
time = 2.62, size = 199, normalized size = 0.67

$$\frac{\frac{(8a^2+37ab+24b^2-4(4a^2+11ab+8b^2)\cos(2(e+fx))+b(7a+8b)\cos(4(e+fx)))\cot(e+fx)\csc^2(e+fx)}{2\sqrt{2}}+2a(7a+8b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-8a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)}{6a^3f\sqrt{2a+b-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (((8*a^2 + 37*a*b + 24*b^2 - 4*(4*a^2 + 11*a*b + 8*b^2)*Cos[2*(e + f*x)] +
b*(7*a + 8*b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/(2*Sqrt[2]) +
2*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)])/(6*a^3*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A]
time = 9.99, size = 353, normalized size = 1.19

method	result
default	$-\frac{4\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2(\sin^3(fx+e))+4b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+4*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-7*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3*EllipticE(sin
(f*x+e),(-1/a*b)^(1/2))-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)
*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+7*a*b*sin(f*x+e)^6+8
*b^2*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4-8*b^2*sin(f*x+e)^4-
5*a^2*sin(f*x+e)^2-4*a*b*sin(f*x+e)^2+a^2)/a^3/sin(f*x+e)^3/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains complex when optimal does not.
time = 0.20, size = 1465, normalized size = 4.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*((7*I*a*b^3 + 8*I*b^4)*cos(f*x + e)^4 + 7*I*a^2*b^2 + 15*I*a*b^3 +
8*I*b^4 + (-7*I*a^2*b^2 - 22*I*a*b^3 - 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*s
qrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((-14*I*a^2*b^2 - 23*I*a*b^3 - 8*I*b^4)
*cos(f*x + e)^4 - 14*I*a^3*b - 37*I*a^2*b^2 - 31*I*a*b^3 - 8*I*b^4 + (14*I*
a^3*b + 51*I*a^2*b^2 + 54*I*a*b^3 + 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sin(
f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(s
qrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e)
)), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2
*((-7*I*a*b^3 - 8*I*b^4)*cos(f*x + e)^4 - 7*I*a^2*b^2 - 15*I*a*b^3 - 8*I*b^
4 + (7*I*a^2*b^2 + 22*I*a*b^3 + 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^
2 + a*b)/b^2)*sin(f*x + e) - ((14*I*a^2*b^2 + 23*I*a*b^3 + 8*I*b^4)*cos(f*x
 + e)^4 + 14*I*a^3*b + 37*I*a^2*b^2 + 31*I*a*b^3 + 8*I*b^4 + (-14*I*a^3*b -
 51*I*a^2*b^2 - 54*I*a*b^3 - 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e
))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*
a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*((3*
I*a^2*b^2 + 11*I*a*b^3 + 8*I*b^4)*cos(f*x + e)^4 + 3*I*a^3*b + 14*I*a^2*b^2
 + 19*I*a*b^3 + 8*I*b^4 + (-3*I*a^3*b - 17*I*a^2*b^2 - 30*I*a*b^3 - 16*I*b^
4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + ((-6*I*a^3
*b - 11*I*a^2*b^2 - 4*I*a*b^3)*cos(f*x + e)^4 - 6*I*a^4 - 17*I*a^3*b - 15*I
*a^2*b^2 - 4*I*a*b^3 + (6*I*a^4 + 23*I*a^3*b + 26*I*a^2*b^2 + 8*I*a*b^3)*co
s(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
+ b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(co
s(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt(
(a^2 + a*b)/b^2))/b^2) - 2*(2*((-3*I*a^2*b^2 - 11*I*a*b^3 - 8*I*b^4)*cos(f*
x + e)^4 - 3*I*a^3*b - 14*I*a^2*b^2 - 19*I*a*b^3 - 8*I*b^4 + (3*I*a^3*b + 1
7*I*a^2*b^2 + 30*I*a*b^3 + 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a
*b)/b^2)*sin(f*x + e) + ((6*I*a^3*b + 11*I*a^2*b^2 + 4*I*a*b^3)*cos(f*x + e
)^4 + 6*I*a^4 + 17*I*a^3*b + 15*I*a^2*b^2 + 4*I*a*b^3 + (-6*I*a^4 - 23*I*a^
3*b - 26*I*a^2*b^2 - 8*I*a*b^3)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt
((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8
*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*((7*a*b^3 + 8*
b^4)*cos(f*x + e)^5 - 2*(2*a^2*b^2 + 9*a*b^3 + 8*b^4)*cos(f*x + e)^3 + (3*a
^2*b^2 + 11*a*b^3 + 8*b^4)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(
(a^3*b^3*f*cos(f*x + e)^4 - (a^4*b^2 + 2*a^3*b^3)*f*cos(f*x + e)^2 + (a^4*b
^2 + a^3*b^3)*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cot(e+f\,x)}^4}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.532 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=218

$$\frac{(8a^2-24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f}-\frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(8a+b)\,\mathrm{se}}{8(a+b)^2f\,(a+b}$$

```
[Out] 1/8*(8*a^2-24*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b
)^(9/2)/f+1/24*(-8*a^2+24*a*b-3*b^2)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(3/2)-1/8
*(8*a+b)*sec(f*x+e)^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)+1/4*sec(f*x+e)^4/(
a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+1/8*(-8*a^2+24*a*b-3*b^2)/(a+b)^4/f/(a+b*si
n(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 53, 65, 214}

$$-\frac{8a^2-24ab+3b^2}{8f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}-\frac{8a^2-24ab+3b^2}{24f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{(8a^2-24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}+\frac{\sec^4(e+fx)}{4f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(8a+b)\sec^2(e+fx)}{8f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/
(8*(a + b)^(9/2)*f) - (8*a^2 - 24*a*b + 3*b^2)/(24*(a + b)^3*f*(a + b*Sin[e
 + f*x]^2)^(3/2)) - ((8*a + b)*Sec[e + f*x]^2)/(8*(a + b)^2*f*(a + b*Sin[e
+ f*x]^2)^(3/2)) + Sec[e + f*x]^4/(4*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)
) - (8*a^2 - 24*a*b + 3*b^2)/(8*(a + b)^4*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)}{4(a+b)f\,(a+b\sin^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a-3b)+2(a+b)x}{(1-x)^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+f\right.}{4(a+b)f}$$

$$= -\frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} + \frac{\sec^4(e+fx)}{4(a+b)f\,(a+b\sin^2(e+fx))^{3/2}} + \frac{(8}{}$$

$$= -\frac{8a^2-24ab+3b^2}{24(a+b)^3f\,(a+b\sin^2(e+fx))^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} +$$

$$= -\frac{8a^2-24ab+3b^2}{24(a+b)^3f\,(a+b\sin^2(e+fx))^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} +$$

$$= -\frac{8a^2-24ab+3b^2}{24(a+b)^3f\,(a+b\sin^2(e+fx))^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} +$$

$$= \frac{(8a^2-24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} - \frac{8a^2-24ab}{24(a+b)^3f\,(a+b\sin}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.34, size = 107, normalized size = 0.49

$$\frac{(-8a^2+24ab-3b^2)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin^2(e+fx)}{a+b}\right) - \frac{3}{2}(a+b)(4a-3b+(8a+b)\cos(2(e+fx)))\sec^4(e+fx)}{24(a+b)^3f\,(a+b\sin^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-8*a^2 + 24*a*b - 3*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[e +
f*x]^2)/(a + b)] - (3*(a + b)*(4*a - 3*b + (8*a + b)*Cos[2*(e + f*x)])*Sec[
e + f*x]^4)/2)/(24*(a + b)^3*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1908 vs. 2(194) = 388.
time = 63.34, size = 1909, normalized size = 8.76

method	result	size
default	Expression too large to display	1909

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*b^4*a*(a-2*b)/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(-a*b)^(1/2)/(si
n(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/2*b^4*a*(a-2
*b)/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^
(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/12*b^2*(-a*b)^(1/2)/(b+(-a*b
)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2
+(a*b+b^2)/b)^(1/2)-1/12*b^2*a/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(
f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12*b^2*(-a*b
)^(1/2)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*x+e)-(-a*b)^(1/2)/b)*
(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/2*b^4*a^2/(b+(-a*b)^(1/2))^4/(-b+(-a*
b)^(1/2))^4/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*si
n(f*x+e)+2*a)/(sin(f*x+e)-1))-b^5*a/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/
(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a
)/(sin(f*x+e)-1))-1/12*b^2*a/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*
x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/2*b^4*a^2/(b+(
-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))-b^5*a/(b+(-a*b)^(1/2))
^4/(-b+(-a*b)^(1/2))^4/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))-1/16*b^4/(b+(-a*b)^(1/2))^3/(-b+(-
a*b)^(1/2))^3/(a+b)/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)+7/16*b^4/(b+(
-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))+1/16*b^2/(b+(-a*b)^(
1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(sin(f*x+e)-1)^2*(a+b-b*cos(f*x+e)^2)^(1/
2)-3/16*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^2/(sin(f*x+e)-1)*(
a+b-b*cos(f*x+e)^2)^(1/2)+1/16*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(
a+b)/(1+sin(f*x+e))^2*(a+b-b*cos(f*x+e)^2)^(1/2)+3/16*b^3/(b+(-a*b)^(1/2))^
2/(-b+(-a*b)^(1/2))^2/(a+b)^2/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)+7/1
6*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(a+b)^(3/2)*ln((2*(a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1))+1/16*b^4/(
b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)/(sin(f*x+e)-1)*(a+b-b*cos(f*x+e
)^2)^(1/2)-7/16*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(a+b)/(sin(f*x
+e)-1)*(a+b-b*cos(f*x+e)^2)^(1/2)+7/16*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1
/2))^3*a/(a+b)/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)+3/16*b^4/(b+(-a*b)
^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^(5/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1))-1/16*b^3/(b+(-a*b)^(1/2))^2
/(-b+(-a*b)^(1/2))^2/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1))+3/16*b^4/(b+(-a*b)^(1/2))^2/(-b+(-a*
b)^(1/2))^2/(a+b)^(5/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*si
```

```
n(f*x+e)+2*a)/(1+sin(f*x+e)))-1/16*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))
^2/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+
2*a)/(1+sin(f*x+e)))-1/16*b^5/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^
(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin
(f*x+e)-1))-1/16*b^5/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(3/2)*ln(
(2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)
)))/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(201) = 402$.
time = 0.56, size = 432, normalized size = 1.98

$$-\frac{\frac{3\left(8a^2b^3-24ab^4+3b^5\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{a+b}}+\frac{2\left(8a^5b^3+24a^4b^4+24a^3b^5+8a^2b^6+3\left(8a^2b^3-24ab^4+3b^5\right)\left(b\sin(fx+e)^2+a\right)^3-5\left(8a^3b^3-16a^2b^4-21ab^5+3b^6\right)\left(b\sin(fx+e)^2+a\right)^2+8\left(a^4b^3-4a^3b^4-11a^2b^5-6ab^6\right)\left(b\sin(fx+e)^2+a\right)\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\left(b\sin(fx+e)^2+a\right)^{\frac{7}{2}}-2\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}+\left(a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6\right)\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}}{48b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(8*a^2*b^3 - 24*a*b^4 + 3*b^5)*log((sqrt(b*sin(f*x + e)^2 + a) - s
qrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a + b)) + 2*(8*a^5*b^3 + 24*a^4*b^4 + 24*a^3
*b^5 + 8*a^2*b^6 + 3*(8*a^2*b^3 - 24*a*b^4 + 3*b^5)*(b*sin(f*x + e)^2 + a)^
3 - 5*(8*a^3*b^3 - 16*a^2*b^4 - 21*a*b^5 + 3*b^6)*(b*sin(f*x + e)^2 + a)^2
+ 8*(a^4*b^3 - 4*a^3*b^4 - 11*a^2*b^5 - 6*a*b^6)*(b*sin(f*x + e)^2 + a))/((
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*sin(f*x + e)^2 + a)^(7/2) - 2
*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(b*sin(f*x + e)^
2 + a)^(5/2) + (a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*
b^5 + b^6)*(b*sin(f*x + e)^2 + a)^(3/2)))/(b^3*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(194) = 388$.
time = 0.64, size = 995, normalized size = 4.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2*b^2 - 24*a*b^3 + 3*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b - 16*a
^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 + (8*a^4 - 8*a^3*b - 37*a^2*b^2 -
 18*a*b^3 + 3*b^4)*cos(f*x + e)^4)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*
(3*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 - 4*(8*a^4 - 8*
a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4 + 6*a^4 + 24*a^3*b +
36*a^2*b^2 + 24*a*b^3 + 6*b^4 - 3*(8*a^4 + 25*a^3*b + 27*a^2*b^2 + 11*a*b^3
```

```
 + b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2 + 5*a^4*
b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*cos(f*x + e)^8 - 2*(a^6*b
+ 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*f*cos(f
*x + e)^6 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*
b^5 + 7*a*b^6 + b^7)*f*cos(f*x + e)^4), -1/24*(3*((8*a^2*b^2 - 24*a*b^3 + 3
*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x
+ e)^6 + (8*a^4 - 8*a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4)*
sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) -
 (3*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 - 4*(8*a^4 - 8
*a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4 + 6*a^4 + 24*a^3*b +
 36*a^2*b^2 + 24*a*b^3 + 6*b^4 - 3*(8*a^4 + 25*a^3*b + 27*a^2*b^2 + 11*a*b^
3 + b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2 + 5*a^4
*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*cos(f*x + e)^8 - 2*(a^6*b
 + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*f*cos(
f*x + e)^6 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2
*b^5 + 7*a*b^6 + b^7)*f*cos(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3826 vs. $2(201) = 402$.
time = 3.09, size = 3826, normalized size = 17.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(3*(8*a^2 - 24*a*b + 3*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*sqrt(-a - b)) + 4*((((4*a^17*b^2 + 51*a^16*b^3 + 294*a^15
*b^4 + 1001*a^14*b^5 + 2184*a^13*b^6 + 3003*a^12*b^7 + 2002*a^11*b^8 - 1287
*a^10*b^9 - 5148*a^9*b^10 - 7007*a^8*b^11 - 6006*a^7*b^12 - 3549*a^6*b^13 -
 1456*a^5*b^14 - 399*a^4*b^15 - 66*a^3*b^16 - 5*a^2*b^17)*tan(1/2*f*x + 1/2
*e)^2/(a^18*b^2 + 18*a^17*b^3 + 153*a^16*b^4 + 816*a^15*b^5 + 3060*a^14*b^6
```

+ 8568*a^13*b^7 + 18564*a^12*b^8 + 31824*a^11*b^9 + 43758*a^10*b^10 + 4862
0*a^9*b^11 + 43758*a^8*b^12 + 31824*a^7*b^13 + 18564*a^6*b^14 + 8568*a^5*b^
15 + 3060*a^4*b^16 + 816*a^3*b^17 + 153*a^2*b^18 + 18*a*b^19 + b^20) + 3*(4
*a^17*b^2 + 55*a^16*b^3 + 342*a^15*b^4 + 1253*a^14*b^5 + 2912*a^13*b^6 + 40
95*a^12*b^7 + 2002*a^11*b^8 - 5291*a^10*b^9 - 15444*a^9*b^10 - 22451*a^8*b^
11 - 22022*a^7*b^12 - 15561*a^6*b^13 - 8008*a^5*b^14 - 2947*a^4*b^15 - 738*
a^3*b^16 - 113*a^2*b^17 - 8*a*b^18)/(a^18*b^2 + 18*a^17*b^3 + 153*a^16*b^4
+ 816*a^15*b^5 + 3060*a^14*b^6 + 8568*a^13*b^7 + 18564*a^12*b^8 + 31824*a^1
1*b^9 + 43758*a^10*b^10 + 48620*a^9*b^11 + 43758*a^8*b^12 + 31824*a^7*b^13
+ 18564*a^6*b^14 + 8568*a^5*b^15 + 3060*a^4*b^16 + 816*a^3*b^17 + 153*a^2*b
^18 + 18*a*b^19 + b^20))*tan(1/2*f*x + 1/2*e)^2 + 3*(4*a^17*b^2 + 55*a^16*b
^3 + 342*a^15*b^4 + 1253*a^14*b^5 + 2912*a^13*b^6 + 4095*a^12*b^7 + 2002*a^
11*b^8 - 5291*a^10*b^9 - 15444*a^9*b^10 - 22451*a^8*b^11 - 22022*a^7*b^12 -
15561*a^6*b^13 - 8008*a^5*b^14 - 2947*a^4*b^15 - 738*a^3*b^16 - 113*a^2*b^
17 - 8*a*b^18)/(a^18*b^2 + 18*a^17*b^3 + 153*a^16*b^4 + 816*a^15*b^5 + 3060
*a^14*b^6 + 8568*a^13*b^7 + 18564*a^12*b^8 + 31824*a^11*b^9 + 43758*a^10*b^
10 + 48620*a^9*b^11 + 43758*a^8*b^12 + 31824*a^7*b^13 + 18564*a^6*b^14 + 85
68*a^5*b^15 + 3060*a^4*b^16 + 816*a^3*b^17 + 153*a^2*b^18 + 18*a*b^19 + b^2
0))*tan(1/2*f*x + 1/2*e)^2 + (4*a^17*b^2 + 51*a^16*b^3 + 294*a^15*b^4 + 100
1*a^14*b^5 + 2184*a^13*b^6 + 3003*a^12*b^7 + 2002*a^11*b^8 - 1287*a^10*b^9
- 5148*a^9*b^10 - 7007*a^8*b^11 - 6006*a^7*b^12 - 3549*a^6*b^13 - 1456*a^5*
b^14 - 399*a^4*b^15 - 66*a^3*b^16 - 5*a^2*b^17)/(a^18*b^2 + 18*a^17*b^3 + 1
53*a^16*b^4 + 816*a^15*b^5 + 3060*a^14*b^6 + 8568*a^13*b^7 + 18564*a^12*b^8
+ 31824*a^11*b^9 + 43758*a^10*b^10 + 48620*a^9*b^11 + 43758*a^8*b^12 + 318
24*a^7*b^13 + 18564*a^6*b^14 + 8568*a^5*b^15 + 3060*a^4*b^16 + 816*a^3*b^17
+ 153*a^2*b^18 + 18*a*b^19 + b^20))/(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) - 6*(8*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 - 8*(sqrt(a)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))^7*a*b - 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
+ a))^6*a^(5/2) - 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6
*a^(3/2)*b - 29*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt
(a)*b^2 - 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 -
296*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b + 43*(sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 12*(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e

```
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^3 + 136*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) - 168*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b - 629*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 + 60*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^4*sqrt(a)*b^3 + 344*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))^3*a^4 + 872*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))^3*a^3*b - 151*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1...
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

3.533 $\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f}+\frac{2a-3b}{6(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}}+\frac{\sec^2(e+fx)}{2(a+b)f\,(a+b\sin^2(e+fx))^{3/2}}$$

```
[Out] -1/2*(2*a-3*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f+
1/6*(2*a-3*b)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)+1/2*sec(f*x+e)^2/(a+b)/f/(
a+b*sin(f*x+e)^2)^(3/2)+1/2*(2*a-3*b)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 53, 65, 214}

$$\frac{2a-3b}{2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{2a-3b}{6f(a+b)^2\,(a+b\sin^2(e+fx))^{3/2}}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}+\frac{\sec^2(e+fx)}{2f(a+b)\,(a+b\sin^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/2*((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/((a + b)
^(7/2)*f) + (2*a - 3*b)/(6*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) + Sec[
e + f*x]^2/(2*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*a - 3*b)/(2*(a +
 b)^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a-3b)\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{5/2}}\,dx, x, \sin^2\right.}{4(a+b)f}$$

$$= \frac{2a-3b}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a}{}$$

$$= \frac{2a-3b}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{2(a}$$

$$= \frac{2a-3b}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{2(a}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} + \frac{2a-3b}{6(a+b)^2f\left(a+b\sin^2(e+f\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 76, normalized size = 0.50

$$\frac{(2a-3b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin^2(e+fx)}{a+b}\right)+3(a+b)\sec^2(e+fx)}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a - 3*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[e + f*x]^2)/(a + b
)] + 3*(a + b)*Sec[e + f*x]^2)/(6*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(133) = 266.
time = 47.82, size = 1065, normalized size = 6.96

method	result

default	$\frac{\frac{b^3(a-b)\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab+b^2}{b}}}{2\left(b+\sqrt{-ab}\right)^3\left(-b+\sqrt{-ab}\right)^3\sqrt{-ab}\left(\sin(fx+e)+\frac{\sqrt{-ab}}{b}\right)}-\frac{b\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab+b^2}{b}}}{12\left(b+\sqrt{-ab}\right)^2\left(-b+\sqrt{-ab}\right)^2\left(\sin(fx+e)-\frac{\sqrt{-ab}}{b}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*b^3*(a-b)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(-a*b)^(1/2)/(sin(f*x
+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12*b/(b+(-a*b)^(1
/2))^2/(-b+(-a*b)^(1/2))^2/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(
a*b+b^2)/b)^(1/2)-1/12*b*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^
2/a/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12*b/
(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*co
s(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/12*b*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^2/(-b+(-
a*b)^(1/2))^2/a/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(
1/2)-1/4*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(sin(f*x+e)-1)*(a
+b-b*cos(f*x+e)^2)^(1/2)+1/4*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+
b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(
sin(f*x+e)-1))+1/4*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(1+sin(
f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)+1/4*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1
/2))^2/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x
+e)+2*a)/(1+sin(f*x+e)))-1/2*b^3*(a-b)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))
^3/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(
1/2)+1/2*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1))*a-1/2
*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1))+1/2*b^3/(b+(-
a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))*a-1/2*b^4/(b+(-a*b)^(1/
2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e))))/f
```

<u>Maxima</u> [A]

time = 0.59, size = 268, normalized size = 1.75

$$\frac{\frac{3\left(2ab^2-3b^3\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{a+b}}-\frac{2\left(2a^3b^2+4a^2b^3+2ab^4-3\left(2ab^2-3b^3\right)\left(b\sin(fx+e)^2+a\right)^2+2\left(2a^2b^2-ab^3-3b^4\right)\left(b\sin(fx+e)^2+a\right)\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}-\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}}{12b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*a*b^2 - 3*b^3)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a + b))/(s
qrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
sqrt(a + b)) - 2*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 - 3*(2*a*b^2 - 3*b^3)*(b*
sin(f*x + e)^2 + a)^2 + 2*(2*a^2*b^2 - a*b^3 - 3*b^4)*(b*sin(f*x + e)^2 + a
))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*sin(f*x + e)^2 + a)^(5/2) - (a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*sin(f*x + e)^2 + a)^(3/2)))/(b^2*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(133) = 266$.
time = 0.52, size = 769, normalized size = 5.03

$$\left[-\frac{3\,{\left({\left(2\,a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{6} - 2\,{\left(2\,a^{2} b - a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{4} + {\left(2\,a^{3} + a^{2} b - 4\,a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{2}\right)} \sqrt{a + b} \log\left(\frac{b \cos\left(f x + e\right)^{2} - 2\,\sqrt{-b \cos\left(f x + e\right)^{2} + a + b} \sqrt{a + b} - 2\,a - 2\,b}{\cos\left(f x + e\right)^{2}}\right) + 2\,{\left(3\,{\left(2\,a^{2} b - a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{4} - 3\,a^{3} - 9\,a^{2} b - 9\,a b^{2} - 3\,b^{3} - 4\,{\left(2\,a^{3} + a^{2} b - 4\,a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{2}\right)} \sqrt{-b \cos\left(f x + e\right)^{2} + a + b}}{12\,{\left({\left(a^{4} b^{2} + 4\,a^{3} b^{3} + 6\,a^{2} b^{4} + 4\,a b^{5} + b^{6}\right)} f \cos\left(f x + e\right)^{6} - 2\,{\left(a^{5} b + 5\,a^{4} b^{2} + 10\,a^{3} b^{3} + 10\,a^{2} b^{4} + 5\,a b^{5} + b^{6}\right)} f \cos\left(f x + e\right)^{4} + {\left(a^{6} + 6\,a^{5} b + 15\,a^{4} b^{2} + 20\,a^{3} b^{3} + 15\,a^{2} b^{4} + 6\,a b^{5} + b^{6}\right)} f \cos\left(f x + e\right)^{2}\right)}}, \frac{3\,{\left({\left(2\,a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{6} - 2\,{\left(2\,a^{2} b - a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{4} + {\left(2\,a^{3} + a^{2} b - 4\,a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{2}\right)} \sqrt{-a - b} \arctan\left(\frac{\sqrt{-b \cos\left(f x + e\right)^{2} + a + b} \sqrt{-a - b}}{a + b}\right) - {\left(3\,{\left(2\,a^{2} b - a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{4} - 3\,a^{3} - 9\,a^{2} b - 9\,a b^{2} - 3\,b^{3} - 4\,{\left(2\,a^{3} + a^{2} b - 4\,a b^{2} - 3\,b^{3}\right)} \cos\left(f x + e\right)^{2}\right)} \sqrt{-b \cos\left(f x + e\right)^{2} + a + b}}{6\,{\left({\left(a^{4} b^{2} + 4\,a^{3} b^{3} + 6\,a^{2} b^{4} + 4\,a b^{5} + b^{6}\right)} f \cos\left(f x + e\right)^{6} - 2\,{\left(a^{5} b + 5\,a^{4} b^{2} + 10\,a^{3} b^{3} + 10\,a^{2} b^{4} + 5\,a b^{5} + b^{6}\right)} f \cos\left(f x + e\right)^{4} + {\left(a^{6} + 6\,a^{5} b + 15\,a^{4} b^{2} + 20\,a^{3} b^{3} + 15\,a^{2} b^{4} + 6\,a b^{5} + b^{6}\right)} f \cos\left(f x + e\right)^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a*b^2 - 3*b^3)*cos(f*x + e)^6 - 2*(2*a^2*b - a*b^2 - 3*b^3)*c
os(f*x + e)^4 + (2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(a +
b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) -
2*a - 2*b)/cos(f*x + e)^2) + 2*(3*(2*a^2*b - a*b^2 - 3*b^3)*cos(f*x + e)^4
- 3*a^3 - 9*a^2*b - 9*a*b^2 - 3*b^3 - 4*(2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*c
os(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^4*b^2 + 4*a^3*b^3 + 6*a
^2*b^4 + 4*a*b^5 + b^6)*f*cos(f*x + e)^6 - 2*(a^5*b + 5*a^4*b^2 + 10*a^3*b^
3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*f*cos(f*x + e)^4 + (a^6 + 6*a^5*b + 15*a^4*
b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)*f*cos(f*x + e)^2), 1/6*(3*((
2*a*b^2 - 3*b^3)*cos(f*x + e)^6 - 2*(2*a^2*b - a*b^2 - 3*b^3)*cos(f*x + e)^
4 + (2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(s
qrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - (3*(2*a^2*b - a*b^2
- 3*b^3)*cos(f*x + e)^4 - 3*a^3 - 9*a^2*b - 9*a*b^2 - 3*b^3 - 4*(2*a^3 + a^
2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a
^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*cos(f*x + e)^6 - 2*(a^5*b
 + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*f*cos(f*x + e)^4 +
(a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)*f*co
s(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}{\left(e + f x \right)}}{\left(a + b \sin^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1792 vs. $2(138) = 276$.
time = 1.55, size = 1792, normalized size = 11.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a - 3*b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a) - sqrt(a))/sqrt(-a - b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a -
 b)) + 2*((((2*a^13*b^2 + 21*a^12*b^3 + 99*a^11*b^4 + 275*a^10*b^5 + 495*a^
9*b^6 + 594*a^8*b^7 + 462*a^7*b^8 + 198*a^6*b^9 - 55*a^4*b^11 - 33*a^3*b^12
 - 9*a^2*b^13 - a*b^14)*tan(1/2*f*x + 1/2*e)^2/(a^14*b^2 + 14*a^13*b^3 + 91
*a^12*b^4 + 364*a^11*b^5 + 1001*a^10*b^6 + 2002*a^9*b^7 + 3003*a^8*b^8 + 34
32*a^7*b^9 + 3003*a^6*b^10 + 2002*a^5*b^11 + 1001*a^4*b^12 + 364*a^3*b^13 +
 91*a^2*b^14 + 14*a*b^15 + b^16) + 3*(2*a^13*b^2 + 23*a^12*b^3 + 119*a^11*b
^4 + 363*a^10*b^5 + 715*a^9*b^6 + 924*a^8*b^7 + 726*a^7*b^8 + 198*a^6*b^9 -
 264*a^5*b^10 - 385*a^4*b^11 - 253*a^3*b^12 - 97*a^2*b^13 - 21*a*b^14 - 2*b
^15)/(a^14*b^2 + 14*a^13*b^3 + 91*a^12*b^4 + 364*a^11*b^5 + 1001*a^10*b^6 +
 2002*a^9*b^7 + 3003*a^8*b^8 + 3432*a^7*b^9 + 3003*a^6*b^10 + 2002*a^5*b^11
 + 1001*a^4*b^12 + 364*a^3*b^13 + 91*a^2*b^14 + 14*a*b^15 + b^16))*tan(1/2*
f*x + 1/2*e)^2 + 3*(2*a^13*b^2 + 23*a^12*b^3 + 119*a^11*b^4 + 363*a^10*b^5
+ 715*a^9*b^6 + 924*a^8*b^7 + 726*a^7*b^8 + 198*a^6*b^9 - 264*a^5*b^10 - 38
5*a^4*b^11 - 253*a^3*b^12 - 97*a^2*b^13 - 21*a*b^14 - 2*b^15)/(a^14*b^2 + 1
4*a^13*b^3 + 91*a^12*b^4 + 364*a^11*b^5 + 1001*a^10*b^6 + 2002*a^9*b^7 + 30
03*a^8*b^8 + 3432*a^7*b^9 + 3003*a^6*b^10 + 2002*a^5*b^11 + 1001*a^4*b^12 +
 364*a^3*b^13 + 91*a^2*b^14 + 14*a*b^15 + b^16))*tan(1/2*f*x + 1/2*e)^2 + (
2*a^13*b^2 + 21*a^12*b^3 + 99*a^11*b^4 + 275*a^10*b^5 + 495*a^9*b^6 + 594*a
^8*b^7 + 462*a^7*b^8 + 198*a^6*b^9 - 55*a^4*b^11 - 33*a^3*b^12 - 9*a^2*b^13
 - a*b^14)/(a^14*b^2 + 14*a^13*b^3 + 91*a^12*b^4 + 364*a^11*b^5 + 1001*a^10
*b^6 + 2002*a^9*b^7 + 3003*a^8*b^8 + 3432*a^7*b^9 + 3003*a^6*b^10 + 2002*a^
5*b^11 + 1001*a^4*b^12 + 364*a^3*b^13 + 91*a^2*b^14 + 14*a*b^15 + b^16))/(a
*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a)^(3/2) - 6*(2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))^3*a + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b + 2*(sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) + 5*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b - 2*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
```

```
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))*b^2 - 2*a^(5/2) - 5*a^(3/2)*b - 4*sqrt(a)*b^2)/((a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^2))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^3}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.534 $\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f}-\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{1}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/3/(a+b)/f/(a+
b*sin(f*x+e)^2)^(3/2)-1/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 53, 65, 214}

$$-\frac{1}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{1}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(5/2)*f) - 1/(3*(a
 + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) - 1/((a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \sin^2(e+f
}{2(a+b)f}$$

$$= -\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{1}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\mathrm{Subs}}{}$$

$$= -\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{1}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\mathrm{Subs}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{}{(a+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 56, normalized size = 0.62

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{b\cos^2(e+fx)}{a+b}\right)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*Cos[e + f*x]^2)/(a + b)]/((a +
 b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(79) = 158$.
time = 38.53, size = 895, normalized size = 9.84

method	result
default	$\frac{-8a^3b^3\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{a\,b^2+b^3}{b^2}}\,\sqrt{a+b}-8a^2b^4\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{a\,b^2+b^3}{b^2}}\,\sqrt{a+b}+3a^4b^3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b^3/a^2/(a+b)^(1/2)/(a^2*b^2*cos(f*x+e)^4+2*a*b^3*cos(f*x+e)^4+b^4*cos(
f*x+e)^4-2*a^3*b*cos(f*x+e)^2-6*a^2*b^2*cos(f*x+e)^2-6*a*b^3*cos(f*x+e)^2-2
*b^4*cos(f*x+e)^2+a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(-8*a^3*b^3*(-b*cos(f*
x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)-8*a^2*b^4*(-b*cos(f*x+e)^2+(a*b^2
+b^3)/b^2)^(1/2)*(a+b)^(1/2)+3*a^4*b^3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+3*a^4*b^3*ln(2/(1+sin(f*x+e))*((a+
b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+3*ln(2/(sin(f*x+e)-1)*
((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^5+3*ln(2/(1+
sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^
5+6*a^3*b^4*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*s
in(f*x+e)+a))+6*a^3*b^4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))+3*a^2*b^5*(ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)))*cos(f*x+e)^4-6*cos(f*x+e)^2*a^2*b
^4*(ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e
)+a))*a+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f
*x+e)+a))*b-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)+ln(2/(1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a+ln(2/(1
+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b))/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(82) = 164$.
time = 0.56, size = 214, normalized size = 2.35

$$-\frac{\frac{2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a+\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b}+\frac{6}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a^2+2\sqrt{b\sin\left(fx+e\right)^2+a}\,ab+\sqrt{b\sin\left(fx+e\right)^2+a}\,b^2}+\frac{3\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)+1)}\right)}{(a+b)^{\frac{5}{2}}}-\frac{3\,\mathrm{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}\,(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}\,(\sin(fx+e)-1)}\right)}{(a+b)^{\frac{5}{2}}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*(2/((b*sin(f*x + e)^2 + a)^(3/2)*a + (b*sin(f*x + e)^2 + a)^(3/2)*b) +
 6/(sqrt(b*sin(f*x + e)^2 + a)*a^2 + 2*sqrt(b*sin(f*x + e)^2 + a)*a*b + sqr
t(b*sin(f*x + e)^2 + a)*b^2) + 3*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x
 + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) + 1)))/(a + b)^(5/2) - 3*arcsinh(-
b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e)
- 1)))/(a + b)^(5/2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(79) = 158$.
time = 0.54, size = 521, normalized size = 5.73

$$\left[\frac{3\left(b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}\right)\sqrt{a+b}\log\left(\frac{b\cos(fx+e)^{2}-2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{a+b}-2a-2b}{\cos(fx+e)^{2}}\right)+2\left(3\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}-4\,a^{2}-8\,ab-4\,b^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{6\left(\left(a^{3}b^{2}+3\,a^{2}b^{3}+3\,ab^{4}+b^{5}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{4}b+4\,a^{3}b^{2}+6\,a^{2}b^{3}+4\,ab^{4}+b^{5}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5}\right)f\right)},-\frac{3\left(b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{-a-b}}{a+b}\right)-\left(3\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}-4\,a^{2}-8\,ab-4\,b^{2}\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{3\left(\left(a^{3}b^{2}+3\,a^{2}b^{3}+3\,ab^{4}+b^{5}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{4}b+4\,a^{3}b^{2}+6\,a^{2}b^{3}+4\,ab^{4}+b^{5}\right)f\cos\left(fx+e\right)^{2}+\left(a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*
sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(3*(a*b + b^2)*cos(f*x + e)^2
- 4*a^2 - 8*a*b - 4*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b^2 + 3*a^2
*b^3 + 3*a*b^4 + b^5)*f*cos(f*x + e)^4 - 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
 + 5*a*b^4 + b^5)*f), -1/3*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x +
 e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a +
 b)*sqrt(-a - b)/(a + b)) - (3*(a*b + b^2)*cos(f*x + e)^2 - 4*a^2 - 8*a*b -
 4*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 +
b^5)*f*cos(f*x + e)^4 - 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f
*cos(f*x + e)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. 2(82) = 164.

time = 1.00, size = 842, normalized size = 9.25

$$-\frac{\frac{\left(\left(\left(\frac{(4a^9b^2+33a^8b^3+120a^7b^4+252a^6b^5+336a^5b^6+294a^4b^7+168a^3b^8+60a^2b^9+12ab^{10}+b^{11})\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^{10}b^2+10a^9b^3+45a^8b^4+120a^7b^5+210a^6b^6+252a^5b^7+210a^4b^8+120a^3b^9+45a^2b^{10}+10ab^{11}+b^{12}}+\frac{3(4a^9b^2+37a^8b^3+152a^7b^4+364a^6b^5+560a^5b^6+574a^4b^7+392a^3b^8+172a^2b^9+44ab^{10}+5b^{11})}{a^{10}b^2+10a^9b^3+45a^8b^4+120a^7b^5+210a^6b^6+252a^5b^7+210a^4b^8+120a^3b^9+45a^2b^{10}+10ab^{11}+b^{12}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\frac{3(4a^9b^2+37a^8b^3+152a^7b^4+364a^6b^5+560a^5b^6+574a^4b^7+392a^3b^8+172a^2b^9+44ab^{10}+5b^{11})}{a^{10}b^2+10a^9b^3+45a^8b^4+120a^7b^5+210a^6b^6+252a^5b^7+210a^4b^8+120a^3b^9+45a^2b^{10}+10ab^{11}+b^{12}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\frac{4a^9b^2+33a^8b^3+120a^7b^4+252a^6b^5+336a^5b^6+294a^4b^7+168a^3b^8+60a^2b^9+12ab^{10}+b^{11}}{a^{10}b^2+10a^9b^3+45a^8b^4+120a^7b^5+210a^6b^6+252a^5b^7+210a^4b^8+120a^3b^9+45a^2b^{10}+10ab^{11}+b^{12}}\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a\right)^{\frac{3}{2}}}+\frac{6\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}-\sqrt{a}}{2\sqrt{-a-b}}\right)}{(a^2+2ab+b^2)\sqrt{-a-b}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((((4*a^9*b^2 + 33*a^8*b^3 + 120*a^7*b^4 + 252*a^6*b^5 + 336*a^5*b^6
+ 294*a^4*b^7 + 168*a^3*b^8 + 60*a^2*b^9 + 12*a*b^10 + b^11)*tan(1/2*f*x +
1/2*e)^2/(a^10*b^2 + 10*a^9*b^3 + 45*a^8*b^4 + 120*a^7*b^5 + 210*a^6*b^6 +
252*a^5*b^7 + 210*a^4*b^8 + 120*a^3*b^9 + 45*a^2*b^10 + 10*a*b^11 + b^12) +
 3*(4*a^9*b^2 + 37*a^8*b^3 + 152*a^7*b^4 + 364*a^6*b^5 + 560*a^5*b^6 + 574*
a^4*b^7 + 392*a^3*b^8 + 172*a^2*b^9 + 44*a*b^10 + 5*b^11)/(a^10*b^2 + 10*a^
9*b^3 + 45*a^8*b^4 + 120*a^7*b^5 + 210*a^6*b^6 + 252*a^5*b^7 + 210*a^4*b^8
+ 120*a^3*b^9 + 45*a^2*b^10 + 10*a*b^11 + b^12))*tan(1/2*f*x + 1/2*e)^2 + 3
*(4*a^9*b^2 + 37*a^8*b^3 + 152*a^7*b^4 + 364*a^6*b^5 + 560*a^5*b^6 + 574*a^
4*b^7 + 392*a^3*b^8 + 172*a^2*b^9 + 44*a*b^10 + 5*b^11)/(a^10*b^2 + 10*a^9*
b^3 + 45*a^8*b^4 + 120*a^7*b^5 + 210*a^6*b^6 + 252*a^5*b^7 + 210*a^4*b^8 +
120*a^3*b^9 + 45*a^2*b^10 + 10*a*b^11 + b^12))*tan(1/2*f*x + 1/2*e)^2 + (4*
a^9*b^2 + 33*a^8*b^3 + 120*a^7*b^4 + 252*a^6*b^5 + 336*a^5*b^6 + 294*a^4*b^
7 + 168*a^3*b^8 + 60*a^2*b^9 + 12*a*b^10 + b^11)/(a^10*b^2 + 10*a^9*b^3 + 4
5*a^8*b^4 + 120*a^7*b^5 + 210*a^6*b^6 + 252*a^5*b^7 + 210*a^4*b^8 + 120*a^3
*b^9 + 45*a^2*b^10 + 10*a*b^11 + b^12))/(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan
(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) + 6*arctan(-1/2
*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b
))/((a^2 + 2*a*b + b^2)*sqrt(-a - b)))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.535 $\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sin(f*x+e
)^2)^(3/2)+1/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3273, 53, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
b*Sin[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2af}$$

$$= \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{1}{a^2 f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+}\ldots}\right.}{}$$

$$= \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{1}{a^2 f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\ldots\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{a^2 f\sqrt{\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 49, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\sin^2(e+fx)}{a}\right)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin[e + f*x]^2)/a]/(3*a*f*(a + b*Si
n[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(71) = 142$.
time = 20.12, size = 260, normalized size = 3.13

method	result
default	$\frac{-\frac{7\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab+b^2}{b}}}{12a^2\sqrt{-ab}\left(\sin(fx+e)+\frac{\sqrt{-ab}}{b}\right)}-\frac{\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab+b^2}{b}}}{12a^2b\left(\sin(fx+e)+\frac{\sqrt{-ab}}{b}\right)^2}+\frac{7\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab+b^2}{b}}}{12a^2\sqrt{-ab}\left(\sin(fx+e)-\frac{\sqrt{-ab}}{b}\right)}-\cdots}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-7/12/a^2/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b
^2)/b)^(1/2)-1/12/a^2/b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b
+b^2)/b)^(1/2)+7/12/a^2/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*
x+e)^2+(a*b+b^2)/b)^(1/2)-1/a^(5/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1
/2))/sin(f*x+e))-1/12/a^2/b/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+
(a*b+b^2)/b)^(1/2))/f
```

Maxima [A]
time = 0.34, size = 69, normalized size = 0.83

$$-\frac{\frac{3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}}-\frac{3}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a^2}-\frac{1}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) - 3/(sqrt(b*sin(f*
x + e)^2 + a)*a^2) - 1/((b*sin(f*x + e)^2 + a)^(3/2)*a))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(71) = 142$.
time = 0.50, size = 382, normalized size = 4.60

$$\left[\frac{3\left(b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{a}\,\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}-2\,a-b\right)}{\cos(fx+e)^2-1}\right)-2\left(3\,ab\cos\left(fx+e\right)^2-4\,a^2-3\,ab\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{6\left(a^3b^2f\cos\left(fx+e\right)^4-2\left(a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^5+2\,a^4b+a^3b^2\right)f\right)},\ \frac{3\left(b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a}}{a}\right)-\left(3\,ab\cos\left(fx+e\right)^2-4\,a^2-3\,ab\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{3\left(a^3b^2f\cos\left(fx+e\right)^4-2\left(a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^5+2\,a^4b+a^3b^2\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(3*a*b*cos(f*x + e)^2 - 4*a^2 -
3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*b^2*f*cos(f*x + e)^4 - 2*(a^4*
b + a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f), 1/3*(3*(b^2*c
os(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a)*
arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - (3*a*b*cos(f*x + e)^2
- 4*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*b^2*f*cos(f*x + e)^4
 - 2*(a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [A]
time = 0.46, size = 74, normalized size = 0.89

$$\frac{\arctan\left(\frac{\sqrt{b\sin(fx+e)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2f}+\frac{3\,b\sin(fx+e)^2+4\,a}{3\,\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*f) + 1/3*(3*b*sin
(f*x + e)^2 + 4*a)/((b*sin(f*x + e)^2 + a)^(3/2)*a^2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)}{\left(b\sin(e+f\,x)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.536 $\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{}{2a^3}$$

```
[Out] 1/2*(2*a+5*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2)/f+1/6*(-2*a
-5*b)/a^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1/2*csc(f*x+e)^2/a/f/(a+b*sin(f*x+e)^2
)^(3/2)+1/2*(-2*a-5*b)/a^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 79, 53, 65, 214}

$$\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) - (
2*a + 5*b)/(6*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) - Csc[e + f*x]^2/(2*a*f*(
a + b*Sin[e + f*x]^2)^(3/2)) - (2*a + 5*b)/(2*a^3*f*Sqrt[a + b*Sin[e + f*x]
^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a+5b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sin^2(e+f\right.}{4af}$$

$$= -\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a+5b)\text{Subs}}{}$$

$$= -\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2a+}{2a^3f\sqrt{a+bs}}$$

$$= -\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2a+}{2a^3f\sqrt{a+bs}}$$

$$= \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.18, size = 69, normalized size = 0.48

$$-\frac{3a\csc^2(e+fx) + (2a+5b)\,{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; 1+\frac{b\sin^2(e+fx)}{a}\right)}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*(3*a*Csc[e + f*x]^2 + (2*a + 5*b)*Hypergeometric2F1[-3/2, 1, -1/2, 1 +
 (b*Sin[e + f*x]^2)/a])/(a^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. 2(123) = 246.
time = 30.44, size = 1038, normalized size = 7.26

method	result	size

default	Expression too large to display	1038

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^(13/2)/b^2/(b^2*cos(f*x+e)^6-2*a*b*cos(f*x+e)^4-3*b^2*cos(f*x+e)^4+a^
2*cos(f*x+e)^2+4*a*b*cos(f*x+e)^2+3*b^2*cos(f*x+e)^2-a^2-2*a*b-b^2)*(3*a^(1
1/2)*b^2*(a+b-b*cos(f*x+e)^2)^(1/2)-6*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)+a))*a^6*b^2+3*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b^4+8*a^(11
/2)*b^2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)+20*(-b*cos(f*x+e)^2+(a*b^2+
b^3)/b^2)^(1/2)*a^(9/2)*b^3+6*a^(9/2)*b^3*(a+b-b*cos(f*x+e)^2)^(1/2)+12*(-b
*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^(7/2)*b^4-27*ln(2/sin(f*x+e)*(a^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^5*b^3-36*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b
*cos(f*x+e)^2)^(1/2)+a))*a^4*b^4-15*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)+a))*a^3*b^5+3*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+a))*a^3*b^4*(2*a+5*b)*cos(f*x+e)^6-3*cos(f*x+e)^4*b^3*(-2*(-b*cos(f*x+e
)^2+(a*b^2+b^3)/b^2)^(1/2)*a^(9/2)-(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b-4*(
-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^(7/2)*b+4*ln(2/sin(f*x+e)*(a^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^5+16*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+a))*a^4*b+15*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)+a))*a^3*b^2)+cos(f*x+e)^2*b^2*(-8*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(
1/2)*a^(11/2)-6*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(9/2)*b-26*(-b*cos(f*x+e)^2+(a
*b^2+b^3)/b^2)^(1/2)*a^(9/2)*b-6*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b^2-24*
(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^(7/2)*b^2+6*ln(2/sin(f*x+e)*(a^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^6+39*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)+a))*a^5*b+78*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)+a))*a^4*b^2+45*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
+a))*a^3*b^3))/f
```

Maxima [A]
time = 0.36, size = 164, normalized size = 1.15

$$\frac{\frac{6 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}}+\frac{15\,b \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{7}{2}}}-\frac{6}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{2}}-\frac{2}{\left(b\sin(fx+e)^{2}+a\right)^{\frac{3}{2}}a}-\frac{15\,b}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{3}}-\frac{5\,b}{\left(b\sin(fx+e)^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{3}{\left(b\sin(fx+e)^{2}+a\right)^{\frac{3}{2}}a\sin(fx+e)^{2}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(6*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) + 15*b*arcsinh(a/(s
qrt(a*b)*abs(sin(f*x + e))))/a^(7/2) - 6/(sqrt(b*sin(f*x + e)^2 + a)*a^2) -
 2/((b*sin(f*x + e)^2 + a)^(3/2)*a) - 15*b/(sqrt(b*sin(f*x + e)^2 + a)*a^3)
 - 5*b/((b*sin(f*x + e)^2 + a)^(3/2)*a^2) - 3/((b*sin(f*x + e)^2 + a)^(3/2)
*a*sin(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(123) = 246$.

time = 0.50, size = 666, normalized size = 4.66

$$\left[\frac{3\left((2ab^2+5b^3)\cos(fx+e)^6-(4a^2b+16ab^2+15b^3)\cos(fx+e)^4-2a^3-9a^2b-12ab^2-5b^3+(2a^3+13a^2b+26ab^2+15b^3)\cos(fx+e)^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right)+2\left(3(2a^2b+5ab^2)\cos(fx+e)^4+11a^3+26a^2b+15ab^2-2(4a^3+16a^2b+15ab^2)\cos(fx+e)^2\right)\sqrt{-b\cos(fx+e)^2+a+b}}{12\left(a^4b^2f\cos(fx+e)^6-(2a^5b+3a^4b^2)f\cos(fx+e)^4+(a^6+4a^5b+3a^4b^2)f\cos(fx+e)^2-(a^6+2a^5b+a^4b^2)f\right)}, -\frac{3\left((2ab^2+5b^3)\cos(fx+e)^6-(4a^2b+16ab^2+15b^3)\cos(fx+e)^4-2a^3-9a^2b-12ab^2-5b^3+(2a^3+13a^2b+26ab^2+15b^3)\cos(fx+e)^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a}}{a}\right)-\left(3(2a^2b+5ab^2)\cos(fx+e)^4+11a^3+26a^2b+15ab^2-2(4a^3+16a^2b+15ab^2)\cos(fx+e)^2\right)\sqrt{-b\cos(fx+e)^2+a+b}}{6\left(a^4b^2f\cos(fx+e)^6-(2a^5b+3a^4b^2)f\cos(fx+e)^4+(a^6+4a^5b+3a^4b^2)f\cos(fx+e)^2-(a^6+2a^5b+a^4b^2)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((2*a*b^2 + 5*b^3)*cos(f*x + e)^6 - (4*a^2*b + 16*a*b^2 + 15*b^3)*
cos(f*x + e)^4 - 2*a^3 - 9*a^2*b - 12*a*b^2 - 5*b^3 + (2*a^3 + 13*a^2*b + 2
6*a*b^2 + 15*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*(3*
(2*a^2*b + 5*a*b^2)*cos(f*x + e)^4 + 11*a^3 + 26*a^2*b + 15*a*b^2 - 2*(4*a^
3 + 16*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/(
a^4*b^2*f*cos(f*x + e)^6 - (2*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^4 + (a^6 +
4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^2 - (a^6 + 2*a^5*b + a^4*b^2)*f), -1/6*
(3*((2*a*b^2 + 5*b^3)*cos(f*x + e)^6 - (4*a^2*b + 16*a*b^2 + 15*b^3)*cos(f*
x + e)^4 - 2*a^3 - 9*a^2*b - 12*a*b^2 - 5*b^3 + (2*a^3 + 13*a^2*b + 26*a*b^
2 + 15*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(-a)/a) - (3*(2*a^2*b + 5*a*b^2)*cos(f*x + e)^4 + 11*a^3 + 26*a^2*b +
15*a*b^2 - 2*(4*a^3 + 16*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(-b*cos(f*x
+ e)^2 + a + b))/(a^4*b^2*f*cos(f*x + e)^6 - (2*a^5*b + 3*a^4*b^2)*f*cos(f*
x + e)^4 + (a^6 + 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^2 - (a^6 + 2*a^5*b +
a^4*b^2)*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(128) = 256$.

time = 0.99, size = 751, normalized size = 5.25

$$-\frac{\frac{\left(\left(\left(\frac{3\left(a^{12}b^2+2a^{11}b^3+a^{10}b^4\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^{13}b^2+2a^{12}b^3+a^{11}b^4}+\frac{4\left(11a^{12}b^2+42a^{11}b^3+51a^{10}b^4+20a^9b^5\right)}{a^{13}b^2+2a^{12}b^3+a^{11}b^4}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\frac{6\left(19a^{12}b^2+90a^{11}b^3+163a^{10}b^4+132a^9b^5+40a^8b^6\right)}{a^{13}b^2+2a^{12}b^3+a^{11}b^4}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\frac{12\left(9a^{12}b^2+42a^{11}b^3+73a^{10}b^4+56a^9b^5+16a^8b^6\right)}{a^{13}b^2+2a^{12}b^3+a^{11}b^4}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\frac{7\left(5a^{12}b^2+18a^{11}b^3+21a^{10}b^4+8a^9b^5\right)}{a^{13}b^2+2a^{12}b^3+a^{11}b^4}}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a\right)^{\frac{3}{2}}}+\frac{6\left(2a^{\frac{3}{2}}+5\sqrt{a}b\right)\log\left(\left|-\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)a-a^{\frac{3}{2}}-2\sqrt{a}b\right|\right)}{a^4}-\frac{6\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)a^{\frac{3}{2}}+2\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)\sqrt{a}b+a^2\right)}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\sqrt{a}-a^{\frac{3}{2}}\right)a^3}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(((((3*(a^12*b^2 + 2*a^11*b^3 + a^10*b^4)*tan(1/2*f*x + 1/2*e)^2/(a^1
3*b^2 + 2*a^12*b^3 + a^11*b^4) + 4*(11*a^12*b^2 + 42*a^11*b^3 + 51*a^10*b^4
 + 20*a^9*b^5)/(a^13*b^2 + 2*a^12*b^3 + a^11*b^4))*tan(1/2*f*x + 1/2*e)^2 +
 6*(19*a^12*b^2 + 90*a^11*b^3 + 163*a^10*b^4 + 132*a^9*b^5 + 40*a^8*b^6)/(a
^13*b^2 + 2*a^12*b^3 + a^11*b^4))*tan(1/2*f*x + 1/2*e)^2 + 12*(9*a^12*b^2 +
 42*a^11*b^3 + 73*a^10*b^4 + 56*a^9*b^5 + 16*a^8*b^6)/(a^13*b^2 + 2*a^12*b^
3 + a^11*b^4))*tan(1/2*f*x + 1/2*e)^2 + 7*(5*a^12*b^2 + 18*a^11*b^3 + 21*a^
10*b^4 + 8*a^9*b^5)/(a^13*b^2 + 2*a^12*b^3 + a^11*b^4))/(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2)
 + 6*(2*a^(3/2) + 5*sqrt(a)*b)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a^4 - 6*((sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^(3/2) + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))*sqrt(a)*b + a^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))^2*sqrt(a) - a^(3/2))*a^3))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^3}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.537 $\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}} \, dx$

Optimal. Leaf size=208

$$-\frac{(8a^2+40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{8a^2+40ab+35b^2}{24a^3f\,(a+b\sin^2(e+fx))^{3/2}}+\frac{(8a+7b)\csc^2(e\cdots}{8a^2f\,(a+b\sin^2(e+\cdots}$$

```
[Out] -1/8*(8*a^2+40*a*b+35*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(9/2
)/f+1/24*(8*a^2+40*a*b+35*b^2)/a^3/f/(a+b*sin(f*x+e)^2)^(3/2)+1/8*(8*a+7*b)
*csc(f*x+e)^2/a^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1/4*csc(f*x+e)^4/a/f/(a+b*sin(
f*x+e)^2)^(3/2)+1/8*(8*a^2+40*a*b+35*b^2)/a^4/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3273, 91, 79, 53, 65, 214}

$$\frac{(8a+7b)\csc^2(e+fx)}{8a^2f\,(a+b\sin^2(e+fx))^{3/2}}-\frac{(8a^2+40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{8a^2+40ab+35b^2}{8a^4f\sqrt{a+b\sin^2(e+fx)}}+\frac{8a^2+40ab+35b^2}{24a^3f\,(a+b\sin^2(e+fx))^{3/2}}-\frac{\csc^4(e+fx)}{4af\,(a+b\sin^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/8*((8*a^2 + 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]
)/(a^(9/2)*f) + (8*a^2 + 40*a*b + 35*b^2)/(24*a^3*f*(a + b*Sin[e + f*x]^2)^
(3/2)) + ((8*a + 7*b)*Csc[e + f*x]^2)/(8*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2)
) - Csc[e + f*x]^4/(4*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (8*a^2 + 40*a*b +
 35*b^2)/(8*a^4*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)}{4af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-8a-7b)+2ax}{x^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{4af}$$

$$= \frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^4(e+fx)}{4af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(8a^2+40ab+3}{}$$

$$= \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^4(e}{4af\,(a+b\sin}$$

$$= \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^4(e}{4af\,(a+b\sin}$$

$$= \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^4(e}{4af\,(a+b\sin}$$

$$= -\frac{(8a^2+40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{8a^2+40ab\,-}{24a^3f\,(a+b\sin^2(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.57, size = 117, normalized size = 0.56

$$\frac{3a\csc^4(e+fx)\,(8a+7b-2a\csc^2(e+fx)) + (8a^2+40ab+35b^2)\csc^2(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\sin^2(e+fx)}{a}\right)}{24a^3f\,(b+a\csc^2(e+fx))\,\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*a*Csc[e + f*x]^4*(8*a + 7*b - 2*a*Csc[e + f*x]^2) + (8*a^2 + 40*a*b + 35
*b^2)*Csc[e + f*x]^2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin[e + f*x]^2
)/a])/(24*a^3*f*(b + a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 987 vs. 2(184) = 368.
time = 41.83, size = 988, normalized size = 4.75

method	result
default	$\frac{-\frac{(a^2+4ab+3b^2)\sqrt{-b(\cos^2(fx+e))+\frac{ab+b^2}{b}}}{2a^4\sqrt{-ab}\left(\sin(fx+e)+\frac{\sqrt{-ab}}{b}\right)}-\frac{\sqrt{-b(\cos^2(fx+e))+\frac{ab+b^2}{b}}}{12a^2b\left(\sin(fx+e)+\frac{\sqrt{-ab}}{b}\right)^2}-\frac{\sqrt{-b(\cos^2(fx+e))+}}{6a^3\left(\sin(fx+e)+\frac{\sqrt{-ab}}{b}\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(a^2+4*a*b+3*b^2)/a^4/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*co
s(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/a^2/b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*
cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/6/a^3/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*c
os(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/a^4*b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b
*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/a^2/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(
1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/6/a^3/(-a*b)^(1/2)*b/(sin(f*x
+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/a^4/(-a*b)^(1/
2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)*b^2+1/a^
3/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-5/a^(7/2)*b*ln((2*a+2*a^(1/2)*(a+b*
sin(f*x+e)^2)^(1/2))/sin(f*x+e))+11/8/a^4*b/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)
^(1/2)-35/8/a^(9/2)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x
+e))+1/2*(a^2+4*a*b+3*b^2)/a^4/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b
*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/a^(5/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2))/sin(f*x+e))-1/4/a^3/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)-1/12/
a^2/b/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/6
/a^3/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12
/a^4*b/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/
12/a^2/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/
b)^(1/2)+1/6/a^3/(-a*b)^(1/2)*b/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^
2+(a*b+b^2)/b)^(1/2)+1/12/a^4/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*
cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)*b^2)/f
```

Maxima [A]
time = 0.38, size = 295, normalized size = 1.42

$$-\frac{\frac{24\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}}+\frac{120\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{7}{2}}}+\frac{105\,b^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{9}{2}}}-\frac{24}{\sqrt{b\sin(fx+e)^2+a}\,a^2}-\frac{8}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}a}-\frac{120\,b}{\sqrt{b\sin(fx+e)^2+a}\,a^3}-\frac{40\,b}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}a^2}-\frac{105\,b^2}{\sqrt{b\sin(fx+e)^2+a}\,a^4}-\frac{35\,b^2}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}a^3}-\frac{24}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}a\sin(fx+e)^2}-\frac{21\,b}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}a^2\sin(fx+e)^2}+\frac{6}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}a\sin(fx+e)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(24*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) + 120*b*arcsinh(
a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(7/2) + 105*b^2*arcsinh(a/(sqrt(a*b)*abs
(sin(f*x + e))))/a^(9/2) - 24/(sqrt(b*sin(f*x + e)^2 + a)*a^2) - 8/((b*sin(
```

```
f*x + e)^2 + a)^(3/2)*a) - 120*b/(sqrt(b*sin(f*x + e)^2 + a)*a^3) - 40*b/((
b*sin(f*x + e)^2 + a)^(3/2)*a^2) - 105*b^2/(sqrt(b*sin(f*x + e)^2 + a)*a^4)
 - 35*b^2/((b*sin(f*x + e)^2 + a)^(3/2)*a^3) - 24/((b*sin(f*x + e)^2 + a)^(
3/2)*a*sin(f*x + e)^2) - 21*b/((b*sin(f*x + e)^2 + a)^(3/2)*a^2*sin(f*x + e
)^2) + 6/((b*sin(f*x + e)^2 + a)^(3/2)*a*sin(f*x + e)^4))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(184) = 368$.
time = 0.56, size = 984, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2*b^2 + 40*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b + 56*
a^2*b^2 + 115*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (8*a^4 + 88*a^3*b + 323*a^2*
b^2 + 450*a*b^3 + 210*b^4)*cos(f*x + e)^4 + 8*a^4 + 56*a^3*b + 123*a^2*b^2
+ 110*a*b^3 + 35*b^4 - 2*(8*a^4 + 64*a^3*b + 171*a^2*b^2 + 185*a*b^3 + 70*b
^4)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(3*(8*a^3*b + 40*
a^2*b^2 + 35*a*b^3)*cos(f*x + e)^6 - (32*a^4 + 232*a^3*b + 500*a^2*b^2 + 31
5*a*b^3)*cos(f*x + e)^4 - 50*a^4 - 205*a^3*b - 260*a^2*b^2 - 105*a*b^3 + (8
8*a^4 + 413*a^3*b + 640*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*
x + e)^2 + a + b))/(a^5*b^2*f*cos(f*x + e)^8 - 2*(a^6*b + 2*a^5*b^2)*f*cos(
f*x + e)^6 + (a^7 + 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 - 2*(a^7 + 3*a^6*
b + 2*a^5*b^2)*f*cos(f*x + e)^2 + (a^7 + 2*a^6*b + a^5*b^2)*f), 1/24*(3*((8
*a^2*b^2 + 40*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b + 56*a^2*b^2 + 11
5*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (8*a^4 + 88*a^3*b + 323*a^2*b^2 + 450*a*
b^3 + 210*b^4)*cos(f*x + e)^4 + 8*a^4 + 56*a^3*b + 123*a^2*b^2 + 110*a*b^3
+ 35*b^4 - 2*(8*a^4 + 64*a^3*b + 171*a^2*b^2 + 185*a*b^3 + 70*b^4)*cos(f*x
+ e)^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - (3*(8
*a^3*b + 40*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^6 - (32*a^4 + 232*a^3*b + 500*
a^2*b^2 + 315*a*b^3)*cos(f*x + e)^4 - 50*a^4 - 205*a^3*b - 260*a^2*b^2 - 10
5*a*b^3 + (88*a^4 + 413*a^3*b + 640*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^2)*sq
rt(-b*cos(f*x + e)^2 + a + b))/(a^5*b^2*f*cos(f*x + e)^8 - 2*(a^6*b + 2*a^5
*b^2)*f*cos(f*x + e)^6 + (a^7 + 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 - 2*(
a^7 + 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^2 + (a^7 + 2*a^6*b + a^5*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. $2(191) = 382$.
time = 1.25, size = 1411, normalized size = 6.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/192*(((((3*((a^17*b^2 + 2*a^16*b^3 + a^15*b^4)*tan(1/2*f*x + 1/2*e)^2/(a
^18*b^2 + 2*a^17*b^3 + a^16*b^4) - (9*a^17*b^2 + 32*a^16*b^3 + 37*a^15*b^4
+ 14*a^14*b^5)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))*tan(1/2*f*x + 1/2*e)^2 -
 2*(197*a^17*b^2 + 1106*a^16*b^3 + 2181*a^15*b^4 + 1832*a^14*b^5 + 560*a^13
*b^6)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))*tan(1/2*f*x + 1/2*e)^2 - 6*(165*a
^17*b^2 + 1072*a^16*b^3 + 2761*a^15*b^4 + 3526*a^14*b^5 + 2232*a^13*b^6 + 5
60*a^12*b^7)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))*tan(1/2*f*x + 1/2*e)^2 - 3
*(307*a^17*b^2 + 1958*a^16*b^3 + 4835*a^15*b^4 + 5792*a^14*b^5 + 3376*a^13*
b^6 + 768*a^12*b^7)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))*tan(1/2*f*x + 1/2*e
)^2 - (295*a^17*b^2 + 1552*a^16*b^3 + 2859*a^15*b^4 + 2242*a^14*b^5 + 640*a
^13*b^6)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))/(a*tan(1/2*f*x + 1/2*e)^4 + 2*
a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) - 24*(8*a^
2 + 40*a*b + 35*b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 12*(8*a^(5/2) + 40*a^(3/2)*b + 35*sqrt(a)
*b^2)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(
3/2) - 2*sqrt(a)*b))/a^5 + 12*(6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))^3*a^2 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
3*a*b + 22*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 8*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b - 8*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 - 28*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))*a^2*b - 26*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))*a*b^2 - 7*a^(7/2) - 12*a^(5/2)*b)/(((sqrt(a)*tan(1/2*f*x + 1/
```

```
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - a)^2*a^4))/f
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

3.538 $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=348

$$\frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}}+\frac{8(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\mathrm{si}\right.}{3(a+b)^4}$$

```
[Out] 1/3*(5*a-3*b)*b*cos(f*x+e)*sin(f*x+e)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(3/2)+8/
3*(a-b)*b*cos(f*x+e)*sin(f*x+e)/(a+b)^4/f/(a+b*sin(f*x+e)^2)^(1/2)+8/3*(a-b
)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*s
in(f*x+e)^2)^(1/2)/(a+b)^4/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(5*a-3*b)*Ellip
ticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+
e)^2/a)^(1/2)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(2*a-b)*tan(f*x+e)/(a+
b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*sec(f*x+e)^2*tan(f*x+e)/(a+b)/f/(a+b*si
n(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 481, 541, 538, 437, 435, 432, 430}

$$-\frac{(5a-3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{8(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\left|-\frac{b}{a}\right.\right)}{3f(a+b)^4\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2(2a-b)\tan(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{8b(a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}+\frac{b(5a-3b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((5*a - 3*b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^3*f*(a + b*Sin[e + f*x
]^2)^(3/2)) + (8*(a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^4*f*Sqrt[a
 + b*Sin[e + f*x]^2]) + (8*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Si
n[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^4*
f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((5*a - 3*b)*Sqrt[Cos[e + f*x]^2]*Ellip
ticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)
/a])/(3*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2]) - (2*(2*a - b)*Tan[e + f*x]
)/(3*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (Sec[e + f*x]^2*Tan[e + f*
x])/(3*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^{5/2}(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\right.}{3(a+b}$$

$$= -\frac{2(2a-b)\tan(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} -$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)\tan(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{3(}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} - \frac{}{3(}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} - \frac{}{3(}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} - \frac{}{3(}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} + \frac{8(}{}$$

Mathematica [A]
time = 2.33, size = 235, normalized size = 0.68

$$\frac{2ab\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}\left(8a(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)+\left(-5a^2-2ab+3b^2\right)F\left(e+fx\middle|-\frac{b}{a}\right)\right)+\sqrt{2}\,b\left(2ab(a+b)\sin(2(e+fx))+4(a-b)b(2a+b-b\cos(2(e+fx)))\sin(2(e+fx))-4(a-b)(2a+b-b\cos(2(e+fx)))^2\tan(e+fx)+(a+b)(2a+b-b\cos(2(e+fx)))^2\sec^2(e+fx)\tan(e+fx)\right)}{6b(a+b)^4f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*a*b*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(8*a*(a - b)*EllipticE[e +
f*x, -(b/a)] + (-5*a^2 - 2*a*b + 3*b^2)*EllipticF[e + f*x, -(b/a)]) + Sqrt[
```

```
2]*b*(2*a*b*(a + b)*Sin[2*(e + f*x)] + 4*(a - b)*b*(2*a + b - b*Cos[2*(e +
f*x)])*Sin[2*(e + f*x)] - 4*(a - b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Tan[e
+ f*x] + (a + b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Sec[e + f*x]^2*Tan[e + f*
x]))/(6*b*(a + b)^4*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(318) = 636$.
time = 26.05, size = 667, normalized size = 1.92

method	result
default	$-\frac{-8\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b^2(a-b)\sin(fx+e)\left(\cos^6(fx+e)\right)+\sqrt{-b\left(\cos^4\left(fx+e\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-8*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(a-b)*sin(f*x+e)*co
s(f*x+e)^6+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(13*a^2+2*a*b-11*b^
2)*cos(f*x+e)^4*sin(f*x+e)-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*
a^3+3*a^2*b-b^3)*cos(f*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)*(a^3+3*a^2*b+3*a*b^2+b^3)*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*
x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*b*(5*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b-3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2+8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e)^4
-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(
f*x+e)^2+(a+b)/a)^(1/2)*(5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*
a*b^2-3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3-8*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a^3+8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)*cos(f*x+e)^2
)/(1+sin(f*x+e))/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/
(sin(f*x+e)-1)/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^4/cos(f*x+e)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains complex when optimal does not.
time = 0.36, size = 1730, normalized size = 4.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")

```
[Out] -1/3*(4*(2*((-I*a*b^4 + I*b^5)*cos(f*x + e)^7 + 2*(I*a^2*b^3 - I*b^5)*cos(f
*x + e)^5 + (-I*a^3*b^2 - I*a^2*b^3 + I*a*b^4 + I*b^5)*cos(f*x + e)^3)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) + ((-2*I*a^2*b^3 + I*a*b^4 + I*b^5)*cos(f*x + e)
^7 + 2*(2*I*a^3*b^2 + I*a^2*b^3 - 2*I*a*b^4 - I*b^5)*cos(f*x + e)^5 + (-2*I
*a^4*b - 3*I*a^3*b^2 + I*a^2*b^3 + 3*I*a*b^4 + I*b^5)*cos(f*x + e)^3)*sqrt(
-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((
2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (
8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 4*(2*((
I*a*b^4 - I*b^5)*cos(f*x + e)^7 + 2*(-I*a^2*b^3 + I*b^5)*cos(f*x + e)^5 + (
I*a^3*b^2 + I*a^2*b^3 - I*a*b^4 - I*b^5)*cos(f*x + e)^3)*sqrt(-b)*sqrt((a^2
 + a*b)/b^2) + ((2*I*a^2*b^3 - I*a*b^4 - I*b^5)*cos(f*x + e)^7 + 2*(-2*I*a^
3*b^2 - I*a^2*b^3 + 2*I*a*b^4 + I*b^5)*cos(f*x + e)^5 + (2*I*a^4*b + 3*I*a^
3*b^2 - I*a^2*b^3 - 3*I*a*b^4 - I*b^5)*cos(f*x + e)^3)*sqrt(-b))*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 +
 a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b +
 b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*((-3*I*a^2*b^3 + 2*
I*a*b^4 + 5*I*b^5)*cos(f*x + e)^7 - 2*(-3*I*a^3*b^2 - I*a^2*b^3 + 7*I*a*b^4
 + 5*I*b^5)*cos(f*x + e)^5 + (-3*I*a^4*b - 4*I*a^3*b^2 + 6*I*a^2*b^3 + 12*I
*a*b^4 + 5*I*b^5)*cos(f*x + e)^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-6*I*a
^3*b^2 + 17*I*a^2*b^3 + 4*I*a*b^4 - 3*I*b^5)*cos(f*x + e)^7 + 2*(6*I*a^4*b
- 11*I*a^3*b^2 - 21*I*a^2*b^3 - I*a*b^4 + 3*I*b^5)*cos(f*x + e)^5 + (-6*I*a
^5 + 5*I*a^4*b + 32*I*a^3*b^2 + 22*I*a^2*b^3 - 2*I*a*b^4 - 3*I*b^5)*cos(f*x
 + e)^3)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f
(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin
(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/
b^2) - (2*((3*I*a^2*b^3 - 2*I*a*b^4 - 5*I*b^5)*cos(f*x + e)^7 - 2*(3*I*a^3*
b^2 + I*a^2*b^3 - 7*I*a*b^4 - 5*I*b^5)*cos(f*x + e)^5 + (3*I*a^4*b + 4*I*a^
3*b^2 - 6*I*a^2*b^3 - 12*I*a*b^4 - 5*I*b^5)*cos(f*x + e)^3)*sqrt(-b)*sqrt((
a^2 + a*b)/b^2) - ((6*I*a^3*b^2 - 17*I*a^2*b^3 - 4*I*a*b^4 + 3*I*b^5)*cos(f
*x + e)^7 + 2*(-6*I*a^4*b + 11*I*a^3*b^2 + 21*I*a^2*b^3 + I*a*b^4 - 3*I*b^5
)*cos(f*x + e)^5 + (6*I*a^5 - 5*I*a^4*b - 32*I*a^3*b^2 - 22*I*a^2*b^3 + 2*I
*a*b^4 + 3*I*b^5)*cos(f*x + e)^3)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2)
 + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2
)*sqrt((a^2 + a*b)/b^2))/b^2) + (8*(a*b^4 - b^5)*cos(f*x + e)^6 - a^3*b^2 -
 3*a^2*b^3 - 3*a*b^4 - b^5 - (13*a^2*b^3 + 2*a*b^4 - 11*b^5)*cos(f*x + e)^4
 + 2*(2*a^3*b^2 + 3*a^2*b^3 - b^5)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 +
 a + b)*sin(f*x + e))/((a^4*b^4 + 4*a^3*b^5 + 6*a^2*b^6 + 4*a*b^7 + b^8)*f*
cos(f*x + e)^7 - 2*(a^5*b^3 + 5*a^4*b^4 + 10*a^3*b^5 + 10*a^2*b^6 + 5*a*b^7
 + b^8)*f*cos(f*x + e)^5 + (a^6*b^2 + 6*a^5*b^3 + 15*a^4*b^4 + 20*a^3*b^5 +
 15*a^2*b^6 + 6*a*b^7 + b^8)*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**4/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(e+f\,x)^4}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.539 $\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2 f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3 f\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\mathrm{si}\right.}{3a(a+b)^3}$$

```
[Out] -4/3*b*cos(f*x+e)*sin(f*x+e)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1/3*(7*a-b)
*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(7*a-b)*E
llipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(
f*x+e)^2)^(1/2)/a/(a+b)^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)+4/3*EllipticF(sin(f*
x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/
2)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)
^(3/2)
```

Rubi [A]

time = 0.20, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3275, 482, 541, 538, 437, 435, 432, 430}

$$\frac{4\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\mathrm{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\mathrm{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3af(a+b)^3\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{\tan(e+fx)}{f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^3\sqrt{a+b\sin^2(e+fx)}}-\frac{4b\sin(e+fx)\cos(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-4*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2
)) - ((7*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^3*f*Sqrt[a + b*Si
n[e + f*x]^2]) - ((7*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f
*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*(a + b)^3*f*Sqr
t[1 + (b*Sin[e + f*x]^2)/a]) + (4*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin
[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)
^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + Tan[e + f*x]/((a + b)*f*(a + b*Sin[e + f
*x]^2)^(3/2))
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{(1-x^2)^{3/2}(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\,(a+b\sin^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \cdots\right.}{(a+b)\cdots}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} + \frac{\tan(e+fx)}{(a+b)f\,(a+b\sin^2(e+fx))^{3/2}} + \cdots$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \cdots$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \cdots$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \cdots$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \cdots$$

Mathematica [A]
time = 1.83, size = 199, normalized size = 0.68

$$\frac{-2a^2(7a-b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+8a^2(a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)+\frac{(24a^3+4a^2b+5ab^2+b^3-4ab(11a+3b)\cos(2(e+fx))+(7a-b)b^2\cos(4(e+fx)))\tan(e+fx)}{\sqrt{2}}}{6a(a+b)^3f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(7*a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*
x, -(b/a)] + 8*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Ellipti
cF[e + f*x, -(b/a)] + ((24*a^3 + 4*a^2*b + 5*a*b^2 + b^3 - 4*a*b*(11*a + 3*
b)*Cos[2*(e + f*x)] + (7*a - b)*b^2*Cos[4*(e + f*x)])*Tan[e + f*x])/Sqrt[2]
)/(6*a*(a + b)^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(268) = 536$.

time = 20.93, size = 851, normalized size = 2.91

method	result	size
default	Expression too large to display	851

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(7*a-b)*sin(f*x+e)*cos(
f*x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(11*a^2+10*a*b-b^2)*c
os(f*x+e)^2*sin(f*x+e)+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a^2+
2*a*b+b^2)*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2
)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*b*(4*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*
x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2
+4*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^3+8*(cos(f
*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a
*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2*b+4*(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*b^2-7*(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b
*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^3-6*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+
e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2*b+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^
2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*EllipticE(sin(f
*x+e),(-1/a*b)^(1/2))*a*b^2)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x
+e)))^(1/2)/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^3/a/cos(f*x+e)/f
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.28, size = 1632, normalized size = 5.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*((-7*I*a*b^4 + I*b^5)*cos(f*x + e)^5 - 2*(-7*I*a^2*b^3 - 6*I*a*b^4
+ I*b^5)*cos(f*x + e)^3 + (-7*I*a^3*b^2 - 13*I*a^2*b^3 - 5*I*a*b^4 + I*b^5)
*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((14*I*a^2*b^3 + 5*I*a*b^4
- I*b^5)*cos(f*x + e)^5 + 2*(-14*I*a^3*b^2 - 19*I*a^2*b^3 - 4*I*a*b^4 + I*b
^5)*cos(f*x + e)^3 + (14*I*a^4*b + 33*I*a^3*b^2 + 23*I*a^2*b^3 + 3*I*a*b^4
- I*b^5)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*
x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2
 + a*b)/b^2))/b^2) + (2*((7*I*a*b^4 - I*b^5)*cos(f*x + e)^5 - 2*(7*I*a^2*b^
3 + 6*I*a*b^4 - I*b^5)*cos(f*x + e)^3 + (7*I*a^3*b^2 + 13*I*a^2*b^3 + 5*I*a
*b^4 - I*b^5)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-14*I*a^2*b^
3 - 5*I*a*b^4 + I*b^5)*cos(f*x + e)^5 + 2*(14*I*a^3*b^2 + 19*I*a^2*b^3 + 4*
I*a*b^4 - I*b^5)*cos(f*x + e)^3 + (-14*I*a^4*b - 33*I*a^3*b^2 - 23*I*a^2*b^
3 - 3*I*a*b^4 + I*b^5)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
 b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*((-3*I*a^2*b^3 - 2*I*a*b^4 + I*b^5)
*cos(f*x + e)^5 + 2*(3*I*a^3*b^2 + 5*I*a^2*b^3 + I*a*b^4 - I*b^5)*cos(f*x +
 e)^3 + (-3*I*a^4*b - 8*I*a^3*b^2 - 6*I*a^2*b^3 + I*b^5)*cos(f*x + e))*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) + ((6*I*a^3*b^2 - 7*I*a^2*b^3 - 5*I*a*b^4)*cos(f
*x + e)^5 + 2*(-6*I*a^4*b + I*a^3*b^2 + 12*I*a^2*b^3 + 5*I*a*b^4)*cos(f*x +
 e)^3 + (6*I*a^5 + 5*I*a^4*b - 13*I*a^3*b^2 - 17*I*a^2*b^3 - 5*I*a*b^4)*cos
(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic
_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*s
in(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2)
)/b^2) - 2*(2*((3*I*a^2*b^3 + 2*I*a*b^4 - I*b^5)*cos(f*x + e)^5 + 2*(-3*I*a
^3*b^2 - 5*I*a^2*b^3 - I*a*b^4 + I*b^5)*cos(f*x + e)^3 + (3*I*a^4*b + 8*I*a
^3*b^2 + 6*I*a^2*b^3 - I*b^5)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2)
+ ((-6*I*a^3*b^2 + 7*I*a^2*b^3 + 5*I*a*b^4)*cos(f*x + e)^5 + 2*(6*I*a^4*b -
 I*a^3*b^2 - 12*I*a^2*b^3 - 5*I*a*b^4)*cos(f*x + e)^3 + (-6*I*a^5 - 5*I*a^4
*b + 13*I*a^3*b^2 + 17*I*a^2*b^3 + 5*I*a*b^4)*cos(f*x + e))*sqrt(-b))*sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((
a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*
a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(3*a^3*b^2 + 6*
```

```
a^2*b^3 + 3*a*b^4 + (7*a*b^4 - b^5)*cos(f*x + e)^4 - (11*a^2*b^3 + 10*a*b^4
 - b^5)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^4
*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)*f*cos(f*x + e)^5 - 2*(a^5*b^3 + 4*a^4
*b^4 + 6*a^3*b^5 + 4*a^2*b^6 + a*b^7)*f*cos(f*x + e)^3 + (a^6*b^2 + 5*a^5*b
^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(e+f\,x)^2}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.540 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin}}{3a^2(a+b)^2f\sqrt{1+\frac{b\sin^2(e+}{a}}}$$

```
[Out] 1/3*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(2*a+b
)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(2*a+b)*
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/a^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(cos(f*x+e)^2
)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^
(1/2)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3263, 3252, 3251, 3257, 3256, 3262, 3261}

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
```

```
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a-2b+b\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \cdots$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \cdots$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \cdots$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(\cdots}{\cdots}$$

Mathematica [A]
time = 0.92, size = 172, normalized size = 0.77

$$\frac{2a^2(2a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-a^2(a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,b(-5a^2-5ab-b^2+b(2a+b)\cos(2(e+fx)))\sin(2(e+fx))}{3a^2(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[
e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])
^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(245) = 490.
time = 10.52, size = 547, normalized size = 2.45

method	result

default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\ \text{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b(\sin^2(fx+e))+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*sin(f*x+
e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-4*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*b^3*sin(f*x+
e)^3-5*sin(f*x+e)*b*a^2-3*sin(f*x+e)*b^2*a)/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a
+b)^2/cos(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.22, size = 1531, normalized size = 6.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*(2*I*a^3*b^2 + 5*I*a^2*b^3 + 4*I*a*b^4 + I*b^5 + (2*I*a*b^4 + I*b^5
)*cos(f*x + e)^4 - 2*(2*I*a^2*b^3 + 3*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) - (-4*I*a^4*b - 12*I*a^3*b^2 - 13*I*a^2*b^3 - 6*
I*a*b^4 - I*b^5 + (-4*I*a^2*b^3 - 4*I*a*b^4 - I*b^5)*cos(f*x + e)^4 + 2*(4*
```

```
I*a^3*b^2 + 8*I*a^2*b^3 + 5*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sqrt(-b))*sqrt
((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8
*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-2*I*a^3*b^2
 - 5*I*a^2*b^3 - 4*I*a*b^4 - I*b^5 + (-2*I*a*b^4 - I*b^5)*cos(f*x + e)^4 -
2*(-2*I*a^2*b^3 - 3*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a
*b)/b^2) - (4*I*a^4*b + 12*I*a^3*b^2 + 13*I*a^2*b^3 + 6*I*a*b^4 + I*b^5 + (
4*I*a^2*b^3 + 4*I*a*b^4 + I*b^5)*cos(f*x + e)^4 + 2*(-4*I*a^3*b^2 - 8*I*a^2
*b^3 - 5*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a
*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a
*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^4*b - 11*I*a^3*b^2 - 15*
I*a^2*b^3 - 9*I*a*b^4 - 2*I*b^5 + (-3*I*a^2*b^3 - 5*I*a*b^4 - 2*I*b^5)*cos(
f*x + e)^4 - 2*(-3*I*a^3*b^2 - 8*I*a^2*b^3 - 7*I*a*b^4 - 2*I*b^5)*cos(f*x +
 e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-6*I*a^5 - 17*I*a^4*b - 17*I*a^3*b
^2 - 7*I*a^2*b^3 - I*a*b^4 + (-6*I*a^3*b^2 - 5*I*a^2*b^3 - I*a*b^4)*cos(f*x
 + e)^4 + 2*(6*I*a^4*b + 11*I*a^3*b^2 + 6*I*a^2*b^3 + I*a*b^4)*cos(f*x + e)
^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcs
in(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x
+ e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)
+ (2*(3*I*a^4*b + 11*I*a^3*b^2 + 15*I*a^2*b^3 + 9*I*a*b^4 + 2*I*b^5 + (3*I*
a^2*b^3 + 5*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^4 - 2*(3*I*a^3*b^2 + 8*I*a^2*b^
3 + 7*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (
6*I*a^5 + 17*I*a^4*b + 17*I*a^3*b^2 + 7*I*a^2*b^3 + I*a*b^4 + (6*I*a^3*b^2
+ 5*I*a^2*b^3 + I*a*b^4)*cos(f*x + e)^4 + 2*(-6*I*a^4*b - 11*I*a^3*b^2 - 6*
I*a^2*b^3 - I*a*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
 b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(2*a*b^4 + b^5)*cos(f*x + e)^3 - (5*a
^2*b^3 + 7*a*b^4 + 2*b^5)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sin
(f*x + e))/((a^4*b^4 + 2*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^4 - 2*(a^5*b^3 +
 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^2 + (a^6*b^2 + 4*a^5*b^3 +
 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*f)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(-5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.541 $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}} dx$

Optimal. Leaf size=287

$$\frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3(a+b)f}-$$

```
[Out] 1/3*cot(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*(3*a+4*b)*cot(f*x+e)/a^2/(a
+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(7*a+8*b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^
(1/2)/a^3/(a+b)/f-1/3*(7*a+8*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+
e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/(a+b)/f/(1+b*sin(f*x+e
)^2/a)^(1/2)+4/3*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^
2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3275, 480, 593, 597, 538, 437, 435, 432, 430}

$$-\frac{(7a+8b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f(a+b)}+\frac{4\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(3a+4b)\cot(e+fx)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] Cot[e + f*x]/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + ((3*a + 4*b)*Cot[e + f*
x])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((7*a + 8*b)*Cot[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)*f) - ((7*a + 8*b)*Sqrt[Cos[e +
 f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Si
n[e + f*x]^2])/(3*a^3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*Sqrt[C
os[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1
+ (b*Sin[e + f*x]^2)/a])/(3*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{}{x^2\sqrt{}}\right.}{3af}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(\right.}{}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b}{}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b}{}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b}{}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b}{}$$

Mathematica [A]
time = 1.82, size = 209, normalized size = 0.73

$$\frac{-\frac{\left(24a^3+68a^2b+69ab^2+24b^3-4b\left(11a^2+19ab+8b^2\right)\cos(2(e+fx))+b^2(7a+8b)\cos(4(e+fx))\right)\cot(e+fx)}{\sqrt{2}} - 2a^2(7a+8b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2} E\left(e+fx\middle|-\frac{b}{a}\right) + 8a^2(a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2} F\left(e+fx\middle|-\frac{b}{a}\right)}{6a^3(a+b)f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(((24*a^3 + 68*a^2*b + 69*a*b^2 + 24*b^3 - 4*b*(11*a^2 + 19*a*b + 8*b^2)*
Cos[2*(e + f*x)] + b^2*(7*a + 8*b)*Cos[4*(e + f*x)])*Cot[e + f*x])/Sqrt[2])
```

```
- 2*a^2*(7*a + 8*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e +
 f*x, -(b/a)] + 8*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Elli
pticF[e + f*x, -(b/a)])/(6*a^3*(a + b)*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/
2))
```

Maple [A]
time = 14.22, size = 411, normalized size = 1.43

method	result
default	$-\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,ab\left(4\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a+4\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*b*(4*Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*
EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*b)*sin(f*x+e)*cos(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)
/a)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+8*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a*b+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-7*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-15*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*
a*b-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*sin(f*x+e)+(-7*a*b^2-8*b^3)
*cos(f*x+e)^6+(11*a^2*b+26*a*b^2+16*b^3)*cos(f*x+e)^4+(-3*a^3-14*a^2*b-19*a
*b^2-8*b^3)*cos(f*x+e)^2)/a^3/(a+b)/(a+b*sin(f*x+e)^2)^(3/2)/sin(f*x+e)/cos
(f*x+e)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.25, size = 1595, normalized size = 5.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*(-7*I*a^3*b^2 - 22*I*a^2*b^3 - 23*I*a*b^4 - 8*I*b^5 + (-7*I*a*b^4 -
 8*I*b^5)*cos(f*x + e)^4 - 2*(-7*I*a^2*b^3 - 15*I*a*b^4 - 8*I*b^5)*cos(f*x
+ e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (14*I*a^4*b + 51*I*a^
3*b^2 + 68*I*a^2*b^3 + 39*I*a*b^4 + 8*I*b^5 + (14*I*a^2*b^3 + 23*I*a*b^4 +
8*I*b^5)*cos(f*x + e)^4 + 2*(-14*I*a^3*b^2 - 37*I*a^2*b^3 - 31*I*a*b^4 - 8*
I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^
2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b
^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(7*I*a^3*b^2 + 22*I*a^2*b^3 + 23*I*a*b
^4 + 8*I*b^5 + (7*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^4 - 2*(7*I*a^2*b^3 + 15*I
*a*b^4 + 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x +
e) - (-14*I*a^4*b - 51*I*a^3*b^2 - 68*I*a^2*b^3 - 39*I*a*b^4 - 8*I*b^5 + (-
14*I*a^2*b^3 - 23*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^4 + 2*(14*I*a^3*b^2 + 37*
I*a^2*b^3 + 31*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqr
t((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 +
 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*(-3*I*a^4
*b - 17*I*a^3*b^2 - 33*I*a^2*b^3 - 27*I*a*b^4 - 8*I*b^5 + (-3*I*a^2*b^3 - 1
1*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^4 + 2*(3*I*a^3*b^2 + 14*I*a^2*b^3 + 19*I*
a*b^4 + 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e
) + (6*I*a^5 + 23*I*a^4*b + 32*I*a^3*b^2 + 19*I*a^2*b^3 + 4*I*a*b^4 + (6*I*
a^3*b^2 + 11*I*a^2*b^3 + 4*I*a*b^4)*cos(f*x + e)^4 + 2*(-6*I*a^4*b - 17*I*a
^3*b^2 - 15*I*a^2*b^3 - 4*I*a*b^4)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*s
qrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sq
rt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2
+ 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*(3*I*a^4
*b + 17*I*a^3*b^2 + 33*I*a^2*b^3 + 27*I*a*b^4 + 8*I*b^5 + (3*I*a^2*b^3 + 11
*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^4 + 2*(-3*I*a^3*b^2 - 14*I*a^2*b^3 - 19*I*
a*b^4 - 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e
) + (-6*I*a^5 - 23*I*a^4*b - 32*I*a^3*b^2 - 19*I*a^2*b^3 - 4*I*a*b^4 + (-6*
I*a^3*b^2 - 11*I*a^2*b^3 - 4*I*a*b^4)*cos(f*x + e)^4 + 2*(6*I*a^4*b + 17*I*
a^3*b^2 + 15*I*a^2*b^3 + 4*I*a*b^4)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2
 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*((7*a*b^4
+ 8*b^5)*cos(f*x + e)^5 - (11*a^2*b^3 + 26*a*b^4 + 16*b^5)*cos(f*x + e)^3 +
 (3*a^3*b^2 + 14*a^2*b^3 + 19*a*b^4 + 8*b^5)*cos(f*x + e))*sqrt(-b*cos(f*x
+ e)^2 + a + b))/(((a^4*b^4 + a^3*b^5)*f*cos(f*x + e)^4 - 2*(a^5*b^3 + 2*a^
4*b^4 + a^3*b^5)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*
b^5)*f)*sin(f*x + e))
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(e+f\,x)^2}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.542 $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=348

$$\frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}}+\frac{8(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e}{3a^4f}$$

```
[Out] 1/3*(a+b)*cot(f*x+e)*csc(f*x+e)^2/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*(a+3*b
)*cot(f*x+e)*csc(f*x+e)^2/a^2/b/f/(a+b*sin(f*x+e)^2)^(1/2)+8/3*(a+2*b)*cot(
f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^4/f-1/3*(3*a+8*b)*cot(f*x+e)*csc(f*x+e)^2
*(a+b*sin(f*x+e)^2)^(1/2)/a^3/b/f+8/3*(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(
1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^4/f/(1+b*s
in(f*x+e)^2/a)^(1/2)-1/3*(5*a+8*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f
*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a^3/f/(a+b*sin(f*x+e)
^2)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3275, 479, 593, 597, 538, 437, 435, 432, 430}

$$\frac{8(a+2b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,-\frac{b}{a}\right)}{3a^4f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{8(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^4f}-\frac{(5a+8b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\text{ArcSin}(\sin(e+fx))\,\middle|\,-\frac{b}{a}\right)}{3a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(3a+8b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3bf}+\frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + b)*Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2)
) + (2*(a + 3*b)*Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b*Sin[e +
 f*x]^2]) + (8*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^4*f)
 - ((3*a + 8*b)*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*
a^3*b*f) + (8*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]]
, -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^4*f*Sqrt[1 + (b*Sin
[e + f*x]^2)/a]) - ((5*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a^3*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
```

```
a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{x^4(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\,(a+b\sin^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\right.}{3ab}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{(\sqrt{}}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{(3a}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a}{}$$

Mathematica [A]
time = 2.09, size = 226, normalized size = 0.65

$$\frac{2a^2b\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}\left(8(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)-(5a+8b)F\left(e+fx\middle|-\frac{b}{a}\right)\right)+\sqrt{2}\,b\left(4(a+2b)(2a+b-b\cos(2(e+fx)))^2\cot(e+fx)-a(2a+b-b\cos(2(e+fx)))^2\cot(e+fx)\csc^2(e+fx)+2ab(a+b)\sin(2(e+fx))+4b(a+2b)(2a+b-b\cos(2(e+fx)))\sin(2(e+fx))\right)}{6a^4bf(2a+b-b\cos(2(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*a^2*b*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(8*(a + 2*b)*EllipticE[e
+ f*x, -(b/a)] - (5*a + 8*b)*EllipticF[e + f*x, -(b/a)]) + Sqrt[2]*b*(4*(a
+ 2*b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Cot[e + f*x] - a*(2*a + b - b*Cos[2
*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^2 + 2*a*b*(a + b)*Sin[2*(e + f*x)]
 + 4*b*(a + 2*b)*(2*a + b - b*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(6*a^4*b
*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [A]
time = 12.11, size = 633, normalized size = 1.82

method	result
default	$-\frac{5\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b(\sin^5(fx+e))+8\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^5+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^5-8*(
cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*b*sin(f*x+e)^5-16*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a
)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^5+8*a*b^2*sin
(f*x+e)^8+16*b^3*sin(f*x+e)^8+5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)
^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*sin(f*x+e)^3+8*(cos(f*x+e)^
2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*
a^2*b*sin(f*x+e)^3-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elli
pticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*sin(f*x+e)^3-16*(cos(f*x+e)^2)^(1/2)*(
(a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b*sin(
f*x+e)^3+13*a^2*b*sin(f*x+e)^6+16*a*b^2*sin(f*x+e)^6-16*b^3*sin(f*x+e)^6+4*
a^3*sin(f*x+e)^4-7*a^2*b*sin(f*x+e)^4-24*a*b^2*sin(f*x+e)^4-5*a^3*sin(f*x+e
)^2-6*a^2*b*sin(f*x+e)^2+a^3)/a^4/(a+b*sin(f*x+e)^2)^(3/2)/sin(f*x+e)^3/cos
(f*x+e)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains complex when optimal does not.
time = 0.27, size = 1971, normalized size = 5.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*(2*((-I*a*b^4 - 2*I*b^5)*cos(f*x + e)^6 + I*a^3*b^2 + 4*I*a^2*b^3 +
 5*I*a*b^4 + 2*I*b^5 + (2*I*a^2*b^3 + 7*I*a*b^4 + 6*I*b^5)*cos(f*x + e)^4 +
 (-I*a^3*b^2 - 6*I*a^2*b^3 - 11*I*a*b^4 - 6*I*b^5)*cos(f*x + e)^2)*sqrt(-b)
*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + ((-2*I*a^2*b^3 - 5*I*a*b^4 - 2*I*b^5)
*cos(f*x + e)^6 + 2*I*a^4*b + 9*I*a^3*b^2 + 14*I*a^2*b^3 + 9*I*a*b^4 + 2*I*
b^5 + (4*I*a^3*b^2 + 16*I*a^2*b^3 + 19*I*a*b^4 + 6*I*b^5)*cos(f*x + e)^4 +
(-2*I*a^4*b - 13*I*a^3*b^2 - 28*I*a^2*b^3 - 23*I*a*b^4 - 6*I*b^5)*cos(f*x +
 e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)
*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x +
 e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 +
a*b)/b^2))/b^2) + 4*(2*((I*a*b^4 + 2*I*b^5)*cos(f*x + e)^6 - I*a^3*b^2 - 4*
I*a^2*b^3 - 5*I*a*b^4 - 2*I*b^5 + (-2*I*a^2*b^3 - 7*I*a*b^4 - 6*I*b^5)*cos(
f*x + e)^4 + (I*a^3*b^2 + 6*I*a^2*b^3 + 11*I*a*b^4 + 6*I*b^5)*cos(f*x + e)^
2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + ((2*I*a^2*b^3 + 5*I*a*b^4
+ 2*I*b^5)*cos(f*x + e)^6 - 2*I*a^4*b - 9*I*a^3*b^2 - 14*I*a^2*b^3 - 9*I*a*
b^4 - 2*I*b^5 + (-4*I*a^3*b^2 - 16*I*a^2*b^3 - 19*I*a*b^4 - 6*I*b^5)*cos(f*
x + e)^4 + (2*I*a^4*b + 13*I*a^3*b^2 + 28*I*a^2*b^3 + 23*I*a*b^4 + 6*I*b^5)
*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2
*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*
(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sq
rt((a^2 + a*b)/b^2))/b^2) - (2*((-3*I*a^2*b^3 - 16*I*a*b^4 - 16*I*b^5)*cos(
f*x + e)^6 + 3*I*a^4*b + 22*I*a^3*b^2 + 51*I*a^2*b^3 + 48*I*a*b^4 + 16*I*b^
5 + (6*I*a^3*b^2 + 41*I*a^2*b^3 + 80*I*a*b^4 + 48*I*b^5)*cos(f*x + e)^4 + (
-3*I*a^4*b - 28*I*a^3*b^2 - 89*I*a^2*b^3 - 112*I*a*b^4 - 48*I*b^5)*cos(f*x
+ e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((-6*I*a^3*b^2 - 19*I
*a^2*b^3 - 8*I*a*b^4)*cos(f*x + e)^6 + 6*I*a^5 + 31*I*a^4*b + 52*I*a^3*b^2
+ 35*I*a^2*b^3 + 8*I*a*b^4 + (12*I*a^4*b + 56*I*a^3*b^2 + 73*I*a^2*b^3 + 24
*I*a*b^4)*cos(f*x + e)^4 + (-6*I*a^5 - 43*I*a^4*b - 102*I*a^3*b^2 - 89*I*a^
2*b^3 - 24*I*a*b^4)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((
a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*((3*I*a^2*b^3 + 16*I*a*b^
4 + 16*I*b^5)*cos(f*x + e)^6 - 3*I*a^4*b - 22*I*a^3*b^2 - 51*I*a^2*b^3 - 48
*I*a*b^4 - 16*I*b^5 + (-6*I*a^3*b^2 - 41*I*a^2*b^3 - 80*I*a*b^4 - 48*I*b^5)
*cos(f*x + e)^4 + (3*I*a^4*b + 28*I*a^3*b^2 + 89*I*a^2*b^3 + 112*I*a*b^4 +
48*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((6
*I*a^3*b^2 + 19*I*a^2*b^3 + 8*I*a*b^4)*cos(f*x + e)^6 - 6*I*a^5 - 31*I*a^4*
```

```
b - 52*I*a^3*b^2 - 35*I*a^2*b^3 - 8*I*a*b^4 + (-12*I*a^4*b - 56*I*a^3*b^2 -
 73*I*a^2*b^3 - 24*I*a*b^4)*cos(f*x + e)^4 + (6*I*a^5 + 43*I*a^4*b + 102*I*
a^3*b^2 + 89*I*a^2*b^3 + 24*I*a*b^4)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))
*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^
2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (8*(a*b^4 +
 2*b^5)*cos(f*x + e)^7 - (13*a^2*b^3 + 48*a*b^4 + 48*b^5)*cos(f*x + e)^5 +
4*(a^3*b^2 + 8*a^2*b^3 + 18*a*b^4 + 12*b^5)*cos(f*x + e)^3 - (3*a^3*b^2 + 1
9*a^2*b^3 + 32*a*b^4 + 16*b^5)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b
))/((a^4*b^4*f*cos(f*x + e)^6 - (2*a^5*b^3 + 3*a^4*b^4)*f*cos(f*x + e)^4 +
(a^6*b^2 + 4*a^5*b^3 + 3*a^4*b^4)*f*cos(f*x + e)^2 - (a^6*b^2 + 2*a^5*b^3 +
 a^4*b^4)*f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**4/(a + b*sin(e + f*x)**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(e+f\,x)^4}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.543 $\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$

Optimal. Leaf size=120

$$\frac{F_1\left(\frac{1+m}{2};\frac{1+m}{2},-p;\frac{3+m}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^2(e+fx)^{\frac{1+m}{2}}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{df(1+m)}$$

```
[Out] AppellF1(1/2+1/2*m,1/2+1/2*m,-p,3/2+1/2*m,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(
cos(f*x+e)^2)^(1/2+1/2*m)*(a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^(1+m)/d/f/(1+
m)/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3276, 525, 524}

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^{m+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(Cos[e + f*x]^2)^((1 + m)/2)*(a + b*Sin[e + f*x]^2)^p*(d*Tan[e
 + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3276

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[f
```

```
f*(d*Tan[e + f*x])^(m + 1)*((Cos[e + f*x]^2)^((m + 1)/2)/(d*f*Sin[e + f*x]^
(m + 1))), Subst[Int[(ff*x)^m*((a + b*ff^2*x^2)^p/(1 - ff^2*x^2)^((m + 1)/2
)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !In
tegerQ[m]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx = \frac{\left(\cos^2(e+fx)^{\frac{1+m}{2}}\sin^{-1-m}(e+fx)(d\tan(e+fx))^{1+m}\right)\text{Sub}}{df}$$

$$= \frac{\left(\cos^2(e+fx)^{\frac{1+m}{2}}\sin^{-1-m}(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1+m}{2};\frac{1+m}{2},-p;\frac{3+m}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^2(e+}{a}$$

Mathematica [A]
time = 0.37, size = 121, normalized size = 1.01

$$\frac{F_1\left(\frac{1+m}{2};\frac{1+m}{2},-p;\frac{3+m}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^2(e+fx)^{\frac{1+m}{2}}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+fx)(d\tan(e+fx))^m}{f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(Cos[e + f*x]^2)^((1 + m)/2)*(a + b*Sin[e + f*x]^2)^p*Tan[e +
f*x]*(d*Tan[e + f*x])^m)/(f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Maple [F]
time = 2.01, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^2(fx+e)\right)\right)^p (d\tan(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [F]
time = 0.63, size = 29, normalized size = 0.24

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}(d\tan\left(fx+e\right))^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^m\,\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int((d*tan(e + f*x))^m*(a + b*sin(e + f*x)^2)^p, x)
```

3.544 $\int (a+b\sin^2(c+dx))^p \tan^3(c+dx)\,dx$

Optimal. Leaf size=102

$$-\frac{(a+b+bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^2(c+dx)}{a+b}\right)(a+b\sin^2(c+dx))^{1+p}}{2(a+b)^2d(1+p)}+\frac{\sec^2(c+dx)\,(a+b\sin^2(c+dx))^{1+p}}{2(a+b)d}$$

```
[Out] -1/2*(b*p+a+b)*hypergeom([1, 1+p],[2+p],(a+b*sin(d*x+c)^2)/(a+b))*(a+b*sin(
d*x+c)^2)^(1+p)/(a+b)^2/d/(1+p)+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)^2)^(1+p)/(
a+b)/d
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3273, 79, 70}

$$\frac{\sec^2(c+dx)\,(a+b\sin^2(c+dx))^{p+1}}{2d(a+b)}-\frac{(a+bp+b)\,(a+b\sin^2(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^3,x]
```

```
[Out] -1/2*((a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^
2)/(a + b)]*(a + b*Sin[c + d*x]^2)^(1 + p))/((a + b)^2*d*(1 + p)) + (Sec[c
+ d*x]^2*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*(a + b)*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
```

```
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^p}{(1-x)^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2(a+b)d} - \frac{(a+b+bp)\text{Subst}\left(\int \frac{(a+bx)}{1-x}\right.}{2(a+b}$$

$$= -\frac{(a+b+bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^2(c+dx)}{a+b}\right)\left(a+b\sin^2(c+dx\right.}{2(a+b)^2d(1+p)}$$

Mathematica [A]
time = 0.17, size = 83, normalized size = 0.81

$$\frac{\left(-\left((a+b+bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^2(c+dx)}{a+b}\right)\right)+(a+b)(1+p)\sec^2(c+dx)\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2(a+b)^2d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^3,x]
[Out] ((-((a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)
/(a + b)]) + (a + b)*(1 + p)*Sec[c + d*x]^2)*(a + b*Sin[c + d*x]^2)^(1 + p)
)/(2*(a + b)^2*d*(1 + p))
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \left(a+\left(\sin^2(dx+c)\right)b\right)^p \left(\tan^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^3,x)
[Out] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^3, x)
```

Fricas [F]
time = 0.40, size = 27, normalized size = 0.26

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^2+a+b\right)^p\tan\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="fricas")
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^3\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^2)^p,x)
[Out] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^2)^p, x)
```

3.545 $\int (a+b\sin^2(c+dx))^p \tan(c+dx)\,dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^2(c+dx)}{a+b}\right)(a+b\sin^2(c+dx))^{1+p}}{2(a+b)d(1+p)}$$

```
[Out] 1/2*hypergeom([1, 1+p],[2+p],(a+b*sin(d*x+c)^2)/(a+b))*(a+b*sin(d*x+c)^2)^(
1+p)/(a+b)/d/(1+p)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3273, 70}

$$\frac{(a+b\sin^2(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)/(a + b)]*(a + b*
Sin[c + d*x]^2)^(1 + p))/(2*(a + b)*d*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1-x}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^2(c+dx)}{a+b}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2(a+b)d(1+p)}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 1.03

$$\frac{\left(a+b-b\cos^2(c+dx)\right)^{1+p} {}_2F_1\left(1,1+p;2+p;1-\frac{b\cos^2(c+dx)}{a+b}\right)}{2(a+b)d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x],x]
```

```
[Out] ((a + b - b*Cos[c + d*x]^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 -
(b*Cos[c + d*x]^2)/(a + b)])/(2*(a + b)*d*(1 + p))
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int \left(a+\left(\sin^2\left(dx+c\right)\right)b\right)^p \tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c),x)
```

```
[Out] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c), x)
```

Fricas [F]
time = 0.39, size = 25, normalized size = 0.42

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^2+a+b\right)^p \tan\left(dx+c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^2\left(c+dx\right)\right)^p\tan\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)**p*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(c+d\,x)\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(tan(c + d*x)*(a + b*sin(c + d*x)^2)^p, x)
```

3.546 $\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=54

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^2(c+dx)}{a}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad(1+p)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^2/a)*(a+b*sin(d*x+c)^2)^(1+p)/
a/d/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3273, 67}

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*S
in[c + d*x]^2)^(1 + p))/(a*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,\sin^2(c+dx)\right)}{2d}$$
$$=-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^2(c+dx)}{a}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad(1+p)}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.00

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^2(c+dx)}{a}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*S
in[c + d*x]^2)^(1 + p))/(a*d*(1 + p))
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int \cot(dx+c)\left(a+\left(\sin^2(dx+c)\right)b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+sin(d*x+c)^2*b)^p,x)
```

```
[Out] int(cot(d*x+c)*(a+sin(d*x+c)^2*b)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c), x)
```

Fricas [F]
time = 0.41, size = 25, normalized size = 0.46

$$\text{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p\cot(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^{2}\left(c+dx\right)\right)^{p}\cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)**p*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\cot(c+d\,x)\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(cot(c + d*x)*(a + b*sin(c + d*x)^2)^p, x)
```

3.547 $\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=95

$$-\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad}+\frac{(a-bp)\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^2(c+dx)}{a}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2a^2d(1+p)}$$

```
[Out] -1/2*csc(d*x+c)^2*(a+b*sin(d*x+c)^2)^(1+p)/a/d+1/2*(-b*p+a)*hypergeom([1, 1
+p],[2+p],1+b*sin(d*x+c)^2/a)*(a+b*sin(d*x+c)^2)^(1+p)/a^2/d/(1+p)
```

Rubi [A]

time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3273, 79, 67}

$$\frac{(a-bp)\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -1/2*(Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2)^(1 + p))/(a*d) + ((a - b*p)*Hyp
ergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*Sin[c + d*
x]^2)^(1 + p))/(2*a^2*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
```

```
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)(a+bx)^p}{x^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= -\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad} - \frac{(a-bp)\text{Subst}\left(\int \frac{(a+bx)^p}{x}\right.}{2ad}$$
$$= -\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad} + \frac{(a-bp)\,{}_2F_1\left(1,1+p;2\right.}{}$$

Mathematica [A]
time = 0.30, size = 73, normalized size = 0.77

$$-\frac{\left(a\csc^2(c+dx)+\frac{(-a+bp)\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^2(c+dx)}{a}\right)}{1+p}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -1/2*((a*Csc[c + d*x]^2 + ((-a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
+ (b*Sin[c + d*x]^2)/a])/(1 + p))*(a + b*Sin[c + d*x]^2)^(1 + p))/(a^2*d)
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int \left(\cot^3(dx+c)\right)\left(a+\left(\sin^2(dx+c)\right)b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+sin(d*x+c)^2*b)^p,x)
```

```
[Out] int(cot(d*x+c)^3*(a+sin(d*x+c)^2*b)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.49, size = 27, normalized size = 0.28

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\cot\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^3\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^2)^p, x)
```

3.548 $\int (a+b\sin^2(c+dx))^p \tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\sin^4(c+dx)\,(a+b\sin^2(c+dx))^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)^{-p}\tan(c+dx)}{5d}$$

```
[Out] 1/5*AppellF1(5/2,5/2,-p,7/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*sin(d*x+c)^4*(a
+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)*tan(d*x+c)/d/((1+b*sin(d*x+c)^2/a)^
p)
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3275, 525, 524}

$$\frac{\sin^4(c+dx)\sqrt{\cos^2(c+dx)}\,\tan(c+dx)\,(a+b\sin^2(c+dx))^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^4,x]
```

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(5*d*
(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^4(c+dx)\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\ \sec(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^p}{\left(1-x^2\right)^{5/2}}\,dx, x, \sin(c+dx\right.}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\ \sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right.\right.}{d}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\ \sin^4(c+}{5d}$$

Mathematica [A]
time = 5.32, size = 102, normalized size = 1.01

$$\frac{F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\ \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}\tan(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^4,x]
```

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(5*d*
((a + b*Sin[c + d*x]^2)/a)^p)
```

Maple [F]
time = 0.63, size = 0, normalized size = 0.00

$$\int \left(a+\left(\sin^2\left(dx+c\right)\right)b\right)^p\left(\tan^4\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^4,x)
```

```
[Out] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^4, x)
```

Fricas [**F**]
time = 0.45, size = 27, normalized size = 0.27

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^2+a+b\right)^p\tan\left(dx+c\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^4\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^2)^p, x)
```

3.549 $\int (a+b\sin^2(c+dx))^p \tan^2(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\sin^2(c+dx)\,(a+b\sin^2(c+dx))^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)}{3d}$$

```
[Out] 1/3*AppellF1(3/2,3/2,-p,5/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*sin(d*x+c)^2*(a
+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)*tan(d*x+c)/d/((1+b*sin(d*x+c)^2/a)^
p)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3275, 525, 524}

$$\frac{\sin^2(c+dx)\sqrt{\cos^2(c+dx)}\,\tan(c+dx)\,(a+b\sin^2(c+dx))^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^2,x]
```

```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(3*d*
(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int (a+b\sin^2(c+dx))^p \tan^2(c+dx)\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\right)\text{Subst}\left(\int \frac{x^2(a+bx^2)^p}{(1-x^2)^{3/2}}\,dx, x, \sin(c+$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\,(a+b\sin^2(c+dx))^p\left(1+\frac{b\sin^2(c+d}{a}\right.\right.}{d}$$

$$= \frac{F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\sin^2(c}{3d}$$

Mathematica [A]
time = 0.22, size = 102, normalized size = 1.01

$$\frac{F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\sin^2(c+dx)\,(a+b\sin^2(c+dx))^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}\tan(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^2,x]
```

```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(3*d*
((a + b*Sin[c + d*x]^2)/a)^p)
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$\int \left(a+\left(\sin^2\left(dx+c\right)\right)b\right)^p\left(\tan^2\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^2,x)
```

```
[Out] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.39, size = 27, normalized size = 0.27

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\tan\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^2)^p, x)
```

3.550 $\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=97

$$-\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p}{d}$$

```
[Out] -AppellF1(-1/2,-1/2,-p,1/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*csc(d*x+c)*sec(d
*x+c)*(a+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)/d/((1+b*sin(d*x+c)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3275, 525, 524}

$$-\frac{\sqrt{\cos^2(c+dx)}\,\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(d*
(1 + (b*Sin[c + d*x]^2)/a)^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
```

```
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\,(a+bx^2)^p}{x^2}\,dx, x, \text{si}\right.}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right.\right.}{d}$$

$$= -\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\text{cs}}{d}$$

Mathematica [A]
time = 0.14, size = 98, normalized size = 1.01

$$-\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p,x]
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(d*
((a + b*Sin[c + d*x]^2)/a)^p))
```

Maple [F]
time = 0.56, size = 0, normalized size = 0.00

$$\int \left(\cot^2(dx+c)\right)\left(a+\left(\sin^2(dx+c)\right)b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+sin(d*x+c)^2*b)^p,x)
[Out] int(cot(d*x+c)^2*(a+sin(d*x+c)^2*b)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.41, size = 27, normalized size = 0.28

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\cot\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^2\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^2)^p, x)
```

3.551 $\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=101

$$-\frac{F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)}{3d}$$

```
[Out] -1/3*AppellF1(-3/2,-3/2,-p,-1/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*csc(d*x+c)^
3*sec(d*x+c)*(a+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)/d/((1+b*sin(d*x+c)^2
/a)^p)
```

Rubi [A]

time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3275, 525, 524}

$$-\frac{\sqrt{\cos^2(c+dx)}\,\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -1/3*(AppellF1[-3/2, -3/2, -p, -1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a
)]*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^3*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^
p)/(d*(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3275

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1
```

```
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Subst[Int[x^m*((a + b*ff^2*x^2)^
p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^p}{x^4}\,dx,x,\text{si}\right.}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+d}{a}\right.\right.}{d}$$

$$= -\frac{F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}}{}$$

Mathematica [A]
time = 4.99, size = 102, normalized size = 1.01

$$-\frac{F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -1/3*(AppellF1[-3/2, -3/2, -p, -1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a
)]*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^3*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^
p)/(d*((a + b*Sin[c + d*x]^2)/a)^p)
```

Maple [F]
time = 0.56, size = 0, normalized size = 0.00

$$\int \left(\cot^4(dx+c)\right)\left(a+\left(\sin^2(dx+c)\right)b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+sin(d*x+c)^2*b)^p,x)
```

```
[Out] int(cot(d*x+c)^4*(a+sin(d*x+c)^2*b)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^4, x)
```

Fricas [F]
time = 0.44, size = 27, normalized size = 0.27

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\cot\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^4\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^2)^p, x)
```

3.552 $\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx$

Optimal. Leaf size=153

$$\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{6a}$$

```
[Out] -1/2*csc(x)^2/a-ln(sin(x))/a-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*sin(x))/a^(5/3)
+1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(x)+b^(2/3)*sin(x)^2)/a^(5/3)+1/
3*ln(a+b*sin(x)^3)/a+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(x))/a^(1
/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3309, 1848, 1885, 206, 31, 648, 631, 210, 642, 266}

$$\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(x)+b^{2/3}\sin^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}-\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/(a + b*Sin[x]^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a
^(5/3)) - Csc[x]^2/(2*a) - Log[Sin[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3)*S
in[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[x] + b^(2/
3)*Sin[x]^2])/(6*a^(5/3)) + Log[a + b*Sin[x]^3]/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{x^3\left(a+bx^3\right)}\,dx, x, \sin(x)\right) \\
&= \text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{ax}+\frac{b(-1+x^2)}{a\left(a+bx^3\right)}\right)\,dx, x, \sin(x)\right) \\
&= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}+\frac{b\text{Subst}\left(\int \frac{-1+x^2}{a+bx^3}\,dx, x, \sin(x)\right)}{a} \\
&= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(x)\right)}{a}+\frac{b\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sin(
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(x)\right)}{3a^{5/3}} \\
&= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}+\frac{b^{2/3}\text{Su}}{} \\
&= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin\right.}{6a^{5/3}} \\
&= \frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(x)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 143, normalized size = 0.93

$$
\frac{-3a^{2/3}\csc^2(x)-6a^{2/3}\log(\sin(x))+2\left(a^{2/3}-(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(x)\right)+2\left(a^{2/3}-b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)+2\left(a^{2/3}+\sqrt[3]{-1}\,b^{2/3}\right)\log\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(x)\right)}{6a^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/(a + b*Sin[x]^3),x]
```

```
[Out] (-3*a^(2/3)*Csc[x]^2 - 6*a^(2/3)*Log[Sin[x]] + 2*(a^(2/3) - (-1)^(2/3)*b^(2
/3))*Log[-((-1)^(2/3)*a^(1/3)) - b^(1/3)*Sin[x]] + 2*(a^(2/3) - b^(2/3))*Lo
g[a^(1/3) + b^(1/3)*Sin[x]] + 2*(a^(2/3) + (-1)^(1/3)*b^(2/3))*Log[a^(1/3)
+ (-1)^(2/3)*b^(1/3)*Sin[x]])/(6*a^(5/3))
```

Maple [A]
time = 0.77, size = 132, normalized size = 0.86

method	result
risch	$\frac{2\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}-1)^2 a} - i\left(\sum_{\_R=\mathrm{RootOf}(27\_Z^3a^5-27ia^4\_Z^2-9\_Za^3+ia^2-ib^2)} \_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{6a^2\_R}{b}+\frac{2ia}{b}\right)\mathrm{e}^{ix}-1\right)\right)$
default	$\frac{\left(-\frac{\ln\left(\sin(x)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(\sin^2(x)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(x)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(a+b\left(\sin^3(x)\right)\right)}{3b}\right)b}{a} - \frac{1}{2a\sin(x)^2} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*sin(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3/b/(1/b*a)^(2/3)*ln(sin(x)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(2/3)*ln(sin(x
)^2-(1/b*a)^(1/3)*sin(x)+(1/b*a)^(2/3))-1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(x)-1))+1/3/b*ln(a+b*sin(x)^3))/a*b-1/2/a/s
in(x)^2-ln(sin(x))/a
```

Maxima [A]
time = 0.53, size = 152, normalized size = 0.99

$$-\frac{\sqrt{3}\left(b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)+2a\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(x)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2}+\frac{\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)\log\left(\sin(x)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(x)\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log(\sin(x))}{a}-\frac{1}{2a\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(b*(3*(a/b)^(1/3) - 2*a/b) + 2*a)*arctan(-1/3*sqrt(3)*((a/b)^(
1/3) - 2*sin(x))/(a/b)^(1/3))/a^2 + 1/6*(2*(a/b)^(2/3) + 1)*log(sin(x)^2 -
(a/b)^(1/3)*sin(x) + (a/b)^(2/3))/(a*(a/b)^(2/3)) + 1/3*((a/b)^(2/3) - 1)*l
og((a/b)^(1/3) + sin(x))/(a*(a/b)^(2/3)) - log(sin(x))/a - 1/2/(a*sin(x)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 38.73, size = 1764, normalized size = 11.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(6*sqrt(1/3)*(a*cos(x)^2 - a)*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1
/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) +
1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a + 4)/a^
2)*arctan(-1/8*(2*sqrt(1/3)*sqrt((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a
^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^4 - 4*b^2*cos(x)^2 - 4*a*b*sin(
x) - 2*(a^2*b*sin(x) - 2*a^3)*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5
+ 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a) + 4*a^2 + 4*b^2)*((3*(I*sqrt(3) + 1)*(
-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a^3 + 2*a^2)*
sqrt(((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^
(1/3) - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*
(a^2 - b^2)/a^5)^(1/3) - 2/a)*a + 4)/a^2) + sqrt(1/3)*((3*(I*sqrt(3) + 1)*(
-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^5 - 8*a^2
*b*sin(x) + 4*a^3 - 4*(a^3*b*sin(x) - a^4)*(3*(I*sqrt(3) + 1)*(-1/54/a^3 +
1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a))*sqrt(((3*(I*sqrt(3) + 1)
*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 + 4*(
3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) -
 2/a)*a + 4)/a^2))/b^2) - 6*sqrt(1/3)*(a*cos(x)^2 - a)*sqrt(((3*(I*sqrt(3)
+ 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 +
 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/
3) - 2/a)*a + 4)/a^2)*arctan(-1/8*(2*sqrt(1/3)*sqrt((3*(I*sqrt(3) + 1)*(-1/
54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^4 - 4*b^2*co
s(x)^2 - 4*a*b*sin(x) - 2*(a^2*b*sin(x) - 2*a^3)*(3*(I*sqrt(3) + 1)*(-1/54/
a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a) + 4*a^2 + 4*b^2)*((
3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) -
 2/a)*a^3 + 2*a^2)*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/5
4*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1
/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a + 4)/a^2) - sqrt(1/3)*((
3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) -
 2/a)^2*a^5 - 8*a^2*b*sin(x) + 4*a^3 - 4*(a^3*b*sin(x) - a^4)*(3*(I*sqrt(3)
 + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a))*sqrt(
((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3)
 - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2
- b^2)/a^5)^(1/3) - 2/a)*a + 4)/a^2))/b^2) + (a*cos(x)^2 - a)*(3*(I*sqrt(3)
 + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*log(1/
4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3
) - 2/a)^2*a^4 - b^2*cos(x)^2 + 2*a*b*sin(x) + (a^2*b*sin(x) + a^3)*(3*(I*s
qrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)
+ a^2 + b^2) - ((a*cos(x)^2 - a)*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a
^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a) + 6*cos(x)^2 - 6)*log((3*(I*sqrt(3)
 + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^4
- 4*b^2*cos(x)^2 - 4*a*b*sin(x) - 2*(a^2*b*sin(x) - 2*a^3)*(3*(I*sqrt(3) +
1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a) + 4*a^2 +
 4*b^2) + 12*(cos(x)^2 - 1)*log(-1/2*sin(x)) - 6)/(a*cos(x)^2 - a)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(x)}{a + b\sin^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*sin(x)**3),x)
```

```
[Out] Integral(cot(x)**3/(a + b*sin(x)**3), x)
```

Giac [A]
time = 0.50, size = 144, normalized size = 0.94

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(x)\right|\right)}{3\,a^2}+\frac{\log\left(\left|b\sin(x)^3+a\right|\right)}{3\,a}-\frac{\log\left(\left|\sin(x)\right|\right)}{a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(x)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\sin(x)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(x)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2}-\frac{1}{2\,a\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(x)))/a^2 + 1/3*log(abs(b*sin
(x)^3 + a))/a - log(abs(sin(x)))/a - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*
sqrt(3)*((-a/b)^(1/3) + 2*sin(x))/(-a/b)^(1/3))/a^2 - 1/6*(-a*b^2)^(1/3)*lo
g(sin(x)^2 + (-a/b)^(1/3)*sin(x) + (-a/b)^(2/3))/a^2 - 1/2/(a*sin(x)^2)
```

Mupad [B]
time = 17.54, size = 2003, normalized size = 13.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + b*sin(x)^3),x)
```

```
[Out] symsum(log(-(256*(64*b^7*tan(x/2) + 32*a*b^6 - 44*a^3*b^4 + 15*a^5*b^2 - 10
24*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*b^8*tan(x/2)^2
 - 84*a^2*b^5*tan(x/2) + 26*a^4*b^3*tan(x/2) + 48*a*b^6*tan(x/2)^2 - 16*roo
t(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^2*b^6 + 328*root(2
7*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^4*b^4 - 165*root(27*a
^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^6*b^2 - 70*a^3*b^4*tan(x
/2)^2 + 25*a^5*b^2*tan(x/2)^2 - 48*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e -
 a^2 + b^2, e, k)^2*a^3*b^6 - 915*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e -
a^2 + b^2, e, k)^2*a^5*b^4 + 630*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a
^2 + b^2, e, k)^2*a^7*b^2 + 873*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^
2 + b^2, e, k)^3*a^6*b^4 - 810*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2
 + b^2, e, k)^3*a^8*b^2 + 864*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2
+ b^2, e, k)^4*a^7*b^4 - 405*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 +
 b^2, e, k)^4*a^9*b^2 - 1296*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 +
```

b^2, e, k)^5*a^8*b^4 + 1215*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^5*a^10*b^2 - 608*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a*b^7*tan(x/2) - 8880*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^3*b^6*tan(x/2)^2 + 5067*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^5*b^4*tan(x/2)^2 + 1050*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^7*b^2*tan(x/2)^2 + 27648*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3*a^4*b^6*tan(x/2)^2 - 15543*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3*a^6*b^4*tan(x/2)^2 - 1350*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3*a^8*b^2*tan(x/2)^2 - 27648*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^4*a^5*b^6*tan(x/2)^2 + 10800*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^4*a^7*b^4*tan(x/2)^2 - 675*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^4*a^9*b^2*tan(x/2)^2 + 9072*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^5*a^8*b^4*tan(x/2)^2 + 2025*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^5*a^10*b^2*tan(x/2)^2 + 1566*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^3*b^5*tan(x/2) - 610*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^5*b^3*tan(x/2) + 1760*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^2*b^6*tan(x/2)^2 - 260*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^4*b^4*tan(x/2)^2 - 275*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^6*b^2*tan(x/2)^2 + 1536*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^2*b^7*tan(x/2) - 9870*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^4*b^5*tan(x/2) + 5238*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^6*b^3*tan(x/2) + 31968*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3*a^5*b^5*tan(x/2) - 21150*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3*a^7*b^3*tan(x/2) - 57888*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^4*a^6*b^5*tan(x/2) + 40824*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^4*a^8*b^3*tan(x/2) + 41472*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^5*a^7*b^5*tan(x/2) - 30456*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^5*a^9*b^3*tan(x/2)))/a^3)*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k), k, 1, 3) - 1/(8*a*tan(x/2)^2) - tan(x/2)^2/(8*a) - log(tan(x/2))/a

3.553 $\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx$

Optimal. Leaf size=45

$$-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\sin^3(x)}$$

```
[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sin(x)^3)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3309, 272, 52, 65, 214}

$$\frac{2}{3}\sqrt{a+b\sin^3(x)}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Sin[x]^3],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sin[x]
^3])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \sin(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^3(x)\right)\\
&= \frac{2}{3}\sqrt{a+b\sin^3(x)} + \frac{1}{3}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^3(x)\right)\\
&= \frac{2}{3}\sqrt{a+b\sin^3(x)} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^3(x)}\right)}{3b}\\
&= -\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right) + \frac{2}{3}\sqrt{a+b\sin^3(x)}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right) + \frac{2}{3}\sqrt{a+b\sin^3(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Sin[x]^3],x]
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sin[x]
^3])/3
```

Maple [A]
time = 6.68, size = 34, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sin^{3}\left(x\right)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{a+b\left(\sin^{3}\left(x\right)\right)}}{3}$	34
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sin^{3}\left(x\right)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{a+b\left(\sin^{3}\left(x\right)\right)}}{3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sin(x)^3)^(1/2)
```

Maxima [A]
time = 0.50, size = 52, normalized size = 1.16

$$\frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{b\sin\left(x\right)^{3}+a}-\sqrt{a}}{\sqrt{b\sin\left(x\right)^{3}+a}+\sqrt{a}}\right)+\frac{2}{3}\sqrt{b\sin\left(x\right)^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(a)*log((sqrt(b*sin(x)^3 + a) - sqrt(a))/(sqrt(b*sin(x)^3 + a) + sq
rt(a))) + 2/3*sqrt(b*sin(x)^3 + a)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^{3}\left(x\right)}\,\cot\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(x)**3)*cot(x), x)
```

Giac [A]
time = 0.43, size = 38, normalized size = 0.84

$$\frac{2\,a\arctan\left(\frac{\sqrt{b\sin\left(x\right)^{3}+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2}{3}\,\sqrt{b\sin\left(x\right)^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a*arctan(sqrt(b*sin(x)^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*sin(x)^3
+ a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(x)\,\sqrt{b\sin\left(x\right)^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*sin(x)^3)^(1/2),x)
```

```
[Out] int(cot(x)*(a + b*sin(x)^3)^(1/2), x)
```

3.554 $\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3309, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Sin[x]^3],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^3/Sqrt[a]])/(3*Sqrt[a])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^3}}\,dx,x,\sin(x)\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\sin^3(x)\right)\\ &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sin^3(x)}\right)}{3b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Sin[x]^3],x]
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Maple [A]
time = 2.30, size = 21, normalized size = 0.75

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sin^3(x)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$	21

default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sin^{3}\left(x\right)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.52, size = 39, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{b\sin\left(x\right)^{3}+a}-\sqrt{a}}{\sqrt{b\sin\left(x\right)^{3}+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log((sqrt(b*sin(x)^3 + a) - sqrt(a))/(sqrt(b*sin(x)^3 + a) + sqrt(a)))/
sqrt(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+3*?+1)),fail
ed) ca
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(x\right)}{\sqrt{a+b\sin^{3}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*sin(x)**3), x)
```

Giac [A]
time = 0.43, size = 24, normalized size = 0.86

$$\frac{2 \arctan\left(\frac{\sqrt{b \sin(x)^3 + a}}{\sqrt{-a}}\right)}{3 \sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="giac")
[Out] 2/3*arctan(sqrt(b*sin(x)^3 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cot(x)}{\sqrt{b \sin(x)^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*sin(x)^3)^(1/2),x)
[Out] int(cot(x)/(a + b*sin(x)^3)^(1/2), x)
```

3.555 $\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d}+\frac{\sqrt{a+b\sin^4(c+dx)}}{2d}$$

```
[Out] -1/2*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))*a^(1/2)/d+1/2*(a+b*sin(d*x+c
)^4)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3308, 272, 52, 65, 214}

$$\frac{\sqrt{a+b\sin^4(c+dx)}}{2d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]])/d + Sqrt[a + b*S
in[c + d*x]^4]/(2*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx, x, \sin^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^4(c+dx)\right)}{4d} \\
&= \frac{\sqrt{a+b\sin^4(c+dx)}}{2d} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^4(c+dx)\right)}{4d} \\
&= \frac{\sqrt{a+b\sin^4(c+dx)}}{2d} + \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^4(c+dx)}\right)}{2bd} \\
&= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d} + \frac{\sqrt{a+b\sin^4(c+dx)}}{2d}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.93

$$-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)-\sqrt{a+b\sin^4(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - Sqrt[a + b*Sin[
c + d*x]^4])/d
```

Maple [F]
time = 2.60, size = 0, normalized size = 0.00

$$\int \cot \left(dx+c\right) \sqrt{a+b \left(\sin^{4}\left(dx+c\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [A]
time = 0.52, size = 68, normalized size = 1.15

$$\frac{\sqrt{a} \log\left(\frac{\sqrt{b \sin\left(dx+c\right)^{4}+a}-\sqrt{a}}{\sqrt{b \sin\left(dx+c\right)^{4}+a}+\sqrt{a}}\right)+2 \sqrt{b \sin\left(dx+c\right)^{4}+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(a)*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c
)^4 + a) + sqrt(a))) + 2*sqrt(b*sin(d*x + c)^4 + a))/d
```

Fricas [A]
time = 0.55, size = 195, normalized size = 3.31

$$\left[\frac{\sqrt{a} \log\left(\frac{8 \left(b \cos\left(dx+c\right)^{4}-2 b \cos\left(dx+c\right)^{2}-2 \sqrt{b \cos\left(dx+c\right)^{4}-2 b \cos\left(dx+c\right)^{2}+a+b}\,\sqrt{a}+2 a+b\right)}{\cos\left(dx+c\right)^{4}-2 \cos\left(dx+c\right)^{2}+1}\right)+2 \sqrt{b \cos\left(dx+c\right)^{4}-2 b \cos\left(dx+c\right)^{2}+a+b}}{4\,d},\ \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{b \cos\left(dx+c\right)^{4}-2 b \cos\left(dx+c\right)^{2}+a+b}\,\sqrt{-a}}{a}\right)+\sqrt{b \cos\left(dx+c\right)^{4}-2 b \cos\left(dx+c\right)^{2}+a+b}}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4
- 2*cos(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a
 + b))/d, 1/2*(sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 +
 a + b)*sqrt(-a)/a) + sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b))/
d]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^4(c + dx)} \cot(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x)**4)*cot(c + d*x), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c + d\,x)\, \sqrt{b\sin(c + d\,x)^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.556 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=89

$$-\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2(a+b)^{3/2}d}+\frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d}$$

```
[Out] -1/2*a*arctanh((a+b*sin(d*x+c)^2)/(a+b)^(1/2)/(a+b*sin(d*x+c)^4)^(1/2))/(a+
b)^(3/2)/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)^4)^(1/2)/(a+b)/d
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3308, 821, 739, 212}

$$\frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2d(a+b)}-\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*(a*ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]
^4])])/((a + b)^(3/2)*d) + (Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(2*(
a + b)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d} - \frac{a\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \sin^2(c-$$

$$= \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d} + \frac{a\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\sin^2(c+d}{\sqrt{a+b\sin^4(c+}}$$

$$= -\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2(a+b)^{3/2}d} + \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+}}{2(a+b)d}$$

Mathematica [A]
time = 0.13, size = 85, normalized size = 0.96

$$-\frac{\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{(a+b)^{3/2}} - \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{a+b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*((a*ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x
]^4])])/(a + b)^(3/2) - (Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(a + b)
)/d
```

Maple [F]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(77) = 154.
time = 0.56, size = 361, normalized size = 4.06

$$\left[\frac{\sqrt{a+b}\,a\cos(dx+c)^2\log\left(\frac{(ab+2b^2)\cos(dx+c)^4-4(ab+b^2)\cos(dx+c)^2+2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\,(b\cos(dx+c)^2-a-b)\sqrt{a+b}+2a^2+4ab+2b^2}{\cos(dx+c)^4}\right)+2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\,(a+b)}{4(a^2+2ab+b^2)d\cos(dx+c)^2},\ -\frac{a\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\,(b\cos(dx+c)^2-a-b)\sqrt{-a-b}}{(ab+b^2)\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right)\cos(dx+c)^2-\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\,(a+b)}{2(a^2+2ab+b^2)d\cos(dx+c)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*a*cos(d*x + c)^2*log(((a*b + 2*b^2)*cos(d*x + c)^4 - 4*(a
*b + b^2)*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a
 + b)*(b*cos(d*x + c)^2 - a - b)*sqrt(a + b) + 2*a^2 + 4*a*b + 2*b^2)/cos(d
*x + c)^4) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(a + b))
/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2), -1/2*(a*sqrt(-a - b)*arctan(sqrt(b
*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(b*cos(d*x + c)^2 - a - b)*sq
rt(-a - b)/((a*b + b^2)*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
 + 2*a*b + b^2))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^
2 + a + b)*(a + b))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^3}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(tan(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.557 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2\sqrt{a+b}\,d}$$

```
[Out] 1/2*arctanh((a+b*sin(d*x+c)^2)/(a+b)^(1/2)/(a+b*sin(d*x+c)^4)^(1/2))/d/(a+b
)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3308, 739, 212}

$$\frac{\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^4])]/(2
*Sqrt[a + b]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
```

```
ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\right)}{2d}$$

$$= \frac{\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 1.27

$$\frac{\tanh^{-1}\left(\frac{a+b-b\cos^2(c+dx)}{\sqrt{a+b}\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2\sqrt{a+b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
[Out] ArcTanh[(a + b - b*Cos[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b - 2*b*Cos[c + d*
x]^2 + b*Cos[c + d*x]^4])]/(2*Sqrt[a + b]*d)
```

Maple [A]
time = 0.49, size = 72, normalized size = 1.41

method	result	size
default	$\frac{\ln\left(\frac{2a+2b-2b\left(\cos^2(dx+c)\right)+2\sqrt{a+b}\,\sqrt{a+b-2b\left(\cos^2(dx+c)\right)+b\left(\cos^4(dx+c)\right)}}{\cos(dx+c)^2}\right)}{2d\sqrt{a+b}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/(a+b)^(1/2)*ln((2*a+2*b-2*b*cos(d*x+c)^2+2*(a+b)^(1/2)*(a+b-2*b*cos(d
*x+c)^2+b*cos(d*x+c)^4)^(1/2))/cos(d*x+c)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(43) = 86.
time = 0.54, size = 240, normalized size = 4.71

$$\left[\frac{\log\left(\frac{(ab+2b^2)\cos(dx+c)^4-4(ab+b^2)\cos(dx+c)^2-2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\left(b\cos(dx+c)^2-a-b\right)\sqrt{a+b}+2a^2+4ab+2b^2}{\cos(dx+c)^4}\right)}{4\sqrt{a+b}\,d}, \frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\left(b\cos(dx+c)^2-a-b\right)\sqrt{-a-b}}{(ab+b^2)\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right)}{2(a+b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a*b + 2*b^2)*cos(d*x + c)^4 - 4*(a*b + b^2)*cos(d*x + c)^2 - 2*s
qrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(b*cos(d*x + c)^2 - a -
b)*sqrt(a + b) + 2*a^2 + 4*a*b + 2*b^2)/cos(d*x + c)^4)/(sqrt(a + b)*d), 1/
2*sqrt(-a - b)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(
b*cos(d*x + c)^2 - a - b)*sqrt(-a - b)/((a*b + b^2)*cos(d*x + c)^4 - 2*(a*b
 + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))/((a + b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(c+d\,x)}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(tan(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.558 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

```
[Out] -1/2*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3308, 272, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^4(c+dx)\right)}{4d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^4(c+dx)}\right)}{2bd}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Mathematica [**A**]
time = 0.02, size = 35, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(Sqrt[a]*d)
```

Maple [**F**]
time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
[Out] int(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [A]
time = 0.52, size = 50, normalized size = 1.43

$$\frac{\log\left(\frac{\sqrt{b\sin\left(dx+c\right)^4+a}-\sqrt{a}}{\sqrt{b\sin\left(dx+c\right)^4+a}+\sqrt{a}}\right)}{4\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] 1/4*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 + a)
 + sqrt(a)))/(sqrt(a)*d)
```

Fricas [A]
time = 0.52, size = 140, normalized size = 4.00

$$\left[\frac{\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2-2\,\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b}\,\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)}{4\,\sqrt{a}\,d},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b}\,\sqrt{-a}}{a}\right)}{2\,ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] [1/4*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c)^4
 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1))/(sqrt(a)*d), 1/2*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*
b*cos(d*x + c)^2 + a + b)*sqrt(-a)/a)/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(c+dx\right)}{\sqrt{a+b\sin^4\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(cot(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(c+d\,x)}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(cot(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.559 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

[Out] 1/2*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))/d/a^(1/2)-1/2*csc(d*x+c)^2*(a +b*sin(d*x+c)^4)^(1/2)/a/d

Rubi [A]
time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3308, 821, 272, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]

[Out] ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(2*Sqrt[a]*d) - (Csc[c + d*x]^2 *Sqrt[a + b*Sin[c + d*x]^4])/(2*a*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= -\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sin^2(c+d\ldots\right.}{2d}$$

$$= -\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^4(c+dx\ldots\right.}{4d}$$

$$= -\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^4(c\ldots}\right.}{2bd}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d} - \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

Mathematica [A]

time = 0.05, size = 66, normalized size = 0.94

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)-\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - Csc[c + d*x]^2*Sqrt[
a + b*Sin[c + d*x]^4])/(2*a*d)
```

Maple [F]
time = 2.70, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [A]
time = 0.56, size = 79, normalized size = 1.13

$$-\frac{\frac{\log\left(\frac{\sqrt{b\sin(dx+c)^4+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^4+a}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\sqrt{b\sin(dx+c)^4+a}}{a\sin(dx+c)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 + a
) + sqrt(a)))/sqrt(a) + 2*sqrt(b*sin(d*x + c)^4 + a)/(a*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.55, size = 247, normalized size = 3.53

$$\left[\frac{(\cos(dx+c)^2-1)\sqrt{a}\log\left(\frac{8\left(b\cos(dx+c)^4-2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\sqrt{a}+2a+b\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}\right)+2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}}{4\left(ad\cos(dx+c)^2-ad\right)}, -\frac{(\cos(dx+c)^2-1)\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\sqrt{-a}}{a}\right)-\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}}{2\left(ad\cos(dx+c)^2-ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cos(d*x + c)^2 - 1)*sqrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x +
c)^2 + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a
+ b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + a + b))/(a*d*cos(d*x + c)^2 - a*d), -1/2*((cos(d*x + c)
^2 - 1)*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
*sqrt(-a)/a) - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b))/(a*d*co
s(d*x + c)^2 - a*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cot(c+d\,x)}^3}{\sqrt{b\,{\sin\left(c+d\,x\right)}^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(cot(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.560 $\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=108

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}+\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad}$$

```
[Out] -1/4*(2*a-b)*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))/a^(3/2)/d+csc(d*x+c)
^2*(a+b*sin(d*x+c)^4)^(1/2)/a/d-1/4*csc(d*x+c)^4*(a+b*sin(d*x+c)^4)^(1/2)/a
/d
```

Rubi [A]
time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3308, 1821, 821, 272, 65, 214}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad}+\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/4*((2*a - b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4/Sqrt[a]])/(a^(3/2)*d) +
(Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(a*d) - (Csc[c + d*x]^4*Sqrt[a
+ b*Sin[c + d*x]^4])/(4*a*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= -\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} - \frac{\text{Subst}\left(\int \frac{4a-(2a-b)x}{x^2\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx\right.}{4ad}$$

$$= \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} + \frac{(2a}{}$$

$$= \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} + \frac{(2a}{}$$

$$= \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} + \frac{(2a}{}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} + \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx}}{ad}$$

Mathematica [A]
time = 1.99, size = 141, normalized size = 1.31

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right) - 4a\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)} + b\sqrt{a+b\sin^4(c+dx)}\left(\frac{a\csc^4(c+dx)}{b} - \frac{\tanh^{-1}\left(\sqrt{1+\frac{b\sin^4(c+dx)}{a}}\right)}{\sqrt{1+\frac{b\sin^4(c+dx)}{a}}}\right)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/4*(2*a^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - 4*a*Csc[c + d
*x]^2*Sqrt[a + b*Sin[c + d*x]^4] + b*Sqrt[a + b*Sin[c + d*x]^4]*((a*Csc[c +
 d*x]^4)/b - ArcTanh[Sqrt[1 + (b*Sin[c + d*x]^4)/a]]/Sqrt[1 + (b*Sin[c + d*
x]^4)/a]))/(a^2*d)
```

Maple [F]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)

[Out] int(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)

Maxima [A]
time = 0.50, size = 166, normalized size = 1.54

$$-\frac{\frac{2\sqrt{b\sin\left(dx+c\right)^4+a}\,b}{\left(b\sin\left(dx+c\right)^4+a\right)a-a^2}-\frac{2\log\left(\frac{\sqrt{b\sin\left(dx+c\right)^4+a}-\sqrt{a}}{\sqrt{b\sin\left(dx+c\right)^4+a}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{b\log\left(\frac{\sqrt{b\sin\left(dx+c\right)^4+a}-\sqrt{a}}{\sqrt{b\sin\left(dx+c\right)^4+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{8\sqrt{b\sin\left(dx+c\right)^4+a}}{a\sin\left(dx+c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")

[Out] -1/8*(2*sqrt(b*sin(d*x + c)^4 + a)*b/((b*sin(d*x + c)^4 + a)*a - a^2) - 2*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 + a) + sqrt(a)))/sqrt(a) + b*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 + a) + sqrt(a)))/a^(3/2) - 8*sqrt(b*sin(d*x + c)^4 + a)/(a*sin(d*x + c)^2))/d

Fricas [A]
time = 0.52, size = 371, normalized size = 3.44

$$\left[-\frac{\left((2a-b)\cos\left(dx+c\right)^4-2(2a-b)\cos\left(dx+c\right)^2+2a-b\right)\sqrt{a}\log\left(\frac{8\left(b\cos\left(dx+c\right)^4-2b\cos\left(dx+c\right)^2+2\sqrt{b\cos\left(dx+c\right)^4-2b\cos\left(dx+c\right)^2+a+b}\,\sqrt{a}+2a+b\right)}{\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2+1}\right)+2\sqrt{b\cos\left(dx+c\right)^4-2b\cos\left(dx+c\right)^2+a+b}\,\left(4a\cos\left(dx+c\right)^2-3a\right)}{8\left(a^2d\cos\left(dx+c\right)^4-2a^2d\cos\left(dx+c\right)^2+a^2d\right)},\ \frac{\left((2a-b)\cos\left(dx+c\right)^4-2(2a-b)\cos\left(dx+c\right)^2+2a-b\right)\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)^4-2b\cos\left(dx+c\right)^2+a+b}\,\sqrt{-a}}{a}\right)-\sqrt{b\cos\left(dx+c\right)^4-2b\cos\left(dx+c\right)^2+a+b}\,\left(4a\cos\left(dx+c\right)^2-3a\right)}{4\left(a^2d\cos\left(dx+c\right)^4-2a^2d\cos\left(dx+c\right)^2+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")

[Out] [-1/8*(((2*a - b)*cos(d*x + c)^4 - 2*(2*a - b)*cos(d*x + c)^2 + 2*a - b)*sqrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(4*a*cos(d*x + c)^2 - 3*a))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d), 1/4*(((2*a - b)*cos(d*x + c)^4 - 2*(2*a - b)*cos(d*x + c)^2 + 2*a - b)*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(-a)/a) - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(4*a*cos(d*x + c)^2 - 3*a))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^5\left(c+dx\right)}{\sqrt{a+b\sin^4\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**5/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^5}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(cot(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.561 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=411

$$\frac{\cos(c+dx)\sin(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{\sqrt{a+b}\,d\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)}-\frac{\sqrt[4]{a}\cos^2(c+dx)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+\cdots}}{}\right.\right.}{}$$

```
[Out] -a^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/
2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arctan((a+
b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+2*a*
tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2
)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/(a+b)^(3/4)/d/(a+b*sin(d*x+c)^4)^(1/2)
+1/2*a^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)
^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan
((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+
2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^
(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/(a+b)^(3/4)/d/(a+b*sin(d*x+c)^4)^(
1/2)+cos(d*x+c)*sin(d*x+c)*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/d/(a+b)^
(1/2)/(a+b*sin(d*x+c)^4)^(1/2)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)
```

Rubi [A]
time = 0.27, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3311, 1153, 1117, 1209}

$$\frac{\sqrt[4]{a}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2d(a+b)^{3/4}\sqrt{a+b\sin^4(c+dx)}}-\frac{\sqrt[4]{a}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{d(a+b)^{3/4}\sqrt{a+b\sin^4(c+dx)}}+\frac{\sin(c+dx)\cos(c+dx)\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}{d\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]*Sin[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4
))/(Sqrt[a + b]*d*Sqrt[a + b*Sin[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c +
 d*x]^2)) - (a^(1/4)*Cos[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c
 + d*x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[
c + d*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a]
 + Sqrt[a + b]*Tan[c + d*x]^2)^2])/((a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]
^4]) + (a^(1/4)*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*
x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sq
rt[a + b]*Tan[c + d*x]^2)^2])/(2*(a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]
)
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 3311

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_)*((d_.)*tan[(e_.) + (f_.)*
(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff
*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f*x]^2)^(2*p)/(f*Apart[a*(1 + Tan[e + f
*x]^2)^2 + b*Tan[e + f*x]^4]^p)), Subst[Int[(d*ff*x)^m*(ExpandToSum[a*(1 +
ff^2*x^2)^2 + b*ff^4*x^4, x]^p/(1 + ff^2*x^2)^(2*p + 1)), x], x, Tan[e + f*
x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\Big(\int \frac{}{\sqrt{a}}}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\left(\sqrt{a}\,\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\Big(\int}{\sqrt{a+b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\cos(c+dx)\sin(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{\sqrt{a+b}\,d\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)} - \frac{\sqrt[4]{a}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 54.38, size = 291, normalized size = 0.71

$$-\frac{2i\sqrt{2}\sqrt{a}\cos^2(c+dx)\left(E\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)|\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)-F\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)|\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)\right)\sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}}{\left(\sqrt{a}+i\sqrt{b}\right)\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\,d\sqrt{8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ((-2*I)*Sqrt[2]*Sqrt[a]*Cos[c + d*x]^2*(EllipticE[I*ArcSinh[Sqrt[1 - (I*Sqr
t[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]
- EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a]
 + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[1 + (1 - (I*Sqrt[b])/Sqrt[a])*Ta
n[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2])/((Sqrt[a]
 + I*Sqrt[b])*Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Cos[2*(c
 + d*x)] + b*Cos[4*(c + d*x)]])
```

Maple [F]
time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F]
time = 0.11, size = 37, normalized size = 0.09

$$\text{integral}\left(\frac{\tan\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] integral(tan(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{2}\left(c+dx\right)}{\sqrt{a+b\sin^{4}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(tan(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^2}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(tan(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.562 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{\cos^2(c+dx)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\,\tan^2(c+dx)\right)\sqrt{\frac{a+2a\tan\cdots}{\left(\sqrt{a}\cdots\right.}}}{2\sqrt[4]{a}\,\sqrt[4]{a+b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] 1/2*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/co
s(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan((a+b)^(1
/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+2*a*tan(d
*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^
(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(1/4)/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1
/2)
```

Rubi [A]
time = 0.06, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3289, 1117}

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\,\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\,\tan^2(c+dx)+\sqrt{a}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}\,d\sqrt[4]{a+b}\,\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (
1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a
+ 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c
 + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 3289

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f
*x]^2)^(2*p)/(f*(a + 2*a*Tan[e + f*x]^2 + (a + b)*Tan[e + f*x]^4)^p)), Subs
```

```
t[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x],
 x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{}{\sqrt{a}\cdots}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\cos^2(c+dx)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a}\cdots\right.}{2\sqrt[4]{a}\sqrt[4]{a+b}\,d\sqrt{a+b\,\text{s}\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.90, size = 304, normalized size = 1.88

$$\frac{2\sqrt{2}\left(i\sqrt{a}+\sqrt{b}\right)\left(2\sqrt{a}-i\sqrt{b}+i\sqrt{b}\cos(2(c+dx))\right)\left(2i\sqrt{a}-\sqrt{b}+\sqrt{b}\cos(2(c+dx))\right)\sqrt{\left(1-\frac{2i\sqrt{a}}{\sqrt{b}}-\cos(2(c+dx))\right)\csc^2(c+dx)}\sqrt{\frac{\cot^2(c+dx)\left(i\sqrt{a}\sqrt{b}-a\csc^2(c+dx)\right)}{\left(\sqrt{a}-i\sqrt{b}\right)^2}}F\left(\sin^{-1}\left(\sqrt{\frac{-i\sqrt{b}+\sqrt{a}\csc^2(c+dx)}{\sqrt{a}-i\sqrt{b}}}\right)|\frac{1}{2}+\frac{i\sqrt{a}}{2\sqrt{b}}\right)\sin^2(c+dx)\tan(c+dx)}{\sqrt{a}\,d(8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (2*Sqrt[2]*(I*Sqrt[a] + Sqrt[b])*(2*Sqrt[a] - I*Sqrt[b] + I*Sqrt[b]*Cos[2*(
c + d*x)])*((2*I)*Sqrt[a] - Sqrt[b] + Sqrt[b]*Cos[2*(c + d*x)])*Sqrt[(1 - (
(2*I)*Sqrt[a])/Sqrt[b] - Cos[2*(c + d*x)])*Csc[c + d*x]^2]*Sqrt[(Cot[c + d*
x]^2*(I*Sqrt[a]*Sqrt[b] - a*Csc[c + d*x]^2))/(Sqrt[a] - I*Sqrt[b])^2]*Ellip
ticF[ArcSin[Sqrt[((-I)*Sqrt[b] + Sqrt[a]*Csc[c + d*x]^2)/(Sqrt[a] - I*Sqrt[
b])]], 1/2 + ((I/2)*Sqrt[a])/Sqrt[b]]*Sin[c + d*x]^2*Tan[c + d*x])/(Sqrt[a]
*d*(8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(181) = 362.
time = 58.51, size = 396, normalized size = 2.44

method	result

default	$-\frac{\sqrt{(4a+b(\cos^2(2dx+2c))+b-2b\cos(2dx+2c))(\sin^2(2dx+2c))}\,\sqrt{-ab}\,\sqrt{\frac{\left(-b+\sqrt{-ab}\right)(-\cdots}{\sqrt{-ab}\,(\cos(2\cdots}}}{\left(-b+\sqrt{-ab}\right)\sqrt{\frac{(-1+\cos(2dx+2c))(\cos(2dx+2c)+1)\left(-b\cos(2dx+2c)+2\cdots\right.}{b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((4*a+b*cos(2*d*x+2*c)^2+b-2*b*cos(2*d*x+2*c))*sin(2*d*x+2*c)^2)^(1/2)*(-a
*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*
c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a
*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-
a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*EllipticF(((-b+(-a*b)^(1/2))*(-1+cos(2
*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),((b+(-a*b)^(1/2))/(-b+(-a
*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d*x+2*c))*(cos(2*d*x+2*
c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)
-b))^(1/2)/sin(2*d*x+2*c)/(4*a+b*cos(2*d*x+2*c)^2+b-2*b*cos(2*d*x+2*c))^(1/
2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F]
time = 0.09, size = 28, normalized size = 0.17

$$\text{integral}\left(\frac{1}{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sin^4(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(1/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,{\sin\left(c+d\,x\right)}^4+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(1/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.563 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=477

$$-\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}}+\frac{\sqrt{a+b}\,\cos(c+dx)\sin(c+dx)\,(a}{ad\sqrt{a+b\sin^4(c+dx)}\,(}$$

```
[Out] -(a+b)^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)
^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arctan
((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+
2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^
(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/d/(a+b*sin(d*x+c)^4)^(1/2)
+1/2*(a+b)^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4))
)^2)^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*ar
ctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*
((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)
^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/d/(a+b*sin(d*x+c)^4)^(
1/2)-cos(d*x+c)^2*cot(d*x+c)*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/a/d/(a
+b*sin(d*x+c)^4)^(1/2)+cos(d*x+c)*sin(d*x+c)*(a+b)^(1/2)*(a+2*a*tan(d*x+c)^
2+(a+b)*tan(d*x+c)^4)/a/d/(a+b*sin(d*x+c)^4)^(1/2)/(a^(1/2)+(a+b)^(1/2)*tan
(d*x+c)^2)
```

Rubi [A]
time = 0.25, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3311, 1137, 12, 1153, 1117, 1209}

$$\frac{\sqrt[4]{a+b}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2a^{3/4}d\sqrt{a+b\sin^4(c+dx)}}-\frac{\sqrt[4]{a+b}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{a^{3/4}d\sqrt{a+b\sin^4(c+dx)}}+\frac{\sqrt{a+b}\sin(c+dx)\cos(c+dx)\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}{ad\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)}-\frac{\cos^2(c+dx)\cot(c+dx)\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}{ad\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((Cos[c + d*x]^2*Cot[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4))/(a*d*Sqrt[a + b*Sin[c + d*x]^4])) + (Sqrt[a + b]*Cos[c + d*x]*Sin[c
+ d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))/(a*d*Sqrt[a + b*S
in[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) - ((a + b)^(1/4)*Cos
[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 -
Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*
a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)^2])/(a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((a + b)^(1/4)*Cos[c +
d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 - Sqrt[
```

```
a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*a*Tan
[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2
)^2])/(2*a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 3311

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_)*((d_.)*tan[(e_.) + (f_.)*
(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff
*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f*x]^2)^(2*p)/(f*Apart[a*(1 + Tan[e + f
*x]^2)^2 + b*Tan[e + f*x]^4]^p)), Subst[Int[(d*ff*x)^m*(ExpandToSum[a*(1 +
ff^2*x^2)^2 + b*ff^4*x^4, x]^p/(1 + ff^2*x^2)^(2*p + 1)), x], x, Tan[e + f*
```

```
x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{}{x^2\sqrt{a}\ldots}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\left(\text{co}\ldots\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\left((a\ldots\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\left(\sqrt{\ldots}\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\sqrt{a}\ldots}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 58.31, size = 372, normalized size = 0.78

$$-\frac{\sqrt{2}\sqrt{8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx))}\cot(c+dx)+\frac{4\cos^4(c+dx)\left(a\sec^4(c+dx)\tan(c+dx)+b\tan^5(c+dx)+\frac{\left(ia+\sqrt{a}\sqrt{b}\right)\left(E\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle|\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)-F\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle|\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)\right)\sec^2(c+dx)\sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}}{\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\left(a+b+2a\cot^2(c+dx)+a\cot^4(c+dx)\right)\sin^4(c+dx)}}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/4*(Sqrt[2]*Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*C
ot[c + d*x] + (4*Cos[c + d*x]^4*(a*Sec[c + d*x]^4*Tan[c + d*x] + b*Tan[c +
d*x]^5 + ((I*a + Sqrt[a]*Sqrt[b])*(EllipticE[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])
/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])] - Ell
```

```
ipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*
Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sec[c + d*x]^2*Sqrt[1 + (1 - (I*Sqrt[b])/S
qrt[a])*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2])
/Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]))/Sqrt[(a + b + 2*a*Cot[c + d*x]^2 + a*Cot[c
 + d*x]^4)*Sin[c + d*x]^4])/(a*d)
```

Maple [F]
time = 1.86, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
[Out] int(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F]
time = 0.12, size = 37, normalized size = 0.08

$$\text{integral}\left(\frac{\cot(dx+c)^2}{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] integral(cot(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.564 $\int (a+b\sin^4(c+dx))^p\tan^m(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((a+b\sin^4(c+dx))^p\tan^m(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^4(c+dx))^p\tan^m(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m, x]
```

Rubi steps

$$\int (a+b\sin^4(c+dx))^p\tan^m(c+dx)\,dx = \int (a+b\sin^4(c+dx))^p\tan^m(c+dx)\,dx$$

Mathematica [A]
time = 6.75, size = 0, normalized size = 0.00

$$\int (a+b\sin^4(c+dx))^p\tan^m(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m, x]
```

Maple [A]
time = 1.74, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4(dx+c)\right)\right)^p \left(\tan^m(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^m, x)
```

Fricas [A]
time = 1.79, size = 37, normalized size = 1.42

$$\text{integral}\Big(\big(b\cos{(dx+c)}^4 - 2\,b\cos{(dx+c)}^2 + a + b\big)^p \tan{(dx+c)}^m\,, x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^m,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^m\,\left(b\sin{(c+d\,x)}^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^4)^p, x)
```

3.565 $\int (a+b\sin^4(c+dx))^p \tan^3(c+dx)\,dx$

Optimal. Leaf size=279

$$-\frac{(a+b+2bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^4(c+dx)}{a+b}\right)(a+b\sin^4(c+dx))^{1+p}}{4(a+b)^2d(1+p)}+\frac{\sec^2(c+dx)\,(a+b\sin^4(c+dx))^{1+p}}{2(a+b)d}$$

```
[Out] -1/4*(2*b*p+a+b)*hypergeom([1, 1+p],[2+p],(a+b*sin(d*x+c)^4)/(a+b))*(a+b*si
n(d*x+c)^4)^(1+p)/(a+b)^2/d/(1+p)+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)^4)^(1+p)
/(a+b)/d-1/2*(2*b*p+a+b)*AppellF1(1/2,1,-p,3/2,sin(d*x+c)^4,-b*sin(d*x+c)^4
/a)*sin(d*x+c)^2*(a+b*sin(d*x+c)^4)^p/(a+b)/d/((1+b*sin(d*x+c)^4/a)^p)+1/2*
b*(1+2*p)*hypergeom([1/2, -p],[3/2],-b*sin(d*x+c)^4/a)*sin(d*x+c)^2*(a+b*si
n(d*x+c)^4)^p/(a+b)/d/((1+b*sin(d*x+c)^4/a)^p)
```

Rubi [A]

time = 0.20, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3308, 849, 858, 252, 251, 771, 441, 440, 455, 70}

$$-\frac{(a+2bp+b)\sin^2(c+dx)\,(a+b\sin^4(c+dx))^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d(a+b)}-\frac{(a+2bp+b)\,(a+b\sin^4(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)+a}{a+b}\right)}{4d(p+1)(a+b)^2}+\frac{b(2p+1)\sin^2(c+dx)\,(a+b\sin^4(c+dx))^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^4(c+dx)}{a}\right)}{2d(a+b)}+\frac{\sec^2(c+dx)\,(a+b\sin^4(c+dx))^{p+1}}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^3,x]
```

```
[Out] -1/4*((a + b + 2*b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x
]^4)/(a + b)]*(a + b*Sin[c + d*x]^4)^(1 + p))/((a + b)^2*d*(1 + p)) + (Sec[
c + d*x]^2*(a + b*Sin[c + d*x]^4)^(1 + p))/(2*(a + b)*d) - ((a + b + 2*b*p)
*AppellF1[1/2, 1, -p, 3/2, Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c +
 d*x]^2*(a + b*Sin[c + d*x]^4)^p)/(2*(a + b)*d*(1 + (b*Sin[c + d*x]^4)/a)^p
) + (b*(1 + 2*p)*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[c + d*x]^4)/a)]*S
in[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p)/(2*(a + b)*d*(1 + (b*Sin[c + d*x]^4
)/a)^p)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p\tan^3(c+dx)\,dx=\frac{\text{Subst}\left(\int\frac{x(a+bx^2)^p}{(1-x)^2}\,dx,x,\sin^2(c+dx)\right)}{2d}$$

$$=\frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d}-\frac{\text{Subst}\left(\int\frac{(a+b(1+2p)x)(a+bx^2)^p}{1-x}\right.}{2(a+b)}$$

$$=\frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d}+\frac{(b(1+2p))\text{Subst}\left(\int(a+bx\right.}{2(a+}$$

$$=\frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d}-\frac{(a+b+2bp)\text{Subst}\left(\int\left(\frac{(a+}{1}\right.\right.}{}$$

$$=\frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d}+\frac{b(1+2p)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bs}{}\right.}{}$$

$$=\frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d}+\frac{b(1+2p)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bs}{}\right.}{}$$

$$=-\frac{(a+b+2bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^4(c+dx)}{a+b}\right)\left(a+b\sin^4(c+d\right.}{4(a+b)^2d(1+p)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 875 vs. 2(279) = 558.

time = 17.37, size = 875, normalized size = 3.14

$$\frac{\left(-b+\sqrt{-ab}\right)\left(b+\sqrt{-ab}\right)\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(-\frac{(1-2p)^2F_1\left(-2p;-p,-p;1-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)\left(a+b+\left(a-\sqrt{-ab}\right)\cot^2(c+dx)\right)\left(a+b+\left(a+\sqrt{-ab}\right)\cot^2(c+dx)\right)\sin^4(2(c+dx))}{8p\left(\left(\left(b+\sqrt{-ab}\right)pF_1\left(1-2p;1-p,-p;2-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)-\left(-b+\sqrt{-ab}\right)pF_1\left(1-2p;-p,1-p;2-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)+b(-1+2p)F_1\left(-2p;-p,-p;1-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)\cos^2(c+dx)\right)(8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))\right)}+\frac{(-1+p)F_1\left(1-2p;-p,-p;2-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)\left(a-\sqrt{-ab}+(a+b)\tan^2(c+dx)\right)\left(a+\sqrt{-ab}+(a+b)\tan^2(c+dx)\right)}{\left(2b(-1+p)F_1\left(1-2p;-p,-p;2-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)+p\left(\left(b+\sqrt{-ab}\right)F_1\left(2-2p;1-p,-p;3-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)+\left(b-\sqrt{-ab}\right)F_1\left(2-2p;-p,1-p;3-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)\right)\sec^2(c+dx)\right)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}\right)}{(a+b)^2d(-1+2p)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^3,x]
```

```
[Out] ((-b + Sqrt[-(a*b)])*(b + Sqrt[-(a*b)])*Sec[c + d*x]^2*(a + b*Sin[c + d*x]^
4)^p*(-1/8*((1 - 2*p)^2*AppellF1[-2*p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c +
d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*
(a + b + (a - Sqrt[-(a*b)])*Cot[c + d*x]^2)*(a + b + (a + Sqrt[-(a*b)])*Cot
[c + d*x]^2)*Sin[2*(c + d*x)]^4)/(p*((b + Sqrt[-(a*b)])*p*AppellF1[1 - 2*p,
 1 - p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a +
 b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] - (-b + Sqrt[-(a*b)])*p*AppellF1[1
- 2*p, -p, 1 - p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])),
 ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + b*(-1 + 2*p)*AppellF1[-2*p,
 -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)
*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]^2)*(8*a + 3*b - 4*b*Cos[2
*(c + d*x)] + b*Cos[4*(c + d*x)])) + ((-1 + p)*AppellF1[1 - 2*p, -p, -p, 2
- 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*
x]^2)/(b + Sqrt[-(a*b)])]*(a - Sqrt[-(a*b)] + (a + b)*Tan[c + d*x]^2)*(a +
Sqrt[-(a*b)] + (a + b)*Tan[c + d*x]^2))/((2*b*(-1 + p)*AppellF1[1 - 2*p, -p
, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Se
c[c + d*x]^2)/(b + Sqrt[-(a*b)])] + p*((b + Sqrt[-(a*b)])*AppellF1[2 - 2*p,
 1 - p, -p, 3 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a +
 b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + (b - Sqrt[-(a*b)])*AppellF1[2 - 2
*p, -p, 1 - p, 3 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((
a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])])*Sec[c + d*x]^2)*(a + 2*a*Tan[c
+ d*x]^2 + (a + b)*Tan[c + d*x]^4))))/((a + b)^2*d*(-1 + 2*p))
```

Maple [F]
time = 0.76, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p \left(\tan^3\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^3, x)
```

Fricas [F]
time = 0.48, size = 37, normalized size = 0.13

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\tan\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^3,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^3\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^4)^p,x)
```

```
[Out] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^4)^p, x)
```

3.566 $\int (a+b\sin^4(c+dx))^p \tan(c+dx)\,dx$

Optimal. Leaf size=141

$$\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^4(c+dx)}{a+b}\right)(a+b\sin^4(c+dx))^{1+p}}{4(a+b)d(1+p)}+\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)\sin^2(c+ \dots}{2d \dots}$$

```
[Out] 1/4*hypergeom([1, 1+p],[2+p],(a+b*sin(d*x+c)^4)/(a+b))*(a+b*sin(d*x+c)^4)^(
1+p)/(a+b)/d/(1+p)+1/2*AppellF1(1/2,1,-p,3/2,sin(d*x+c)^4,-b*sin(d*x+c)^4/a
)*sin(d*x+c)^2*(a+b*sin(d*x+c)^4)^p/d/((1+b*sin(d*x+c)^4/a)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3308, 771, 441, 440, 455, 70}

$$\frac{\sin^2(c+dx)\,(a+b\sin^4(c+dx))^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d}+\frac{(a+b\sin^4(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)+a}{a+b}\right)}{4d(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^4)/(a + b)]*(a + b*
Sin[c + d*x]^4)^(1 + p))/(4*(a + b)*d*(1 + p)) + (AppellF1[1/2, 1, -p, 3/2,
 Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d*x]^2*(a + b*Sin[c + d*x
]^4)^p)/(2*d*(1 + (b*Sin[c + d*x]^4)/a)^p)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sin^4(c+dx)\right)^p \tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{1-x}\,dx,x,\sin^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+bx^2)^p}{1-x^2}-\frac{x(a+bx^2)^p}{-1+x^2}\right)\,dx,x,\sin^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{1-x^2}\,dx,x,\sin^2(c+dx)\right)}{2d}-\frac{\text{Subst}\left(\int \frac{x(a+bx^2)^p}{-1+x^2}\,dx,x,\sin\right.}{2d} \\
&= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{-1+x}\,dx,x,\sin^4(c+dx)\right)}{4d}+\frac{\left(\left(a+b\sin^4(c+dx)\right)^p\left(1\right.\right.}{} \\
&= \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^4(c+dx)}{a+b}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4(a+b)d(1+p)}+\frac{F_1\left(\frac{1}{2};\right.}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 463 vs. 2(141) = 282.

time = 7.69, size = 463, normalized size = 3.28

$$\frac{2\left(-b+\sqrt{-ab}\right)\left(b+\sqrt{-ab}\right)(-1+2p)F_1\left(-2p;-p,-p;1-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)\cos^2(c+dx)\left(a+b+\left(a-\sqrt{-ab}\right)\cot^2(c+dx)\right)\left(a+b+\left(a+\sqrt{-ab}\right)\cot^2(c+dx)\right)\sin^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p}{(a+b)^2dp\left(\left(\left(b+\sqrt{-ab}\right)pF_1\left(1-2p;1-p,-p;2-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)-\left(-b+\sqrt{-ab}\right)pF_1\left(1-2p;-p,1-p;2-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)+b(-1+2p)F_1\left(-2p;-p,-p;1-2p;-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)\cos^2(c+dx)\right)(8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x],x]
```

```
[Out] (2*(-b + Sqrt[-(a*b)])*(b + Sqrt[-(a*b)])*(-1 + 2*p)*AppellF1[-2*p, -p, -p,
 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c +
 d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]^2*(a + b + (a - Sqrt[-(a*b)])*Cot
[c + d*x]^2)*(a + b + (a + Sqrt[-(a*b)])*Cot[c + d*x]^2)*Sin[c + d*x]^4*(a
+ b*Sin[c + d*x]^4)^p)/((a + b)^2*d*p*((b + Sqrt[-(a*b)])*p*AppellF1[1 - 2*
p, 1 - p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a
 + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] - (-b + Sqrt[-(a*b)])*p*AppellF1[
1 - 2*p, -p, 1 - p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])
), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + b*(-1 + 2*p)*AppellF1[-2*
p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a +
b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]^2)*(8*a + 3*b - 4*b*Cos
[2*(c + d*x)] + b*Cos[4*(c + d*x)]))
```

Maple [F]
time = 1.68, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c), x)
```

Fricas [F]
time = 0.44, size = 35, normalized size = 0.25

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\tan\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x)^4)^p,x)
```

```
[Out] int(tan(c + d*x)*(a + b*sin(c + d*x)^4)^p, x)
```

3.567 $\int \cot(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx$

Optimal. Leaf size=54

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)}$$

```
[Out] -1/4*hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^4/a)*(a+b*sin(d*x+c)^4)^(1+p)/
a/d/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3308, 272, 67}

$$-\frac{\left(a+b\sin^4(c+dx)\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] -1/4*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*S
in[c + d*x]^4)^(1 + p))/(a*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx^2)^p}{x}\,dx,x,\sin^2(c+dx)\right)}{2d}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,\sin^4(c+dx)\right)}{4d}$$
$$=-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 1.00

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^4)^p,x]
[Out] -1/4*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*S
in[c + d*x]^4)^(1 + p))/(a*d*(1 + p))
```

Maple [F]
time = 1.62, size = 0, normalized size = 0.00

$$\int\cot\left(dx+c\right)\left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x)
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c), x)
```

Fricas [F]
time = 0.42, size = 35, normalized size = 0.65

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\cot\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**4)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c+d\,x)\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^p, x)
```

3.568 $\int \cot^3(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=127

$$\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)}-\frac{\csc^2(c+dx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sin^4(c+dx)}{a}\right)(a+b}{2d}$$

```
[Out] 1/4*hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^4/a)*(a+b*sin(d*x+c)^4)^(1+p)/a
/d/(1+p)-1/2*csc(d*x+c)^2*hypergeom([-1/2, -p],[1/2],-b*sin(d*x+c)^4/a)*(a+
b*sin(d*x+c)^4)^p/d/((1+b*sin(d*x+c)^4/a)^p)
```

Rubi [A]

time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3308, 778, 372, 371, 272, 67}

$$\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sin^4(c+dx)}{a}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*Sin[c
+ d*x]^4)^(1 + p))/(4*a*d*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-1/2
, -p, 1/2, -((b*Sin[c + d*x]^4)/a)]*(a + b*Sin[c + d*x]^4)^p)/(2*d*(1 + (b*
Sin[c + d*x]^4)/a)^p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cot^3(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)(a+bx^2)^p}{x^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2}\,dx, x, \sin^2(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x}\,dx, x, \text{s}\right.}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sin^4(c+dx)\right)}{4d} + \frac{\left(\left(a+b\sin^4(c+dx)\right)^p\right.}{}$$

$$= \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)} - \frac{\text{cs}}{}$$

Mathematica [A]
time = 0.46, size = 119, normalized size = 0.94

$$\frac{\left(a+b\sin^4(c+dx)\right)^p\left(\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)}{a(1+p)} - 2\csc^2(c+dx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sin^4(c+dx)}{a}\right)\left(1+\frac{b\sin^4(c+dx)}{a}\right)^{-p}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] ((a + b*Sin[c + d*x]^4)^p*((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c
 + d*x]^4)/a]*(a + b*Sin[c + d*x]^4))/(a*(1 + p)) - (2*Csc[c + d*x]^2*Hyper
geometric2F1[-1/2, -p, 1/2, -((b*Sin[c + d*x]^4)/a)])/(1 + (b*Sin[c + d*x]^
4)/a)^p))/(4*d)
```

Maple [F]
time = 1.63, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(dx+c\right)\right) \left(a+b\left(\sin^{4}\left(dx+c\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^3, x)
```

Fricas [F]
time = 0.48, size = 37, normalized size = 0.29

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\cot\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^3,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^3\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^4)^p, x)
```

3.569 $\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((a+b\sin^4(c+dx))^p \tan^4(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4, x]
```

Rubi steps

$$\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx = \int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx$$

Mathematica [A]
time = 45.62, size = 0, normalized size = 0.00

$$\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4, x]
```

Maple [A]
time = 0.77, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p \left(\tan^4\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^4, x)
```

Fricas [A]
time = 0.49, size = 37, normalized size = 1.42

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\tan\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^4,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^4\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^4)^p, x)
```

3.570 $\int (a+b\sin^4(c+dx))^p \tan^2(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((a+b\sin^4(c+dx))^p \tan^2(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^4(c+dx))^p \tan^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2, x]
```

Rubi steps

$$\int (a+b\sin^4(c+dx))^p \tan^2(c+dx)\,dx = \int (a+b\sin^4(c+dx))^p \tan^2(c+dx)\,dx$$

Mathematica [A]
time = 2.32, size = 0, normalized size = 0.00

$$\int (a+b\sin^4(c+dx))^p \tan^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2, x]
```

Maple [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4(dx+c)\right)\right)^p \left(\tan^2(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^2, x)
```

Fricas [A]
time = 0.48, size = 37, normalized size = 1.42

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\tan\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^2,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**2,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^2\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^4)^p, x)
```

3.571 $\int \left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^4)^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx = \int \left(a+b\sin^4(c+dx)\right)^p\,dx$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p,x]
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p, x]
```

Maple [A]
time = 1.67, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p, x)
```

Fricas [A]
time = 0.42, size = 28, normalized size = 1.65

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^4 + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int\left(b\sin\left(c+d\,x\right)^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^4)^p,x)
[Out] int((a + b*sin(c + d*x)^4)^p, x)
```

3.572 $\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p,x]
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi steps

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Mathematica [A]
time = 2.02, size = 0, normalized size = 0.00

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p,x]
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p, x]
```

Maple [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \left(\cot^2(dx+c)\right)\left(a+b\left(\sin^4(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
[Out] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^2, x)
```

Fricas [**A**]
time = 0.44, size = 37, normalized size = 1.42

$$\mathrm{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\cot\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^2,
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**4)**p,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(c+d\,x)^2\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^4)^p, x)
```

3.573 $\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p,x]
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi steps

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Mathematica [A]
time = 52.18, size = 0, normalized size = 0.00

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p,x]
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p, x]
```

Maple [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \left(\cot^4(dx+c)\right)\left(a+b\left(\sin^4(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
[Out] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^4, x)
```

Fricas [A]
time = 0.46, size = 37, normalized size = 1.42

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\cot\left(dx+c\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^4,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**4)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(c+d\,x)^4\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^4)^p, x)
```

3.574 $\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx$

Optimal. Leaf size=306

$$\frac{a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{3a^2b\cos^2(c+dx)^{\frac{1+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m\right.}{d(1+m+}$$

```
[Out] a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/
(1+m)+3*a^2*b*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+1/
2*n],[3/2+1/2*m+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^n*tan(d*x+c)^(1+m)/d/(1+m+n
)+3*a*b^2*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+n],[3/
2+1/2*m+n],sin(d*x+c)^2)*sin(d*x+c)^(2*n)*tan(d*x+c)^(1+m)/d/(1+m+2*n)+b^3*
(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+3/2*n],[3/2+1/2*
m+3/2*n],sin(d*x+c)^2)*sin(d*x+c)^(3*n)*tan(d*x+c)^(1+m)/d/(1+m+3*n)
```

Rubi [A]
time = 0.29, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3313, 3557, 371, 2682, 2657}

$$\frac{a^3\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{3a^2b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(c+dx)\right)}{d(m+n+1)}+\frac{3ab^2\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^{2n}(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+2n+1);\frac{1}{2}(m+2n+3);\sin^2(c+dx)\right)}{d(m+2n+1)}+\frac{b^3\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^{3n}(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+3n+1);\frac{1}{2}(m+3n+3);\sin^2(c+dx)\right)}{d(m+3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^n)^3*Tan[c + d*x]^m,x]
```

```
[Out] (a^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]^(1 + m))/(d*(1 + m)) + (3*a^2*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeomet
ric2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x
]^n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (3*a*b^2*(Cos[c + d*x]^2)^((1 +
 m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2, Sin[c
 + d*x]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n)) + (b^
3*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 3*n)/2
, (3 + m + 3*n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3*n)*Tan[c + d*x]^(1 + m))
/(d*(1 + m + 3*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3313

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx &= \int \left(a^3\tan^m(c+dx)+3a^2b\sin^n(c+dx)\tan^m(c+dx)+3ab^2\sin\right.\\ &= a^3\int \tan^m(c+dx)\,dx+\left(3a^2b\right)\int \sin^n(c+dx)\tan^m(c+dx)\,dx\\ &= \frac{a^3\text{Subst}\left(\int \frac{x^m}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}+\left(3a^2b\cos^{1+m}(c+dx)\sin^{-}\right.\\ &= \frac{a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{3a^2b\cos^2}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 17.74, size = 3544, normalized size = 11.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^3*Tan[c + d*x]^m,x]

[Out] (2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^3*AppellF1[(1 + m)/2, m, 1, (3 +
 m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)
/2]^2)^n*Sin[c + d*x]^n*((3*a^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m +
n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d
*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 +
m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*Appel
lF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan
[(c + d*x)/2]*Tan[c + d*x]^m*(a^3*Tan[c + d*x]^m + 3*a^2*b*Sin[c + d*x]^n*T
an[c + d*x]^m + 3*a*b^2*Sin[c + d*x]^(2*n)*Tan[c + d*x]^m + b^3*Sin[c + d*x
]^(3*n)*Tan[c + d*x]^m))/(d*(2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Sec[c
 + d*x]^2*((a^3*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a^
2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3
*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*
n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)
/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan[(c + d*x)/2]*Tan[c + d*x]^(-1
 + m) + Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^3*Appell
F1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1
 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a^2*AppellF1[(1 + m + n
)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1
+ m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2
 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/
(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]
^n)/(1 + m + 3*n))))*Tan[c + d*x]^m + 2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(-1 + m)*((a^3*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a
^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((
3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3
*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x
)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan[(c + d*x)/2]*(-(Sec[(c + d*x
)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*Ta
n[c + d*x]^m + 2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(b*n*
Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n)*((3*a^2*AppellF1[
(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1
[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m +
```

```
3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Si
n[c + d*x]^n)/(1 + m + 3*n))) + b*n*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(
(3*a^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^
n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1
 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c +
 d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n)))*Tan[(c + d*x)/2] + (a^3*(-(((
1 + m)*AppellF1[1 + (1 + m)/2, m, 2, 1 + (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m)) + (m*(1 + m
)*AppellF1[1 + (1 + m)/2, 1 + m, 1, 1 + (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m)))/(1 + m) + b
*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(b*n*Cos[c + d*x]*(Sec[(c + d*x)/2]^
2)^n*Sin[c + d*x]^(-1 + n)*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 +
m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*Appel
lF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n)) + b*n
*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 +
 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n
) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m +
 3*n))*Tan[(c + d*x)/2] + (3*a^2*(-(((1 + n)*(1 + m + n)*AppellF1[1 + (1 +
m + n)/2, m, 2 + n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + ...
```

Maple [F]
time = 2.29, size = 0, normalized size = 0.00

$$\int \left(a+b(\sin^{n}\left(dx+c\right))\right)^{3}\left(\tan^{m}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x)
[Out] int((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^3*tan(d*x + c)^m, x)
```

Fricas [F]
time = 0.43, size = 59, normalized size = 0.19

$$\mathrm{integral}\left(\left(b^{3} \sin\left(dx+c\right)^{3\,n}+3\,ab^{2} \sin\left(dx+c\right)^{2\,n}+3\,a^{2}b \sin\left(dx+c\right)^{n}+a^{3}\right) \tan\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b^3*sin(d*x + c)^(3*n) + 3*a*b^2*sin(d*x + c)^(2*n) + 3*a^2*b*sin
(d*x + c)^n + a^3)*tan(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^{n}\left(c+dx\right)\right)^{3}\tan^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**3*tan(d*x+c)**m,x)
```

```
[Out] Integral((a + b*sin(c + d*x)**n)**3*tan(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^3*tan(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{tan}(c+d\,x)^{m}\,{\left(a+b\,{\sin\left(c+d\,x\right)}^{n}\right)}^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^3,x)
```

```
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^3, x)
```

3.575 $\int (a+b\sin^n(c+dx))^2 \tan^m(c+dx)\,dx$

Optimal. Leaf size=215

$$\frac{a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{2ab\cos^2(c+dx)^{\frac{1+m}{2}}\,{}_2F_1\left(\frac{1+m}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m}{d(1+m}$$

```
[Out] a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/
(1+m)+2*a*b*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+1/2*
n],[3/2+1/2*m+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^n*tan(d*x+c)^(1+m)/d/(1+m+n)+
b^2*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+n],[3/2+1/2*
m+n],sin(d*x+c)^2)*sin(d*x+c)^(2*n)*tan(d*x+c)^(1+m)/d/(1+m+2*n)
```

Rubi [A]

time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3313, 3557, 371, 2682, 2657}

$$\frac{a^2\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{2ab\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(c+dx)\right)}{d(m+n+1)}+\frac{b^2\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^{2n}(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+2n+1);\frac{1}{2}(m+2n+3);\sin^2(c+dx)\right)}{d(m+2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^n)^2*Tan[c + d*x]^m,x]
```

```
[Out] (a^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]^(1 + m))/(d*(1 + m)) + (2*a*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometri
c2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^
n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (b^2*(Cos[c + d*x]^2)^((1 + m)/2)
*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3313

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx &= \int \left(a^2\tan^m(c+dx)+2ab\sin^n(c+dx)\tan^m(c+dx)+b^2\sin^{2n}(c\right.\\ &= a^2\int \tan^m(c+dx)\,dx+(2ab)\int \sin^n(c+dx)\tan^m(c+dx)\,dx+\\ &= \frac{a^2\text{Subst}\left(\int \frac{x^m}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}+\left(2ab\cos^{1+m}(c+dx)\sin^{-1-}\right.\\ &= \frac{a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{2ab\cos^2(c}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 14.74, size = 2368, normalized size = 11.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^2*Tan[c + d*x]^m,x]
```

```
[Out] (2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^2*AppellF1[(1 + m)/2, m, 1, (3 +
 m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)
/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)
```

```
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2
 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/2
]*Tan[c + d*x]^m*(a^2*Tan[c + d*x]^m + 2*a*b*Sin[c + d*x]^n*Tan[c + d*x]^m
+ b^2*Sin[c + d*x]^(2*n)*Tan[c + d*x]^m))/(d*(2*m*(Cos[c + d*x]*Sec[(c + d*
x)/2]^2)^m*Sec[c + d*x]^2*((a^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[
c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m
, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c
+ d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/2]*Tan[c + d*x
]^(-1 + m) + Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^2*A
ppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m
+ n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/
(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1
 + m + 2*n)))*Tan[c + d*x]^m + 2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 +
m)*((a^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a*Appell
F1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 +
n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c +
d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/2]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]
) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*Tan[c + d*x]^m + 2*(C
os[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(b*n*Cos[c + d*x]*(Sec[(
c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n)*((2*a*AppellF1[(1 + m + n)/2, m, 1 +
 n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) +
(b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)) +
 b*n*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m,
 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n
) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n
))*Tan[(c + d*x)/2] + (a^2*(-(((1 + m)*AppellF1[1 + (1 + m)/2, m, 2, 1 + (3
 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/(3 + m)) + (m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m, 1, 1 + (3
+ m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2])/(3 + m)))/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((b*
n*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n
))/(1 + m + 2*n) + (b*n*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x
]^n*Tan[(c + d*x)/2])/(1 + m + 2*n) + (b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x
]^n*(-(((1/2 + m/2 + n)*(1 + 2*n)*AppellF1[3/2 + m/2 + n, m, 2 + 2*n, 5/2 +
```

```
m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])/(3/2 + m/2 + n)) + (m*(1/2 + m/2 + n)*AppellF1[3/2 + m/2 + n,
1 + m, 1 + 2*n, 5/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3/2 + m/2 + n)))/(1 + m + 2*n) + (2*a*(-
(((1 + n)*(1 + m + n)*AppellF1[1 + (1 + m + n)/2, m, 2 + n, 1 + (3 + m + n)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])/(3 + m + n)) + (m*(1 + m + n)*AppellF1[1 + (1 + m + n)/2, 1 + m, 1 +
 n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/(3 + m + n)))/(1 + m + n)))*Tan[c + d*x]^m))
```

Maple [F]
time = 0.88, size = 0, normalized size = 0.00

$$\int (a+b(\sin^n(dx+c)))^2 (\tan^m(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x)
[Out] int((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^2*tan(d*x + c)^m, x)
```

Fricas [F]
time = 0.40, size = 41, normalized size = 0.19

$$\mathrm{integral}\left(\left(b^2\sin(dx+c)^{2n}+2ab\sin(dx+c)^n+a^2\right)\tan(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="fricas")
[Out] integral((b^2*sin(d*x + c)^(2*n) + 2*a*b*sin(d*x + c)^n + a^2)*tan(d*x + c)
^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^2 \tan^m(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**2*tan(d*x+c)**m,x)
```

```
[Out] Integral((a + b*sin(c + d*x)**n)**2*tan(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^2*tan(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(c+d\,x)^m\,{\left(a+b\sin\left(c+d\,x\right)^n\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^2,x)
```

```
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^2, x)
```

3.576 $\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx$

Optimal. Leaf size=124

$$\frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{b\cos^2(c+dx)^{\frac{1+m}{2}}\ {}_2F_1\left(\frac{1+m}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n\right.}{d(1+m+n}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/(1
+m)+b*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+1/2*n],[3/
2+1/2*m+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^n*tan(d*x+c)^(1+m)/d/(1+m+n)
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3313, 3557, 371, 2682, 2657}

$$\frac{a\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(c+dx)\right)}{d(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^n)*Tan[c + d*x]^m,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]
^(1 + m))/(d*(1 + m)) + (b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1[(
1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^n*Tan[
c + d*x]^(1 + m))/(d*(1 + m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
```

```
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3313

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx = \int (a\tan^m(c+dx)+b\sin^n(c+dx)\tan^m(c+dx))\,dx$$

$$= a\int \tan^m(c+dx)\,dx + b\int \sin^n(c+dx)\tan^m(c+dx)\,dx$$

$$= \frac{a\text{Subst}\left(\int \frac{x^m}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d} + \left(b\cos^{1+m}(c+dx)\sin^{-1-m}(c$$

$$= \frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)} + \frac{b\cos^2(c+$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 12.46, size = 1065, normalized size = 8.59

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)*Tan[c + d*x]^m,x]
```

```
[Out] (2*(a + b*Sin[c + d*x]^n)*(a*(1 + m + n)*AppellF1[(1 + m)/2, m, 1, (3 + m)/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + b*(1 + m)*AppellF1[(1 + m + n
)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec
[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Tan[(c + d*x)/2]*Tan[c + d*x]^m)/(d*(Sec
[(c + d*x)/2]^2*(a*(1 + m + n)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] + b*(1 + m)*AppellF1[(1 + m + n)/2, m, 1
```

```
+ n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)
/2]^2)^n*Sin[c + d*x]^n) - 16*m*Cos[(c + d*x)/2]*Csc[c + d*x]^3*Sec[c + d*x
]*Sin[(c + d*x)/2]^5*(a*(1 + m + n)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + b*(1 + m)*AppellF1[(1 + m + n)/2,
m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c +
 d*x)/2]^2)^n*Sin[c + d*x]^n) + 2*m*Csc[c + d*x]*Sec[c + d*x]*(a*(1 + m + n
)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + b*(1 + m)*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Tan[
(c + d*x)/2] + 2*(1 + m)*Tan[(c + d*x)/2]*(b*n*AppellF1[(1 + m + n)/2, m, 1
 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x]*
(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n) + (a*(1 + m + n)*(-AppellF1[(3
 + m)/2, m, 2, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + m*Appe
llF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m) + b*n*AppellF1[(1 + m + n)
/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[
(c + d*x)/2]^2)^n*Sin[c + d*x]^n*Tan[(c + d*x)/2] + (b*(1 + m + n)*(-((1 +
n)*AppellF1[(3 + m + n)/2, m, 2 + n, (5 + m + n)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]) + m*AppellF1[(3 + m + n)/2, 1 + m, 1 + n, (5 + m + n)/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Sec[(c + d*x)/2]^2)^(1 + n)*Sin
[c + d*x]^n*Tan[(c + d*x)/2])/(3 + m + n))))
```

Maple [F]
time = 0.73, size = 0, normalized size = 0.00

$$\int \left(a+b(\sin^{n}\left(dx+c\right))\right)\left(\tan^{m}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x)
[Out] int((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

Fricas [F]
time = 0.43, size = 23, normalized size = 0.19

$$\mathrm{integral}\left(\left(b\sin\left(dx+c\right)^{n}+a\right)\tan\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin^n(c + dx))\tan^m(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)*tan(d*x+c)**m,x)
```

```
[Out] Integral((a + b*sin(c + d*x)**n)*tan(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + d\,x)^m\,\left(a + b\sin\left(c + d\,x\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n),x)
```

```
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n), x)
```

3.577 $\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)},x\right)$$

```
[Out] Unintegrable(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n),x]
[Out] Defer[Int][Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n), x]
```

Rubi steps

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx = \int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$$

Mathematica [A]
time = 5.99, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n),x]
[Out] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n), x]
```

Maple [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(dx+c)}{a+b\left(\sin^n(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

```
[Out] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

Fricas [A]
time = 0.40, size = 25, normalized size = 0.96

$$\text{integral}\left(\frac{\tan\left(dx+c\right)^{m}}{b\sin\left(dx+c\right)^{n}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{m}\left(c+dx\right)}{a+b\sin^{n}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+b*sin(d*x+c)**n),x)
```

```
[Out] Integral(tan(c + d*x)**m/(a + b*sin(c + d*x)**n), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(c+d\,x)^m}{a+b\sin\left(c+d\,x\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n),x)
[Out] int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n), x)
```

3.578 $\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2},x\right)$$

```
[Out] Unintegrable(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2,x]
[Out] Defer[Int][Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2, x]
```

Rubi steps

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx = \int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$$

Mathematica [A]
time = 54.86, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2,x]
[Out] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2, x]
```

Maple [A]
time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(dx+c)}{(a+b(\sin^n(dx+c)))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

```
[Out] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a)^2, x)
```

Fricas [A]
time = 0.41, size = 43, normalized size = 1.65

$$\text{integral}\left(\frac{\tan\left(dx+c\right)^{m}}{b^{2}\sin\left(dx+c\right)^{2\,n}+2\,ab\sin\left(dx+c\right)^{n}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(tan(d*x + c)^m/(b^2*sin(d*x + c)^(2*n) + 2*a*b*sin(d*x + c)^n + a^
2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}\left(c+dx\right)}{\left(a+b\sin^{n}\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+b*sin(d*x+c)**n)**2,x)
```

```
[Out] Integral(tan(c + d*x)**m/(a + b*sin(c + d*x)**n)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(c + d\,x)^m}{\left(a + b\sin\left(c + d\,x\right)^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n)^2,x)
```

```
[Out] int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n)^2, x)
```

3.579 $\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx$

Optimal. Leaf size=47

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{a+b\sin^n(x)}}{n}$$

```
[Out] -2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*sin(x)^n)^(1/2)/n
```

Rubi [A]

time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3309, 272, 52, 65, 214}

$$\frac{2\sqrt{a+b\sin^n(x)}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/n + (2*Sqrt[a + b*Sin[x]
^n])/n
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, \sin(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^n(x)\right)}{n}\\
&= \frac{2\sqrt{a+b\sin^n(x)}}{n} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^n(x)\right)}{n}\\
&= \frac{2\sqrt{a+b\sin^n(x)}}{n} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^n(x)}\right)}{bn}\\
&= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n} + \frac{2\sqrt{a+b\sin^n(x)}}{n}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.96

$$\frac{-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\sin^n(x)}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sin[x]^n])
/n
```

Maple [A]
time = 3.09, size = 38, normalized size = 0.81

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\left(\sin^{n}\left(x\right)\right)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\sin^{n}\left(x\right)\right)}}{\sqrt{a}}\right)}{n}$	38
default	$\frac{2\sqrt{a+b\left(\sin^{n}\left(x\right)\right)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\sin^{n}\left(x\right)\right)}}{\sqrt{a}}\right)}{n}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*sin(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2*(a+b*sin(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))
)
```

Maxima [A]
time = 0.51, size = 57, normalized size = 1.21

$$\frac{\sqrt{a}\log\left(\frac{\sqrt{b\sin\left(x\right)^{n}+a}-\sqrt{a}}{\sqrt{b\sin\left(x\right)^{n}+a}+\sqrt{a}}\right)}{n}+\frac{2\sqrt{b\sin\left(x\right)^{n}+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*log((sqrt(b*sin(x)^n + a) - sqrt(a))/(sqrt(b*sin(x)^n + a) + sqrt(a
)))/n + 2*sqrt(b*sin(x)^n + a)/n
```

Fricas [A]
time = 0.41, size = 97, normalized size = 2.06

$$\left[\frac{\sqrt{a}\log\left(\frac{b\sin\left(x\right)^{n}-2\sqrt{b\sin\left(x\right)^{n}+a}\sqrt{a}+2a}{\sin\left(x\right)^{n}}\right)+2\sqrt{b\sin\left(x\right)^{n}+a}}{n},\frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{b\sin\left(x\right)^{n}+a}\sqrt{-a}}{a}\right)+\sqrt{b\sin\left(x\right)^{n}+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*sin(x)^n - 2*sqrt(b*sin(x)^n + a)*sqrt(a) + 2*a)/sin(x)^n)
 + 2*sqrt(b*sin(x)^n + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*sin(x)^n + a)*sqrt(
-a)/a) + sqrt(b*sin(x)^n + a))/n]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^{n}(x)}\, \cot(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(x)**n)*cot(x), x)
```

Giac [**A**]
time = 0.52, size = 46, normalized size = 0.98

$$\frac{2\left(\frac{ab\arctan\left(\frac{\sqrt{b\sin(x)^{n}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b\sin(x)^{n}+a}\,b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(a*b*arctan(sqrt(b*sin(x)^n + a)/sqrt(-a))/sqrt(-a) + sqrt(b*sin(x)^n + a
)*b)/(b*n)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(x)\, \sqrt{a + b\sin(x)^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*sin(x)^n)^(1/2),x)
```

```
[Out] int(cot(x)*(a + b*sin(x)^n)^(1/2), x)
```

3.580 $\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

[Out] -2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3309, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/Sqrt[a + b*Sin[x]^n],x]

[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/(Sqrt[a]*n)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
```

```
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, \sin(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^n(x)\right)}{n}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^n(x)}\right)}{bn}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A]
time = 0.52, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sin^n(x)\right)}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sin^n(x)\right)}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*sin(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [A]
time = 0.52, size = 41, normalized size = 1.41

$$\frac{\log\left(\frac{\sqrt{b\sin(x)^n+a}-\sqrt{a}}{\sqrt{b\sin(x)^n+a}+\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*sin(x)^n + a) - sqrt(a))/(sqrt(b*sin(x)^n + a) + sqrt(a)))/(sqr
t(a)*n)
```

Fricas [A]
time = 0.42, size = 74, normalized size = 2.55

$$\left[\frac{\log\left(\frac{b\sin(x)^n-2\sqrt{b\sin(x)^n+a}\sqrt{a}+2a}{\sin(x)^n}\right)}{\sqrt{a}\,n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\sin(x)^n+a}\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*sin(x)^n - 2*sqrt(b*sin(x)^n + a)*sqrt(a) + 2*a)/sin(x)^n)/(sqrt(a)
*n), 2*sqrt(-a)*arctan(sqrt(b*sin(x)^n + a)*sqrt(-a)/a)/(a*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)**n)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*sin(x)**n), x)
```

Giac [A]
time = 0.52, size = 27, normalized size = 0.93

$$\frac{2\arctan\left(\frac{\sqrt{b\sin\left(x\right)^{n}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="giac")
[Out] 2*arctan(sqrt(b*sin(x)^n + a)/sqrt(-a))/(sqrt(-a)*n)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(x)}{\sqrt{a+b\sin\left(x\right)^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*sin(x)^n)^(1/2),x)
[Out] int(cot(x)/(a + b*sin(x)^n)^(1/2), x)
```

3.581 $\int (a+b\sin^n(c+dx))^p\tan^m(c+dx)\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}((a+b\sin^n(c+dx))^p\tan^m(c+dx),x)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p\tan^m(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p\tan^m(c+dx)\,dx=\int (a+b\sin^n(c+dx))^p\tan^m(c+dx)\,dx$$

Mathematica [A]
time = 6.88, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p\tan^m(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m,x]
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m, x]
```

Maple [A]
time = 1.30, size = 0, normalized size = 0.00

$$\int \left(a+b(\sin^n(dx+c))\right)^p\left(\tan^m(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

Fricas [A]
time = 0.41, size = 25, normalized size = 0.96

$$\text{integral}((b\sin{(dx+c)}^n + a)^p\tan{(dx+c)}^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**m,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^m\,(a+b\sin{(c+d\,x)}^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^p, x)
```

3.582 $\int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((a+b\sin^n(c+dx))^p\tan^3(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p\tan^3(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p\tan^3(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p\tan^3(c+dx)\,dx$$

Mathematica [A]
time = 51.65, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p\tan^3(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3, x]
```

Maple [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(a+b(\sin^n(dx+c))\right)^p\left(\tan^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

Fricas [A]
time = 0.43, size = 25, normalized size = 0.96

$$\text{integral}\left((b\sin(dx+c)^n + a)^p \tan(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**3,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^3\,\left(a+b\sin\left(c+d\,x\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^n)^p, x)
```

3.583 $\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$

Optimal. Leaf size=24

$$\text{Int}((a+b\sin^n(c+dx))^p \tan(c+dx),x)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x],x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x], x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$$

Mathematica [A]
time = 0.71, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x],x]
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x], x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int (a+b(\sin^n(dx+c)))^p \tan(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

Fricas [A]
time = 0.41, size = 23, normalized size = 0.96

$$\text{integral}((b\sin{(dx+c)}^n+a)^p\tan{(dx+c)},x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin^n{(c+dx)})^p\tan{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c),x)
[Out] Integral((a + b*sin(c + d*x)**n)**p*tan(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)\,\left(a+b\sin{(c+d\,x)}^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x)^n)^p,x)
```

```
[Out] int(tan(c + d*x)*(a + b*sin(c + d*x)^n)^p, x)
```

3.584 $\int \cot(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=55

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)\left(a+b\sin^n(c+dx)\right)^{1+p}}{adn(1+p)}$$

```
[Out] -hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^n/a)*(a+b*sin(d*x+c)^n)^(1+p)/a/d/
n/(1+p)
```

Rubi [A]

time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3309, 272, 67}

$$-\frac{\left(a+b\sin^n(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[
c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \cot(c+dx)\,(a+b\sin^n(c+dx))^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sin^n(c+dx)\right)}{dn}$$
$$= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)(a+b\sin^n(c+dx))^{1+p}}{adn(1+p)}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 1.00

$$-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)(a+b\sin^n(c+dx))^{1+p}}{adn(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[
c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \cot(dx+c)\,(a+b(\sin^n(dx+c)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

Fricas [F]
time = 0.45, size = 23, normalized size = 0.42

$$\text{integral}((b\sin(dx+c)^n + a)^p \cot(dx+c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin^n(c+dx))^p \cot(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**n)**p,x)
[Out] Integral((a + b*sin(c + d*x)**n)**p*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(c+d\,x)\,\left(a+b\sin(c+d\,x)^n\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(cot(c + d*x)*(a + b*sin(c + d*x)^n)^p, x)
```

3.585 $\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=136

$$\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)\left(a+b\sin^n(c+dx)\right)^{1+p}}{adn(1+p)}-\frac{\csc^2(c+dx)\,{}_2F_1\left(-\frac{2}{n},-p;-\frac{2-n}{n};-\frac{b\sin^n(c+dx)}{a}\right)(\ldots}{2d}$$

[Out] hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^n/a)*(a+b*sin(d*x+c)^n)^(1+p)/a/d/n/(1+p)-1/2*csc(d*x+c)^2*hypergeom([-p, -2/n],[(-2+n)/n],-b*sin(d*x+c)^n/a)*(a+b*sin(d*x+c)^n)^p/d/((1+b*sin(d*x+c)^n/a)^p)

Rubi [A]
time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3309, 1858, 372, 371, 272, 67}

$$\frac{(a+b\sin^n(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}-\frac{\csc^2(c+dx)\,(a+b\sin^n(c+dx))^p\left(\frac{b\sin^n(c+dx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{2}{n},-p;-\frac{2-n}{n};-\frac{b\sin^n(c+dx)}{a}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^n)^p,x]

[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-2/n, -p, -((2 - n)/n), -((b*Sin[c + d*x]^n)/a)]*(a + b*Sin[c + d*x]^n)^p)/(2*d*(1 + (b*Sin[c + d*x]^n)/a)^p)

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cot^3(c+dx)\,(a+b\sin^n(c+dx))^p\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^n)^p}{x^3}\,dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(a+bx^n)^p}{x^3}-\frac{(a+bx^n)^p}{x}\right)dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x^3}\,dx,x,\sin(c+dx)\right)}{d}-\frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx,x,\text{si}\right.}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,\sin^n(c+dx)\right)}{dn}+\frac{\left((a+b\sin^n(c+dx))^p\right.}{}\\
&= \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)(a+b\sin^n(c+dx))^{1+p}}{adn(1+p)}-\frac{\text{cs}}{}
\end{aligned}
$$

Mathematica [A]

time = 0.73, size = 129, normalized size = 0.95

$$
\frac{(a+b\sin^n(c+dx))^p\left(\frac{2\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)(a+b\sin^n(c+dx))}{an(1+p)}-\csc^2(c+dx)\,{}_2F_1\left(-\frac{2}{n},-p;\frac{-2+n}{n};-\frac{b\sin^n(c+dx)}{a}\right)\left(1+\frac{b\sin^n(c+dx)}{a}\right)^{-p}\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] ((a + b*Sin[c + d*x]^n)^p*((2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin
[c + d*x]^n)/a]*(a + b*Sin[c + d*x]^n))/(a*n*(1 + p)) - (Csc[c + d*x]^2*Hyp
ergeometric2F1[-2/n, -p, (-2 + n)/n, -((b*Sin[c + d*x]^n)/a)])/(1 + (b*Sin[
c + d*x]^n)/a)^p))/(2*d)
```

Maple [**F**]
time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(dx+c\right)\right) \left(a+b(\sin^{n}\left(dx+c\right))\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.41, size = 25, normalized size = 0.18

$$\text{integral}\left(\left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\cot\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**n)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^3\,\left(a+b\sin\left(c+d\,x\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^n)^p, x)
```

3.586 $\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((a+b\sin^n(c+dx))^p \tan^4(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$$

Mathematica [A]
time = 61.88, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4, x]
```

Maple [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(a+b(\sin^n(dx+c))\right)^p \left(\tan^4(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

Fricas [A]
time = 0.46, size = 25, normalized size = 0.96

$$\text{integral}\left((b\sin(dx+c)^n+a)^p\tan(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^4\,\left(a+b\sin\left(c+d\,x\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^n)^p, x)
```

3.587 $\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((a+b\sin^n(c+dx))^p\tan^2(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p\tan^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p\tan^2(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p\tan^2(c+dx)\,dx$$

Mathematica [A]
time = 5.63, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p\tan^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2, x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int (a+b(\sin^n(dx+c)))^p\left(\tan^2(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

Fricas [A]
time = 0.42, size = 25, normalized size = 0.96

$$\operatorname{integral}\left((b\sin\left(dx+c\right)^{n}+a)^{p}\tan\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^2\,\left(a+b\sin\left(c+d\,x\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^n)^p,x)
```

```
[Out] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^n)^p, x)
```

3.588 $\int (a+b\sin^n(c+dx))^p\,dx$

Optimal. Leaf size=17

$$\text{Int}\left((a+b\sin^n(c+dx))^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p\,dx = \int (a+b\sin^n(c+dx))^p\,dx$$

Mathematica [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p,x]
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p, x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^n(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p,x)
```

[Out] int((a+b*sin(d*x+c)^n)^p,x)

Maxima [A]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")

[Out] integrate((b*sin(d*x + c)^n + a)^p, x)

Fricas [A]

time = 0.42, size = 16, normalized size = 0.94

$$\text{integral}((b\sin{(dx+c)}^n+a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")

[Out] integral((b*sin(d*x + c)^n + a)^p, x)

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin^n{(c+dx)})^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c)**n)**p,x)

[Out] Integral((a + b*sin(c + d*x)**n)**p, x)

Giac [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="giac")

[Out] integrate((b*sin(d*x + c)^n + a)^p, x)

Mupad [A]

time = 0.00, size = -1, normalized size = -0.06

$$\int (a+b\sin{(c+d\,x)}^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(c + d*x)^n)^p,x)

[Out] int((a + b*sin(c + d*x)^n)^p, x)

3.589 $\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p,x]
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi steps

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx = \int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Mathematica [A]
time = 4.73, size = 0, normalized size = 0.00

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p,x]
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p, x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(\cot^2\left(dx+c\right)\right)\left(a+b\left(\sin^n\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
[Out] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

Fricas [**A**]
time = 0.44, size = 25, normalized size = 0.96

$$\text{integral}\left((b\sin(dx+c)^n + a)^p \cot(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**n)**p,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(c+d\,x)^2\,(a+b\sin(c+d\,x)^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^n)^p, x)
```

3.590 $\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p,x]
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi steps

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx = \int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Mathematica [A]
time = 80.63, size = 0, normalized size = 0.00

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p,x]
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p, x]
```

Maple [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \left(\cot^4(dx+c)\right)\left(a+b(\sin^n(dx+c))\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
[Out] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

Fricas [A]
time = 0.45, size = 25, normalized size = 0.96

$$\mathrm{integral}\left((b\sin\left(dx+c\right)^{n}+a)^{p}\cot\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**n)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \mathrm{cot}(c+d\,x)^4\,{\left(a+b\,{\sin\left(c+d\,x\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^n)^p, x)
```

3.591 $\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=107

$$\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}+\frac{(2a-b)F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{3fg^2\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}$$

```
[Out] 2/3*(a+b)*(d*sin(f*x+e))^(1/2)/d/f/g/(g*cos(f*x+e))^(3/2)-1/3*(2*a-b)*(sin(
e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2
))*sin(2*f*x+2*e)^(1/2)/f/g^2/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3281, 468, 335, 243, 342, 281, 238}

$$\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}-\frac{2(2a-b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+fx))^{3/2}F\left(\frac{1}{2}\csc^{-1}(\sin(e+fx))\middle|2\right)}{3d^2fg(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]),x]
```

```
[Out] (2*(a + b)*Sqrt[d*Sin[e + f*x]])/(3*d*f*g*(g*Cos[e + f*x])^(3/2)) - (2*(2*a
 - b)*(1 - Csc[e + f*x]^2)^(3/4)*EllipticF[ArcCsc[Sin[e + f*x]]/2, 2]*(d*Si
n[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 3281

```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[ff*c^(2*IntPart[(m - 1)/2] + 1)*((c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]
)), Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x],
 x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !Int
egerQ[m]
```

Rubi steps

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{a+bx^2}{\sqrt{dx}\,(1-x^2)^{7/4}}dx, x, \sin(e+fx)\right)}{fg(g\cos(e+fx))^{3/2}}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{\left((-2a+b)\cos^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \right.}{3fg(g\cos(e}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{\left(2(-2a+b)\cos^2(e+fx)^{3/4}\right)\text{Subst}\left(\right.}{3dfg(g\cos(}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{\left(2(-2a+b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\right.}{}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{\left(2(-2a+b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\right.}{}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{\left((-2a+b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\,\text{s}\right.}{3dfg}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{2(2a-b)\left(1-\csc^2(e+fx)\right)^{3/4}F\left(\frac{1}{2}\right.}{3d^2fg(g\,\text{co}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.14, size = 102, normalized size = 0.95

$$\frac{2\cos^2(e+fx)^{3/4}\left(5a\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\sin^2(e+fx)\right)\sin(e+fx)+b\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\sin^2(e+fx)\right)\sin^3(e+fx)\right)}{5fg(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*(5*a*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[e + f*x
]^2]*Sin[e + f*x] + b*Hypergeometric2F1[5/4, 7/4, 9/4, Sin[e + f*x]^2]*Sin[
e + f*x]^3))/(5*f*g*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(118) = 236.
time = 183.30, size = 324, normalized size = 3.03

method	result
default	$-\frac{\left(2\sin(fx+e)\cos(fx+e)\sqrt{\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\ \text{EllipticF}\left(\sqrt{\frac{1-c}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/3/f*(2*sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))
*a-sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*b-cos(
f*x+e)*2^(1/2)*a-cos(f*x+e)*2^(1/2)*b+2^(1/2)*a+2^(1/2)*b)*sin(f*x+e)*cos(f
*x+e)/(-1+cos(f*x+e))/(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.11, size = 125, normalized size = 1.17

$$-\frac{\sqrt{i\,dg}\,(2\,a-b)\cos{(fx+e)}^2\,F(\arcsin{(\cos{(fx+e)}+i\,\sin{(fx+e)})}\,|-1)+\sqrt{-i\,dg}\,(2\,a-b)\cos{(fx+e)}^2\,F(\arcsin{(\cos{(fx+e)}-i\,\sin{(fx+e)})}\,|-1)-2\,\sqrt{g\cos{(fx+e)}}\,\sqrt{d\sin{(fx+e)}}\,(a+b)}{3\,dfg^3\cos{(fx+e)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] -1/3*(sqrt(I*d*g)*(2*a - b)*cos(f*x + e)^2*elliptic_f(arcsin(cos(f*x + e) +
 I*sin(f*x + e)), -1) + sqrt(-I*d*g)*(2*a - b)*cos(f*x + e)^2*elliptic_f(ar
csin(cos(f*x + e) - I*sin(f*x + e)), -1) - 2*sqrt(g*cos(f*x + e))*sqrt(d*si
n(f*x + e))*(a + b))/(d*f*g^3*cos(f*x + e)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)/(g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\sin\left(e+f\,x\right)^2+a}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int((a + b*sin(e + f*x)^2)/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2)),
 x)
```

3.592 $\int (c\cos(e+fx))^m (d\sin(e+fx))^n \left(a+b\sin^2(e+fx)\right)^p$

Optimal. Leaf size=137

$$\frac{cF_1\left(\frac{1+n}{2};\frac{1-m}{2},-p;\frac{3+n}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(c\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{1+n}(}{df(1+n)}$$

```
[Out] c*AppellF1(1/2+1/2*n,1/2-1/2*m,-p,3/2+1/2*n,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)
*(c*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*(d*sin(f*x+e))^(1+n)*(a+b
*sin(f*x+e)^2)^p/d/f/(1+n)/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3281, 525, 524}

$$\frac{c\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}(d\sin(e+fx))^{n+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2};\frac{1-m}{2},-p;\frac{n+3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (c*AppellF1[(1 + n)/2, (1 - m)/2, -p, (3 + n)/2, Sin[e + f*x]^2, -((b*Sin[e
 + f*x]^2)/a)]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(d*Si
n[e + f*x])^(1 + n)*(a + b*Sin[e + f*x]^2)^p)/(d*f*(1 + n)*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3281

```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
```

```
 FreeFactors[Sin[e + f*x], x]}, Dist[ff*c^(2*IntPart[(m - 1)/2] + 1)*((c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]
)), Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x],
 x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !Int
egerQ[m]
```

Rubi steps

$$\int (c\cos(e+fx))^m (d\sin(e+fx))^n \left(a+b\sin^2(e+fx)\right)^p \, dx = \frac{\left(c(c\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right) \mathrm{S}}{}$$

$$= \frac{\left(c(c\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}} (c}{}$$

$$= \frac{cF_1\left(\frac{1+n}{2};\frac{1-m}{2},-p;\frac{3+n}{2};\sin^2(e+fx),-\frac{b\sin^2(e-}{a}}{}$$

Mathematica [**A**]
time = 0.64, size = 135, normalized size = 0.99

$$\frac{F_1\left(\frac{1+n}{2};\frac{1-m}{2},-p;\frac{3+n}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(c\cos(e+fx))^m\cos^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\tan(e+fx)}{f(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + n)/2, (1 - m)/2, -p, (3 + n)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(c*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m)/2)*(d*Sin[e + f*x
])^n*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + n)*(1 + (b*Sin[e + f*x]
^2)/a)^p)
```

Maple [**F**]
time = 1.01, size = 0, normalized size = 0.00

$$\int \left(\cos\left(fx+e\right)c\right)^{m} \left(d\sin\left(fx+e\right)\right)^{n} \left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(f*x+e)*c)^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((cos(f*x+e)*c)^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algorit
hm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^n, x
)
```

Fricas [F]
time = 1.21, size = 39, normalized size = 0.28

$$\text{integral}\Big(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\left(c\cos\left(fx+e\right)\right)^m\left(d\sin\left(fx+e\right)\right)^n,x\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algorit
hm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^
n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(d*sin(f*x+e))**n*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algorit
hm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^n, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\cos(e+fx))^m (d\sin(e+fx))^n \left(b\sin(e+fx)^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(d*sin(e + f*x))^n*(a + b*sin(e + f*x)^2)^p,x)
[Out] int((c*cos(e + f*x))^m*(d*sin(e + f*x))^n*(a + b*sin(e + f*x)^2)^p, x)
```

3.593 $\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{E\left(e+fx+\tan^{-1}(b,c)|-\frac{b^2+c^2}{a}\right)\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}{f\sqrt{1+\frac{(c\cos(e+fx)+b\sin(e+fx))^2}{a}}}$$

```
[Out] (cos(e+f*x+arctan(b,c))^2)^(1/2)/cos(e+f*x+arctan(b,c))*EllipticE(sin(e+f*x
+arctan(b,c)),((-b^2-c^2)/a)^(1/2))*(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2)
/f/(1+(c*cos(f*x+e)+b*sin(f*x+e))^2/a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3320, 3319, 3256}

$$\frac{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}\,E\left(e+fx+\tan^{-1}(b,c)|-\frac{b^2+c^2}{a}\right)}{f\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] (EllipticE[e + f*x + ArcTan[b, c], -((b^2 + c^2)/a)]*Sqrt[a + (c*Cos[e + f*
x] + b*Sin[e + f*x])^2])/(f*Sqrt[1 + (c*Cos[e + f*x] + b*Sin[e + f*x])^2/a]
)
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3319

```
Int[((a_) + (b_.)*(cos[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)*sin[(e_.) + (f_.)*
(x_)])^2)^(p_), x_Symbol] :> Int[(a + b*(Sqrt[c^2 + d^2]*Sin[ArcTan[c, d] +
 e + f*x])^2)^p, x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[p^2, 1/4] && GtQ
[a, 0]
```

Rule 3320

```
Int[((a_) + (b_.)*(cos[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)*sin[(e_.) + (f_.)*
(x_)])^2)^(p_), x_Symbol] :> Dist[(a + b*(c*Sin[e + f*x] + d*Cos[e + f*x])^
2)^p/(1 + (b*(c*Sin[e + f*x] + d*Cos[e + f*x])^2)/a)^p, Int[(1 + (b*(c*Sin[
```

```
e + f*x] + d*Cos[e + f*x])^2)/a)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& EqQ[p^2, 1/4] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{2(i-x)}+\frac{i\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{2(i+x)}\right)dx, x, t\right.}{f}$$

$$= \frac{i\text{Subst}\left(\int \frac{\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{i-x}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{i\text{Subst}\left(\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 325 vs. 2(79) = 158.
time = 1.20, size = 325, normalized size = 4.11

$$-\frac{E\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{(b^2+c^2)^2}+(b^2-c^2)\cos(2(e+fx))-2bc\sin(2(e+fx))}{\sqrt{(b^2+c^2)^2}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{(b^2+c^2)^2}}{2a+b^2+c^2+\sqrt{(b^2+c^2)^2}}\right)\sqrt{2a+b^2+c^2+(-b^2+c^2)\cos(2(e+fx))+2bc\sin(2(e+fx))}\,(2bc\cos(2(e+fx))+(b^2-c^2)\sin(2(e+fx)))}{\sqrt{2}\sqrt{(b^2+c^2)^2}\,f\sqrt{\frac{2a+b^2+c^2+(-b^2+c^2)\cos(2(e+fx))+2bc\sin(2(e+fx))}{2a+b^2+c^2+\sqrt{(b^2+c^2)^2}}}\sqrt{\frac{(2bc\cos(2(e+fx))+(b^2-c^2)\sin(2(e+fx)))^2}{(b^2+c^2)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] -((EllipticE[ArcSin[Sqrt[(Sqrt[(b^2 + c^2)^2] + (b^2 - c^2)*Cos[2*(e + f*x)
] - 2*b*c*Sin[2*(e + f*x)])/Sqrt[(b^2 + c^2)^2]]/Sqrt[2]], (2*Sqrt[(b^2 + c
^2)^2])/(2*a + b^2 + c^2 + Sqrt[(b^2 + c^2)^2])]*Sqrt[2*a + b^2 + c^2 + (-b
^2 + c^2)*Cos[2*(e + f*x)] + 2*b*c*Sin[2*(e + f*x)]]*(2*b*c*Cos[2*(e + f*x)
] + (b^2 - c^2)*Sin[2*(e + f*x)]))/(Sqrt[2]*Sqrt[(b^2 + c^2)^2]*f*Sqrt[(2*a
 + b^2 + c^2 + (-b^2 + c^2)*Cos[2*(e + f*x)] + 2*b*c*Sin[2*(e + f*x)])/(2*a
 + b^2 + c^2 + Sqrt[(b^2 + c^2)^2])]*Sqrt[(2*b*c*Cos[2*(e + f*x)] + (b^2 -
c^2)*Sin[2*(e + f*x)])^2/(b^2 + c^2)^2]))
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 4.48, size = 4067171, normalized size = 51483.18

method	result	size
default	Expression too large to display	4067171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(cos(f*x+e)*c+b*sin(f*x+e))^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

Fricas [**F**]
time = 0.46, size = 44, normalized size = 0.56

$$\mathrm{integral}\left(\sqrt{2\,bc\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(b^2-c^2\right)\cos\left(fx+e\right)^2+b^2+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*b*c*cos(f*x + e)*sin(f*x + e) - (b^2 - c^2)*cos(f*x + e)^2
+ b^2 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\left(c\,\cos\left(e+f\,x\right)+b\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2),x)
```

```
[Out] int((a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2), x)
```

3.594 $\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx$

Optimal. Leaf size=79

$$\frac{F\left(e+fx+\tan^{-1}(b,c)|-\frac{b^2+c^2}{a}\right)\sqrt{1+\frac{(c\cos(e+fx)+b\sin(e+fx))^2}{a}}}{f\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}$$

```
[Out] (cos(e+f*x+arctan(b,c))^2)^(1/2)/cos(e+f*x+arctan(b,c))*EllipticF(sin(e+f*x
+arctan(b,c)),((-b^2-c^2)/a)^(1/2))*(1+(c*cos(f*x+e)+b*sin(f*x+e))^2/a)^(1/
2)/f/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3320, 3319, 3261}

$$\frac{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}\,F\left(e+fx+\tan^{-1}(b,c)|-\frac{b^2+c^2}{a}\right)}{f\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] (EllipticF[e + f*x + ArcTan[b, c], -((b^2 + c^2)/a)]*Sqrt[1 + (c*Cos[e + f*
x] + b*Sin[e + f*x])^2/a])/(f*Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2]
)
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3319

```
Int[((a_) + (b_.)*(cos[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)*sin[(e_.) + (f_.)*
(x_)])^2)^(p_), x_Symbol] :> Int[(a + b*(Sqrt[c^2 + d^2]*Sin[ArcTan[c, d] +
 e + f*x])^2)^p, x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[p^2, 1/4] && GtQ
[a, 0]
```

Rule 3320

```
Int[((a_) + (b_.)*(cos[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)*sin[(e_.) + (f_.)*
(x_)])^2)^(p_), x_Symbol] :> Dist[(a + b*(c*Sin[e + f*x] + d*Cos[e + f*x])^
```

```
2)^p/(1 + (b*(c*Sin[e + f*x] + d*Cos[e + f*x])^2)/a)^p, Int[(1 + (b*(c*Sin[
e + f*x] + d*Cos[e + f*x])^2)/a)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& EqQ[p^2, 1/4] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}+\frac{i}{2(i+x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\right.\right.}{f} \\
&= \frac{i\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{i\text{Su}}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 1.13, size = 529, normalized size = 6.70

$$
\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{2a+b^2+c^2+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\sin\left(2(e+fx)+\tan^{-1}\left(\frac{-b^2+c^2}{2bc}\right)\right)}{2a+b^2+c^2-bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}},\frac{2a+b^2+c^2+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\sin\left(2(e+fx)+\tan^{-1}\left(\frac{-b^2+c^2}{2bc}\right)\right)}{2a+b^2+c^2+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}}\right)\sec\left(2(e+fx)+\tan^{-1}\left(\frac{-b^2+c^2}{2bc}\right)\right)\sqrt{-\frac{bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\left(-1+\sin\left(2(e+fx)+\tan^{-1}\left(\frac{-b^2+c^2}{2bc}\right)\right)\right)}{2a+b^2+c^2+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}}}\sqrt{-\frac{bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\left(1+\sin\left(2(e+fx)+\tan^{-1}\left(\frac{-b^2+c^2}{2bc}\right)\right)\right)}{2a+b^2+c^2-bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}}}\sqrt{2a+b^2+c^2+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\sin\left(2(e+fx)+\tan^{-1}\left(\frac{-b^2+c^2}{2bc}\right)\right)}}{bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\,f}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 1/2, 3/2, (2*a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^
2)^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]])/(2*a + b^2
 + c^2 - b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]), (2*a + b^2 + c^2 + b*c*Sqrt[(b
^2 + c^2)^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]])/(2*
a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)])]*Sec[2*(e + f*x) + ArcTa
n[(-b^2 + c^2)/(2*b*c)]]*Sqrt[-((b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]*(-1 + Si
n[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]))/(2*a + b^2 + c^2 + b*c*Sqrt
[(b^2 + c^2)^2/(b^2*c^2)]))]*Sqrt[-((b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]*(1 +
 Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]))/(2*a + b^2 + c^2 - b*c*S
```

```
qrt[(b^2 + c^2)^2/(b^2*c^2)]))]*Sqrt[2*a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^2)
^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]])/(b*c*Sqrt[(
b^2 + c^2)^2/(b^2*c^2)]*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.21, size = 258013, normalized size = 3265.99

method	result	size
default	Expression too large to display	258013

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(cos(f*x+e)*c+b*sin(f*x+e))^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))**2)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\left(c\,\cos\left(e+f\,x\right)+b\,\sin\left(e+f\,x\right)\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2),x)
[Out] int(1/(a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```